Towards the Integration of Power-Indexed Formulations in Multi-Architecture Connected Facility Location Problems for the Optimal Design of Hybrid Fiber-Wireless Access Networks*

Fabio D'Andreagiovanni[1], Fabian Mett[2], and Jonad Pulaj[3]

1 Department of Mathematical Optimization, Zuse Institute Berlin (ZIB), Takustraße 7, 14195 Berlin, Germany; and
DFG Research Center MATHEON and Einstein Center for Mathematics, Straße des 17 Juni, 10623 Berlin, Germany; and
Institute for System Analysis and Computer Science, National Research Council of Italy (IASI-CNR), via dei Taurini 19, 00185 Roma, Italy
d.andreagiovanni, mett, pulaj@zib.de

2 Department of Mathematical Optimization, Zuse Institute Berlin (ZIB), Takustraße 7, 14195 Berlin, Germany
mett@zib.de

3 Department of Mathematical Optimization, Zuse Institute Berlin (ZIB), Takustraße 7, 14195 Berlin, Germany; and
DFG Research Center MATHEON and Einstein Center for Mathematics, Straße des 17 Juni, 10623 Berlin, Germany
pulaj@zib.de

Abstract

Urban access networks are the external part of worldwide networks that make telecommunication services accessible to end users and represent a critical part of the infrastructures of modern cities. An important recent trend in urban access networks is the integration of fiber and wireless networks, leading to so-called fiber-wireless (Fi-Wi) networks. Fi-Wi networks get the best of both technologies, namely the high capacity offered by optical fiber networks and the mobility and ubiquity offered by wireless networks. The optimal design of fiber and wireless networks has been separately extensively studied. However, there is still a lack of mathematical models and algorithms for the integrated design problem. In this work, we propose a new Power-Indexed optimization model for the 3-architecture Connected Facility Location Problem arising in the design of urban telecommunication access networks. The new model includes additional power-indexed variables and constraints to represent the signal-to-interference formulas expressing wireless signal coverage. To solve the problem, which can prove very hard even for a state-of-the art optimization solver, we propose a new heuristic that combines a probabilistic variable fixing procedure, guided by (tight) linear relaxations, with an MIP heuristic, corresponding to an exact very large neighborhood search. Computational experiments on realistic instances show that our heuristic can find solutions of much higher quality than a state-of-the-art solver.

1998 ACM Subject Classification G.1.6 Optimization

* The work of Fabio D'Andreagiovanni and Jonad Pulaj was partially supported by the *Einstein Center for Mathematics Berlin* (ECMath) through Project MI4 (ROUAN) and by the *German Federal Ministry of Education and Research* (BMBF) through Project VINO (Grant 05M13ZAC) and Project *ROBUKOM* (Grant 05M10ZAA).

5th Student Conference on Operational Research (SCOR'16).
Editors: Bradley Hardy, Abroon Qazi, and Stefan Ravizza; Article No. 8; pp. 8:1–8:11
OASICS Open Access Series in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

Keywords and phrases Telecommunications Access Networks, Connected Facility Location, Mixed Integer Linear Programming, Power-Indexed Formulations, MIP Heuristics

Digital Object Identifier 10.4230/OASIcs.SCOR.2016.8

1 Introduction

The volume of data exchanged over telecommunications networks has enormously increased in the last two decades and telecommunication companies predict that such increase will relentlessly continue. This has originated the need for more technologically advanced and complex telecommunications networks. Within this context, *access networks*, namely the "external" part of a telecommunication network that connects users to their service providers, have experienced a deep technological evolution and have become a vital part of modern smart cities. Last generation access networks heavily rely on the use of optical fiber connections, which provide much higher capacity and better transmission rates than the traditional copper-based connections. Since the deployment of a pure optical fiber access network is nowadays considered impractical and uneconomical, in recent times, different types of hybrid optical fiber deployments have been proposed to provide broadband access. Taken as a whole, these several deployments, usually called *architectures*, are commonly referred to by the acronym *FTTX* (Fiber-To-The-X): here, the *X* specifies to which point of the network the optical fiber is brought. Major examples of architectures are: *Fiber-To-The-Home (FTTH)*, which brings a fiber directly to the final user; *Fiber-To-The-Cabinet (FFTC)* and *Fiber-To-The-Building* (FTTB), which bring a fiber to a street cabinet or to the building of the user, respectively (the fiber termination point is then connected to the user typically through a copper-based connection). We refer the reader to [11] for an exhaustive introduction to FTTX networks and their design. A recent and promising trend in FTTX has been represented by the integration of wired and wireless connections, leading to *3-architecture* networks that include also the so-called *Fiber-To-The-Air* (FTTA) architecture [10, 11]. Such 3-architecture represents an evolution of mixed-wired *2-architecture* networks like FTTH and FTTC/FTTB (see e.g., [14]). A 3-architecture network is aimed at getting the best of both wired and wireless worlds: the high capacity offered by optical fiber networks and the mobility and ubiquity offered by wireless networks [10]. Additionally, it grants a determinant cost advantage, since deploying wireless transmitters is cheaper and faster than deploying optical fibers, which requires costly and time-consuming excavations.

In this paper, we present a new optimization model based on Power-Indexed formulations for the design of 3-architecture access networks that integrate wired fiber/copper connections with wireless connections. With respect to state-of-the-art literature (we refer the reader to [11] and [14] for an overview), our model has the merit of including the formulas that are recommended by international telecommunications regulatory bodies to evaluate service coverage in wireless networks. Such formulas are the Signal-to-Interference Ratios (SIRs) [15], which evaluate the strength of the wireless signal providing service with respect to the total strength of the wireless interfering signals. The inclusion of SIRs is critical in wireless network design problem that consider wireless signal coverage: their exclusion may indeed lead to wrong design solutions (see [6, 7] for a discussion). This work represents also a refinement of the first study that we made in [4] and that we improve here by using a more-advanced power-indexed model for wireless network design [6]. In this work, our main original contributions are:

1. we propose a power-indexed formulation for optimally designing a 3-architecture access network, modelling the *signal-to-interference formulas* that express wireless signal coverage through discrete power emission decision variables;
2. we strengthen the basic power-indexed formulation of the problem by using a set of tight valid inequalities that model forbidden power configurations of the wireless transmitters;
3. since the problem can result difficult even for a state-of-the-art MIP solver, we propose to solve it by a heuristic based on the combination of a probabilistic procedure for fixing variables, guided by Linear Programming (LP) relaxations of the problem, with a Mixed Integer Programming (MIP) heuristic, which executes an *exact very large neighborhood search* (by the term "exact", we mean that the search is formulated as an MIP problem that is then solved exactly by an MIP solver);
4. we present computational results obtained for realistic network instances, showing that our new algorithm can return solutions of much higher quality than those provided by a state-of-the-art MIP solver.

2 A Power-Indexed model for 3-architecture access networks

In order to derive a power-indexed formulation for hybrid fiber-wireless network design, we first need to define a generalization of a *Connected Facility Location Problem* (ConFL) that includes three types of architectures. For a thorough introduction to concepts of graph and network flow theory and to the ConFL, we refer the reader to the book [1] and to the paper [12]. Given a set of users and a set of openable facilities that may serve the users, we can essentially describe the ConFL as the problem of deciding: (a) which facilities to open; (b) how to assign served users to open facilities; (c) how to connect open facilities through a Steiner tree; in order to minimize the total cost deriving from opening and connecting facilities and the assignment of facilities to users. The canonical ConFL considers a *single* network architecture and has been introduced and proven to be NP-Hard in [13].

A 3-architecture ConFL (3-ConFL) representing a network integrating fiber, copper and wireless technologies can be obtained by properly generalizing a 2-architecture version of the ConFL, which has been first introduced in [14]. The 3-ConFL associated with access network design involves a set of potential telecommunications facilities that can provide services to a set of potential users by installing one of the three available technologies. Each facility that is opened must be connected to a central office and each served user must be assigned to exactly one open facility. The objective of the design problem is to minimize the total cost of deployment of the network, while guaranteeing a minimum user coverage by each technology.

We denote the set of available technologies by $T = \{1, 2, 3\}$ and conventionally we assume that $t = 1$ is the optical fiber technology, $t = 2$ the copper technology and $t = 3$ the wireless technology. As first step to derive an optimization model, we introduce a directed graph $G(V, A)$ to model the network. In $G(V, A)$:

- the set of nodes V corresponds to the disjoint union of:
  1. a set of users U - each user $u \in U$ is associated with a weight $w_u \geq 0$ expressing its importance;
  2. a set of facilities F - each facility $f \in F$ can be opened at a cost $c_f^t \geq 0$ that depends upon the technology $t \in T$ that it installs;
  3. a set of central offices Γ - each office $\gamma \in \Gamma$ can be opened at a cost $c_\gamma \geq 0$;
  4. a set of Steiner nodes S.

  We call *core nodes* the subset of nodes $V^{\mathrm{C}} = F \cup \Gamma \cup S$ that does not include the user nodes. Additionally, we denote by F_u^t the subset of facilities using technology t that may serve user u and by U_f^t the subset of users that may be served by facility f when using technology t. We also denote by $F_u = \cup_{t \in T}$ the set of all the facilities that can serve u;

- the set of arcs A is the disjoint union of:
  1. a set of *core arcs* $A^{\mathrm{C}} = \{(i,j) : i,j \in V^{\mathrm{C}}\}$ that represent connections only between core nodes and are associated with a cost of realization $c_{ij} \geq 0$;
  2. a set of *assignment arcs* $A^{\mathrm{ASS}} = \{(f,u) \in A : u \in U, f \in F_u\}$ representing connection of facilities to users and associated with a cost of realization c_{fu}^t that depends upon the used technology.

We call *core graph* the subgraph $G^{\mathrm{C}}(V^{\mathrm{C}}, A^{\mathrm{C}})$ of $G(V,A)$ that represents the potential topology of the *core network*, namely the fiber-based network that interconnects the facilities and the central offices. In order to consider the cost of opening central offices in the optimization model, we adopt the modeling expedient of adding an artificial root node r to $G(V,A)$. We then introduce a set of (artificial) *root arcs* $A^{\mathrm{R}} = \{(r,\gamma) : \gamma \in \Gamma\}$ to represent the connection of the root node to every central office $\gamma \in \Gamma$. Each arc $(r,\gamma) \in A^{\mathrm{R}}$ has a cost $c_{r\gamma}$ set equal to the cost c_γ of opening the office γ. The set A^{R} is included in $G(V,A)$ and we use the notation $A^{\text{R-C}} = A^{\mathrm{R}} \cup A^{\mathrm{C}}$ to denote the union of the root and the core arcs.

One requirement in the design problem is to guarantee a minimum weighted coverage of users for each architecture. Specifically, with the total weight of users denoted by $W = \sum_{u \in U} w_u$, we express the coverage requirement for technology $t \in T$ by defining thresholds $W_t \in [0, W]$, $t \in T$. The total cost of a design solution of the access network is equal to the sum of the cost of opening central offices and facilities, the cost of connections activated in the core graph and the cost of connecting open facilities to served users.

Modeling wireless coverage. Until now, we have introduced all the elements that allow us to define an optimization model for 3-ConFL that does not include the formulas used to assess wireless coverage. In order to include such formulas, we must first briefly discuss basic concepts from wireless network design related to configuring wireless transmitters. For an introduction to the concepts of wireless network design, we refer the reader to [6, 15]. In our case, a wireless transmitter is a facility installing the technology $t = 3$. Each wireless transmitter is characterized by a number of radio-electrical parameters to set (e.g., the power emission, the tilt of the antenna and the frequency used to transmit). All these parameters could be in principle set in an optimal way, by solving an appropriate mathematical optimization problem, but in practice it is typical to optimize just a subset of them [5, 6, 9]. The vast majority of the models available in literature includes the setting of power emissions of the transmitters, since these are critical parameters that deeply affect the service coverage of the users. Such power emissions are commonly modelled by semi-continuous power variable. However, as shown in [6], it is better to consider a set of discrete power values both from a theoretical and an applied point of view: we can indeed derive effective (strong) valid inequalities and be in line with the practice of professionals, who commonly consider a (small) set of discrete power values for each transmitter. In order to model power emissions in a range $[P_{\min}, P_{\max}]$, we thus introduce a set of discrete power values $\mathcal{P} = \{P_1, \ldots, P_{|\mathcal{P}|}\}$, with $P_1 = P_{\min}$ and $P_{|\mathcal{P}|} = P_{\max}$ and $P_i > P_{i-1}$, for $i = 2, \ldots, |\mathcal{P}|$. Then, for each $f \in F$, we introduce one binary variable φ_{fl} (*power variable*) that is equal to 1 if f emits power P_l and 0 otherwise. The power emitted by a facility f can be thus denoted by $p_f = \sum_{l \in L} P_l \, \varphi_{fl}$, where $L = \{1, \ldots, |\mathcal{P}|\}$ is the set of power value indices or simply *power levels* (we must then add the constraint that the emission of f can be a single power value).

Every user $u \in U$ may pick up signals from each facility $f \in F$ installing a wireless transmitter and the power P_{fu} that u gets from f is proportional to the emitted power p_f

by a factor $a_{fu} \in [0,1]$, i.e. $P_{fu} = a_{fu}\, p_f$. The factor a_{fu} is a coefficient that summarizes the reduction in power experienced by a signal propagating from f to u [15]. A user $u \in U$ is said *covered* or *served* if it receives the wireless service signal within a minimum level of quality. The service is provided by one single wireless facility, chosen as *server* of the user, while all the other wireless facilities interfere with the server and reduce the quality of service. The minimum quality condition can be expressed through the *Signal-to-Interference Ratio (SIR)*, a measure comparing the power received from the server with the sum of the power received by the interfering transmitters [15]:

$$\frac{a_{fu}\ \left(\sum_{l \in L} P_l\ \varphi_{fl}\right)}{N + \sum_{k \in F \setminus \{f\}} a_{ku}\ \left(\sum_{l \in L} P_l\ \varphi_{kl}\right)}\ \geq\ \delta\ . \tag{1}$$

The user is served if the SIR is at least equal to a threshold $\delta > 0$ that expresses the minimum wanted quality of service. In the denominator, the coefficient $N > 0$ represents the noise of the system. The inequality (1) can be reorganized by simple operations in the so-called *SIR inequality*: $a_{fu}\ \left(\sum_{l \in L} P_l\ \varphi_{fl}\right) - \delta \sum_{k \in F \setminus \{f\}} a_{ku}\ \left(\sum_{l \in L} P_l\ \varphi_{kl}\right)\ \geq\ \delta\ N\ .$

Deciding which wireless facility $f \in F$ is the server of some user $u \in U$ is part of the decision process. As a consequence, we must activate or deactivate the SIR inequalities depending upon the wireless facility-user assignment. We thus face a disjunction of constraints, which we can model by modifying the SIR inequality. To this end, we must first define the set of *assignment arc variables* $y^t_{fu} \in \{0,1\}\ \forall (f,u) \in A^{\text{ASS}}\ \forall u \in U,\ f \in F^t_u,\ t \in T$: the generic variable y^t_{fu} is equal to 1 if facility f is connected to user u by technology t and is 0 otherwise. Using the assignment variable y^3_{fu}, representing the service connection of u through facility f by the wireless technology $t = 3$, and by defining a sufficiently large positive constant M (the so-called *big-M coefficient*), we define the modified SIR constraint:

$$a_{fu} \left(\sum_{l \in L} P_l\ \varphi_{fl} \right) - \delta \sum_{k \in F \setminus \{f\}} a_{ku} \left(\sum_{l \in L} P_l\ \varphi_{kl} \right) + M(1 - y^3_{fu}) \geq \delta N \tag{2}$$

It is straightforward to check that if $y^3_{fu} = 1$, then u is served by f through wireless technology and (2) reduces to a SIR inequality to satisfy. On the contrary, if $y^3_{fu} = 0$, then M activates, thus making (2) satisfied by any power configuration and therefore redundant.

Putting together all the elements that we have introduced, we can finally define a *Mixed Integer Linear Programming* (MILP) problem for modelling the 3-ConFL. To this end, we first introduce the following additional families of variables:

1. facility opening variables $z^t_f \in \{0,1\}\ \forall f \in F, t \in T$ (z^t_f is equal to 1 if facility f is open and uses technology t and is 0 otherwise);
2. arc installation variables $x_{ij} \in \{0,1\}\ \forall (i,j) \in A^{\text{R-C}}$ (x_{ij} is equal to 1 if the root or core arc (i,j) is installed and is 0 otherwise);
3. user variables $v^t_u \in \{0,1\}$, $\forall u \in U,\ t \in T$ (v^t_u is equal to 1 if user u is served by technology t and is 0 otherwise);
4. flow variables ϕ^f_{ij}, $\forall (i,j) \in A^{\text{R-C}}$, $f \in F$ that represent the amount of flow sent on a root or core arc (i,j) for facility f and are introduced to model the connectivity among facilities and central offices in the root-core network.

The MILP problem for 3-ConFL, that we denote as 3-ConFL-MILP, is described by (4)–(12).

In 3-ConFL-MILP, the objective function aims at minimizing the total cost, expressed as the sum of the cost of activating root and core arcs (note that the corresponding summation includes the cost of activated central offices, opened facilities and of activated assignment arcs). The constraints (4) impose that each facility is opened using a single technology, whereas constraints (5) impose that if a user u is served by technology t, exactly one of the assignment arcs coming from a facility that can serve u is activated on technology t. The constraints (6) link the opening of a facility f on technology t to the activation of assignment arcs involving f and t. The constraints (7) impose the coverage requirement for each technology (we remark that here the weighted sum of users getting a better technology like fiber contributes to satisfying the requirement for the coverage of worse technology like copper). The constraints (8) and (9) jointly model the fiber connectivity within the core network as a multicommodity flow problem that includes one commodity per facility. Specifically, the constraints (8) represent flow conservation in root and core nodes, while (9) are variable upper bound constraints that express the linking between the activation of a root or core arc and the activation of the arc. The constraints (10) impose to activate each wireless facility on at most one power level. Finally, (11) are power-indexed SIR constraints and (12) link the power emission variables to the opening of a wireless facility.

We can obtain a *tighter formulation* of 3-ConFL-MILP using a reformulation of the SIR constraints (11) and (10), which exploit the binary power variables φ_{fl} to derive a special family of power-indexed valid inequalities. These valid inequalities were introduced in [6], as a peculiar family of lifted Generalized Upper Bound (GUB) cover inequalities, and we refer the reader to that paper for an exhaustive description of them. In our case, for a given user u, a serving wireless facility f and a subset of interfering wireless facilities K, these inequalities identify joint power configurations of serving and interfering facilities that deny the service coverage of u and thus correspond with violated SIR constraints. Their form is:

$$y^3_{fu} + \sum_{l=1}^{\lambda} \varphi_{fl} + \sum_{k=1}^{|K|} \sum_{l=q_i}^{|L|} \varphi_{kl} \leq |K| + 1 \; , \tag{3}$$

with $u \in U, \;\; \lambda \in L, \;\; K \subseteq F \backslash \{f\}, \, (q_1, \ldots, q_{|K|}) \in L^I(t, \Delta, \lambda, \Gamma)$, with $L^I(u, f, \lambda, K) \subseteq L^{|K|}$ representing the subset of interfering power levels of facilities in K that deny the service coverage of u provided by wireless facility f, emitting with power level λ. Such inequalities can be separated and added at the root node to obtain a remarkable strengthening of the linear relaxation of the 3-ConFL-MILP. We denote by *Strong-3-ConFL-MILP*, the problem 3-ConFL-MILP strengthened by inequalities (3).

$$\min \sum_{(i,j) \in A^{\text{R-C}}} c_{ij}\, x_{ij} + \sum_{f \in F} \sum_{t \in T} c^t_f\, z^t_f + \sum_{u \in U} \sum_{t \in T} \sum_{f \in F^t_u} c^t_{fu}\, y^t_{fu} \tag{3-ConFL-MILP}$$

$$\sum_{t \in T} z^t_f \leq 1 \qquad f \in F \tag{4}$$

$$\sum_{f \in F^t_u} y^t_{fu} = \; v^t_u \qquad u \in U, t \in T \tag{5}$$

$$y^t_{fu} \leq z^t_f \qquad u \in U, f \in F, t \in T \tag{6}$$

$$\sum_{u \in U} \sum_{\tau=1}^{t} w_u \; v^\tau_u \geq W_t \qquad t \in T \tag{7}$$

$$\sum_{(j,i) \in A^{\text{R-C}}} \phi^f_{ji} - \sum_{(i,j) \in A^{\text{R-C}}} \phi^f_{ij} = \begin{cases} -\sum_{t \in T} z^t_f & \text{if } i = r \\ 0 & \text{if } i \neq r, f \\ +\sum_{t \in T} z^t_f & \text{if } i = f \end{cases} \qquad i \in V^{\text{C}} \cup \{r\}, f \in F \tag{8}$$

$$0 \leq \phi^f_{ij} \leq x_{ij} \qquad (i,j) \in A^{\text{R-C}}, f \in F \tag{9}$$

$$\sum_{l \in L} \varphi_{fl} \leq 1 \qquad f \in F \tag{10}$$

$$a_{fu}\left(\sum_{l\in L} P_l\ \varphi_{fl}\right) - \delta \sum_{k\in F\setminus\{f\}} a_{ku}\left(\sum_{l\in L} P_l\ \varphi_{kl}\right) +$$

$$+ M(1-y^3_{fu}) \geq \delta\ N \qquad f\in F, u\in U \qquad (11)$$

$$\varphi_{fl} \leq y^3_{fu} \qquad f\in F, u\in U, l\in L \qquad (12)$$

$$v^t_u,\ z^t_f,\ x_{ij},\ y^t_{fu},\ \varphi_{fl} \in \{0,1\} \qquad (i,j)\in A, u\in U, f\in F, t\in T, l\in L$$

3 A fast heuristic for solving the 3-ConFL-MILP

The 3-ConFL-MILP can in principle be solved by a state-of-the-art MIP solver, such as IBM ILOG CPLEX [2]. However, the introduction of the SIR constraints (11) make 3-ConFL-MILP a very challenging generalization of the ConFL: we experienced that in the case of realistic instances CPLEX has big difficulties in finding feasible solutions of good quality even after hours of computations. In order to tackle these computational difficulties, we propose to solve the problem by a heuristic that mixes a *probabilistic variable fixing procedure*, guided by the information retrieved by solving (tighter) linear relaxations of 3-ConFL-MILP, with an MIP heuristic based on the execution of an *exact very large neighborhood search*. Our heuristic, formalized in Algorithm 1, is based on considerations about the use of linear relaxations in a variable fixing procedure that have been first made in [3], the paper to which we refer for a more detailed discussion about the mechanisms and features of the heuristic concisely presented here. By solving (tight) linear relaxations, we are able to derive dual bounds for the problem that we can use to compute an *optimality gap* measuring how far the best solution returned from our heuristic is from the best lower bound given by Strong-3-ConFL-MILP. In order to explain how we construct a feasible solution for 3-ConFL-MILP, we first introduce the concept of *facility opening state*:

▶ **Definition 1.** Facility Opening State (FOS): an FOS specifies an opening of a subset of facilities $\bar{F} \subseteq F$ on some technologies such that no facility is open with more than one technology. Formally: $FOS \subseteq F \times T :\ \nexists (f_1,t_1),(f_2,t_2) \in FOS :\ f_1 = f_2 \wedge t_1 \neq t_2$.

Given a FOS and a facility-technology couple $(f,t) \in FOS$, we denote by W^{POT}_{ft} the total weight of users that can be potentially served by f using technology t, i.e. $W^{\mathrm{POT}}_{ft} = \sum_{u\in U^t_f} w_u$. Using this measure, we say that a FOS is *partial for technology* t when the total weight of potential users that can be served by facilities appearing in the FOS using technology t does not reach the minimum coverage requirements W_t for t, i.e. $\sum_{f\in F:(f,t)\in FOS} W^{\mathrm{POT}}_{ft} < W_t$. We also say that a FOS is *complete for technology* t when the total weight is not lower than W_t. Additionally, we call *fully complete* a FOS that is complete for all technologies $t \in T$. We use the completeness concepts to guide the probabilistic fixing of facility opening variables during the construction phase of feasible solutions.

Given a *partial* FOS for technology t, the probability p^{FOS}_{ft} of operating an additional fixing $(f,t) \notin FOS$, thus making a further step towards reaching a complete FOS, is set according to the formula:

$$p^{\mathrm{FOS}}_{ft} = \frac{\alpha\ \tau_{ft} + (1-\alpha)\ \eta_{ft}}{\sum_{(k,t)\notin FOS} \alpha\ \tau_{kt} + (1-\alpha)\ \eta_{kt}}\ , \qquad (13)$$

which convexly combines through factor $\alpha \in [0,1]$ two measures: τ_{ft}, measuring a-priori the attractiveness of operating a variable fixing, and η_{ft}, measuring a-posteriori the attractiveness of operating a fixing (see [4] for more details). In our case, we set τ_{ft} equal to the optimal value of the linear relaxation Strong-3-ConFL-MILP including the additional fixing $z_f^t = 1$, whereas η_{ft} is equal to the optimal value of the linear relaxation of 3-ConFL-MILP, obtained for a partial fixing of the facility opening variables z.

At the end of a solution construction phase, which is aimed at constructing Σ feasible solutions, the a-priori fixing measures τ are updated, evaluating how good were the fixings made in built solutions. The formula that we use for updates is based on the concept of *optimality gap* (*Gap*) (for a feasible solution of value v and a lower bound L that is available on the optimal value v^* of the problem, we set $Gap(v, L) = (v - L)/v$) and is:

$$\tau_{ft}(h) = \tau_{ft}(h-1) + \sum_{\sigma=1}^{\Sigma} \Delta\tau_{ft}^{\sigma} \text{ with } \Delta\tau_{ft}^{\sigma} = \tau_{ft}(0) \cdot \left(\frac{Gap(\bar{v}, L) - Gap(v_\sigma, L)}{Gap(\bar{v}, L)} \right) \tag{14}$$

where $\tau_{ft}(h)$ is the a-priori attractiveness of fixing (f, t) at fixing iteration h, v_σ is the value of the σ-th feasible solution built in the last construction phase and $\bar{v}$ is the (moving) average of the values of the Σ solutions produced in the previous construction phase. $\Delta\tau_{ft}^{\sigma}$ is a penalization/reward factor for a fixing and depends upon the initialization value $\tau_{ft}(0)$ of τ, combined with the relative variation in the optimality gap that v_σ implies with respect to $\bar{v}$.

Once a *fully complete FOS* is built, we have characterized an opening of facilities that can *potentially* satisfy the requirements on the weighted coverage for each technology. We say "potentially" because the activation of facilities specified by the FOS may not have a feasible completion in terms of connectivity variables and assignment of users of facilities: it is indeed possible that not all the SIR constraints (11) activated by the probabilistic fixing procedure can be satisfied together because of interference phenomena. As a consequence, a complete FOS may be infeasible. To tackle this risk of infeasibility, after the construction of a complete FOS, we execute a *check-and-repair phase*, in which the feasibility of the FOS is checked and, if not verified, we make an attempt to repair and make it feasible. The reparation attempt is based on the same MIP heuristic that we introduce below, with the name MOD-RINS, and that we adopt at the end of the construction phase to possibly improve a feasible solution.

Given a FOS that is complete for all technologies, we check its feasibility and try to find a feasible solution for the complete problem 3-ConFL-MILP by defining a restricted version of 3-ConFL-MILP, where we set $z_f^t = 1$ if $(f, t) \in FOS$. We solve this restricted problem by the MIP solver running with a time limit: if this problem is recognized as infeasible by the solver, we execute the MIP heuristic for reparation. Otherwise, we run the solver to possibly find a solution that is better than the best incumbent solution.

To try to improve an incumbent feasible solution or to repair an infeasible partial fixing of the variables z induced by a complete FOS, we rely on an MIP heuristic that operates a very large neighborhood search *exactly*, by formulating the search as an MILP problem solved through an MIP solver. Specifically, as we did in [3], we rely on a modified version of the RINS Heuristic [8], denoted by MOD-RINS, where the neighborhood is defined combining information from the linear relaxation of 3-ConFL-MILP with that of the current incumbent solution. Due to lack of space, we refer the reader to [3] for a description of the modified RINS algorithm that we have adopted.

The complete algorithm that we used for solving the 3-ConFL-MILP is shown in Algorithm 1. It is based on two nested loops: the outer loop runs until a global time limit is reached; the inner loop is aimed at building Σ feasible solutions, by first defining complete FOSs and then executing the modified heuristic to repair or complete the fixing associated with the FOS. We denote by X^* and X^B the best solutions found by the algorithm in the outer and in the inner loop, respectively. Each run of the inner loop provides for building a complete

Algorithm 1 Heuristic for 3-ConFL-MILP

solve the linear relaxation of Strong-3-ConFL-MILP for every single fixing $z_f^t = 1$ and initialize the values $\tau_{ft}(0)$ with the corresponding optimal values
while a global time limit is not reached **do**
  for $\sigma := 1$ to Σ **do**
    build a complete FOS
    solve 3-ConFL-MILP imposing the fixing $\bar{z}$ specified by the FOS
    if 3-ConFL-MILP with fixing $\bar{z}$ is infeasible **then**
      run MOD-RINS for repairing the fixing $\bar{z}$
    end if
    if a feasible solution $\bar{X}$ is found by the MIP solver and $c(\bar{X}) < c(X^B)$ **then**
      update the best solution found in the inner loop $X^B := \bar{X}$
    end if
  end for
  update τ according to (14)
  if $c(X^B) < c(X^*)$ **then**
    update the best solution found $X^* := X^B$
  end if
end while
run MOD-RINS for improving X^*
return X^*

FOS by considering, in order, fiber, copper and wireless technologies. The complete FOS is built according to the procedure using the probability measures (13) and update formulas (14). The complete FOS provides a (partial) fixing of the facility opening variables $\bar{z}$ and the MIP solver uses it as a basis for finding a complete feasible solution X^* to the problem. If $\bar{z}$ is recognized as an infeasible fixing by the MIP solver, then we run MOD-RINS to try to find a repaired solution. Otherwise, if $\bar{z}$ is feasible and gives rise to a feasible solution that is better than X^B in the current execution of the inner loop, then X^B is updated and the inner loop is iterated. At the end of each run of the inner loop, the a-priori measures τ are updated according to (14) and the best solution X^* is updated, if necessary. When the global time limit is reached, MOD-RINS is applied to X^* in an attempt to improve it.

4 Preliminary computational results

The algorithm was tested on 15 realistic networks instances, which refer to a urban district of the Italian city of Rome that has been discretized into a raster of about 450 elementary small-sized areas. The code was written in C/C++ using IBM ILOG CPLEX Concert Technology. The experiments were performed on a 2.70 GHz Windows machine with 8 GB of RAM and using CPLEX 12.5 as MIP solver. The experiments were run with a time limit of 3600 seconds. The instances consider different traffic generation and user location scenarios and in the considered district 5 central offices and 30 facilities are supposed to be available for deployment. Concerning the setting of the parameters of the heuristic, on the basis of past experience and preliminary tests, we imposed: $\alpha = 0.5$, $\Sigma = 5$, a time limit of 3000 seconds for the execution of the outer loop of Alg. 1 and a limit of 600 seconds for the final execution of the improvement heuristic MOD-RINS. The computational results are presented in Table 1: here, for each instance, we report its ID, the best percentage optimality gap *Gap-CPLEX%* reached by CPLEX within the time limit, the best percentage optimality gap *Gap-Heu%* reached by our heuristic within the time limit. In the case of the heuristic, we note that the gap is obtained combining the best feasible solution found by Algorithm 1 with the best known lower bound obtained by CPLEX using the strengthened formulation Strong-3-ConFL-MILP. The power-indexed version of 3-ConFL-MILP appears to

Table 1 Experimental results

ID	Gap-CPLEX%	Gap-Heu%	ΔGap%
I1	126.69	102.35	-19.21
I2	118.13	82.55	-30.11
I3	136.44	104.94	-23.08
I4	178.11	130.37	-26.80
I5	125.76	86.88	-30.91
I6	109.38	75.00	-31.43
I7	121.66	67.16	-44.79
I8	103.21	57.73	-44.06
I9	163.42	129.01	-21.00
I10	156.29	115.15	-26.32
I11	105.58	82.62	-21.74
I12	101.86	68.98	-32.27
I13	132.21	94.09	-28.83
I14	134.28	89.32	-33.48
I15	123.57	84.87	-31.31

be very difficult to solve for a state-of-the-art solver like CPLEX and the best optimality gap obtained for all instances is (well) over 100%. We believe that this is due to combining two distinct network design problems, wired and wireless, which are challenging already when taken separately. In contrast, our heuristic presents a very good performance, granting an average reduction of 29% in the optimality gaps (the best reduction reaches 44%). This is a very promising outcome and as future work we plan to better investigate the mechanisms of the heuristic and its integration within a branch-and-cut algorithm, which could also more effectively exploit the strength of power-indexed lifted GUB cover inequalities.

References

1. Ahuja, R. K., Magnanti, T., Orlin, J.: Network Flows: Theory, Algorithms, and Applications. Prentice Hall, Upper Saddle River (1993).
2. IBM ILOG CPLEX, `http://www-01.ibm.com/software/integration/optimization`.
3. D'Andreagiovanni, F., Krolikowski, J., Pulaj, J.: A fast hybrid primal heuristic for Multiband Robust Capacitated Network Design with Multiple Time Periods. Applied Soft Computing 26, pp. 497–507 (2015). 10.1016/j.asoc.2014.10.016.
4. D'Andreagiovanni, F., Mett, F., Pulaj, J.: An (MI)LP-based Primal Heuristic for 3-Architecture Connected Facility Location in Urban Access Network Design. Applications of Evolutionary Computation. LNCS 9597, pp. 283–298. Springer, Heidelberg (2016)
5. D'Andreagiovanni, F., Mannino, C., Sassano, A.: Negative Cycle Separation in Wireless Network Design. Network Optimization. LNCS 6701, 51–56. Springer, Heidelberg (2011).
6. D'Andreagiovanni, F., Mannino, C., Sassano, A.: GUB Covers and Power-Indexed Formulations for Wireless Network Design. Management Science 59 (1), pp. 142–156 (2013).
7. D'Andreagiovanni, F.: Revisiting wireless network jamming by SIR-based considerations and multiband robust optimization. Optimization Letters 9(8):1495–1510 (2015).
8. Danna, E., Rothberg, E., Le Pape, C.: Exploring relaxation induced neighborhoods to improve MIP solutions. Math. Program. 102, pp. 71–90 (2005).
9. Dely, P., D'Andreagiovanni, F., Kassler, A.: Fair optimization of mesh-connected WLAN hotspots. Wireless Communications and Mobile Computing 15 (5), pp. 924–946 (2015).

10 Ghazisaidi, N., Maier, M., Assi, C. M.: Fiber-wireless (FiWi) access networks: A survey. IEEE Comm. Mag. 47 (2), pp. 160–167 (2009).

11 Grötschel, M., Raack, C., Werner A.: Towards optimizing the deployment of optical access networks. EURO J. Comp. Opt. 2 (1):17–53 (2014).

12 Gollowitzer, S., Ljubic, I.: MIP models for connected facility location: A theoretical and computational study. Computers & OR 38 (2):435–449 (2011).

13 Gupta, A., Kleinberg, J., Kumar, A., Rastogi, R., Yener, B.: Provisioning a virtual private network: a network design problem for multicommodity flow. Proc. STOC'01, pp. 389–398, ACM, New York (2001)

14 Leitner, M., Ljubic, I., Sinnl, M., Werner, A.: On the Two-Architecture Connected Facility Location Problem. Electronic Notes in Discrete Math. 41, pp. 359–366 (2013).

15 Rappaport, T.S.: Wireless Communications: Principles and Practice, 2nd Edition. Prentice Hall, Upper Saddle River, USA (2001).