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Review

Nonlinear dynamics of Bevel Gears: A comprehensive review

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ABSTRACT

This article provides a detailed review of research on the nonlinear dynamics of bevel gears. It summarizes more than 220 scientific papers on bevel gears dynamics up to August 2024, covering various aspects of the dynamics of such complex mechanical components, namely: modeling, testing, dynamic scenarios (e.g. modal properties, nonlinearities, complex dynamics); the effects of geometric parameters and influence of imperfections. In the past decades, investigations on bevel gears have been increasing significantly. The review highlights the technical challenges associated with understanding and predicting the dynamic behavior of bevel gear systems, including the impact of nonlinearities arising from backlash, bearings, and Hertzian contact stresses, as well as time dependencies resulting from geometric transmission error, mesh stiffness, and input torque. Additionally, it discusses practical aspects such as tooth imperfections, profile modifications, and the influence of operational conditions such as damping ratio, lubrications, and thermal conditions on vibrations and dynamics of the systems.

1. Introduction - historical perspective

The history of classical bevel gear usage dates back to the 3rd century B.C., when the water-lifting devices, in the form of 'Persian wheels', were used [1,2] moving heavy loads. This concept later imported by ancient Egyptians, where the need for irrigation led to the invention of the Sakia, a machine that used wooden gears to lift water. Then, the concept of gears spread to other ancient civilizations, including Mesopotamia, India, Greeks, and China, where they were adopted for various purposes, such as milling and water lifting. The Romans expanded the knowledge of gears inherited from the Greeks and Egyptians and introduced water-powered mills and saws with wooden gears [3,4]. Before the seventeenth century, beginning of the industrial era, it was possible to use belt drives to successfully transmit power, but with steam engines achieving greater power and higher rotational speeds, the need for more effective drives became a necessity. From the middle of the nineteenth century, a gear manufacturing industry began to grow. Heinrich Schicht took up the idea of hobbing cylindrical gears and transferred it to bevel gears, using a conical hob instead of a cylindrical hob to manufacture spiral bevel gears [5]. Manufacturing techniques and geometry of bevel gear has been undergoing continues evolution giving rise to the actual sophisticated tooling techniques and gear geometry. Indeed, extensive research has been conducted on geometric characteristics, encompassing imperfections and tolerance control, leading to a well-established understanding of these aspects; however, the impact of geometric deviations and tolerance variations on vibrational behavior remains an underexplored area, necessitating further investigation.

Bevel gears first acquired substantial significance with the development of the automotive industry at the beginning of the twentieth century. Nowadays bevel gears with different geometries are employed to transmit mechanical power in a wide range of

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applications, such as vehicle transmissions, aircraft engines, aircraft turbines, flap drives of aircraft wings, and marine drives, see Fig. 1.

The initial in-deep investigation of bevel gear goes back to the 1900s, when Oscar Beale developed a generating method to produce bevel gears using two disc-shaped cutters, which could machine both flanks at the same time. Paul Bottcher improved this concept and, in 1910, presented a face mill cutter system which produced spiral shaped teeth on bevel gears. Taking the idea of hobbing cylindrical gears to bevel gears, Schicht and Preis patented a new technology to manufacturing spiral bevel gears in 1921 [5]. Since then, numerous studies have appeared on dynamics and vibrations of bevel gear, ranging from vibration mode properties to nonlinear dynamics and bifurcations. Given the significant growth in bevel gear research activity and the lack of prior literature surveys, there is value to the research community in reviewing the actual state of the art. Table 3 of Appendix “B” provides a comprehensive list of review papers in the field of gears, highlighting key contributions and advancements made by various researchers over the years; what we can conclude from the aforementioned list is that, despite the importance of bevel gears in various engineering applications, there is a lack of comprehensive literature reviews focusing on their dynamics. This review paper aims to fill the existing gap in the literature on bevel gear dynamics, providing an insightful resource for researchers to identify unexplored areas and effectively guide their future investigations.

The present review evaluates more than 220 journal and conference papers focused on dynamics and vibrations of bevel gears up to August 2024; the 220 papers have been selected from a list of about 3000 papers focused on bevel gears, by eliminating those not related to vibrations. Fig. 2 shows a histogram of papers published on bevel gear dynamics and vibration cited in the present work. During the 1980s and 1990s, there were relatively few publications, with each decade producing less than ten papers. However, advancements in the knowledge of complex dynamical systems as well as more powerful computational techniques and hardware, have provided researchers with the tools to model gear systems more effectively. As technology progressed, the number of publications increased significantly. In the 2000s, the number of publications nearly doubled compared to the previous decades. In the last four years, the number of publications has nearly matched the total output from the entire decade of 2010 to 2020, indicating a significant surge in research interest, which is likely related to the new challenging NVH problems arising in electric vehicles.

The increasing utilization of bevel gears across diverse industrial applications has driven growing research interest, particularly in optimizing their performance, efficiency, and durability. This notable increase in research activities reflects a growing recognition of the complex dynamics inherent in bevel gear systems, driving a deeper exploration into their unique non-linear behaviors and instability phenomena. Bevel gears are an interesting engineering system with rich dynamic phenomena. Their governing equations are non-smooth and non-autonomous as the dynamic system is directly and parametrically excited by the combination of the fluctuating Mesh Stiffness (MS) and the transmitted torque (power). During operation the gears rotate, causing the contact conditions at the tooth mesh interface to vary periodically. This gives rise to periodic stiffness fluctuations at each gear mesh period. Furthermore, near the resonances the vibrations and inertial forces can become large enough to cause the contacting teeth to lose contact. This introduces a non-smooth nonlinearity and softening type nonlinearity due to the combination of backlash and static loading. When the principal parametric instability takes place, i.e. twice the natural frequency, sub-harmonic response take place and the level of vibration reaches the threshold where the system may experience a backside impact; this phenomenon induces a hardening nonlinearity, clearly detectable from the amplitude-frequency diagram. Due to complexity of bevel gear geometry, having a small level of errors might lead to a catastrophic phenomenon in the system such non-steady response, amplitude modulation and chaos. These are among the topics discussed in this review.

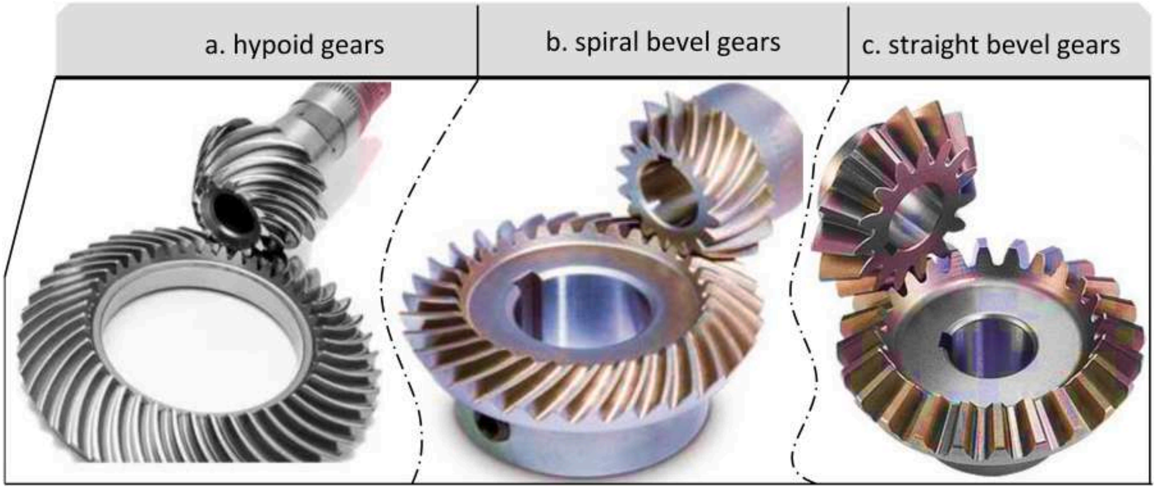

Fig. 1. Three different types of bevel gears: a) hypoid gears, b) spiral bevel gears, and c) straight bevel gears.

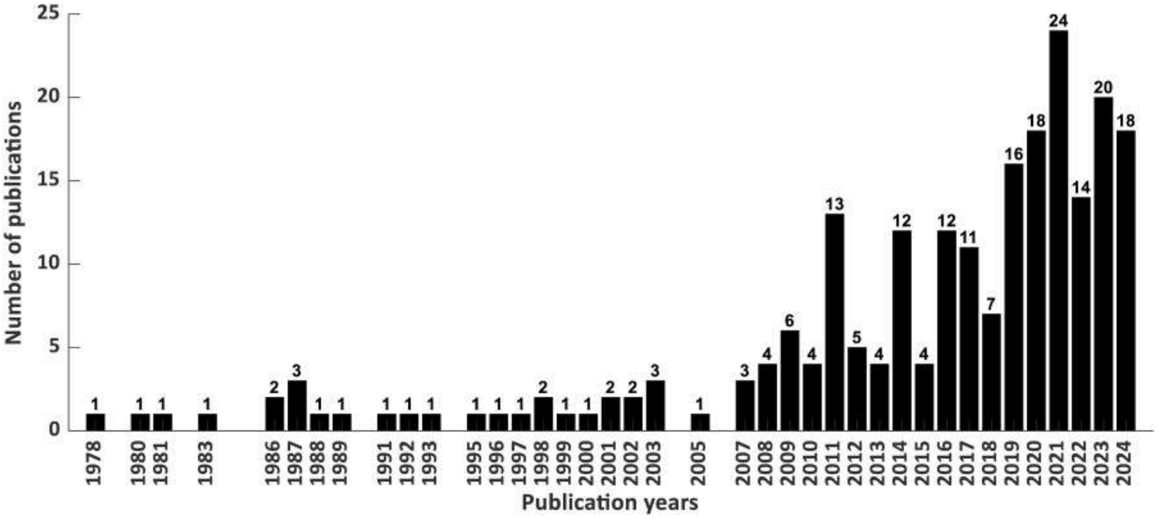

Fig. 2. Histogram of research papers on dynamics of bevel gear.

2. Dynamical models

In mechanical application, bevel gears stand as a marvel of design and function in two different category of internal and external model [6]. In the early stages of research on bevel gears, the primary focus was on the geometry and manufacturing processes. Researchers dedicated significant effort to understanding the intricate design aspects and developing precise methods for producing these gears. However, as the field matured, attention increasingly shifted towards predicting the dynamic response. The dynamical model of bevel gears allows for the formalization of the complex interactions and response characteristics of their components under different working conditions. In investigating the dynamics of bevel gears, it is crucial to comprehend the concept of transmission errors, which has a key role as it mainly affects the dynamic response of the system [7]. Transmission errors in gear systems may be categorized into different concepts addressing different aspects of gear performance; however, three categories can be identified: Kinematics, Statics, and Dynamics. Kinematic transmission error (KTE), also called geometric transmission error (GTE) - see Fig. 3 and Eq. (1), refers to deviations from the ideal angular displacement or velocity ratio between gears, due to geometric imperfections and assembly inaccuracies under unloaded conditions [8,9]. Static transmission error (STE), see Eq. (2), includes KTE and adds the effects of elastic deformations under static loads, measuring deviations when gears are statically loaded [9,10]. Indeed, the distinction between KTE and STE lies in a method used to determine the rotational motion of the mating gear elements when the rotation of one element—either the pinion or the gear—is prescribed. In the case of KTE, the resulting rotation of the mating component is derived using the Unloaded Tooth Contact Analysis (UTCA). This analysis assumes ideal conditions, neglecting external forces or elastic deformations, and it is governed purely by the geometric characteristics of the gear pair. In contrast, STE is determined through a Loaded Tooth Contact Analysis (LTCA), which accounts for the effects of applied forces, elastic deformations, and other compliance factors in the computation. Dynamic transmission error (DTE), see Eq. (3), covers deviations during actual operation, accounting for kinematic errors, static load-induced deformations, and factors like operational forces, vibrations, and inertial effects.

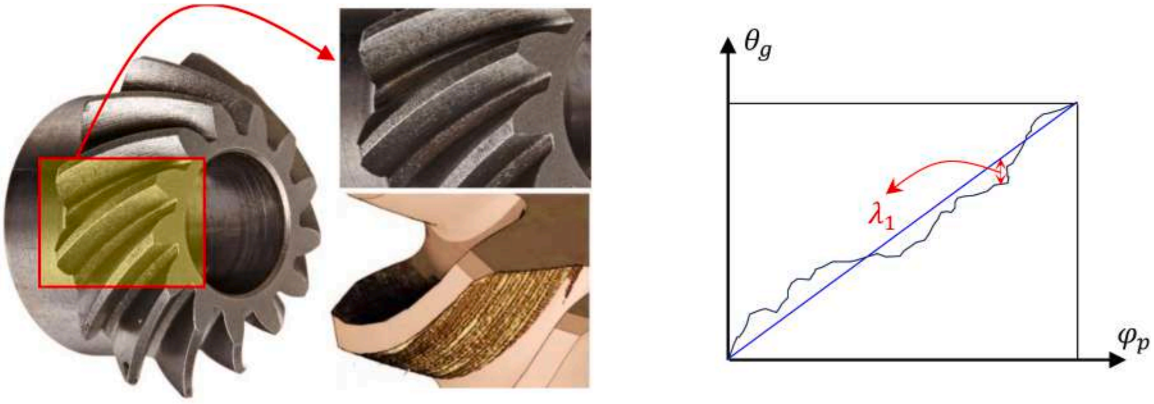

Fig. 3. Kinematic transmission error due to imperfection on the tooth surface [8].

$$\textbf{KTE}:\ {}^{\lambda_1}|UTCA\ \ = \theta_{g,K} - \theta_g^0 - \frac{N_p\left(\varphi_{p,K} - \varphi_p^0\right)}{N_g} \tag{1}$$

$$\textbf{STE}:\ {}^{\lambda_2}|LTCA\ \ = \theta_{g,S} - \theta_g^0 - \frac{N_p\left(\varphi_{p,S} - \varphi_p^0\right)}{N_g} \tag{2}$$

$$\textbf{DTE}:\lambda_3 = \theta_{g,D} - \theta_g^0 - \frac{N_p\left(\varphi_{p,D} - \varphi_p^0\right)}{N_g} + f(x,y,z) \tag{3}$$

φ_p^0 and θ_g^0 are the initial angular positions of the pinion and the gear, the same for all operating conditions: purely kinematic KTE, statically loaded STE and dynamic state DTE; $\theta_{g,K}$ and $\varphi_{p,K}$ are the instantaneous angular position of the gear and the pinion respectively, only due to the kinematics of the mating gears; $\theta_{g,S}$ and $\varphi_{p,S}$ are the instantaneous angular position of the gear and the pinion respectively, due to the kinematics and static deflection (external load) of the mating gears; $\theta_{g,D}$ and $\varphi_{p,D}$ are the instantaneous angular position of the gear and the pinion respectively, due to the combination of kinematics, external load and inertial forces; N_p and N_g are the numbers of teeth of pinion and gear, respectively; $f(x,y,z)$ represents the torsional displacement due to vibration in x,y, and z. Depending on the number of degrees of freedom (DOF), we may have different expressions for $f(x,y,z)$, as explained in detail in Section 2.3, "Mathematical Approach.

The first step of the research on the dynamic response of the system began with some experimental tests, as recognition always starts with real evidence [11,12]. In 1986, an experimental test was conducted by NASA [13]; they analyzed the performance and vibrations of the YUH-61A helicopter transmission and identified the vibration sources; they focused the attention on gear meshing frequencies and the impact of torque as well as the effect of the speed variations on the vibrations. The initial researches have made significant advancements in understanding the dynamics of bevel gear systems under different conditions and evaluating the effects of parameters variations, which may affect the dynamic response of the system, e.g.: the housing elasticity [14], torque fluctuation [15], tooth surface modifications [16], and effects of uncertain parameters on wind turbine gear systems [17,18]. These models typically integrate factors such as time-varying mesh stiffness, backlash, and GTE, which are essential for accurately predicting dynamic responses. Then researchers tended to further understand how the dynamic responses, under different operating conditions, can be modeled to improve gear design and functionality [19,20]. The comprehensive analysis of the dynamic behavior allows for the optimization of gear systems, enhancing their durability and performance.

The dynamical model in bevel gear systems can be categorized into three detailed subsections: Finite Element Method (FEM) models, lumped mass models, and experimental models. FEM models utilize advanced computational techniques to simulate the dynamical behavior of gear systems under various conditions. On the other hand, lumped mass models employ mathematical algorithms to solve the governing equations characterized by a reduced number of degrees of freedom (DOFs), allowing for the analysis of complex concepts such as nonlinear characteristics, gear meshing stiffness fluctuation, torque variation and GTE. Experimental models involve practical testing and data collection from gear systems, typically such data is used to validate numerical predictions and refine computational models, ensuring that the simulations accurately represent real-world behaviors.

2.1. Finite element model

The finite element method has revolutionized the investigation of power transmission systems, particularly in simulating the dynamics of complex systems such as bevel gears. This method allows engineers to predict how gears would perform under various conditions, ensuring their reliability and efficiency. As technology advanced, FEM has evolved to become more sophisticated, providing even greater accuracy and insights into gear dynamics. In the field of bevel gear vibrations, a series of studies have collectively advanced our understanding of the dynamic behaviors of bevel and hypoid gears. In the early 2000s, Wang et al. [21] pioneered a nonlinear dynamic model for spatial geared systems with intersecting axes, specifically focusing on bevel gears. Their approach, grounded in finite element theory, effectively captured 3D motions in axial, lateral, and torsional directions. Through numerical simulations and experimental comparisons, they demonstrated the significant impact of stiffness variations on vibration and resonance, providing crucial insights for optimizing gear designs to reduce vibrations. As the research progressed, Liang and Xin [22] integrated SolidWorks® and MSC-ADAMS® to create accurate 3D models of spiral bevel gears and simulate their dynamic behavior. This study employed numerical methods to analyze angular speed, torque, and meshing forces, validating their approach with theoretical calculations. The results underscored the reliability of this method for analyzing and optimizing gear dynamics, which is essential for applications in automotive and machinery industries.

In 2011, a team of researchers of Romax® [23] employed advanced CAE techniques to study hypoid gearset vibrations. By combining FEM with multibody dynamics, they modeled the dynamic responses of gears under various operating conditions. This comprehensive approach highlighted the necessity of accurate gear modeling to predict vibration and noise, ultimately improving gear design. Meanwhile, Wang et al. [24] focused on the 3D dynamic contact and impact analysis of spiral bevel gears. They developed dynamic models that accounted for friction, gear clearance, and time-varying stiffness. Using FEM, they simulated vibration behavior and stress distribution, revealing the importance of understanding the dynamic responses to avoid resonance and ensure reliable gear performance. In another significant contribution, Hua et al. [25] combined FEM with enhanced lumped parameter models to analyze the dynamic behavior of spiral bevel geared rotor systems. Their study validated numerical results against experimental data, demonstrating the effectiveness of this integrated approach for designing robust geared rotor systems. Further advancements came with the development of a nonlinear FEM model for double circular arc spiral bevel gear nutation drives [26]. This study explored vibration displacement, dynamic meshing stress, and contact forces at different rotational speeds, showcasing the high transmission accuracy and load-bearing capacity of nutation drives.

In Aerospace, Zhu et al. [30] modeled the dynamic behavior of spiral bevel gear coupled systems in helicopter gearboxes. Utilizing FEM-based simulations, they investigated various gear parameters and demonstrated the critical role of precise modeling in enhancing gearbox performance and reducing noise. In another effort, a study in 2022 delved into the dynamic behavior of high-speed spiral bevel gears, considering the effects of web thicknesses and angles. They considered three different web angles, i.e., 55°, 70°, and 85°, and three different web thicknesses, i.e., 6.5 mm, 5.0 mm, and 3.5 mm. Using the finite element method for dynamic meshing analysis, the researchers discovered how these factors critically influenced gear stability and performance. The average values of DTE for the models with web support angles of 55°, 70°, and 85° are −35.73 arcs, −44.24 arcs, and −38.00 arcs, respectively. This indicates that a mid-sized web support angle increases deformation by more than 20 %. Additionally, variations in the web support angle have a minimal impact on DTE fluctuations [31].

To address the challenges of high-speed gear dynamics, Hou et al. [32] proposed a modified damping model using the Vector Form Intrinsic Finite Element Method (VFIFE). Their lumped mass model showed the model's effectiveness in reducing vibration amplitudes and improving gear performance, highlighting the potential of VFIFE in high-speed applications. In the same year, in 2022 Ref [33], it was published a study on the dynamic behavior of cracked spiral bevel gears under assembly errors; FEM and lumped mass model were used to examine the impact of cracks and assembly inaccuracies on gear dynamics. The results emphasized the need for precise assembly and regular inspection to ensure long-term reliability, providing valuable recommendations for gear design and maintenance. In 2022, Ding et al. [34] delved into the dynamic behavior of spiral bevel and hypoid gears focusing on the size, direction and position of dynamic meshing impact in the system by using a semi-FEM. Their method integrated FEM with numerical integration to predict whether there exists the meshing impact or not under the given dynamic condition. Another significant effort of the same authors was focused on sensitive misalignment-based dynamic loaded meshing impact diagnosis for aviation spiral bevel gear transmission [27], see Fig. 4-a. This research applied FEM techniques to simulate the effects of misalignment on gear performance, providing insights into how even minor deviations could impact the overall system. The conclusions emphasized the necessity of precise alignment (3–4 grade) in high-performance applications like aviation, where the working speed is 18,000–40,000 rpm, to ensure optimal gear function. Zhang et al. [35] investigated the dynamics of a rotor-stator coupling systems in coaxial contra-rotating gearboxes. This study combined FEM with experimental tests to explore the complex interactions between rotating components. The results demonstrated how FEM could be used to predict and mitigate potential issues in these intricate systems, highlighting its crucial role in modern gear design.

Further advancements came with the research on active pre-control strategies for the shape and performance of helicopter spiral bevel gears. In Ref [36], using FEM-based simulations, the authors explored various design modifications to enhance gear performance by reducing the impact force and impact velocity of the optimized gear by 27.02 % and 25.81 %, respectively. The findings showed that proactive design adjustments could significantly improve the operational efficiency and longevity of helicopter gear systems. In 2023, Chen et al. [28] focused their attention on the vibration attenuation characteristics of squeeze film dampers (SFDs) in spiral bevel gear system, see Fig. 4-b. This study used FEM to model the damping effects of squeezing film dampers, revealing their effectiveness in reducing gear vibrations. The conclusions highlighted the potential of these dampers to enhance the stability and noise performance of gear systems. In a comprehensive analysis, Tian et al. [29] developed a model for flexible bevel gear rotor systems, examining their modal and dynamic characteristics. The study employed FEM to analyze the effects of flexibility on gear dynamics, providing valuable insights for the design of more resilient and efficient gear systems. A particularly interesting study on noise reduction in drive axle hypoid gears introduced a method of tooth surface mismatch modification by Nie et al. [37]. This work used FEM to simulate the effects of tooth surface mismatch modifications on gear performance, showing how specific adjustments, such as cutting and machining setting parameters of teeth surface, could reduce noise and improve overall gear efficiency. The study's conclusions underscored the importance of tooth surface designing parameters, i.e., crown coefficients, spiral angle, and pressure angle, in achieving optimal gear performance. In 2024, Talakesh et al. [38] published a study focused on crack identification in single-stage straight bevel gear systems through dynamic analysis. They used FEM to detect and analyze cracks in gear teeth, providing a robust method for early detection and maintenance. The conclusions emphasized the critical role of FEM in ensuring the reliability and safety of gear systems.

2.2. Experimental approach

Experimental approaches in analyzing the dynamics of bevel gears are invaluable for validating theoretical models and simulations, offering comprehensive real-world data, and identifying practical issues such as manufacturing defects and environmental influences. They excel in capturing the multifaceted nature of gear behavior under various operational conditions, particularly in terms of noise and vibration analysis. Despite the complexity and cost of experimental setups, and challenges in ensuring repeatability and consistency, their integration with advanced sensing technologies, automated data analysis, and simulations can significantly enhance their efficacy. These methods provide crucial insights that complement theoretical models, ensuring an understanding of gear dynamics and the development of reliable diagnostic techniques.

The body of work in gear dynamics reveals a robust exploration of the methods and conclusions drawn across various studies, each contributing a unique perspective to the field of gear transmission error (TE), vibration characteristics, and fault diagnosis. The study by Hongbin and Yu [39] delved into the transmission error of modified spiral bevel gears through both theoretical and experimental

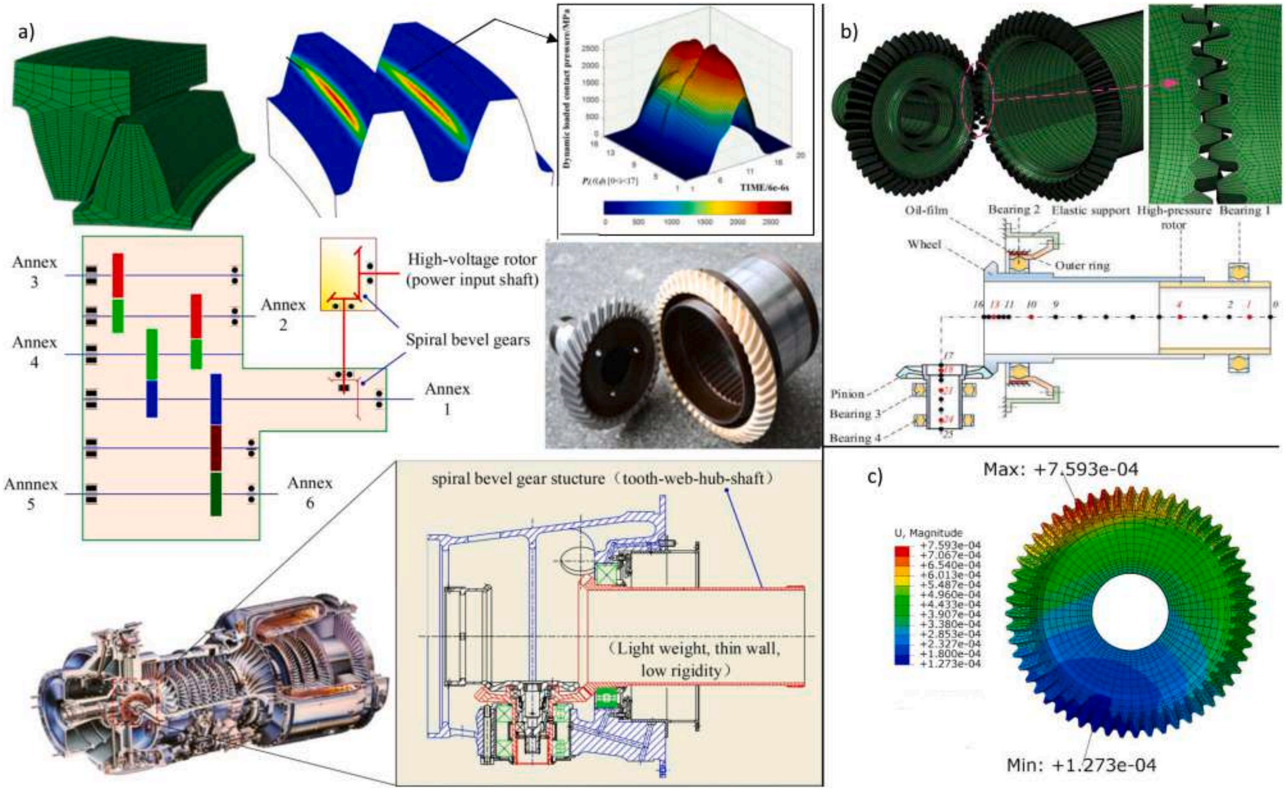

Fig. 4. Finite element model used in some researches: a) A schematic diagram of spiral bevel gear drive in aviation engine transmission system [27], b) real model and Finite element model of the spiral bevel gear pair [28] c) Comparison of loaded deformation cloud map [29].

lenses. Using meshing theory, they simulated TE curves for both general and modified gears and validated these curves through comparative experiments under different loads (50 Nm, 100 Nm). Their findings highlighted that the modified gears, where the module and pinion tangential displacement varied from 5.59 and 0 to 5.5 and 0.0226, respectively, exhibited lower TE amplitudes and improved dynamic behavior, effectively reducing vibration and noise. Wu et al. [40] approached the issue of tooth surface wear in spiral bevel gears using modulation signal bispectrum (MSB) analysis. They conducted a run-to-failure experiment under accelerated wear conditions and used MSB to monitor gearbox vibrations. Their research showed that MSB was sensitive to gear defects and effective in monitoring wear progression, particularly in the early stages.

In 2022, Dewangan et al. [41] focused on fault diagnosis in bevel gearboxes through a dynamic model incorporating time-varying mesh stiffness (TVMS). Their model simulates the dynamic response of a gearbox with a missing-tooth fault, and experimental validation showed that the model accurately identified such faults based on time series analysis. The study by Yang et al. [42] investigated the torsional vibration characteristics of spiral bevel gear systems with broken teeth by developing a dynamic model that incorporates TVMS and friction excitation, validated through both lumped mass model and experimental tests. The findings revealed that the crest factor is sensitive to minor tooth failures, while kurtosis is sensitive to severe failures, and that the energy in the low-frequency band (0–1000 Hz) increased with the severity of the failure, highlighting the significant role of frictional force.

Han et al. [43] extended the investigation into gear vibration by analyzing modulation sidebands in a coupled bevel gear and planetary gear train system. They employed a numerical approach to model the dynamic response and carried out experimental tests to validate their findings. This research revealed that sidebands due to amplitude modulation could effectively indicate gear faults and

Fig. 5. The experiment test bench, a) Hypoid gear system for automotive transmission [45], b) Layout of the high-speed test platform [46], c) Experimental rig and vibration-test system [47].

the dynamic interplay between gear pairs. Meanwhile, the paper published by Rana et al. [44] addressed noise and vibration reduction in gears through abrasive flow finishing. They considered three different gear pairs: spur, helical, and straight bevel gears with two distinct surface roughness (unfinished and finished cases). The average surface roughness of unfinished and finished cases is 1.2 μm and 0.7 μm for spur gear pairs, 0.8 μm and 0.5 μm for helical gear pairs, and 1.7 μm and 0.7 μm for straight bevel gears, respectively.

By experimentally testing cylindrical and conical gears, they demonstrated that this finishing process significantly reduced operational noise and vibration, suggesting a practical application for improving gear performance in various industrial contexts. In another contribution, Liu et al. [47] utilized parametric modeling to analyze the vibration response of high-speed gear transmission systems, see Fig. 5-c. Their research integrated FEM with experimental validation (the maximum error of 7.8 %) to explore the dynamic characteristics. Their conclusions advocated for the model's robustness in predicting gear dynamics. Xu et al. [46] introduce a resonance attractor evaluation method for thin-walled gears, where the traveling wave vibration is a severe threat to thin-walled gears in high-speed systems. Since it is an external manifestation of significant changes in the system state, it can be described by attractors theoretically. To solve this issue, they proposed a model to identify the traveling wave vibration from the evolution of attractor features—which are quantified by indicators: the boundary radius and the phase point expansion rate. Through a combination of modeling and experiments, shown in Fig. 5-b, they assessed the vibration characteristics and proposed new diagnostic indicators for early fault detection, emphasizing the importance of resonance behavior in gear diagnostics. Another study in 2024 by Kumar et al. [48] explored electromechanical modeling and multi-fault diagnosis in straight bevel gear pairs. Their experimental validation underlined the model's effectiveness in diagnosing multiple faults: chipped tooth and missing tooth, simultaneously, offering a comprehensive tool for maintaining gear health in complex mechanical systems.

2.3. Mathematical approach

The mathematical approach to bevel gear dynamics is grounded in governing equations that constitute a non-smooth and non-autonomous model. These equations capture the interactions and motion within the gear system, accounting for the complexities introduced by real-world operating conditions. As additional components, such as bearings or shafts, Fig. 6, are incorporated into the models of bevel gear pairs, the degree of freedoms of dynamical model increases correspondingly.

A wide variety of dynamic models might be used to investigate gear systems, and similar predictive accuracy could be achieved, depending on the system's dynamic properties. For instance, when the shaft's torsional and flexural stiffnesses are decoupled from other vibrational modes, a SDOF model may provide sufficiently accurate predictions, in this case considering a more complex MDOF model is unnecessary. However, if the dynamic properties result in strong coupling between meshing vibrations and other modes, a SDOF model, even with advanced features such as nonlinear elements, gear error excitation, time-varying mesh stiffness, and damping, may be inadequate. In such cases, a more sophisticated MDOF model is required to accurately capture the complex interactions and ensure reliable response predictions [49,50]. This section contains a comprehensive list of studies on dynamic models with different DOF, highlighting how each model has been applied to analyze the dynamic behavior of gear systems under several operating conditions.

2.3.1. Single degrees-of-freedom model

The early investigations of the dynamics of spiral bevel gears began with simple single-degree-of-freedom (SDOF) models. This approach allowed researchers to analyze specific dynamic characteristics, such as the effects of MS, damping ratio, and backlash, providing a foundational understanding that would later be expanded upon. Considering only two mated gears, i.e., pinion and gear, led to the development of two-degree-of-freedom (2-DOF) models, represented by two governing equations, Eqs. (4) and (5). These

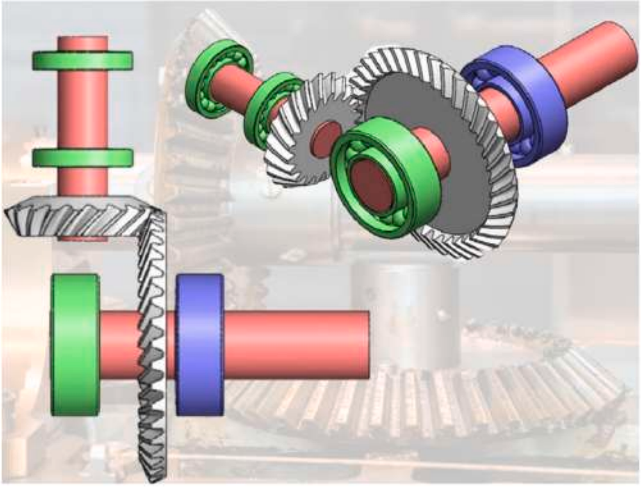

Fig. 6. General view of components in a dynamical model.

equations accounted for the rotational movements of both the pinion and the gear, capturing the interactions between the two components. By introducing the concept of dynamic transmission error (Eq. (6)), the two governing equations of the 2-DOFs model could be reduced to a single governing equation. This reduction effectively transformed the 2-DOFs model back into a SDOF model, i.e. Eq. (7), where the dynamic transmission error encapsulated the essential dynamics of the system. The dynamic equations of motion of the pure rotational model of pinion and gear (Fig. 7) are given by:

$$I_g^y \ddot{\theta}_g = -r_g F_z - T_l \tag{4}$$

$$I_p^x \ddot{\varphi}_p = r_p F_z + T_m \tag{5}$$

The linear dynamic transmission error, λ, along the line of action is defined as:

$$\lambda = \left(r_p \varphi_p - r_g \theta_g\right) a \tag{6}$$

where, $a = \cos\alpha \times \cos\beta$, β is the spiral angle and α is the normal pressure angle. Finaly, the nonlinear differential equation with time-varying parameters is derived as follows:

$$\bar{\lambda}'' + \bar{K}_m f(\bar{\lambda} - \bar{e}) + 2\xi\bar{\lambda}' = \bar{T}_{eq} \tag{7}$$

In 2012, Karagiannis et al. [52] developed a SDOF model to delve into the effects of lubrication on the dynamic behavior of hypoid gear set, recognizing that the thin film of lubricant between teeth could significantly influence vibration and stability. Their findings underscored the importance of proper lubrication in maintaining the smooth operation of gear systems, a critical insight for industries relying on these components for high-performance applications. As the research progressed, a deeper understanding of the nonlinear dynamics of spiral bevel gears began to emerge. TANG et al. [53] carried out a study in 2013 investigating the effect of static transmission error, sine STE and pre-design parabolic STE, on the dynamic responses of the gear set. By modeling the system with varying stiffness and backlash, they analyzed the chaotic dynamics that could arise at low speeds. This work highlighted the delicate balance required in gear design—how deviations in transmission error could lead to significant changes in system behavior such as impact phenomenon or jumping into chaos. In 2014, Karagiannis and Theodossiades [54] proposed a SDOF model for DTE, accounting for the variation of the effective mesh position. However, gears do not operate independently from the other powertrain components, indeed, they are typically part of a large mechanical system where interactions with other components can introduce additional

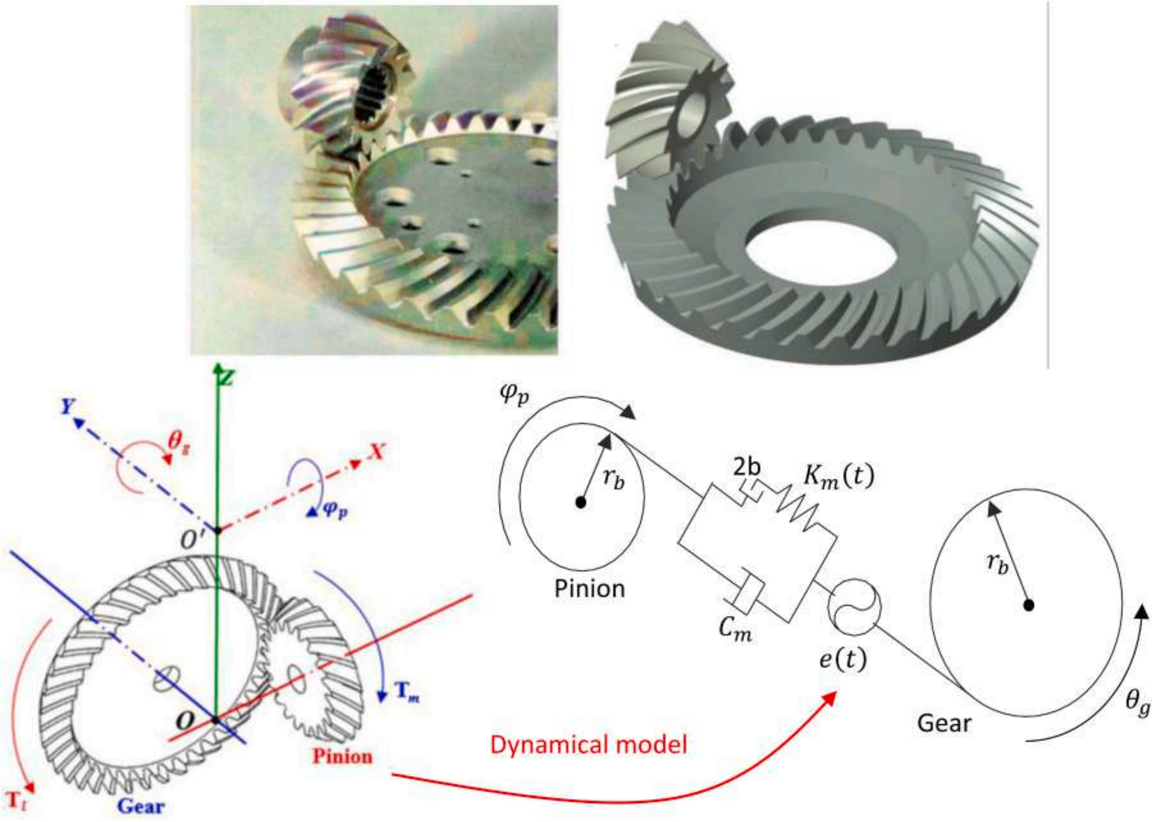

Fig. 7. The dynamical system with single Degree-of- freedom [51].

complexities. By 2016, Li et al. [55] examined the nonlinear vibrations of hypoid gears with backlash, revealing how these small clearances between gear teeth could lead to chaotic vibrations under certain conditions. In parallel, Motahar et al. [56] started a study focused on the impact of teeth profile modifications on bevel gears, showing how specific changes to the gear's shape, i.e., tooth profile modifications, could reduce vibrations and enhance performance. In 2021, Samani et al. [57] explored how the stiffness of the gear system and the location of bearing supports could influence the nonlinear vibration responses of spiral bevel gears. They discovered that the position of the supports and the type of the input torque—whether constant or periodic—could drastically affect the system's behavior, sometimes leading to chaotic responses. An investigation done by Molaie et al. [58] took a closer look at the effects of misalignments—both radial and axial—on the vibration characteristics of spiral bevel gears. The findings were clear: even small misalignments could destabilize the system, leading to increased vibrations and potential gear failure. This research further emphasized the importance of precision in gear alignment and the need for robust designs that could tolerate minor deviations without compromising performance. In 2023, the same authors [51] focused on the internal characteristics of the gears themselves. By incorporating accurate static stiffness evaluations into the dynamic models, they were able to improve predictions of gear behavior under various conditions. Their work represented a culmination of years of research, bringing together insights from previous studies to create more reliable and efficient gear systems to evaluate the effect of different parameters such as backlash and damping ratio.

2.3.2. Four degree-of-freedoms model

The number of DOFs in a system is determined by the constraints we impose on its components or from considerations on elastic and inertial properties of the elements of the powertrain, as well as the dynamic interactions. For example, incorporating the torsional stiffness of the shaft into the model adds two additional degrees of freedom, Fig. 8, this can be important when the flexibility of the shaft cannot be neglected or when inertial masses are relevant. Similarly, including the effects of translational vibrations introduces other degrees of freedoms, reflecting the system's ability to move back and forth along a particular axis, this is related to the bearing stiffness. In 2008, Li [59] simplified the dynamic model of a rotor-bearing system by neglecting gear teeth errors, rotor stiffness, and backlash. He proposed a model with four degrees of freedom, incorporating the translational motion of the pinion and torsional vibrations around their axes.

In 2023, Molaie et al. [50] conducted an in-depth analysis of the nonlinear dynamic behavior of spiral bevel gears within a purely torsional system. The study focused on the effects of torsional shaft stiffness, backlash, and TVMS on the dynamics of spiral bevel gears. The dynamic model used in the analysis incorporates non-smooth nonlinearities and time-dependent stiffness, which were evaluated using a nonlinear FEM. The study investigated various dynamic phenomena, including periodic, quasi-periodic, and chaotic responses, using tools such as bifurcation diagrams, amplitude-frequency diagrams, and Poincaré maps. The research highlighted the complex dynamic interactions within the system, including trapping phenomena and boom-and-bust cycles, and emphasized the importance of accurately modeling these dynamics to lead design and control of spiral bevel gear systems. They also compared the SDOF model with a three-DOFs model, demonstrating that reducing the degrees of freedom could lead to different dynamic behavior. The findings suggested that while simpler models could be sometimes sufficient, they might miss critical aspects of the gear system's behavior such as existence of nonlinear phenomena or chaos, particularly when strong coupling between modes is present. Indeed, incorporating the torsional stiffness of a shaft into the dynamic model of a gear pair, the system evolved from a simple SDOF model, i.e., Fig. 7, into a more complex system with multiple degrees of freedom, i.e., Fig. 8. By including the torsional stiffness of the shaft, we must also consider the rotational dynamics of both the driving (motor) and driven (load) components. This inclusion results in two additional degrees of freedom: one associated with the load and the other with the motor. The dynamic equations of motion, written in terms of angles θ_g for the gear and φ_p for the pinion, are:

$$\theta_g\mathrm{DOF}:\ \theta_g^{''} = \frac{m_{eq} b r_g a_3}{I_g^y}\left(\overline{K}_m(t) f(\overline{\lambda} - \overline{e}) + 2\xi\overline{\lambda}^{'}\right) + \overline{K}_t^g(\theta_l - \theta_g) + 2\overline{C}_t^g\left(\theta_l^{'} - \theta_g^{'}\right) \tag{8}$$

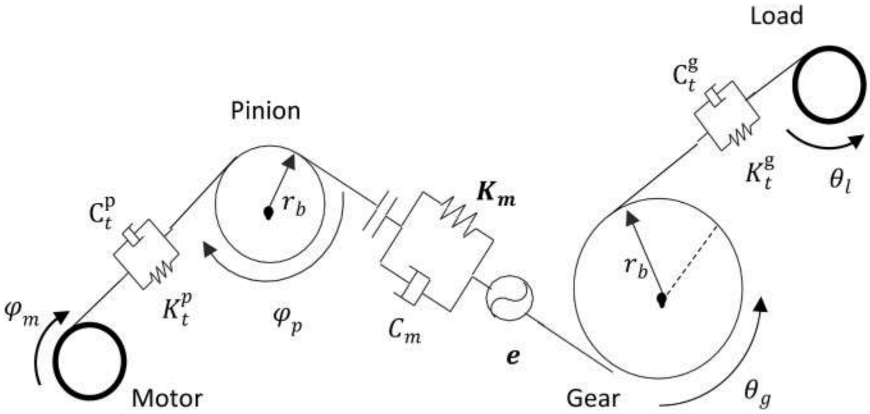

Fig. 8. The dynamical model of the system with consideration of torsional shaft stiffness [50].

$$\varphi_p\text{DOF}:\ \varphi_p^{''} = \overline{K}_t^p(\varphi_m - \varphi_p) + 2\overline{C}_t^p\left(\varphi_m^{'} - \varphi_p^{'}\right) - \frac{m_{eq}br_pa_3}{I_p^x}(\overline{K}_m(t)f(\overline{\lambda} - \overline{e}) + 2\xi\overline{\lambda}^{'}) \tag{9}$$

Where: K_t^p, and K_t^g are the torsional shaft stiffness pinion and gear respectively; r_p and r_g are the mean radii at the meshing points; C_t^p and C_t^g are the material damping of the pinion and gear shafts. The gear system is simulated taking the motor and load masses (e.g. flywheel) into consideration as rotational masses; their relative governing equations for the motor (φ_m) and the load (θ_l) are given by:

$$\varphi_m\text{DOF}:\ \varphi_m^{''} = \overline{T}_m + \overline{K}_t^m(\varphi_p - \varphi_m) + 2\overline{C}_t^m\left(\varphi_p^{'} - \varphi_m^{'}\right) \tag{10}$$

$$\theta_l\text{DOF}:\ \theta_l^{''} = \overline{K}_t^l(\theta_g - \theta_l) + 2\overline{C}_t^l\left(\theta_g^{'} - \theta_l^{'}\right) - \overline{T}_l \tag{11}$$

Typically, in gearboxes the driven shaft is connected to big inertial mass (flywheel); therefore, θ_l can be neglected in Eqs. (8)–(11). The four-DOFs system is then reduced to a three-DOFs system as represented in the following equations:

$$\theta_g^{''} = \frac{m_{eq}br_ga_3}{I_g^y}(\overline{K}_m(t)f(\overline{\lambda} - \overline{e}) + 2\xi\overline{\lambda}^{'}) - \overline{K}_t^g(\theta_g) - 2\overline{C}_t^g\left(\theta_g^{'}\right) \tag{12}$$

$$\varphi_p^{''} = \overline{K}_t^p(\varphi_m - \varphi_p) + 2\overline{C}_t^p\left(\varphi_m^{'} - \varphi_p^{'}\right) - \frac{m_{eq}br_pa_3}{I_p^x}(\overline{K}_m(t)f(\overline{\lambda} - \overline{e}) + 2\xi\overline{\lambda}^{'}) \tag{13}$$

$$\varphi_m^{''} = \overline{T}_m + \overline{K}_t^m(\varphi_p - \varphi_m) + 2\overline{C}_t^m\left(\varphi_p^{'} - \varphi_m^{'}\right) \tag{14}$$

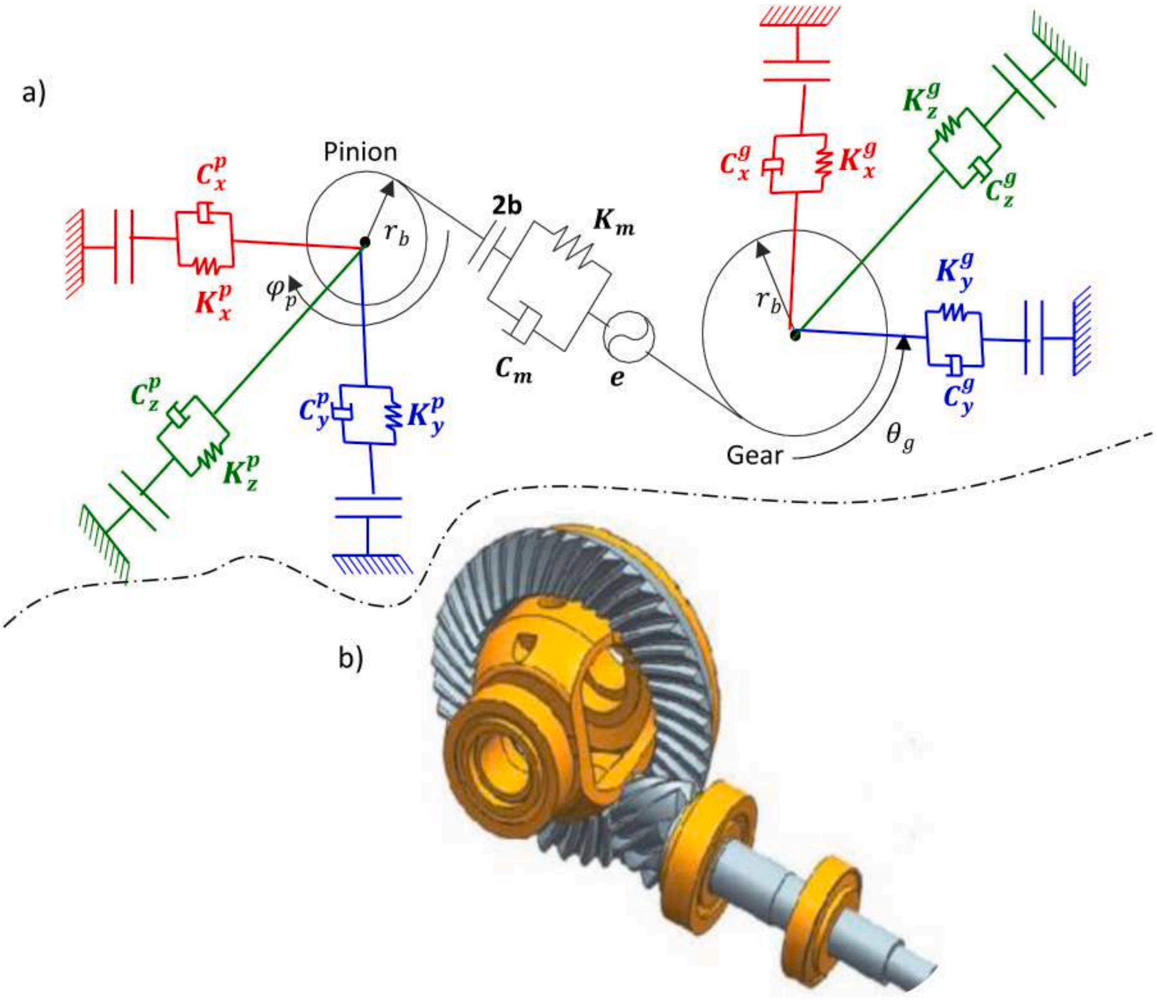

Fig. 9. The dynamical model with seven-DOFs, a) a general dynamical model with eight-DOFs, b) 3D model of the dynamical model [66].

2.3.3. Eight degrees-of-freedom model

A further improvement of the mathematical model of bevel gears considers translational degrees of freedom in addition to the simpler SDOF that focus the analysis on the transmission error only. This expanded model reflects a more comprehensive approach to cover the effects of other elastic components, such as bearings, on the dynamics of gear trains. This section focuses on studies where scientists have developed eight-DOFs models to investigate the dynamic response of bevel gear systems under various conditions. Fig. 9 represents a dynamic model for a system with eight-DOFs governed by Eqs. (15)–(21). This model comprehensively captures the system's dynamics by including both translational and torsional degrees of freedom. However, by defining the concept of DTE, the rigid mode is removed, ultimately resulting in a seven-DOFs model. This reduction occurs because the dynamic transmission error accounts for one degree of freedom, effectively encapsulating the relative motion between the gears and thus eliminating the need to separately model one of the torsional DOFs.

$$m_{eq}\ddot{\lambda} + (a_3)^2 K_m(t) f(\lambda) + (a_3)^2 C_m \dot{\lambda} - m_{eq}\left(\left(-\ddot{X}_p + \ddot{X}_g\right)a_1 + \left(\ddot{Y}_p - \ddot{Y}_g\right)a_2 + \left(\ddot{Z}_p - \ddot{Z}_g\right)a_3 + \ddot{e}(t)\right) = \frac{T_m}{r_p}a_3 \tag{15}$$

$$z_p\mathrm{DOF} : m_p\ddot{z}_p = -K_{zp}^T z_p - C_{zp}^T \dot{z}_p - a_3\left(K_m(t)f(\lambda) + C_m\dot{\lambda}\right) \tag{16}$$

$$z_g\mathrm{DOF} : m_g\ddot{z}_g = -K_{zg}^T z_g - C_{zg}^T \dot{z}_g + a_3\left(K_m(t)f(\lambda) + C_m\dot{\lambda}\right) \tag{17}$$

$$y_p\mathrm{DOF} : m_p\ddot{y}_p = -K_{yp}^T y_p - C_{yp}^T \dot{y}_p - a_2\left(K_m(t)f(\lambda) + C_m\dot{\lambda}\right) \tag{18}$$

$$y_g\mathrm{DOF} : m_g\ddot{y}_g = -K_{yg}^T y_g - C_{yg}^T \dot{y}_g + a_2\left(K_m(t)f(\lambda) + C_m\dot{\lambda}\right) \tag{19}$$

$$x_p\mathrm{DOF} : m_p\ddot{x}_p = -K_{xp}^T x_p - C_{xp}^T \dot{x}_p + a_1\left(K_m(t)f(\lambda) + C_m\dot{\lambda}\right) \tag{20}$$

$$x_g\mathrm{DOF} : m_g\ddot{x}_g = -K_{xg}^T x_g - C_{xg}^T \dot{x}_g - a_1\left(K_m(t)f(\lambda) + C_m\dot{\lambda}\right) \tag{21}$$

Where we can define the dynamic transmission error as follows:

$$\lambda = \left(-X_p + X_g\right)a_1 + \left(Y_p - Y_g\right)a_2 + \left(Z_p - Z_g + r_p^m\varphi_p - r_g^m\theta_g\right)a_3 + e(t) \tag{22}$$

Let's review the literature to understand the advancements made in the dynamics of bevel gears, particularly focusing on models with eight-DOFs —or seven-DOFs after accounting for DTE. In 2010, Yinong et al. [60] investigated the impact of asymmetric mesh stiffness on the dynamics of spiral bevel gear systems, developing an eight-DOFs model. This study revealed that asymmetric stiffness can significantly affect vibration amplitudes, leading to increased noise and wear. Mohammadpour et al. [61] developed an eight-DOFs model to examine the multi-physics interactions in differential hypoid gears, including thermal and lubrication factors. Their study highlighted the importance of considering thermal and lubrication effects alongside mechanical dynamics to ensure reliable gear performance under varying operational conditions. In another study [62], they analyzed how tapered roller bearing supports affect the dynamic behavior of hypoid gear pairs in differentials. They found that bearing stiffness and damping characteristics significantly impact vibration levels and the overall stability of the gear system. In 2016, Li et al. [63] carried out an investigation on the dynamic behaviors of face gears with different tooth profile modifications by considering an eight-DOFs model. This study showed that specific modifications could enhance load distribution and reduce vibrations, leading to improved gear performance. In Ref [64] a study on the nonlinear vibration characteristics of spiral bevel gears was published, focusing on the complex behaviors such as periodic and chaotic vibrations that these systems could exhibit under various conditions. Cao et al. [65] focused on the relationship between gear dynamics and tribology. They developed a tribo-dynamic model for spiral bevel gears, which integrated an eight-DOFs nonlinear dynamic model with a mixed lubrication model. This innovative approach allowed them to consider the complex interactions between gear contact geometry, entraining flow angles, and real surface roughness. Their tribo-dynamic model revealed how tribology significantly influenced gear dynamics, particularly near the resonance points, where the gear vibrations were most pronounced. By simulating the fatigue life of the gears, using a rolling-contact fatigue model, they demonstrated that traditional static models often underestimate the impact of dynamic forces on gear wear and fatigue. The results from their model showed that dynamic conditions could significantly reduce the fatigue life of gears, especially under high-stress conditions. For instance, the contact fatigue lives at 10400 rpm and 21200 rpm under the tribo-dynamic condition decreased 31.2% and 46.5% compared to the lives under quasi-static conditions.

Further studies devoted to coupled bending-torsional vibrations, in spiral bevel gears, used an eight-DOFs model to identify bifurcation points where the system's dynamic behavior changed significantly, which was vital for predicting and preventing stability issues [66]. The main novelty that set their dynamic model was the inclusion of bearing clearances functions, which introduce additional nonlinearity into the dynamic model alongside the traditionally considered backlash function between mating teeth, Fig. 9-a. In 2017, Cai et al. [67] investigated the influence of system parameters on the performance of a nutation drive with double circular arc spiral bevel gears. Their results provided valuable insights into how variations in parameters like nutation angle and spiral angle affect transmission efficiency and stability. In 2019, Chen et al. [68] proposed a dynamical model incorporating elastic ring squeeze film dampers. They showed that these dampers effectively reduce vibrations and improve the system stability, especially under high-load conditions. In the same year, Dong et al. [69] conducted a study to analyze the dynamic characteristics of power split spiral bevel gear systems. A linear time-varying torsional vibration model was developed, eliminating rigid body displacement and verifying dynamic and static power flows. The study found that increasing the meshing damping ratio reduces vibration, while higher speeds lead to an increment of vibrations. Experimental strain gauge measurements at the gear tooth root closely matched theoretical calculations, validating their model despite minor errors due to low loading and measurement accuracy.

Recently, researchers have made significant advancements in improving the vibration control of bevel gear systems. Chen et al. [70] proposed a dynamic model which considers finite length squeeze film dampers (FLSFD) as a method of passive vibration control. Their goal was to improve vibration attenuation in bevel gear systems using FLSFD, which were typically not widely used in such applications. In another study, they compared the dynamic behaviors of the system with and without the FLSFD [71]. The stability and bifurcation characteristics were evaluated using Lyapunov exponents, characteristic multipliers, bifurcation diagrams, phase portraits, and Poincaré sections. Their results showed that the application of FLSFD significantly reduces the occurrence of various types of bifurcations, such as saddle-node, Hopf, and period-doubling bifurcations. In 2021, Mu et al. [72,73] proposed ease-off flank modification method to optimize the gear tooth surface to minimize LTE and meshing impact, which are the main sources of vibration in gear transmissions. Their optimization variables, in their proposed approach, are machine settings of pinion working surface such as radial setting, roll ratio, machine root angle, and so on. In 2022, Gou et al. [74] developed an eight-DOFs dynamic model for a straight bevel gear system, considering multi-state meshing and time-varying parameters. They identified three primary meshing states in the gear system: drive-side meshing, back-side contacting, and tooth disengagement.

Another study in 2023 focused on a spiral bevel gear and planetary gear train, which is part of a high-power-density bucket elevator system [75]. They investigated how changes in rotational speed and input power affect the dynamic mesh forces. Higher rotational speeds tended to reduce the mesh force, while higher power levels increased it. Another application of bevel gears, where a high level of power is required, is the Helicopter transmission systems. Mu et al. [36] introduced an active pre-control strategy to optimize the performance of bevel gears. An eight-DOFs model was employed to simulate the effects of different control strategies on the gear's dynamics. They performed an optimization to reduce the root mean square of vibration acceleration of the spiral bevel gear transmission. The optimization process used genetic algorithms to adjust the pinion machining parameters and improve the meshing performance. Their results proved that, integrating both mechanical and control system dynamics, provides insights into how pre-control strategies can minimize wear and enhance gear longevity under varying operational conditions. In the same year, Fei et al. [76] investigated a nonlinear dynamic characteristics of spiral bevel gears, particularly focusing on the impact of time-varying thermal deformation and friction. They considered the interaction between thermal effects and mechanical dynamics, which is crucial for

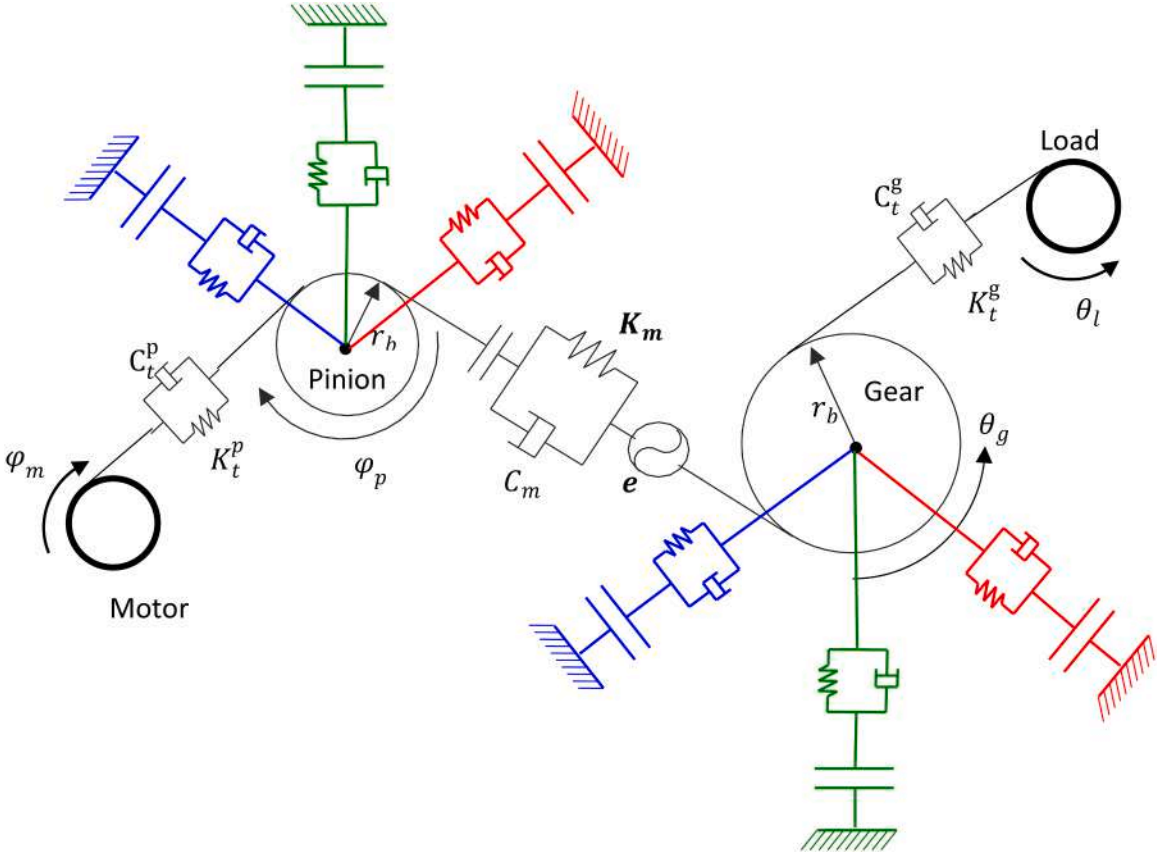

Fig. 10. The dynamical model with ten-DOFs.

understanding how temperature variations influence gear performance. Their results showed that thermal deformation reduces system stability by decreasing backlash, leading to potential gear jamming, and time-varying friction increased the system's nonlinearity and made it more prone to chaotic behavior, reducing overall stability. Finally, Cai et al. [77] addressed the challenge of modeling the TVMS of spiral bevel gears with varying surface roughness. They studied how different surface conditions affect the gear's dynamic behavior by considering three different cases: unground machining (surface roughness of 0.8 μm), grinding machining (surface roughness of 0.4 μm), and polishing (surface roughness of 0.2 μm). They found that higher surface roughness leads to lower meshing stiffness, which in turn increased the dynamic transmission error and reduced the system's stability. This indicates that smoother gear surfaces are beneficial for reducing vibrations and improving gear performance.

2.3.4. Tenth degrees-of-freedom model

Incorporating the degrees of freedom associated with the motor and the load into an eight-DOFs dynamical model extends the system to a comprehensive ten-DOFs framework. This enhancement enables a more robust analysis, allowing for the evaluation of not only the translational vibrations of the gear system but also the significant impact of motor and load inertia on dynamics. Moreover, a model with ten-DOFs enables us to account for the effects of bearing stiffness and damping, as well as the torsional shaft stiffness of both the pinion and the gear. By incorporating these factors, we can better understand their influence on vibration characteristics, resonance conditions, and overall system stability, leading to more accurate predictions and improved design optimization. In 2023, Jorani et al. [78] proposed such a linear ten-DOFs dynamical model with TVMS, emphasizing the understanding and predicting the system's response to gear cracks. In the following year, Talakesh et al. [38] developed a nonlinear dynamical model including both translational and rotational motions of the pinion, gear, and the input/output shafts.

Fig. 10 represents a dynamic model for a ten-DOFs system [38], this model comprehensively captures the system's dynamics by including both translational and torsional degrees of freedom. However, by defining the concept of dynamic transmission error, the model simplifies, ultimately resulting in a nine-DOFs model. The 10dofs model is represented by the following equations:

$$\varphi_p\text{DOF}:\ I_p\varphi_p'' = F_{gz}r_p + K_t^m(\varphi_m - \varphi_p) + C_t^m\left(\varphi_m' - \varphi_p'\right) \tag{23}$$

$$\theta_g\text{DOF}:\ I_g\theta_g'' = -F_{gz}r_g + K_t^l(\theta_l - \theta_g) + C_t^l\left(\theta_l' - \theta_g'\right) \tag{24}$$

$$\varphi_m\text{DOF}:\ I_m\varphi_m'' = T_m + K_t^m(\varphi_p - \varphi_m) + C_t^m\left(\varphi_p' - \varphi_m'\right) \tag{25}$$

$$\theta_l\text{DOF}:\ I_l\theta'_l = -T_l + K_t^l(\theta_g - \theta_l) + C_t^l(\theta'_g - \theta'_l) \tag{26}$$

$$z_p\text{DOF}:\ m_p\ddot{z}_p + K_{zp}z_p + C_{zp}\dot{z}_p = F_{gz} \tag{27}$$

$$z_g\text{DOF}:\ m_g\ddot{z}_g + K_{zg}z_g + C_{zg}\dot{z}_g = -F_{gz} \tag{28}$$

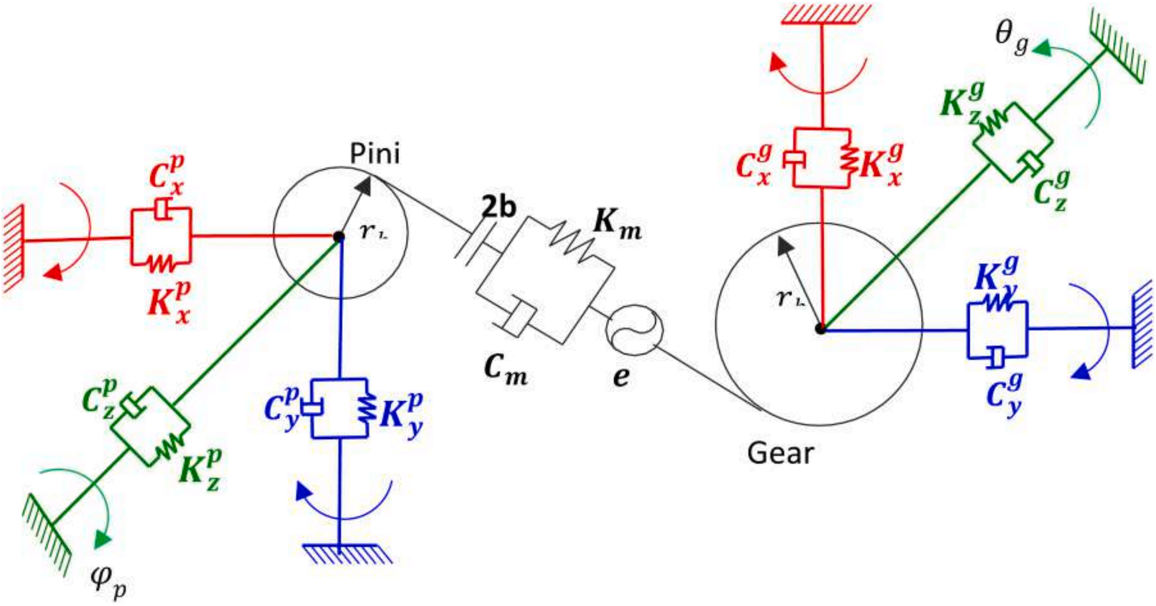

Fig. 11. The dynamical model with all 12 degrees of freedoms.

$$y_p\text{DOF}: \; m_p\ddot{y}_p + K_{yp}y_p + C_{yp}\dot{y}_p = F_{gy} \tag{29}$$

$$y_g\text{DOF}: \; m_g\ddot{y}_g + K_{yg}y_g + C_{yg}\dot{y}_g = -F_{gy} \tag{30}$$

$$x_p\text{DOF}: \; m_p\ddot{x}_p + K_{xp}x_p + C_{xp}\dot{x}_p = F_{gx} \tag{31}$$

$$x_g\text{DOF}: \; m_g\ddot{x}_g + K_{xg}x_g + C_{xg}\dot{x}_g = -F_{gx} \tag{32}$$

By defining the dynamic transmission error:

$$\lambda = (-X_p + X_g)a_1 + (Y_p - Y_g)a_2 + \left(Z_p - Z_g + r_p^m\varphi_p - r_g^m\theta_g\right)a_3 + e(t) \tag{33}$$

2.3.5. Twelve degrees-of-freedom model

In a gear system, each individual component—both the gear and the pinion—has originally six degrees of freedom. These DOFs include: 3 Translational DOFs, corresponding to the potential movement of the gear or pinion along the three spatial axes (X, Y, and Z); 3 Rotational DOFs, corresponding to the potential rotation of the gear or pinion around the same axes, (φ, θ, and ψ). Even though no interactions of the remaining parts of the powertrain are considered, the gear and the pinion system have twelve-DOFs. However, in practice, not all these DOFs are independent or active. When we introduce constraints into the system, some of these degrees of freedom are effectively reduced or coupled together. Defining DTE, allows to rewrite the initial system of twelve second order ordinary differential equations into an eleven second order differential equations, without loose of accuracy when one is interested in the vibrations, i.e. perturbations of the operating state. This reduction reflects a more realistic model where certain motions are restricted or coupled, allowing for more accurate analysis and prediction of the system's behavior in practical scenarios, see Fig. 11. The key point is that developing a dynamical system with twelve-DOFs allows us to investigate the effect of gyroscopic forces on the system's dynamic response where we can observe the effect of torsional speed on the system [79],

The first investigation to develop a twelve-DOFs dynamic model for bevel gears dates back to the 1980s, when Chao and Cheng [80] proposed a model to evaluate the dynamics of bevel gears. However, their model neglected the effect of backlash, which introduces nonlinearity into the system. This model is used for a while [81] until in 2003, when Cheng and Lim [82] considered the nonlinearity due to the backlash in the dynamical model for a hypoid gear. Their focus was on understanding how the time-varying mesh

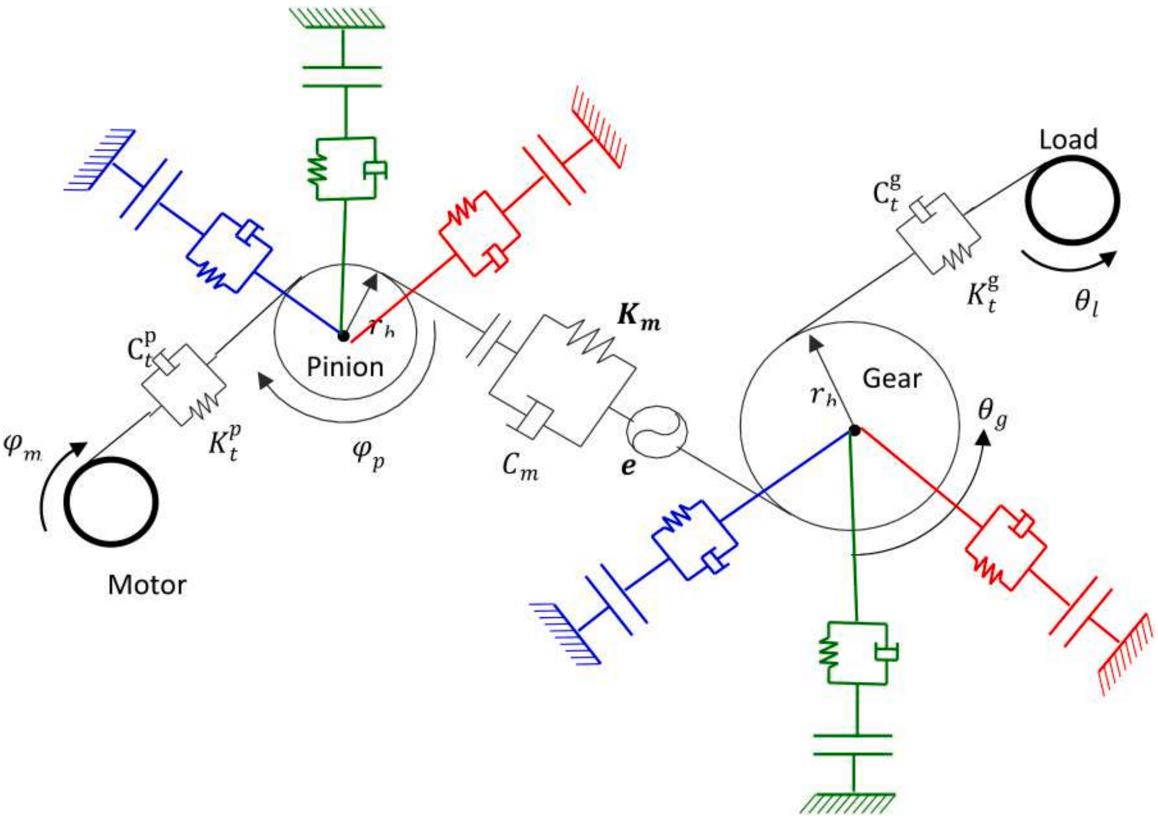

Fig. 12. The dynamical model with all 14 degree of freedoms.

parameters and backlash influence the dynamic response, particularly under different loading conditions. They observed that, at low loads, the system exhibited rich nonlinear behavior, including chaotic and sub-harmonic responses. As the load increased, the system became more linear, with fewer jump discontinuities and tooth impacts. Wang et al. [83,84] continued their work on the previous model by introducing a new approach to incorporate the concept of MS in the dynamic model, which was “exact time-varying harmonic mesh model” compared with the fundamental harmonic method. In general, the mesh stiffness is expressed by Fourier series; when they mentioned fundamental harmonic method means only the fundamental harmonic, i.e., first harmonic in the Fourier expansion, is considered. In 2012, Hua et al. [25] noted that the traditional lumped parameter models are computationally efficient but are limited in their ability to accurately represent the dynamics of the shaft-bearing structures. Therefore, they proposed a synthesis approach that integrates finite element data. This approach used effective mass and inertia elements derived from finite element analysis to improve the accuracy of the lumped parameter model.

Yavuz [85] introduced a nonlinear time-varying dynamic model of a drivetrain that included a spiral bevel gear pair, shafts, and bearings. Indeed, they simulated FE modeling of the shaft-bearing assembly with the mesh coupling by commercial FE software ANSYS®. The gear shafts were modeled using Timoshenko beam finite elements, which allowed for an accurate representation of their dynamic behavior. They used the Multi-Term Harmonic Balance Method to transform the system’s nonlinear differential equations into a set of nonlinear algebraic equations. The continuous-time Fourier transform was used to calculate Fourier coefficients, avoiding convergence issues that arise in large nonlinear systems. The recent development of dynamical model for bevel gear systems are conducted to evaluate the effect of parameters such as: bearings configurations [86], complex interactions between different gear types [30,43,79], lubrication conditions [87], torque fluctuations and external disturbances, self-excited vibration and rubbing impact [88], and multiple excitation sources, including aerodynamic forces and structural vibrations [89].

2.3.6. *Fourteen degrees-of-freedom model*

The dynamic model with fourteen degrees of freedom, Fig. 12, offers an additional framework enhancement for analyzing the behavior of gear systems, where not only all the 12 degrees of freedom of the gear and pinion are considered, but also those associated with the load and motor are integrated into the equations. This extensive modeling approach allows for a detailed examination of the system, capturing the intricate interactions and influences that various factors exert on the overall dynamics. By including these additional DOFs, the model enables a more accurate prediction of system behavior under various operational conditions, providing valuable insights into the performance and reliability of the gear system. However, using a dynamic model with a high number of DOFs, such as a fourteen-DOFs model, requires a robust solver capable avoiding numerical instabilities. This increased computational demand can make the use of such a detailed model less time-effective, especially in situations where the added complexity does not provide proportionally greater insights. In these cases, a model with fewer degrees of freedom may be more practical, offering a reasonable trade-off between accuracy and computational efficiency, making it a more appropriate choice for certain applications.

The study, done by Cheng and Lim [90], in 1998 was an early investigation into the noise issues associated with high-speed hypoid gears, where authors considered a linear fourteen-DOFs dynamical model. The focus here was on understanding the sources of vibration and noise, particularly in automotive applications. The study emphasized the role of gear transmission error as a significant contributor to noise. Later in 1999, Lim and Cheng [91] investigated the specific impact of the pinion offset on the system dynamics with the same dynamical model. Their study highlighted how variations in the pinion offset could alter the dynamic response, particularly affecting the vibration characteristics and the overall stability of the gear system.

In 2007, Peng and Lim [92] brought the effect of backlash into their dynamical model; so, they developed a nonlinear model where the dynamic mesh force did not transfer, while the level of vibration was in the threshold of positive and negative half of backlash. Later in 2009, the same authors [93] published another study to evaluate the effect of gyroscopic forces on the system dynamics. The gyroscopic effect in power transmission systems becomes increasingly significant at high rotational speeds. As components like gears and shafts spin rapidly, gyroscopic forces—arising due to the variation of the angular momentum of the rotating parts—can significantly influence the dynamic behavior of the system. These forces act perpendicular to the axis of rotation, introducing additional stress and vibrations. Their results demonstrated that, while the gyroscopic effects might not always be significant, they could lead to noticeable changes in the vibratory response at certain high frequencies, particularly in systems where the shape and inertial properties of the gear components emphasized gyroscopic effects. Yang and Lim [94] developed their model to evaluate the effect of bearing stiffness on the nonlinear dynamics of a hypoid gear set, neglecting the effect of bearing clearance but considering the time-varying bearing stiffness. It is shown that, while the variation in bearing stiffness did not significantly alter the dynamic mesh forces between the gear teeth, it had a substantial effect on the dynamic loads experienced by the bearings themselves. This was particularly evident under heavy load conditions, where the impact of these variations was more pronounced. Meanwhile, Hua et al. [95] investigated how different shaft-bearing configurations impact on the dynamics and the gear mesh characteristics of spiral bevel gears. Their results showed that the configuration of the shaft and bearings could greatly affect the dynamic behavior of the gear system. Feng et al. [96] introduced a time dependent friction force into the dynamical model to highlight how the friction influences the dynamic response, particularly in high-load conditions. Their results proved that time-varying friction coefficient contributions were significant at low-to-medium operating speed (mesh frequency), and relatively negligible at higher speed. In the process of improving the dynamic model, Wang et al. [97], in 2013, modified the previous model to account for multi-point mesh. This study introduced a multi-point mesh model that captured the detailed dynamic behavior of each gear tooth pair. This model facilitated the investigation of gear surface wear and fatigue, which are critical for the longevity and reliability of hypoid gears.

In 2016, Jinli et al. [98] carried out a study to evaluate the bearing stiffness on a shaft-final drive system. They found out that increasing bearing stiffness could initially reduce the vibration response of the system. However, once a critical value of stiffness was reached, a further increment of the stiffness diminishes the effect on vibration reduction, indicating that there was an optimal range of bearing stiffness that minimizes vibration. Feng and Song [99] investigated how different geometric design parameters, such as pressure angle, root fillet radius, and tooth thickness ratio, influence both the static strength and dynamic behavior of spiral bevel gears. They varied the geometric parameters, i.e., increasing the pressure angle, root fillet radius, and tooth thickness ratio, to enhance the gear's performance. The optimized gears showed a 44 % increase in the number of cycles before failure, indicating a substantial improvement in durability. Shi and Lim [100] developed a dynamical model where they considered not only time-varying bearing stiffness but also the nonlinearity raised from gear backlash and bearing clearance. It is shown that the interaction between gear backlash and bearing clearance can either be strong or weak, depending on the relative stiffness of the gear mesh and bearing supports. Strong coupling led to significant changes in the system's dynamic response, including shifts in resonance frequencies and jump discontinuities, while weak coupling allowed these nonlinearities to be analyzed separately. Yang et al. [101], in 2018, proposed a new gear surface modification methodology based on curvature synthesis by generated high-order transmission error for spiral bevel and hypoid gear systems. Indeed, they evaluated the impact of different types of transmission errors, i.e., Parabolic TE, Fourth-Order TE, and Sixth-Order TE. It was shown that the parabolic TE had higher peak to peak amplitude compared to the other two types of TE. Therefore, higher-order TEs resulted in better dynamic performance due to their smoother transitions and lower fluctuation amplitudes.

2.3.7. Multi degree- of-freedom model

Numerous studies have been conducted considering systems with a higher number of degrees of freedom to examine multi mesh gear systems, i.e. more than one gear pair. Fig. 13 represents some of these MDOF models, where researchers evaluated the influence of some parameters such as geometric accuracy deviations [102] and crowning parameters [103] on the dynamics of powertrains. Besides, the effects of geometric parameters, such as offsets of unbalance and pitch cone angles, on the dynamic performance of spiral bevel gears were evaluated, showing that these parameters affect the critical speeds and the unbalance in the responses [104,105]. Additionally, a study on the influence of gear mesh fluctuations and defects, i.e., eccentricity defect, profile errors, and cracked teeth, on the dynamic behavior of a two-stage straight bevel system, found that these factors can lead to a reduction in the fatigue life [106]. In 2021, Song et al. [45] presented a detailed study on the dynamic effects of bearing roller diameter errors in hypoid gear systems. As shown in Fig. 13-c, the overall dynamic model has eight nodes, and each node has six DOFs, resulting in a total of 48 DOFs in the coupled model. Hua and Chen [107] developed a 92-DOFs model to investigate how the elasticity of roller bearings influences the dynamic behavior of spiral bevel gears, with two different configurations, straddle mounted pinion with three bearings and overhung mounted pinion with two bearings. It is found out that the radial stiffness of bearing directly influences the pinion shaft bending modes, which leads to a change in the pinion torsional modes coupled to the pinion shaft bending modes and then leads to a change in the dynamic mesh force responses. Besides, the axial stiffness of bearings plays a significant role in the frequencies where the system may experience a resonance, lower axial stiffness value, lower frequency regarding the resonance. Yang and Lim [108] investigated the effects of propeller shaft bending flexibility on the dynamics of hypoid geared rotor systems, particularly focusing on off-highway vehicle drivelines. Their results showed that the effect of the shaft bending flexibility depends on the mode shape. The radial reaction forces and moments of the pinion-supporting bearings are significantly influenced by the propeller shaft's bending elasticity, whereas the axial reaction forces are only minimally affected. In 2024, Liu et al. [47] introduced a parametric probabilistic regression (p-PR) model based on data-driven and parametric modeling theories. The p-PR model was trained using probabilistic regression and maximum likelihood estimation, allowing it to accurately predict vibration signals in both time and frequency domains.

3. Dynamical parameters

In this section, we introduce the key parameters influencing the nonlinear dynamic behavior of bevel gears, including geometric transmission error, mesh stiffness, backlash, and friction. Each of these factors is discussed in depth to highlight their origins, and dynamic effects. The aim of this section is to provide a clear understanding of how these parameters interact and contribute to the excitation sources and nonlinearities in gear systems, thereby setting the stage for dynamic modeling and analysis.

The main goal behind designing a gear pair is transferring power through the transmission systems, where at least two gears mate each other; the driver is called pinion and the driven is called gear. The torque that is transferred between two mated gears, i.e., dynamic mesh torque (DMT), is not exactly the same input torque, which is applied to the pinion, due to the existence of energy dissipation in the system and also vibrations that induce variable contact force and consequently torque. Indeed, the transferred force, i.e., dynamic mesh force (DMF), is calculated by the following equation:

$$DMF = K_m(t,\lambda)f(\lambda) + C_m\dot{\lambda}$$

$$DMT = (K_m(t,\lambda)f(\lambda) + C_m\dot{\lambda})r_b$$

Where $K_m(t,\lambda)$ is the mesh stiffness, $f(\lambda)$ is the backlash function, C_m is the damping ratio, r_b is the base radius, and λ is the dynamic transmission error. The key point about damping ratio is that researchers have mainly defined it as a viscous damping; Shi and Lim [109] conducted a study to evaluate the difference between a viscous damping model and non-viscous damping model. The dynamic mesh force is applied alongside the line of action, which is characterized by pressure and spiral angles, see Fig. 14.

The characteristics parameters are explained in detail further, to provide a deeper understanding of their impact on the system's behavior.

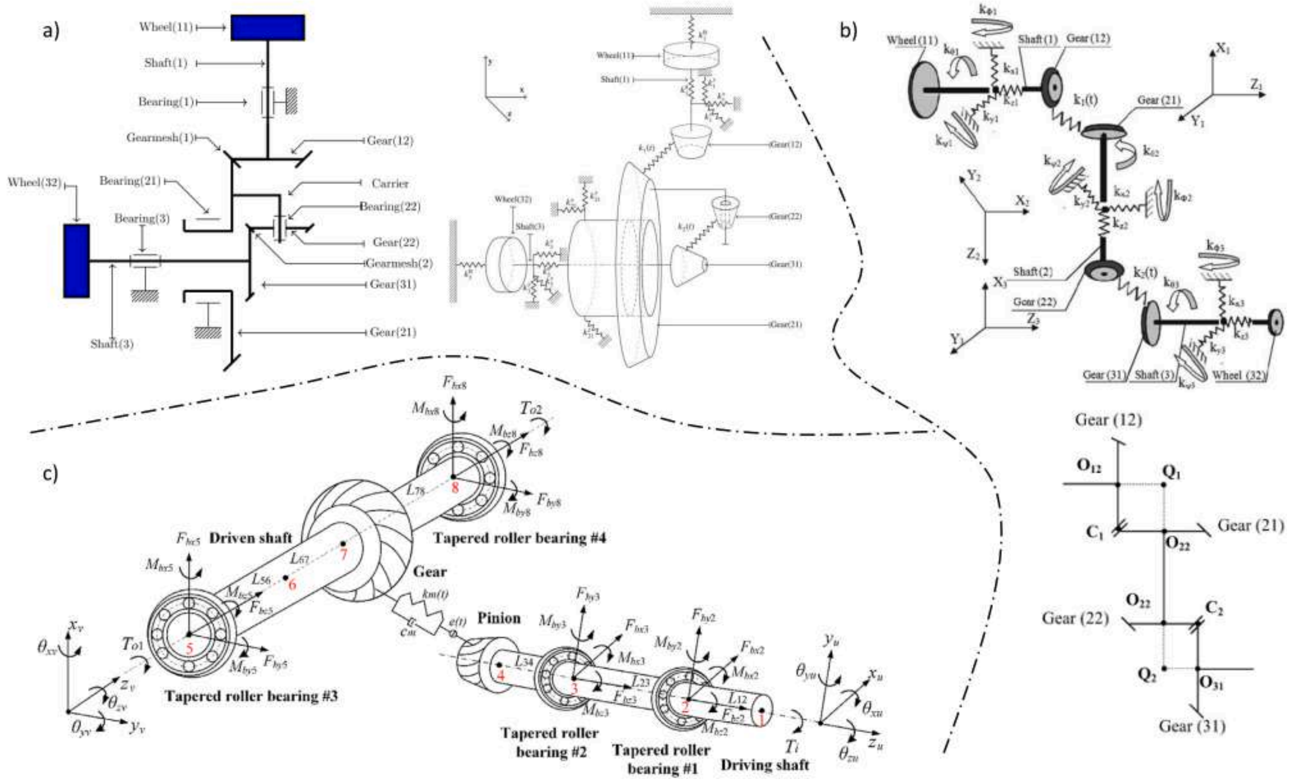

Fig. 13. The dynamical model with multi degrees of freedom, a) Dynamic modelling of differential bevel gear system with 18-DOF [102], b) Dynamic modelling of two stages bevel gear system with 21-DOF [106], c) Dynamic modelling of system with 48-DOFs [45].

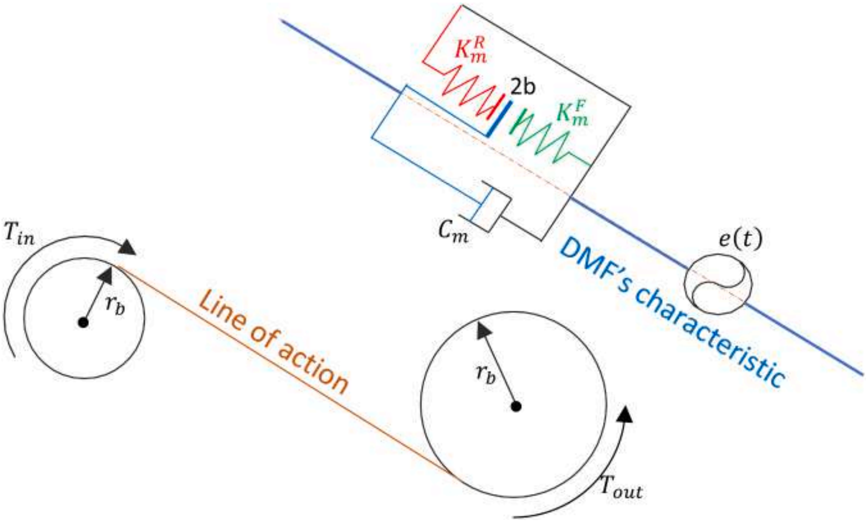

Fig. 14. The line of action and DMF's characteristic elements.

3.1. Geometric transformation error

The geometric transmission error, also known as kinematic transmission error, refers in bevel gears to the local gaps or deviations between mating teeth in a gear pair, see Fig. 15. These errors can result from various factors, including mounting and manufacturing errors, as well as modifications to the teeth profiles [30,51]. The presence of GTE impacts on the smoothness of the motion transmission along the line of action between the teeth. In spiral bevel gears, the GTE is often an inherent characteristic due to intentional mismatches introduced during the manufacturing process. These mismatches are designed to allow the gear mesh to perform effectively even in the presence of misalignment, manufacturing errors, or high torque levels that cause the gears to deflect to positions different from their intended design [51].

The measurement and analysis of GTE are typically conducted through unloaded tooth contact analysis. A very low torque is applied to the gear pair, allowing the measurement of the error without significant elastic deformation [89]. This analysis reflects the rigid body transmission error, which is primarily influenced by geometric factors rather than elastic deformation. The GTE is a critical factor in determining the mesh stiffness of the gear pair, which directly affects the dynamic behavior of the system [50,51,105].

When addressing the changes in the GTE over time, it necessitates a periodic function to express its variation as time progresses. The Fourier series is commonly employed to represent periodic functions such as the GTE, it incorporates the mesh frequency (fundamental harmonics of the series), $\omega_m = \frac{2\pi}{60} N_p n_s$, where n_s is the input shaft speed and N_p is the number of pinion teeth; the GTE for both forward and reverse motions, denoted as $e^F(t)$ and $e^R(t)$ respectively, is given by:

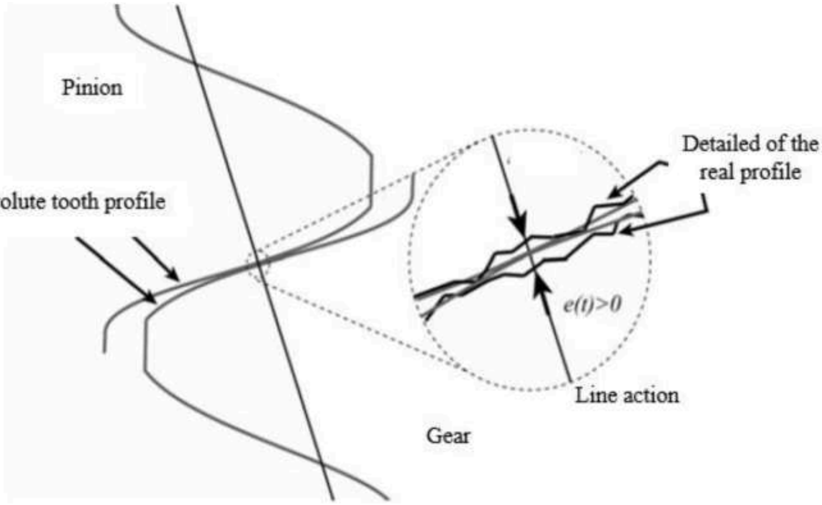

Fig. 15. Geometric transmission error due to tooth profile imperfections [51].

$$e(t) = \begin{cases} e^F(t) = e_0^F + \sum_{j=1}^{s} ea_j^F \cos(j\omega_m t) + \sum_{j=1}^{s} ea_j^F \sin(j\omega_m t) & \text{forward motion} \\ e^R(t) = e_0^R + \sum_{j=1}^{s} ea_j^R \cos(j\omega_m t) + \sum_{j=1}^{s} ea_j^R \sin(j\omega_m t) & \text{reverse motion} \end{cases} \tag{34}$$

Where, s is the number of harmonics of the Fourier series and $s = (N_1 - 1)/2$. N_1 is the number of time steps over the period, which is identified by the instant when one tooth comes to contact and leave the contact [50].

3.2. Mesh stiffness

Mesh stiffness is a critical parameter in the dynamic analysis of gear systems. It represents the resistance of gear teeth to the deformation under load, influencing the transmission error, vibration, and noise levels of gear systems. The mesh stiffness is inherently time-varying [51,53,110], asymmetric [111], and force-dependent [112,113].

The mesh stiffness of gear pairs varies over time as different teeth come into and out of contact during the rotation, see Fig. 16. This variation is influenced by factors such as gear geometry, material properties, and load distribution. The mesh stiffness variation causes fluctuations in the dynamic mesh force, which in turn excites vibrations in the gear system. Such parametric fluctuation might lead resonance and dynamic instability in the system and may contribute to the premature failure of gear teeth due to fatigue [50]. To calculate the MS of the system, one needs to obtain the elastic deflection of the gear pair. Conducting a loaded tooth contact analysis provides us with loaded static transmission errors, which contains not only the elastic deflection, but also the geometric transmission error. Then, the loaded static transmission errors must be subtracted from the geometric transmission error, obtained from unloaded tooth contact analysis, in order to evaluate the elastic deformation to be used for calculating the elastic stiffness and/or the elastic contact force. A periodic function is required to express stiffness variation vs. time. Again, due to the periodicity of the stiffness variation, which coincides with the periodicity of the transmission error, it can be expanded Fourier series, which is given by:

$$K_m(t) = k_0 + \sum_{j=1}^{S} a_j \cos(j\omega_m t) + \sum_{j=1}^{S} b_j \sin(j\omega_m t) \tag{35}$$

Due to the unique curvilinear geometry and kinematics of spiral bevel and hypoid gears, Fig. 17, the mesh stiffness for the drive and coast sides is inherently different, leading to asymmetry in the system [51,111]. Consequently, the mesh stiffness must be defined for both forward and reverse motions, denoted as $K_m^F(t)$ and $K_m^R(t)$ respectively:

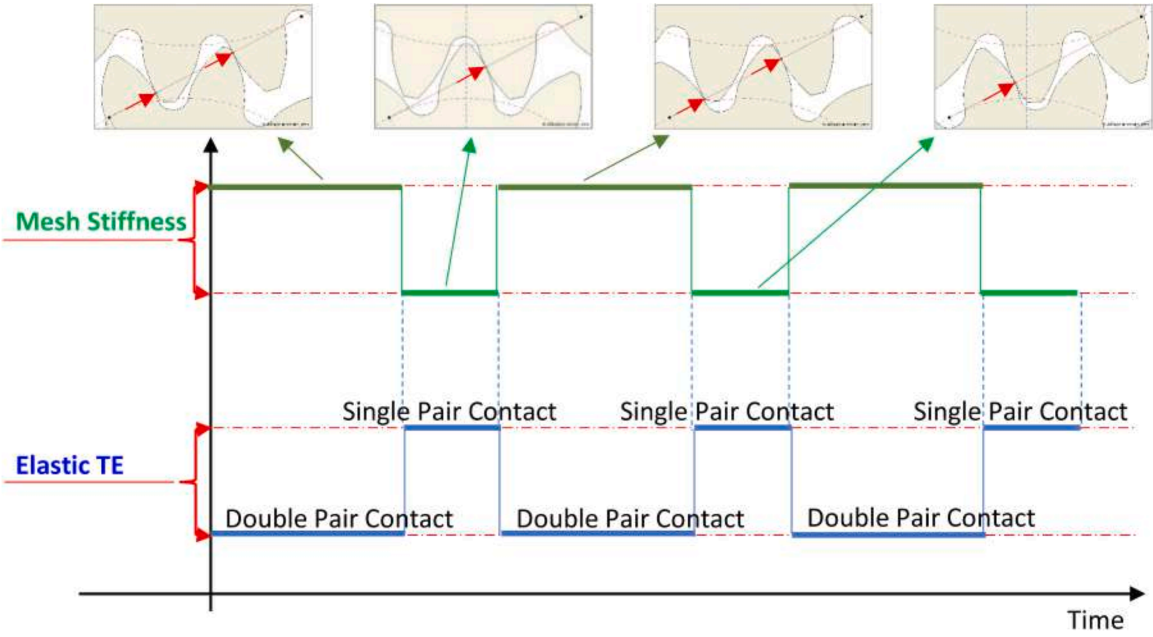

Fig. 16. The time varying mesh stiffness.

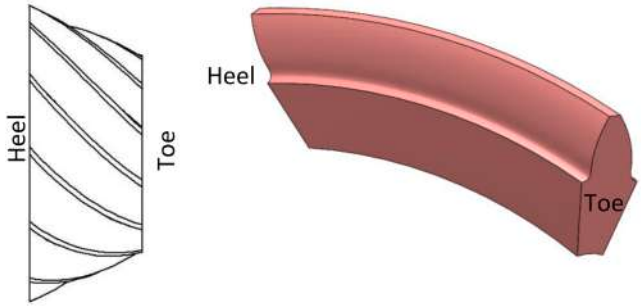

Fig. 17. Representing the tooth trace of a spiral bevel gear where the MS is different in driven and coast sides.

$$K_m\ (t) = \begin{cases} K_m^F(t), & \lambda - e > b \\ \\ K_m^R(t), & \lambda - e < -b \end{cases} \tag{36}$$

Due to the Hertzian deflection, there is a nonlinear and non-smooth relation between the applied force and the relative deflection; consequently, the mesh stiffness would be force dependent, and it brings a nonlinear characteristic into the dynamical model where the dynamic interaction between the meshing teeth is considered [112]. In general, there are three approaches to calculate the mesh stiffness: **Average Secant Mesh Stiffness;** this traditional method calculates the stiffness as the ratio of mesh force to mesh deflection along the line of action. It is widely used due to its simplicity, but it may not capture the nonlinear behavior of the gear contact adequately, especially under varying load conditions [51]. **Local Tangent Mesh Stiffness;** this approach provides a more precise evaluation of stiffness by considering the local slope of the force-deflection curve at each mesh position. It is particularly useful in dynamic analysis as it reflects the instantaneous changes in stiffness due to variations in load and gear contact conditions [114]. **Mesh Force Interpolation Function;** this method interpolates the dynamic mesh forces based on precomputed force-deflection relationships for different mesh positions. It provides a more accurate representation of the non-linear behavior of gear contact forces,

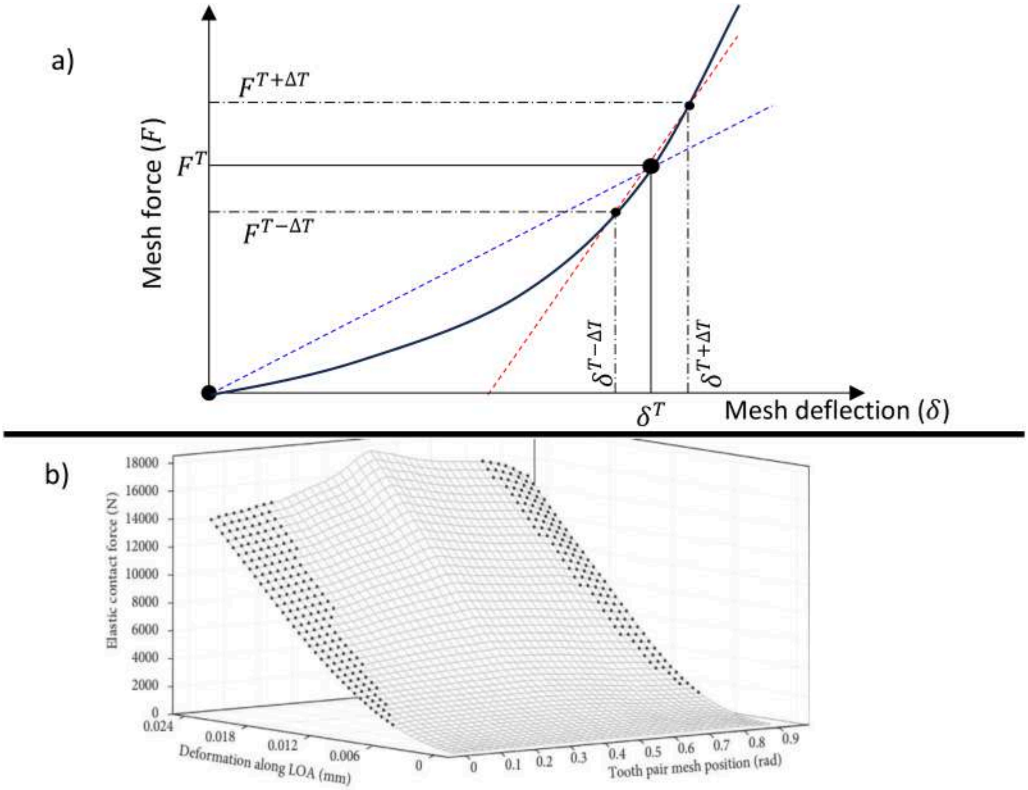

Fig. 18. Mesh stiffness approaches: a) illustration of the local tangent mesh stiffness and the average secant mesh stiffness, and b) a typical fitted force-deflection curve [112].

especially in cases of significant mesh deflection, see Fig. 18 [112].

$$K_{average\ secant} = \frac{F^T}{\delta^T} \tag{37}$$

$$K_{local\ tangent} = \left.\frac{\mathrm{d}F}{\mathrm{d}\delta}\right|_{\delta=\delta^T} \approx \frac{1}{2}\left[\frac{F^T - F^{T-\Delta T}}{\delta^T - \delta^{T-\Delta T}} + \frac{F^{T+\Delta T} - F^T}{\delta^{T+\Delta T} - \delta^T}\right] \tag{38}$$

To accurately capture the TVMS, a multi-point mesh modeling has been proposed. Traditional single-point models may not sufficiently capture the complexity of the contact interactions in hypoid gears, particularly under high torque and high-contact-ratio conditions. Wang et al. [97] developed a multi-point mesh model that integrates three-dimensional loaded tooth contact analysis into a coupled multi-body dynamic framework. This model allows for the detailed simulation of dynamic behaviors for each tooth pair, providing insights into gear surface wear and the prediction of fatigue life. The mesh stiffness, along with STE, plays a crucial role in determining the dynamic behavior of gears; therefore, finding a way to reduce the level of vibration means to optimize the mesh stiffness fluctuations, for example by decreasing the peak-to-peak mesh stiffness [56,101,115].

3.3. Backlash

The backlash in gear systems refers to the small gap or clearance between the meshing teeth of gears. This gap results from manufacturing tolerances or intentional design choices, allowing for slight relative movement between gears before engagement occurs. While a certain amount of backlash is necessary to prevent gears from locking due to thermal expansion, misalignment, or any other reasons, excessive backlash can lead to undesirable effects. It can cause inaccuracies in motion control systems leading to positioning errors, vibration, and noise. Moreover, in high-speed applications, excessive backlash may result in shock loads when the gears suddenly engage, potentially leading to increased wear or early failure. Engineers must carefully balance the need for backlash to accommodate tolerances with the need for precision and smooth operations. In order to simulate the aforementioned backlash, a

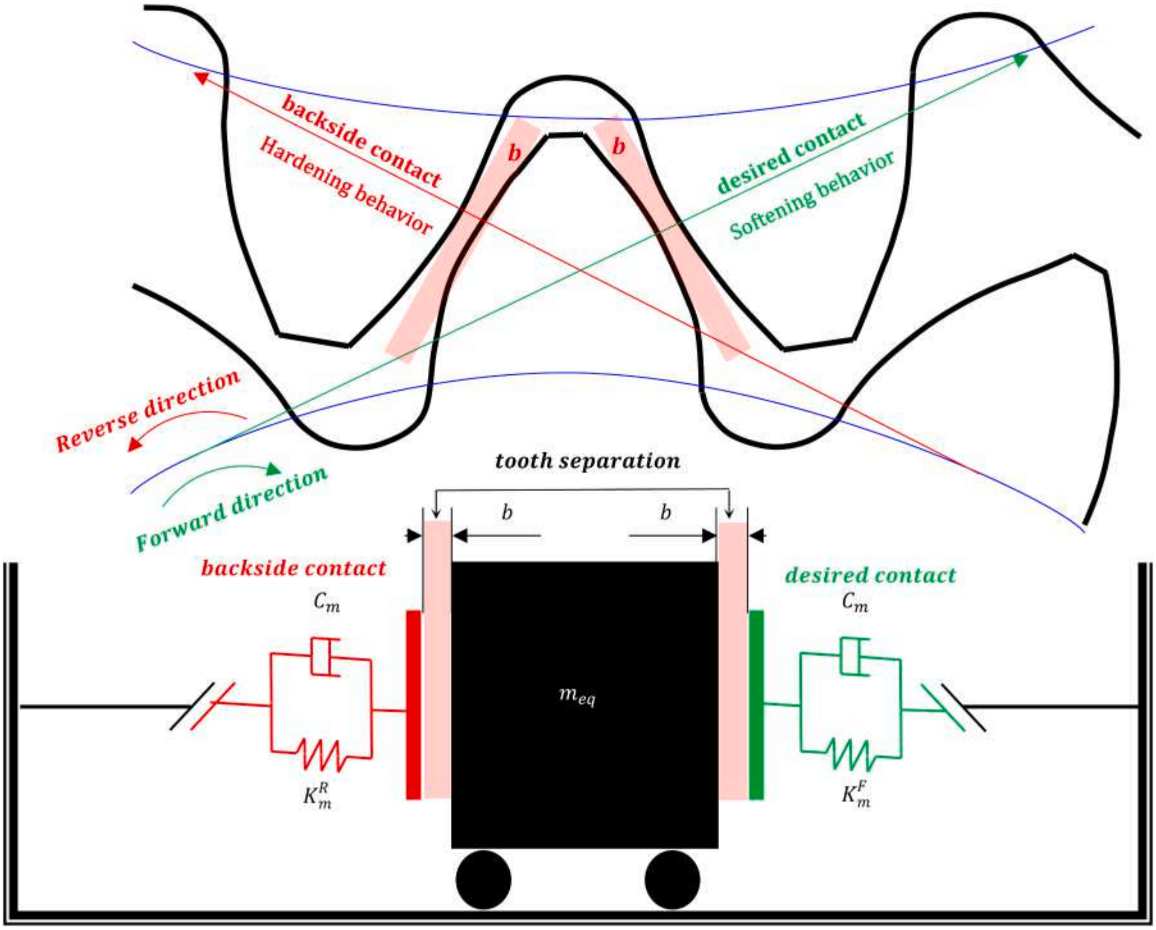

Fig. 19. Equivalent gear model and backlash function.

backlash function is often used, it is denoted here as $f(\lambda - e)$, it is a piecewise linear function described in Eq. (39) and represented in Fig. 19.

$$f(\lambda - e) = \begin{cases} \lambda - b - e, & \lambda - e > b \\ 0, & -b \leq \lambda - e \leq b \\ \lambda + b - e, & \lambda - e < -b \end{cases} \tag{39}$$

When this function is multiplied by the stiffness, it produces the elastic restoring force. If the value of $\lambda - e$ lies within the range $-b$ to $+b$, it indicates a loss of contact between the teeth, which is an undesirable situation. Ideally, when $\lambda - e > b$, the gear teeth are in forward contact, which is the desired scenario, and the stiffness, K_m^F, is determined from static nonlinear finite element analysis during this forward motion. However, if $\lambda - e < -b$, the teeth experience unwanted backside contact, leading to a double-sided impact, where the stiffness, K_m^R, is derived from static simulations during reverse motion. Tooth separation is likely to appear in resonance conditions and when the torque is very low with respect to the nominal value, this explains the importance of considering gear backlash non-linearity in dynamic models. The aforementioned method is a common approach to introduce the backlash into the dynamical system; however, it is worthwhile noting that most authors considered a constant value for backlash. In some studies, due to the complexity of the system, for example in the case of wind turbines [116–118], authors neglected the effect of backlash in their analyses to simplify the modeling and focus on other critical dynamics. However, this simplification may introduce errors into the dynamic model by eliminating one of the primary sources of nonlinearity in the system, which determines the type of contact: forward contact, tooth separation, or backside impact. In 2014, Qiu et al. [119] developed a dynamical model where backlash is expressed as a Fourier series, Eq. (40), incorporating both the mean value and harmonic components:

$$b(t) = b_0 + \sum_{j=1}^{Nb} b_j \cos\left(j\omega_b t + \varphi_b\right) \tag{40}$$

Their study revealed that the primary harmonic amplitude of gear backlash had a significant effect on the nonlinear characteristics of the gear system. As the amplitude increased, the system transitioned from quasi-periodic motion to chaotic motion, thereby increasing the degree of nonlinearity and reducing system stability.

3.4. *Friction*

Friction plays a critical role in the operation and performance of gear systems and strongly influences vibrations. During operations, friction occurs at the contact surfaces between the gear teeth. This frictional force can impact on the efficiency, the wear, and the lifespan of the gear system. While some level of friction is inevitable, understanding the role and effects of the friction is essential for optimizing the gear performance, as a thin lubricant film forms between meshing teeth, reducing friction and improving efficiency [120]. Factors such as lubrication, surface finishing, and material properties influence the amount of friction of the system. This section delves into the mechanics of friction in gear systems, examining its causes, effects, and the methods used to mitigate its adverse impacts.

In 2011, Feng et al. [96] introduced an advanced friction model. This model departed from the traditional constant friction coefficient approach by incorporating a physics-based, time-varying friction coefficient that accounted for mixed elasto-hydrodynamic lubrication conditions. The study revealed that the frictional behavior in gear systems was significantly influenced by factors such as torque load, surface roughness, and lubrication properties. The results demonstrated that under heavier torque loads, friction had a more pronounced impact on the dynamic response of the gear system. Their results showed that the effects of time-varying friction coefficients were particularly significant at low to medium operating speeds, while their influence diminished at higher speeds due to the dominance of full-film lubrication. In 2012, Karagiannis et al. [52] developed a dynamical model for a hypoid gear pair, which are known for their intricate geometry and challenging lubrication conditions, to consider the interaction between gear teeth under a mixed thermo-elasto-hydrodynamic regime of lubrication. The lubricant behavior was modeled considering non-Newtonian shear properties under high-pressure and temperature conditions, which significantly affects the frictional behavior of the gears. They observed a significant impact of non-Newtonian shear and boundary friction on the dynamic behavior of hypoid gear pairs. Their results showed that under high-speed conditions, the lubricant film undergoes substantial thinning, leading to increased asperity interactions. This effect is particularly pronounced near resonant conditions, where the loss of teeth contact may occur, exacerbating noise and vibration issues such as axle whine. They noted that, apparently, the friction torque only marginally affects the torsional dynamics of the system; note that their dynamical model was single degree of freedom. Later in 2014, Mohammadpour et al. [120] examined the elasto-hydrodynamic lubrication regime, considering both viscous and boundary friction, with the lubricant's non-Newtonian behavior. They showed that transmission efficiency was higher at higher vehicle speeds, due to better lubricant film formation. However, at low speeds, the frictional losses are more pronounced, leading to lower efficiency. Cao et al. [65] used a tribo-dynamic model to evaluate the effect of different parameters such as the film thickness, the flash temperature, and the friction coefficient under different operating conditions. Their study emphasized the complex interaction between tribology and gear dynamics, particularly under high-speed and heavy-load conditions.

A comprehensive discussion on friction and gear dynamics was due to Wang et al. [121,122], 2021, they investigated the dynamic contact stiffness in lubricated interfaces. They developed a transient mixed lubrication model to analyze how these fluctuations affect the contact stiffness in a spiral bevel gear. The model accounted for various factors, including the non-Newtonian behavior of lubricants, the squeezing effect, and the interaction between asperity contact and lubrication film. Their results showed that surface roughness played a critical role; increasing the roughness within a certain range enhanced stiffness but also led to more significant fluctuations. Indeed, they expressed that “the contact stiffness shows an increasing trend as roughness increases within a certain range, along with drastic fluctuations when the roughness is too serious.”, although Cai et al. [77] found that smoother gear surfaces, i.e., lower surface roughness, increase meshing stiffness and reduce vibrations;

They noted that traditional models, which rely on Hertzian contact or full-film lubrication assumptions, significantly underestimate the complexity of real-world gear dynamics. Cao et al. [123] investigated how different contact paths (heel, middle, and toe) affect the dynamics and the efficiency of spiral bevel gears under lubricated conditions, besides to predicting the friction and film thickness. Their theory was based on a mixed elasto-hydrodynamic lubrication model. Their results revealed that the toe contact path resulted in higher meshing efficiency compared to the heel and middle paths, primarily due to lower sliding velocity and smaller contact radii. In 2022, Lafi et al. [124] proposed a new approach to consider the uncertainty in friction coefficients, which could vary due to changes in lubrication quality and temperature fluctuations over time. Further discussion on the dynamic behavior of bevel gears were due to Tian et al. [125], which investigated the implementation and effects of SFDs in bevel gear systems; in particular, they were focused on the influence of the SFD’s nonlinear oil film forces on the system’s dynamics. Their results showed that the operating state of the system can change from a chaotic state to a periodic or quasi-period motion after using SFD, which prolongs the system’s life. However, using unsuitable squirrel cage elastic support may deteriorate the system’s operating state and increase the system’s vibration. To conclude this comprehensive discussion on the dynamics and friction of spiral bevel gears, the study by Li et al. [76] represented the effects of time-varying thermal deformation and friction. The friction between meshing tooth surfaces was modeled as a time-varying factor that was influenced by several variables, including the position of the meshing point, load, and temperature. The study showed that the thermal deformation significantly impacts on the gear dynamics by reducing backlash as the gear teeth heated up. This reduction in backlash affected the vibration characteristics of the system, generally leading to decreased vibration displacement. The friction, which varied over time due to changes in temperature and other factors, plays a crucial role in the system’s dynamics. The study found that when time-varying friction was considered, the gear system was more prone to chaotic behavior. Therefore, friction could destabilize the gear system, particularly under dry running conditions. When both time-varying thermal deformation and friction were considered together, the system exhibited complex nonlinear behaviors. While thermal deformation tended to stabilize the system by reducing vibration, the addition of time-varying friction could push the system into chaotic states.

4. Effect of imperfections

The aim of this section was to investigate the influence of geometric and manufacturing imperfections—namely eccentricity, misalignment, surface defects, and wear—on the dynamic behavior and vibration characteristics of bevel gears. These deviations, whether due to production tolerances, assembly errors, or operational degradation, can significantly alter the meshing conditions and load distribution, leading to undesirable phenomena such as modulation, increased dynamic loads, chaotic responses, and noise emissions.

Imperfections, mounting and manufacturing errors in gear pair sets are critical factors that influence the performance, efficiency, and durability of mechanical systems. Various types of imperfections can arise during the manufacturing process, assembly, and even during operation, each of which can significantly affect the gear set’s functionality. In this section, we explore the research conducted to investigate the role of gear imperfections and errors, e.g., crack, eccentricity, misalignment, and tooth breakage, along with the phenomena that may arise from them, such as whining noise, and wear. The key consideration when analyzing imperfections in a dynamic gear pair system is that mesh stiffness and geometric transmission error inherently capture and represent the effects of these imperfections or errors. Indeed, these two parameters are affected by the existence of errors in the system, so that the dynamic responses of the system react to the changes appeared in the behavior of mesh stiffness or geometric transmission errors.

4.1. Crack and tooth breakage

Cracks in mechanical systems are structural defects that can develop over the time due to fatigue, overloading, misalignment, poor material quality, surface defects, corrosion, or thermal stresses. Cracks can reduce a gear’s load-carrying capacity, increase noise and vibration, accelerate wear, and potentially lead to catastrophic failure. The crack typically initiates at the point of maximum stress at the tooth root and propagates in a direction influenced by the tooth’s geometry and loading conditions. The presence of a crack reduces the mesh stiffness; indeed, the deeper the crack progresses, the more the MS is reduced [38]. This MS reduction leads to additional peaks in the frequency spectrum associated with the defect’s rotational frequency [33,78]. The presence of the crack introduces sidebands around the meshing frequencies and their harmonics in the frequency spectrum. The rotational frequency of the gear with the crack appears prominently, and as more teeth crack, the vibration levels rise further [106]. In 2013, Karray et al. [126], investigated the dynamic behavior of bevel gear transmissions, specifically focusing on how local damages such as cracks affected the system’s performance. They modeled the tooth cracks as reductions in mesh stiffness, approximately 50 %, which occurs every time the cracked tooth engages. The presence of a crack increases the vibration levels and introduces amplitude modulation, which can be detected through spectral analysis. Han et al. [33] carried out a study which proved that cracks introduced new frequencies into the system’s vibration profile, which could be detected through spectral analysis, useful for early fault detection. Indeed, they used a time-domain waveform and amplitude spectrum to illustrate the differences between a healthy condition and a cracked fault condition. In the healthy state, the acceleration waveforms remain consistent across all meshing cycles, and the amplitude spectrum contains only the primary meshing frequency and its harmonics. Under fault conditions, periodic fluctuations appear in the time-domain waveform due to the faulty tooth engaging in the meshing process. This leads to variations in the waveform across multiple meshing cycles. In the corresponding amplitude spectrum, sidebands emerge around the meshing frequency and its harmonics, influenced by the rotational frequency of the faulty component. Fig. 20 represents their gear model where there was a crack on the root of gear tooth.

It is worth mentioning that, in presence of faults like a chipped or missing tooth, the FFT revealed additional sidebands around the gear mesh frequency. Indeed, the FFT analysis is crucial for detecting and diagnosing faults in gear systems. The appearance of sidebands and increased amplitudes at specific frequencies are reliable indicators of gear faults. Tooth breakage leads to a significant reduction in time-varying mesh stiffness. The degree of stiffness reduction correlates directly with the severity of the tooth breakage. For example, a minor breakage results in a slight decrease in stiffness, while more severe breakages cause a substantial drop [42]. Dewangan et al. [41] during their research observed that, as the gear shaft rotated, the missing tooth caused a shift from double tooth pair contact to single tooth pair contact, reducing the overall MS during these periods. Additionally, there were instants when no teeth were in contact, resulting in zero MS. This fluctuation in stiffness occurred repeatedly with each revolution of the shaft. In 2024, Kumar et al. [48] investigated the impact of different faults, i.e., chipped and missing teeth, on the dynamic behavior of the gear system. They observed that in faulty gears, particularly with missing teeth, the sidebands around the gear mesh frequency became more pronounced. This was due to the repeated impact each time the missing tooth part of the gear came into contact with the other gear, causing a periodic disturbance in the vibration signal.

4.2. Eccentricity

The eccentricity refers to a deviation or offset between the actual axis of the gear and the intended rotational axis. This can occur when the center of the gear does not perfectly aligns with the center of the shaft or the axis is meant to rotate around; such factor leads to noise and amplitude modulation in the vibration signal [127]. The eccentricity creates additional forces that modulate the meshing frequencies, leading to sidebands around these frequencies in the frequency spectrum [106]. Peng et al. [128] investigated the radial deviation of the gear's geometric center from its rotational axis on the dynamics of bevel gear. They illustrated how the presence of eccentricity (considered two offsets 10 μm and 50 μm) causes additional peaks in the dynamic mesh force. These peaks corresponded to the jump phenomena, where the system's response changed rapidly, particularly when the mesh frequency reached certain critical values. They observed that, depending on the eccentricity value, the system might experience softening or hardening behavior. The eccentricity in bevel gear sets is a critical factor that can affect the performance, noise, vibration, and longevity of the gears. It is important to minimize the eccentricity through precise manufacturing, careful assembly, and regular maintenance to ensure optimal gear performance.

4.3. Misalignment

Misalignment refers to the lack of proper alignment between two or more components within a system. Misalignment can lead to reduced efficiency, increased wear and tear, unwanted vibrations, and eventually, the failure of the machine. There are three main kinds of misalignments in a bevel gear system, see Fig. 21: **Angular misalignment**; this occurs when the angle between the gear and the pinion faces deviates from the intended design. Since bevel gears are designed to mesh at a specific angle, even a small deviation can lead to improper contact between the gear teeth, causing uneven wear and increased noise. **Radial misalignment;** it happens when the centerlines of the gear and pinion are not aligned correctly, causing an offset. **Axial misalignment;** it refers to a displacement along the axis of the gear or pinion. For bevel gears, this type of misalignment alters the contact pattern between the gear teeth, leading to improper meshing and increased wear.

In 2017, Kolivand et al. [129] analyzed the effect of different misalignments on the NVH performance. Later in 2019, Shi and Lim [130] evaluated the effect of angular misalignment on the dynamics of hypoid gear set. They understood that angular misalignment caused an additional bending moment on the pinion shaft, which increased the dynamic load on the bearings supporting the shaft. Molaie et al. [58] investigated a dynamical model for spiral bevel gear, they considered three different misalignments: axial misalignment, radial misalignment, and combination of these two misalignments. They observed that, among three cases study, axial misalignment alone is the most detrimental, often leading to chaotic behavior at low excitation frequencies ($\frac{\omega}{\omega n} < 1$). Moreover, radial

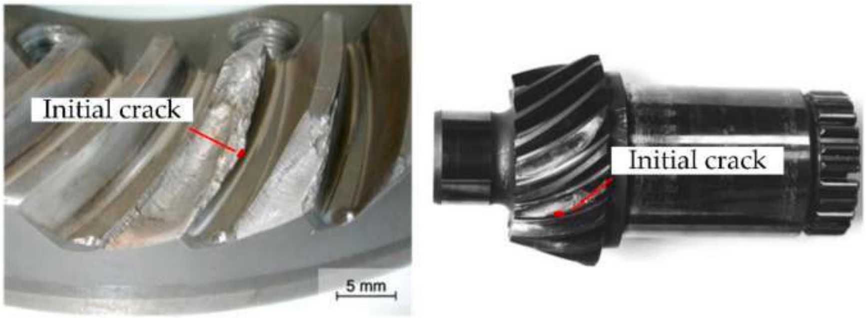

Fig. 20. Crack and tooth breakage in bevel gears [33].

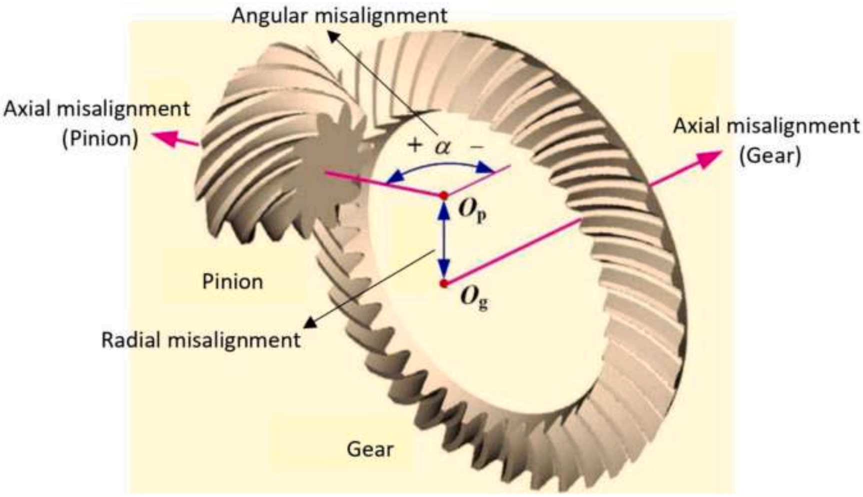

Fig. 21. Different types of misalignments in gear transmission systems.

misalignment affects gear dynamics but is less severe than the other two types of misalignments.

4.4. Surface imperfections

The precision in the manufacturing of bevel gears is critical, as even minor surface imperfections can lead to notable deviations in their performance. Surface imperfections, such as roughness, profile deviation, waviness, or localized defects like pits and scratches, can arise due to several factors, including machining errors, material inhomogeneities, and wear over time, see Fig. 22. These imperfections disturb the ideal tooth contact conditions, which are designed to ensure smooth transmission of motion and force between the gears. When surface defects are present, the gear teeth do not mesh perfectly, leading to irregularities in the contact patterns. These irregularities manifest as fluctuations in the contact force and mesh stiffness, which in turn induce vibrations and noise, altering the dynamic response of the gear system [36,131].

A wide range of research has been conducted to evaluate the effect of some imperfections such as crowning [103], profile deviation [103], surface roughness [44,77,122], tooth surface error distribution for tool errors [131], tooth surface mismatch [37], and runout [44]. Profile errors arise from deviations between the theoretical and actual tooth profiles, which could be due to manufacturing imperfections or wear. In terms of spectra of exciting force, these defects lead to an increment of the frequency level at the meshing frequency and its harmonics, but unlike eccentricity, they do not cause modulation [103,106]. The surface roughness of the gear teeth significantly influenced the dynamic response, with the asperity contact and the lubricant's squeezing effect playing a key role. High levels of roughness intensify the vibrations and can lead to double side impacts between the teeth [122]. Indeed, an increased roughness leads to greater deformation and lower meshing stiffness, which in turn affects the vibration characteristics of the gears. The rougher the surface, the greater the impact on the gear's dynamic performance, leading to increased vibration and noise levels [77].

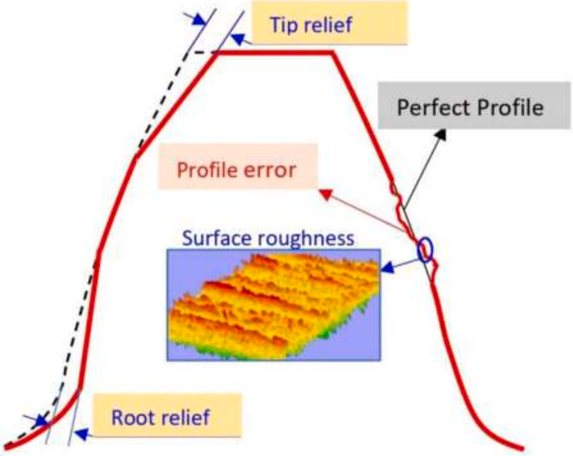

Fig. 22. Different types of tooth profile imperfection might appear in gear pairs [121].

Unfinished gears have high surface roughness peaks, which lead to inefficient and noisy gear operation. Abrasive flow finishing could improve the surface quality and reduced roughness peaks, resulting in a smoother surface that enhances the meshing efficiency between gears [44]. Moreover, the manufacturing process might apply a catastrophic error on tooth surface such as errors in the cutting tools. Positive and negative errors in cutter radius cause the effective mesh points to shift towards different directions on the gear tooth, altering the line-of-action and increasing the likelihood of dynamic issues under high torque. Like the cutter radius error, positive and negative blade angle errors shift the mesh points along the height of the tooth. The positive blade angle error tends to push the contact towards the tooth tip, potentially leading to edge contact and higher vibrations [131].

4.5. Wear and Whine noise

Wear phenomenon in bevel gears refers to the gradual deterioration of the gear teeth surfaces due to the relative motion and contact stress experienced during operation, especially in hypoid and spiral bevel gear sets. Wear can manifest in various forms, such as abrasive wear, adhesive wear, and surface fatigue. Abrasive wear occurs when hard particles or asperities on the gear surface cause material removal, leading to rough and pitted surfaces. Adhesive wear, on the other hand, arises when the metal surfaces of the gear teeth come into direct contact under high loads, leading to material transfer between the gears and the formation of wear particles. Surface fatigue, often seen as pitting or spalling [132], is caused by repeated cyclic loading, which induces cracks under the surface and material loss from the gear teeth. Wear is not uniformly distributed along the meshing path. The wear tracks are asymmetric [40], with the highest wear occurring on the side of the contact area that is closer to the exit of the meshing path. This asymmetry is due to the varying directions of the sliding velocities of the contacting surfaces [133]. It is important to monitor wear progression in gear systems. This can be done, for example, using the modulation signal bispectrum approach, a technique that helps identify errors in the system [40]. The modulation signal bispectrum method is based on detecting modulations in signals caused by mechanical faults. The modulations often occur due to irregularities or defects in rotating machinery, such as gears or bearings, which cause periodic variations in the vibration signal.

The whine phenomenon is characterized by a continuous, steady-state tonal sound that is emitted due to the meshing of gears under specific conditions, such as high torque or resonant frequencies. It is likely to occur at lower oil temperatures at specified coasting operating conditions, usually at higher speeds [134]. Whine noise might be as a result of transmission errors that occur when there is a deviation in the ideal meshing of gear teeth under load conditions [135]. Choi et al. [136] conducted a study which proved that the whine noise was linked to transmission errors and high temperatures. In 2014, Guo and Sun [137] proposed a model to evaluate and control the whine noise by considering both gear tooth contact analysis and vibroacoustic analysis.

5. Modal analysis

Modal analysis is a key tool which gives us the ability to determine the natural frequencies, mode shapes (Fig. 23), and damping characteristics. A significant amount of research has been conducted on modal analysis, as its findings are both important and unavoidable for the design and optimization of gear systems. Alongside developments in dynamic modeling, modal analysis itself is evolving, providing more precise and comprehensive insights into gear system behavior over time.

Resonant modes are characterized by their natural frequencies, and if gears are excited near these frequencies, stresses can escalate quickly, causing failures. In 1992, Drago and Brown [138] carried out a study to find a way to shift resonant frequencies away from operating excitation frequencies. Also, they conducted frequency response analyses to predict dynamic response levels. Meanwhile, Ramamurti et al. [139] proposed a modal analysis approach with a combination of finite element method and cyclic symmetry concepts to accurately predict the natural frequencies and mode shapes with reduced computational resources. In 2001, Cheng and Lim [140] conducted a study to identify the critical elastic modes that significantly contribute to gear-mesh-induced vibrations by

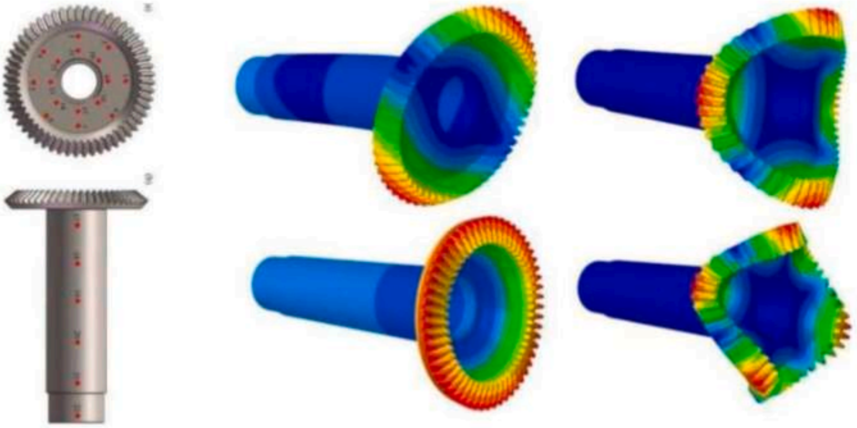

Fig. 23. Vibration mode shapes of a bevel gear shaft obtained in Ref [125].

analyzing the free and forced vibration responses of the system. Their results showed that critical out-of-phase torsional modes are particularly sensitive to geometric transmission error excitations and are critical in determining the severity of the system's vibrational response. They observed that the translation or torsion compliances tended to affect the lower modes, while the bending stiffness controlled the higher order ones.

A wide range of studies have been carried out to analyze and evaluate the effect of different parameters on the natural frequencies and mode shapes, such as: gyroscopic effects [141–143], rocking motion [143], gear meshing [144], support stiffness [81],

Table 1
List of studies done on diagnostics and prognostics.

Year	Subject	Authors [Ref.]	Approach	Defects
1996	Evaluation of a Vibration Diagnostic System for the Detection of Spiral Bevel Gear Pitting Failures	Townsend et al. [149]	Time synchronous averaging of the vibration signal	Pitting, spalls, eccentric
2008	Classification of Features by SVM and PSVM Extracted Using Morlet Wavelet for Fault Diagnosis of Spur Bevel Gear Box	Saravanan et al. [150]	Support Vector Machines; Proximal Support Vector Machines	Tooth breakage, cracks, face wear
2008	Root Cause and Vibration Signal Analysis for Gearbox Condition Monitoring	Bartelmus [151]	Coherence Gearbox Condition Monitoring Method	Pitting, scuffing, erosion
2009	Classification of Features by Fast Single-Shot Multiclass PSVM Using Morlet Wavelet for Fault Diagnosis of Spur Bevel Gear Box	Saravanan and Ramachandran [152]	Proximal Support Vector Machine	Tooth breakage, cracks, face wear
2009	Fault Diagnosis of Spur Bevel Gear Box Using Discrete Wavelet Features and Decision Tree Classification	Saravanan, Ramachandran [153]	Discrete Wavelet Transform	Tooth breakage, cracks, face wear
2009	Vibration Detection of Local Gear Damage by Advanced Demodulation and Residual Techniques	Combet et al. [154]	Advanced demodulation technique, the advanced residual technique	Tooth missing, cracked teeth
2009	Vibration-based Fault Diagnosis of Spur Bevel Gear Box Using Fuzzy Technique	Saravanan et al. [155]	Fuzzy logic-based classification technique	Tooth breakage, cracks, face wear
2010	Fault Diagnosis of Spur Bevel Gear Box Using Artificial Neural Network and Proximal Support Vector Machine	Saravanan et al. [156]	Artificial Neural Networks, Proximal Support Vector Machines	Tooth breakage, cracks, face wear
2011	Component-level Fault Diagnostics of a Bevel Gear Using a Wavelet Packet Transform	Hong et al. [157]	Wavelet packet transform	Cracked teeth, broken teeth
2013	Case study: the Failure of Bevel Gears in an Aircraft Engine	Siddiqui et al. [158]	Visual inspection, chemical analysis, energy dispersive spectroscopy	Cracks, wear, pitting
2014	Approach to Extracting Gear Fault Feature Based on Dominant Intrinsic Mode Function	Liu et al. [159]	Dominant Intrinsic Mode Function	Gear crack, localized wear
2014	Fault Diagnosis of Rotating Machinery Based on Kernel Density Estimation and Kullback-Leibler Divergence	Zhang et al. [160]	Data-driven fault diagnosis method	Worn gear, crack, missing/ broken tooth
2015	Fault Diagnosis on Bevel Gearbox with Neural Networks and Feature Extraction	Waqar et al. [161]	Feature extraction techniques, Back-Propagation Neural Network	Teeth faults
2016	Diagnostic Features for the Condition Monitoring of Hypoid Gear Utilizing the Wavelet Transform	Skrickij et al. [162]	Vibro-acoustic signal measurement, acoustic emission analysis	Teeth faults
2016	spectral kurtosis technology for adaptive vibration condition monitoring of multi-stage gearboxes	Gelman et al. [163]	Wavelet spectral kurtosis	Pitting gear faults
2018	Spur Bevel Gearbox Fault Diagnosis Using Wavelet Packet Transform and Rough Set Theory	Huang et al. [164]	Wavelet Packet Transform, Rough Set theory	Tooth breakage, tooth surface wear
2020	Fault Diagnosis of Bevel Gears Using Neural Pattern Recognition and MLP Neural Network Algorithms	Keleşoğlu et al. [165]	Sound signal analysis combined with machine learning techniques	Wear, chipped tooth, broken tooth
2021	A Novel MPE-LPP-ELM Recognition Method for the Fault Diagnosis of Spiral Bevel Gears	Lingli et al. [166]	Multiscale Permutation Entropy, Locality Preserving Projection, Learning Machine algorithms	Broken teeth, severe scratches
2021	Automated Gearbox Fault Diagnosis Using Entropy-Based Features in Flexible Analytic Wavelet Transform Domain	Ramteke et al. [167]	Entropy-based features extracted in the Flexible Analytic Wavelet Transform domain	Wear faults
2021	Fault Detection and Severity Level Identification of Spiral Bevel Gears Under Using Artificial Intelligence Techniques	Tayyab et al. [168]	Artificial Neural Networks, and K-Nearest Neighbors	Tooth broken
2022	Sensitive Misalignment-Based Dynamic Loaded Meshing Impact Diagnosis Mechanism for Aviation Spiral Bevel Gear	Ding et al. [27]	Data-driven finite element model	Misalignment
2023	A Dynamic Mode Decomposition-Based Deep Learning Technique for Prognostics	Akkad, and He [169]	Dynamic Mode Decomposition, deep learning techniques	Remaining Useful Life
2023	A New Framework for Intelligent Fault Diagnosis of Spiral Bevel Gears with Unbalanced Data	Wei et al. [170]	Data-level and algorithm-level strategies	Cracks, pitting, missing tooth, missing foot, wear and tear
2024	Variational Attention-Based Interpretable Transformer Network for Rotary Machine Fault Diagnosis	Li et al. [171]	Variational Attention-Based Transformer Network	Bearing faults, gear faults

shaft-bearing configuration [95], and flexibility of the gear components [29], through different approaches like theoretical approach [141,142], experiment [145], and finite element method [139,145]. Wang et al [141,142] carried out modal analyses to consider the interaction between gear-shaft coupling and gyroscopic effects. Their results showed that neither the gear gyroscopic effects nor the shaft flexibility alone have a significant influence on the dynamic transmission error or mesh force spectra. However, when both effects were considered together, the interaction leads to additional structural modes, indicating that the combining the gyroscopic effect and the shaft flexibility is critical for getting an accurate dynamic analysis. Indeed, their study concluded that the interactions between the gear-shaft coupling dynamics and gyroscopic effects strongly affected the resonance behavior of the system, particularly around pinion and gear bending modes. In another study, they evaluated the effect of the bearings on the modal frequencies, the actual bearing stiffness and simply supported assumption were considered [146]. They noted that the gear torsional modes, which are critical in dynamic response analysis, are found at different frequencies depending on the bearing stiffness.

In 2020, Lin et al. [147] evaluated the effect of MS and bearing stiffness on the natural frequencies and mode shapes of a double circular arc spiral bevel gear pair. They found that the increment of the meshing stiffness significantly influences the higher-order natural frequencies; however, it has a minor effect on lower-order modes. Their results showed that changes in the bearing support stiffness, predominantly affects the intermediate-order natural frequencies, with less impact on the lowest and highest modes. In 2023, Yi et al. [148] explored the dynamic characteristics of a coupled gear system, where a planetary gear train and a bevel gear pair are coupled, the system was part of an helicopter transmissions. Their study found that, coupling the gear sets through a shaft increased all the natural frequencies, indicating that, the addition of the shaft introduces new constraints that alter the system's dynamic behavior. They identified a phenomenon called "modal transition," which occurs as the coupling stiffness increases. It means that some natural frequencies remain unchanged when the shaft stiffness is varied, while other natural frequencies show significant changes, indicating a transition between different modes. Their study concluded that the coupling altered both the natural frequencies and vibration modes, which could influence the system's overall vibration response and potential resonance issues.

6. Fault diagnosis

In this final section of the literature review, we present a comprehensive collection of scholars' research focused on diagnostics in bevel gear systems. This compilation serves as a testament to the collective efforts of researchers who have dedicated their expertise to developing innovative methods for early defect prediction in gear systems. Diagnosing errors in gear systems has become a focal point within the research community due to its critical role in maintaining the performance, reliability, and safety of these systems. The importance of early detection cannot be overstated, as it plays a pivotal role in preventing catastrophic failures, such as gear breakage, which can lead to costly downtime and significant operational risks. By identifying potential issues before they escalate, researchers aim to enhance the longevity of gear systems, while ensuring their optimal efficiency and safety.

This part follows a systematic approach, categorizing research based on several key parameters, Table 1: **Year of Publication;** tracking the evolution and trends in diagnostic research over time. **Subject of Research;** identifying the specific aspects of bevel gear diagnostics that each study addresses. **Authors;** highlighting the contributions of leading researchers and institutions in the field. **Techniques Used for Diagnosis;** examining the various diagnostic techniques employed, including machine learning algorithms, signal processing methods, and condition monitoring technologies. **Main Faults Investigated;** detailing the specific types of faults or defects that the research aims to predict, such as pitting, wear, misalignment, and tooth breakage.

7. Conclusion

The dynamic characteristics of bevel gears, such as vibration modes, nonlinearities, and time dependency, have been extensively studied using a combination of experimental, numerical, and analytical approaches. The progression from simple single-degree-of-freedom models to complex multi-degree-of-freedom models reflects the growing of sophistication in gear analysis, addressing the need for accurate prediction and control of dynamic responses in practical applications. The findings highlight the critical influence of factors such as transmission errors, backlash, and gear mesh stiffness on the overall performance and reliability of gear systems. Particularly, the integration of finite element methods with traditional dynamic modeling has proven effective in capturing the nuanced interactions within gear systems, offering valuable insights for optimizing gear design and enhancing durability. Moreover, advancements in experimental techniques have complemented theoretical models, provided a robust validation framework and uncovered practical issues that could affect gear operation.

Despite the significant strides made in this field, challenges remain, particularly in the areas of gear noise reduction, fault diagnosis, and the development of more resilient gear systems under high-stress conditions. Future research should focus on further refining dynamic models, incorporating more realistic operating conditions such as lubrication effects, thermal influences, and material imperfections. Additionally, there is a need for advanced diagnostic tools that can predict more accurately the gear failures, ensuring the continued reliability and efficiency of bevel gears in increasingly demanding applications.

A promising future direction lies in the development of multi-physics models that integrate thermal and elastohydrodynamic effects into dynamic simulations. Such coupling is increasingly important for accurately capturing the real-world behavior of bevel gears operating under high-speed and high power-density conditions, where lubrication regimes and temperature fluctuations can significantly influence the vibrational response. As pointed out in Ref [172], a future direction of gear design suggests a large use of digital-twin technology, see e.g. Ref [173], which offers a transformative approach to intelligent fault diagnosis and RUL prediction by combining real-time monitoring, deep learning, and high-fidelity modeling, enabling predictive maintenance and improved reliability across various industrial sectors such as energy and aerospace. Additionally, combining data-driven techniques with physics-based models may provide powerful tools for enhancing diagnostics and predictive maintenance, particularly in safety-critical applications. These hybrid approaches could improve early fault detection and refine model-based control, better aligning theoretical insights with the practical demands of modern gear systems. Data-driven models hold great promise for enabling accurate damage prediction, remaining useful life (RUL) estimation, and performance optimization in spiral bevel gears. However, one of the fundamental challenges in developing such models is the need for large volumes of high-quality data, which are often difficult to obtain solely through experimental means. Experimental campaigns are limited by cost, time, and the impracticality of reproducing a wide range of low-probability but safety-critical operating conditions. To overcome this limitation, digital twins are increasingly employed to synthetically generate datasets across a variety of simulated conditions, thereby expanding the domain of training data available for data-driven model development. Nevertheless, the reliability of any data-driven model is strongly dependent on the fidelity of the digital twin used to generate the training data. It is therefore essential that digital twins be rigorously validated against experimental data, particularly test-bench measurements, to ensure their predictive accuracy and physical realism, i.e. a digital twin-based monitoring framework is essential [173]; for the specific case of gears, real test-bench data must be used to capture transmission error, dynamic response, and fatigue life with high fidelity and used for validating the digital twins, see e.g [174] where experiments were carried out to validate tooth contact patterns under quasi-static and large-load conditions, showing the importance of physical measurements for confirming simulation accuracy. Digital twin models for tooth surface grinding of non-orthogonal spiral bevel gears can integrate sensitivity analysis and measured data to improve the robustness of manufacturing outcomes and ensure performance under uncertainties [175]. Comprehensive digital twin frameworks for spiral bevel gears that couple tooth contact analysis with structural dynamic modeling and validate simulations with experimental vibration and transmission data, demonstrate high consistency between physical tests and digital predictions [176].

To summarize, while digital twins play a key role in overcoming the limitations of experimental data availability, their effectiveness in supporting data-driven modeling depends on continuous and rigorous validation against real-world test data. Future developments in spiral bevel gear digital twin technology will benefit from hybrid frameworks that integrate physics-based simulation, machine learning, and empirical validation to enhance reliability, generalizability, and predictive performance. Table 2

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CRediT authorship contribution statement

Moslem Molaie: Writing – review & editing, Writing – original draft, Visualization, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Antonio Zippo:** Writing – review & editing, Writing – original draft, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Data curation. **Francesco Pellicano:** Writing – review & editing, Writing – original draft, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Francesco Pellicano reports financial support was provided by Italian Ministry of University and Research. Francesco Pellicano reports financial support was provided by Emilia-Romagna Region. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Abbreviation

Table 2
Used abbreviation in the current paper.

Abbreviation	Meaning
MS	mesh stiffness
KTE	Kinematic transmission error
GTE	geometric transmission error
STE	Static transmission error
UTCA	unloaded Tooth Contact Analysis
LTCA	loaded tooth contact analysis
DTE	Dynamic transmission error

(*continued on next page*)

Table 2 (*continued*)

Abbreviation	Meaning
FEM	Finite Element Method
VFIFE	Intrinsic Finite Element Method
TE	transmission error
DOFs	degree of freedoms
MDOF	multi-degree-of-freedom
SDOF	single degree-of-freedom
FLSFD	finite length squeeze film dampers
SFD	Squeeze Film Damper
DMT	dynamic mesh torque
DMF	dynamic mesh force
TVMS	time-varying mesh stiffness
p-PR	parametric probabilistic regression

Appendix B. written review papers on gear systems

Table 3
List of literature articles written on power transmission systems.

Year	Key points [Ref.]	Year	Key points [Ref.]
2024	Dynamics and fault diagnosis of railway vehicle gearboxes [177]	2021	Bending strength of asymmetric gear [178]
	The advancements in 3d printing for gear design and analysis [179]		Classification of Design Methodologies to Minimize Vibrations in Gears and Bearings [180]
	Mathematical complexities in modelling damage in spur gears [181]		Optimization of cylindrical gear pairs [182]
	Thermal failure analysis of gear transmission system [183]		Mesh stiffness models for cylindrical gears [184]
	Deep learning in planetary gearbox health state recognition [185]		Gear fault diagnosis of planetary gearboxes using acoustic emissions [186]
	Gearbox condition monitoring and signal analysis techniques [187]		Diagnostic and prognostic approaches for gears [188]
	Gear transmission system dynamics [189]		Design and efficiency improvement of worm and worm wheel of a gear motor [190]
	Dynamics of gear systems with gravity effects [191]		The performance of gear with backlash [192]
	Spiral bevel gear manufacturing technology [193]	2020	Classifying, predicting, and reducing strategies of the mesh excitations of gear whine [194]
	Gear scuffing studies [195]		Gear fault models and dynamics-based modelling for gear fault detection [196]
2023	Concentric magnetic gears [197]		Effects of lubrication on gear performance [198]
	Magnetic gear technologies used in mechanical power transmission [199]		Latest developments in gear defect diagnosis and prognosis [200]
	Vibration-based gear wear monitoring and prediction techniques [201]		Logarithmic spiral bevel gear [202]
	Design and analysis of spur gears [203]		Compact gearboxes for modern robotics [204]
Year	Key points [Ref.]	Year	Key points [Ref.]
2022	Vibration signal processing towards data-driven gear fault diagnosis [205]	2020	A practical approach to gear design and lubrication [206]
	Planet load-sharing and phasing in planetary gear sets [207]		Sources of excitation and models for cylindrical gear dynamics [208]
2019	Magnetic gears: topologies, computational models, and design aspects [209]	2018	Polymer spur gears behaviors under different loading conditions [210]
	Frictional contact stress distribution in involute gears [211]		Design optimization of gearbox [212]
	Micro pitting studies of steel gears [213]		Dynamic modeling of gearbox faults [214]
	Failure characteristics of polymeric gears [215]	2017	The dynamic transmission error of a spur gear pair with eccentricities by fem [216]
2016	Decoupling diagnosis of hybrid failures in gear systems using vibration sensor signal [217]	2015	Dynamics of cracked gear systems [218]
	Physics-based models in prognostics: application to gears and bearings [219]	2014	Dynamics modelling of friction draft gear [220]
2012	Different types failure in gears [221]		The Mathematical Model of Spiral Bevel Gears [222]
2011	Study of a spur gear dynamic behavior in transient regime [223]		Planetary and epicyclic gear dynamics and vibrations research [224]
2008	Finite element analyses and simulations of gears and gear drives [225]	2007	Mathematical models used in gear dynamics [226]
2003	Nonlinear vibration of gear transmission systems [227]	1989	Gear housing dynamics and acoustics literature [228]
2002	Gear lubrication [229]	1988	Mathematical models used in gear dynamics [230]
1995	Modeling of automotive gear rattle phenomenon [231]	1978	Friction predictions in gear teeth [232]

Data availability

No data was used for the research described in the article.

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