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Optimal design criteria for form-finding of double-curved surfaces

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Abstract

The development of new digital design tools and fabrication technologies stimulated a large research interest in the design and construction of free-form architecture. Free-form architecture indicates the symbolic act of freeing architecture from the limitations of pure form. During the form-finding process, the priority is on identifying the geometry that enables the optimum force flow within the structure.

This study focuses on the problem of the form-finding problem of concrete double-curved surfaces. First, a suitable form-finding optimization framework to optimize shell surfaces based on the surface Stress Density method is established. This framework is based on the use of different software such as Rhinoceros, Grasshopper, and Matlab. The stress density method is chosen because it allows obtaining an optimized shape starting by few parameters: the geometric characteristics of the model, the surface density factor and the magnitude of the load. In a second step, the study is focused on a single panel of the structure. Structural analyses of this panel are carried out using the commercial finite element software SAP2000 to demonstrate that it is a shape resistant structure. Finally, a new production process for concrete double-curved surfaces is presented showing a prototype at a small scale. This process is trying to satisfy the needs of new shapes within architectural design. The proposed solution is the improvement of an existing flexible mould formwork technology and represents the first attempt to reach a reusable, reconfigurable and affordable procedure.

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1. Introduction

In the history of European engineering, the second post-war period was characterized by an exceptional development of the construction industry [1] and the progress of reinforced and pre-stressed concrete technologies encouraged the spreading of new structural forms never seen before. New production methods shaped the formal appearance of concrete structures, which were characterized by projects of great originality such as the large vaults by Pier Luigi Nervi, the stayed bridges by Riccardo Morandi, the sober slab bridges by Silvano Zorzi, the roofings by Aldo Favini and Angelo Mangiarotti, the hyperbolic paraboloids by Felix Candela. Emerging engineers tended to combine aesthetics requirements and efficient use of materials: Frei Otto pioneered lightweight tensile structures, Eugène Freyssinet introduced parabolic-arched roofs, Robert Maillart optimized arch-bridges, Eduardo Torroja and Heinz Isler reinvented dome structures, Buckminster Fuller and Kenneth Snelson invented the tensegrity structures. Between the early 1920s and the late 1960s, the use of this structural typology was very common especially for the construction of large span roofs and bridges. In addition to the names above recalled, additional mentionable examples of this structural typology can be found in the works by Ildefonso Sanchez del Rio Pison, Ulrich Müther or Antón Tedesko. However, shells lost their popularity compared to their heyday in the 1950s and 1960s, when architects eagerly adopted them as a new mean for artistic expression [2]. A large number of studies investigated the reasons for their disappearance [3] [4] [5]. Possible reasons for the fading interest in thin concrete shell structures are probably related to a combination of multiple factors concerning the difficulties in the numerical analysis and form-finding techniques.

Inherently characterized by the interaction of geometry and forces, the unique nature of shell structures readily allows collaboration between architects and engineers in the examination of their optimal form. Through the elimination of bending and shear forces in the structure, less material and reinforcement is needed. By minimizing the use of materials, a form that is economical, sustainable and aesthetically attractive emerges. Mathematical shapes could be generated and analytically studied, whereas funicular and freeform shells required more complex modelling. Today the landmark examples of contemporary architecture use large span shells such as the Heydar Aliyev Center in Baku designed by Zaha Hadid Architects, Gehry's Guggenheim Museum in Spain, or the big shells of Jörg Schlaich. This has been possible thanks to the development of new technologies for the design and construction of this type of structure. The factor making less attractive the application of double-curved thin concrete shells is that their final cost is very high due to the costs of formworks and scaffolding requiring considerable labor. However, even today, with the standardized formwork systems, the overall cost of formwork represents 30-60% of the total costs for concrete structures [6].

The development of techniques able to reduce the formworks costs is crucial to guarantee a large diffusion of thin concrete shells. The simplest technique adopted is the reuse of a specific formwork, just suitable for the erection of identical shells or shell sections, so limiting the geometry of structures to extrusion surfaces. Moreover, the construction sequence must allow the erection or the casting of a stable patch before the formwork can be de-centered and relocated. Due to its geometrical limitations, this method is not generally applicable to funicular shells. Another strategy to increase the efficiency of shell construction could be the use of prefabricated elements.

Recently, to save labor and material, new methods have been developed using flexible formworks [7]. In contrast to the rigid formworks, flexible formwork is a system that uses lightweight, high strength sheets of fabric to take advantage of the fluidity of concrete and create highly optimized, architecturally interesting, building forms. The two basic types of these formworks were pneumatic and textile-covered cable-net formworks for the erection of concrete shells. Many types of pneumatic forms have been designed, as presented by [8], but their physical limitations and geometrical constraints prohibit their application as a construction method for funicular shapes in general. On the other hand, formworks based on funicular form-finding methods are under development [9] [10].

This study deals with the problem of form-finding of concrete double-curved surfaces. First, a suitable form-finding optimization framework to optimize shell surfaces based on the surface Stress Density method is established. This framework is based on the use of different software. The stress density method is chosen because it allows to obtain an optimized shape starting by few parameters: the geometric characteristics of the model, the surface density factor and the magnitude of the load. In a second step, the study is focused on a single panel of the structure. FEM analysis of this panel is carried out to better understand the structural behaviour under different loads. Finally, a new

production process for concrete double-curved surfaces is presented showing a prototype at a small scale. This process is trying to satisfy the needs of new shapes within architectural design.

2. Form finding

2.1 Geometrical model with Grasshopper

The 2D geometrical model of the structure (base grid) is first designed by Rhinoceros combined with the parametric design plug-in Grasshopper [12, 13]. Grasshopper is a visual programming language and environment that runs within the Rhinoceros 3D computer-aided design application. The models with this plug-in are dynamic systems that can be modified in real-time by changing some parameters defined during the creation of the model. The design geometry, shown in Figure 1, is composed of the offset of twelve rectangles represented by 549 points. The model is then exported in the Matlab environment for the form-finding procedure. After the form-finding process, the 3D model can be generated in Rhinoceros for a better graphical view.

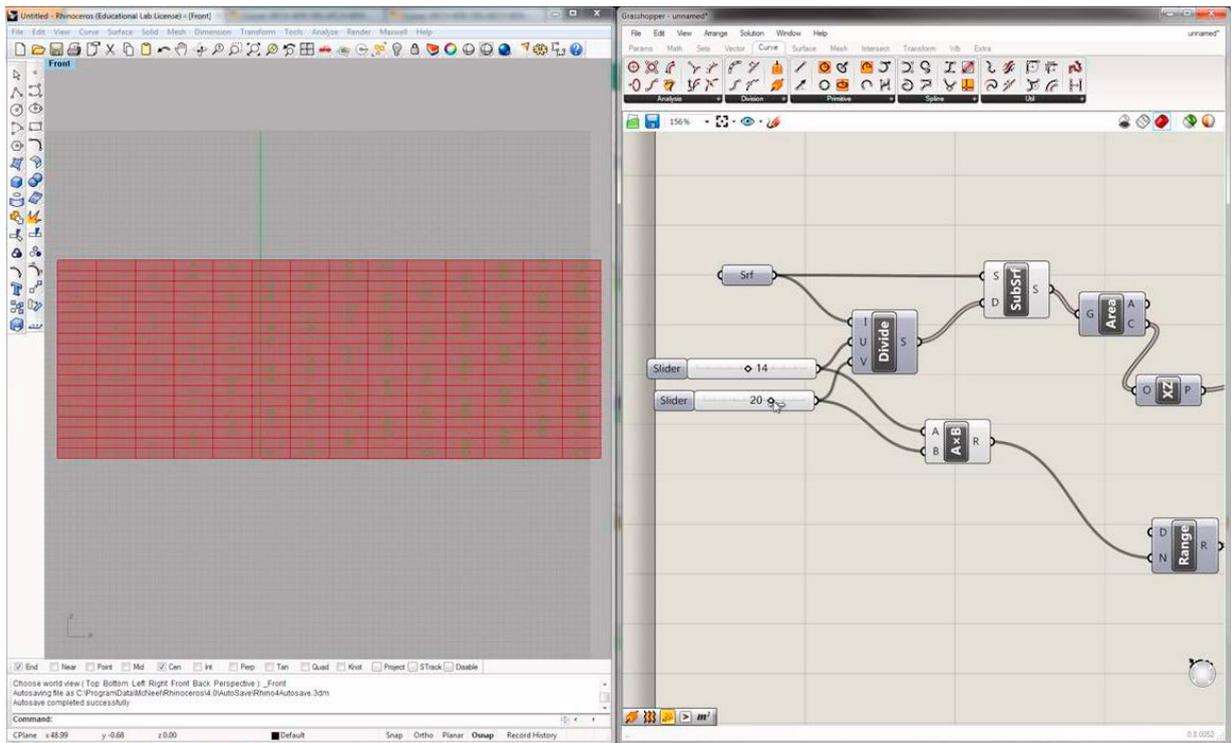


Figure 1 Geometry of the model in Rhino (left) and graphical programming to design the geometry in Grasshopper (right).

2.2 Form-finding with Matlab

The surface stress density method [11] is implemented in a Matlab script to obtain an optimized surface characterized by the elimination of bending and shear forces in the structure, i.e., a form resistant structure. The surface stress density method allows finding the optimal shape corresponding to the minimum stress configuration on the surface. By introducing the coordinates of the points that compose the structure imported by Grasshopper, many resistant shapes of the structure can be obtained by only changing the values of the stress density coefficient and the external loads. The four nodes located at the vertices are considered the constraints and are fully-fixed.

The differential geometry of surfaces deals with the differential geometry of smooth surfaces with various additional structures, most often, a Riemannian metric. Surfaces have been extensively studied from various

perspectives. In this example, the Delaunay triangulation method is implemented to have the topology description of the geometry for complex structures. Delaunay triangulation is a proximal method that satisfies the requirement that a circle drawn through the three nodes of a triangle will contain no other node. For modelling terrain or other objects given a set of sample points, the Delaunay triangulation gives a nice set of triangles to use as polygons in the model. In particular, the Delaunay triangulation avoids narrow triangles (as they have large circumcircles compared to their area).

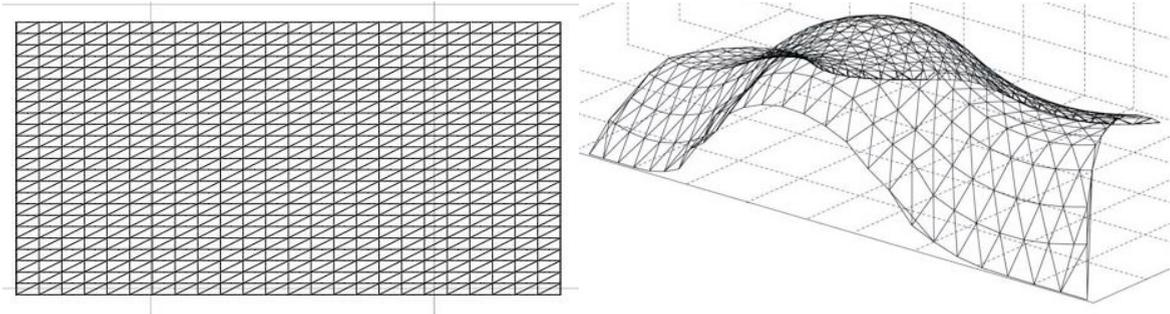


Figure 2 Delaunay triangulation of the design structure (left); Best shape found by the Surface Stress Density Method (right).

The assigned value of external force, to be applied at each node, is given by $W = 4,5 \text{ kN/m}^2$ considering a concrete weight $W_c = 30 \text{ kN/m}^3$ and a structure thickness equal to 0,1 m. Nodes are divided into 3 groups: some are subject to a force equal to 6 kN, other ones to a force equal to 4 kN and others are fixed (i.e., the four at the vertices). The final configuration obtained by the described procedure is shown in Figure 2 (right). The dimensions of the shell are $20 \times 10 \text{ m}^2$ and the thickness is equal to 15 cm.

Figure 3 shows the convergence of points subjected to T_z and N_z forces. At the beginning of the convergence, the points are not in equilibrium, while after eight hundred loops the lines that represent the positions of points are rectilinear. This indicates that all points are in equilibrium and in each node the sum of the applied forces is equal to zero.

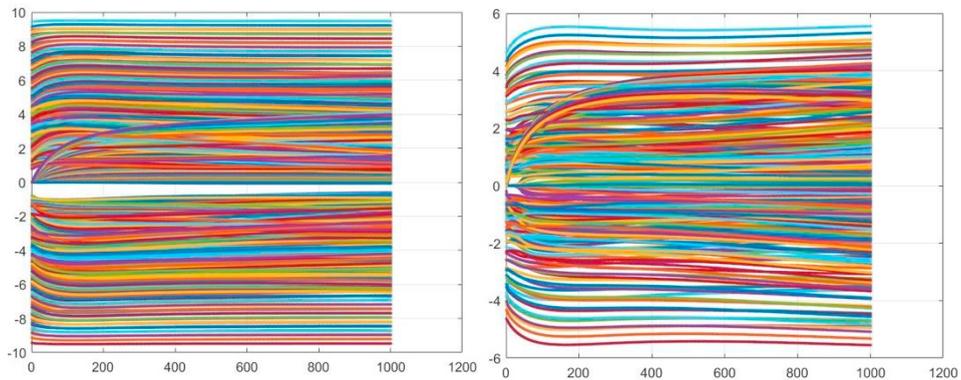


Figure 3 Convergence of points subjected to: T_z (left); N_z (right).

3. Finite Element Analysis

Structural analyses of the surface obtained by the Surface Stress Density Method are carried out by the commercial finite element software SAP2000.

3.1. Definition of loads

The first design step is to identify the loads acting on the structure. Load factors were calculated according to the Italian Building Code 2018 [14]. In the following, the assumed loads acting on the structure are presented. The symbols adopted are the same as those reported by the Italian Building Code 2018 [14]. The loads are assumed to be equally distributed on the structure. The thickness of the shell is assumed equal to 15 cm.

The permanent structural loads are assumed starting from a specific self-weight equal to 25 kN/m^3 , while the permanent non-structural load is set equal to $G_2 = 2 \text{ kN/m}^2$. The structure is considered installed at a site quote equal to 330 m. The characteristic values of snow load on the ground for a return period of $T_R=50$ years are assumed equal to $q_{sk}=1,25 \text{ kN/m}^2$ and $q_s=1,00 \text{ kN/m}^2$. The latter is obtained by considering Zone II and normal topographic conditions, thus $C_E=1$ and $C_T=1$. The wind loads are calculated for the city of Chieti. The following parameters are considered in the calculations:

- $V_0=27 \text{ m/s}$
- $a_0=500,00 \text{ m}$; $k_a=0,02 \text{ s}^{-1}$
- Zone=3; Roughness class=A; Distance from the coast=20,00 km; Exposure category=IV
- $k_r=0,22$; $z_0=0,30 \text{ m}$; $z_{min}=8,00 \text{ m}$
- Height of the construction=3,90 m; $T_R=50$ years; $C_T=1,00$; $c_d=1,00$; $c_e=1,48$; $v_b=27,00 \text{ m/s}$

Using these parameters, the wind loads are given by: $c_{pe1} = 0,8$; $p_1 = 0,415 \text{ kN/m}^2$; $c_{pe2} = -0,4$; $p_2 = -0,363 \text{ kN/m}^2$.

The seismic loads are calculated by using the response spectrum (Figure 4) defined according to the following parameters:

- Time-averaged shear-wave velocity to 30 m depth: $V_{s30} = 620 \text{ ms}^{-1}$
- Subsoil category: B
- Topographic amplification: T_1
- Characteristics of the structure: $V_n=50$ yrs; Class=III; $C_u=1$; $V_R=50$ yrs; structure factor $q=1,5$.

The load combinations for the ultimate limit states (SLU) and the exercise limit states (SLE) were considered.

3.2. Linear dynamic analysis of the shell

The participating mass ratios and the fundamental periods and associated mode shapes of the structure are first found solving the eigenvalue problem of the structure. The maximum force under the load combinations prescribed by the Italian Building Code 2018 [14] and given in the previous section are then calculated by applying the linear dynamic analysis. The thickness of the shell is assumed equal to 15 cm.

The results (Figures 4-6) indicate that the deflection is very low for the RC shell, due to the previously found shape with strong structural efficiency. A predominance of green areas indicates deflection between 0 and 5 mm in the largest part of the shell (Figure 4 left). The highest deflection regions are found at the extremities of opening edges with a deflection equal to 20 mm. Bending moments for the shell in x and y directions are rather low, resulting in the range -0.1 – 0.1 kNm/m (Figure 4 right and 5 left). This indicates the good optimization obtained. The results indicate that the most vulnerable are the free edges. Compression is in the range of -40 to 20 kN/m , see Figure (right). Small areas in the valleys (red) and at points along the free edge (red) indicate that compression is between 0 kN/m and 20 kN/m . Finally, at maximum tension modes, tensile forces mainly assume values in the range 0 – 21 kN/m (Figure 6).

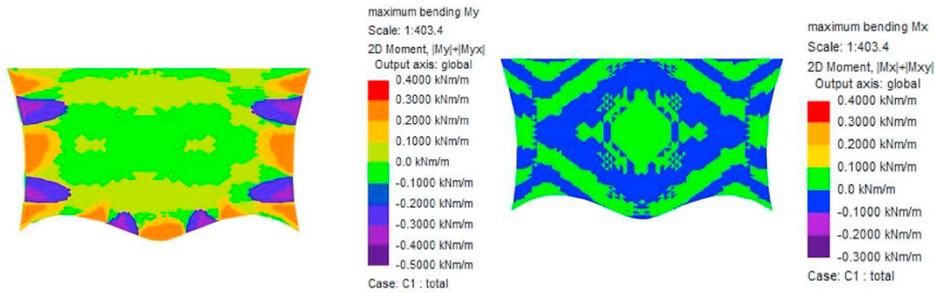


Figure 4 Deformed shape with-Deflection (left) and deformed shape with maximum bending M_x (right).

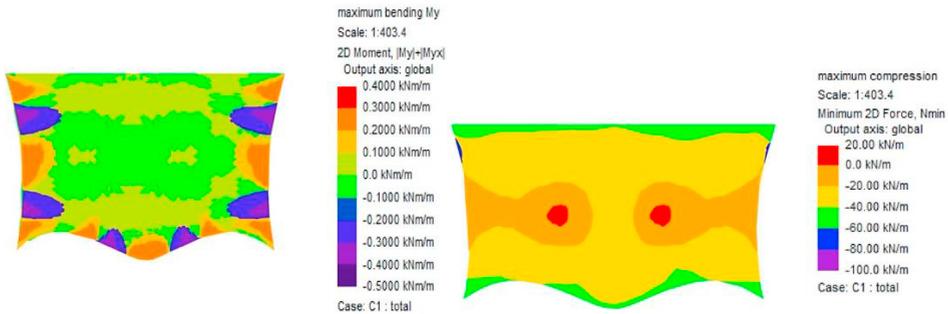


Figure 5 Deformed shape with maximum bending M_y (left) and deformed shape with maximum compression (right).

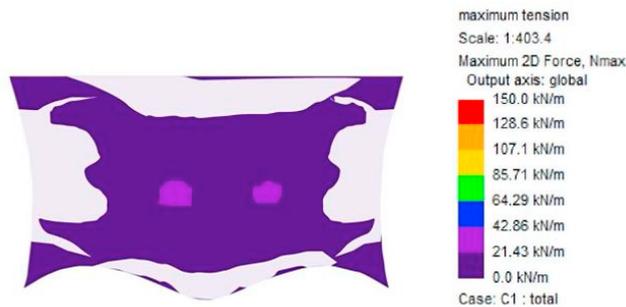


Figure 6 Deformed shape with maximum bending Tension.

3.3. Finite Element Analysis of a single panel

On the basis of the structural analysis of the double-curved structure, a more detailed structural analysis was carried out for a single panel having dimension equal to $1 \times 1 \text{ m}^2$, to validate the obtained shape and demonstrate that it is a shape resistant structure (i.e., characterized by low values of the shear forces and bending moments).

The model of the single panel accounts that the boundary conditions are applied on the nodes in correspondence of the four edges (Figure 7 left). The simple support conditions are assumed by blocking the horizontal displacements parallel to the shell edges and the vertical displacements. On the other hand, the horizontal deformations perpendicular to the edge are allowed (Figure 8 left). The maximum principal stresses (tensile) in the panels are displayed in Figure 7 (right).

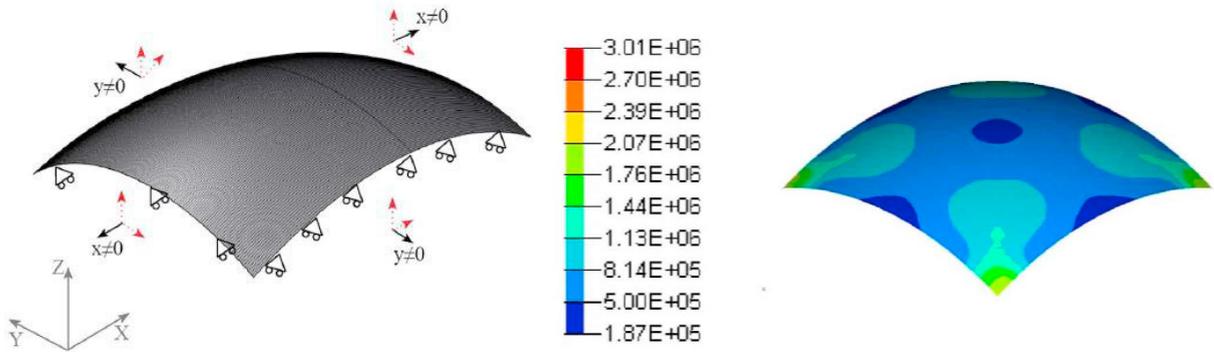


Figure 7 Boundary conditions for singly-curved panel (left) and Stress plot showing maximum principal stresses (tensile) (right).

4. Production process

A new production process for concrete double-curved surfaces is proposed in the following. The prototype machine for the production of the double-curved shell is composed of three parts (Figure 8): the structure, the pin pad, and the mould. The mainframe is the structure that supports all the elements used to shape the shell. 13 pins are sufficient to create a large variety of forms. The pin spacing is equal to 20 cm and the pin pad is suitable to make double-curved panels with dimensions equal to $50 \times 50 \text{ cm}^2$.



Figure 8 Prototype machine for the production of the double-curved shell with the pin pad system (left), mould (right) and concrete casting (bottom).

The mould has to be built with a material that is: *i*) flexible, to allow the design curvature of the shell; *ii*) resistant, to allow the realization of the concrete shell; *iii*) light, for not burden the supporting structure. The silicon rubber was chosen as mould materials since it well meets the above requirements.

The proposed production process can be summarized into four main steps:

- setting the shaping machine;
- casting concrete in the mould;
- setting the design shape on the machine and positioning the pins at the correct height;
- after 24 hours un-moulding the concrete shell and waiting the curing time.

The fiber-reinforced concrete Sika Grout-212 was used in the tests, having the following characteristics: water percent equal to 13÷15% of water for 28 kg powder; layer thickness equal to 10÷50mm; Compressive Strength 24 h > 31 MPa, 7 days > 42 MPa, 28 days > 51,7 MPa (UNI 196-1); Modulus of Elasticity in Compression: 28.8 GPa. This type of high-performance concrete was chosen to avoid the use of steel bars in the tension areas.

Using this prototype machine, a single panel of the surface was realized and the shape was compared with the designed one finding a good agreement. A similar machine can be built at a large scale. The proposed approach represents a first attempt to reach a reusable, reconfigurable and affordable procedure.

5. Conclusions

Research on the design and construction of free-form architecture has significantly developed in recent years, due to the introduction of new digital design and fabrication technologies. In this framework, this study proposes a procedure dealing with the form-finding of concrete double-curved surfaces. The procedure consists of the combined implementation of the parametric design plug-in Grasshopper and of the surface Stress Density method implemented in Matlab. In order to validate the form found using this approach and demonstrate that it is a shape resistant structure, a structural analysis was carried out using SAP2000 software. Finally, a new production process for concrete double-curved surfaces was investigated and described, representing a first attempt to reach a reusable, reconfigurable and affordable procedure.

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