

# Project valuation and investment decisions: CAPM versus arbitrage

**Abstract.** This paper shows that (i) project valuation via CAPM contradicts valuation via arbitrage pricing, (ii) CAPM-minded decision makers may fail to profit from arbitrage opportunities, (iii) CAPM-based valuation violates value additivity. As a consequence, the use of CAPM for project valuation and decision making should be reconsidered.

**Keywords and phrases.** Investment, valuation, CAPM, arbitrage.

The Capital Asset Pricing Model (CAPM) is a bedrock for project valuation and is widely used for investment decisions (see Rubinstein, 1973; Copeland and Weston, 1988; Damodaran, 1999; Ross, Westerfield and Jaffe, 1999; Brealey and Myers, 2000, Fernández, 2002). Arbitrage choice theory as well is a fundamental tool for valuing risky projects (see Nau and McCardle, 1991; Smith and Nau, 1995). The principle of arbitrage is a cornerstone in financial economics (Modigliani and Miller, 1958; Black and Scholes, 1973; Varian, 1987), and is equivalent to the notion of “Pareto optimality” (Nau, 2004) and to noncooperative game theory (Nau and McCardle, 1990). Recently, it has been shown that this principle is the fundamental principle of economic rationality, unifying theories of subjective probability, expected utility, and subjective expected utility, as well as competitive equilibrium (Nau and McCardle, 1991; Nau, 1999).

This paper provides some simple but hopefully enlightening examples showing that if CAPM is used for project valuation and decision making the principle of arbitrage is violated, as well as the property of value additivity. The analysis is confined to one period and it is supposed that a security market exists, described in Table 2, where three securities are traded, numbered 1, 2, 3, the latter being a risk-free asset. The market is complete (the asset span equals the whole space  $\mathbb{R}^3$ ) and is assumed to be in equilibrium so that all assets lie on the Security Market Line (SML).<sup>1</sup> Three states of nature may occur and cash flows vary across these states according to the probabilities 0.5, 0.1, and 0.4 respectively. All numbers are rounded off to the second (or third) decimal. Table 1 collects the notations employed throughout the paper (the term ‘asset’ therein includes both projects and securities).

The examples just rely on standard relations among variables. As for CAPM, the value of any asset  $l$  is given by

$$V_0^l = \frac{C_1^l}{1 + r_f + \beta_l(r_m - r_f)} \quad (1)$$

and the beta is given by

$$\beta_l = \frac{\text{cov}(\tilde{r}_l, \tilde{r}_m)}{\sigma_m^2}. \quad (2)$$

where

$$\tilde{r}_l = \frac{\tilde{C}_1^l}{C_0^l} - 1. \quad (3)$$

As for arbitrage pricing technique, let  $t$  be a security lying on the SML such that  $\tilde{C}_1^l = \theta \tilde{C}_1^t$  for some nonzero  $\theta$  ( $t$  is then a twin security). We have that the value of  $l$  is the price it would have

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<sup>1</sup>If a security did not lie on the SML, then its value would differ from its price.

if it were traded:

$$v_0^l = \theta v_0^t = \frac{\theta C_1^t}{1+r_t} = \frac{C_1^l}{1+r_t} = \frac{C_1^l}{1+r_f + \beta_t(r_m - r_f)}. \quad (4)$$

Assume a decision maker faces project  $A$  whose cost is 738.48 and whose cash flows are 1200, 1000, 800 in the three states of nature respectively. Simple calculations show that the beta of  $A$  is the same as the beta of security 1 ( $\beta_A = \beta_1 = 1.094$ ). This reflects in a cost of capital  $i_A = 0.0433 + 1.094(0.1547 - 0.0433) = 0.1652$ , which implies

$$V_0^A = \frac{0.5(1200) + 0.1(1000) + 0.4(800)}{1 + 0.1652} = 875.33.$$

But note that project  $A$ 's payoff may be replicated by purchasing two shares of security 2 ( $\tilde{C}_1^A = 2\tilde{C}_1^2$ ). Arbitrage pricing then implies that project  $A$ 's value is

$$v_0^A = 2v_0^2 = 2 \frac{0.5(600) + 0.1(500) + 0.4(400)}{1 + 0.1443} = 2(445.66) = 891.32.$$

We have then  $V_0^A \neq v_0^A$ . This fact is striking, since we have two different valuations for project  $A$  depending on whether we use arbitrage theory or CAPM. This simple counterexample allows us to claim that CAPM-based valuations are not consistent with arbitrage-based valuations.

Formally, this difference derives from the following fact: if a project's payoffs are proportional to the payoffs of a security traded in the security market, then project and security have different beta (provided that the project does not lie on the SML). Equivalently, if project and security have equal beta, then their payoffs are not proportional (i.e. the security at hand is not a twin security of the project).

To prove the above claim, let  $A$  be a project and let  $t$  be a security such that  $t$  lies on the SML and replicates  $A$ 's cash flows  $\tilde{C}_1^A$  in every state of nature ( $\tilde{C}_1^A = \theta \tilde{C}_1^t$  for some nonzero  $\theta$ ), and assume  $A$  does not lie on the SML, i.e.  $C_0^A \neq V_0^A$ . If we had  $\beta_A = \beta_t$  we would have  $\text{cov}\left(\frac{\tilde{C}_1^A}{C_0^A}, \tilde{r}_m\right) = \text{cov}\left(\frac{\tilde{C}_1^t}{C_0^t}, \tilde{r}_m\right)$  which implies  $\text{cov}\left(\frac{\theta \tilde{C}_1^t}{C_0^A}, \tilde{r}_m\right) = \text{cov}\left(\frac{\tilde{C}_1^t}{C_0^t}, \tilde{r}_m\right)$  which in turn entails  $\frac{\theta}{C_0^A} \text{cov}(\tilde{C}_1^t, \tilde{r}_m) = \frac{1}{C_0^t} \text{cov}(\tilde{C}_1^t, \tilde{r}_m)$  whence

$$C_0^A = \theta C_0^t = \theta V_0^t \quad (5)$$

(the last equality holds since security  $t$  lies on the SML). On the other hand,  $\beta_A = \beta_t$  would also imply  $V_0^A = \frac{C_1^A}{1+r_f + \beta_A(r_m - r_f)} = \frac{C_1^A}{1+r_f + \beta_t(r_m - r_f)}$ . As  $t$  lies on the SML, this would in

turn mean  $V_0^A = \frac{C_1^A}{1+r_t}$ . Hence,  $V_0^A = \frac{\theta C_1^t}{1+r_t} = \theta C_0^t = \theta V_0^t$  so that, using (5),  $C_0^A = V_0^A$ . But this would contradict the assumption  $C_0^A \neq V_0^A$ .

A project's value in the CAPM depends on the beta of the project (see eq. (1)), whereas a project's value in arbitrage pricing depends on the beta of the twin security (see eq. (4)). As just shown, a project and its twin security have different beta, therefore values in the two paradigms are different.

This contrast does not only make valuation different, but may lead to behavioral anomalies. The following example shows that decision makers may fail to take advantage of arbitrage opportunities if they comply with the CAPM paradigm.

Assume a CAPM-minded decision maker comes across an investment opportunity, say  $D$ , consisting of two different projects (to be both selected or both rejected): Project  $B$  costs 926 and generates, at time 1, the certain sum 935; project  $C$  costs 64 and generates a random payoff equal to 466, 338.6, and  $-73$  in the three respective states of nature. Given the security market of Table 2 and looking at eqs. (2) and (3), the betas are easily computed:  $\beta_B=0$  (the project is risk-free) and  $\beta_C=17.21$ , and the costs of capital are then  $i_B = r_f = 4.33\%$  and  $i_C = r_f + \beta_C(r_m - r_f) = 196.08\%$  respectively. The NPV of alternative  $D$  for a CAPM-minded decision maker is

$$\left( -926 + \frac{935}{1 + 0.0433} \right) + \left( -64 + \frac{0.5(466) + 0.1(338.6) + 0.4(-73)}{1 + 0.0433 + 17.21(0.1547 - 0.0433)} \right) = -13.46.$$

The CAPM-minded evaluator rejects investment  $D$ , because its NPV is negative. But this decision conflicts with the decision taken by an arbitrageur. The latter accepts to invest in  $D$  because it gives arbitrage opportunities. Indeed, security 1 replicates the investment's payoff: an arbitrageur would sell short 0.77 shares of securities 1 receiving  $1006.65=0.77(1307.34)$  and use the sum to buy  $D$  at a total cost of  $990=926+64$ , so gaining 16.65. At time 1, the arbitrageur will use the payoffs from  $D$  to close off the position on security 1 (i.e. final net cash flow is zero).

It is now worth reminding that Dybvig and Ingersoll (1982) show that if (i) the CAPM pricing relation holds for all securities in the market, (ii) the market is complete, (iii) the probability that  $\tilde{r}_m > r_m + \frac{\sigma_m^2}{r_m - r_f}$  is positive, then arbitrage opportunities arise. In the security market described in Table 2 condition (iii) is not satisfied, so arbitrage opportunities do not arise *within the security market*. However, we have shown that an arbitrage opportunity does arise for the investor facing investment  $D$  and that the CAPM-based valuation of  $D$  does not signal it (Dybvig

and Ingersoll's results do not refer to project selection, but to pricing of financial assets; in other terms they only refer to arbitrage opportunities arising *within a security market* where CAPM pricing holds).

Finally, it is easy to see that additivity is not preserved in a CAPM-based valuation. Referring again to investment  $D$ , our CAPM-minded investor may aggregate the two projects' payoffs and sum them to compute the NPV. This boils down to saying that he is (framing and) valuing  $D$  as a single project.<sup>2</sup> A simple calculation shows that the beta of  $D$  is  $\beta_D=1.11$ , and its NPV is then

$$-990 + \frac{0.5(1401) + 0.1(1273.6) + 0.4(862)}{1 + 0.0433 + 1.11(0.1547 - 0.0433)} = 14.88.$$

Additivity is then violated, since  $14.88=\text{NPV}(D)=\text{NPV}(B + C)\neq\text{NPV}(B)+\text{NPV}(C)=-13.46$ ; the same is obviously true for the values:  $V_0(D) = V_0(B + C)=976.53\neq 1004.88=V_0(B) + V_0(C)$ . In other terms, the CAPM-minded evaluator undergoes framing effects (see Magni, 2002, sec. 4). By contrast, it is evident that additivity is not violated in arbitrage-based valuation: Modigliani and Miller's (1958) Proposition 1 just shows that the value of an asset (in particular, a firm) does not change irrespective of whether one sees it as a unique asset or as a two-asset (equity-and-debt) portfolio.

To sum up the results, this paper uses simple numerical (counter)examples to show some anomalies in the use of CAPM for valuation and decision making. In particular:

- value in the CAPM conflicts with value in arbitrage theory
- CAPM-minded decision makers may fail to take advantage of arbitrage opportunities
- value in the CAPM is not additive.

As an interesting byproduct, deviations of decision makers' behaviors from the CAPM prescriptions, massively recorded in the current literature (e.g. Brigham, 1975; Gitman and Mercurio, 1982; Summers, 1987; Graham and Harvey, 2001, 2002; Jagannathan and Meier, 2002; Brounen, de Jong and Koedijk, 2004) should be seen under a new light: they are just violations of a benchmark that contradicts the principle of arbitrage and infringes the property of value additivity.

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<sup>2</sup>The choice of how to frame the investment (single investment or two-project investment) depends on "the economic conditions giving rise to that particular net cash flow and on the psychological factors that influence the cognitive perception of the decision maker" (Magni, 2002, p. 211).

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**Table 1. Notations**

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$C_0^l$	Cost/price of project/security $l$ (i.e. outlay for undertaking/buying $l$ )
$\tilde{C}_1^l$	Payoff released by asset $l$ at time 1
$C_1^l$	Expected payoff released by asset $l$ at time 1
$\tilde{r}_l$	Rate of return of asset $l$
$r_l$	Expected rate of return of asset $l$
$\tilde{r}_m$	Market rate of return
$r_m$	Expected market rate of return
$\sigma_m^2$	Variance of market rate of return
$r_f$	Risk-free rate in the security market
$\beta_l$	Beta of asset $l$
$V_0^l$	Value of asset $l$ obtained from CAPM
$v_0^l$	Value of asset $l$ obtained from arbitrage theory
$i_l$	Cost of capital of asset $l$ from CAPM
cov	covariance

$l=1, 2, 3, A, B, C, D$

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**Table 2. The security market**

	Security			Market
	1	2	3	
Outstanding shares	10,000	21,000	3,000	34,000
Price	1307.34	445.66	119.80	
Cash Flow	1820	600	125	31,175,000
	1654	500	125	27,415,000
	1120	400	125	19,975,000
Rate of return (%) (see eq. (3))	39.21	34.63	4.33	36.78
	26.51	12.19	4.33	20.28
	-14.33	-10.24	4.33	-12.35
Exp. rate of return (%)	16.52	14.43	4.33	15.47
Beta (see eq. (2))	1.094	0.906	0.00	1.00
Value (see eq. (1))	1307.34	445.66	119.80	22,791,792