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Macroeconomic uncertainty and vector autoregressions

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ABSTRACT

A procedure to estimate measures of macroeconomic uncertainty and compute the effects of uncertainty shocks based on standard VARs is proposed. Uncertainty and its effects are estimated using a single model so to ensure internal consistency. Under suitable assumptions, the procedure is equivalent to using the square of the VAR forecast error as an external instrument in a proxy SVAR. The procedure allows to add orthogonality constraints to the standard proxy SVAR identification scheme. The method is applied to a US data set; results show that macroeconomic uncertainty is responsible of a large fraction of business-cycle fluctuations while financial uncertainty plays a modest role.

1. Introduction

Since Bloom (2009), a vast literature studying the role of uncertainty shocks for macroeconomic fluctuations has developed. A partial list of contributions includes (Fernandez-Villaverde et al., 2011; Bachmann et al., 2013; Bekaert et al., 2013; Caggiano et al., 2014; Rossi and Sekhposyan, 2015; Jurado et al., 2015; Scotti, 2016; Baker et al., 2016; Caldara et al., 2016; Leduc and Liu, 2016; Basu and Bundick, 2017; Fajgelbaum et al., 2017; Nakamura et al., 2017; Piffer and Podstawski, 2018; Bloom et al., 2018; Carriero et al., 2018a; Jo and Sekkel, 2019; Angelini and Fanelli, 2019; Ludvigson et al., 2021; Shin and Zhong, 2020; Dew-Baker and Giglio, 2020; Brianti, 2021). For more references, see the survey articles in Cascaldi-Garcia et al. (2020) and Fernandez-Villaverde and Guerrón-Quintana (2020). A growing consensus points out that uncertainty shocks are important drivers of the business cycle, although the magnitude of the effects varies across studies.

From a theoretical perspective, there are various theoretical channels through which increases in uncertainty can depress economic activity, see Bloom (2014) for a survey. Perhaps the most notable one is the real option channel: when investment is not reversible, higher uncertainty induces agents to postpone investment, thus producing a downturn in economic activity.

From an econometric perspective, two main approaches have been used in the literature to measure the effects of uncertainty shocks. The first is based on Structural Vector Autoregressive (SVAR) models. Within this approach, the common practice is to include in a VAR a measure of uncertainty, derived from outside the model, as an additional endogenous variable. The uncertainty shock and its effects are then identified by imposing suitable restrictions.

Several papers have proposed proxies of uncertainty which are not model-based but exploit different sources of information, such as stock market volatility (Bloom, 2009; Bekaert et al., 2013; Caldara et al., 2016), forecast disagreement in survey

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data (Bachmann et al., 2013), the frequency of selected keywords in journal articles (Baker et al., 2016), the unconditional distribution of forecast errors (Jo and Sekkel, 2019). Other papers (e.g. Jurado et al. (2015); Ludvigson et al. (2021)) start from a rigorous statistical definition of uncertainty as the conditional volatility of a forecast error and specify and estimate a stochastic volatility model by using sophisticated time series techniques.

The second approach is based on Stochastic Volatility VAR (SV-VAR) models, see for instance (Carriero et al., 2018b). In these models, an explicit dynamic process for uncertainty is specified, and uncertainty and its effects on economic variables are jointly estimated. The advantage of this second approach, relative to the first, is that the estimates of uncertainty and its effects, being obtained within a single framework, are internally consistent.

Indeed, as will become clear later, the VAR model has its own implications about uncertainty, so that the first approach opens the door to a potential problem of inconsistency between the external measure of uncertainty and the VAR model itself.

The cost of using SV-VAR is represented by the imposition of restrictive a priori assumptions, which might not necessarily fulfilled by the data, and a more complicated estimation procedure.

A new econometric procedure is proposed here. The procedure measures uncertainty and estimates the effects of uncertainty shocks based on a single standard VAR model. Throughout this work, the focus is on the definition of uncertainty adopted in Jurado et al. (2015), JLN henceforth: uncertainty is the forecast error conditional variance or, equivalently, the conditional expectation of the forecast error squared. The procedure unfolds in four steps: (i) estimating a VAR and the associated reduced form impulse response functions; (ii) computing the implied squared forecast error for the variable and horizon of interest; (iii) regressing the squared forecast error onto the current and past values of the VAR variables to get an estimate of uncertainty; (iv) using the coefficients of the regression in (ii) to combine the VAR impulse response functions obtained in (i), together with the desired restrictions to identify the uncertainty shock and the related IRFs. This procedure, similarly to SV-VAR models, ensures consistency between the estimate of uncertainty and the estimate of the effects of uncertainty shocks. It is worth noting that, while the focus here in on a VAR model, the method can easily be adapted to a FAVAR model or a factor model.

Under suitable conditions, steps (iii-iv) are equivalent to using the squared forecast error as the instrument within a proxy SVAR (Mertens and Ravn (2013); Stock and Watson (2018); Plagborg-Møller and Wolf (2021). To put it differently, the method proposed can be thought of as a proxy SVAR, where the proxy, instead of being an external variable, is a function of the estimated forecast error. The relevance condition of the instrument is clearly satisfied since the squared forecast error is, by definition, correlated with the uncertainty shock. However, in order for the exogeneity condition to hold, an additional assumption is needed, namely that uncertainty (or, more precisely, the squared prediction error) is not affected on impact by other structural shocks. This assumption is of course questionable. This assumption is relaxed by imposing orthogonality constraints with respect to other structural shocks.

Since Bloom (2009), it has become quite common to use a recursive ordering to identify the uncertainty shock. Noticeable recent exceptions are Ludvigson et al. (2021) and Brianti (2021), where the important problem of the exogeneity of uncertainty is studied. In this work only standard zero impact and long-run constraints are used, since exogeneity of uncertainty is not the main focus.

The method has a few noticeable advantages. First, it is simple to implement. Second, there is a clear and rigorous definition of uncertainty for each variable and horizon in the VAR. Third, it avoids the problematic choice of an external uncertainty measure.

There are many measures of uncertainty, which deliver different effects on the macro economy. For instance, stock market volatility measures (VIX and VXO) and the index developed by Rossi and Sekhposyan (2015) have small and barely significant effects on output, whereas other measures, such as the Economic Policy Uncertainty index of Baker et al. (2016), and the measure of Jurado et al. (2015) have large and significant effects.

Fourth, internal consistency between the estimate of uncertainty and its effects is ensured, as in SV-VAR, since they are both obtained with a single model. Fifth, unlike in SV-VARs, assumptions on the functional form of the conditional distribution of the shocks are not needed.

The procedure is applied to a US macroeconomic data set. Results show that (a) the estimates of uncertainty are reliable, in that (a.1) the squared prediction errors are significantly predicted by a linear combination of the VAR variables, with sizable explained variances; (a.2) uncertainty estimates obtained with the linear approximation employed are strongly correlated with comparable estimates in the literature; (a.3) price uncertainty and interest-rate uncertainty are related to recognizable economic events. As for the impulse response functions and variance decomposition, results show that (b) exogenous macroeconomic uncertainty shocks explain a large fraction of business-cycle fluctuations while financial uncertainty plays a modest role; (c) results are robust with respect to the choice of the uncertainty horizon and variable, the number of lags and the choice of the variables included in the VAR.

The remainder of this work is organized as follows. Section 2 discusses the econometric approach. Section 3 presents the results. Section 4 concludes.

2. Econometric approach

This section discusses the econometric approach to estimate uncertainty and identify the effects of the uncertainty shock in a simple VAR model.

2.1. The VAR model

The starting point is the assumption that the macroeconomic variables in the n-dimensional vector y_t follow the VAR model

$$A(L)y_t = \mu + \varepsilon_t, \tag{1}$$

$A(L) = I - \sum_{k=1}^{p} A_k L^k$ is a matrix of degree-p polynomials in the lag operator L. The statistical properties of the vector innovation ε_t deserve some discussion. It is assumed that ε_t is a zero-mean serially uncorrelated process with covariance Σ_ε. However the process is not assumed to be serially independent. Indeed, the procedure works, as shown below, only when ε_t is serially dependent and conditionally heteroschedastic. This might appear an atypical requirement, since it is common in the literature to assume normal distribution for the shock process. Notice however that serial dependence is embedded in ARCH and GARCH models or Stochastic-Volatility VAR models since the conditional variance depends on past information. It is shown below a simple model where ε_t is serially dependent and conditionally heteroschedastic. In the empirical section it is shown that serial independence and homoschedasticity are largely rejected in the data. Notice that the fact that the shocks are serially uncorrelated but not independent implies that the shocks cannot be normally distributed since if they were gaussian and serially uncorrelated, they would be serially independent.

By inverting the VAR, the VMA representation is obtained

$$y_t = \delta + B(L)\varepsilon_t, \tag{2}$$

where $B(L) = \sum_{k=0}^{\infty} B_k L^k = A(L)^{-1}$, with $B_0 = I_n$, is the matrix of reduced form impulse response functions and $\delta = B(1)\mu$. The implied h-step ahead prediction error is

$$e_{t+h} = \sum_{k=0}^{h-1} B_k \varepsilon_{t+h-k}. \tag{3}$$

2.2. VAR-based uncertainty

Following JLN, uncertainty is defined as the conditional volatility of the forecast error. Formally, for variable i and horizon h uncertainty is defined as

$$U_{ht}^i = E_t e_{i,t+h}^2. \tag{4}$$

The conditional expectation cannot be computed without introducing additional assumptions about the conditional distribution of the VAR residuals, for instance a stochastic volatility model. However, it can be approximated by means of linear projections. More precisely, the logarithm of uncertainty is approximated by taking the orthogonal projection of the log of the squared prediction error onto the linear space spanned by the constant and the present and past values of the y's:

Note that log uncertainty is approximated here, rather than uncertainty itself, to avoid negative estimates of uncertainty. However by approximating directly uncertainty very similar results are obtained.

$$\log(U_{ht}^i) \approx P_{ht}^i = Proj\big(\log(e_{i,t+h}^2)|y_{i,t-k}, i = 1, \dots, n; k = 0, \dots, q\big). \tag{5}$$

Hence

$$P_{ht}^i = \theta + c_0' y_t + \dots + c_q' y_{t-q} = \theta + c(L)' y_t, \tag{6}$$

where θ is a scalar, c_k, $k = 0, 1, \dots, q$, is an n-dimensional column vector and $c(L)' = \sum_{k=1}^{q} c_k' L^k$ is a row vector of polynomials in the lag operator L.

For notational simplicity, the dependence of θ, c_k and $c(L)$ on i and h is omitted.

Notice that, if the VAR residuals were serially independent (and therefore independent of lagged y's), then $\log(e_{i,t+h}^2)$ would be orthogonal to the predictors, implying $c(L) = 0$. Hence the procedure requires that the VAR residuals, while being serially uncorrelated, are not serially independent as observed above.

2.3. A simple reference model

While uncertainty is not explicitly modelled, relying on approximation (5), to fix ideas it can be useful to have a simple reference model with conditional heteroschedasticity, where (5) holds exactly for $h = 1$. Let

$$\varepsilon_{it} = \exp(c_{0i}' y_{t-1}/2) v_{it}, \tag{7}$$

$v_t = (v_{1t} \ \cdots \ v_{nt})'$ being zero-mean, serially independent and independent of y_{t-k}, $k > 0$, with identity covariance matrix.

In turn, $E(\varepsilon_{it}\varepsilon_{j,t-k}) = E v_{it}\, E[\exp(c_{0i}' y_{t-1}/2)\exp(c_{0j}' y_{t-k-1}/2) v_{j,t-k}] = 0$, so that ε_t is a vector white noise, consistently with (1).

Moreover, $U_{1t}^i = E_t(\varepsilon_{i,t+1}^2) = \exp(c_{0i}' y_t) E_t(v_{i,t+1}^2) = \exp(c_{0i}' y_t)$, since $E_t(v_{i,t+1}^2) = E(v_{i,t+1}^2) = 1$. Hence $\log U_{1t}^i = c_{0i}' y_t$. On the other hand, the linear projection of $\log(e_{i,t+1}^2)$ onto the space spanned by y_{t-k}, $k \geq 0$ is

$$\log(e_{i,t+1}^2) = \log(\varepsilon_{i,t+1}^2) = c_{0i}' y_t + w_{it}, \tag{8}$$

where $w_{it} = \log(v_{i,t+1}^2)$ is serially independent and independent of y_t, so that (5) holds with the equality sign.

2.4. Estimation

The model is estimated in two steps. First, the VAR in Eq. (1) and the implied in-sample prediction error e_{t+h} are computed using Eq. (3). Second, a variable i, an horizon h are chosen and the OLS regression

$$\log(e^2_{i,t+h}) = P^i_{ht} + \nu_t = \theta + c_0' y_t + \cdots + c_q' y_{t-q} + \nu_t, \tag{9}$$

is estimated, where the error ν_t is orthogonal to y_t and its past history.

Notice that if other variables, in addition to those included in y_t, are important to forecast the log square errors, these variables should be also included in the VAR. Otherwise the procedure would not work, see Eq. (10).

In the empirical section it is documented that the estimated coefficients are significantly different from zero (thus rejecting serial independence).

Uncertainty can then be estimated as the exponential of the fitted values:

$$\hat{U}^i_{ht} = \exp\left(\hat{\theta} + \hat{c}(L)' y_t\right).$$

Notice that an uncertainty index can be estimated for any variable in the VAR and any horizon.

To check whether the linear approximation is good, it is possible to test for the significance of additional regressors, such as squared y's or $\log(\epsilon^2_{it})$, $i = 1, \dots n$. Moreover, it is possible to verify whether the residual of the projection Eq. (9) is serially uncorrelated (as is the case for the reference model above, see Eq. (8)).

The standard approach at this point would be to include this measure of uncertainty as an additional endogenous variable in the VAR and re-estimate the model. This route is not pursued here. Instead, Eqs. (2) and (6) are directly combined. Precisely,

$$\begin{aligned} P^i_{ht} &= \theta + c(L)' y_t, \\ &= \omega + c(L)' B(L) \varepsilon_t \\ &\text{and} \\ &= \omega + g(L)' \varepsilon_t, \end{aligned} \tag{10}$$

where $g(L) = \sum_{k=0}^{\infty} g_k L^k = c(L)' B(L)$ and $\omega = \theta + c(1)' \delta$. According to the above equation, the estimate of $g(L)$ is obtained as $\hat{g}(L) = \hat{c}(L) \hat{B}(L)$ and an estimate of ω is obtained as $\hat{\omega} = \hat{\theta} + \hat{c}(1)' \hat{\delta}$. Notice that, according to (10), log uncertainty P^i_{ht} is an exact linear combination of the y's. Hence direct estimation of a VAR including both P^i_{ht} and y_t would be impossible, since the regressors would be perfectly collinear.

The reduced form of the full model can be written as

$$\begin{pmatrix} P^i_{ht} \\ y_t \end{pmatrix} = \begin{pmatrix} \omega \\ \delta \end{pmatrix} + \begin{pmatrix} g(L)' \\ B(L) \end{pmatrix} \varepsilon_t. \tag{11}$$

This representation will prove useful in the next subsection.

2.5. Identification

In this subsection it is discussed how to identify the uncertainty shock as a combination of the VAR residuals ε_t and study its impulse response functions. Two identification schemes are used. The first identification simply postulates that the uncertainty shock is the innovation in log uncertainty. In the second, zero short-run and zero long-run restrictions are used to account for potential endogeneity of uncertainty. With this identification the shock is the residual of the regression of the innovation of log uncertainty onto suitable regressors.

Eq. (10) makes clear why there is no need of relying on external measures of uncertainty. Uncertainty is directly derived from the forecast errors obtained from the data generating process for y_t. This ensures the internal consistency between the model and the uncertainty measure.

2.5.1. Innovation

To begin with, consider the simple case in which the uncertainty shock is identified as the innovation of log uncertainty, normalized to have unit variance. From Eq. (10) the innovation is

$$g_0' \varepsilon_t = c_0' B_0 \varepsilon_t = c_0' \varepsilon_t$$

(recall that $B_0 = I_n$). Then the normalized innovation u^*_t is

$$u^*_t = \frac{c_0'}{\sqrt{c_0' \Sigma_\varepsilon c_0}} \varepsilon_t = \nu' \varepsilon_t, \tag{12}$$

where Σ_ε is the variance-covariance matrix of ε_t. The corresponding vector of impulse response functions can be computed by the formula

$$d^*(L) = B(L) \Sigma_\varepsilon \nu, \tag{13}$$

with contemporaneous effects equal to $d^*(0) = \Sigma_\varepsilon \nu$, being $B(0) = I_n$ (see Appendix A for details on the derivation of the impulse response functions). Similarly, the impulse-response function for log uncertainty is $g(L)'\Sigma_\varepsilon \nu = c(L)'d^*(L)$. Hence the structural MA representation is

$$\begin{pmatrix} P_t^i \\ y_t \end{pmatrix} = \begin{pmatrix} \omega \\ \delta \end{pmatrix} + \begin{pmatrix} c(L)'d^*(L) \\ d^*(L) \end{pmatrix} u_t^* + \Psi^*(L)w_t^*, \tag{14}$$

where $\Psi^*(L)w_t^*$ is the term containing the $n-1$ remaining unidentified shocks times their impulse response functions.

2.5.2. Short-run and long-run zero restrictions

The identification procedure in the previous section imposes that on impact only the uncertainty shock affects log uncertainty since the shock is just the innovation. While quite common in the literature, this assumption is questionable, see for instance Bachmann et al. (2013) since there could be other shocks which might affect uncertainty contemporaneously. Here it is shown how to relax this assumption and impose other restrictions. More specifically, in this subsection it is discussed how to impose both short-run and long-run restrictions to zero. The discussion of the specific restrictions used to identify the uncertainty shock it is postponed to section 3.3, after having discussed model specification,

Suppose a researcher wants to impose that the uncertainty shock has a zero impact effect on variable y_{1t}. To impose the restriction it suffices to impose orthogonality of the uncertainty shock with respect to $\varepsilon_{1t} = D_1'\varepsilon_t$, where $D_1' = [1\ 0\ \cdots\ 0]$, i.e. the innovation in the first variable. This can be done by taking the residual of the projection of the uncertainty innovation $c_0'\varepsilon_t$ onto $D_1'\varepsilon_t$. It is easily seen that such residual is $[c_0' - c_0'\Sigma_\varepsilon D_1 \Gamma D_1']\varepsilon_t$, where $\Gamma = (D_1'\Sigma_\varepsilon D_1)^{-1}$. The uncertainty shock can then be obtained by normalizing the residual to have unit variance.

Similarly, to impose that the shock has no long run effect on some variables, for instance GDP, a long run shock is identified as the only shock affecting GDP in the long run. Letting this shock be $D_2'\varepsilon_t$, the residual of the projection of the uncertainty innovation $c_0'\varepsilon_t$ onto this shock can be found as $[c_0' - c_0'\Sigma_\varepsilon D_2 \Gamma D_2']\varepsilon_t$.

More generally, let D be the $m \times n$ matrix having on the rows the vectors $D_1', D_2', \ldots, D_m'$, with $m < n$. To impose orthogonality with respect to the corresponding m shocks $D_1'\varepsilon_t, D_2'\varepsilon_t, \ldots, D_m'\varepsilon_t$, one has to take the residual of the orthogonal projection of the uncertainty innovation $c_0'\varepsilon_t$ onto $D\varepsilon_t$, normalized to have unit variance. The corresponding uncertainty shock, call it u_t, can then be computed from the VAR coefficients by applying the formulas

$$u_t = \gamma \varepsilon_t,$$

$$\gamma = \frac{\beta}{\sqrt{\beta'\Sigma_\varepsilon \beta}},$$

and

$$\beta = c_0' - c_0'\Sigma_\varepsilon D'(D\Sigma_\varepsilon D')^{-1}D. \tag{15}$$

The impulse-response functions for the variables included in the VAR corresponding to the shock $u_t = \gamma\varepsilon_t$ are

$$d(L) = B(L)\Sigma_\varepsilon \gamma. \tag{16}$$

The full model becomes

$$\begin{pmatrix} P_t^i \\ y_t \end{pmatrix} = \begin{pmatrix} \omega \\ \delta \end{pmatrix} + \begin{pmatrix} c(L)'d(L) \\ d(L) \end{pmatrix} u_t + \Psi(L)w_t, \tag{17}$$

where $\Psi(L)w_t$ is again the term containing the $n-1$ remaining unidentified shocks times their impulse response functions.

2.5.3. Exogenous and endogenous components of uncertainty

Notice that the term $d_u(L) = c(L)'d(L)$ identifies the effect of the uncertainty shock on log uncertainty. Hence the component of log uncertainty driven by the uncertainty shock is $d_u(L)u_t = d_u(L)\gamma\varepsilon_t$, let us call it the *exogenous component*. The part of uncertainty not driven by the uncertainty shock, i.e. the *endogenous component*, is therefore $c(L)'y_t - d_u(L)u_t = [c(L)'B(L) - d_u(L)\gamma]\varepsilon_t$. Since the two components are orthogonal, it is possible to compute a variance decomposition both for the total variance and for the prediction errors at all horizons.

2.6. Equivalence with a proxy SVAR

In this section global invertibility of the structural representation of y_t is assumed, i.e. it is assumed that all of the n structural shocks are contemporaneous linear combinations of the VAR residuals in the vector ε_t. It is possible to show that, under the assumption that the uncertainty shock is simply the innovation of uncertainty, (i) $z_t = \log(e_{i,t+h}^2)$ is a valid instrument for the identification of uncertainty shocks, provided that $c_0 = g_0 \neq 0$, and (ii) the proposed estimation procedure is equivalent in population to a proxy SVAR using $z_t = \log(e_{i,t+h}^2)$ as the instrument for the uncertainty shock.

See Mertens and Ravn (2013); Stock and Watson (2018) and Plagborg-Møller and Wolf (2021) on the proxy SVAR approach.

When the number of lags in Eq. (6) is the same as the number of lags in the VAR, the results of the two procedures are identical even in the sample.

Let us begin from point (i), i.e. the validity of $z_t = \log(e^2_{i,t+h})$ as an instrument for the uncertainty shock. Under invertibility, the validity conditions for the instrument (Stock and Watson, 2018) are:

I (relevance) $E(z_t u^*_t) \neq 0$
II (exogeneity) $E(z_t w^*_t) = 0$,

where w^*_t, see (14), is the vector collecting the other $n-1$ structural shocks. Combining Eqs. (9), (10) and (12) it is possible to get

$$\log(e^2_{i,t+h}) = \omega + g_0'\varepsilon_t + \sum_{k=1}^{\infty} g_k'\varepsilon_{t-k} + \nu_t = \omega + \alpha u^*_t + \sum_{k=1}^{\infty} g_k'\varepsilon_{t-k} + \nu_t,$$

where $\alpha = \sqrt{g_0'\Sigma_\varepsilon g_0}$. Hence $E(z_t u^*_t) = \alpha \neq 0$, provided that $g_0 \neq 0$, so that condition I is fulfilled. Moreover, w^*_t is orthogonal to u^*_t, and, being a linear transformation of ε_t, is also orthogonal to both ν_t, see Eq. (9), and the summation on the right side, implying that condition II is fulfilled.

Let us come now to point (ii), i.e. the equivalence between the estimation procedure proposed and the proxy-SVAR procedure of Mertens and Ravn (2013). Such procedure consists in projecting the VAR residuals ε_t onto the proxy z_t. The population parameters are $\phi = Ez_t\varepsilon_t/Ez_t^2$ (see Mertens and Ravn (2013)). The impact effects ϕ are therefore proportional to $Ez_t\varepsilon_t$. It is easily seen that the population impact effects are also proportional to $Ez_t\varepsilon_t$, so that they are equal to those of the proxy SVAR when the same normalization is imposed. If the proxy z_t is $\log(e^2_{i,t+h})$, from Eqs. (9) and (10), it is possible to get

$$z_t = \omega + c(L)'B(L)\varepsilon_t + \nu_t, \tag{18}$$

and ν_t is orthogonal to y_{t-k}, $k \geq 0$ and therefore to ε_{t-k}, $k \geq 0$. Post-multiplying by ε_t' and taking expected values, $Ez_t\varepsilon_t' = c_0'\Sigma_\varepsilon$ is obtained, since $B(0) = I$. But, as seen above, the impact effects are $\Sigma_\varepsilon v = \Sigma_\varepsilon c_0/\alpha$ with $\alpha = \sqrt{c_0'\Sigma_\varepsilon c_0}$ (see Eqs. (12) and (13)). Hence the impact effects are $Ez_t\varepsilon_t/\alpha$.

In Appendix B it is shown also show that the OLS estimates are equal to those of Mertens and Ravn (2013) if $q = p$, i.e. when the number of lags of y_t included in the regression of z_t is equal to the number of lags of the VAR. Hence, as far as the estimation of the effects of uncertainty are concerned, the approach proposed here and the standard proxy SVAR approach produce the same results.

The advantage of the method proposed is that it allows us to get an estimate of uncertainty itself, besides the uncertainty shock and its impulse-response functions. On the other hand, the above discussion clarifies that, for the identification of the uncertainty shock, it is not necessary that the linear approximation of uncertainty in Eq. (6) is good: it is sufficient to just show the standard assumptions of relevance and exogeneity are fulfilled.

2.7. Summary of the procedure

Summing up, the procedure is the following.

1. Estimate by OLS the VAR in Eq. (1) to get $\hat{B}(L) = \hat{A}(L)^{-1}$, the vector of residuals $\hat{\varepsilon}_t$ and its sample variance-covariance matrix $\hat{\Sigma}_\varepsilon$. Compute $\hat{e}_{t+h}$ according to Eq. (3).
2. Compute $\hat{z}_t = \log(\hat{e}^2_{i,t+h})$. Estimate by OLS Eq. (9) to get $\hat{\theta}$ and $\hat{c}(L)$ and compute $\hat{U}^i_{ht}$ according to Eq. (6) as $\hat{U}^i_{ht} = \exp(\hat{\theta} + \hat{c}(L)'y_t)$.
3. Compute $\hat{u}^*_t$ and $\hat{d}^*(L)$ according to Eqs. (12) and (13) by replacing c_0 and Σ_ε with the corresponding estimates. Alternatively:

3′. Specify the relevant orthogonality restrictions by choosing the matrix D. Compute the estimates $\hat{u}_t$ and $\hat{d}(L)$ according to Eqs. (15) and (16) by replacing c_0 and Σ_ε with the corresponding estimates.

4. Get the estimate of the IRFs of uncertainty, either $d^*_u(L)$ or $d_u(L)$, according to (13) or (16).

Appendix C describes in detail the bootstrap procedure to construct confidence bands.

3. Empirics

In this section the main results of the empirical analysis are discussed.

3.1. Specification

US quarterly data spanning the period from 1960:Q1 to 2019:Q3 are employed. The benchmark VAR includes seven variables: the log of real per-capita GDP, the unemployment rate, CPI inflation, the federal funds rate, the log of the S&P500

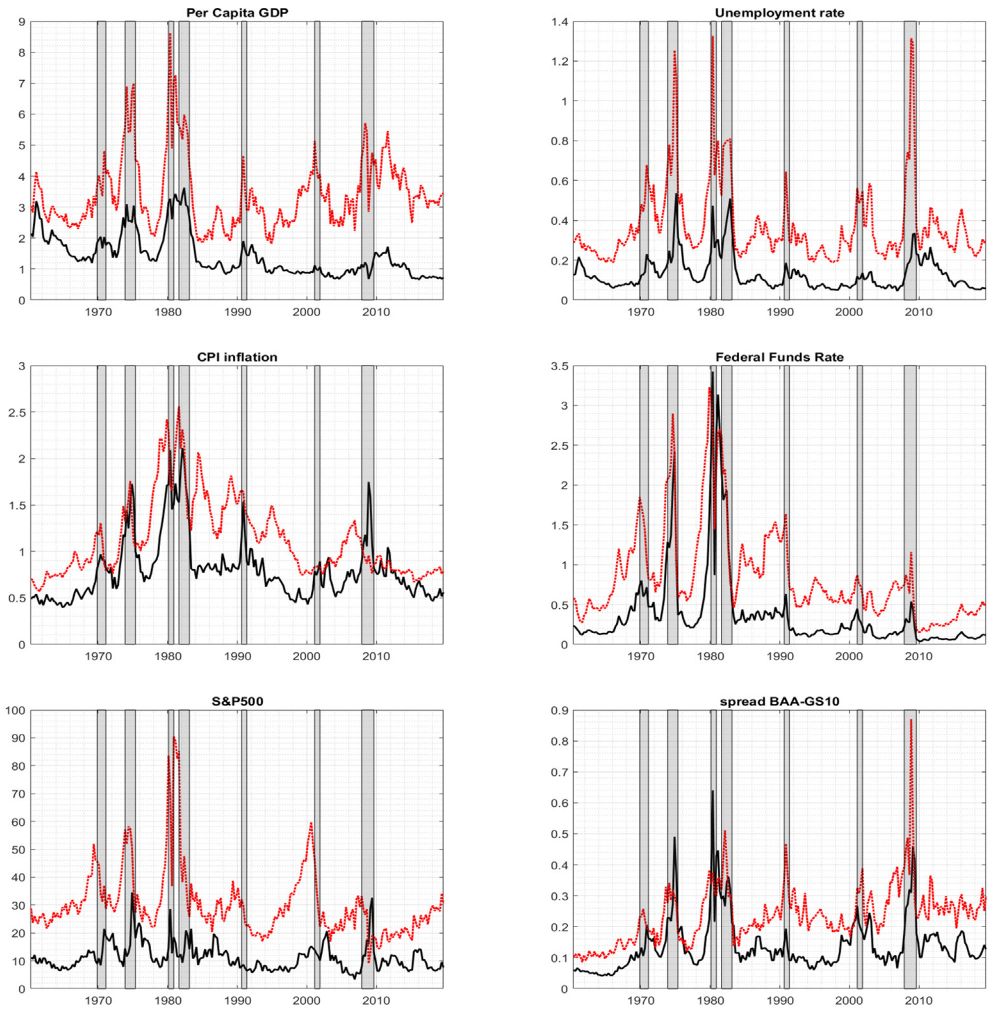

Fig. 1. Estimated uncertainties. Black line 1-quarter ahead. Red line 4-quarter ahead. Gray vertical bands NBER recessions dates.

stock price index, a component of the Michigan Consumer Confidence Index, i.e. expected business conditions for the next 12 months (E1Y), and the spread between BAA corporate bond yield and GS10 (BAA-GS10).

GDP and stock prices are taken in log levels to take into account potential cointegration relations.

The last four variables are included essentially because they are supposed to quickly react to shocks and therefore are hopefully able to better capture the information necessary to reveal uncertainty. In the robustness section, stock prices and the spread BAA-GS10 are replaced with a different set of forward-looking variables.

The VAR is assumed to have only one lag, as suggested by the BIC criterion. In the robustness section results for 2 and 4 lags are shown.

Eq. (9) is estimated for all the variables included in the model and considering 1, 4 and 8 quarters ahead forecast horizons. In all cases, following the BIC criterion, y_t is included without further lags on the right-hand side (i.e. $q = 0$ and $c(L) = c_0$). In the robustness section, y_{t-1} is also included, so that $p = q$ and the method proposed produce exactly the same result as the proxy-SVAR method discussed above.

3.2. Estimated uncertainty: diagnostic checks

In this section the validity of the linear approximation of uncertainty is analyzed in three different ways. First, the overall significance of the regressors in Eq. (9) is documented. Table 1 shows the R^2 statistic along with the F-test for the overall significance of the regression, for all variables and horizons, when using just the contemporaneous VAR variables as regressors ($q = 0$). All regressions but the one for stock price uncertainty at horizon 8 are significant at the 5% level, and 16 regressions out of 21 are significant at the 1% level. The VAR variables predict the squared prediction errors implied by the VAR itself. This result, to our knowledge, was not found before and, as already observed, implies that the VAR residuals are

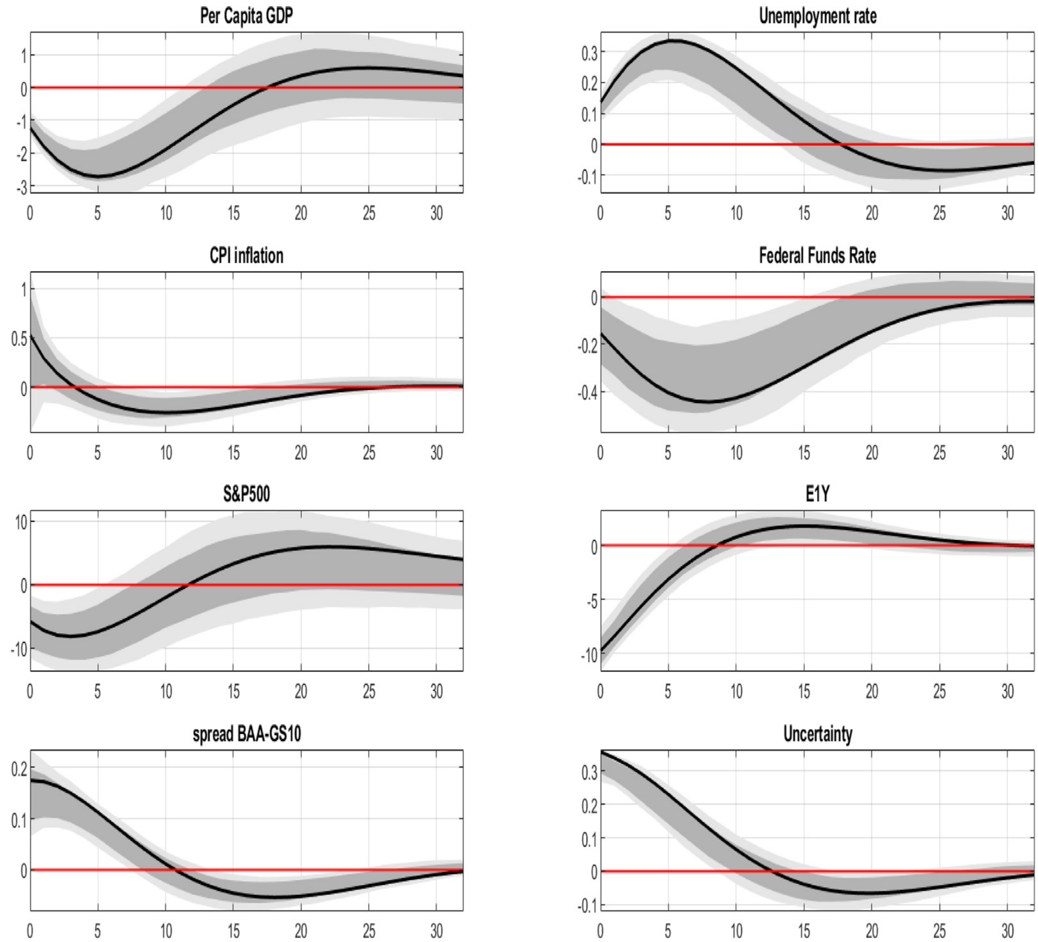

Fig. 2. Impulse response functions of the unemployment rate uncertainty shock, 1-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

Table 1
R^2 of regression (6) and p-values of the F-test of the significance of the regression for different horizons h.

	R^2			p-value (F-test)		
	$h=1$	$h=2$	$h=4$	$h=1$	$h=2$	$h=4$
Per Capita GDP	0.15	0.10	0.08	0.00	0.00	0.01
Unemployment rate	0.19	0.26	0.15	0.00	0.00	0.00
CPI inflation	0.09	0.15	0.08	0.00	0.00	0.01
Federal Funds Rate	0.43	0.37	0.25	0.00	0.00	0.00
S&P500	0.10	0.09	0.09	0.00	0.00	0.00
E1Y	0.08	0.08	0.08	0.01	0.01	0.01
spread BAA-GS10	0.21	0.11	0.09	0.00	0.00	0.00

not serially independent. This preliminary step lends support to the validity of the approximation procedure proposed and the idea that shocks, although white noise, are far from being independent.

As a second check, additional regressors are included in Eq. (9) to verify whether such regressors are significant or not. Two different sets of regressors are tried, one at a time. First, $\log(e_{it}^2)$, $i = 1, \dots, n$ is icluded. Second, y_{it}^2, $i = 1, \dots, n$ is added. In both cases, there are seven regressors for each equation. To test for the significance of the additional regressors an F-test is performed, equation by equation, in addition to the seven contemporaneous VAR variables ($q = 0$). The p-values of the tests are reported in Table 2. For the horizon $h = 1$, which is the benchmark in the structural VARs below, both sets of additional regressors are not significant at the 5% level, except for the federal funds rate.

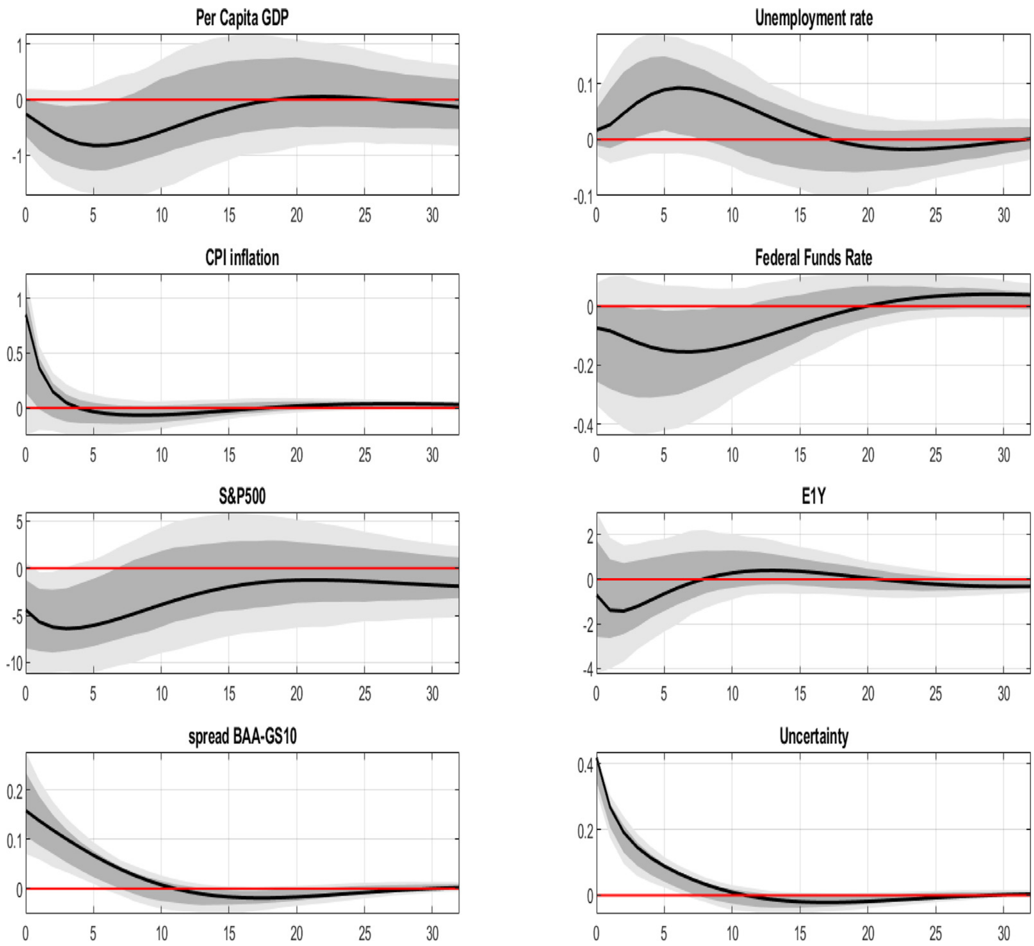

Fig. 3. Impulse response functions of the S&P500 uncertainty shock, 1-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

Table 2
p-values of the F-test for the joint significance of additional regressors in Eq. (6) for different horizons h.

	Additional regressors: $\log(\varepsilon^2_{it})$, $i = 1, \ldots, n$			Additional regressors: y^2_{it}, $i = 1, \ldots, n$		
	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$
Per Capita GDP	0.12	0.02	0.00	0.13	0.05	0.00
Unemployment rate	0.21	0.13	0.00	0.24	0.32	0.00
CPI inflation	0.89	0.28	0.04	0.63	0.38	0.08
Federal Funds Rate	0.03	0.00	0.00	0.02	0.00	0.00
S&P500	0.56	0.06	0.00	0.66	0.04	0.00
E1Y	0.74	0.18	0.01	0.91	0.38	0.01
spread BAA-GS10	0.51	0.52	0.01	0.62	0.75	0.06

As a final check, it is verified whether the residuals of Eq. (6) with ($q = 0$) are autocorrelated, by running the Ljung-Box Q-test with 8 and 12 lags. Results are reported in Table 3. Again, for the horizon $h = 1$ the null hypothesis of no autorrelation cannot be reject at the 5% level, except for the federal funds rate (8 lags) and E1Y (12 lags).

3.3. Estimated uncertainty: results

This subsection shows the uncertainty measures. Figure 1 plots the estimated uncertainty indexes for the 1-quarter and the 4-quarter ahead horizons. Real economic activity uncertainty, i.e. GDP and unemployment uncertainty, behave as already largely documented in the literature. The indexes tend to lead recessions, they begin to increase right before the onset of

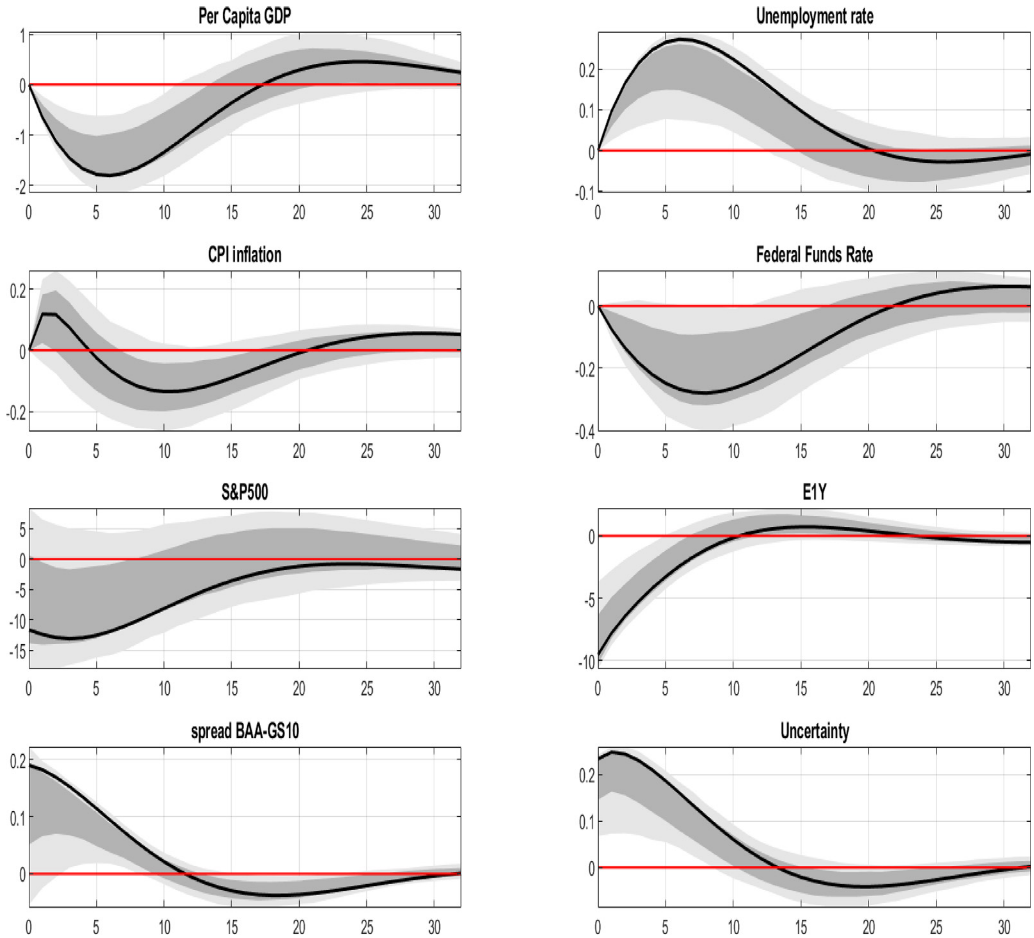

Fig. 4. Impulse response functions of the unemployment rate uncertainty shock, 1-quarter ahead. The shock is identified as the residual of the projection of the uncertainty innovation onto the long-run shock, the GDP innovation, the unemployment rate innovation, the CPI innovation and the federal funds rate innovation (Identification II). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

Table 3
p-values of the Ljung-Box *Q*-test for autocorrelation of the residuals of regression (6) with 8 and 12 lags for different horizons *h*.

	8 lags			12 lags		
	$h=1$	$h=2$	$h=4$	$h=1$	$h=2$	$h=4$
Per Capita GDP	0.32	0.88	0.08	0.94	0.08	0.06
Unemployment rate	0.99	0.70	0.01	0.44	0.52	0.02
CPI inflation	0.50	0.64	0.11	0.09	0.21	0.44
Federal Funds Rate	0.00	0.98	0.91	0.10	0.71	0.20
S&P500	0.41	0.02	0.10	0.56	0.15	0.11
E1Y	0.24	0.12	0.49	0.01	0.51	0.00
spread BAA-GS10	0.28	0.60	0.24	0.80	0.37	0.35

the recessions and to reduce right before the end of the recessions. The results are quite similar across horizons except that the 4-quarter ahead uncertainty is much larger.

Stock prices uncertainty, 1-quarter ahead, behaves very similarly to real economic activity uncertainty, with correlations around 0.7, see Table 4. The 4-quarter ahead uncertainty however displays an opposite behavior, especially since the early 80s. Uncertainty steadily increases during expansionary periods and suddenly drops right before the recession remaining relatively low during recession. Stock prices become hard to forecast at long horizon compared to short horizons in periods of booms while in recession the forecast error variances are very similar at both horizons. The result seems to suggest that the longer is the period of economic expansion, the larger is the probability assigned to a fall in prices. As prices drop, uncertainty suddenly reduces because good outcomes are no longer expected. The result shows that protracted period

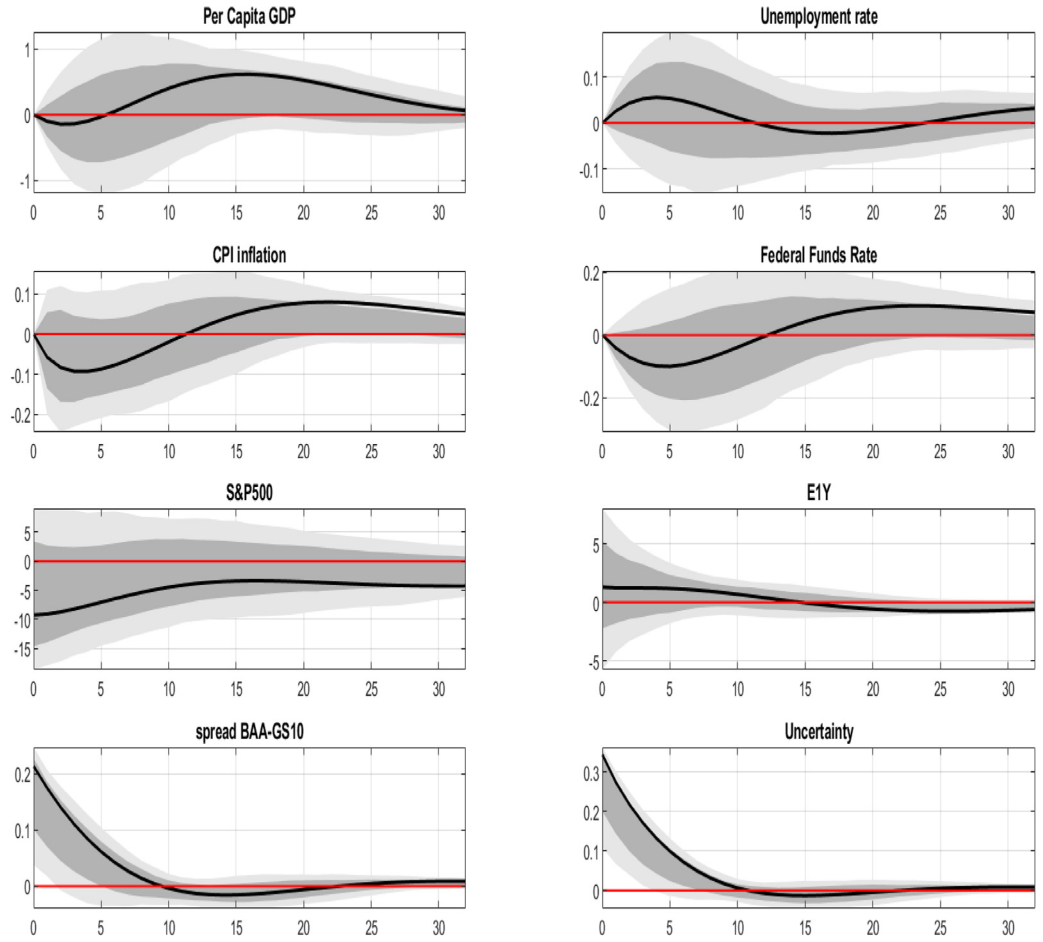

Fig. 5. Impulse response functions of the S&P500 uncertainty shock, 1-quarter ahead. The shock is identified as the residual of the projection of the uncertainty innovation onto the long-run shock, the GDP innovation, the unemployment rate innovation, the CPI innovation and the federal funds rate innovation (Identification II). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

Table 4
Correlation of unemployment uncertainty 1-quarter ($\hat{U}^{UN}_{1,t}$) and 4-quarter ahead ($\hat{U}^{UN}_{4,t}$) and S&P uncertainty 1-quarter ($\hat{U}^{S\&P}_{1,t}$) and 4-quarter ahead ($\hat{U}^{S\&P}_{4,t}$) with existing measures: VXO, LMN financial 12-month ahead (LMN F12m), JLN 12-month ahead (JLN 12m), LMN real 12-month ahead (LMN R12m), economic policy uncertainty (EPU), and Rossi and Sekhposyan (2015) 4-quarter ahead (RS 4q).

	$\hat{U}^{UN}_{1,t}$	$\hat{U}^{S\&P}_{1,t}$	$\hat{U}^{UN}_{4,t}$	$\hat{U}^{S\&P}_{4,t}$	VXO	LMN F3m	JLN 12m	LMN R12m	EPU	RS 4Q
$\hat{U}^{UN}_{1,t}$	1.00	-	-	-	-	-	-	-	-	-
$\hat{U}^{S\&P}_{1,t}$	0.63	1.00	-	-	-	-	-	-	-	-
$\hat{U}^{UN}_{4,t}$	0.80	0.69	1.00	-	-	-	-	-	-	-
$\hat{U}^{S\&P}_{4,t}$	0.25	0.27	0.37	1.00	-	-	-	-	-	-
VXO	0.34	0.43	0.56	0.12	1.00	-	-	-	-	-
LMN F12m	0.39	0.50	0.60	0.32	0.78	1.00	-	-	-	-
JLN 12m	0.66	0.48	0.79	0.58	0.47	0.52	1.00	-	-	-
LMN R12m	0.68	0.49	0.76	0.57	0.28	0.44	0.82	1.00	-	-
EPU	0.71	0.33	0.48	-0.41	0.35	0.38	0.29	0.25	1.00	-
RS 4q	-0.02	0.16	0.13	0.36	0.28	0.31	0.14	0.12	-0.14	1.00

of increasing stock price uncertainty are followed by economic downturns. It would be interesting to understand whether medium-run stock prices uncertainty is able to predict in real time economic recessions. This issue is left for future research.

Inflation uncertainty 4-quarters ahead and federal funds rate uncertainty 1-quarter ahead also present some differences compared to real economic activity uncertainty. Interestingly they do not exhibit a peak corresponding to the Great Recession. Inflation uncertainty is large during periods of high inflation, with peaks corresponding roughly to oil shocks. Federal

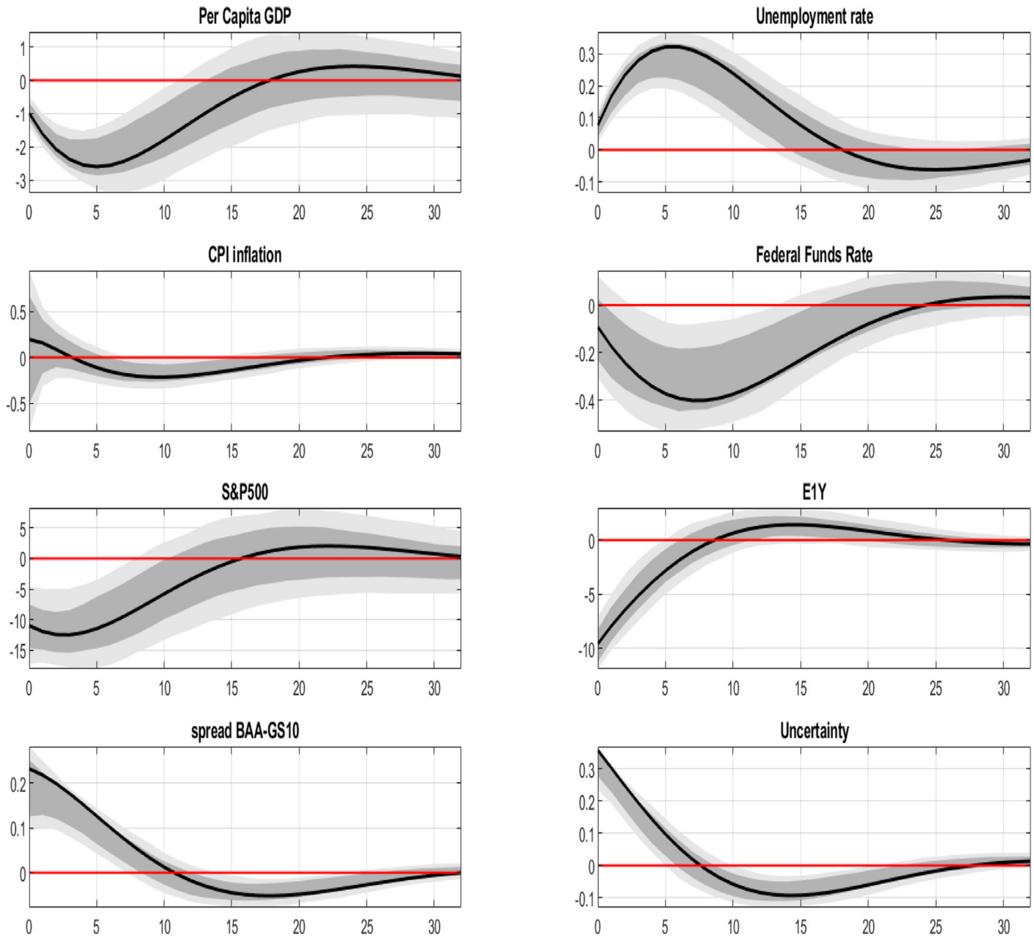

Fig. 6. Impulse response functions of the unemployment rate uncertainty shock, 4-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

funds rate uncertainty is high during the so-called "stop and go" monetary policy of the 70s, and during the Volcker era, when the federal funds rate was very high; it is very low at the end of the sample, when interest rates are close to zero.

Table 4 shows the correlation coefficients between four selected uncertainty indexes, computed according to Eq. (6), namely the unemployment rate uncertainty index, 1-quarter ad 4-quarter ahead ($\hat{U}^{UN}_{1,t}$ and $\hat{U}^{UN}_{4,t}$) and the stock price uncertainty index, 1-quarter ahead and 4-quarter ahead ($\hat{U}^{S\&P}_{1,t}$ $\hat{U}^{S\&P}_{4,t}$), and (a) the VXO index, extended as in Bloom (2009), (b) the (Ludvigson et al., 2021)) financial uncertainty index 12-months (LMN F12m), (c) the JLN (2015) macroeconomic uncertainty index 12 months (JLN 12m), (d) the Ludvigson et al. (2021) real uncertainty index 12-months (LMN R12m), (e) the Becker et al. (2016) US Economic Policy Uncertainty index (EPU) and (f) the (Rossi and Sekhposyan, 2015) 4-quarters ahead uncertainty index (RS 4q).

The indexes, except $\hat{U}^{S\&P}_{4,t}$, are highly positively correlated with each other and with JLN and LMN indexes, which use the same definition of uncertainty used here. In particular, $\hat{U}^{UN}_{4,t}$ exhibits correlation coefficients with LMN R12m and JLN 12m as high as 0.76 and 0.79, respectively. On the contrary, and in line with the above discussion $\hat{U}^{S\&P}_{4,t}$, displays substantially smaller correlations, even negative with US Economic Policy Uncertainty index.

3.4. Uncertainty shocks: Identification I

The effects of shocks in unemployment uncertainty and stock prices uncertainty are studied here. The former is undoubtedly a proxy for macroeconomic uncertainty. The latter, being a measure of volatility present in the stock market, reflects, to some extent, also uncertainty arising from financial conditions.

Of course stock prices uncertainty will be also driven by macroeconomic factors.

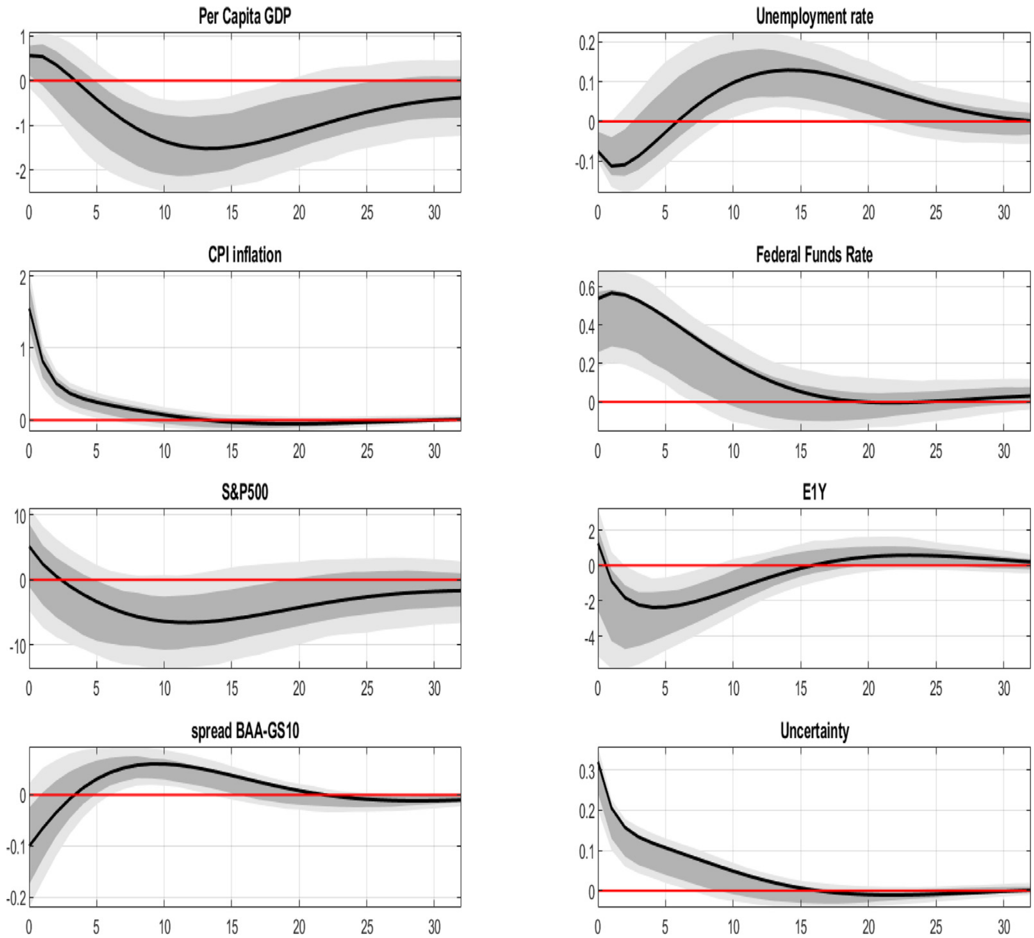

Fig. 7. Impulse response functions of the S&P500 uncertainty shock, 4-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

Unemployment is chosen rather than GDP as a benchmark for macroeconomic uncertainty mainly because the R^2 reported in Table 1 are larger and more significant than those for GDP. In the robustness section, results for GDP uncertainty are shown. As for the horizon, 1-quarter ahead is displayed here. In Section 3.6, results for $h = 4$ are shown.

The literature does not provide a widespread consensus about a set of identification restrictions for the exogenous uncertainty shock. In this section, the uncertainty shock (Identification I) is identified as the VAR innovation of uncertainty u_t^*, see Section 2.5.1. Therefore, the only shock affecting uncertainty on impact is the uncertainty shock. As already observed, this scheme is questionable. On the other hand, it is common in the literature hence results may be useful for comparison.

Figure 2 shows results obtained for macroeconomic uncertainty under Identification I. The uncertainty shock is contractionary for real economic activity, significantly reducing output and increasing unemployment. The effects are very large, as in JLN (2015), but not that much persistent, since they vanish after about 4 years. This result is different from those in JLN and Carriero et al. (2018b). Inflation is not significantly affected. The federal funds rate reduces, reacting to the slowdown of real activity and prices. Stock prices reduce on impact. The confidence index goes down on impact, reflecting consumers' expectations. The BAA-GS10 spread increases, reflecting the increased risk premium of Baa Corporate bonds.

Figure 3 plots the results for the financial uncertainty shock. The shock is again contractionary, although the effects are much smaller in magnitude than those obtained for macroeconomic uncertainty and barely significant.

Table 5 shows the variance decomposition. The macroeconomic uncertainty shock accounts for a very high fraction of GDP and especially the unemployment rate. The shock explains more than half of the fluctuations in unemployment at the one-year horizon. The effect on the risk premium is also large: according to this identification, the uncertainty shock explains about three quarters of the spread variance at the one-year horizon. On the contrary, the financial uncertainty shock generates very small effects. It essentially plays no role for fluctuations in real economic activity variables and stock prices. The only variable which seems to be driven by the shock is the spread, the explained variance being around 30%.

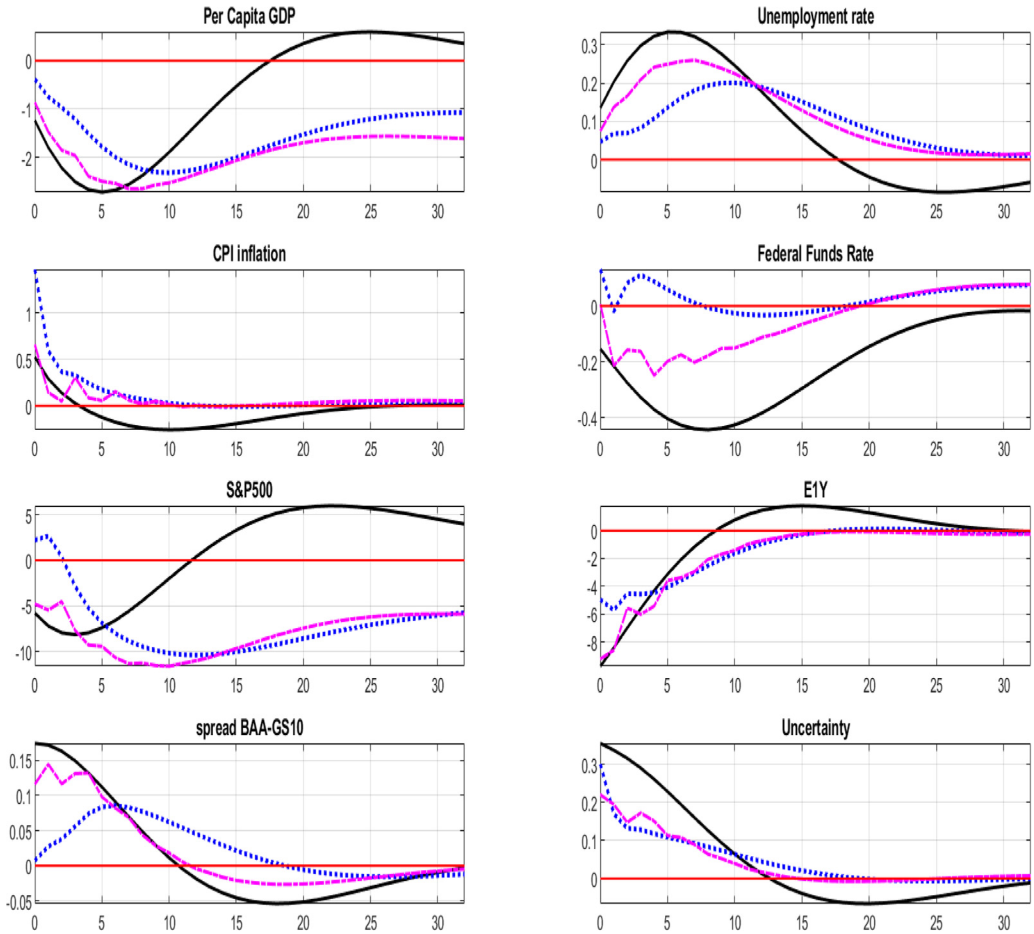

Fig. 8. Comparison between the benchmark impulse response functions of Identification I (solid black lines), obtained with 1 lag in the VAR and the corresponding impulse response functions obtained with 2 lags (dotted blue lines) and 4 lags (dashed-dotted magenta lines).

3.5. Uncertainty shocks: Identification II

This section repeats the analysis using a different identification strategy: the uncertainty shock is imposed to be orthogonal to the long-run shock, identified as the only one shock affecting GDP at the 40 quarter horizon, and, in addition, is orthogonal to the innovations of GDP, unemployment, CPI and the federal funds rate (hence, five rows in the matrix D are included). Therefore, the identifying assumptions are: (i) the uncertainty shock has transitory effects on output; (ii) the slow-moving variables (output, unemployment and prices) do not react to uncertainty on impact, as is assumed for the monetary policy shock *à la* Christiano et al. (1999); in addition, (iii) the federal funds rate does not react to uncertainty on impact. The last constraint is imposed because, given (ii), (iii) entails that the uncertainty shock is orthogonal to a monetary policy shock which moves on impact the federal funds rate and therefore cannot be confused with it. On the other hand, the monetary policy shock, as well as the long-run shock and, possibly, other unidentified transitory shocks, may affect uncertainty on impact. Again, the focus is on 1-quarter ahead uncertainty of unemployment and stock prices.

Figure 4 shows results for macroeconomic uncertainty. Results are qualitatively similar to those of Identification I: the uncertainty shock significantly depresses real economic activity but without any significant effect on the inflation rate. The variance explained by the uncertainty shock (see Table 5) is now much smaller. Still, at the one-year and the 4-year horizons, uncertainty shocks explains about 10-15% of output volatility and about 30-40% of unemployment volatility. Finally, the shock explains now only 43% of uncertainty itself on impact, leaving a large role for other shocks, including the long-term shock and the monetary policy shock. With Identification II, exogenous uncertainty is about 40-50% at all horizons.

Figure 5 shows results for financial uncertainty. The effects are negligible and not significant for all of the variables except the spread. By imposing the additional restrictions the conclusion for the financial uncertainty shock are reinforced: the shock plays a negligible role for economic fluctuations, even if it explains a large fraction of the risk premium (Table 5).

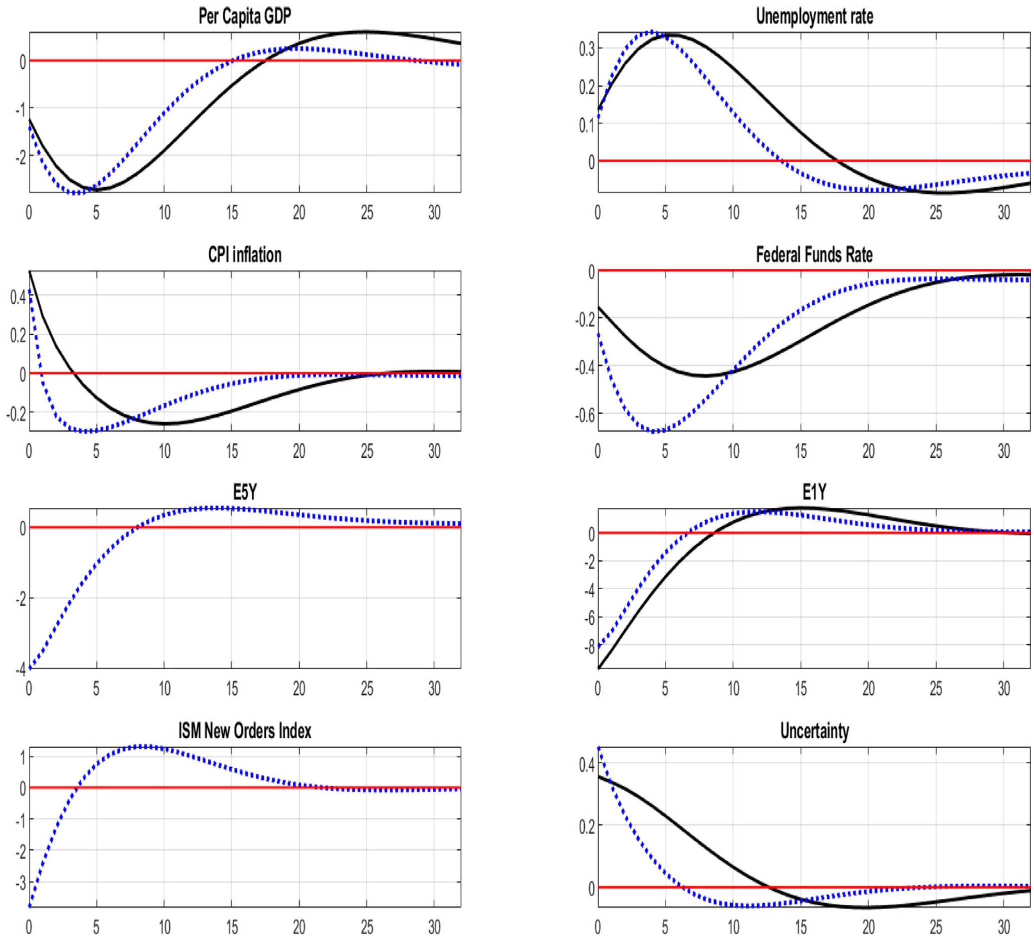

Fig. 9. Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained with a different VAR specification, including E5Y (a component of the Michigan University Consumer Confidence Index) and the ISM New Order Index in place of S&P500 and the spread BAA-GS10 (dotted blue lines).

3.6. Uncertainty shocks: 4-quarter ahead

This section repeats the analysis using Identification I, focusing on the 4-quarter ahead uncertainty. Figure 6 and Figure 7 plot the impulse response functions for the macroeconomic uncertainty shock and financial uncertainty shock respectively. Macroeconomic uncertainty generates effects which are very similar to those obtained for the 1-quarter ahead: a significant and protracted downturn of economic activity and stock prices which triggers an expansionary response of monetary policy authorities. As far as financial uncertainty is concerned, the results are different from before. The shock generates a significant but very delayed contraction. Indeed the trough of the downturn occurs four years after the shock. Going back to the discussion in Section 3.2, the result seems to suggest that there is, at least to some extent, a causal link between the protracted increase of 4-quarter ahead stock prices uncertainty and economic recessions.

3.7. Two caveats

A drawback of Identification I is that uncertainty is assumed to be unaffected by other macroeconomic shocks contemporaneously. This assumption has been criticized in several recent works, see Bachmann et al. (2013); Carriero et al. (2018a); Angelini et al. (2019); Ludvigson et al. (2021); Brianti (2021). Identification II mitigates this problem, since with this identification scheme other shocks can in principle affect uncertainty on impact. Other identification schemes could be more effective in identifying truly exogenous uncertainty shocks, but there is no consensus in the literature on what is the correct way to do it.

An additional, related problem is that financial uncertainty and macroeconomic uncertainty might interact to each other, see Angelini et al. (2019) and Ludvigson et al. (2021). For instance, the financial shock might affect the economy through its effect on macroeconomic uncertainty, as suggested in Angelini et al. (2019). To verify this, a macroeconomic and a financial

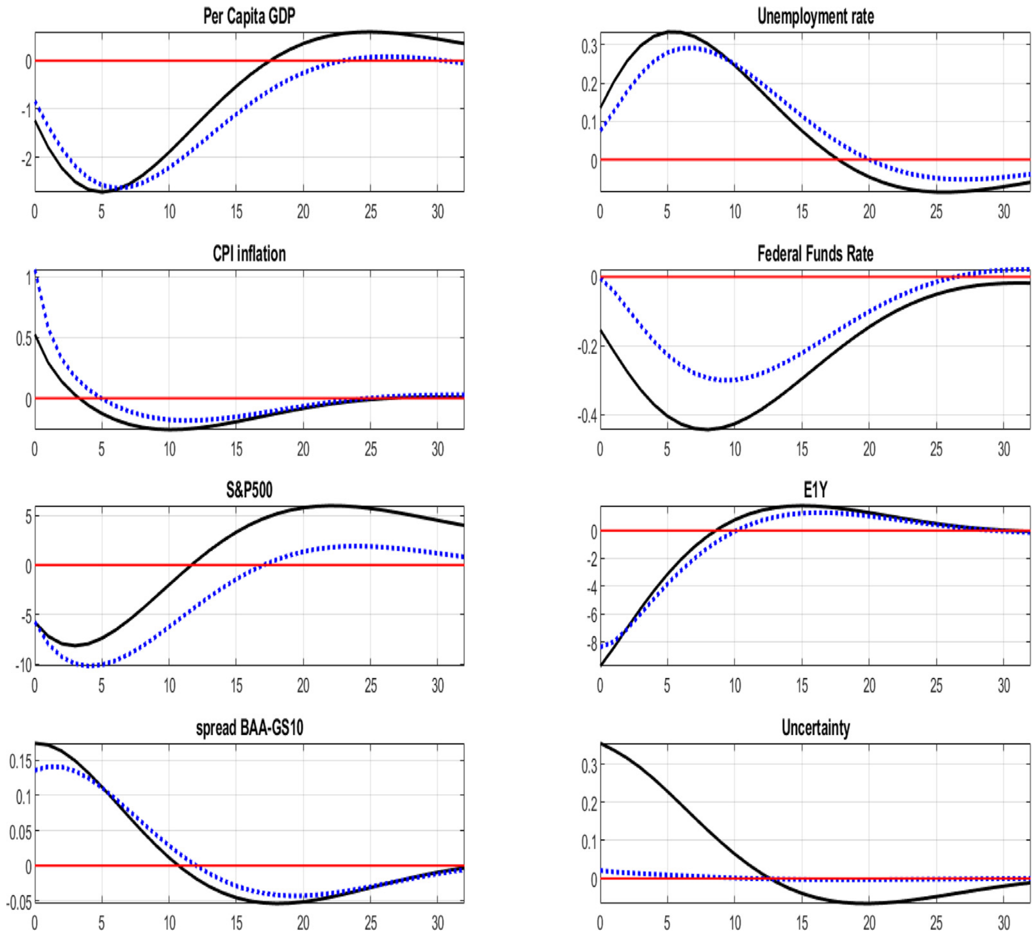

Fig. 10. Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained when using the squared predictions error in place of the log of the squared prediction error to compute uncertainty (dotted blue lines).

uncertainty shock should be identified simultaneously, rather than being analyzed once at a time. This however is left for future research.

3.8. Robustness checks

Several robustness checks are perfomed. In the first exercise, reported in Figure 8, the number of lags is changed and 2 lags (blue dotted lines) and 4 lags (magenta dotted-dashed lines) instead of 1 lag (benchmark case, black solid lines) are used. Results are somewhat different from those obtained in the baseline model, particularly because the effects on GDP and stock prices are more persistent. However, both the sign and the size of the responses are similar to those of the baseline specification.

In the second exercise, the VAR specification is changed by removing stock prices and the spread BAA-GS10, and including two different forward-looking variables: the ISM New Order Index and another component of the Michigan Consumer Confidence Index, the expected business conditions for the next five years (E5Y). The latter variable is studied in depth in Barsky and Sims (2012).

The spread is removed mainly to avoid a possible contamination of uncertainty shocks with credit market shocks (Gilchrist and Zakrajsek, 2012; Caldara et al., 2016). Results are reported in Figure 9. The effects of uncertainty shocks on the variables which are included in both specifications are similar.

In the last two exercises, the baseline specification for the VAR is retained, but the way in which uncertainty is estimated changes. First, the squares of the prediction error in place of their logs are used, i.e. Eq. (6) is not used, but the conditional expectation appearing in Eq. (4) is replaced with the linear projection. The effects of the implied uncertainty shock are very similar to those of the baseline model (Figure 10). Second, in Eq. (6) it is assumed $q = 1$ instead of $q = 0$, so that $q = p$ and the results are identical to those obtained with the proxy SVAR approach. The results are reported in Figure 11. The effects on GDP and stock prices are larger and more persistent than in the benchmark model, whereas those on unemployment

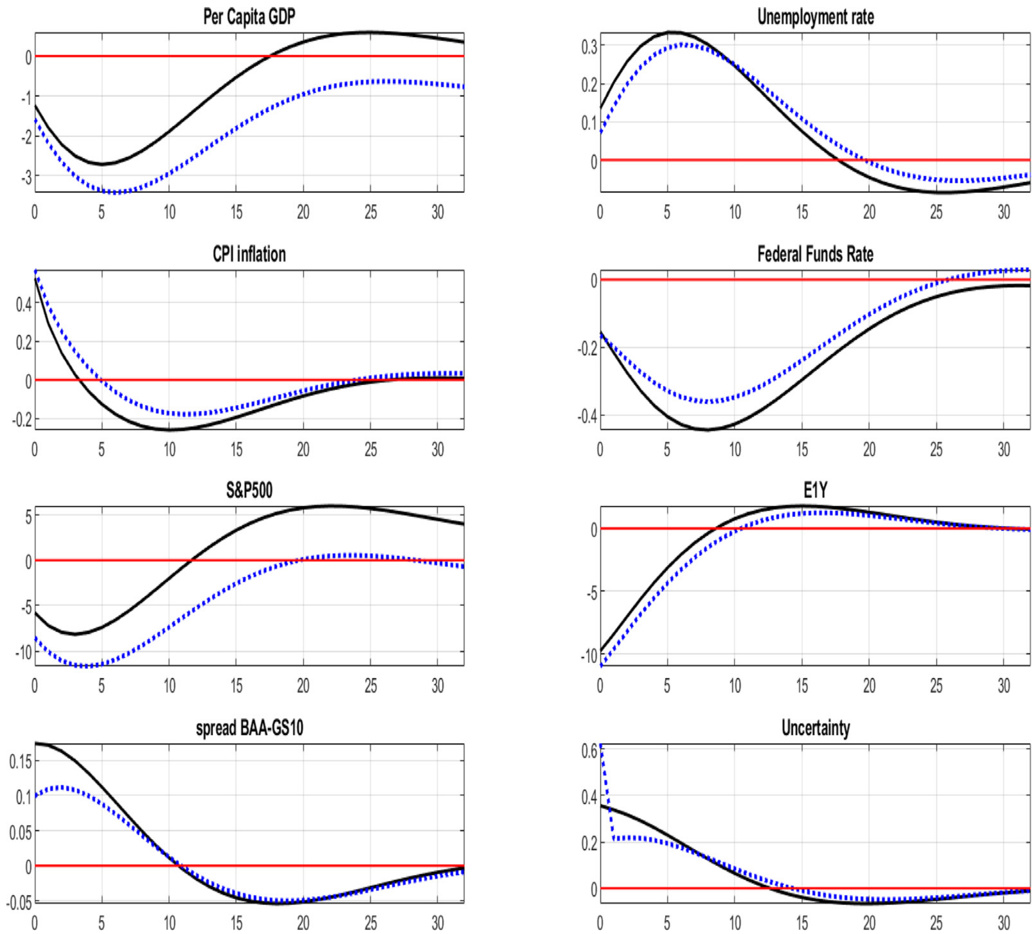

Fig. 11. Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained when using 1 lag of the variables, in addition to the current values, to compute uncertainty (dotted blue lines).

Table 5
Variance decomposition. Identification I: uncertainty innovation. Identification II: no long-run effect (40 quarters) on GDP and zero contemporaneous effects on GDP, unemployment rate, CPI and federal funds rate. The Uncertainty variable in the last row of the table is the uncertainty measure proposed.

Identification I	$\hat{U}^{UN}_{1,t}$				$\hat{U}^{S\&P}_{1,t}$			
	$h = 0$	$h = 4$	$h = 16$	$h = 40$	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	20.1	44.5	30.7	16.9	0.9	3.3	2.7	1.5
Unemployment rate	30.9	66.4	61.6	48.8	0.5	2.9	4.1	3.1
CPI inflation	8.2	7.0	13.5	13.2	21.7	16.1	12.8	12.0
Federal Funds Rate	3.3	11.6	29.7	26.2	0.8	1.7	3.2	2.8
S&P500	6.0	9.5	4.7	5.1	3.5	5.9	3.6	2.2
E1Y	64.3	56.3	44.2	42.1	0.3	1.5	1.2	1.3
spread BAA-GS10	34.7	49.2	46.7	48.6	28.8	29.5	24.8	23.5
Uncertainty	100.0	94.2	71.7	69.0	100.0	79.1	65.1	61.0
Identification II	$\hat{U}^{UN}_{4,t}$				$\hat{U}^{S\&P}_{4,t}$			
	$h = 0$	$h = 4$	$h = 16$	$h = 40$	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	0.0	12.8	12.5	7.0	0.0	0.1	1.1	1.2
Unemployment rate	0.0	29.7	40.9	30.5	0.0	1.8	1.1	1.6
CPI inflation	0.0	0.6	2.4	2.7	0.0	0.5	0.8	1.9
Federal Funds Rate	0.0	3.1	10.4	9.2	0.0	0.7	0.9	2.3
S&P500	24.1	27.0	15.9	8.6	15.3	12.6	6.4	5.2
E1Y	61.3	50.6	38.8	36.1	1.1	1.7	2.1	3.1
spread BAA-GS10	41.3	54.3	50.0	48.8	52.1	44.9	35.2	32.8
Uncertainty	43.0	51.8	41.7	39.4	67.2	71.2	59.3	55.3

are smaller. However, the main results are confirmed: a positive uncertainty shock has large negative effects on economic activity.

All in all the results appear to be robust to changes in several features of the model specification.

4. Conclusions

It is possible to produce reliable uncertainty estimates using a standard VAR model, without modeling time-varying volatility and using only OLS. The basic idea is to compute the squares of the prediction errors implied by the VAR model and replace expected values with linear projections. The estimate of uncertainty is a linear combination of the VAR variables. Therefore, the uncertainty shock is a linear combination of the VAR residuals and its effects can be computed by applying simple formulas to the reduced form impulse response functions. In this way, the same VAR model is used to estimate both uncertainty and its effects on the economy.

Simple formulas that can be used to impose suitable orthogonality constraints on the uncertainty shock have been provided.

The advantage of the procedure is twofold: on the one hand, the problematic choice of an external uncertainty measure is avoided; on the other hand, imposing restrictive assumption about the structure of conditional volatility is also avoided.

The procedure can be regarded as a variant of a proxy SVAR with the log of the squared prediction error taken as the relevant proxy. Under suitable conditions, the two methods yield the same results.

The procedure described here can easily be adapted to a factor model or a factor-augmented VAR. Moreover, it can be applied to survey-based forecast errors associated with local projection impulse-response functions estimation.

The procedure has been applied to a US macroeconomic quarterly data set. The main empirical conclusion is that macroeconomic uncertainty explains a large part of business cycle fluctuations while short-run financial uncertainty plays a minor role.

Acknowledgements

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Appendix A. A useful formula

If the unit-variance structural shock is $\nu'\varepsilon_t$, its impact effects are $d = \Sigma_\varepsilon \nu$. To see this, consider first the Cholesky representation with orthonormal shocks: $y_t = B(L)CC^{-1}\varepsilon_t$, where C is such that $CC' = \Sigma_\varepsilon$. Any other fundamental representation with orthogonal, unit-variance shocks will be given by

$$y_t = B(L)CUU'C^{-1}\varepsilon_t,$$

where U is a unitary matrix (i.e. $UU' = I$). Assuming, without loss of generality, that the structural shock of interest is the first one, the impact effects are $d = CU_1$, where U_1 is the first column of U, and the vector identifying the structural shock is $\nu' = U_1'C^{-1}$. Hence $U_1 = C'\nu$ and $d = CC'\nu = \Sigma_\varepsilon \nu$.

Appendix B. The relation with standard proxy SVAR

In the main text it has been shown that in population the procedure proposed is equivalent to the proxy-SVAR methodology.

Here it is shown that the OLS estimates are identical to those of Mertens and Ravn (2013) if the number of lags of y_t included in the regression of z_t is equal to the number of lags of the VAR for y_t (see Eq. (18)).

Starting from the OLS estimation of the VAR in Eq. (1), reported here for convenience:

$$y_t = \mu - A_1 y_{t-1} - \cdots - A_p y_{t-p} + \varepsilon_t. \tag{B.19}$$

Additional notation is needed. Let

$$Y_k = \begin{pmatrix} y'_{p+1-k} \\ y'_{p+2-k} \\ \vdots \\ y'_{T-k} \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad X = \begin{pmatrix} \mathbf{1} & Y_1 & \cdots & Y_p \end{pmatrix}, \quad \mathcal{E} = \begin{pmatrix} \varepsilon'_{p+1-k} \\ \varepsilon'_{p+2-k} \\ \vdots \\ \varepsilon'_{T-k} \end{pmatrix}.$$

Moreover, let $Y = Y_0$. Hence the VAR equation can be written as

$$Y = XA + \mathcal{E},$$

where $A = \begin{pmatrix}\mu & -A_1 & \cdots & -A_p\end{pmatrix}'$. The OLS estimates of A and $\mathcal{E}$ are

$$\hat{A} = (X'X)^{-1}X'Y, \qquad \hat{\mathcal{E}} = Y - X(X'X)^{-1}X'Y.$$

Of course, $X'\hat{\mathcal{E}} = 0$.

Mertens and Ravn (2013) focuses on the effects of the structural shock. Such effects are estimated by performing the OLS regression of $\hat{\varepsilon}_t$ onto the proxy z_t, which for ease of exposition and without loss of generality it is assumed to be zero-mean. Precisely, let $z = \begin{pmatrix}z'_{p+1} & z'_{p+2} & \cdots & z'_T\end{pmatrix}'$, and consider the regression equation

$$\hat{\mathcal{E}} = z\phi' + V.$$

The vector of the impact effects is obtained as the OLS estimator of ϕ, suitably normalized (for instance to get unit variance for the corresponding structural shock). The OLS estimator of ϕ is

$$\hat{\phi} = \hat{\mathcal{E}}'z/z'z. \tag{B.20}$$

The vector of the impact effects is then obtained by normalizing the above vector in the desired way.

The proposed procedure focuses on the estimation of the structural shock, rather than the estimation of the corresponding impulse-response functions. Computing the OLS regression of z onto the columns of Y and X:

$$z = Yc_0 + Xb + \nu,$$

where $b = (\theta' \ c'_1 \ \cdots \ c'_p)'$ (see Eq. 6). Letting $W = \begin{pmatrix}Y & X\end{pmatrix}$, the fitted value of z (which in this case is the estimate of uncertainty) is $W(W'W)^{-1}W'z$ and the residual is $\hat{\nu} = z - W(W'W)^{-1}W'z$. Clearly, $W'\hat{\nu} = 0$, so that $Y'\hat{\nu} = 0$ and $X'\hat{\nu} = 0$. Hence $\hat{\mathcal{E}}'\hat{\nu} = 0$. Pre-multiplying the above equation by $\hat{\mathcal{E}}'$, it is seen that

$$\hat{c}_0 = (\hat{\mathcal{E}}'Y)^{-1}\hat{\mathcal{E}}'z = (\hat{\mathcal{E}}'\hat{\mathcal{E}})^{-1}\hat{\mathcal{E}}'z,$$

where the last equality is obtained by observing that $\hat{\mathcal{E}}'Y = \hat{\mathcal{E}}'\left(X(X'X)^{-1}X'Y + \hat{\mathcal{E}}\right) = \hat{\mathcal{E}}'\hat{\mathcal{E}}$. Hence $\hat{c}_0$ could be obtained equivalently by OLS regression of z_t onto ε_t. This makes sense: the estimated structural shock is nothing else than the OLS projection of the proxy z_t onto the VAR residuals. The reason why this way is not followed is that it would not enable us to get an estimate of uncertainty.

It has been shown above that the impact effects of $c'_0\varepsilon_t$ are proportional to $\Sigma_\varepsilon c_0$. Hence, such impact effects can be estimated as $\hat{\mathcal{E}}'\hat{\mathcal{E}}\hat{c}_0 = \hat{\mathcal{E}}'z$, up to a multiplicative constant which is fixed by the unit variance normalization. These effects are proportional to the ones in Eq. (B.20) and are equal once the same normalization is imposed.

Appendix C. The bootstrap procedure

Confidence bands are computed as follows: draw randomly $T - p$ times (with replacement) from the uniform discrete distribution with possible values $p+1, \ldots, T$, to get the sequence $t(\tau)$, $\tau = p+1, \ldots, T$ and the corresponding sequences $\varepsilon_\tau = \hat{\varepsilon}_{t(\tau)}$, $r_\tau = \hat{r}_{t(\tau)}$, $\tau = p+1, \ldots, T$. Then set $y_\tau = y_t$ for $\tau = 1, \ldots, p$. Moreover, according to (B.19), set $y_\tau = \hat{\mu} - \hat{A}_1 y_{\tau-1} - \cdots, -A_p y_{\tau-p} + \varepsilon_\tau$, and, according to (9), $z_\tau = \hat{\theta} + \hat{c}'_0 y_\tau + \cdots + \hat{c}'_p y_{\tau-p} + r_\tau$, for $\tau = p+1, \ldots, T$. Having the artificial series y_τ, $\tau = 1, \ldots, T$, and z_τ, $\tau = p+1, \ldots, T$, re-estimate the relevant impulse-response functions. The procedure is repeated N times to get a distribution of IRFs and the desired point-wise percentiles to form the confidence bands.

The above procedure takes into account the parameter estimate uncertainty of both the VAR and the proxy Eq. (9). On the other hand, z_t is treated as an observed variable, whereas it is estimated. This cannot be avoided since a fully specified stochastic volatility model that would enable us to reproduce the correct covariances between the squared prediction errors and the lagged variables is not specified.

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