

Finite element model updating of a tied-arch bridge using Douglas-Reid method and Rosenbrock optimization algorithm

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Abstract: Condition assessment of bridges has become increasingly important. In order to accurately simulate the real bridge, finite element (FE) model updating method is often applied. This paper presents the calibration of the FE model of a reinforced concrete tied-arch bridge using Douglas-Reid method in combination with Rosenbrock optimization algorithm. Based on original drawings and topographic survey, a FE model of the investigated bridge is created. Eight global modes of vibration of the bridge are identified by ambient vibration tests and the frequency domain decomposition technique. Then, eight structural parameters are selected for FE model updating procedure through sensitivity analysis. Finally, the optimal structural parameters are identified using Rosenbrock optimization algorithm. Results show that although the identified parameters lead to a perfect agreement between approximate and measured natural frequencies, they may not be the optimal variables which minimize the differences between numerical and experimental modal data. However, a satisfied agreement between them is still presented. Hence, FE model updating based on Douglas-Reid method and Rosenbrock optimization algorithm could be used as an alternative to other complex updating procedures.

Key words: tied-arch bridge; finite element model updating; ambient vibration test; experimental modal data; Douglas-Reid method; Rosenbrock algorithm

1 Introduction

As major transport infrastructures, bridges are of great importance to modern society. During the life cycle, they are exposed to various types of loads such as winds, traffics, earthquakes and so on. As time goes

by, the aging of the bridges cannot be avoided. Therefore, condition assessment of bridge structures has become increasingly necessary, which is often carried out through finite element (FE) method. Of course, FE models can be created based on technical design data, as-built drawings and engineering judg-

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ment. However, these FE models usually cannot predict the exact response of the real structures due to structural uncertainties. A possible practice to fill the lack between the real structures and the corresponding FE models is to employ the FE model updating technique.

FE model updating had emerged in the 1990s as a subject of immense importance to the design, construction and maintenance of structures (Jin 2011). The overview of FE model updating was summarized fundamentally by Friswell and Mottershead (Friswell and Mottershead 1995; Mottershead et al. 2011). In general, methodologies developed to update the FE model fall into two categories: direct and iterative. The direct methods (Imregun and Visser 1991; Mottershead and Friswell 1993; Friswell et al. 1998; Carvalho et al. 2007; Yang and Chen 2009) update the FE model without any regard to changes in physical parameters, which directly update the stiffness and mass matrices of the system in a one-step procedure. The iterative methods (Farhat and Hemez 1993; Maia and Silva 1997; Levin and Lieven 1998; Fritzen et al. 1998; Teughels et al. 2003;) update physical parameters until the FE model reproduces the measured data to a sufficient degree of accuracy, where a penalty function (objective function) is typically used. Because of this nature of iterative methods, they give FE models that ensure the connectivity of nodes, and have mass and stiffness matrices that have physical meaning. This approach is more flexible in its application as the physical properties of the FE model can be updated (Ribeiro et al. 2012). Due to the increased applications, this paper only focuses on iterative updating technique.

The success of FE model updating is depending on the use of experimental data, the selection of updating variables and the application of optimization methods. Experimental modal data, such as natural frequencies and mode shapes are often identified based on ambient vibration tests. The most sensitive variables could be selected by sensitivity analysis. Regarding the optimization algorithm used in FE model updating, several methods are available to solve the optimization problem, such as gradient-based methods (quasi-Newton, sequential quadratic programming, augmented La-

grangian, etc.) (Teughels 2003), response surface methods (Ren and Chen 2010; Deng and Cai 2010; Zhou et al. 2013) and nature inspired algorithms (e.g., genetic algorithm, evolutionary strategies, particle swarm optimization) (Levin and Lieven 1998; Jafarkhani and Masri 2011).

For iterative updating procedure, a large number of analyses need to be performed. In addition, the investigated FE models are usually very large. Therefore, it will take much time to carry out the FE model updating and approximate methods will be necessary to reduce the computational time. One of these approximate methods is the procedure proposed by Douglas and Reid (Douglas and Reid 1982), which approximates the natural frequencies of FE model with a specified function of the unknown structural parameters.

This paper presents the FE model updating of a tied-arch bridge using MATLAB and MIDAS/CIVIL. The former is used for sensitivity analysis and optimization analysis while the later is responsible for structural modeling and eigenvalue analysis. The outline of this study is as follows. Section 2 presents the detailed FE model updating procedure based on Douglas-Reid method and Rosenbrock optimization algorithm. Description of a three-dimensional FE model of the bridge is shown in section 3. The modal parameters of the bridge are identified in section 4 by ambient vibration tests, such as the natural frequencies and the mode shapes. In section 5 a sensitivity analysis is performed to select the structural parameters used for model updating. Section 6 calibrates the FE model of the bridge, and conclusions are drawn in section 7.

2 Considered FE model updating technique

In FE model updating, an optimization problem is often set-up in which the differences between the experimental and numerical modal data have to be minimized. Assuming the experimental modal data, i.e., the natural frequencies and the mode shapes, have been obtained from ambient vibration tests, the FE model updating technique is carried out in this study by developing MATLAB codes interfaced with MIDAS/CIVIL. The key aspects of FE model updating

procedure considered in this paper are the implementations of Douglas-Reid method and Rosenbrock optimization algorithm, which are further described in the following subsections.

2.1 Douglas-Reid method

In Douglas-Reid method (Douglas and Reid 1982), the relationship between the i -th natural frequency of the model and the unknown structural parameters $X_k (k=1, 2, \dots, n)$ of the model is approximated around the current values of X_k , by the following:

$$f_i^* (X_1, X_2, \dots, X_n) = \sum_{k=1}^n (A_{ik}X_k + B_{ik}X_k^2) + C_i \quad (1)$$

where f_i^* represents the approximated i -th natural frequency of the FE model.

To satisfy the expression and solve the problem, $(2n + 1)$ constants (A_{ik} , B_{ik} , and C_i) must be determined before to compare each f_i^* to its experimental counterpart. In order to evaluate these constants, engineering judgment is first used to estimate a base value of the structural parameters $X_k^B (k=1, 2, \dots, n)$ and the range in which such variables can vary. Let denote the lower and upper limits of the unknown parameters as X_k^L and $X_k^U (k=1, 2, \dots, n)$, respectively:

$$X_k^L \leq X_k \leq X_k^U \quad (2)$$

Then, the $2n + 1$ constants on the right-hand side of Eq. (1) can be determined by computing the i -th natural frequency f_i^{num} of the finite element model for $2n + 1$ choices of the unknown parameters. The first choice of the structural parameters corresponds to the base values; then each structural unknown is varied, one at a time, from the base value to the upper and lower limit, respectively.

Thus, the $2n + 1$ conditions used to evaluate the constants in Eq. (1) are the following:

$$\begin{cases} f_i^{num} (X_1^B, X_2^B, \dots, X_n^B) = f_i^* (X_1^B, X_2^B, \dots, X_n^B) \\ f_i^{num} (X_1^L, X_2^B, \dots, X_n^B) = f_i^* (X_1^L, X_2^B, \dots, X_n^B) \\ f_i^{num} (X_1^U, X_2^B, \dots, X_n^B) = f_i^* (X_1^U, X_2^B, \dots, X_n^B) \\ \vdots \\ f_i^{num} (X_1^B, X_2^B, \dots, X_n^L) = f_i^* (X_1^B, X_2^B, \dots, X_n^L) \\ f_i^{num} (X_1^B, X_2^B, \dots, X_n^U) = f_i^* (X_1^B, X_2^B, \dots, X_n^U) \end{cases} \quad (3)$$

The constants A_{ik} , B_{ik} , and C_i can be easily calculated by the above stated equations. Once these constants have been computed, the approximation (i. e., Eq. (1)) is completely defined and it can be used to

update the structural parameters. The optimal parameter estimates are defined to be the values which minimize the following:

$$J = \sum_{i=1}^m \omega_i \varepsilon_i^2$$

$$\varepsilon_i = f_i^{exp} - f_i^* (X_1, X_2, \dots, X_n) \quad (4)$$

where f_i^{exp} represents the i -th experimentally identified natural frequency; ω_i is a weighting constant.

Since the natural frequencies of FE model are approximated using functions of the unknown structural parameters, FE model updating procedure can be performed based on any optimization algorithm. Moreover, it is obvious that the computational efforts are much less than the procedures using nature inspired algorithms, such as genetic algorithm, evolutionary strategies and particle swarm optimization. In recent years, Douglas-Reid method has been widely adopted by many researchers (Gentile 2006; Gentile and Saisi 2007; Eusani and Benedettini 2009; Ramos 2011) to perform FE model updating of different structures. However, one should have in mind that the quadratic approximation (Eq. (1)) is as better as the base values are closer to the solution. Indeed, the accuracy and stability of the optimal estimates may be readily checked either by the complete correlation with the experimental data or by repeating the procedure with new base values. For complex systems, especially for arch bridges or cable-stayed bridges that often exhibit similar mode shapes, the use of Douglas-Reid method should prevent misleading correlation between numerical and experimental mode shapes (Gentile 2006).

2.2 Rosenbrock optimization algorithm

In this study, an optimization algorithm with adaptive sets of search directions proposed by Rosenbrock (Rosenbrock 1960) is used to solve the optimization problem. Rosenbrock method proceeds by a series of stages, each of which consists of a number of exploratory searches along a set of directions that are fixed for the given stage, but which are updated from stage to stage by using information about the curvature of the objective obtained during the course of the search. In addition, Rosenbrock method is a 0th order search algorithm and it does not require gradient of the target function. Only simple evaluations of the objective

function are used. But, this algorithm approximates a gradient search thus combining advantages of 0th order and 1st order strategies. Flowchart of Rosenbrock method is presented in Fig. 1, which can be also described by the following steps:

1) Initialize the selected variables (parameters), and the lower and upper limits of variables.

2) Select an initial set of orthogonal vectors, i. e. , the orthogonal vectors of the unit base in n -dimensional space, and step lengths.

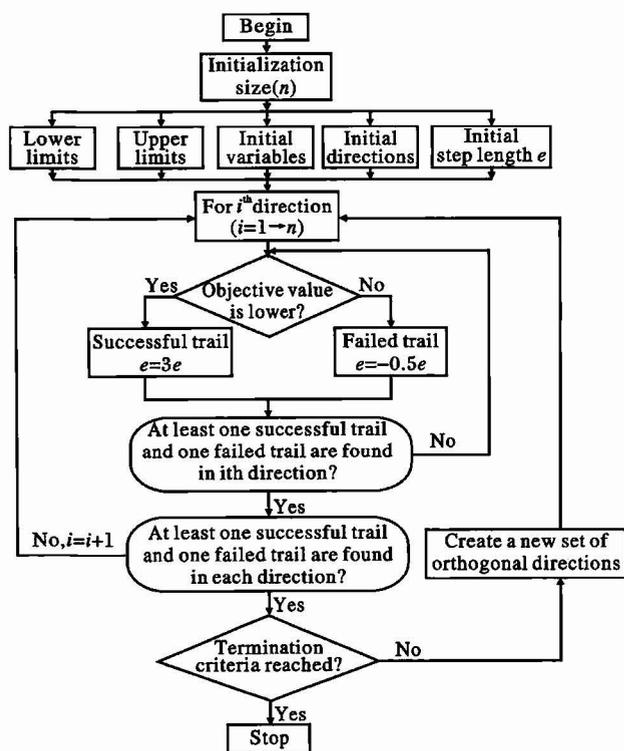


Fig. 1 Flowchart of Rosenbrock optimization algorithm

3) Conduct searches along these directions, cycling over each in turn, moving to new iterates that yield successful steps (an unsuccessful step being one that leads to a less desirable value of the objective). If the trail is successful, step length is multiplied by 3, otherwise multiplied by -0.5 .

4) Continue until there has been at least one successful and one unsuccessful step in each search direction. Once this occurs, the current stage terminates. If the objective at any of these steps is perceived as being an improvement over the objective at the current best point, the new point is then considered. Be care-

ful not to exceed the upper or lower limits. If the limits are exceeded, replace the calculated values of coordinates by the limit value which is surpassed.

5) If the change of the objective function is within the limits of error, stop the calculation and end the optimization.

6) Generate a new set of orthonormal vectors using the Gram-Schmidt orthonormalization procedure, with the “promising” direction from the just-completed stage used as the first vector in the orthonormalization process. Naturally, the direction from the start point to the final point in the current exploratory iterative phase may be a direction which can optimize the function. So a new direction group should include this direction. Detailed procedures to generate these orthonormal vectors can be found in many research studies (Votruba 1975 ; Chen 2006).

7) Using these new orthonormal vectors, compute a new calculation (beginning with item 3) until the optimum is reached.

3 Canonica bridge

The investigated bridge was built around 1950, as shown in Fig. 2. It was designed by Giulio Krall, one of the most eminent Italian bridge engineers of the 20th century, to replace a former iron bridge on the same span. The deck of the bridge, with a longitudinal slope of 2.5%, is a four-cell concrete box girder (Fig. 3); the total width of the girder is 12.69 m for two traffic lanes and two pedestrian walkways. The girder is 1.23 m deep so that a good transparency of the deck is attained from an aesthetic standpoint. The two lateral cells suspend the deck by means of inclined ties made by conventional reinforcement bars immersed in a cast-in-place grout. The parabolic arch structure consists of two solid R. C. arch ribs, transversally connected together with cross struts; the arches are characterized by a rise/span ratio of 1/6 and suspend the deck on a length of 75.50 m so that the bridge represents one of the most interesting examples of Nielsen structure still in service in Italy.

Figure 4 presents the 3D FE model of Canonica bridge developed based on the following assumptions and the preliminary guess of unknown structural parameters:

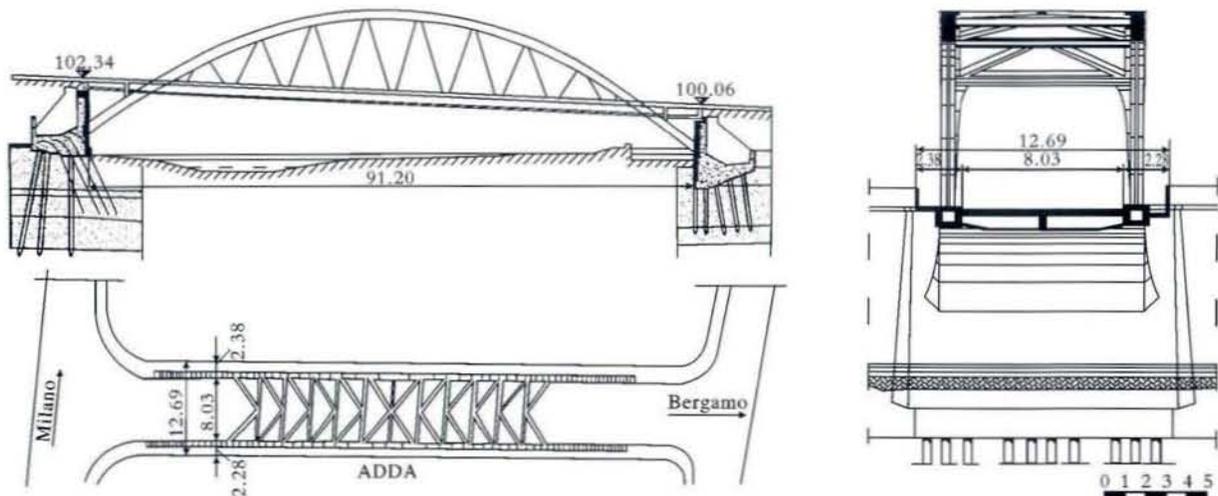


Fig. 2 View of investigated bridge (m)

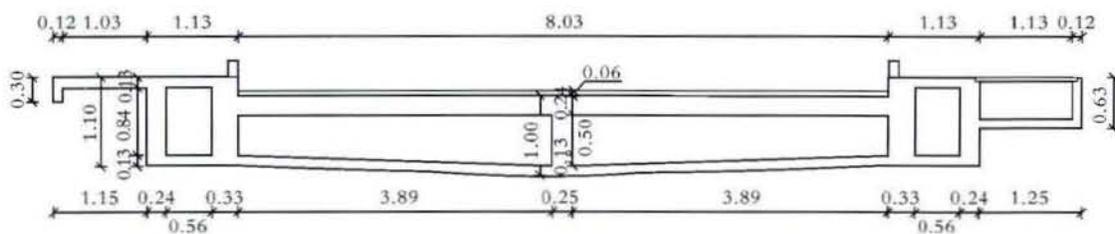


Fig. 3 Cross-section of the deck (m)



Fig. 4 3D FE model of Canonica bridge

1) Unit weight of concrete is set to be 24.0 kN/m^3 and that of the steel is assigned to be 78.0 kN/m^3 .

2) The Poisson's ratio of concrete is held constant and equal to 0.20 and that of the steel is assumed to be 0.25.

3) Yong's modulus of the concrete is 34 GPa and that of the steel is 210 GPa.

4) Four-node shell elements are used to model the upper and lower concrete slabs of the deck and the lower parts of the arches.

5) The two lateral box stringers and the transverse cross-beams of the deck are modeled by two-node 3D beam elements. Rigid links are used between the concrete slabs and the grid of lateral stringers and transverse cross-beams.

6) The arches and bracing members are modeled as beam elements.

7) The arch footings are considered as fixed.

8) The ties are modeled as truss elements.

9) The effects of the abutments and the foundations, as restraints to the movements of the bridge, are taken into account by introducing a series of springs oriented in the different directions and attached to the ends of the bridge along each node. Specifically, the resultant stiffness of all springs is assumed to be $5E+8 \text{ N/m}$.

Finally, the 3D FE model of Canonica bridge contains a total of 3096 nodes, 1856 beam elements, 36 truss elements, 1896 shell elements.

In this study, only the global modes of the bridge are considered, which involve global modal deflections of the bridge deck or arches. Based on the initial FE model, the natural frequencies of the first ten global modes of the bridge and the corresponding mode shapes are presented in Fig. 5. Mode 1G involves the transversal bending of the arches. Modes 2G, 3G, 7G and 9G are flexural modes of the deck. Mode 4G essentially involves the transversal bending

of the deck. Modes 5G, 6G, 8G and 10G are torsional modes of the deck. In addition, it can be found that some global modes are coupled together. These particular modes are characterized by common move-

ments, with similar amplitude, of the deck or arches. As described in section 2.1, special attention must be directed when correlating mode shapes between numerical and experimental modal data.

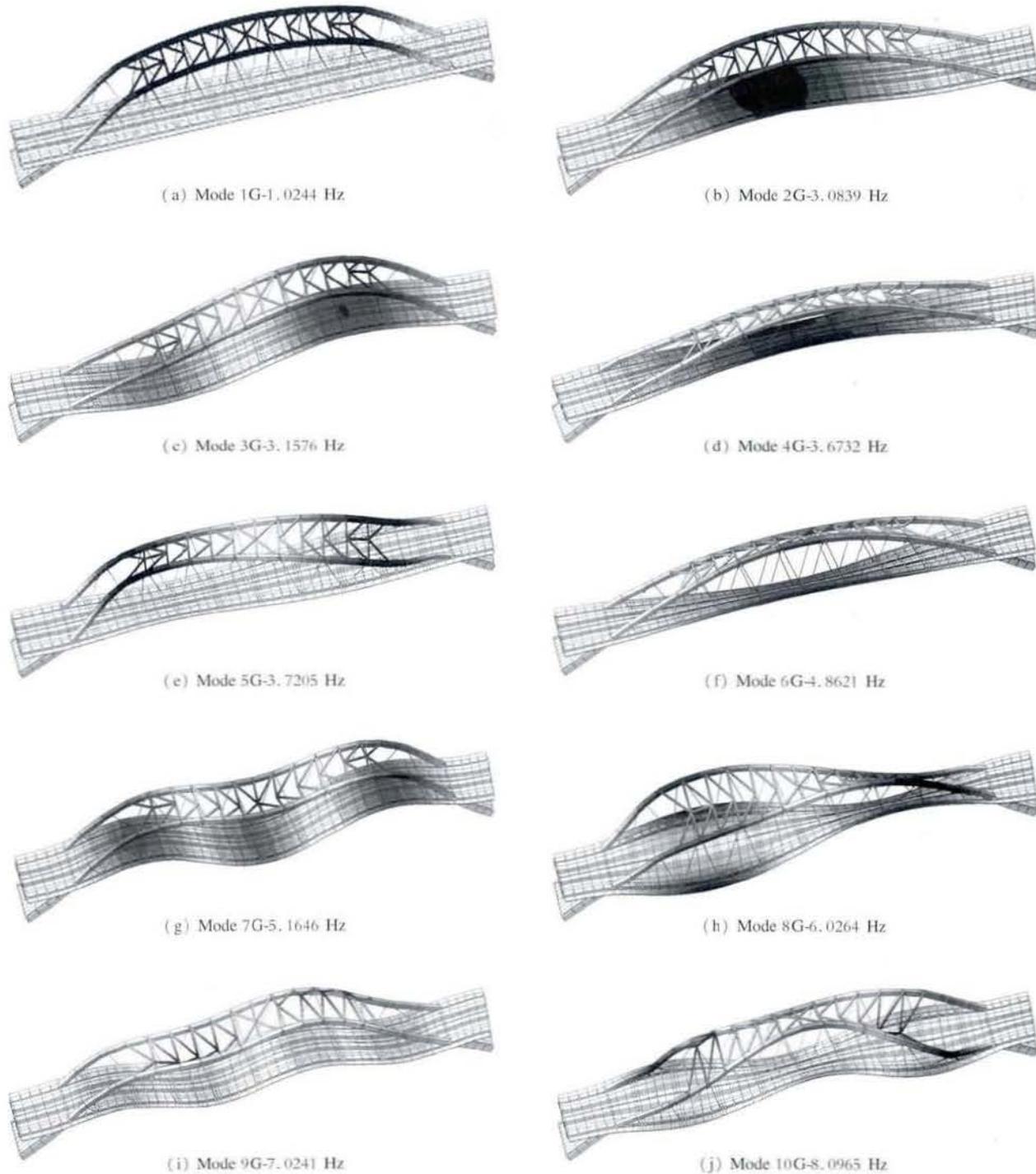


Fig. 5 First ten global modes of initial FE model of Canonica bridge

4 Ambient vibration test

In this study, experimental test is basically focused on the characterization of the overall dynamic behavior of the bridge, in particular of the deck. The full-scale tests are conducted on the bridge using a 16-channel data acquisition system with 14 uniaxial piezoelectric

accelerometers (WR model 731A), each with a battery power unit. For each channel, the ambient acceleration-time histories are recorded for 3600 s at an interval of 0.005 s. The schematic of the sensor layout is presented in Fig. 6, and the installed accelerometer is shown in Fig. 7.

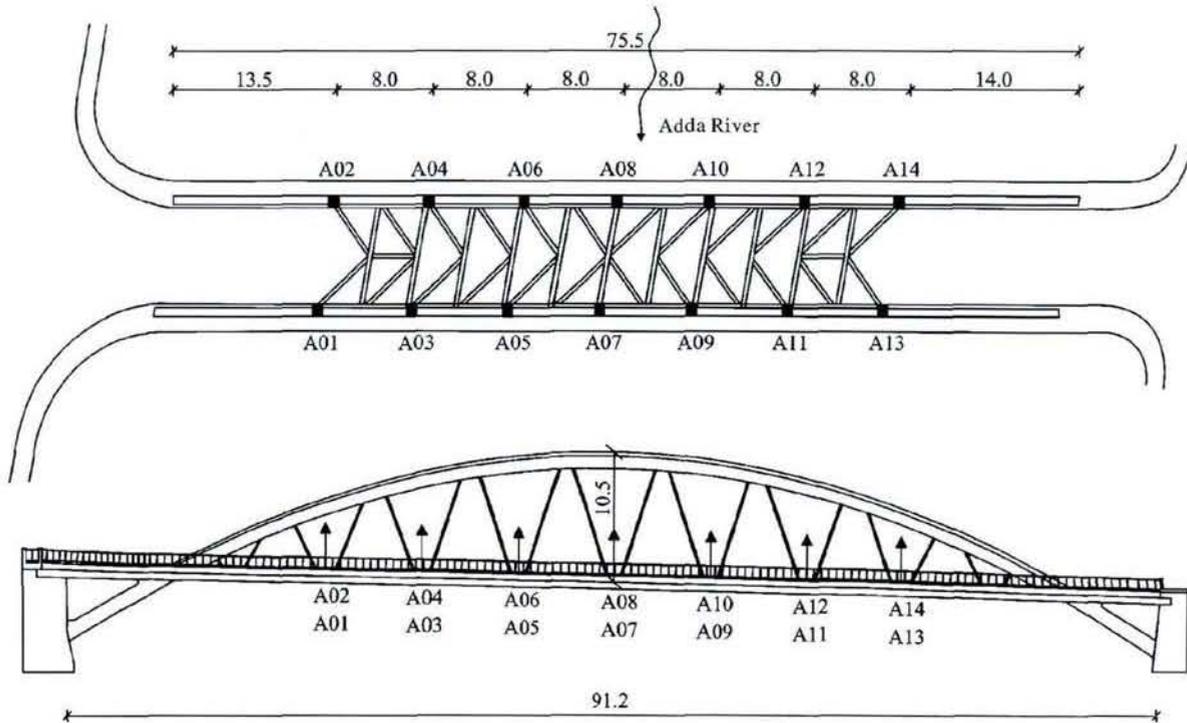


Fig.6 Schematic of sensor layout (m)

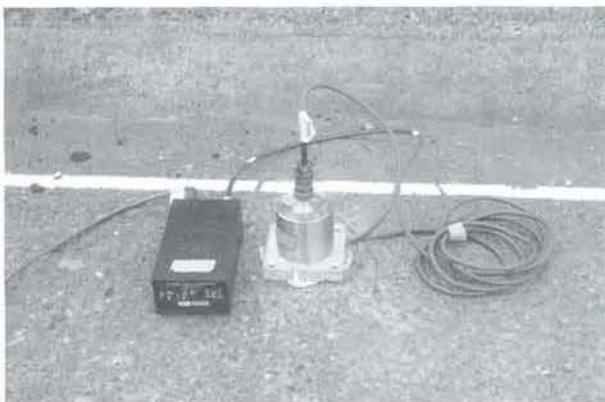


Fig.7 Installation of accelerometers on deck

The operational modal analysis, which identifies the modal parameters from output-only time-histories, is carried out using the Frequency Domain Decomposition (FDD) (Brincker et al. 2001) method imple-

mented in the ARTeMIS software. The FDD technique involves the following main steps:

1) Evaluation of the spectral matrix $G(f)$, i. e., the matrix where the diagonal terms are the auto-spectral densities (ASD) while the other terms are the cross-spectral densities (CSD). In the present application, the ASDs and the CSDs were estimated, after decimating the data 5 times, from 2048-points Hanning-windowed periodograms that are transformed and averaged with 66.7% overlapping. Since the re-sampled time interval is 0.025 s, the resulting frequency resolution is $1/(2048 \times 0.025) \approx 0.0195$ Hz.

2) Singular value decomposition (SVD) of the matrix $G(f)$ at each frequency, according to:

$$G(f) = U(f) \times S \times U^H(f) \quad (5)$$

where the diagonal matrix S collects the real positive

singular values in descending order; U is a complex matrix containing the singular vectors as columns; the superscript H denotes complex conjugate matrix transpose.

3) Inspection of the curves representing the singular values to identify the resonant frequencies and estimate the corresponding mode shape using the information contained in the singular vectors of the SVD.

The principle in the FDD techniques is easily understood by recalling that any response can be written in modal co-ordinates and that the spectral matrix of a linear dynamic system subjected to a white-noise random excitation may be expressed as:

$$G(f_r) \approx \phi \times G_{qq}(f) \times \phi^H \quad (6)$$

where ϕ is the matrix of mode shapes; $G_{qq}(f)$ is the spectral matrix of the modal co-ordinates. Since the modal co-ordinates are un-correlated, the matrix $G_{qq}(f)$ is diagonal; hence, if the mode shapes are orthogonal, Eq. (6) is a SVD of the response spectral matrix. As a consequence, if only one mode is important at a given frequency f_r , as it has to be expected for well-separated modes, the spectral matrix can be approximated by a rank-one matrix:

$$G(f_r) \approx \sigma_1(f_r) \times u_1(f_r) \times u_1^H(f_r) \quad (7)$$

The first singular vector $u_1(f_r)$ is an estimate of the mode shape. On the other hand, the first singular value $\sigma_1(f_r)$ at each frequency represents the strength of the dominating vibration mode at that frequency so that the first singular function can be suitably used as a modal indication function (yielding the resonant frequencies as local maxima) whereas the successive singular values contain either noise or modes close to a strong dominating one.

Figure 8 shows the averages of the first 3 normalized singular values associated with the spectral matrices of all data sets. As can be observed, eight global modes of the bridge are identified in the investigated frequency interval of 0-9 Hz. In the following, the identified modes are marked with B, indicating the bending modes of the deck, and with T, indicating torsion modes of the deck. The identified modes are illustrated in Fig. 9. It can be noted that all these modes can be found in Fig. 5 when the initial FE model is investigated, although some of them do not emerge in the same order.

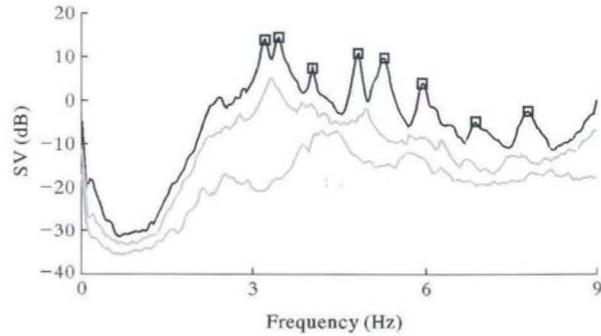


Fig. 8 Average of normalized singular values of spectral matrix of all data sets and selected modes

5 Sensitivity analysis

The main uncertainties in FE modeling of the bridge can be selected based on the engineering experience and judgment. As shown in Fig. 10, the bridge is divided into three regions, in each of which the Young's modulus and density of the concrete are assumed as constant. Furthermore, different supports of the deck slab and the lower parts of the arches are assigned in this study, i. e. , supports 1 and 2, respectively. Both of them include three different degrees of freedom, i. e. , two degrees of freedom translation and one degree of freedom rotation. In addition, the Young's modulus and the mass density of the steel are also considered as unknown variables. As a consequence, a set of 14 uncertain independent variables of the model are selected and listed in Tab. 1.

To determine the sensitive variables that most influence the modal parameters of the bridge, sensitivity analysis (Maia and Silva 1997) is performed in this section. Here, both natural frequencies and mode shapes of the FE model are considered as structural responses. The sensitivity analysis computes the sensitivity coefficient, which represents the percentage change in modal parameter R_i^{num} per 100% change in the variable X_j , as the following form:

$$S_{i,j} = 100 \times \frac{X_j}{R_i^{num}} \times \frac{\Delta R_i^{num}}{\Delta X_j} \quad (8)$$

Note that in sensitivity analyses, only one parameter is varied at one time and others keep the same as the values in initial FE model. The change in each variable is assigned to be 1% of its value.

Figure 11 shows the computed sensitivity coefficients against the selected structural parameters and

modal responses. In order to identify the updating parameters, sensitivity coefficients less than 3 are excluded in this figure. It can be observed in this figure, the modal parameters of the bridge are significantly affected by the concrete properties involved in

regions 1 and 2 as well as the Young's modulus of the steel. Furthermore, the Young's modulus of concrete in region 3 and the horizontal stiffness of both supports also influence the modal parameters to some extent.

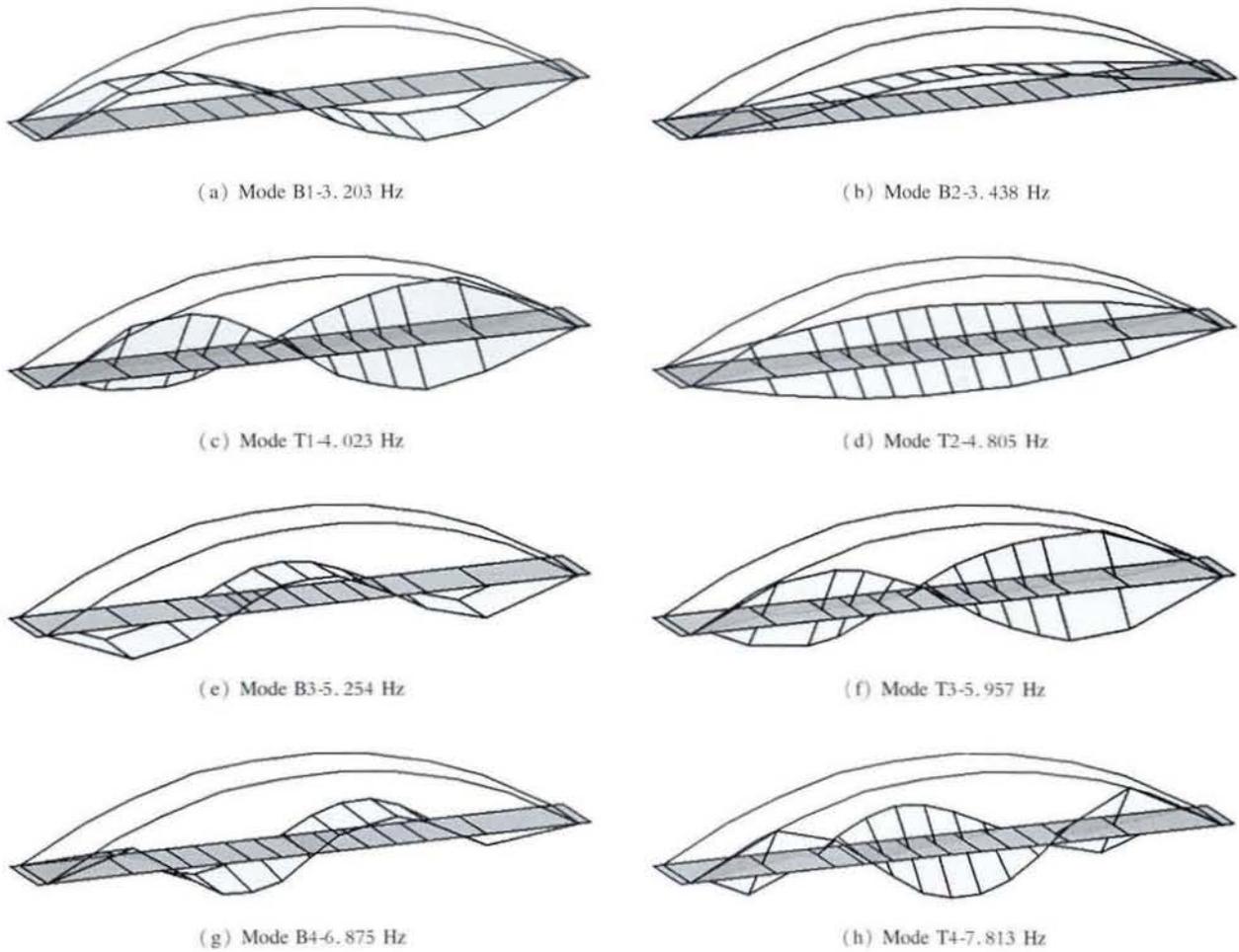


Fig. 9 Experimental natural frequencies and mode shapes

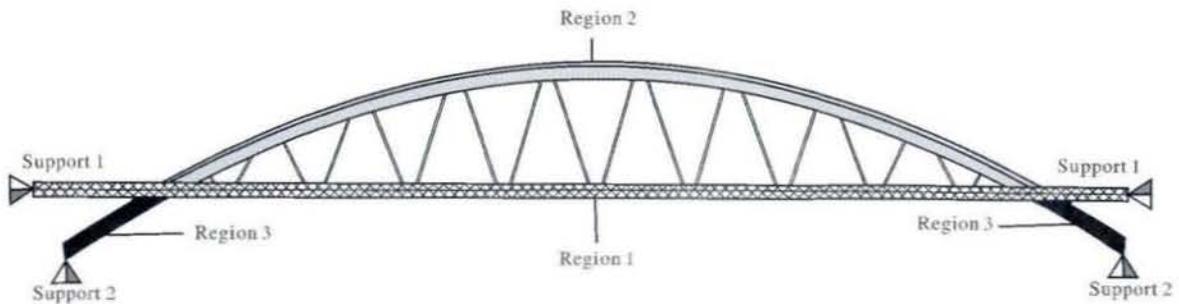


Fig. 10 Parameters selected for sensitivity analysis

Tab. 1 Notation, lower bounds, and upper bounds for input variables

No.	Parameter	Designation	Lower bound	Upper bound	Unit
1	E_{c1}	Concrete Young's modulus in region 1	2.0E+10	4.5E+10	N/m ²
2	ρ_{c1}	Concrete density in region 1	2.2E+3	2.7E+3	kg/m ³
3	E_{c2}	Concrete Young's modulus in region 2	2.0E+10	4.5E+10	N/m ²
4	ρ_{c2}	Concrete density in region 2	2.2E+3	2.7E+3	kg/m ³
5	E_{c3}	Concrete Young's modulus in region 3	2.0E+10	4.5E+10	N/m ²
6	ρ_{c3}	Concrete density in region 3	2.2E+3	2.7E+3	kg/m ³
7	E_s	Steel Young's modulus	2.0E+11	2.3E+11	N/m ²
8	ρ_s	Steel density	7.7E+3	8.0E+3	kg/m ³
9	K_{h1}	Horizontal stiffness of support 1	1.0E+8	1.0E+9	N/m
10	K_{v1}	Vertical stiffness of support 1	1.0E+8	1.0E+9	N/m
11	$K_{\theta 1}$	Rotate stiffness of support 1	1.0E+8	1.0E+9	N/m
12	K_{h2}	Horizontal stiffness of support 2	1.0E+8	1.0E+9	N/m
13	K_{v2}	Vertical stiffness of support 2	1.0E+8	1.0E+9	N/m
14	$K_{\theta 2}$	Rotate stiffness of support 2	1.0E+8	1.0E+9	N/m

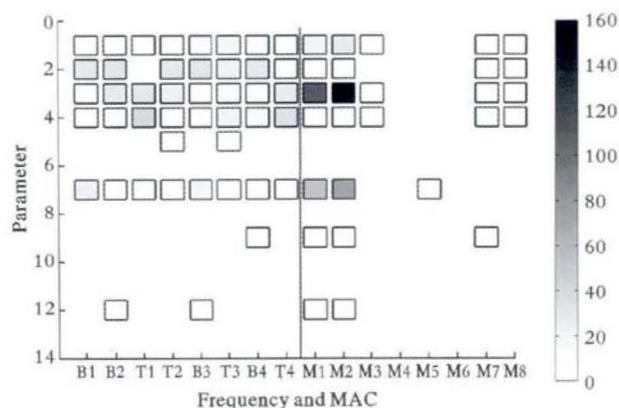


Fig. 11 Sensitivity coefficients matrix

6 Calibration of FE model

According to the sensitivity analysis, eight structural parameters are considered to be the dominant factors affecting the numerical modal data. The base values of eight parameters are assumed to be the values in the initial FE model, and the limits of each variable can be seen in Tab. 1. The calibration of FE model is conducted in this section based on Douglas-Reid method and Rosenbrock optimization algorithm described in section 2. In the present application, unit weighting constants are assumed in the objective function, i. e., Eq. (4).

The ratios of the optimal parameters relative to the limits indicated in Tab. 1 are represented in Fig. 12. A ratio of 0 means that the parameter coincides with

the lower limit. A ratio of 100% means that it coincides with the upper limit. As can be observed, all the optimal parameters are in between the lower and the upper limits.

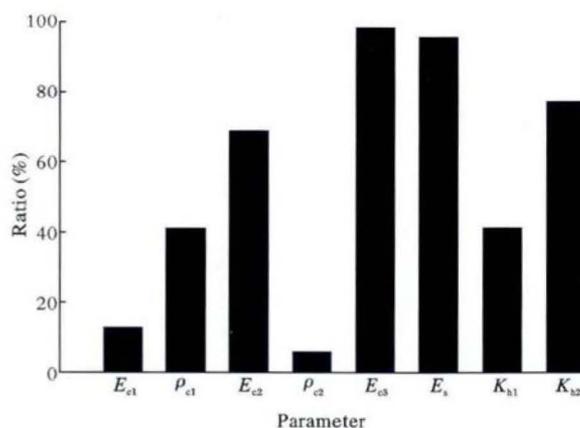


Fig. 12 Optimal estimates of structural parameters

It can be expected that the order of first two bending modes (B1 and B2) is significantly affected by the ratio of Young's modulus of the deck (E_{c1}) to that of the arch (E_{c2}). In this study, the first measured bending mode B1 is antisymmetric rather than symmetric (Fig. 9), so E_{c1} should be much less than E_{c2} which can be also seen in Fig. 12. From an engineering standpoint, the stiffness of arch ribs is seldom affected under service loads because of the compression dominant internal force, but the stiffness of the deck is significantly influ-

enced by the potential cracked section caused by reciprocal traffic loads during the life cycle of the bridge, explaining why a relative low Young's modulus is found in the deck.

Table 2 shows the error values between numerical and experimental data, taking as reference the values of the experimental data. It can be observed that the maximum relative error (RE) between natural frequencies, which is 8.16% for initial model before updating, becomes 3.50% after optimization. But, one should have in mind that the approximate formu-

la Eq. (1) is not exact for any given structural parameters and it cannot provide the information of mode shapes. Hence, natural frequencies and mode shapes of the updated FE model are computed again through eigenvalue analysis based on the optimal structural parameters. After calibration, the maximum relative error between natural frequencies becomes 4.48% and the minimum MAC value passes from 0.0169, before the calibration, to a value of 0.9930. Fig. 13 presents the mode shapes of the FE model after calibration.



Fig. 13 Mode shapes of updated FE model

Tab. 2 Correlation between modal behavior of updated model and experimental results

Experiment		Initial model			Optimal approximation			Updated model		
Mode	f^{exp} (Hz)	f^{num} (Hz)	RE (%)	MAC	f^* (Hz)	RE (%)	MAC	f^{num} (Hz)	RE (%)	MAC
B1	3.203	3.0839	3.72	0.0169	3.2030	0.00	–	3.1913	0.37	0.9066
B2	3.438	3.1576	8.16	0.0628	3.3175	3.50	–	3.2841	4.48	0.9544
T1	4.023	3.7205	7.52	0.9086	3.9134	2.73	–	3.9068	2.89	0.9576
T2	4.805	4.8621	1.19	0.9937	4.8050	0.00	–	4.9068	2.12	0.9916
B3	5.254	5.1646	1.70	0.9800	5.2088	0.86	–	5.1816	1.38	0.9880
T3	5.957	6.0264	1.17	0.9927	5.9572	0.00	–	5.9782	0.36	0.9930
B4	6.875	7.0241	2.17	0.9107	6.8750	0.00	–	6.8912	0.24	0.9247
T4	7.813	8.0965	3.63	0.9787	7.8238	0.14	–	7.9487	1.74	0.9765

It is noted that although the optimal structural parameters lead to a very good agreement between approximate and measured natural frequencies (RE = 3.50%), they may not be the variables which minimize the differences between numerical and experimental modal data. However, a satisfied match (RE = 4.48%) between them can be still presented when compared with the initial FE model. Therefore, FE model updating based on Douglas-Reid method and Rosenbrock optimization algorithm could be used as an alternative to other complex updating procedures.

7 Conclusions

This paper described the calibration of a FE model of a tied-arch bridge using Douglas-Reid method in combination with Rosenbrock optimization algorithm. The considered FE model updating procedure is first introduced. Then, based on the preliminary guess of unknown structural parameters, initial FE model of the studied bridge is created. After that, eight global modes of vibration of the bridge, four bending modes and four torsion modes, are clearly identified within the frequency range of 0-9 Hz. To perform the FE model updating, eight structural parameters are selected as updating variables through a sensitivity analysis of modal parameters.

The updating of the numerical model involves 8 numerical parameters and 16 modal responses. Based on the described updating procedure, the optimal structural parameters are identified. After optimization, natural frequencies estimated by the approximate formula have a very good agreement with experimentally measured values. The maximum difference changes from

8.16% for initial model before updating to 3.50% after optimization and for some modes (B1, T2, T3 and B4) the relative errors are even equal to zero. For the real updated FE model, the maximum relative error is found to be 4.48%, and the minimum MAC value passes from 0.0169, before calibration, to a value of 0.9930 after calibration. With regard to the average MAC value, it changes from 0.7305, before calibration, to 0.9615 after calibration.

Although the optimal parameters obtained based on Douglas-Reid method and Rosenbrock optimization algorithm are not the variables which minimize the differences between numerical and experimental modal data, a satisfied match between them can be still presented and the updated FE model can be used to evaluate the structural safety of the bridge under dynamic loads.

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