Solution of the Cumulative Assignment Problem with a Well-Structured Tabu Search Method

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Abstract

The *Cumulative Assignment Problem* is an NP-complete problem obtained by substituting the linear objective function of the classic Linear Assignment Problem, with a non-linear cumulative function. In this paper we present a first attempt to solve the Cumulative Assignment Problem with metaheuristic techniques. In particular we consider two standard techniques, namely the Simulated Annealing and the Multi-Start methods, and we describe the *eXploring Tabu Search*: a new structured Tabu Search algorithm which uses an iterative multi-level approach to improve the search. The new method is analyzed through extensive computational experiments and proves to be more effective than the standard methods.

Key Words: Metaheuristics, Linear Assignment, Cumulative Functions, Tabu Search

1. Introduction

Any problem in *combinatorial optimization* can be described by a pair (S, f) where S is a finite set of feasible solutions and $f : S \rightarrow \mathcal{R}$ is a given objective function. The goal is to find the solution $s^* \in S$ which minimizes (or maximizes) the objective function over S. It is well known that the computational complexity of a combinatorial problem can change when we maintain the solutions set S, but we change the objective function. One of the simplest examples of such behavior is given by the problem of finding a spanning tree of a graph. If f is linear, then the problem is equivalent to finding a base of a single matroid, so it is solvable in polynomial time with a greedy algorithm (see e.g. Lawler, 1976). However if we adopt different objective functions we often have NP-complete problems (see e.g. Camerimi, Galbiati and Maffioli, 1980; Dell'Amico, Labbé and Maffioli, 1996). When the problem is a two matroid intersection, i.e. S contains all common subsets of two matroids, we can still solve the problem in polynomial time. A classic example of two matroids intersection is the *Linear Assignment Problem* (AP) which consists of finding n elements of a given $n \times n$ cost matrix $C = [c_{ij}]$, such that no two elements belong to the same row or column, and the sum

of the elements chosen is a minimum. (Several polynomial time algorithms for solving AP have been given in the last thirty years, see e.g. Dell'Amico and Martello, 1997b.)

In this paper we consider a generalization of AP obtained by replacing its linear objective function with a *cumulative function*. The resulting problem, called *Cumulative Assignment Problem* (*CumAP*), is NP-complete and no specific technique has been proposed before now to solve it. To be more specific, let a be an n dimensional vector of *penalties*. *CumAP* asks for an assignment of each row to exactly one column and for an ordering of the elements of C involved, such that the scalar product of a times the ordered vector of the chosen elements of C is minimized.

An immediate application of *CumAP* is the following. We associate to each row i of matrix C an *operator* and to each column j a *job*. The value c_{ij} is the time spent by operator i to perform job j. Moreover, each penalty a_i $(i = 1, \ldots, n)$ is the cost of operator i for one time unit. Using these definitions, *CumAP* is to determine the assignment of each operator to a job in such a way that the cost paid to perform all the jobs is minimized. *CumAP* is also interesting since it is a relaxation of more general problems as the Delivery Man Problem (Fischetti, Laporte and Martello, 1993). Lastly *CumAP* is a special case of the Three-Dimensional Axial Assignment Problem (see, e.g. Balas and Saltzman, 1989).

In this paper we study the solution of *CumAP* by means of metaheuristic techniques. We consider a Multistart heuristic, a Simulated Annealing algorithm and a new well-structured Tabu Search approach that we have called the *eXploring Tabu Search* ($\mathcal{X}$-TS). The new method is described in detail and the effect of each of the strategies adopted is studied by means of extensive computational experiments. A preliminary version of $\mathcal{X}$-TS has been used with success in Dell'Amico and Maffioli, 1996; Dell'Amico and Trubian, 1997; Dell'Amico, Maffioli and Trubian, 1997.

In Section 2 we give a mathematical model of *CumAP*, we discuss its complexity status and overview results from the literature on related problems. In Section 3 we present lower bounds on the optimal solution value. Section 4 is devoted to introducing the neighborhood we have used for all the metaheuristic algorithms, and to presenting an efficient implementation of the exploration of the neighborhood. In Section 5 we introduce the metaheuristic techniques we adopted and, in particular, we describe the $\mathcal{X}$-TS approach. Extensive computational results are presented in Section 6, whilst the last section summarizes the work and suggests some interesting directions for future research.

2. Mathematical model and complexity

Let C be an integer square cost matrix with n rows and columns, and let a be an n dimensional integer vector of penalties. Without loss of generality we will assume that

$$a_1 \geq a_2 \geq \ldots \geq a_n. \tag{1}$$

CumAP asks for two permutations of the integers $1, 2, \ldots, n$, say ϕ and ξ, which minimize

$$z(\phi, \xi) = \sum_{k=1}^{n} a_k c_{\phi(k),\xi(\phi(k))}. \tag{2}$$

Permutation ξ defines the assignment of each row to a column, whereas permutation ϕ gives the ordering of the elements chosen.

Observe that if we are given the "assignment" permutation ξ, then the optimal ordering ϕ is obtained by associating to the smallest $c_{i,\xi(i)}$ element the largest penalty, then associating to the second smallest element the second largest penalty, and so on, i.e.

$$\phi(k) = \text{argkmin}\{c_{i,\xi(i)}, i = 1, \ldots, n\} \quad k = 1, \ldots, n, \tag{3}$$

where argkmin denotes the argument of the k-th smallest element.

If we define the boolean variables

$$x_{ijk} = \begin{cases} 1 & \text{if row } i \text{ is assigned to column j} \\ & \text{and } c_{ij} \text{ is the } k\text{-th element chosen} \\ 0 & \text{otherwise} \end{cases} \qquad i, j, k = 1, \ldots, n$$

then we can give the following linear programming model for $CumAP$:

$$(CumAP) \quad \min z = \sum_{k=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n} a_k c_{ij} x_{ijk} \tag{4}$$

$$\sum_{k=1}^{n}\sum_{j=1}^{n} x_{ijk} = 1 \qquad i = 1, \ldots, n, \tag{5}$$

$$\sum_{k=1}^{n}\sum_{i=1}^{n} x_{ijk} = 1 \qquad j = 1, \ldots, n, \tag{6}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ijk} = 1 \qquad k = 1, \ldots, n, \tag{7}$$

$$x_{ijk} \in \{0, 1\} \quad i, j, k = 1, \ldots, n. \tag{8}$$

Equations (5) and (6) are the classic "row" and "column" constraints of AP (for each fixed k value), whereas (7) impose that exactly one element of C is assigned to each ordering position k. Observe that (4)–(8) define a special case of the *Three-Dimensional Axial Assignment Problem* (3AP) which is given by the same constraints and by the more general objective function

$$\min z = \sum_{k=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ijk} x_{ijk}$$

3AP is well known to be NP-hard and many of its special cases remain NP-hard too. For example Garey and Johnson (1979) have shown that 3AP is NP-hard even if the costs d_{ijk} can only assume two distinct values. Another interesting special case is 3AP with *decomposable* costs (D3AP) in which $d_{ijk} = \alpha_i \beta_j \gamma_k$ where α, β and γ are n dimensional vectors of nonnegative numbers. Burkard, Rudolf and Woeginger (1996) have shown that also 3DAP is NP-hard.

It is immediately seen that $CumAP$ is more general than D3AP. Indeed, any instance of D3AP can be polinomially transformed into an equivalent instance of $CumAP$ by setting $a_k = \gamma_k$, for $k = 1, \ldots, n$ and $c_{ij} = \alpha_i \beta_j$ for $i, j = 1, \ldots, n$, thus proving $CumAP$ to be NP-hard.

3AP has been attacked with implicit enumeration methods (Pierskalla, 1968; Burkard and Rudolf 1993) and with cutting plane techniques (Balas and Saltzman, 1989,1991; Qi, Balas and Gwan, 1994). The most effective method is a cutting plane algorithm which solves instances with up to 28 rows and columns in about 2.000 seconds on a Sequent Hydra multiprocessor. It seems therefore to be quite unlikely that the general techniques developed for the exact solution of 3AP can solve large instances of $CumAP$, and hence heuristic techniques are needed to obtain good solutions within reasonable running times.

3. Lower bounds

In this section we describe lower bounding procedures from the literature and our adaptation to $CumAP$ of a lagrangean bound developed for 3AP.

The first two lower bounds for $CumAP$ have been introduced by Haas (1995) and consist of relaxations by constraints elimination.

The first lower bound LE1 considers two problems obtained from $CumAP$ by removing, respectively, constraints (5) and (7), and constraints (6) and (7). In the first case the elements of matrix C chosen are the minima of each column, whereas in the second case the elements are the minima of each row. The lower bound value is given by the maximum of the two solution values.

The second lower bound LE2 is obtained with a reformulation of the problem. We first define the "differences" of the penalties

$$d_k = a_k - a_{k+1}, \quad k = 1, \ldots, n$$

where $a_{n+1} = 0$. Then we define the n matrices $C^k = [c^k_{ij}]$ with $c^k_{ij} = d_k c_{ij}$, for $k = 1, \ldots, n$ and we ask for an assignment of exactly k elements from each matrix C^k, with the additional constraint that if an element (i, j) is chosen in matrix C^k it must be chosen also in the next matrices C^l with $l > k$. More formally:

$$(CumAP') \quad \min \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} c^k_{ij} x_{ijk} \tag{9}$$

$$\sum_{j=1}^{n} x_{ijk} \leq 1 \qquad i, k = 1, \ldots, n, \tag{10}$$

$$\sum_{i=1}^{n} x_{ijk} \leq 1 \qquad j, k = 1, \ldots, n, \tag{11}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijk} = k \qquad k = 1, \ldots, n, \tag{12}$$

$$x_{ijk} - x_{ijl} \leq 0 \qquad i, j = 1, \ldots, n, \ 1 \leq k < l \leq n \tag{13}$$

$$x_{ijk} \in \{0, 1\} \quad i, j, k = 1, \ldots, n. \tag{14}$$

Constraints (10) and (11) ensure that at most one element from each row and column is chosen from each matrix C^k. Equations (12) impose that exactly k elements be chosen from each matrix C^k, whereas (13) impose that the elements chosen in a matrix must also be chosen in all the following matrices.

Note that due to constraints (12) and (13) exactly one element is chosen for the first time from each matrix C^k, whereas the other $k-1$ elements have been chosen in the previous matrices. Moreover from the definition of the differences d_k, the element chosen for the first time in the k-th matrix "cumulates" the penalties $d_k, d_{k+1}, \ldots, d_n$. But $\sum_{h=k}^{n} d_h = a_k$, so choosing a new element from a matrix is equivalent to defining its ordering in the permutation ϕ (see (3)). Looking at this model, the meaning of the attribute "cumulative" given to our problem is clear.

Lower bound LE2 is obtained by eliminating constraints (13), thus the problem separates into n independent assignment problems with cardinalities $1, 2, \ldots, n$. Each AP with fixed cardinality k (with $k \leq n$) has been efficiently solved with the procedure of Dell'Amico and Martello (1997a).

Another effective lower bound can be obtained by adapting to $CumAP$ the lagrangean bound proposed by Balas and Saltzman (1991), for 3AP. This bound is obtained by embedding in a Lagrangean fashion constraints (7) in the objective function. Applying this technique to $CumAP$ we obtain the following:

$$\text{LCAP}(\lambda) = \min \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} a_k c_{ij} x_{ijk} + \sum_{k=1}^{n} \lambda_k (1 - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijk}) \tag{15}$$

subject to (5), (6) and (8). Observing that the above objective function can be rewritten as

$$\text{LCAP}(\lambda) = \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} (a_k c_{ij} - \lambda_k) x_{ijk} + \sum_{k=1}^{n} \lambda_k, \tag{16}$$

one can see that the optimal solution to LCAP(λ) can be obtained by solving a linear assignment on the reduced costs $\bar{c}_{ij} = \min_k \{a_k c_{ij} - \lambda_k\}$, for $i, j = 1, \ldots, n$.

The lagrangean dual LAG $= \max_\lambda$ LCAP(λ) is then solved with the modified subgradient technique introduced by Camerini, Fratta and Maffioli (1975).

Our computational experiments with these three lower bounds show that bounds LE1 and LE2 give better results than LAG only when few iterations of the subgradient optimization are allowed. Since our final aim is to use the better lower bound value only to evaluate the quality of the solutions obtained with the heuristic algorithms, we therefore gave LAG a large time limit (one hour of CPU time on a Sun Sparc Ultra 2 workstation) so that the subgradient optimization converged, in almost all cases, to its maximum. With this time limit LAG always turned out to be the winner versus LE1 and LE2. Therefore the comparisons of Section 6 are made with the lower bound value computed by LAG. (We do not report our computational experiments with the three lower bounding procedures, since we are only interested in the final lower bound value and not in studying the relative performances of the three methods, when the time limit changes.)

4. A Neighborhood

In this section we introduce and discuss one of the basic elements, common to the metaheuristic methods: the neighborhood.

A metaheuristic algorithm is a strategy based on a *local search* (LS). Any LS presupposes the definition of a *neighborhood function*, $\mathcal{N} : S \rightarrow 2^{|S|}$, i.e. a mapping of the solution space

which associates with each solution $s \in S$ a subset $\mathcal{N}(s) \subset S$. The LS method starts with a solution s, moves to an *adjacent* solution $s' \in \mathcal{N}(s)$, then defines $s = s'$ and iterates the process until a given stopping criterion holds. Roughly speaking, the various metaheuristics differ in the choice of the neighborhood and in the strategy used to select the next solutions.

We are interested in studying the effect of different strategies, when the same neighborhood is used. Therefore we need a 'good' neighborhood and an efficient procedure for its exploration, which can be used for all the metaheuristics we consider.

One of the problems in the design of an algorithm based on LS is the trade-off between the width of the neighborhood and the time used for its exploration. The larger a neighborhood, the more accurate the search, for a single iteration, but the longer the time spent in the exploration. Both the accuracy of the search and the total number of iterations help in finding good solutions, therefore one has to determine a suitable compromise between accuracy and speed. In order to have both the above advantages we adopted a wide neighborhood, but we carefully studied the implementation of its exploration, so that the resulting code is fast.

More precisely, let $\xi = (\xi(i), \ldots, \xi(n))$ be the permutation defining the current assignment. Since any permutation defines a unique solution of $CumAP$, let us call ξ a "solution". Given two integers i, j with $1 \leq i < j \leq n$ we can define a new permutation ξ' by *swapping* $\xi(i)$ with $\xi(j)$, i.e. setting $\xi'(i) = \xi(j)$, $\xi'(j) = \xi(i)$ and $\xi'(l) = \xi(l)$ for $l \neq i, l \neq j$. Our neighborhood $\mathcal{N}(\xi)$ consists of all the permutations generated with all the possible choices of pair i, j.

The value of the given solution ξ can be computed in $O(n \log n)$ time by reordering the elements $c_{l,\xi(l)}$ $(l = 1, \ldots, n)$ by nondecreasing weights and defining $\phi(l) = l$ for $l = 1, \ldots, n$. The next solution values can be computed in $O(n)$ time, since only two elements are changed, at each iteration. Each neighborhood contains $O(n^2)$ different solutions, thus exploring $\mathcal{N}$ with a standard implementation requires $O(n^3)$ computational time. But this is a very long time, for any effective heuristic algorithm, so we need to speed up the exploration with an improved implementation. In particular we describe how to reduce the time required to compute each solution value, from $O(n)$ to $O(\log n)$.

When a swap is performed the assignments $(i, \xi(i))$ and $(j, \xi(j))$ are removed from the current solution and the two new assignments $(i, \xi(j))$ and $(j, \xi(i))$ are added to the solution. This determines the substitution of two values from the set $A = \{c_{l,\xi(l)}, l = 1, \ldots, n\}$ of the assigned elements with two new values. Let us store the elements of A into an n-dimensional vector e such that

$$e_k \leq e_{k+1}, \quad k = 1, \ldots, n-1 \tag{17}$$

Then the value of the current solution is obtained with the scalar product ea (recall that the penalties vector a is ordered according to (1)). Let *out* be the smallest index of an element of e which contains the value $M^o = \max(c_{i,\xi(i)}, c_{j,\xi(j)})$ and let *in* be the smallest index of an element of e such that $e_{in} > M^i = \max(c_{i,\xi(j)}, c_{j,\xi(i)})$ (see Example 1 below). The indices *out* and *in* can be identified in $O(\log n)$ with a binary search. We now consider the variation of the solution obtained by removing the element with value M^o and adding the element of value M^i. We call e' the resulting vector, reordered by nondecreasing values. According to the relative positions of *in* and *out* several cases arise: we describe in detail only the case $in < out$; the other cases are similar and a complete description of them could be rather boring.

The new partial solution value can be efficiently computed by observing that: (a) $e'_k = e_k$ for $k = 1, \ldots, in-1$ and for $k = out+1, \ldots, n$; (b) the elements $e_{in}, \ldots, e_{out-1}$ are shifted one position right when M^i is inserted, i.e. $e'_{k+1} = e_k$ for $k = in, \ldots, out-1$; (c) $e'_{in} = M^i$. Therefore if z denotes the current solution value, then the new partial solution assumes value:

$$z^p = z - \sum_{k=in}^{out} e_k a_k + \sum_{k=in}^{out-1} e_k a_{k+1} + M^i a_{in} \tag{18}$$

Let us define the vectors:

$$\sigma_k = \sum_{l=1}^{k} e_l a_l \quad k = 1, \ldots, n$$

$$\widehat{\sigma}_k = \sum_{l=1}^{k} e_l a_{l+1} \quad k = 1, \ldots, n-1$$

then $\sum_{k=in}^{out} e_k a_k = (\sigma_{out} - \sigma_{in-1})$ and $\sum_{k=in}^{out-1} e_k a_{k+1} = (\widehat{\sigma}_{out-1} - \widehat{\sigma}_{in-1})$. Thus if we have already stored σ and $\widehat{\sigma}$ the computation of (18) can be done in constant time.

Example 1. Let us consider an instance with $n = 10$, penalties a =(25,22,19,13,10,8,6,4,1,0) and the vector of the values currently assigned being $e = (1, 5, 9, 10, 13, 15, 16, 17, 22, 26)$. The current solution value is $z = 872$. Assume that a swap removes the values 9 and 17, and adds the new values 11 and 14. Our implementation first identifies the indices *out* (= 8) and *in* (= 6) associated with the two values M^o (= 17) and M^i (= 14). Vector e' is obtained by removing value 17, by shifting e_6 and e_7 one position right, and by inserting the value 14 into e'_6.

	1	2	3	4	5	6	7	8	9	10
$e =$	1	5	9	10	13	15	16	17	22	26
						↘	↘			
$e' =$	1	5	9	10	13		15	16	22	26
						⇑				
						14				

Using vectors σ and $\widehat{\sigma}$ (18) becomes

$$\begin{aligned} z^p &= z - (\sigma_8 + \sigma_5) + (\widehat{\sigma}_7 - \widehat{\sigma}_5) + M^i a_{in} \\ &= 872 - (850 - 566) + (592 - 438) + 112 = 854 \quad \square \end{aligned}$$

Applying a procedure similar to the above, we can compute the complete value of the new solution ξ', by determining the change of z^p due to the other two elements involved in the swap: $m^o = \min(c_{i,\xi(i)}, c_{j,\xi(j)})$ and $m^i = \min(c_{i,\xi(j)}, c_{j,\xi(i)}))$. We need to consider the vector e' and to find the smallest index out' such that $e'_{out'} = m^o$ and the smallest index in' such that $e'_{in'} > m^i$. Using a straightforward implementation one could determine the two indices in $O(n)$ time, by defining and scanning vector e', but we can speed up the search by avoiding the explicit definition of e'. Let us consider first in': since $m^i \leq M^i$, then

$in' \in \{1, \ldots, in\}$. But in this interval e' and e coincide, so we can search the required value in e instead than in e'. The definition of out' is a little more complicated. If $m^o \leq M^i$, then we can search again the required value in $e_1, \ldots, e_{in-1}$, (which coincides with $e'_1, \ldots, e'_{in-1}$). Otherwise ($m^o > M^i$) we know that $m^o \in \{e_{in}, \ldots, e_{out-1}\}$, but these elements should be stored in e' one position righter than in e. Hence we can simply search m^o in e (instead than in e') and define out' as the index of the element we found, plus one.

Lastly, to compute the complete solution value we should define the final vector e'' obtained from e' by removing the element of value m^o, by inserting the element of value m^i and by shifting the elements between in' and out'. Once again we want to avoid the explicit definition of e'', so we need to know the exact positioning of the elements of e in e''. If $out' \leq in$ then the elements of e with indices between in' and $out' - 1$ are shifted one position to the right when e'' is defined. Hence the final value z can be computed efficiently using vectors σ and $\widehat{\sigma}$. If otherwise $out' > in$ then the elements of e with indices between positions in and out' should be shifted two positions right to obtain vector e''. Therefore to compute z efficiently we need to define a new vector $\overline{\sigma}$ in which a value e_l is associated with the penalty a_{l+2}, i.e.

$$\overline{\sigma}_k = \sum_{l=1}^{k} e_l a_{l+2} \quad k = 1, \ldots, n-2$$

Summarizing, the computational time required to explore a neighborhood is as follows. We need $O(n \log n)$ time to compute the value of the starting solution ξ and to define the vectors $\sigma, \widehat{\sigma}, \overline{\sigma}$ and other similar vectors needed for the cases we have not described explicitly. Then for each solution in $\mathcal{N}(\xi)$ we need $O(\log n)$ to identify the indices in and out and a constant time to compute the value z^p of the partial solution. Then we need again $O(\log n)$ to define in' and out' and a constant time to compute the value of the complete solution ξ'. We have thus proved the following theorem.

Theorem 1. *Neighborhood $\mathcal{N}$ can be explored in $O(n^2 \log n)$ time.*

A further reduction of the average computing time was obtained by applying several simple criteria which, in many cases, allow one to determine if a solution is not improving with respect to the current best solution of the neighborhood, without computing the exact solution value.

5. Metaheuristic algorithms

We considered three metaheuristic methods based on local search: *Multi-Start* algorithm, a *Simulated Annealing* algorithm and an implementation of the *Tabu Search* method, called *eXploring Tabu Search*. As already observed, we will use the neighborhood described in the previous section for all the algorithms we consider. According to the general description of an algorithm based on local search, given at the beginning of Section 4, it remains to present only the search strategy adopted for each method.

5.1. Multi-Start

A Multi-Start algorithm basically consists of two nested loops. At each iteration of the external loop we simply generate a random feasible solution, which is improved by the

operations performed in the internal loop. Given a current solution s at each iteration of the inner loop, the best solution $s' \in \mathcal{N}(s)$ is selected. If $z(s') < z(s)$ then we set $s = s'$ and we start a new iteration of the internal loop, otherwise a local optimum has been found and the inner loop terminates. The most common criteria used to stop the algorithm consist of giving a global time limit, or of fixing the number of iterations of the external loop. The algorithm returns the best solution identified during the search.

The procedure we used to generate the random feasible solutions is an implementation of a *greedy randomized* (GR) algorithm. We start by ordering the entries of the cost matrix C by non-decreasing values, then we build a feasible solution by performing n times the following operations. At each iteration we enlarge a current partial solution by adding a pair (i, j) such that both row i and column j are not assigned in the partial solution. More specifically, given a parameter $K > 0$, the pair to be added is randomly selected among the K pairs with smallest c_{ij} which can be feasibly added to the partial solution. If $K = 1$ this method is a pure (deterministic) greedy algorithm, otherwise it is a randomized method in which the effect of the randomization on the final solution is evident to the extent that the value of K increases. It is well known that the average value of the solutions obtained with a GR algorithm first decreases when K increases, but after a small threshold value it grows rapidly with K. Preliminary computational experiments were used to set the value of K to five.

5.2. Simulated annealing

The Simulated Annealing method (SA) we implemented starts with a random feasible solution s, generated with procedure GR, and iteratively applies the following steps:

1) randomly select a solution $s' \in \mathcal{N}(s)$
2) **if** $z(s') \leq z(s)$ **then** set $s = s'$
   else set $s = s'$ with probability $e^{(z(s)-z(s'))/temp}$

where *temp* is the current *temperature*.

The overall algorithm starts by setting the value of *temp* to an initial temperature T_{start}. Then it repeats the above two steps a number $NITER(n)$ of times, depending on the size of the problem, and decreases the temperature with the *geometric cooling schedule*: $temp = \alpha \cdot temp$ ($\alpha < 1$). When the temperature descends below a minimum value T_{end} it sets again $temp = T_{start}$ and continues as above until a time limit is reached. (See e.g. Aarts, Korst and Laarhoven, 1997 for a complete description of the SA method.)

In our experiments we defined $NITER(n) = 300n$ and set α to 0.95. The starting and ending values of the temperature were determined through a preprocessing phase which generates 100 random solutions and, for each of them, performs 100 random swaps. During this phase we compute the minimum and maximum difference in cost, say δ and Δ, respectively, between two adjacent solutions. The value T_{start} is set to $-\Delta/\ln(0.90)$ whilst the value T_{end} is set to $-\delta/\ln(0.01)$. This implies that if $temp = T_{start}$ (resp. $temp = T_{end}$), during step 2 a solution s' whose value is equal to $z(s) + \Delta$ (resp. $z(s) + \delta$) is accepted with probability

90% (resp. 1%).

5.3. The eXploring Tabu Search

In this section we describe a general structured method for implementing a Tabu Search algorithm (TS). In particular we introduce a combination of strategies from the Tabu Search framework (see e.g. Laguna and Glover, 1993 and Glover and Laguna, 1997) which leads to a new method that we have called the *eXploring Tabu Search* ($\mathcal{X}$-TS). The reason for the choice of this name will be explained later.

An ideal beginning level Tabu Search method starts with a feasible solution $s \in S$ and, at each iteration, it substitutes s with the best solution $s' \in \mathcal{N}(s)$ that has not been visited in a previous iteration. The new solution can have an objective function value smaller, equal to or larger than the current solution. In practice it is not possible to store all the information describing all the visited solutions, so we try to recognize a solution using only some *attributes*, i.e. partial information on the structure of the solution. These attributes are stored in a finite length list and a solution $\hat{s} \in \mathcal{N}(s)$ is said to be *tabu*, i.e. it is not considered as a possible candidate for the next iteration, if its attributes are in the list. The search terminates when a given time limit expires.

A common theme of tabu search is to join the beginning level TS with an intermediate term intensification strategy and a longer term diversification strategy, to create an iterated multi-level approach. In spite of this theme, many implementations are limited to the beginning level component of TS. Our $\mathcal{X}$-TS provides a specific and highly effective pattern of a more advanced multi-level design, which we demonstrate to be dramatically superior to the beginning level TS component by itself.

Before going into the details of the $\mathcal{X}$-TS strategies we must describe our implementation of the beginning level TS. Let ξ and ξ' be, respectively, the permutations defining the current solution and the solution selected in $\mathcal{N}(\xi)$. Moreover, let i and j be the row indices of the elements involved in the swap which transforms ξ into ξ'. When we have moved to ξ', the attributes we use to identify solution ξ are the two pairs $(i, \xi(i))$ and $(j, \xi(j))$. In the following iterations we consider tabu a solution which tries to assign again row i to column $\xi(i)$ or row j to column $\xi(j)$. The attributes are stored in a simple FIFO list of length ℓ: when a new solution is selected the two attributes of the new solution are added on the top of the list and the two oldest attributes are removed, if the list length exceeded ℓ. We also apply a simple *aspiration criterion* which removes the tabu status of a solution if the solution value is smaller than the current best solution value.

We are now ready to introduce the specific strategies we have adopted. We use a multi-level approach consisting of three intensification and/or diversification tools which operate on areas of the solution space growing with the level. The first level tool operates in the neighborhood of the current solution. The second level tool operates on a *local area* which is close to the trajectory in the solution space, followed during the search. The third level tool operates on the whole solution space. The first tool consists of a *tabu list management*, the second one is a *proximate good solutions management*, whilst the third is a *global restarting strategy*.

Tabu list management

This tool is an implementation of a strategy known as *dynamic updating of the tabu list length*. The main idea is to emphasize the intrinsic behavior of the TS method. Indeed TS is a so-called "hill climbing" method, i.e. it descends into a "valley" to find a local minimum, then it climbs one of the faces of the same valley trying to reach a different minimum, placed in another valley. The tabu tenure ℓ, i.e. the length of the tabu list (or the number of iterations a solution maintains its tabu status) is initialized to a given value *start_tenure* and is modified according to the evolution of the search. The aim is to intensify the search when we think we are close to a local minimum, and to accelerate the diversification when we are escaping from an already visited minimum.

The intensification is obtained by shortening the tabu tenure, i.e. allowing more solutions to be considered candidate for the next step. On the contrary the diversification is obtained by increasing the value ℓ, which avoids removing the attributes of the recent solutions from the tabu list. More specifically, let us call *improving phase* a set of Δimp consecutive iterations which lower the objective function value. If we detect an improving phase, then the search is certainly going toward a local minimum, so we reduce the tabu tenure. In particular we use the following updating which guarantees that the tabu tenure remains greater than a reasonable minimum:

$$\ell = \max(\ell - 1, \frac{1}{2} start\_tenure).$$

Now let us consider the case in which the current solution is a local minimum and we start to climb a face of the valley. If the tabu tenure does not change, and the climbing is long enough, then after ℓ iterations the attributes of the local minimum are forgotten and there is the possibility of the search returning toward the already visited minimum. To avoid this, it is important that before ℓ iterations are performed we increase the value ℓ so that we do not forget the attributes of the local minimum. An effective choice is to increase the tabu tenure before ℓ climbing iterations have been performed, so providing that also the attributes of few solutions visited just before descending into the minimum remain in the list. Let us call *worsening phase* a set of $\Delta wor < \ell$ consecutive iterations in which the objective function value does not improve. If a worsening phase is detected, then it is enough to increase the tabu tenure by one, in order to recall the attributes of the minimum and of the last $\ell - \Delta wor$ solutions visited before reaching the minimum. In practice, we adopt the following updating:

$$\ell = \min(\ell + 1, \frac{3}{2} start\_tenure),$$

which limits the tabu tenure, so that the tabu status does not become too binding.

Preliminary computational experiments were used to fix the value of *start_tenure* to 15 and the values of Δimp and Δwor to 3. The same parameters were used for all the remaining experiments, presented in Section 6.

Proximate good solutions management

This is an implementation of another tool from the TS framework which is often neglected: a long term memory which enables the algorithm to learn from its evolution.

The basic idea of this tool is to store some good solutions which have been analyzed, but not visited, during the evolution of the algorithm. These solutions are used, under certain conditions, to continue the search. When we have recourse to one of such solutions, then we jump from the current solution s to a new one which is not in $N(s)$, but is in a promising region close to the trajectory in the solution space followed by the algorithm up to the current iteration. Hence the method implements both an intensification of the search into regions analyzed, but not completely explored, and a diversification from the current solution.

We implemented this tool as follows. We used a fixed length list called *Second* to store l high quality solutions which were analyzed during the search, but whose value was only the second best value in their neighborhood. At each iteration, when we determine, as well as the best solution $s' \in N(s)$, also the second best solution $s'' \in N(s)$, then we add s'' to *Second*, either if we have already stored less than l solutions or if there exists a solution $\hat{s} \in$ *Second* such that $z(s'') < z(\hat{s})$. Note that owing to our implementation not all the solutions in a neighborhood are examined, so it may happen that we compute completely only one solution and none is added to *Second*.

Instead of using a solution of the current neighborhood, we resort to a solution from *Second*, when the behavior of the search indicates that a great effort would be necessary to find an improving solution, if we continue the search from the current neighborhood. We use three conditions to try to detect the above situation:

1. the tabu status prevents all the solutions in the current neighborhood from being used;
2. the current objective function value has not been improved in the last MC iterations;
3. in the last MB iterations there was no improving of the global best solution.

If one of the three above conditions holds, we remove from *Second* the solution s^* with the best objective function value and we continue the search from it. Note that we obtain a correct working of the algorithm only if we can restore the conditions present when s^* was added to *Second*. To do this we need to store in *Second*, within each solution, also a copy of the tabu list and of the other parameters driveing the search.

The meaning of the first condition is obvious, but some explanations are necessary for the other two conditions. The value MC involves using a solution from *Second* when the algorithm is climbing a very deep and high face, or when it is exploring a flat region. In both cases the last local optimum remains the best local solution for many iterations: using a solution from *Second*, we accelerate the search by jumping into a new region. This jump is a diversification from the current solution, but it is also an intensification of the search in the local area, since the solutions stored in *Second* are not too far from the current solution, (they were found along the path leading to the current solution).

From a set of few preliminary experiments we have seen that the value 15 is adequate for MC, for the instances we tested, but a slight growing with n is also useful. Hence we adopted the final formula $MC = \lceil 15 + 7\ln(n/100) \rceil$.

The value MB is used to detect situations in which the exploration of the current area seems not to be fruitful in determineing the global optimum. In this case we need to have a complete but fast exploration of the area. This is achieved with a further recourse to a solution from *Second*, so that we increase the intensification, but we also draw the search toward a restart from a completely new solution (see below the subsection on the global

restarting). Thus the parameter MB reduces the time needed to explore the local area, by inducing further intensification, but also reduces the time between two strong diversification points. For the above reasons the value of MB must not be too small. With preliminary computational experiments we chose to set $MB = 500$ for the smallest instances tested ($n = 50$) and $MB = 800$ for larger instances.

Global restarting strategy

The aim of the previous two tools is to optimize the ratio accuracy/speed in the exploration of a local area in the solution space.

When we are confident enough that no better solution can be found in the current local area, we must move to a new and not yet explored local area. To do this we generate a new starting solution and we re-initialize the search from this new point. This method determines a jump into a new area, so giving a strong diversification.

To apply this tool we obviously need a procedure which generates a different feasible solution at each run. Moreover, it would be most advisable if the procedure is able to define solutions which are "uniformly" distributed in the solution space. In general it is not too difficult to write a procedure satisfying the first requirement, but it is much more difficult to satisfy the second one. In this implementation we used procedure GR to generate the solutions. This is a randomized method which gives different solutions for different runs, with a sufficiently large probability. However this algorithm does not guarantee any uniformity in the distribution of the solutions. The study of greedy algorithms satisfying the two above requirements is a challenge for future research in the metaheuristic area.

The $\mathcal{X}$-TS method restarts the search when there is some evidence that the search in the current local area is no longer profitable. In particular, we adopted the three following inexact criteria to detect such situations: (a) the conditions adopted for the second level tool indicate that it is necessary to use a solution from *Second*, but the list is empty; (b) the value of the best solution found after I iterations from the last global restart is p percent larger than the value of the global best solution; (c) the second level tool has been used for SL consecutive times without improving the best solution from the last restart, or from the beginning. Criterion (a) is an obvious extension of the considerations which define the use of the second level tool; criterion (b) is useful to detect situations in which the last randomly generated solution (after a restart) belongs to a local area with no good solutions; criterion (c) is introduced to avoid fury in the search inside a single local area.

The sets of parameters used for criteria (b) and (c) are as follows. Parameter I is adaptively set to $0.1\lceil n/100 \rceil$ times the number of iterations performed from the starting of the algorithm to the first recourse to a global restart. The percentage value p is initially set to $0.05n$ for instances with 'small' values of the starting solution (less than 10^7), and to 0.1 for the other instances. The value of p is increased of one third of its initial value whenever criterion (b) is applied twice consecutively. The value of the last parameter SL is a little more sensitive to the instance. We set $SL = 5$ for all instances, except for that of class C with $n \leq 100$, (see below Section 6) where we set $SL = 10$.

Why the name ?

Most of the metaheuristic and evolutionary methods owe their name to the resemblance of their behavior with some natural phenomenon. This is also true for $\mathcal{X}$-TS, but the phenomenon it tries to imitate is the way of operating of an expert human, in a particular field. Suppose you are a famous explorer whose main activity is to go to some lost-land to look for ancient treasures. In particular, suppose you are interested in finding a treasure hidden in a very large and intricate jungle. You can use all modern equipment (like helicopters, radios, etc.), but the nature of the environment makes them almost ineffective. For example you can take the helicopter and fly over the jungle, but it is so thick and intricate that nothing can be seen beneath the trees. So you can only choose a point on the map and descend to the ground from the helicopter with a rope. Then you can look around this point and if you find some interesting trace or piece of evidence, you can follow this trace and repeat your observation from the new position. Suppose you continue with this method until you see no interesting trace around your last position. Now you take your radio and call the helicopter that picks you up and delivers you to a new random position. This is exactly the behavior of a *Multi-Start approach.*

This explorer uses the technology, but not his brain. Indeed, he makes no effort to learn from his previous explorations. A more skilled explorer, instead, uses short and long term memory to drive the search. In order to obtain the maximum information from his walk on the ground and to avoid walking round in circles, he tries to remember the places where he has already passed by storing in his memory some peculiar aspects of the various places visited. So he moves from one point to an adjacent one, even if the second is no better than the first. This is the behavior of a beginning level explorer. A more advanced explorer recognizes exceptional interesting situations: for example, a series of consecutive traces. In this case he moves to a new point even if some aspects of the place are similar to those of places already visited. This can be seen as an intensification of the search, into a region close to the current position, obtained by discarding some old tabu status. If, instead, he does not see interesting traces for a certain time interval, then he becomes more cautious and tries to recall all the previously visited places, so that he certainly moves toward new places. This is a local diversification technique. But any explorer with a long experience also knows that if he is in a given point of the jungle and, looking around, he sees more than one interesting trace, then the best local trace does not always lead to the best find. Therefore he tries to recall good traces that he has seen during his walk, but has not followed. When the search fails to give interesting results for a long time, then he returns to the path at the point in which he recalls the first good unexplored trace, and continues the search in this new direction. This is an intensification of the search in the local area, but it is also a diversification from the current situation. Lastly, if the walk does not give good results for a long time he calls the helicopter and moves to a new area. These three ways of approaching the search are the three-level tools of our $\mathcal{X}$-TS method.

6. Computational experiments

We have implemented and tested the lower bounds of Section 3 and the approximating algorithms of Section 5. More precisely we have coded in C language the Multi-Start algorithm

(MS), the Simulated Annealing (SA) and four different Tabu Search algorithms. The first algorithm, called TS, is the beginning level tabu search. The second algorithm, denoted with TS1, is an improved version of TS obtained using the first level tool, algorithm TS2 is obtained from TS1 by adding the second level tool, and $\mathcal{X}$-TS is the complete algorithm which uses all the three-level tools. With these four implementations we aim to study the effect of each strategy on the final performances of the tabu search method. The computational experiments were performed on a Sun Sparc Ultra 2 workstation running under Unix System V 4.0.

To test the algorithms, we generated and solved 320 random instances from four different classes. The instances of *Class A* are obtained by randomly generating the costs $[c_{ij}]$ and the penalties a_k, in the interval $[0, 100]$. The instances of *Classes B, C* and *D* are obtained by using, respectively, the intervals $[0, 1000]$, $[50, 100]$ and $[500, 1000]$. The number of rows and columns was set to $50, 100, 150$ and 200. For each pair $(n, Class)$ 20 random instances were generated and solved. We gave all the algorithms the same time limit: $4n$ seconds.

The columns of the tables corresponding to the heuristic algorithms give: (i) the average percentage error $\Delta\%$ between the solution value and the lower bound value (i.e. $\Delta\%$ = 100(upper bound value - LB) / LB); (ii) the number of times the procedure has found the best solution, among those generated by the heuristic algorithms (bst).

In Table 1 we report, for each value of n, the averages over the 80 instances generated from the four classes. The total number of best solutions found by the beginning level tabu search TS (see Figure 1) is more than double that of MS and SA. This value strictly increases

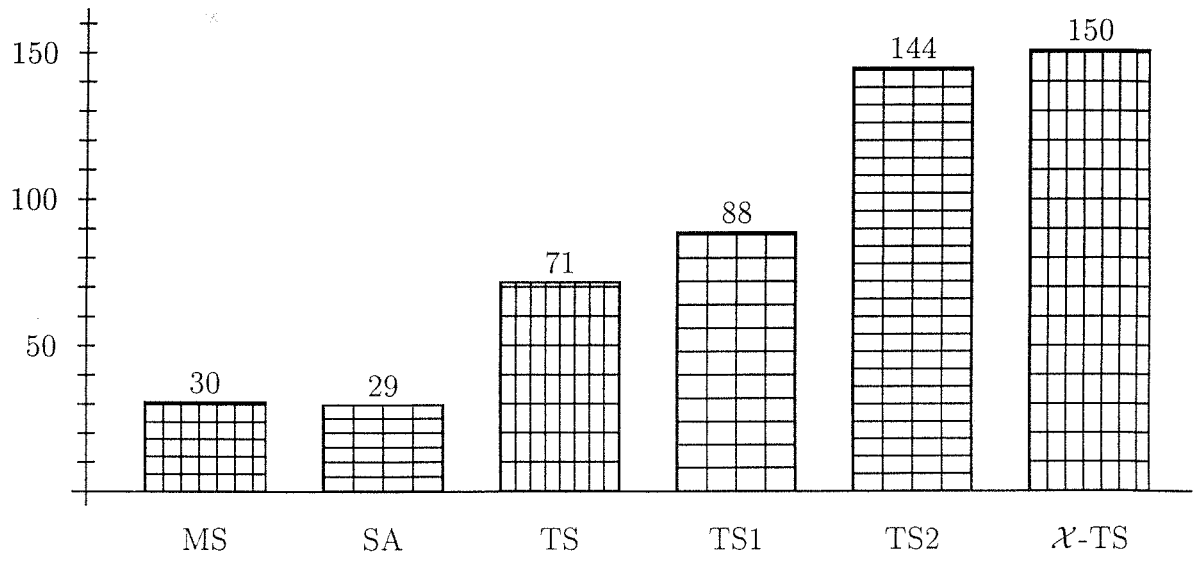

Figure 1: Grand total of the best solution found

when we add the first, the second and the third level tool. In particular, $\mathcal{X}$-TS obtains more than twice best solutions than TS. The average percentage error also decreases when we go from TS to $\mathcal{X}$-TS (see Figure 2). The error of SA, instead, is two times that of TS, while the error of MS is one order of magnitude larger. However, looking at the disaggregate data

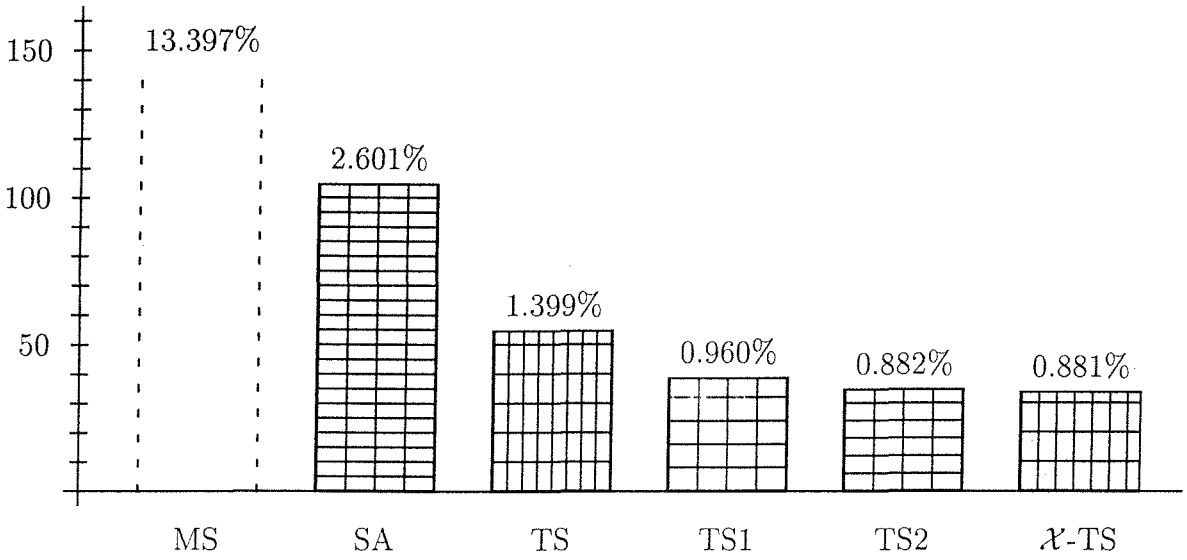

Figure 2: Average errors (grand total)

of Table 2 we can see that the performances of MS change significantly with the class of instances and are not always bad. For class A it has very poor performances (only one best solution and errors up to two orders of magnitude larger than that of the other algorithms), but for the large instances of class D it performs very well and is slightly better than $\mathcal{X}$-TS (for $n = 200$ the average error of MS is 0.287 versus the average error of $\mathcal{X}$-TSwhich is 0.290).

Lastly it is worth noting that the performances of the tabu search improve at each addition of a tool, but the most significant changes are due to the first two tools. The situation is quite different when we consider other problems. For example in the equicut problem (see Dell'Amico and Maffioli, 1996; Dell'Amico and Trubian, 1997) the most important tools are the first and the third (tabu list management and global restarting strategy), while in the SS/TDMA problem considered in Dell'Amico, Maffioli and Trubian, 1997, the importance of the second tool (proximate good solutions management) and of the global restarting is comparable.

From these studies it seems that the tabu list management is a "basic" tool which should be used extensively in all tabu search algorithms. Instead the rules to determine when we have to use the second and the third tool are the object of a parameter tuning. Up to now we have no theory which helps to find these rules, therefore preliminary computational experiments are the only method we can apply to define rules and parameters. In the next section we propose some questions and research direction which should be considered to improve the use of long term memory and restarting strategies.

7. Conclusions and Future Research

We have considered an NP-complete problem which is obtained by substituting the objective function of the classic linear assignment problem with a cumulative function. We have

Table 1: Grand total for the four classes

	MS		SA		TS		TS1		TS2		$\mathcal{X}$-TS	
n	$\Delta\%$	bst	$\Delta\%$	bst	$\Delta\%$	bst	$\Delta\%$	bst	$\Delta\%$	bst	$\Delta\%$	bst
50	1.236	7	0.231	28	0.255	29	0.100	41	0.083	53	0.068	52
100	5.478	1	1.251	1	0.649	19	0.360	20	0.369	35	0.315	34
150	15.198	7	3.496	0	1.266	14	1.255	11	1.091	29	1.083	30
200	31.677	15	5.424	0	3.425	9	2.124	16	1.986	27	2.060	34
gr.tot	13.397	30	2.601	29	1.399	71	0.960	88	0.882	144	0.881	150

Sun Sparc Ultra 2 seconds, averages over 80 instances.

reviewed the relevant results of the literature and we have proposed a study of metaheuristic algorithms for solving the problem. In particular, we have carefully developed a procedure to explore a neighborhood function and have used this procedure to obtain different metaheuristic algorithms using different strategies with zero memory, local memory or long term memory. Lastly we have described a general meta-strategy using three tools from the tabu search framework. The resulting algorithm, called $\mathcal{X}$-TS, is a general well-structured tabu search approach. The computational results show that the systematic use of strategies based on long time memory helps towards improving dramatically the performances of a beginning level tabu search and that $\mathcal{X}$-TS is very effective as against other metaheuristic approaches.

We also observed that the rules and the parameters which determine the recourse to the second and third tools must be defined with preliminary computational experiments. The second level tool is a pure long term memory management, whilst the third one is a restarting procedure. Also the third tool can be considered a long term memory-based approach if we are able to drive the generation of new solutions far from the already visited areas. As already pointed out in the previous section, we believe that the study of procedures to generate feasible solutions which uniformly span the space of the solutions is a very important challenge in the theory of metaheuristic algorithms. But other questions arise. In particular, the use of long term memory would be much more effective if we know the mapping $\mathcal{M}_{\mathcal{N}} : S \to S$ which associates with each solution $s \in S$ the local minimum $\hat{s} \in S$ that we would find by applying a pure local search to s, with a certain neighborhood function $\mathcal{N}$. We believe that the study of properties of this mapping is a second very important challenge for future research. A third pont that should be considered for improving the theory of metaheuristic algorithms is how one can efficiently store the different local minima corresponding to already visited solutions and how one can efficiently check if the mapping of a given solution s corresponds to an already visited solution.

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Table **2**, Sun Sparc Ultra 2 seconds, averages over 20 instances.

		MS		SA		TS		TS1		TS2		$\mathcal{X}$-TS	
Class	n	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst
	50	3.070	1	0.070	16	0.479	13	0.039	19	0.129	18	0.025	20
A	100	15.651	0	2.534	1	1.598	4	0.486	7	0.703	7	0.486	11
	150	50.055	0	8.688	0	3.277	3	2.981	7	2.706	7	2.693	5
	200	111.452	0	15.601	0	11.115	1	6.052	7	5.461	11	5.719	9
	tot	45.057	1	6.723	17	4.117	21	2.390	40	2.250	43	2.231	45
Class	n	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst
	50	1.511	3	0.285	10	0.285	10	0.040	19	0.032	19	0.060	18
B	100	5.603	0	1.408	0	0.364	8	0.355	9	0.280	9	0.280	9
	150	10.092	0	4.121	0	1.019	8	1.382	2	1.094	6	1.091	7
	200	14.668	0	5.040	0	1.886	6	1.785	6	1.887	8	1.962	8
	tot	7.969	3	2.714	10	0.889	32	0.891	36	0.823	42	0.848	42
Class	n	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst
	50	0.186	2	0.324	0	0.119	5	0.134	3	0.096	7	0.106	6
C	100	0.367	0	0.523	0	0.319	6	0.284	3	0.257	8	0.255	6
	150	0.356	2	0.572	0	0.410	1	0.305	1	0.271	11	0.274	10
	200	0.301	4	0.458	0	0.366	1	0.306	3	0.292	4	0.268	10
	tot	0.303	8	0.469	0	0.303	13	0.257	10	0.229	30	0.226	32
Class	n	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst	Δ%	bst
	50	0.178	1	0.245	2	0.138	1	0.187	0	0.076	9	0.080	8
D	100	0.292	1	0.539	0	0.315	1	0.314	1	0.237	11	0.239	8
	150	0.289	5	0.604	0	0.358	2	0.353	1	0.294	5	0.275	8
	200	0.287	11	0.596	0	0.334	1	0.354	0	0.302	4	0.290	7
	tot	0.262	18	0.496	2	0.286	5	0.302	2	0.227	29	0.221	31

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