This is the peer reviewd version of the followng article:

Free and forced morphodynamics of river bifurcations / Redolfi, Marco; Zolezzi, Guido; Tubino, Marco. - In: EARTH SURFACE PROCESSES AND LANDFORMS. - ISSN 0197-9337. - 44:4(2019), pp. 973-987. [10.1002/esp.4561]

Terms of use:

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

28/06/2024 17:43

# Free and forced morphodynamics of river bifurcations

M. Redolfi<sup>\*</sup>, G. Zolezzi<sup>\*</sup>, M. Tubino<sup>\*</sup>.

\* Department of Civil, Environmental and Mechanical Engineering, University of Trento, Italy

#### Abstract

1

Water and sediment distribution by river bifurcations is often highly unbalanced. This 2 may result from a variety of factors, like migration of bars, channel curvature, backwater 3 effects, which promote an uneven partition of flow and sediment fluxes in the downstream 4 branches, which we call "forcings". Bifurcations also display an intrinsic instability mecha-5 nism that leads to unbalanced configurations, as it occurs in the idealized case of a geometrically symmetric bifurcation, which we call "free", provided the width-to-depth ratio of the 7 incoming flow is large enough. Most frequently, these free and forced mechanisms coexist, 8 however their controlling roles on bifurcation dynamics has not been investigated so far. In 9 this paper we address such question by proposing a unified free-forced modelling framework 10 for bifurcation morphodynamics. Upstream channel curvature and different slopes of down-11 stream branches (slope advantage) are specifically investigated as forcing effects typically 12 occurring in bifurcations of alluvial channels. The modelling strategy is based on the widely 13 used two-cell model of Bolla Pittaluga et al. (2003) here extended to account for the spa-14 tially non-uniform fluxes entering the bifurcation node. Results reveal that the relative role 15 of free and forced mechanisms depends on the width to depth ratio falling above or below 16 the resonant threshold that controls the stability of free bifurcations: when the main chan-17 nel is relatively wide and shallow (super-resonant regime) the bifurcation invariably evolves 18 towards unbalanced configurations, whatever the combination of curvature and slope advan-19 tage values, which instead control the bifurcation response under sub-resonant conditions. 20 Detection of the resonant aspect ratio as a key threshold also releases the modelling approach 21 from the need of parameter calibration that characterized previous approaches, and allows 22 for interpreting under a unified framework the opposite behaviours shown by gravel bed and 23 sand bed bifurcations for increasing Shields parameter values. 24

## 25 1 Introduction

Channel bifurcations control the downstream distribution of water and sediments in a variety of 26 fluvial environments, such as deltas, alluvial fans, braided and anabranching rivers (Slingerland 27 and Smith, 2004; Kleinhans et al., 2013). Understanding their dynamics is therefore important for 28 managing water resources and the flooding risk, predicting the long-term morphological evolution 29 of channel networks and evaluating the effectiveness over time of river restoration projects aimed 30 at reactivating a multi-thread configuration (e.g., Habersack and Piéqay, 2007). Bifurcation dy-31 namics also control instream processes in meander bends that mitigate the development of channel 32 sinuosity through the occurrence of short cuts through point bars (Grenfell et al., 2012; van Dijk 33 et al., 2014). 34

River bifurcations have been extensively studied through laboratory-scale physical models (Fed-35 erici and Paola, 2003; Bertoldi and Tubino, 2007; Bertoldi et al., 2009; Le et al., 2018b), and 36 mathematical models based on 1D (Wang et al., 1995; Bolla Pittaluga et al., 2003; Kleinhans 37 et al., 2013; Salter et al., 2018), 2D (Edmonds and Slingerland, 2008; Siviglia et al., 2013; Le 38 et al., 2018b,a) and 3D approaches (Kleinhans et al., 2008). Along with field observations (e.g., 39 Zolezzi et al., 2006; Kleinhans et al., 2012), these studies highlight the almost invariable tendency 40 of bifurcations to produce an uneven distribution of flow and sediment transport, which results in 41 a strong asymmetry of the channel width and bed elevation of downstream anabranches. 42

This type of unbalanced configuration is often promoted by various "forcing" effects that drive 43 the bifurcation towards an unbalanced state, sometimes leading to the complete closure of one 44 of the anabranches. Forcing factors include both upstream and downstream effects. Upstream 45 effects comprise mechanisms that feed the bifurcating node with a topographically-driven uneven 46 distribution of flow and transport rate, like the curvature of the upstream channel (*Kleinhans*) 47 et al., 2008; Hardy et al., 2011; Sloff and Mosselman, 2012) and the occurrence of migrating 48 bars (Bertoldi et al., 2009; Bertoldi, 2012) or steady bars (Le et al., 2018b). Downstream effects 49 include mechanisms that provide, locally or through backwater effects, a slope advantage to one 50 of the distributaries (Edmonds, 2012; van Dijk et al., 2014; Zhang et al., 2017; Salter et al., 2018). 51 For purely illustrative purposes, Figure 1 shows real-world bifurcations with examples of these 52

concepts. Figures 1a and 1b provide examples of bifurcations where one of these forcing factors is 53 likely dominant: the upstream channel curvature (Figure 1a) and the slope advantage for the right 54 bifurcate (Figure 1b). In Figures 1c and 1d these two forcings likely have comparable relevance. 55 They might cooperate in the case of Figure 1c, because the chute channel detaches from the outer 56 bank of the upstream channel bend, thus receiving most of the water and sediment input from 57 upstream, and is also shorter than the other (left) bifurcate, thus having a slope advantage. On 58 the contrary, they likely compete in the case of Figure 1d, because the shorter chute channel occurs 59 on the inner bank of the upstream channel bend. 60

Interestingly, the unbalanced configuration can also result from an inherent instability mecha-61 nism, even in the absence of external forcings, as shown theoretically by Wang et al. (1995) and 62 Bolla Pittaluga et al. (2003), and later demonstrated through laboratory and numerical studies 63 (Bertoldi and Tubino, 2007; Edmonds and Slingerland, 2008; Siviglia et al., 2013; Salter et al., 64 2018). More recently *Redolfi et al.* (2016) provided an interpretation of such "free" bifurcations 65 instability within the framework of the theory of morphodynamic influence of Zolezzi and Sem-66 inara (2001), showing that the unbalanced configuration arises when the bifurcation is able to 67 exert an upstream morphodynamic influence that allows for the formation of an upstream steady 68 bar. Such upstream morphodynamic influence theoretically occurs when the width to depth ratio 69 exceeds a threshold "resonant" value, as originally defined by Blondeaux and Seminara (1985) in 70 the theory of regular meanders. For small (sub-resonant) values of the width to depth ratio a 71 symmetric free bifurcation keeps stable and equally distributes water and sediment fluxes in the 72 downstream branches, while in the super-resonant regime such balanced configuration is no longer 73 stable and the bifurcation invariably evolves towards an unbalanced configuration. 74

The above scenario suggests that in sub-resonant conditions the tendency towards unbalanced states observed in real rivers is mainly driven by external forcing factors, while in super-resonant conditions both free and forced mechanisms are likely interacting, though their respective roles have not been investigated so far. The question therefore arises to which extent the autogenic, freeinstability mechanism or instead the external forcings affect the behaviour of natural bifurcations, and under which conditions those effects cooperate or compete to produce what is observed in <sup>81</sup> such complex settings.

We aim at answering this question by taking the viewpoint of river bifurcations as dynamical 82 systems for which a distinct role of the free and forced responses can be identified. This method-83 ological distinction is based on the recognition that free and forced mechanisms display substantial 84 differences in their evolutionary temporal and spatial scales. Similar approaches have proven to 85 provide thorough insight in the study of other morphodynamic processes, like the dynamics of 86 river bars in curved or meandering channels (Seminara and Tubino, 1989), where migrating free 87 bars develop on a much faster scale than that required to shape the meander planform (Tubino 88 and Seminara, 1990). 89

In this paper we cast within a unified theoretical framework previous results on free and forced bifurcations and consider two main forcing factors: the upstream effect exerted by an incoming curved channel and the downstream effect of slope advantage of one of the distributaries, which can derive from the different length of the distributaries, from differential downstream degradation/deposition (*Salter et al.*, 2018), or from backwater effects (e.g. *Edmonds*, 2012).

The analysis is based on the two-cell model originally proposed by Bolla Pittaluga et al. (2003), 95 as extended by *Kleinhans et al.* (2008) to account for the curvature-driven secondary flow, and on 96 the theoretical results of *Redolfi et al.* (2016). The analytical model prescribes physically-based, 97 simplified nodal point relationships that enables us to explore the basic mechanisms that drive the 98 water and sediment distribution at the node. As highlighted by Wang et al. (1995), the behaviour 99 of the bifurcation depends on how the sediment is distributed with respect to the downstream 100 transport capacity; sediment distribution is in turn determined by the transverse flow-exchange 101 and gravitational effects on bed load transport just upstream the bifurcation node, as explained 102 by Bolla Pittaluga et al. (2003). 103

## $_{104}$ 2 Methods

<sup>105</sup> Our model stems from the *Bolla Pittaluga et al.* (2003) two-cell approach, and is formulated to <sup>106</sup> incorporate both free and forced bifurcation responses, and their interaction. It allows predicting <sup>107</sup> how water and sediment fluxes delivered from the upstream main channel are drained by the



Figure 1: Satellite images showing illustrative examples of river bifurcations: (a) Tigris River near Baghdad (Iraq),  $34^{\circ}16' N$ ,  $43^{\circ}50' E$ , with a curved upstream channel and downstream bifurcates having a nearly symmetrical channel geometry; (b) estuary in the Kamchatka Peninsula (Russia),  $60^{\circ}02' N$ ,  $163^{\circ}40' E$ , where the left bifurcate likely covers a longer distance for the same elevation gap from the bifurcation node to the sea, suggesting a possible slope advantage for the left bifurcate; (c) bends with chute cutoffs in the meandering Siret River (Romania),  $47^{\circ}39' N$ ,  $26^{\circ}30' E$ , with the cutoff channel initiating on the outer bank of an upstream channel bend and being shorter than the left bifurcate; (d) meandering River in the Ust-Chaun area (Russia),  $68^{\circ}42' N$ ,  $170^{\circ}35' E$ , with the cutoff channel initiating on the inner bank of a bend and being shorter than the left bifurcate; from *Google Earth*, *Digital Globe* (2018).

<sup>108</sup> downstream anabranches, under different combination of external forcings.

The bifurcation geometry is sketched in Figure 2a, having an upstream, curved main channel of width  $W_a$ , slope  $S_a$  and radius of curvature R, which bifurcates in two downstream channels having width  $W_b$  and  $W_c$  and slope  $S_b$  and  $S_c$ , respectively. In the model, flow and sediment balances applied to two cells of length  $\alpha W_a$ , which also accounts for transverse exchanges, rules the distribution of water (Q) and sediment (Qs) fluxes between the bifurcates, as represented in Figures 2b and 2c.



Figure 2: Bifurcation geometry and notation used in the mathematical model formulation: (a) planform view, showing the curved main channel, the two distributaries and the two cells; (b) and (c) water and sediment fluxes through the cells.

The model considers the effect of secondary flows associated with streamline curvature within the node cells (*Kleinhans et al.*, 2008), and includes an extension of such previous model to account for the non-uniform distribution for the entering water and sediment fluxes.

### 118 2.1 The free bifurcation

 $\mathbf{s}$ 

The core of the method is a model for a free bifurcation with perfectly symmetrical geometry and 119 boundary conditions (i.e. with no curvature nor slope advantage) that follows the classic approach 120 of Bolla Pittaluga et al. (2003). The flow and bed topography in the three channels result from 121 a 1D mobile bed model, which can be solved once the following five matching conditions at the 122 bifurcation node are specified: conservation of sediment and water fluxes (two conditions), energy 123 conservation (i.e. water surface elevation) along each cell (two conditions), and a physically-based 124 relation that prescribes how sediment fluxes are partitioned at the bifurcation node. This type 125 of nodal point relation is the key to incorporate bifurcation morphodynamics within a simple 1D 126 scheme, and accounts for the exchange of sediment between the two cells through the following 127 relationship: 128

$$\underbrace{\frac{Qs_y/(\alpha W_a)}{Qs_a/W_a}}_{\text{ediment flux direction}} = \underbrace{\frac{Q_y/(\alpha W_a D_{abc})}{Q_a/(W_a D_a)}}_{\text{velocity direction}} - \underbrace{\frac{2r}{\sqrt{\theta_a}} \frac{\eta_b - \eta_c}{W_b + W_c}}_{\text{gravitational effect}},\tag{1}$$

where the direction of the sediment flux is determined by the direction of velocity and by the gravitational effect induced by the transverse gradient of the bed elevation. The last term of Equation (1) is estimated according to the *Ikeda* (1982) formulation, where r is a dimensionless coefficient (e.g., *Baar et al.*, 2018),  $\theta_a$  is the Shields stress in the main Channel a,  $\eta_b$  and  $\eta_c$  indicate the bottom elevation at the inlet of the distributary Channels b and c, respectively. The mean depth within the cell, defined as  $D_{abc} = (2D_a + D_b + D_c)/4$ , can be simplified to  $D_{abc} = D_a$  as proposed by *Salter et al.* (2018).

The flow in the three anabranches is modelled using a classic 1D shallow water and Exner model, whose steady solution is simply an uniform flow. The water flow in each channel  $i = \{a, b, c\}$ is given by:

$$Q_i = W_i c_i \sqrt{gS_i} D_i^{3/2}, \tag{2}$$

<sup>139</sup> where g is the gravitational acceleration,  $D_i$  is the water depth and  $c_i$  is the dimensionless Chézy

<sup>140</sup> coefficient, which can be calculated as (*Engelund and Fredsoe*, 1982):

$$c_i = 6 + 2.5 \log\left(\frac{1}{2.5} \frac{D_i}{d_{50}}\right),\tag{3}$$

with  $d_{50}$  indicating the median grain size.

<sup>142</sup> Similarly, the volumetric sediment flux is computed as:

$$Qs_i = W_i \sqrt{g\Delta d_{50}^3} \Phi\left(\theta_i, \frac{D_i}{d_{50}}\right), \qquad \theta_i = \frac{S_i D_i}{\Delta d_{50}},\tag{4}$$

where  $\Delta$  is the relative submerged density of the sediment,  $\theta_i$  is the Shields stress and  $\Phi$  is a function given by the sediment transport formula. Specifically, we used the *Parker* (1990) formula for gravel bed channels and the *Engelund and Hansen* (1967) formula for sand bed cases.

The free character of the bifurcation manifests itself in the symmetrical configuration of upstream and downstream channels, which determines water and sediment fluxes that enter and exit the cells (Figures 2b and 2c). First, the input fluxes  $Q_i^{IN}$  and  $Qs_i^{IN}$  that are delivered by the upstream channel into the node cells are uniform. Second, the absence of slope advantage (i.e.  $S_b = S_c$ ) leads to the same water and sediment rating curves for the two bifurcates. Therefore, possible asymmetries of the output fluxes are not driven by the upstream/downstream conditions but can only derive from an uneven redistribution by the bifurcation node.

## <sup>153</sup> 2.2 The forced bifurcation

Different forcing effects can be further incorporated in the free bifurcation model, to increase its 154 ability of the model to represent real bifurcation configurations. The key to model those effects 155 is to act on the upstream and boundary conditions imposed at the node cells, which practically 156 implies considering non-uniform water and sediment fluxes entering the two node cells, and/or 157 imposing different water and sediment rating curves for the two bifurcates. Such an approach has 158 been already exploited by *Bertoldi et al.* (2009) when modelling how bifurcations dynamics can 159 be affected by migrating bars, which were schematised as periodic temporal oscillations of water 160 and sediment fluxes delivered to the node cells. Here we consider two different forcing effects: (i) 161

a downstream slope advantage of one bifurcate with respect to the other one and (ii) the presence
of a curved upstream channel.

A downstream slope advantage increases the probability for an unbalanced water and sediment flux towards the "advantaged" distributary. This effect can be taken into account by setting different values of  $S_b$  and  $S_c$  in Equations (2) and (4). This breaks the symmetry in the downstream rating curves, even if the other geometrical parameters remain equal between the two downstream branches. We quantify the slope advantage through the following parameter:

$$\Delta S = \frac{S_b - S_c}{S_b + S_c},\tag{5}$$

with positive values of  $\Delta S$  indicating that the outer bend bifurcate (Channel *b* of Figure 2a) is steeper than the inner bend bifurcate (Channel *c*).

<sup>171</sup> We model the presence of an upstream bend (Channel *a*) feeding the bifurcation as does a <sup>172</sup> channel with constant radius of curvature *R*. The curvature of the main channel leads to the <sup>173</sup> formation of a spiral flow (Figure 3a), which in turn produces a shear stress in the transverse <sup>174</sup> direction. Therefore, the bottom stress ends up being deflected by an angle  $\phi_{\tau}$ , which can be <sup>175</sup> computed as (*Struiksma et al.*, 1985):

$$\tan\left(\phi_{\tau}\right) = -A\frac{D}{R},\tag{6}$$

where D is the local water depth and A is the coefficient that defines the intensity of the secondary flow, given by:

$$A = \frac{2}{\kappa^2} \left( 1 - \frac{1}{\kappa c} \right) \tag{7}$$

with  $\kappa = 0.4$  indicating the Von Karman constant.

In the region where the flow is fully developed (i.e., far enough from the bend entrance), the flow characteristics do not vary along the channel, and the depth-averaged velocity is purely longitudinal. In these conditions, the deflection of the bed shear stress is compensated by a <sup>182</sup> transverse gradient of the bed elevation (see Figure 3a) that is given by:

$$\frac{d\eta}{dy} = \frac{\sqrt{\theta}}{r} \tan\left(\phi_{\tau}\right),\tag{8}$$

(for mathematical details see Appendix A), which in turn generates non-uniform transverse profiles of water depth, longitudinal velocity and shear stress. Consequently, water and sediment fluxes feeding the two cells are not uniform but are mainly delivered towards the cell positioned at the outer bend (Cell B of Figure 3b).



Figure 3: Illustration of the secondary flow solution in the upstream channel: (a) cross-sectional view, indicating the vertical profile of the transverse velocity (arrows), and the transverse profiles of bed elevation and water depth; (b) planimetric view, with the transverse velocity profile (arrows) and the associated sediment fluxes entering the cells  $(Qs_b^{IN} \text{ and } Qs_c^{IN})$ .

Furthermore, as suggested by *Kleinhans et al.* (2008) the deviation of the bed shear stress given by Equation (6) is also active within the cells, so that the nodal point relation (1) needs to be extended as follows:

$$\underbrace{\frac{Qs_y}{\alpha Qs_a}}_{\text{liment flux direction}} = \underbrace{\frac{Q_y}{\alpha Q_a}}_{\text{velocity direction}} - \underbrace{\frac{2r}{\sqrt{\theta_a}} \frac{\eta_b - \eta_c}{W_b + W_c}}_{\text{gravitational effect}} - \underbrace{\frac{A\frac{D_a}{R}}{R}}_{\text{spiral flow effect}}, \qquad (9)$$

where  $D_a$  and  $\theta_a$  are the average values of water depth and Shields stress in the upstream Channel a.

see

The above formulation is generally suitable for modelling bifurcations with arbitrary channel widths. However, for the purpose of analysing the interaction between the different mechanisms, also in comparison with previous works (e.g., *Bolla Pittaluga et al.*, 2015), we restrict our attention to the basic case where both the bifurcates have half the width of the main channel ( $W_b = W_c = W_a/2$ ).

## 197 **3** Results

The model solution can be expressed as a function of a few dimensionless parameters. First, the solution depends on the reference conditions, which are defined as the uniform flow and sediment transport in a straight channel with same slope, width and discharge of the main channel. Specifically, we need to prescribe three main parameters, namely the aspect ratio, the Shields stress and the relative roughness:

$$\beta_0 = \frac{1}{2} \frac{W_a}{D_0}, \quad \theta_0 = \frac{S_a D_0}{\Delta d_{50}}, \quad d_{s0} = \frac{d_{50}}{D_0}, \tag{10}$$

where zero subscript (e.g.,  $D_0$ ) indicates the reference conditions.

Second, the solution depends on the intensity of the forcing effects, which is specified through the normalized curvature  $W_a/R$  and the slope advantage  $\Delta S$ .

In the following, we analyse the model outputs in terms of discharge asymmetry (*Bertoldi et al.*, 2009), which can be taken as a representative indicator of the bifurcation response:

$$\Delta Q = \frac{Q_b - Q_c}{Q_a},\tag{11}$$

which may range from -1 (no flow in Channel b) to +1 (no flow in Channel c), with  $\Delta Q = 0$ indicating balanced bifurcations.

### 210 3.1 The free bifurcation

The free bifurcation configuration is made by three straight channels without any slope advantage nor width or angle asymmetry, thus without external effects that may force an unbalanced configuration. In this condition the intuitive expectation would be a symmetrical bifurcation response, with an even distribution of downstream water and sediment fluxes. As illustrated in Figure 4, the balanced configuration ( $\Delta Q = 0$ ) is indeed an equilibrium solution of the system. However, for relatively high values of the aspect ratio the balanced configuration becomes unstable, and one of the channels, indifferently, tends to dominate.

As highlighted by Bolla Pittaluga et al. (2003) this type of instability of the symmetric equilib-218 rium solution does not necessarily lead to a complete closure of one branch, but new, unbalanced 219 and stable equilibrium states are possible. In Figure 4, this is represented by the formation of a 220 so called "pitchfork bifurcation" (e.g., Wiggins, 2003) in the equilibrium diagram, which occurs 221 at  $\beta_0$  near 13.5. The two unbalanced equilibrium configurations are physically sustained by the 222 formation of an inlet step, i.e. a localized steep reach at the head of the largest flow-carrying bifur-223 cate (Bertoldi et al., 2009), which steers the sediment flux and thus satisfies the balance between 224 sediment supply and transport capacity of the bifurcates. 225



Figure 4: Equilibrium solutions of a free bifurcation (i.e. straight channel with no slope advantage) according to the two-cell model. Solid lines indicates stable solutions, while the dashed line represents an unstable equilibrium configuration. Parameters are  $\theta_0 = 0.1$ ,  $d_{s0} = 0.02$ , r = 0.5; the *Parker* (1990) transport formula is used.

The critical point at which the pitchfork bifurcation appears is determined through a linear stability analysis (*Bolla Pittaluga et al.*, 2015), whose result can be expressed in the following 228 general form:

$$\beta_C = \frac{r\alpha}{\sqrt{\theta_0}} \frac{4}{\Phi_D + \Phi_T - (3/2 + c_D)},\tag{12}$$

<sup>229</sup> whose coefficients are defined as:

$$c_D := \left. \frac{D_0}{c_0} \frac{\partial c}{\partial D} \right|_{D_0}, \quad \Phi_D := \left. \frac{D_0}{\Phi_0} \frac{\partial \Phi}{\partial D} \right|_{D_0,\theta_0}, \quad \Phi_T := \left. \frac{\theta_0}{\Phi_0} \frac{\partial \Phi}{\partial \theta} \right|_{D_0,\theta_0}, \tag{13}$$

and represent the sensitivity of Chézy coefficient and of the dimensionless sediment transport rate to variations of water depth and Shields stress. The algebraic expressions of the coefficients  $c_D$ ,  $\Phi_D$  and  $\Phi_T$  for the used formulae of *Parker* (1990) and of *Engelund and Hansen* (1967), as well as for the classical relation of *Meyer-Peter and Muller* (1948), are reported in Appendix B for the sake of clarity. Equation (12) represents a generalization of the formula proposed by *Bolla Pittaluga et al.* (2015) for arbitrary transport and friction formulae.

According to Equation (12), the critical aspect ratio is directly proportional to the cell length represented by  $\alpha$ . In a 1D formulation, the parameter  $\alpha$  needs to be empirically calibrated, resulting in rather different literature values, ranging from 1 (*Bolla Pittaluga et al.*, 2003) to 6 (*Bertoldi and Tubino*, 2007). This limitation has been solved by *Redolfi et al.* (2016), who developed an analytical linear solution of the fully 2D problem; in that formulation, the length of the upstream cells is resolved by the model itself, so that no specific calibration is needed.

The analysis of *Redolfi et al.* (2016) demonstrated that the emergence of an unbalanced solution 242 in a free bifurcation depends on the formation of an upstream steady bar, which occurs when the 243 bifurcation is able to exert a morphodynamic influence in the upstream direction (see Figure 244 5). As theoretically derived by *Zolezzi and Seminara* (2001) and experimentally observed by 245 Zolezzi et al. (2005) any fixed geometrical disturbance can produce a permanent upstream bed 246 deformation, usually taking the form of a steady bar, when the channel is wide and shallow enough 247 for the aspect ratio of the main channel to exceed the resonant threshold,  $\beta_R$ , as originally defined 248 in the theory of regular meanders by *Blondeaux and Seminara* (1985). Therefore, under super-249 resonant conditions ( $\beta_0 > \beta_R$ ) the bifurcation node - as a fixed geometrical disturbance - can 250 trigger such an upstream morphodynamic influence. This is essentially the physical mechanism 251

<sup>252</sup> behind the mathematical instability of the balanced equilibrium solution, which therefore makes
<sup>253</sup> a free bifurcation unbalanced.



Super-resonant conditions  $\beta_0 > \beta_R$ 

Figure 5: Map of bed elevation and water distribution in a free bifurcation from the numerical simulations of *Edmonds and Slingerland* (2008), adapted from their Figure 5b. The equilibrium configuration is made unbalanced by the formation of an upstream steady bar, which deviates most water and sediment fluxes towards the right bifurcate. According to *Redolfi et al.* (2016) this is an effect of the upstream morphodynamic influence exerted by the bifurcation node, which occurs when the aspect ratio of the upstream channel  $\beta_0$  exceeds the resonant threshold  $\beta_R$ .

The computation of the resonant aspect ratio requires the solution of a fourth-order polynomial, 254 which can be readily obtained using the available Matlab code (see Acknowledgements Section). 255 The resulting values for gravel bed and sand bed river channels as a function of the relevant 256 dimensionless parameters are reported in Figures 6a and 6b, respectively. For sand bed rivers 257 the Chézy coefficient is independently fixed, rather than derived from Equation (3), to account 258 for the higher drag exerted by bedforms, also consistently with previous applications (*Edmonds*) 259 and Slingerland, 2008). In gravel bed channels  $\beta_R$  increases with both the Chézy coefficient and 260 the Shields stress, which explains the tendency of bifurcations to stabilize when increasing  $\theta_0$ 261 (Bolla Pittaluga et al., 2015). Conversely, in sand bed channels the resonant threshold tends to 262 decrease with the Shields stress, which explains the opposite effect of  $\theta_0$  observed by Edmonds 263 and Slingerland (2008). Consistently with Bolla Pittaluga et al. (2015) the different behaviour of 264 sand bed and gravel bed channels is related to the different response of the dimensionless sediment 265 transport rate to variations of the Shields stress (which depends on the transport formula) rather 266

Dataset	Method	Bed material	# of cases	$\beta_0$	$ heta_0$
Bertoldi and Tubino (2007)	Laboratory	gravel	25	4.9 - 26.3	0.042 - 0.099
Edmonds and Slingerland (2008)	Numerical	sand	11	8	0.80 - 2.19
Siviglia et al. (2013)	Numerical	gravel	18	3.5 - 24.0	0.060 - 0.200
Zolezzi et al. (2006)	Field	gravel	6	9.5 - 14.5	0.053 - 0.088
Bolla Pittaluga et al. (2015)	Field	sand	11	16.9 - 77.1	0.30 - 1.16

Table 1: List of datasets used to evaluate the resonance criterion for bifurcation instability.

than an effect of a gradient in the water surface elevation near the bifurcation, as suggested by *Edmonds and Slingerland* (2008).

Alternatively, the resonant aspect ratio can be calculated though the approximate expression by *Camporeale et al.* (2007):

$$\beta_R = \frac{\pi}{2\sqrt{2}} \frac{c_0 \sqrt{r}}{\theta_0^{1/4}} \frac{1}{\sqrt{\Phi_D + \Phi_T - (3/2 + c_D)}},\tag{14}$$

which provides rather accurate estimates of  $\beta_R$  for sand bed channels, while it gives slightly 271 underestimated values for gravel bed cases (up to -13% for the range of parameters in Figure 6a). 272 The resonant threshold provides a simple criterion to determine if a free bifurcation remains 273 balanced or tends to evolve towards unbalanced states. To test this criterion we used the lit-274 erature datasets listed in Table 1, which include gravel and sand bifurcations measured in the 275 field, modelled numerically and reproduced in laboratory-scale physical models. For each of the 276 71 bifurcations the different authors provided observation of their balanced/unbalanced state and 277 the basic flow parameters, which were used to compute the resonant threshold and the parameter 278  $(\beta_0 - \beta_R)/\beta_R$  representing the relative distance to resonance. We set r = 0.5 for all cases except 279 for the Edmonds and Slingerland (2008) experiments, for which an equivalent  $r \simeq 0.35$  value was 280 needed to match the default  $\alpha_{bn} = 1.5$  value of their Delft3D formulation (e.g., Lesser et al., 2004). 281 Results reported in Figures 6c and 6d show that balanced bifurcations (closed markers) tend to 282 stay below the dashed line (i.e. sub-resonant conditions), while all the unbalanced bifurcations 283 (open markers) are located above the dashed line (i.e. super-resonant conditions), independently 284 of the Shields stress. It is worth noting that the resonant criterion also captures the numerical 285 results of *Edmonds and Slingerland* (2008), who demonstrated that balanced solutions in sand bed 286 channels tends to become unstable with increasing  $\theta_0$ . The above analysis confirms that the reso-287



Figure 6: (a) and (b) Resonant aspect ratio  $\beta_R$  for gravel and sand bed channels as a function of Shields stress and Chézy coefficient. Under sub-resonant conditions (i.e.  $\beta_0 < \beta_R$ ) the balanced bifurcation configuration is stable, while in super-resonant channels ( $\beta_0 > \beta_R$ ) the instability mechanism makes the bifurcation unbalanced. (c) and (d) Observations of the balanced (close markers) or unbalanced (open makers) state of the bifurcation as a function of Shields stress and relative distance from the resonant threshold (scaled aspect ratio), for each of the 71 bifurcations listed in Table 1.

<sup>288</sup> nant threshold correctly predicts stability of both gravel and sand bed bifurcations, with the key <sup>289</sup> advantage of avoiding the need of calibrating a specific parameter like the  $\alpha$  required by previous <sup>290</sup> theoretical models (e.g., *Bolla Pittaluga et al.*, 2003, 2015).

The value of  $\alpha$  that makes the nonlinear two-cell model consistent with the 2D theory can be determined by simply setting  $\beta_C = \beta_R$  in Equation (12). Results reported in Figure 7 show a significant dependence of  $\alpha$  on the reference conditions ( $\theta_0$  and  $c_0$ ), which explains the large variability emerging in the literature. We notice that this estimate is strictly valid in the neighbour of the instability threshold, while for larger  $\beta$  it provides an upper limit of the optimal value for predicting the discharge asymmetry, as suggested by both laboratory observations and theoretical considerations (*Bertoldi and Tubino*, 2007; *Redolfi et al.*, 2016).



Figure 7: Values of the parameter  $\alpha$  that make the two-cell model consistent with the linear 2D theory of *Redolfi et al.* (2016): (a) gravel bed channels, (b) sand bed channels.

Data of Table 1 include field measurements of natural bifurcations, which are not necessarily free. In this case the observed asymmetry may be not fully ascribed to the free instability mechanism but it may also be enhanced by the presence of the external forcings. The analysis of forcing effects and their interaction with the free instability mechanism is the main subject of the next section.

### **303 3.2** The forced bifurcation

Natural bifurcations are rarely free because of a number of forcing effects, including curvature of the main channel, different bifurcation angles, slope advantages, migration of bars, presence of obstacles, differential downstream sedimentation (e.g. Van der Mark and Mosselman, 2013; van Dijk et al., 2014; Le et al., 2018b,a; Salter et al., 2018).

In the presence of a forcing effect, for example a curvature of the main channel, the equilibrium 308 diagram of Figure 4 changes its topology. Specifically, as illustrated in Figure 8a, the equilibrium 309 solution at relatively low values of  $\beta_0$  is not balanced (i.e.  $\Delta Q = 0$ ) and as expected more water 310 is flowing towards the outer bend. When  $\beta_0$  increases, the effect of the channel curvature tends to 311 be amplified by the bifurcation, resulting in a more and more unbalanced configuration. However, 312 at a given  $\beta$  value two additional equilibrium solutions form (a so called "imperfect pitchfork") 313 bifurcation", see for example Golubitsky and Schaeffer, 1979). One of them is unstable (dashed 314 line in Figure 8a), while the more unbalanced solution is stable. This suggests the possibility for 315 the bifurcation to attain a different, stable equilibrium point, where most of water and sediment 316 fluxes are deviated towards the inner bend bifurcate. 317

The behaviour of the equilibrium solutions for different values of the dimensionless channel 318 curvature  $W_a/R$  is illustrated in Figure 8b. When  $W_a/R = 0$  (straight channel) we obtain again 319 the solution of Figure 4, here represented in terms of the distance from the resonant point, so that 320 the pitchfork "bifurcation" appears at  $(\beta_0 - \beta_R)/\beta_R = 0$ . By analysing the effect of increasing 321 curvature two relevant aspects emerge. First, the increase of the discharge asymmetry with channel 322 curvature is significantly more pronounced at relatively small values of the channel aspect ratio 323 (i.e. in the sub-resonant regime), where lines corresponding to different curvature values are more 324 spaced apart, while at higher (i.e., super-resonant) aspect ratios the effect of curvature is minimal. 325 Second, the value of the aspect ratio at which the second stable solution forms increases with 326 channel curvature. For example, when  $W_a/R = 0.1$  an aspect ratio 50% higher than the resonant 327 value is needed to allow for the existence of multiple stable solutions. 328

The possibility of obtaining multistable solutions depending on channel curvature and aspect ratio is better illustrated in Figure 9a. While under sub-resonant conditions the equilibrium



Figure 8: Stable (solid lines) and unstable (dashed lines) equilibrium solutions for a bifurcation with a curved upstream channel, as a function of the scaled aspect ratio. (a) Example with fixed curvature ( $W_a/R = 0.01$ ), where the dotted line indicates the discharge ratio at the cell entrance and the grey lines represent the reference, "free" solution. (b) Effect of increasing curvature values. The vertical dashed line separates the sub-resonant (left) from the super-resonant (right) region.

solution is unique, two stable equilibrium solutions exist in super-resonant conditions. However, if the curvature is sufficiently large (depending on  $(\beta_0 - \beta_R)/\beta_R$ ) only the solution for which the outer channel dominates is possible.



Figure 9: Stability diagram indicating regions where two stable equilibrium solutions exist depending on: (a) scaled aspect ratio and channel curvature (no slope advantage); (b) the combined effect of channel curvature and slope advantage, for different values of the scaled aspect ratio.

The above depicted scenario is characteristic of imperfect systems, and turns out to be similar when analysing different kind of forcings. Here we do not specifically report on the effect of the slope advantage but we directly focus on the more interesting analysis of its interaction with the curvature of the upstream channel.

In some cases the effect of main channel curvature can be compensated by a slope advantage that tends to steer water and sediment flows towards the steeper inner bend bifurcate (e.g., *Klein*-

hans et al., 2008; van Dijk et al., 2014). Analysis of the combined effect of the different forcings 340 on the discharge asymmetry gives the results illustrated in Figure 10a, which confirms that for a 341 sub-resonant bifurcation a channel curvature can be compensated by a gradient advantage. When 342  $\Delta S = -0.01$  the discharge asymmetry is the same as the upstream asymmetry independently of 343  $\beta_0$ , while for higher slope advantages the bifurcation tends to distribute water towards the inner 344 bend channel, in a greater proportion when  $\beta_0$  increases. However, the scenario dramatically 345 changes when the aspect ratio exceeds the resonant threshold (i.e. super-resonant conditions). 346 In this case, equilibrium solutions are never balanced, with one of the two bifurcates becoming 347 dominant for any combination of curvature and slope advantage. In Figure 10b one sees that un-348 der sub-resonant conditions the equilibrium  $\Delta Q$  varies smoothly with the slope advantages, while 349 when  $\beta_0$  exceeds  $\beta_R$  sharp transitions and hysteresis are expected when varying  $\Delta S$ . 350

In general the discharge asymmetry depends on slope advantage and curvature as illustrated in Figure 11. Under sub-resonant conditions (Figure 11a) variations of  $\Delta Q$  are always smooth, and the effect of channel curvature can be always compensated by negative values of  $\Delta S$ . Conversely, under super-resonant conditions (Figure 11b) the bifurcation is never balanced, and there is a welldefined region (black oblique stripes) in the forcing parameters space where two stable solutions coexist. The width of such bi-stable region depends on the distance from the resonant point as depicted in Figure 9b.

It is important to remark that all our diagrams have been obtained by considering fixed values of relative roughness ( $d_{s0} = 0.02$ ), Shields stress ( $\theta_0 = 0.1$ ) and *Ikeda* (1982) coefficient (r = 0.5), and the same transport (*Parker*, 1990) and friction (Equation (3)) formulae. Nevertheless, from a qualitative point of view, model results, and therefore their interpretation, are fully independent of the specific choice of flow parameters values and closure relations for sediment transport.

## 363 4 Discussion

The present work has built on previous analyses to propose a theoretical framework for river bifurcations within the context of 1D morphodynamic modelling that accounts for key 2D ingredients at the entrance of the bifurcation node and in the fluxes delivered by the upstream channel.



Figure 10: Effect of the slope advantage on a curved  $(W_a/R = 0.02)$  bifurcation. (a) Equilibrium solutions as a function of the aspect ratio  $\beta_0$ . (b) Equilibrium solutions as a function of the slope advantage under sub-resonant (grey line) and super-resonant (black lines) conditions. Solid and dashed lines indicate stable and unstable solutions respectively, while arrows indicate possible trajectories when increasing (red arrows) or decreasing (blue arrows)  $\Delta S$ .



Figure 11: Discharge asymmetry as a function of curvature and slope asymmetry: (a) subresonant conditions  $((\beta_0 - \beta_R)/\beta_R = -0.3)$ ; (b) super-resonant conditions  $((\beta_0 - \beta_R)/\beta_R = 0.3)$ , with black oblique stripes indicating the region where two stable solutions exists. (c) example of different slope advantages in meanders chute cutoff. Left: steeper inner channel ( $\Delta S < 0$ ); right: steeper outer channel ( $\Delta S > 0$ ).

Here we discuss (i) the main implications of our findings and the potential of the proposed approach for the interpretation of bifurcation dynamics, as it emerges from both observations and modelling; (ii) the significance of the equilibrium analysis for time-dependent processes; (iii) the need to clarify the specific use of the wording "instability" when addressing bifurcation dynamics, in the light of our findings and in the context of previous studies; (iv) applicability and limitations of our approach.

373

#### 374 Enhanced insight on bifurcation morphodynamics

The core of the model lies in incorporating in the 1D scheme of Bolla Pittaluga et al. (2003) 375 (i) the resonant aspect ratio as threshold for bifurcation stability (*Redolfi et al.*, 2016) and (ii) 376 the effect of the forcing factors, through a proper modelling of the water and sediment fluxes 377 delivered from the upstream channel and accepted by the downstream bifurcates. These fluxes 378 are laterally symmetrical in the case of a purely free bifurcation, while the opposite occurs when 379 adding forcing effects, which are almost invariably observed in real settings. Among the broad 380 variety of forcing factors that characterize natural river bifurcations, here we addressed the isolated 381 and the combined effect of an upstream channel curvature and a gradient advantage between the 382 downstream branches. 383

A first important insight allowed by the proposed approach is the confirmation of the key role 384 of the resonant aspect ratio on bifurcation behaviour, and an in-depth quantitative understanding 385 of how the bifurcation response depends on the relative distance of the channel-forming aspect 386 ratio value from such resonant threshold. This is supported with an analysis of an unprecedented 387 number of laboratory, numerical and field data, for both gravel-bed and sand bed streams. The 388 stability criterion based on the resonant threshold explains the loss of stability with increasing 389 Shields stress of balanced sand bed bifurcations observed by Edmonds and Slingerland (2008), 390 which is simply the consequence of adopting different transport and friction formulae. Compared 391 with previous stability criteria, the analysis based on resonance offers the key advantage that it 392 does not require the calibration of a specific parameter like the k exponent of Wang et al. (1995) 393 or the  $\alpha$  length parameter of Bolla Pittaluga et al. (2003, 2015). 394

Interestingly, the role of the free instability mechanism is not limited to ideal, geometrically 395 symmetric bifurcations with symmetrical boundary conditions, but it is also a key controlling 396 factor for complex, forced bifurcations. Consequently, also the response of bifurcation to the 397 external forcings highly depends on channel conditions with respect to resonance. Under sub-398 resonant conditions, the bifurcation behaviour is relatively simple, as it is perfectly balanced for 399 free, symmetrical bifurcations and mostly dominated by the forcing effects when they are present. 400 On the contrary, under super resonant conditions balanced equilibrium configurations are never 401 stable, so that the bifurcation always tends to highly asymmetric equilibrium states. Here, multiple 402 stable solutions are possible, including counter-intuitive configurations where the inner bifurcate 403 prevails, which suggests the possibility of complete shifts and hysteresis in the bifurcation response 404 to changing conditions (Scheffer et al., 2001). 405

The whole picture yields a clear, physically-based key to interpreting results of field observa-406 tions and numerical models, which at times displayed behaviours that could not be given a fully 407 exhaustive explanation. An example is provided by the result of Kleinhans et al. (2008), based on 408 a three-dimensional model of a curved bifurcation with different channel gradients. These results 409 revealed a "dramatic effect" of the width to depth ratio on discharge distribution and overall bed 410 morphology, with the bifurcation switching from a dominant inner-bend bifurcate to a dominant 411 outer-bend bifurcate, which indicates that the bifurcation behaviour "bifurcates" at a certain 412 width between  $W_a = 288 \,\mathrm{m}$  and  $378 \,\mathrm{m}$  (see Section 4.5 of Kleinhans et al., 2008). By applying our 413 proposed modelling framework, and using the same closure relation and flow conditions as in the 414 numerical experiments, it turns out that the observed range of widths correspond to the transition 415 from from sub-resonant (Figure 12a) to super-resonant (Figures 12b and 12c) conditions. This 416 transition can explain the fairly different morphological evolution upstream the bifurcation: under 417 sub-resonant conditions the bifurcation does not significantly affect the upstream bed elevation 418 (dashed line of Figure 12a), while under super-resonant conditions the upstream morphodynamic 419 influence triggered by the bifurcation induces the formation of steady bars in the upstream channel 420 (Figures 12b and 12c), which affect how discharge is downstream distributed. 421

422 Super-resonant conditions are not rare in nature, insofar as gravel bed rivers tends to behave



Figure 12: Profiles of bed elevation at the outer bank of a curved channel that bifurcates at x = 6 km, according to the numerical simulations of *Kleinhans et al.* (2008), adapted from their Figure 12 (long bend series). (a) Sub-resonant conditions; (b) and (c) super-resonant conditions. Lines from light to dark indicate advancing time. The dashed line indicates the final bed profile upstream the bifurcation, which shows significant oscillations only when the upstream channel falls under super-resonant conditions.

<sup>423</sup> super-resonantly, especially for near-threshold (i.e. low Shields stress) channels (*Zolezzi et al.*,
<sup>424</sup> 2009). This confirms the tendency of balanced bifurcations to become unstable when the fixed
<sup>425</sup> bank hypothesis is released so that channels are free to reach their regime width (*Miori et al.*,
<sup>426</sup> 2006).

When different forcing effects are simultaneously acting they can compensate themselves (e.g. 427 Kleinhans et al., 2008; van Dijk et al., 2014; Kleinhans et al., 2013), leading to a nearly balanced 428 configuration. For example a gradient advantage in a scroll-slough chute cutoff of a meander bend 429 (Figure 11c) can balance the opposite effect of the channel curvature. However, this is possible 430 only if the upstream channel is in sub-resonant conditions, so that the bifurcation is intrinsically 431 stable. On the contrary, under super-resonant conditions the bifurcations tends to propagate its 432 morphodynamic influence in the upstream direction, so that the discharge partition is always 433 highly unbalanced, independently of the upstream and downstream conditions. 434

Similar results can be found when other kind of forcing effects are interacting with the free 435 instability mechanism, as emerging in the analysis of *Bertoldi et al.* (2009) about the effect of 436 downstream-migrating alternate bars on the bifurcation dynamics. In that context, different char-437 acteristic regimes have been identified depending on the channel aspect ratio with respect to a 438 critical threshold. For relatively small  $\beta_0$  the bifurcation is essentially balanced with discharge 439 oscillations around  $\Delta Q = 0$  caused by the passage of bars, while for higher  $\beta_0$  unbalanced states 440 are observed. In this latter case the bifurcation can be either "bar perturbed" (small oscilla-441 tions around a stable unbalanced state) or "bar dominated" (frequent switching of the bifurcation 442 between opposite, highly unbalanced states). A diagram similar to Figure 10b can be used to in-443 terpret the different scenarios: essentially balanced solutions occur under sub-resonant conditions, 444 while super resonant conditions yield highly unbalanced solutions, which are "bar perturbed" 445 when the variations of the forcing factor are relatively weak, or "bar dominated" when forcing 446 effect is strong enough to make the solution jumping between opposite states. 447

448

### 449 Quasi-equilibrium and temporal scales: the present work in a broader context

450

Our analysis is focused on steady equilibrium configurations, where both upstream and down-

stream channels are considered in planimetric and altimetric equilibrium. Strictly speaking, this is rarely the case of natural bifurcation, because forcing effects are usually varying in time. Nevertheless, as long as their rate of change is slow as compared with the intrinsic time scale of the bifurcation, the response of the system can be studied as a sequence of quasi-equilibrium states. This allows us to interpret the action of downstream migrating bars, as well as the analogous effect of the migration of the upstream meander (*Kleinhans et al.*, 2011), by means of equilibrium diagrams like those of Figure 10.

Similarly, the quasi-steady analysis can be applied for interpreting the effect of downstream 458 variations, provided they are comparatively slow. For example, in the depositional systems in-459 vestigated by Salter et al. (2018), the interaction between the bifurcation and the downstream 460 bifurcates leads to autogenic temporal oscillations of channel slope and discharge asymmetry. 461 This process evolves on a time scale that is proportional to the length of the downstream bifur-462 cates, which is usually much longer than the intrinsic time scale of the bifurcation evolution (see 463 *Miori et al.*, 2006). Therefore, focusing on the local behaviour of the bifurcation node, such down-464 stream mechanisms can be considered as an external forcing effect, coherently with the definition 465 adopted in the present work. 466

This provides a worthwhile example of how the definition of the forcing factors depends in 467 general on the spatial and temporal scales under consideration. An analogous concept is at the 468 core of classical theoretical studies on bar-bend interactions in river meanders (e.g., Tubino and 469 Seminara, 1990), which pointed out how the dynamics of sediment bars inside meandering chan-470 nels depends on the interaction between the free instability mechanisms that causes spontaneous 471 development of migrating bars, and the effect of the variable meander curvature. In general also 472 the channel curvature is not fixed but changes in time as the meander develops. However, as long 473 as the two mechanisms act at different time scales, the planform evolution being a much slower 474 process, when focusing on bar dynamics the meander curvature can be considered as a fixed forcing 475 factor. 476

477

### 478 The meaning of "instability" within bifurcation dynamics

The outcomes of this work also suggest revisiting the use of the key wording "instability", 479 which has been often used in previous studies of river bifurcation morphodynamics, though in 480 many times with different meanings. "Instability" has been indeed used to indicate: (i) the sit-481 uation whereby an equilibrium bifurcation configuration is unstable; (ii) a systematic change of 482 the discharge distribution over time (e.g., Kleinhans et al., 2013); (iii) a bifurcation that evolves 483 towards an highly unbalanced configuration and eventually produces the complete closure of one 484 of the two bifurcates (e.g., Burge, 2006; Le et al., 2018b). In this paper we have used "instability" 485 in its mathematical meaning (i), thus indicating the mathematical instability of an equilibrium 486 configuration (mathematical solution), which in itself could be either symmetrical or asymmetri-487 cal, and therefore may become inconsistent with meanings (ii) or (iii). Moreover, though meanings 488 (ii) and (iii) might be interchangeable under some circumstances, this does not apply in general, 489 and they may not be of help in disentangling the role of the free and of the forced bifurcation 490 mechanisms when analysing a specific situation. For example, the instability of the balanced so-491 lution in the super-resonant regime does not necessarily lead to the closure of one bifurcate, but 492 often leads to a stable, unbalanced configuration. Similarly, a partial or complete channel closure 493 may be caused by a forcing factor rather than an instability of an equilibrium configuration, which 494 in itself could be symmetrical or asymmetrical. This is, for instance, the case when a localized 495 obstacle deviates the flux towards one preferred bifurcate (Le et al., 2018b). 496

497

#### 498

### Applicability and limitations of the present approach

In this paper we have adopted a local viewpoint of the bifurcation morphodynamics, which focuses on a tile of a complex mosaic of processes where several autogenic mechanisms interact at different spatial and temporal scales (e.g., in the case of bifurcations coupled with aggrading downstream channels or embedded in braided networks).

The methodological approach can be broadly applied to analyse river bifurcations in real settings. The key ingredient required by the two-cell model to account for the upstream forcings is the availability of suitable transverse distributions of flow and sediment transport to compute water and sediment fluxes that enter the bifurcation node. Our results refer to the simple case of a

relatively long channel of constant curvature, and are based on the assumption of fully developed 507 flow, which is not satisfied in short bends and in general when the curvature is spatially varying, as 508 in meandering channels. Extending the model to treat such complex configurations would require 509 coupling the two-cell nodal point conditions with a sound model for flow and bed topography in 510 meandering channels (e.g., Zolezzi and Seminara, 2001). This analysis is beyond the scope of the 511 present paper; however, we may expect that in this case bifurcation stability will depend not only 512 on local curvature, but also on the position of the bifurcation node with respect to the steady 513 pattern of point bars forming in the upstream channel (Le et al., 2018a). 514

Further investigation is needed to understand to what extent the key mechanisms driving the 515 bifurcation instability, and in particular the upstream morphodynamic influence, can be affected 516 by processes that are often not reproduced by mathematical and physical models. Specifically, 517 two fundamental processes would probably need more consideration in future research. The first 518 is sediment sorting: despite some indications of a relatively weak effect of grain sorting on the 519 stability of migrating bars (Lanzoni and Tubino, 1999), its role in determining the bifurcation 520 stability is not clear, especially in gravel bed channels (Burge, 2006). The second process is 521 suspended sediment transport, which is often dominant in large, multi-thread, sand bed rivers 522 (Szupiany et al., 2012): when suspended load is the dominant mode of sediment transport the 523 gravitational pull towards the deeper channel is probably weaker, so that the bifurcation may be 524 even more unstable than currently predicted (*Kleinhans et al.*, 2006). 525

## 526 5 Conclusions

The present work offers a viewpoint of river bifurcations as dynamical systems for which a distinct role of the free and forced responses can be identified. We propose a theoretical framework based on the 1D model with two-cell bifurcation node originally developed by *Bolla Pittaluga et al.* (2003), as extended by *Kleinhans et al.* (2008) to account for the curvature-driven secondary flow. Furthermore, we incorporate the key outcomes of the fully 2D analytical approach of *Redolfi et al.* (2016) within the classical 1D scheme for river bifurcations. Two main forcing factors are considered, the curvature of the upstream channel and a slope advantage of one of the bifurcates, though the approach could be easily extended to account for other factors (e.g., the presence of a local obstacle upstream the bifurcation).

The key advantage of the proposed approach is its ability to clearly isolate the different free and forced mechanisms that may control the bifurcation dynamics in a complex setting, like that of real-world bifurcations, thus resulting in a suitable tool to gain clear insight in the analysis and interpretation of numerical model outcomes and of field observations. The key novel outcomes for bifurcation dynamics can be summarized in the following four items.

1. The bifurcation stability criterion based on the resonant aspect ratio threshold has been 541 successfully tested against data of both gravel bed and sand bed channels, and it allows 542 for capturing the opposite effects of Shields stress, which tend to promote more balanced 543 bifurcations in gravel bed rivers and more unbalanced bifurcations in sand bed streams. 544 This criterion can then be used for predicting the balanced/unbalanced character of free 545 bifurcations, and it allows incorporating the fully-2D solution of *Redolfi et al.* (2016) within 546 the classical 1D theory (Wang et al., 1995; Bolla Pittaluga et al., 2003) with no need to 547 calibrate specific bifurcation parameters. 548

2. The role of the free instability mechanism is not limited to purely free bifurcations, but is also fundamental in the dynamics of the forced bifurcations that are more representative of real-world settings. Therefore, river bifurcations with super-resonant upstream channels are dominated by the free mechanism, characterized by multiple, highly unbalanced equilibrium configurations. This remarkable behaviour might lead to counter-intuitive outcomes, where for example water and sediment fluxes are mainly delivered towards the bifurcate located at the inner bank of a channel bend.

Analysis of the interaction between two of the most common forcing effects (slope advantage and curvature) allows us to quantify the parameters range where free and forcing effects cooperate or compete in determining the overall bifurcation dynamics. Under sub-resonant conditions, the interaction between upstream curvature and slope advantage is smoothly dependent on the relative intensity of the forcings (*Kleinhans et al.*, 2008; *van Dijk et al.*,

31

<sup>561</sup> 2014), while this is not the case under super-resonant conditions, for which abrupt transitions
 <sup>562</sup> between opposite, highly unbalanced equilibrium states are expected.

4. The above results highlight how river bifurcations behave as dynamical systems like many
other eco-morphological processes in rivers and freshwater bodies (e.g. Scheffer et al., 2001),
where the nonlinear interaction among internal and external mechanisms gives rise to a
complex response, characterized by sensitivity to the initial conditions, multistable states
and hysteresis cycles.

## **Acknowledgements**

Marco Redolfi's work is supported by "Agenzia Provinciale per le Risorse Idriche e l'Energia (APRIE) - Provincia Autonoma di Trento". All data are from published works. Matlab codes for the model solution and for the computation of the resonant aspect ratio are available at https:// bitbucket.org/Marco\_Redolfi/freeforced\_bifurcations and at https://bitbucket.org/ Marco\_Redolfi/bars\_res-crit, respectively. This manuscript has highly benefited from the comments of Chris Paola and Gerard Salter.

## <sup>575</sup> Appendix A: Fully developed flow in a constant curvature channel

<sup>576</sup> Here we provide a detailed derivation of the flow field in a channel of constant curvature, obtained
<sup>577</sup> by following the *Struiksma et al.* (1985) approach.

The model is formulated in a curvilinear system of reference  $\{x, y\}$ , where x is pointing in the downstream direction and y represents the transverse coordinate (see Figure 3). Assuming a fully developed flow, all dependent variables vary only with y, and therefore the x-derivatives vanish; in such conditions, the continuity equation gives zero transverse fluxes of water and sediment, while the longitudinal momentum equation reduces to an uniform flow relation for the depth-averaged longitudinal velocity U, namely:

$$U = c \sqrt{g S D},\tag{A1}$$

where the longitudinal slope S depends on y as:

$$S = S_0 \, \frac{R}{R+y},\tag{A2}$$

with  $S_0$  indicating the slope of the channel centreline (i.e. at y = 0).

The longitudinal velocity generates a shear stress  $\tau_x$ , which can be computed as:

$$\tau_x = \rho \, \frac{U^2}{c^2},\tag{A3}$$

while the secondary flow produces a shear stress in the transverse direction, given by (see *Struiksma et al.*, 1985):

$$\tau_y = -\rho A \frac{DU^2}{c^2} \frac{1}{R},\tag{A4}$$

589 where  $\rho$  indicates the water density.

The transverse stress  $\tau_y$  needs to be compensated by a gradient of the bed elevation. Therefore, considering the *Ikeda* (1982) formulation for the effect of gravity on the sediment transport direction, the following relation arises:

$$\frac{r}{\sqrt{\theta}}\frac{d\eta}{dy} = \frac{\tau_y}{\tau_x} = -A\frac{D}{R}.$$
(A5)

Transverse profiles of bed elevation can be obtained by integrating Equation (A5). To this aim, we need to specify how the water depth varies along the cross section through the following geometrical relation:

$$\frac{dD}{dy} = \frac{dH}{dy} - \frac{d\eta}{dy},\tag{A6}$$

where H indicates the water surface elevation.

<sup>597</sup> Under the hypothesis of horizontal free surface, Equation (A6) reduces to  $dD/dy = -d\eta/dy$ , <sup>598</sup> so that the differential equation (A5) can be easily solved in terms of D. More generally, the <sup>599</sup> gradient of free surface elevation can be derived from the equation of transverse momentum:

$$g\frac{dH}{dy} + \frac{\tau_y}{\rho D} = \frac{U^2}{R},\tag{A7}$$

 $_{600}$  which, when combined with Equations (A5) and (A6), gives an expression of the type:

$$\frac{dD}{dy} = fct(y, D),\tag{A8}$$

<sup>601</sup> which can be easily solved by numerical integration.

The effect of the channel curvature on the transverse profiles of bed and water surface elevation is illustrated in Figure 13. The spiral flow induces higher bed elevation and slightly lower water surface elevation at the inner bend. Consequently, water depth, velocity, and water and sediment fluxes are higher at the outer bend. The resulting transverse profiles are clearly nonlinear, especially when the channel is highly curved.



Figure 13: Transverse profiles of (scaled) bed and water surface elevation in the main channel, where  $\overline{\eta}$  is the mean bed elevation. Dashed lines: straight channel; solid lines: curved channel. Parameters are  $\theta_0 = 0.1$ ,  $d_s = 0.02$ , r = 0.5. Background colours indicate the position of the two cells of size  $B_b$  and  $B_c$ .

<sup>607</sup> Once the transverse profiles are known, the input fluxes for the two-cell model can be com-<sup>608</sup> puted by integrating along their respective domain. In the general case of different width of the downstream bifurcates (i.e.  $W_b \neq W_c$ ), the width of the two cells (see Figure 13) can be calculated as:

$$B_b = W_a \frac{W_b}{W_b + W_c}, \qquad B_c = W_a \frac{W_c}{W_b + W_c}, \tag{A9}$$

<sup>611</sup> so that the transverse position of the interface between the two cells reads:

$$y_c = B_b - \frac{W_a}{2} = \frac{W_a}{2} \left( \frac{W_b - W_c}{W_b + W_c} \right),$$
 (A10)

which vanishes when  $W_b = W_c$  as assumed in the paper. Finally, water and sediment fluxes feeding the two cells are given by the following relations:

$$Q_b^{IN} = \int_{-W_a/2}^{y_c} UD \, dy, \qquad Q_c^{IN} = 1 - Q_b^{IN}, \tag{A11}$$

614

$$Qs_b^{IN} = \sqrt{g\Delta d_{50}^3} \int_{-W_a/2}^{y_c} \Phi\left(\theta, \frac{D}{d_{50}}\right) \, dy, \qquad Qs_c^{IN} = 1 - Qs_b^{IN}.$$
(A12)

## <sup>615</sup> Appendix B: Algebraic expression for $c_D$ , $\Phi_D$ and $\Phi_T$ coefficients

In this section we provide an explicit expression of the coefficients arising from linear stability analysis, which are needed to evaluate the critical and the resonant aspect ratio through Equations (12) and (14).

The  $c_D$  coefficient, which defines the response of the Chézy coefficient to variations of water depth, is defined as:

$$c_D := \left. \frac{D_0}{c_0} \frac{\partial c}{\partial D} \right|_{D_0}.$$
 (B1)

<sup>621</sup> When considering the logarithmic formula of *Engelund and Fredsoe* (1982) (Equation (3)), we <sup>622</sup> obtain:

$$c_D = \frac{2.5}{c_0}.$$
 (B2)

where  $c_0$  is the Chézy coefficient evaluated at reference conditions, namely:

$$c_0 = 6 + 2.5 \log\left(\frac{1}{2.5 \, d_s}\right).$$
 (B3)

Similarly, the coefficients  $\Phi_D$  and  $\Phi_T$ , which specify the sensitivity of the sediment transport to variations of water depth and Shields stress, are defined as:

$$\Phi_D := \left. \frac{D_0}{\Phi_0} \frac{\partial \Phi}{\partial D} \right|_{\theta_0, D_0}, \qquad \Phi_T := \left. \frac{\theta_0}{\Phi_0} \frac{\partial \Phi}{\partial \theta} \right|_{\theta_0, D_0}, \tag{B4}$$

<sup>626</sup> and their explicit expression depends on the sediment transport formula used.

<sup>627</sup> The *Engelund and Hansen* (1967) transport formula reads:

$$\Phi = 0.05 \, c^2 \theta^{2.5},\tag{B5}$$

<sup>628</sup> and gives the following coefficients:

$$\Phi_D = 2c_D, \quad \Phi_T = 2.5. \tag{B6}$$

Transport formulae designed for bed load are often expressed in terms of  $\theta$  only, and therefore  $\Phi_D$  vanishes as the bed load function does not depend explicitly on water depth. For example when using the *Meyer-Peter and Muller* (1948) relation

$$\Phi = 8 \left(\theta - \theta_{cr}\right)^{1.5} \tag{B7}$$

632 the coefficients reads:

$$\Phi_D = 0, \quad \Phi_T = 1.5 \frac{\theta_0}{\theta_0 - \theta_{cr}}.$$
 (B8)

<sup>633</sup> The *Parker* (1990) formula can be expressed as:

$$\Phi = 0.00218 \,\theta^{1.5} \,G(\xi), \quad \xi := \frac{\theta}{0.0386}, \tag{B9}$$

634 where  $G(\xi)$  is a piecewise-defined function:

$$G(\xi) = \begin{cases} 5474 \left(1 - 0.853/\xi\right)^{4.5} & \xi > 1.59\\ \exp\left[14.2(\xi - 1) - 9.28(\xi - 1)^2\right] & 1 \le \xi \le 1.59 \\ \xi^{14.2} & \xi < 1 \end{cases}$$
(B10)

635 In this case we obtain:

$$\Phi_D = 0, \quad \Phi_T = 1.5 + \frac{G_{\xi}}{0.0386}, \quad G_{\xi} := \frac{\xi_0}{G_0} \frac{dG}{d\xi}, \tag{B11}$$

 $_{\rm 636}~$  where  $G_{\xi}$  can be expressed by deriving Equation (B10), which gives:

$$G_{\xi} = \begin{cases} \frac{4.5}{\xi_0/0.853 - 1} & \xi_0 > 1.59\\ -18.56\,\xi_0^2 + 32.76\,\xi_0 & 1 \le \xi_0 \le 1.59 \\ 14.2 & \xi_0 < 1 \end{cases}$$
(B12)

### 637 References

- Baar, A. W., J. de Smit, W. S. J. Uijttewaal, and M. G. Kleinhans (2018), Sediment Transport of
  Fine Sand to Fine Gravel on Transverse Bed Slopes in Rotating Annular Flume Experiments,
  Water Resources Research, 54(1), 19–45, doi:10.1002/2017WR020604.
- Bertoldi, W. (2012), Life of a bifurcation in a gravel-bed braided river, Earth Surface Processes
   and Landforms, 37(12), 1327–1336, doi:10.1002/esp.3279.
- Bertoldi, W., and M. Tubino (2007), River bifurcations: Experimental observations on equilibrium
  configurations, *Water Resources Research*, 43(10), 1–10, doi:10.1029/2007WR005907.
- Bertoldi, W., L. Zanoni, S. Miori, R. Repetto, and M. Tubino (2009), Interaction between migrating bars and bifurcations in gravel bed rivers, *Water Resources Research*, 45(6), 1–12,
  doi:10.1029/2008WR007086.
- Blondeaux, P., and G. Seminara (1985), A Unified Bar Bend Theory of River Meanders, Journal
   of Fluid Mechanics, 157, 449–470.
- Bolla Pittaluga, M., R. Repetto, and M. Tubino (2003), Correction to "Channel bifurcation in
  braided rivers: Equilibrium configurations and stability", *Water Resources Research*, 39(3),
  1–13, doi:10.1029/2003WR002754.
- Bolla Pittaluga, M., G. Coco, and M. G. Kleinhans (2015), A unified framework for stability of
  channel bifurcations in gravel and sand fluvial systems, *Geophysical Research Letters*, 42(18),
  7521–7536, doi:10.1002/2015GL065175.
- <sup>656</sup> Burge, L. M. (2006), Stability, morphology and surface grain size patterns of channel bifurcation
  <sup>657</sup> in gravel-cobble bedded anabranching rivers, *Earth Surface Processes and Landforms*, 31(10),
  <sup>658</sup> 1211–1226, doi:10.1002/esp.1325.
- Camporeale, C., P. Perona, A. Porporato, and L. Ridolfi (2007), Hierarchy of models for me andering rivers and related morphodynamic processes, *Reviews of Geophysics*, 45(1), 1–28,
   doi:10.1029/2005RG000185.

- Edmonds, D. A. (2012), Stability of backwater-influenced river bifurcations: A study of
  the Mississippi-Atchafalaya system, *Geophysical Research Letters*, 39(8), 1–5, doi:10.1029/
  2012GL051125.
- Edmonds, D. A., and R. L. Slingerland (2008), Stability of delta distributary networks and their
  bifurcations, Water Resources Research, 44(9), 1–13, doi:10.1029/2008WR006992.
- Engelund, F., and J. Fredsoe (1982), Sediment Ripples and Dunes, Annual Review of Fluid Me *chanics*, 14, 13–37, doi:10.1146/annurev.fl.14.010182.000305.
- Engelund, F., and E. Hansen (1967), A monograph on sediment transport in alluvial streams, 65
  pp., Tekniks Forlag, Copenhagen, Denmark, doi:10.1007/s13398-014-0173-7.2.
- Federici, B., and C. Paola (2003), Dynamics of channel bifurcations in noncohesive sediments,
  Water Resources Research, 39(6), 1162, doi:10.1029/2002WR001434.
- Golubitsky, M., and D. Schaeffer (1979), An analysis of imperfect bifurcation, Annals of the New
  York Academy of Sciences, 316(1), 127–133, doi:10.1111/j.1749-6632.1979.tb29464.x.
- Grenfell, M., R. Aalto, and A. Nicholas (2012), Chute channel dynamics in large, sand-bed meandering rivers, *Earth Surface Processes and Landforms*, 37(3), 315–331, doi:10.1002/esp.2257.
- <sup>677</sup> Habersack, H., and H. Piégay (2007), 27 River restoration in the Alps and their surroundings:
  <sup>678</sup> past experience and future challenges, *Developments in Earth Surface Processes*, 11, 703–735.
- Hardy, R. J., S. N. Lane, and D. Yu (2011), Flow structures at an idealized bifurcation: A
  numerical experiment, *Earth Surface Processes and Landforms*, 36(15), 2083–2096, doi:10.1002/
  esp.2235.
- Ikeda, S. (1982), Incipient motion of sand particles on side slopes, Journal of Hydraulic Division,
   108(1), 95–114.
- Kleinhans, M. G., B. Jagers, E. Mosselman, and K. Sloff (2006), Effect of upstream meanders on
  bifurcation stability and sediment division in 1D, 2D and 3D models, in *River flow 2006 : pro-*

- ceedings of the International Conference on Fluvial Hydraulics, Lisbon, Portugal, 6-8 September
   2006, pp. 1355–1362.
- Kleinhans, M. G., H. R. A. Jagers, E. Mosselman, and C. J. Sloff (2008), Bifurcation dynamics
  and avulsion duration in meandering rivers by one-dimensional and three-dimensional models, *Water Resources Research*, 44(8), 1–31, doi:10.1029/2007WR005912.
- Kleinhans, M. G., K. M. Cohen, J. Hoekstra, and J. M. Ijmker (2011), Evolution of a bifurcation
   in a meandering river with adjustable channel widths, Rhine delta apex, The Netherlands, *Earth Surface Processes and Landforms*, 36(15), 2011–2027, doi:10.1002/esp.2222.
- Kleinhans, M. G., T. de Haas, E. Lavooi, and B. Makaske (2012), Evaluating competing hypotheses for the origin and dynamics of river anastomosis, *Earth Surface Processes and Landforms*, *37*(12), 1337–1351, doi:10.1002/esp.3282.
- Kleinhans, M. G., R. I. Ferguson, S. N. Lane, and R. J. Hardy (2013), Splitting rivers at their
  seams: Bifurcations and avulsion, *Earth Surface Processes and Landforms*, 38(1), 47–61, doi:
  10.1002/esp.3268.
- Lanzoni, S., and M. Tubino (1999), Grain sorting and bar instability, *Journal of Fluid Mechanics*,
   393, 149–174, doi:10.1017/S0022112099005583.
- Le, T., A. Crosato, E. Mosselman, and W. Uijttewaal (2018a), On the stability of river bifurcations
  created by longitudinal training walls. Numerical investigation, *Advances in Water Resources*,
  113, 112–125, doi:10.1016/j.advwatres.2018.01.012.
- Le, T. B., A. Crosato, and W. S. Uijttewaal (2018b), Long-term morphological developments of
  river channels separated by a longitudinal training wall, *Advances in Water Resources*, 113,
  73–85, doi:10.1016/j.advwatres.2018.01.007.
- Lesser, G. R., J. A. Roelvink, J. A. van Kester, and G. S. Stelling (2004), Development and
  validation of a three-dimensional morphological model, *Coastal Engineering*, 51 (8-9), 883–915,
  doi:10.1016/j.coastaleng.2004.07.014.

- Meyer-Peter, E., and R. Muller (1948), Formulas for bed load transport, Proceedings of the 2nd
   congress of the International Association for Hydraulic Research, 2, 39–64.
- <sup>713</sup> Miori, S., R. Repetto, and M. Tubino (2006), A one-dimensional model of bifurcations in <sup>714</sup> gravel bed channels with erodible banks, *Water Resources Research*, 42(11), 1–12, doi: <sup>715</sup> 10.1029/2006WR004863.
- Parker, G. (1990), Surface-based bedload transport relation for gravel rivers, Journal of Hydraulic *Research*, 28(4), 417–436, doi:10.1080/00221689009499058.
- Redolfi, M., G. Zolezzi, and M. Tubino (2016), Free instability of channel bifurcations and morphodynamic influence, *Journal of Fluid Mechanics*, 799, 476–504, doi:10.1017/jfm.2016.389.
- Salter, G., C. Paola, and V. R. Voller (2018), Control of Delta Avulsion by Downstream Sediment Sinks, *Journal of Geophysical Research: Earth Surface*, pp. 1060–1067, doi:10.1002/
  2017JF004350.
- Scheffer, M., S. Carpenter, J. A. Foley, C. Folke, and B. Walker (2001), Catastrophic shifts in
  ecosystems, *Nature*, 413(6856), 591–596, doi:10.1038/35098000.
- Seminara, G., and M. Tubino (1989), Alternate Bars and Meandering : Free , Forced and Mixed
  Interactions, in *River Meandering*, vol. 12, edited by S. Ikeda and G. Parker, pp. 267 320,
  AGU, Washington, D.C.
- Siviglia, A., G. Stecca, D. Vanzo, G. Zolezzi, E. F. Toro, and M. Tubino (2013), Numerical
   modelling of two-dimensional morphodynamics with applications to river bars and bifurcations,
   Advances in Water Resources, 52, 243–260, doi:10.1016/j.advwatres.2012.11.010.
- <sup>731</sup> Slingerland, R., and N. D. Smith (2004), River Avulsions and Their Deposits, Annu. Rev. Earth
   <sup>732</sup> Planet. Sci., 32(1), 257–285, doi:10.1146/annurev.earth.32.101802.120201.
- <sup>733</sup> Sloff, K., and E. Mosselman (2012), Bifurcation modelling in a meandering gravel-sand bed river,
  <sup>734</sup> Earth Surface Processes and Landforms, 37(14), 1556–1566, doi:10.1002/esp.3305.

- Struiksma, N., K. W. Olesen, C. Flokstra, and H. J. De Vriend (1985), Bed deformation in curved
  alluvial channels, *Journal of Hydraulic Research*, 23(1), 57–79, doi:10.1080/00221688509499377.
- Szupiany, R. N., M. L. Amsler, J. Hernandez, D. R. Parsons, J. L. Best, E. Fornari, and A. Trento
  (2012), Flow fields, bed shear stresses, and suspended bed sediment dynamics in bifurcations of
  a large river, *Water Resources Research*, 48(11), 1–20, doi:10.1029/2011WR011677.
- Tubino, M., and G. Seminara (1990), Free–forced interactions in developing meanders and suppression of free bars, *Journal of Fluid Mechanics*, 214, 131–159, doi:10.1017/S0022112090000088.

Van der Mark, C. F., and E. Mosselman (2013), Effects of helical flow in one-dimensional modelling
of sediment distribution at river bifurcations, *Earth Surface Processes and Landforms*, 38(5),
502–511, doi:10.1002/esp.3335.

van Dijk, W. M., F. Schuurman, W. I. V. D. Lageweg, and M. G. Kleinhans (2014), Bifurcation
instability and chute cutoff development in meandering gravel-bed rivers, *Geomorphology*, 213,
277–291, doi:10.1016/j.geomorph.2014.01.018.

Wang, Z., M. De Vries, R. Fokkink, and A. Langerak (1995), Stability of river bifurcations in 1D morphodynamic models, *Journal of Hydraulic Research*, 33(6), 739–750, doi:
10.1080/00221689509498549.

<sup>751</sup> Wiggins, S. (2003), Introduction to Applied Nonlinear Dynamical Systems and Chaos, Texts in
 <sup>752</sup> Applied Mathematics, vol. 2, Springer-Verlag, New York, doi:10.1007/b97481.

Zhang, W., H. Feng, A. J. F. Hoitink, Y. Zhu, F. Gong, and J. Zheng (2017), Tidal impacts on the
subtidal flow division at the main bifurcation in the Yangtze River Delta, *Estuarine, Coastal and Shelf Science*, 196, 301–314, doi:10.1016/j.ecss.2017.07.008.

Zolezzi, G., and G. Seminara (2001), Downstream and upstream influence in river meandering.
Part 1. General theory and application to overdeepening, *Journal of Fluid Mechanics*, 438, 213–230, doi:10.1017/S002211200100427X.

42

- Zolezzi, G., M. Guala, D. Termini, and G. Seminara (2005), Experimental observations of upstream
  overdeepening, *Journal of Fluid Mechanics*, 531, 191–219, doi:10.1017/S0022112005003927.
- Zolezzi, G., W. Bertoldi, and M. Tubino (2006), Morphological analysis and prediction of river
  bifurcations, in *Braided Rivers: Process, Deposits, Ecology and Management*, vol. 36, edited by
  G. H. Best, J. L. Bristow, and C. S. Petts, pp. 233–256, Blackwell, doi:10.1002/9781444304374.
  ch11.
- Zolezzi, G., R. Luchi, and M. Tubino (2009), Morphodynamic regime of gravel bed, singlethread meandering rivers, *Journal of Geophysical Research: Earth Surface*, 114(1), doi:
  10.1029/2007JF000968.