

# Effects of couple tractions on contact problems at the microscale

Enrico Radi<sup>1</sup>, Mehmet Guler<sup>2</sup>

<sup>1</sup>*Department of Science and Methods in Engineering, University of Modena and Reggio Emilia, Reggio Emilia, Italy*  
E-mail: [enrico.radi@unimore.it](mailto:enrico.radi@unimore.it)

<sup>2</sup>*College of Engineering and Technology, American University of the Middle East, Egaila, Kuwait*  
E-mail: [maguler@alum.lehigh.edu](mailto:maguler@alum.lehigh.edu)

**Keywords:** Contact mechanics, Couple stress elasticity, Thin film.

With the advancement of additive manufacturing technologies, thin film/substrate systems gained much attention in micro- and nano-electromechanical systems (MEMS/NEMS) where the length scales become comparable to the intrinsic length of the material. The simplest theory that allows for the introduction of characteristic lengths is the couple stress theory of elasticity, where rotation gradients are introduced in addition to the usual strains. The related constitutive equations then involve a length parameter,  $l$ , which does not exist in the classical theory of elasticity. Moreover, according to the classical theory, only stress tractions are exchanged by bodies in contact. However, at the microscale the interaction between the bodies in contact is described by a richer set of tractions including higher-order stress components, like couple tractions. Recently, the contact problems between an Eulero-Bernoulli microbeam and a couple stress elastic substrate was investigated in [1] by assuming that the interaction occurs by couple tractions also. Under this assumption, a strong size effect was observed on the internal shearing forces and bending moment of the microbeam, for microbeam lengths comparable to the characteristic length of the substrate. Later, the investigation was extended in [2] by considering a receding contact between the beam and the substrate. It was found that including size dependency results in highly peaked contact pressures near the load application point and significantly affects the contact pressure distribution when the length scales of the substrate and contact zone are comparable. In the present work, we also investigate the size effect in a thin film resting on a couple stress elastic substrate [3]. We assume that a tensile load is applied to one edge of the film. The main objective of this work is to study the effects of geometrical parameters and material mismatch between the film and the substrate on the interfacial shear stress. The problem is reduced to a singular integral equation, which is solved by using a Chebyshev series expansion. The results show that accounting for couple tractions yields a remarkable reduction of the axial load and axial stress within the thin film. Furthermore, the couple tractions are significant compared to the shear stress tractions, especially for thick films, and must be necessarily considered in modelling contact problems if the substrate is micropolar and size dependent.

## References

- [1] Radi E. "A loaded beam in full frictionless contact with a couple stress elastic half-plane: Effects of non-standard contact conditions," *Int. J. Solids Struct.*, **232**, 111175 (2021).
- [2] Radi E, Nobili A, Guler M. "Indentation of a free beam resting on an elastic substrate with an internal lengthscale," *Eur. J. Mech.-A/Solids*, **10**, 104804 (2022).
- [3] Guler M. A., Alinia Y., Radi E. "Couple stress effects in a thin film bonded to a half-space," *Math. Mech. Solids*. In press, <https://doi.org/10.1177/10812865231209975> (2024)