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# Instrumental Cardinal Concerns for Social Status in Two-Sided Matching with Non-Transferable Utility

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## Abstract

In this paper we apply the instrumental approach to social preferences in order to distinguish among various shapes of preferences for social status. In particular, we consider the shape of reduced preferences that emerge in the equilibrium of a two-sided matching model with non-transferable utility. [Cole et al. \(1992, 1995\)](#) show that, under full observability of potential mates' attributes, instrumental concerns for social status are ordinal, i.e., only one's own rank in the distribution of attributes matters. We show that when we depart from full observability, instrumental concerns for social status become cardinal, i.e., also other features of the distribution of attributes matter. We also show that the actual shape of cardinal concerns depends on how individuals can deal with the informational asymmetry, alternatively leading to upward concerns – i.e., making comparisons with higher rank people – downward concerns – i.e., making comparisons with lower rank people – or bidirectional concerns – i.e., being both upward and downward.

**JEL classification code:** B40, C78, D10, D82

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# 1 Introduction

Although most economists agree that social preferences are relevant to economic behavior, not many economists systematically incorporate social preferences into their models. As recently argued by [Postlewaite \(2010\)](#), a reason for this is the lack of consensus on a set of principles that can rule out most specifications of social preferences. Indeed, allowing for social preferences without having reasons to prefer one specification rather than many others available can easily lead models to explain almost anything, making them of little use. An instance of this problem is illustrated in [Bilancini and Boncinelli \(2008\)](#) with regard to preferences for social status. They show that whether preferences entail ordinal or cardinal concerns for social status (and which kind of cardinal concerns) can make a great difference for the model predictions. Ordinal concerns for social status mean that what matters is only one's own rank in the distribution of the relevant attributes, while cardinal concerns mean that other characteristics of the relevant distribution matter too – typically some measure of distance with respect to others in the space of relevant attributes. Unfortunately, often there is no evident and compelling reason to prefer an ordinal specification to a cardinal one, or to prefer a certain cardinal specification instead of another cardinal specification.

In this paper we study how to distinguish between various shapes of preferences for social status using the instrumental approach, i.e., deriving preferences for status as reduced form preferences that emerge from an underlying economic interaction. In particular, we focus on the case where reduced form preference for status emerge from a two-sided matching market with non-transferable utility (NTU). We focus on this model of matching because it provides a strategic environment where, under perfect observability of potential mates, concerns for social status naturally emerge in the form of ordinal concerns ([Cole et al., 1995](#)), i.e., concerns for one's own rank only. Further, matching markets are interesting *per se* as they represent a widespread kind of economic interaction.

Our basic finding is that, if we weaken the extreme assumption of perfect observability of potential mates' attributes, then reduced preferences no longer show ordinal concerns for status but instead they show cardinal concerns. In other words, allowing for asymmetric information leads to reduced preferences for social status that induce concerns not only for one's own rank in the distribution of attributes but also for how much higher or lower one's own attribute is. Furthermore, we show that the way in which individuals can deal with the informational asymmetry determines the actual shape of cardinal concerns for status that emerges in equilibrium.

The paper is organized as follows. In section 2 we motivate the present analysis illustrat-

ing how our results can shed light on some issues explored by the literature. In section 3, we briefly review the literature on the theoretical criteria for the modeling of social preferences. In section 4 we present the basic matching framework that will be applied throughout the paper and, more importantly, we describe the instrumental approach by showing how to go from primitive preferences to reduced preferences over a distribution of attributes. Section 5 provides, in an intuitive way, definitions of concerns for social status and their possible shapes. In section 6 we illustrate the emergence of ordinal concerns for social status when individuals face a two-sided matching problem with NTU and perfect observability of partners' attributes. As this result has been already established by [Cole et al. \(1995\)](#), and discussed in further detail in [Postlewaite \(1998\)](#), we skip most technical aspects and omit all proofs. This model provides the benchmark case to be contrasted with the models of the subsequent three sections where the hypothesis of perfect observability of partners' attributes is dropped. In section 7 we extend the model to consider the possibility of imperfect observability of partners' attributes. We prove that imperfect observability leads to the emergence of bidirectional cardinal concerns for social status in the reduced form utility function of the informed side. In section 8 we consider instead the case of unobservable attributes which can be costly signaled to potential mates. To study this case we introduce a signaling stage borrowing from the model with NTU in [Hopkins \(2012\)](#), which we particularize to fit the situation considered. We prove that, in such a setup, the reduced utility function of individuals on the informed side shows downward cardinal concerns for social status. In section 9 we consider the case of unobservable attributes which can be costly revealed to potential mates. To study this case we introduce the option of costly revelation by borrowing the model in [Bilancini and Boncinelli \(2013\)](#), of which we extend the comparative statics analysis to explore the shape of concerns for status. We prove that, in this setup, the reduced form utility function of the informed side shows upward cardinal concerns for social status. In section 10 we summarize the results presented in the paper and we discuss their relevance with respect to the economic analysis of behaviors under preferences for social status. Finally, appendices collect more technical material, which can be skipped without compromising an intuitive understanding of the arguments provided throughout the paper. In particular, appendix A contains formal definitions of concerns for social status and their possible shapes, while appendix B contains the proofs of all the original results that are given in the paper.

## 2 Motivation

Our results demonstrate that the instrumental approach to social preferences can be fruitfully applied to distinguish among different shapes of preferences for social status. This is especially relevant because the analysis of economic behavior under preferences for social status is quite sensitive to the presumed shape of preferences, and therefore a method to discipline our presumptions is very much needed. A few examples in this regard may help to see why the instrumental approach can be a prominent candidate as a preference selection method.

An old-fashioned but important issue is how to explain the evidence that Americans' saving behavior shows a cross-sectional positive correlation with income at any point in time but not over time. The presumption of ordinal concerns for status reconciles theory with facts (Frank, 1985) because the interest in one's rank creates an additional incentive to prefer consumption of positional goods to savings when everybody's income increases. Reconciliation is also obtained for those forms of cardinal concerns that induce similar incentives (e.g., Duesenberry, 1949), but not for others (see the example in Bilancini and Boncinelli, 2008). Since much depends on the shape of status concerns, a natural question arises: can we legitimately employ preferences for status to explain Americans' saving behavior? If concerns for status can be reasonably imputed to competition for mates in a matching market where earning capabilities are crucial, then our results suggest that the answer is a conditional yes. More precisely, our analysis suggests that we should reject explanations based on ordinal concerns and, among those based on cardinal concerns, we should reject the ones which are incompatible with the specific characteristics of the matching market that is considered. In some cases, it might even turn out that no explanation based on preferences for status is viable.

Another important example is provided by the analysis of the relationship between inequality and waste in conspicuous consumption. With ordinal concerns for status a reduction in income inequality increases the waste in conspicuous consumption (Hopkins and Kornienko, 2004); this happens because under ordinal concerns inequality reduces social competition for status by making it more difficult to surpass a peer in terms of conspicuous consumption. Instead, with cardinal concerns for status, less inequality can lead to a smaller waste in conspicuous consumption, even when cardinal concerns are only downward (Bilancini and Boncinelli, 2012); the reason is that under cardinal concerns more inequality means a greater prize for the winners of the social competition, so an individual may want to increase conspicuous consumption when inequality increases. If we agree that concerns

for status can be imputed to competition for mates in a matching market where earning capabilities are crucial, then we can apply our results and conclude that – since concerns that emerge in case of asymmetric information are cardinal – we should be prone to think that greater inequality leads to greater wastes – even when individuals can engage in wasteful signaling activities, i.e., when cardinal concerns emerge in the downward form.

Finally, there are cases where predictions are so sensitive to the actual shape of the concerns for social status that, for instance, knowing that concerns are cardinal and bilateral may well not be enough to pin-down a single prediction. One case is provided by the desirability of a linear labor income tax under concerns for social status. With ordinal concerns, the desirability of the tax negatively depends on the degree of pre-tax inequality (Ireland, 1998), but under cardinal concerns such a relationship varies drastically depending on the details of the cardinal concerns (Bilancini and Boncinelli, 2009). Another case in this regard is the emergence of snobbish or conformist behavior under preferences for social status, where much depends on the interplay between the shape of concerns for status and the shape of the marginal utility from status (Corneo and Jeanne, 1997; Clark and Oswald, 1998). In these cases, the instrumental approach might not give sufficient restrictions on the shape of preferences to allow precise predictions. Nevertheless, it indicates where to search for the needed further constraints. If concerns for social status can be reasonably imputed to competition for mates, then the instrumental approach tells us that more precise predictions could be obtained by introducing into the model finer details about the strategic environment in which agents compete for mates.

One last comment is worth doing. It must be noted that the instrumental approach leads to reduced form preferences that can show any type of concerns for social status: either ordinal or cardinal concerns and, in case of cardinal concerns, either upward or downward or both. This observation might be interpreted as a lack of predictive power of the instrumental approach. We do not think so. Indeed, the instrumental approach helps us explain when concerns for status are ordinal, when they are cardinal and, in such a case, what type of cardinal. In the absence of compelling empirical evidence on the form of concerns for social status, the modeler who takes preferences for status as primitives has to pick the type of concerns on the basis of his own preferences (maybe, because a particular type leads to more interesting results); instead, the modeler who follows the instrumental approach has to derive concerns for social status from the underlying economic situation, whose characteristics are typically easier to observe than preferences, so that there are less degrees of freedom in his modeling choice.

### 3 Modeling criteria for social preferences

At least three possible approaches have been followed to identify suitable modeling criteria for social preferences.

The first approach postulates that preferences are to some degree socially determined in the sense that an individual's preferences partly reflect the preferences of the society she lives in. In this approach the reason why individuals might have social preferences is that the social environment has an incentive to endow individuals with them. And the kind of social preferences that we expect to find can be deduced from the current social environment and the available incentives. This line of reasoning has inspired a stream of literature focusing on how individuals' preferences are shaped by the social environment in which they are raised (e.g., [Bisin and Verdier, 2000](#); [Bisin et al., 2004](#); [Fernández et al., 2004](#); [Tabellini, 2008](#); see also [Corneo and Jeanne, 2009, 2010](#) on values).

The second approach is more parsimonious and assumes that social preferences are hard-wired into human beings. Evolutionary theory is often applied to distinguish between social preferences that can be reasonably assumed to exist and those that cannot ([Robson and Samuelson, 2010](#)). This approach has been recently put forward by [Rayo and Becker \(2007\)](#), [Samuelson \(2004\)](#) and [Samuelson and Swinkels \(2006\)](#) who have shown that evolutionary arguments support the existence of certain social preferences, and among these of preferences for relative standing.

The third approach, advocated in particular by [Postlewaite \(1998, 2010\)](#), is even more parsimonious. Preferences can be deep or reduced. Deep preferences are primitive, and hence unexplained, and are assumed to be fixed and hardwired into human beings. Reduced preferences are instead the ordering over relevant states of the world which incorporates the implications of social and economic interactions (potentially strategic) in a given institutional setting. In this approach social preferences are typically reduced preferences that are endogenously determined from four exogenous data: (i) agents' primitive (deep) preferences over economic outcomes, (ii) a model of the economic interaction which takes into account the institutional setting, (iii) an appropriate equilibrium concept for the model, and (iv) a relevant social norm that can act as an equilibrium selection device. The strength of this approach lies in the fact that, while it allows to accommodate many classes of social preferences, it permits to do so by building on standard economics without any presumption on social preferences (see, e.g., [Cole et al., 1992, 1995, 1998, 2001](#)).

All three approaches are useful to some extent.<sup>1</sup> In this paper we focus on the instrumen-

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<sup>1</sup>*Deep* social preferences that strictly depend on genes take a long time to be shaped but are also extremely

tal approach, abstracting from the possibility of deep social preferences. We stress that this is just for expositional convenience and it is not meant to suggest that deep social preferences are not relevant for economic behavior.

## 4 The instrumental approach applied to preferences for social status

In the following we describe a simple way to map the exogenous elements sketched in the previous section into reduced preferences. Given our focus on preferences for social status, we consider a model where reduced utility functions depend on the distribution of attributes in the population, hence entailing anonymity. Then, building on this kind of reduced utilities, we provide the definition of instrumental concerns for social status and some of their potential shapes.

We consider two populations that we call, for the ease of reference, women and men.<sup>2</sup> The set of women is represented by the interval of the real line  $W = [0, 1]$ , where  $w \in W$  denotes the generic woman. Analogously, the set of men is represented by the interval  $M = [0, 1]$ , where  $m \in M$  denotes the generic man. Women are heterogeneous in their attributes that can take value in  $X = [\underline{x}, \bar{x}]$ , which is also an interval of the real line. Men too are heterogeneous in their attributes that can take value in  $Y = [\underline{y}, \bar{y}]$ , again an interval of the real line. The cumulative distribution functions over  $X$  and  $Y$  represent the distribution of attributes in the two populations and are denoted, respectively, with  $F$  and  $G$ .<sup>3</sup>

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resilient to the social environment, surviving for thousands of years; to single out such social preferences we can apply the second – evolutionary – approach. Instead, *deep* social preferences that mostly depend on culture take a shorter time to be shaped, which might be as short as one or two generations; to single out such social preferences and predict their evolution over the next few generations we can apply the first – socialization – approach. Finally, both kinds of deep social preferences can be taken as primitives within the third – instrumental – approach, which can then be applied to single out reduced social preferences in all sorts of economic situations.

<sup>2</sup>We opt for this modeling choice because in the following we focus on two-sided matching problems with NTU, so a model with a population of women and a population of men perfectly fits our purposes. We remark, however, that interpretation is by no means restricted to women and men (think of, for instance, buyers and sellers or workers and firms).

<sup>3</sup>If we denote with  $f$  and  $g$  the density functions of  $F$  and  $G$  respectively, and if we assume that they are always strictly positive, then a bijection can be established between individuals and attributes, for both women and men. This allows us to refer to  $W$  and  $X$  or to  $M$  and  $Y$  interchangeably. Alternatively, we could define  $X$  as the set of women and  $Y$  as the set of men, and then use  $F(x)$  and  $G(y)$  to compute the rank in the distribution of attributes of woman  $x$  and man  $y$ .



Women and men interact with each other in a given economic and institutional setting which, together with the actual assignment of attributes in the population, i.e., given  $F$  and  $G$ , determines both agents' choice sets and the outcome function assigning economic outcomes to each profile of agents' choices. At this stage of the analysis we model neither the mechanism regulating interaction nor the individuals' choice sets nor the outcome function, as these details do not play any role in illustrating the instrumental approach to preferences for social status.

All women  $w \in W$  have the same utility function  $U^W(F^{-1}(w), z)$ , where  $z$  is the economic outcome that eventually emerges. Similarly, all men  $m \in M$  have the same utility function  $U^M(z, G^{-1}(m))$ . The set of economic outcomes is denoted by  $Z$ . Once a solution concept is selected and applied, a restriction on possible outcomes is obtained, which clearly depends on the distributions of attributes in the two populations,  $F$  and  $G$ . How to select the appropriate solution concept is evidently a crucial issue for any given application of the instrumental approach, but for what matters here it is enough to stress that such a choice should take into account the characteristics of the interaction and the presumed cognitive and epistemological skills of agents.

if the solution concept is always able to select a single economic outcome – which can be denoted with  $z^*(F, G)$  – then agents' utility can be written as  $U^W(x, z^*(F, G))$  and  $U^M(z^*(F, G), y)$ . Each of these functions is called *reduced form utility function* in the sense that it incorporates the dependency of the outcome on the distribution of agents' attributes and, therefore, reduces the dependency to the distributions of attributes only. In case the solution concept admits a multiplicity of economic outcomes, then we have to refine its predictions on the basis of social norms (as done, e.g., in [Cole et al., 1992](#)). This also opens to the idea of social value of attributes, i.e., the additional economic value due to the selection of a particular equilibrium ([Mailath and Postlewaite, 2006](#)). More precisely, in this framework a social norm is understood as an exogenous convention that complements the solution concept to eventually obtain function  $z^*(F, G)$ .

The investigation of the shape of reduced preferences can be carried out by applying comparative statics on the reduced utility functions  $V^W(x, F, G) = U^W(x, z^*(F, G))$  and  $V^M(x, F, G) = U^M(z^*(F, G), y)$ . For the sake of conciseness we only consider women's function  $V^W(x, F, G)$ , as a similar analysis applies to men's reduced function  $V^M(y, F, G)$ . More precisely, we characterize the shape of reduced preferences for social status by studying how  $V^W(x, F, G)$  varies in response to changes in  $F$ . Of course we could also explore how  $V^W(x, F, G)$  varies in response to changes in  $G$ . However, here we focus on induced concerns for social status and their shapes so that the relevant changes are those in one's own

distribution of attributes.<sup>4</sup> For the same reason we will often write  $V^W(x, F)$  in the place of  $V^W(x, F, G)$  when this does not create ambiguities.

## 5 Shapes of concerns for social status

In this section we provide the intuitive definitions of concerns for social status on reduced form preferences, and we discuss some of their potential shapes. We refer the reader to appendix A for the formal definitions.

Intuitively, social status is intimately related to the idea that individuals care about being ahead of (and/or not being behind) others. In line with this, we adopt the following definition: we say that an individual’s reduced utility  $V^W(x, F)$  exhibits *concerns for social status* if  $V^W(x, F)$  never increases (decreases) when other individuals in the same population get their attributes increased (decreased), and that at least in one case  $V^W(x, F)$  strictly decreases (increases).

We say that  $V^W(x, F)$  exhibits *ordinal concerns for social status* when  $V^W(x, F)$  is only sensitive to  $F(x)$ , meaning that having a certain rank in the distribution of attributes entails a level of reduced utility which is otherwise independent of the actual distribution. The intuition behind this definition is simple: concerns are about the fraction of individuals with higher, lower or same attribute. All kinds of concerns for social status that are rank-based fall under this definition (see, e.g., Frank, 1985; Corneo and Jeanne, 1997; Hopkins and Kornienko, 2004). Here we encompass also concerns for social status where only the fraction of individuals with higher or lower attributes matters – so that the fraction of ties is irrelevant (see, e.g., Haagsma and van Mouche, 2010) – and even more extreme shapes of concerns, such as concerns restricted to being in the top 1% or not being in the bottom 1% of the distribution.

We say that  $V^W(x, F)$  exhibits *cardinal concerns for social status* if they are not ordinal, i.e., if  $V^W(x, F)$  is sensitive not only to  $F(x)$ , but also to some cardinal feature of  $F$ . Evidently, many different shapes of cardinal concerns are possible. Therefore, it is useful to identify some relevant features of cardinal concerns for status that help to distinguish among the various shapes.

We speak of *downward cardinal concerns for social status* when an individual’s reduced utility  $V^W(x, F)$  is sensitive to the shift of attributes of individuals below  $F(x)$ , that is, with attribute lower than  $x$ . Intuitively, under downward cardinal concerns individuals benefit

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<sup>4</sup>Going deeper, we might apply further limitations to identify the set of people an individual compares with, making proper sense of the notion of *reference group*.

from differentiating more from those who have lower attributes. An important example of cardinal concerns for status that are evidently downward is relative satisfaction (Runciman, 1966) in the formalization of Yitzhaki (1979).

Symmetrically, we speak of *upward cardinal concerns for social status* when  $V^W(x, F)$  is sensitive to the shift of attributes of individuals above  $F(x)$ , that is, with attribute higher than  $x$ . So, under upward cardinal concerns individuals benefit from becoming more similar to those who have higher attributes. A famous example of upward cardinal concerns for status is relative deprivation (see again Runciman, 1966; Yitzhaki, 1979). Another example is provided by the kind concerns for status theorized by Veblen (1899) (see Bowles and Park, 2005, for a modern formalization).

Finally, we speak of *bidirectional cardinal concerns for social status* whenever  $V^W(x, F)$  is sensitive to the shift of attributes of individuals both below and above  $F(x)$  (see Hayakawa, 2000, for a case of bidirectional concerns that are not necessarily symmetric). For instance, bidirectional concerns are present in all models where individuals care about the ratio or the difference between their income and average income (see Clark et al., 2008, and references therein).

## 6 The emergence of ordinal concerns: Perfect observability of partners' attributes

The emergence of ordinal status when partners' attributes are perfectly observable has been already established by Cole et al. (1992, 1995) and discussed in further detail in Postlewaite (1998). Therefore, we illustrate the analysis skipping most technical details and omitting all proofs. The model of this section provides both a baseline and a benchmark for the models of the next three sections, where we drop the hypothesis of perfect observability of partners' attributes.

Women and men are interested in matching with each other, and by so doing they obtain a positive utility. We denote with  $U^W(x, y)$  the utility accruing to a woman with attribute  $x$  when matched with a man with attribute  $y$ . Similarly, we denote with  $U^M(x, y)$  the utility accruing to a man with attribute  $y$  when matched with a woman with attribute  $x$ .<sup>5</sup> We also

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<sup>5</sup>Primitive utility functions  $U^W(x, y)$  and  $U^M(x, y)$  are defined in a slightly different way with respect to the primitive utility functions  $U^W(x, z)$  and  $U^M(y, z)$  presented in section 4. The reason is that here agents are assumed to care only about their own mate, not others' mates. Hence, although an economic outcome in this model is represented by a matching pattern for all women and men, primitive utilities depend only on one's own and partner's attributes.

assume that not being matched with anyone entails a null utility for both men and women, so that everybody prefers to be matched with someone instead of staying alone. We further assume that both  $U^W$  and  $U^M$  are twice continuously differentiable and strictly increasing in each argument.

A *matching* is an isomorphism  $\mu$  between the measure spaces defined on  $X$  and  $Y$ . In words, a matching function is a rule that assigns each man to a distinct woman and each woman to a distinct man. Hence a matching  $\mu$  is an invertible map and both  $\mu$  and  $\mu^{-1}$  are measurable as well as measure preserving maps.

A *stable matching* is a matching  $\mu$  such that, for all  $x \in X$ , if  $U^W(x, \mu(x)) < U^W(x, y)$  for some  $y \in Y$ , then  $U^M(\mu^{-1}(y), y) > U^M(x, y)$ . In words, a matching  $\mu$  is stable if and only if there is no pair of man and woman who are not matched together according to  $\mu$  and that would be strictly better off by matching with each other.

In this model there exists one and only one stable matching where women and men are matched in a positively assortative way.<sup>6</sup> Formally, the unique stable matching is  $\mu^* = G^{-1}(F)$ . Figure 1 illustrates an example of matching  $\mu^*$  for two distributions of women's attributes, one distribution representing an upward shift of attributes with respect to the other.

We can now construct the reduced form utility functions which take into account the unique stable outcome of the matching game. Since the relevant economic outcome is given by  $\mu^*$ , the reduced form utility function of a generic woman  $w$  is  $V^W(x, F) = U^W(x, G^{-1}(F(x)))$  with  $x = F^{-1}(w)$ . Analogously, the reduced form utility function of the generic man  $m$  is  $V^M(y, G) = U^M(F^{-1}(G(y), y))$  with  $y = G^{-1}(m)$ . Note that for woman  $w$  with attribute  $F^{-1}(w)$  the only feature of the distribution  $F$  that plays a role is her own rank  $F(x) = w$ .

In the light of the definitions given in section 5, we can conclude that reduced utility function exhibits ordinal concerns for social status (indeed,  $V^W(x, F)$  satisfies definition 1 and 2 in appendix A). The following proposition summarizes the result.

**PROPOSITION 1.** *In a two-sided matching with NTU where women's and men's attributes are perfectly observable, reduced utility  $V^W$  exhibits ordinal concerns for status.*

The intuition behind Proposition 1 is illustrated by figure 2, showing different examples of upward shifts in women's attributes that do not affect the rank of the woman considered,

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<sup>6</sup>The first existence result for the marriage problem with general preferences and finite sets of agents is provided by Gale and Shapley (1962). Recent results about uniqueness of two-sided stable matching can be found in Eeckhout (2000), Clark (2006), and Legros and Newman (2010). For specific results in a setup like ours, with a continuum of agents and preferences depending on partner's attribute, we may refer to Cole et al. (1992, 1995).

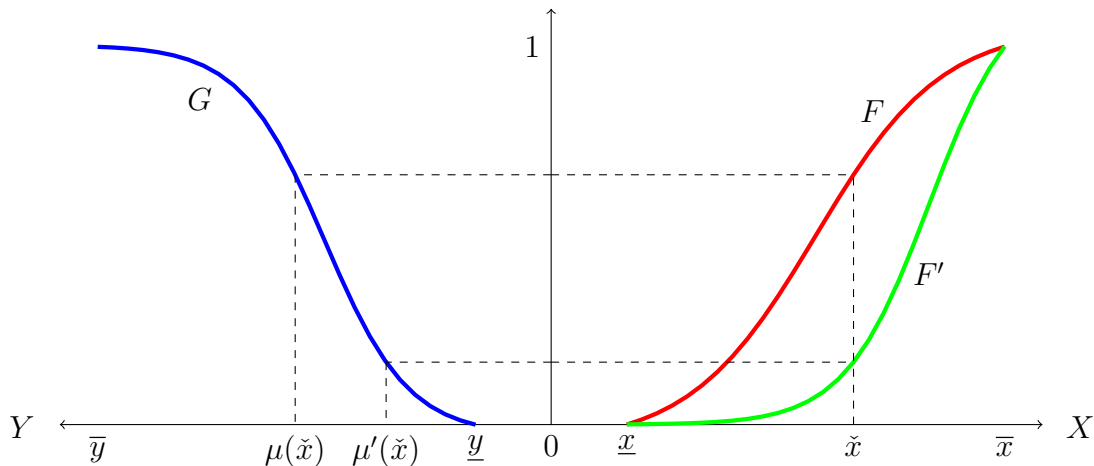


Figure 1: An upward shift of women’s attributes from  $F$  to  $F'$ . Woman with attribute  $\tilde{x}$  is of different rank under  $F$  and  $F'$ . In particular, for any given attribute  $x \in (\underline{x}, \bar{x})$  the associated woman has a lower rank under  $F'$  and, hence, obtains a worse match, i.e.,  $\mu'(x) < \mu(x)$  for all  $x \in (\underline{x}, \bar{x})$ .

and hence leave her utility unaltered.

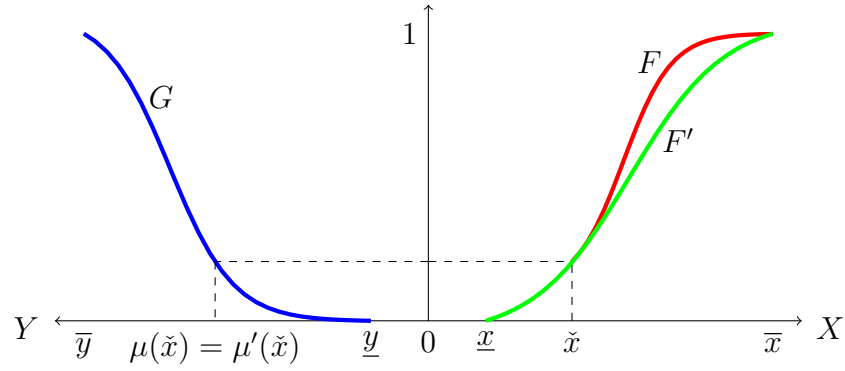
Finally, since women and men have symmetric roles in the model considered, we observe that an analogous result to Proposition 1 can be stated for men, whose reduced utility  $V^M$  exhibits ordinal concerns for status as well.

## 7 The emergence of bidirectional cardinal concerns: Imperfect observability of partners’ attributes

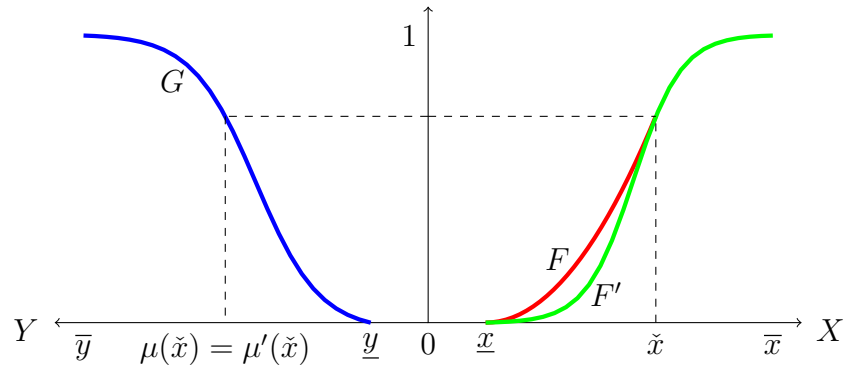
The basic idea we want to put forward in this section is that, when individuals face a matching problem, imperfect observability of attributes is sufficient to provide a rationale for the emergence of bidirectional cardinal concerns in the place of ordinal concerns. To this aim we sketch below a simple model with imperfect observability of women’s attributes. The model, although not fully general, is nevertheless compatible with different specifications of the stochastic process describing the imperfect observability of women’s attributes and men’s inference procedure.<sup>7</sup>

The distribution of women’s attributes  $F$  is public information, but men imperfectly observe the attribute of any specific woman. More precisely, the attribute of woman  $w$

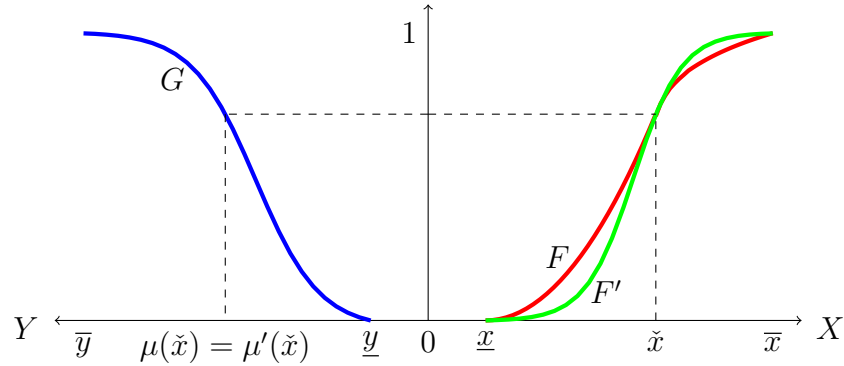
<sup>7</sup>See [Bhaskar and Hopkins \(2013\)](#) for another example of a matching model where attributes are imperfectly observed.



(a)



(b)



(c)

Figure 2: A shift of women's attributes from  $F$  to  $F'$ . In (a) the rank of women with attribute in  $(\tilde{x}, \bar{x})$  reduces for given attribute. In (b) the rank of women with attribute in  $(\underline{x}, \tilde{x})$  reduces for given attribute. In (c) the rank of women with attribute in  $(\tilde{x}, \bar{x})$  increases for given attribute and, at the same time, the rank of women in  $(\underline{x}, \tilde{x})$  reduces for given attribute. In all cases, woman  $F(\tilde{x})$  has the same rank and, hence, the same match and utility under  $F$  and  $F'$ .

is subject to a perception bias and, as a result,  $w$ 's attribute is perceived by all men as  $z = F^{-1}(w) + \epsilon$ , where  $\epsilon$  is a stochastic term which is distributed according to the probability density function  $p$  with cumulative distribution  $P$ . We assume that the support of  $p$  is the real line, that  $p$  is everywhere strictly positive, and that  $p$  satisfies the following property: the ration  $p(z - k)/p(z)$  is increasing in  $z$  for all  $k > 0$ . The latter property yields a family of conditional probability densities that satisfy the strict monotone likelihood ratio property, i.e.,  $p(z' - x')/p(z' - x) > p(z - x')/p(z - x)$  for all pairs of true attributes  $x'$  and  $x$ , with  $x' > x$ , and alla pairs of observed attributes  $z'$  and  $z$ , with  $z' > z$ . We observe that the distribution of woman  $w$ 's observed attribute is  $\bar{F}(z|w) = P(x' - F^{-1}(w))$ , i.e., the probability that  $F^{-1}(w) + \bar{\epsilon} \leq x'$ . It follows that the distribution of observed attributes across the population is  $\bar{F}(z) = \int_0^1 \bar{F}(z|s) ds$ .

Women observe men's true attributes, and prefer to be matched to men with higher attributes. Instead, men observe only women's noisy attributes, but they still prefer to be matched with women with higher observed attributes. This is so because of the strict monotone likelihood ratio property: a higher signal implies that women with higher true attributes are relatively more likely. So, being ranked  $\bar{F}(z)$  leads to be matched with a man also ranked  $\bar{F}(z)$ , and hence being observed with attribute  $z$  leads to match a man with attribute  $G^{-1}(\bar{F}(z))$ . Therefore, for every given  $\bar{F}$  there exists a unique stable matching of the form  $\mu^* = G^{-1}(\bar{F})$  which is positively assortative with respect to the perceived distribution of attributes among women,  $\bar{F}$ , and the actual distribution of attributes among men,  $G$ .

Taking into account the stable matching  $\mu^*$  and the imperfect observability of attributes as described by  $P$ , a woman  $w$  with attribute  $x = F^{-1}(w)$  has the following reduced utility:

$$V^W(x, F) = \int_{-\infty}^{+\infty} U^W \left( x, G^{-1} \left( \int_0^1 P(x + \epsilon - F^{-1}(s)) ds \right) \right) p(\epsilon) d\epsilon \quad (1)$$

The way in which function (1) depends on  $F$  entails both upward and downward cardinal concerns, as summarized by the following proposition.

**PROPOSITION 2.** *In a two-sided matching with NTU where women's attributes are imperfectly observable, reduced utility  $V^W$  exhibits bidirectional cardinal concerns for status.*

The intuition behind Proposition 2 is illustrated by the example reported in figure 3 where an upward shift in women's attributes is shown to affect negatively the expected match of the considered woman even though her rank remains the same. Basically, each woman prefers that those women below her – i.e., with truly lower attribute – are moved farther below because this reduces the probability that these women are wrongly perceived as better mates than herself. Similarly, each woman prefers that those women above her –

i.e., with truly higher attribute – are moved closer since this raises the probability that she is wrongly perceived as a better mate than them.

Finally, we can easily recognize that  $V^M$  exhibits ordinal concerns for social status, as in the benchmark model of the previous section. Indeed, the reduced form utility for man  $m$  with attribute  $y$  is:

$$V^M(y, G) = \int_0^1 U^M(F^{-1}(s), y) p(\tilde{\epsilon}(G(y), s)) ds \quad (2)$$

where  $\tilde{\epsilon}(G(y), s)$  is defined as the value of  $\epsilon$  that solves the equation  $G(y) = \int_0^1 P(s + \epsilon - r) dr$ , for  $G(y) \in (0, 1)$ , and  $\tilde{\epsilon}(0, s) = \tilde{\epsilon}(1, s) = 0$ . Note that such a solution is uniquely determined since the right-hand side is strictly increasing in  $\epsilon$ . As  $V^M(y, G)$  depends on  $G(y)$  but is independent of any other feature of the distribution  $G$ , it satisfies the definition of ordinal concerns for status. We can easily guess that in the case of imperfect observability of both women’s and men’s attributes, the reduced form utility function of both men and women would show bidirectional cardinal concerns.

## 8 The emergence of downward cardinal concerns: Signaling of unobservable attributes

In this section we illustrate how, in a matching market, the possibility to signal one’s own unobservable attribute by means of a costly activity can lead to the emergence of downward cardinal concerns for social status. To this purpose, we apply a particularization of the model with NTU developed in [Hopkins \(2012\)](#) which well extends our baseline model by allowing for both asymmetric information and signaling. In particular, we consider a situation where men cannot observe women’s attributes but women engage in signaling.

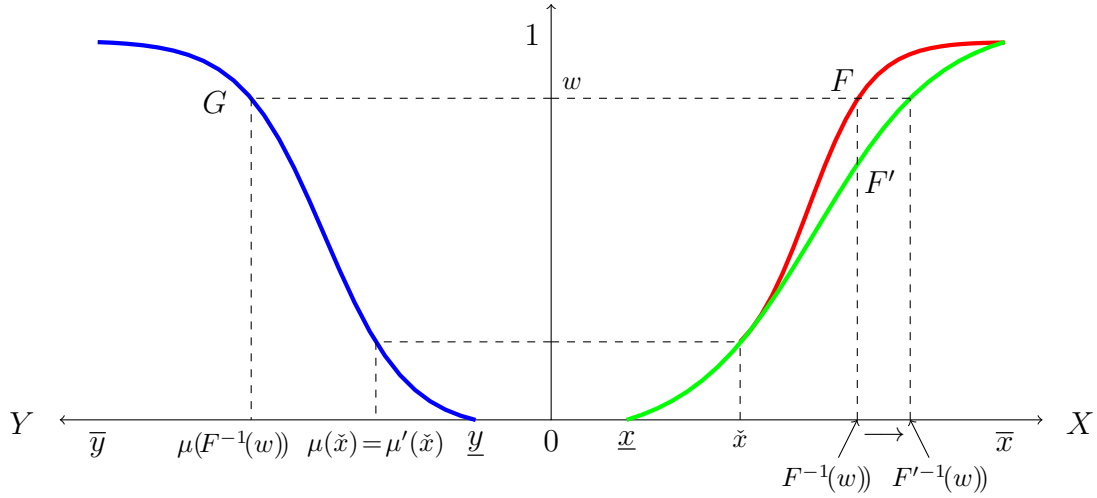
Let  $F$  be public information while, for every  $w \in W$ , let attribute  $F^{-1}(w)$  be a private information of woman  $w$ . Moreover, woman  $w$  can send a costly signal  $\sigma(w) \in \mathfrak{R}_+$  which is freely observable by both men and women. Let also the cross derivative of  $U^W$  with respect to  $x$  and  $y$  be strictly positive.<sup>8</sup> Finally, let the activity  $\sigma$  be costly for women and of no intrinsic value for men, so that a woman with attribute  $x$  and signal  $\sigma$  and a man with attribute  $y$  who are matched together obtain a utility  $U^W(x, y) - \sigma$  and  $U^M(x, y)$ , respectively.<sup>9</sup>

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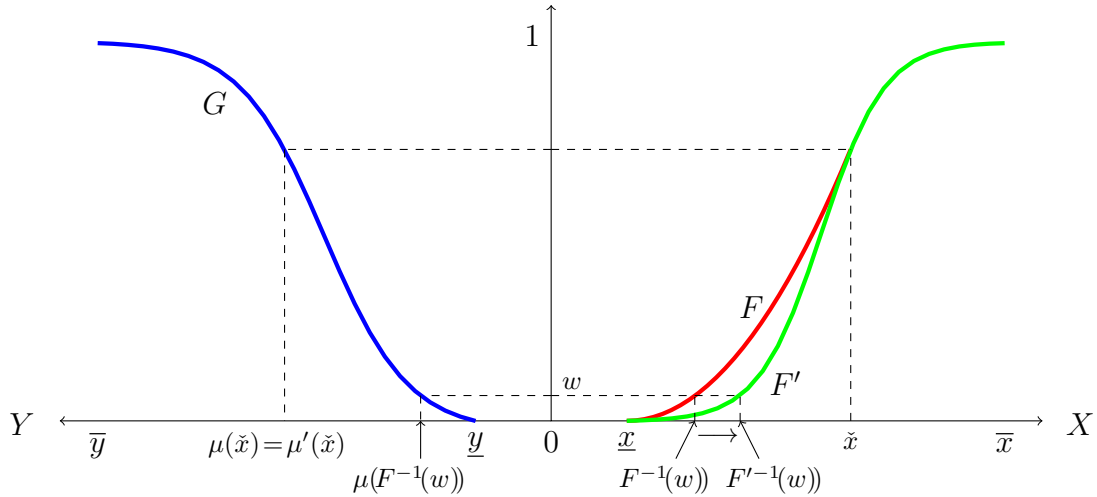
<sup>8</sup>This implies the single-crossing property on women’s preferences, which is required for separation in signaling models.

<sup>9</sup>Allowing  $\sigma$  to be useful to men – as in pre-marital investment (see, e.g., [Peters, 2007](#)) – would not change





(a)



(b)

Figure 3: A shift of women's attributes from  $F$  to  $F'$  when attributes are imperfectly observable. In (a) the rank of women with attribute in  $(\tilde{x}, \bar{x})$  reduces for given attribute. In (b) the rank of women with attribute in  $(\underline{x}, \tilde{x})$  reduces for given attribute. For every realized perception bias affecting woman  $F(\tilde{x})$ , the probability that the generic woman  $w$  is perceived to have an attribute lower than  $F(\tilde{x})$  depends negatively on the distance between  $\tilde{x}$  and  $F^{-1}(w)$ . Hence, even if  $F(\tilde{x}) = F'(\tilde{x})$ , woman  $F(\tilde{x})$  is worse off in expected value.

The strategic interaction induced by the described setup unfolds as follows. In the first stage women play a signaling game in which they signal their attribute to men through the costly activity  $\sigma$ . In the second stage women and men play a matching game where mating is resolved according to the information obtained from women's signals and men's publicly observable attributes. An equilibrium in this model is a pair  $(\sigma^*, \mu^*)$ , where  $\sigma^*$  is an equilibrium of the signaling game in the first stage – taking into account that in the second stage pairs will be formed on the basis of observed signals – and matching  $\mu^*$  is stable given the outcome of the first stage. From Hopkins (2012, Proposition 1) we know that there exists a unique equilibrium where women are fully separated. This equilibrium, denoted with  $(\sigma^*, \mu^*)$  is such that  $\sigma^*$  is both strictly increasing and differentiable,  $\sigma^*(0) = 0$ , and  $\mu^* = G^{-1}(F)$ .<sup>10</sup> In other words, there exists a unique separating equilibrium where the signal increases smoothly in women's attributes, the less attractive woman does not signal, and matching is positively assortative.

In order to derive the reduced utility function of women, it is useful to start from the equilibrium signaling function. This is obtained through integration exploiting the first order condition for the optimal choice of  $\sigma$ :

$$\sigma^*(w) = \int_0^w \frac{\partial U^W(F^{-1}(s), G^{-1}(s))}{\partial y} \frac{1}{g(G^{-1}(s))} ds \quad (3)$$

In the light of (3), the reduced form utility function of woman  $w$  with attribute  $x = F^{-1}(w)$  is:

$$V^W(x, F) = U^W(x, G^{-1}(F(x))) - \int_0^{F(x)} \frac{\partial U^W(F^{-1}(s), G^{-1}(s))}{\partial y} \frac{1}{g(G^{-1}(s))} ds \quad (4)$$

The following proposition establishes that the way in which function (4) depends on  $F$  entails downward cardinal concerns.

**PROPOSITION 3.** *In a two-sided matching with NTU where women's attributes are unobservable but can be signaled, reduced utility  $V^W$  exhibits downward cardinal concerns for status.*

The mechanism giving rise to the result stated in Proposition 3 is illustrated by the example reported in figure 4 where the considered woman is forced to signal more – and,

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the type of status concerns arising in the reduced form utility function of women. The main difference would be a more articulated dependency of men's reduced utility on the distribution of women's attributes.

<sup>10</sup>Function  $\sigma^*$  is the unique solution, starting from the initial condition  $\sigma^*(0) = 0$ , of the differential equation arising from the first order condition for the optimal choice of  $\sigma$ , i.e.,  $\frac{\partial U(F^{-1}(w), G^{-1}(s))}{\partial y} \frac{1}{g(G^{-1}(s))} - \sigma'(s) = 0$ , where  $s$  is the signaled identity.

hence, obtains a lower utility – as a consequence of an upward shift in the attributes of women with lower rank, even if her rank remains the same. We stress that this outcome hinges on two facts: first, each woman signals just enough to distinguish herself from lower types in equilibrium – so that every woman has to adjust her signaling if women with a lower attribute increase their signal – and, second, the net benefit from signaling is increasing in the attribute – because of complementarities. Thus, a woman wants to signal more whenever the women below her have a relatively higher attribute, since this raises their marginal willingness to pay for the signal and, hence, their signaling.

We also stress that reduced utility function  $V^W$  does not exhibit upward cardinal concerns for status. This can be easily seen by considering  $F$ ,  $F'$ , and  $\check{x}$  such that  $F'(x) = F(x)$  for all  $x \in [\underline{x}, \check{x}]$ , and  $F'(x) < F(x)$  in  $(\check{x}, \bar{x})$ . From  $F'(x) = F(x)$  for all  $x \in [\underline{x}, \check{x}]$  follows that, for a woman with attribute  $\check{x}$ , both the first and the second term in (4) are the same under  $F$  and  $F'$ . Hence, a woman who faces an increase in the attribute of the women above her does not have any incentive to change her signal because the signal needed to separate from lower types is exactly as before (indeed, function (4) does not satisfy definition 5 in appendix A).

We note that, as for the model of section 7,  $V^M$  exhibits ordinal concerns for social status. In particular, the reduced form utility for man  $m$  with attribute  $y$  is the same as in section 6. We also note that if both men’s and women’s attributes were not observable and both sides could send costly signals, then both men’s and women’s reduced form utility functions would show cardinal downward concerns for social status. Indeed, in the fully separating equilibrium we would still have a positive assortative matching where individuals with higher attributes sustain a higher cost of signaling. The only substantial difference would be that men have to sustain a cost to get a woman with their same rank in the distribution of attributes.

## 9 The emergence of upward cardinal concerns: Costly revelation of unobservable attributes

In this section we show how, in a matching market, the possibility to sustain a cost to reveal one’s own unobservable attribute can lead to the emergence of upward cardinal concerns. To illustrate how this can happen we borrow the model of two-sided matching with costly reve-

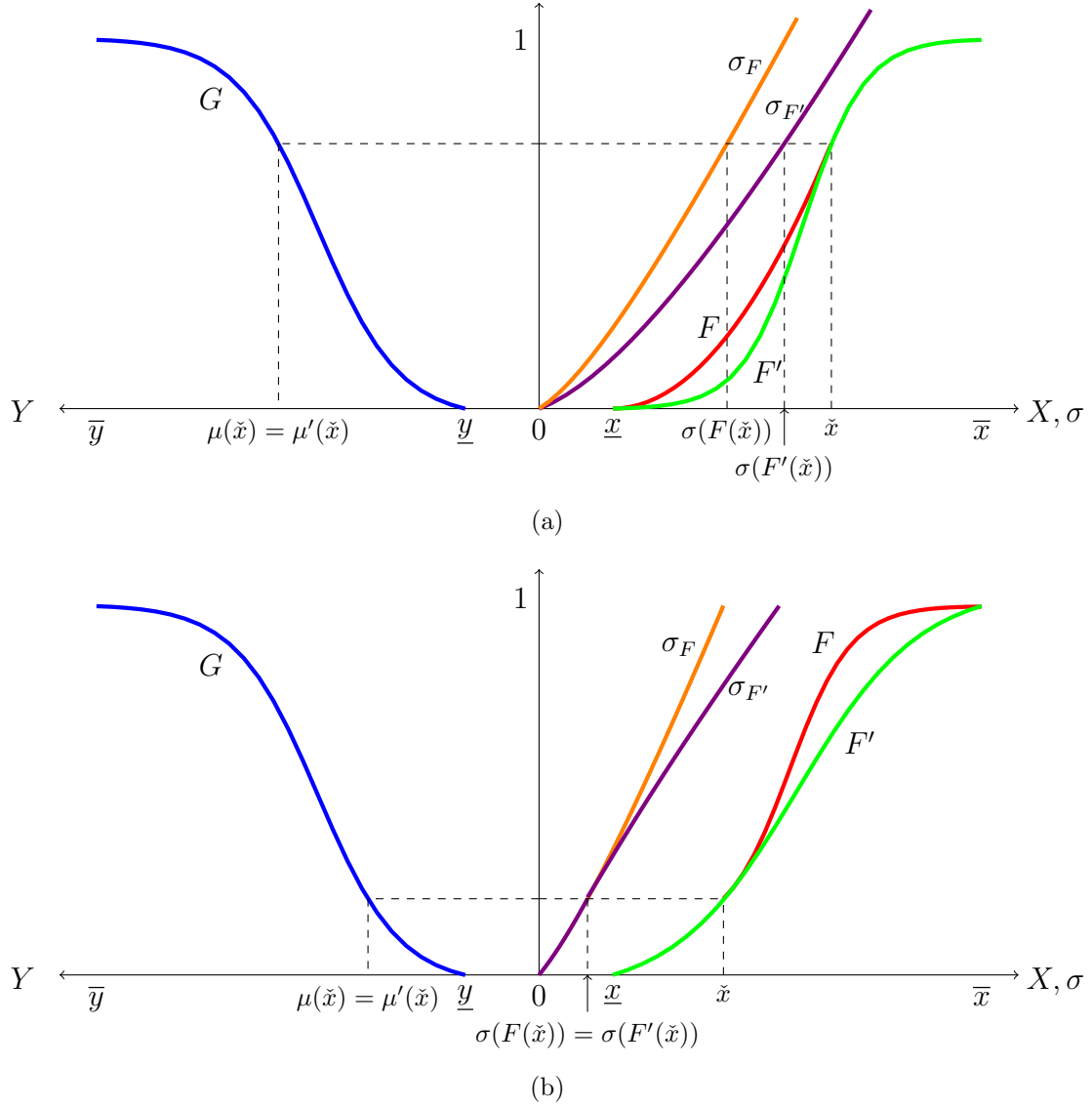


Figure 4: A shift of women's attributes from  $F$  to  $F'$  when attributes are unobservable and women can send costly signals. In (a) the rank of women with attribute in  $(\underline{x}, \tilde{x})$  reduces for given attribute. The level of wasteful signal  $\sigma$  increases for everybody but woman  $w = 0$ . All women  $w \geq F(\tilde{x})$  maintain the same match under both  $F$  and  $F'$  but under  $F'$  they must signal more in equilibrium, so that they are all worse off under  $F'$ . In (b) the rank of women with attribute in  $(\tilde{x}, \bar{x})$  reduces for given attribute. The level of wasteful signal  $\sigma$  increases only for women  $w > F(\tilde{x})$ . All women  $w \leq F(\tilde{x})$  maintain the same match and signal under both  $F$  and  $F'$ , so that they maintain the same utility under  $F'$ .

lation developed in [Bilancini and Boncinelli \(2013\)](#).<sup>11</sup> In particular, we consider a situation

<sup>11</sup>Here we only give the minimum description of the model which is needed for our purposes. We refer the reader to the cited paper for a complete and precise description of the model, its assumptions and results.

where men cannot observe a woman's attribute unless this woman decides to costly reveal it.

Let  $F$  be public information while, for every  $w \in W$ , let attribute  $F^{-1}(w)$  be a private information of woman  $w$ . Moreover, women are given the possibility to pay a fixed cost  $c > 0$  to publicly reveal their type. Women decide whether to reveal or not, while men decide whether to try to match with a revealing woman or randomly match with a non-revealing one. Let  $W_R \subset W$  be the set of women who reveal their attribute and let  $W_H = W \setminus W_R$  be the set of women who hide their attribute. Also, let function  $\rho^W$  describe women's decisions about revealing or hiding, where  $\rho^W(w) = 1$  if  $w \in W_R$  and  $\rho^W(w) = 0$  if  $w \in W_H$ . Furthermore, let  $M_R \subset M$  be the set of men who want to match with a revealing woman and let  $M_H = M \setminus M_R$  be the set of men who want to match with a non-revealing woman. Also, let function  $\rho^M$  describe men's decisions about whether to match with a revealing woman, where  $\rho^M(m) = 1$  if  $m \in M_R$  and  $\rho^M(m) = 0$  if  $m \in M_H$ .

Given agents' decisions, as summarized by  $\rho = (\rho^R, \rho^M)$ , matching is resolved in the following way. Matches between women in  $W_R$  and men in  $M_R$  are treated in the usual way, as described in section 6. Instead, matches between women in  $W_H$  and men in  $M_H$  are resolved randomly. A profile in this setup is a pair  $(\rho, \mu)$  such that  $\mu$  is a matching between attributes of women in  $W_R$  and attributes of men in  $M_R$ .

The revelation cost  $c$  is expressed in terms of utility, so that the utility of woman  $w \in W_R$  with attribute  $x$  who is matched with man  $m \in M_R$  with attribute  $y$  is given by  $U^W(x, y) - c$ , while the utility of man  $m$  for the same match is  $U^M(x, y)$ . The utility of woman  $w \in W_H \neq \emptyset$  and the utility of man  $m \in M_H \neq \emptyset$  are random, in the sense that they depend on the actual match. Hence, decisions are taken on the basis of expected utility, given the distribution on  $x$  and  $y$  in, respectively,  $W_H$  and  $M_H$ . Finally,  $U^W$  is assumed to satisfy increasing differences in  $x$  and  $y$ , i.e., women with higher attribute benefit relatively more by matching with men with higher attribute. We note that this assumption is crucial for many of the following results, as with the single crossing-property in the case of signaling.

An equilibrium of this model is a profile  $(\rho^*, \mu^*)$  where, first, no revealing agent finds it convenient to save the revelation cost and pick up a partner from the pool of hiding individuals and, second, if a woman finds it convenient to be paired with a certain man with respect to her equilibrium status quo then such a man finds it detrimental to match with this woman with respect to his equilibrium status quo.

In this model all equilibria are characterized as follows. First,  $\mu^*$  is positively assortative. Second,  $\rho^*$  is such that there exists a cutoff woman  $\hat{w}$  that lies below all other women who reveal and above those who hide their attribute, i.e., that separates  $W_R$  and  $W_H$ . The same

must occur for the cutoff man  $\hat{m} = G^{-1}(\mu^*(F^{-1}(\hat{w}))) = \hat{w}$ , who separates  $M_R$  and  $M_H$ . Third, the relative gain from revelation must be equal to zero for any equilibrium with an interior cutoff, i.e.,  $\hat{w} \in (0, 1)$ . This amounts to have:

$$U^W(F^{-1}(\hat{w}), G^{-1}(\hat{w})) - c - \frac{1}{\hat{w}} \int_0^{\hat{w}} U^W(F^{-1}(\hat{w}), G^{-1}(m)) dm = 0 \quad (5)$$

We stress that (5) is continuous in  $\hat{w}$ . Figure 5 provides a graphical representation of the relative gain from revelation depicted as a function of the cutoff woman  $\hat{w}$ .<sup>12</sup>

Despite that multiple equilibria can exist in this model (when the function representing the gain from revelation crosses the horizontal axis at multiple points), [Bilancini and Boncinelli \(2013, Proposition 4\)](#) show that for  $c$  positive but sufficiently low, there always exists at least one equilibrium with  $\hat{w} \in (0, 1)$  that is stable with respect to a payoff-based adjustment dynamics of the cutoff. Essentially, we have a stable equilibrium when the function representing the gain from revelation crosses the horizontal axis from below. We focus on stable equilibria and, in case of their multiplicity, we suppose that the ruling social norm is selecting one of them.<sup>13</sup>

We stress that, given a social norm that selects a stable equilibrium, the resulting cutoff  $\hat{w}$  is endogenously determined by  $F$  and  $G$ . To make clear this dependency we write  $\hat{w}(F, G)$ . The reduced utility function of woman  $w$  with attribute  $x$  can therefore be written as:

$$V^W(x, F) = \begin{cases} \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} U^W(x, G^{-1}(s)) ds, & \text{if } F(x) < \hat{w}(F, G), \\ U^W(x, G^{-1}(F(x))) - c, & \text{if } F(x) \geq \hat{w}(F, G), \end{cases} \quad (6)$$

where the upper expression holds for a woman who relies on random matching, while the lower expression holds for a woman who relies on standard assortative matching.

The following proposition establishes that the way in which function (6) depends on  $F$  entails upward cardinal concerns.

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<sup>12</sup>We have chosen to write the gain from revelation in a slightly different way with respect to [Bilancini and Boncinelli \(2013\)](#) in order to simplify the exposition in this section.

<sup>13</sup>This allows us to remark the important role that social norms can have in determining the shape of reduced social preferences, acting as a selection device (see, e.g., [Cole et al., 1992](#); [Mailath and Postlewaite, 2006](#)). In particular, a ruling social norm can be always thought of as the result of a payoff-based evolutionary process; as such, it will necessarily select one of the equilibria that are stable in the payoff-based dynamics of subsection 4.3 in [Bilancini and Boncinelli \(2013\)](#).

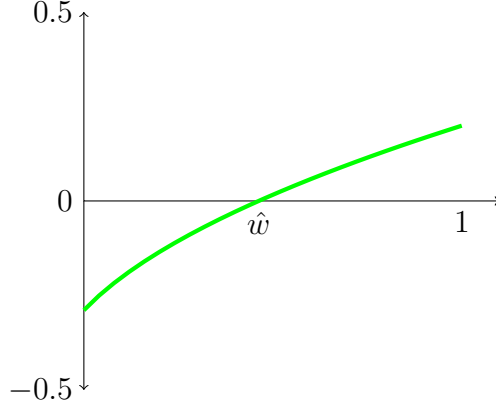


Figure 5: This graph is plotted for the case in which  $F^{-1}(w) = w$ ,  $G^{-1}(m) = m$ ,  $U^W(x, y) = 2\sqrt{x + 0.1}$ , and  $c = 0.5$ . This, by using (5), gives rise to a gain from revelation, which is reported on the vertical axis, equal to  $-0.5 + \frac{2}{3}\sqrt{\hat{w} + 0.1}$ . The equilibrium corresponds to  $\hat{w} \approx 0.42$ , where the gain from revelation is nil.

**PROPOSITION 4.** *In a two-sided matching with NTU where women's attributes are unobservable but can be costly revealed, reduced utility  $V^W$  exhibits upward cardinal concerns for status.*

The intuition underlying the result stated in Proposition 4 is illustrated by the example reported in figure 6 where the woman under consideration sees a reduction in the value of her expected match as a consequence of an upward shift in the attributes of women with higher rank, even if her rank remains the same. Indeed, a non-revealing woman always prefers that those above her are not very far from her in terms of attributes, because women with lower attributes have a smaller incentive to reveal and, hence, tend to increase the pool of non-revealing women which in turn improves the quality of the pool of males that match randomly.

We also stress that reduced utility function  $V^W$  does not exhibit downward cardinal concerns for status. This can be seen by considering  $F$ ,  $F'$ , and  $\tilde{x}$  such that  $F'(x) = F(x)$  for all  $x \in [\tilde{x}, \bar{x}]$ , and  $F'(x) < F(x)$  in  $(\underline{x}, \tilde{x})$ . From the proof of proposition 4 (see appendix B) we know that  $\hat{w}(F', G) \leq \hat{w}(F, G)$ . Hence, if  $F(\tilde{x}) \in W_R$  then  $F'(\tilde{x}) \in W_R$  too, so that from  $F'(\tilde{x}) = F(\tilde{x})$  we get that  $V(\tilde{x}, F') = U^W(x, G^{-1}(F'(x))) - c = U^W(x, G^{-1}(F(x))) - c = V(\tilde{x}, F)$ . If instead  $F(\tilde{x}) \in W_H$  then  $F(\tilde{x}) < \hat{w}(F, G)$  which in turn implies that  $x(\hat{w}(F, G)) = x(\hat{w}(F', G))$  and therefore that  $\hat{w}(F, G) = \hat{w}(F', G)$ . So,  $F'(\tilde{x}) \in W_H$  too, which in the light of  $\hat{w}(F', G) = \hat{w}(F, G)$  implies that  $V(\tilde{x}, F') = \int_0^{\hat{w}(F', G)} U^W(\tilde{x}, G^{-1}(s)) ds / \hat{w}(F', G) = \int_0^{\hat{w}(F, G)} U^W(\tilde{x}, G^{-1}(s)) ds / \hat{w}(F, G) = V(\tilde{x}, F)$  (indeed, function (6) does not satisfy definition 4 in appendix A).

Finally, let us remark that, differently from what seen for the models of section 7 and section 8, in this model men's reduced utility does not exhibit ordinal concerns for social status. Indeed, the reduced utility function function  $V^M$  for man  $m$  with attribute  $y$  is not the same as in section 6, but the following one:

$$V^M(y, G) = \begin{cases} \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} U^M(F^{-1}(G(s)), y) ds, & \text{if } m < \hat{w}(F, G), \\ U^M(F^{-1}(G(y)), y) - c, & \text{if } m \geq \hat{w}(F, G). \end{cases} \quad (7)$$

Not only function (7) does not exhibit ordinal concerns for social status, but it does not

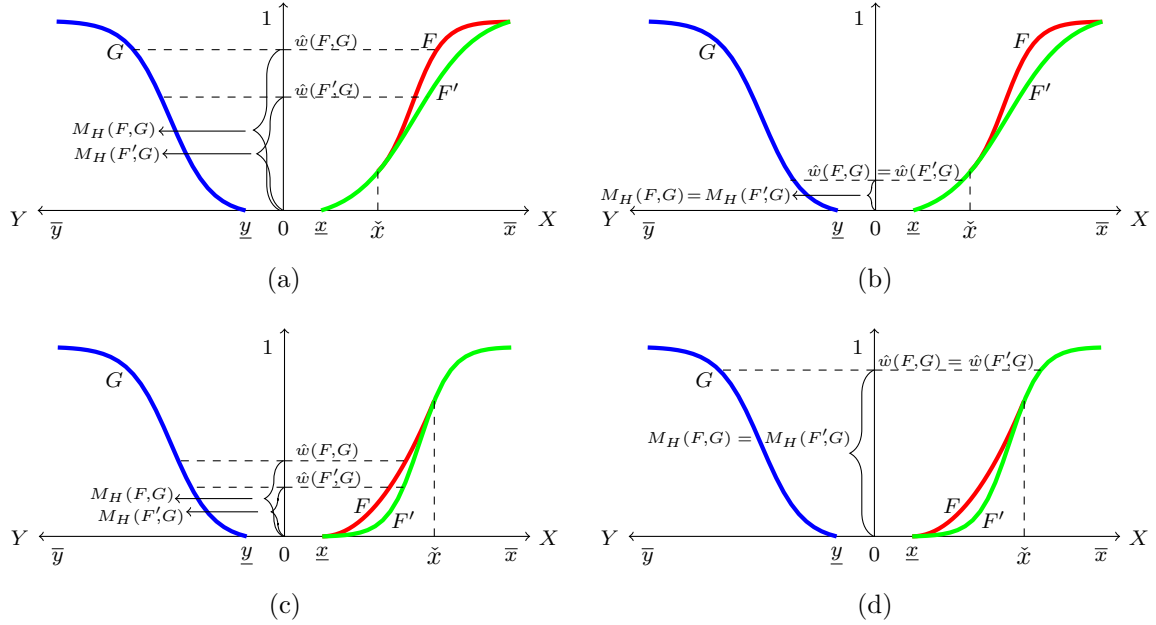


Figure 6: An upward shift of women's attributes from  $F$  to  $F'$  when attributes are unobservable and women can costly disclose them. In (a) the rank of women with attribute in  $(\tilde{x}, \bar{x})$  reduces for given attribute. The equilibrium cutoff woman changes from  $\hat{w}(F, G)$  to  $\hat{w}(F', G)$ , thus the set of men in the random pool shrinks from  $M_H(F, G)$  to  $M_H(F', G)$ , and the expected value of a random match for a woman gets lower. Therefore, all women with attribute in  $[x, \tilde{x}]$  are worse off. In (b) the rank of women with attribute in  $(\tilde{x}, \bar{x})$  reduces for given attribute. The equilibrium cutoff woman does not change so that the utility of women with attribute in  $[x, \tilde{x}]$  is unaffected. In (c) the rank of women with attribute in  $(x, \tilde{x})$  reduces for given attribute. The equilibrium cutoff woman changes from  $\hat{w}(F, G)$  to  $\hat{w}(F', G)$ , but this does not affect the utility of women with attribute in  $[\tilde{x}, \bar{x}]$  since both their match and their attribute remain the same. In (d) the rank of women with attribute in  $(x, \tilde{x})$  reduces for given attribute. The equilibrium cutoff woman does not change so that the utility of women with attribute in  $[\tilde{x}, \bar{x}]$  is unaffected.



exhibit concerns for social status at all. To see why, take a man  $m$  with attribute  $\hat{y} = G^{-1}(m)$  and consider a change from  $G$  to  $G'$  such that the attributes of some men other than  $m$  increase but  $m$ 's rank is unaffected, i.e.,  $G(\hat{y}) = G'(\hat{y}) = m$ , and such that  $m$  matches with a revealing woman under  $G'$  and with a randomly selected mate under  $G$ , i.e.,  $\hat{w}(F, G') < G(\hat{y}) < \hat{w}(F, G)$ . Note that such  $G$ ,  $G'$ , and  $\hat{y}$  possibly exist, since men's attributes can increase from  $G$  to  $G'$  in such a way that the equilibrium cutoff woman (and man) is pushed downward. Note also that, if  $m$  lies below but sufficiently close to the initial cutoff, then under  $G'$  he will end up matched with a woman with attribute very close to the maximum attribute attainable by his random mate under  $G$ , and so  $m$  will be better off under  $G'$ , against the definition of concerns for social status.

## 10 Concluding remarks

In this paper we have applied the instrumental approach to social preferences in order to distinguish between various shapes of preferences for social status when such preferences can be thought of as emerging from a strategic situation characterized by two-sided matching with non-transferable utility. In particular, we have shown that, weakening the extreme assumption of perfect observability of potential mates' attributes – i.e., allowing for asymmetric information – leads in equilibrium to reduced preferences for social status that induce concerns not only for the rank of one's own attribute in the distribution of attributes, but also for *how much* higher or lower such attribute is with respect to those of other agents. In addition, we have shown that the way in which agents can deal with the informational asymmetry determines whether the actual shape of cardinal concerns for status induces agents to care about the attributes of other agents who are above or below them (or both). The following table summarizes:

Shape of reduced form concerns for social status

Characteristics of attributes on side $X$	Ordinal	Cardinal	
		Downward	Upward
perfectly observable	•		
imperfectly observable		•	•
unobservable, can be costly signaled		•	
unobservable, can be costly revealed			•

Let us conclude by indicating a couple of different matching setups that we think are worth studying, and for which we can discuss the possible developments and the relation with the

literature.

One potentially interesting variant of our setup is that one-population matching, i.e., there is just one pool of agents that are matched among themselves. It can be easily understood that, if attributes are perfectly observable, then nobody would accept to match with someone with a lower attribute, so that an agent with attribute  $x$  ends up being matched with a partner that also has attribute  $x$ . Since this happens regardless of one's rank in the distribution of attributes, one's reduced-form utility could be independent of the distribution of attributes. Thus, in such a one-sided setting with perfect observability of attributes, reduced-form preferences would not show concerns for social status. However, we expect that when attributes are imperfectly observable cardinal concerns for social status can emerge, because having closer attributes makes it more likely to be perceived one for the other – analogously to what happens in our two-sided model. Instead, for the case where attributes are unobservable but we allow agents to send costly signals, we know from [Bidner \(2010\)](#) and [Bidner \(2013\)](#) that the distribution of types matters depending on the presence of frictions, and therefore we can infer that the same would hold for the emergence of concerns for social status. Lastly, further research is needed to explore the one-sided case where attributes are unobservable but we let agents reveal them at a cost.

Another interesting variant of our setup would be to allow for transferable utility. The work of [Costrell and Loury \(2004\)](#) on job assignments provides an intuition of what can happen in such a case: the gap in terms of attributes in one's population determines the bargaining power of the agents. So, we can expect that the share of surplus obtained by an agent  $w$  with attribute  $x$  depends on how close to  $x$  are the attributes of agents with whom  $w$  could be replaced by her mate – i.e., the closer the attributes, the weaker agent  $w$ . This, we think, would hold a fortiori when attributes are imperfectly observable, as suggested by the analysis of [Costrell and Loury \(2004\)](#) for imperfectly observable skills. Moreover, the transferable utility model in [Hopkins \(2012\)](#) provides an indication of what could happen when attributes are unobservable but agents can costly signal them: separating from lower types becomes more costly if lower types gets closer in terms of attributes, and this adds to the intuition just described. So, we expect at least downward cardinal concerns to emerge. Finally, for the case of unobservable attributes that can be revealed at a cost things seem to be less straightforward and indeed, to the best of our knowledge, there are no contributions exploring such a case.

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## A Appendix - Definitions

Preliminarily, we have to introduce orderings over distribution functions. Let us indicate with  $\succ_{fod}$  the relation of first order stochastic dominance between two distribution functions. To say that  $F' \succ_{fod} F$  means that the graph of  $F'$  never lies above the graph of  $F$  and it lies strictly below in some interval in  $X$ . Intuitively, in  $F'$  the distribution of women’s attributes

is shifted upward with respect to  $F$ . To indicate a relation of first order stochastic dominance between  $F'$  and  $F$  on  $[x_1, x_2] \subset X$  we write  $F' \succ_{fod} F$  on  $[x_1, x_2]$ . Note also that we keep the support of  $F$  fixed in our comparative statics. This is done to reduce technicalities at the minimum. Finally, let  $\mathcal{F}(X)$  and  $\mathcal{G}(Y)$  be, respectively, the set of all feasible distributions of women's and men's attributes.

We first define instrumental concerns for social status.

**DEFINITION 1** (Concerns for social status). *Reduced utility  $V^W$  is said to exhibit concerns for social status if:*

- (i) for all  $x \in X$ , for all  $F, F' \in \mathcal{F}(X)$ ,  $F' \succ_{fod} F$  implies  $V^W(x, F) \geq V^W(x, F')$ ;
- (ii) there exist  $\check{x} \in X$  and  $F, F' \in \mathcal{F}(X)$  such that  $F' \succ_{fod} F$  and  $V^W(\check{x}, F) > V^W(\check{x}, F')$ .

Definition 1 says that a reduced utility  $V^W$  exhibits concerns for social status if it is both (i) weakly monotone and (ii) non-constant in  $F$  with respect to first order stochastic dominance. In words, (i) means that no upward shifts of the distribution of attributes can increase the reduced utility of an individual with a given attribute, while (ii) means that there exists at least one level of attributes and an upward shift that lead to a strictly lower reduced utility.

We are now ready to introduce ordinal concerns for social status.

**DEFINITION 2** (Ordinal concerns for social status). *Reduced utility  $V^W$  is said to exhibit ordinal concerns if it exhibits concerns for social status and, for all  $x \in X$ , for all  $F, F' \in \mathcal{F}(X)$ ,  $F(x) = F'(x)$  implies  $V^W(x, F) = V^W(x, F')$ .*

Definition 2 says that  $V^W$  exhibits ordinal concerns for status when same rank leads to same utility.

**DEFINITION 3** (Cardinal concerns for social status). *Reduced utility  $V^W$  is said to exhibit cardinal concerns if it exhibits concerns for social status and it does not exhibit ordinal concerns.*

Definition 3 says that all concerns for status that are not ordinal, are cardinal.

**DEFINITION 4** (Downward cardinal concerns for social status). *Reduced utility  $V^W$  is said to exhibit downward cardinal concerns if it exhibits concerns for social status and there exists  $\check{x} \in X$  and  $F, F' \in \mathcal{F}(X)$  such that  $F(x) = F'(x)$  for all  $x \geq \check{x}$ ,  $F' \succ_{fod} F$  on  $[\underline{x}, \check{x}]$ , and  $V^W(\check{x}, F) \neq V^W(\check{x}, F')$ .*

Definition 4 says that  $V^W$  exhibits downward cardinal concerns when there exists an attribute  $\check{x}$  and an upward (downward) shift of the distribution of attributes below  $\check{x}$  that decreases (increases) the reduced utility of the individual with attribute  $\check{x}$ . We note that  $V^W(\check{x}, F) \neq V^W(\check{x}, F')$  in definition 4 actually implies  $V^W(\check{x}, F) > V^W(\check{x}, F')$  when combined with the requirement that the reduced utility exhibits concerns for social status. Note that definition 4 is not exclusive, meaning that  $V^W$  may exhibit downward cardinal concerns together with other cardinal features.

**DEFINITION 5** (Upward cardinal concerns for social status). *Reduced utility  $V^W$  is said to exhibit upward cardinal concerns if it exhibits concerns for social status and there exists  $\check{x} \in X$  and  $F, F' \in \mathcal{F}(X)$  such that  $F(x) = F'(x)$  for all  $x \leq \check{x}$ ,  $F' \succ_{fod} F$  on  $[\check{x}, \bar{x}]$ , and  $V^W(\check{x}, F) \neq V^W(\check{x}, F')$ .*

Definition 5 is specular to definition 4 but with respect to individuals with higher attributes. In particular, definition 5 says that  $V^W$  exhibits upward cardinal concerns when there exists an attribute  $\check{x}$  and an upward (downward) shift of the distribution of attributes above  $\check{x}$  that decreases (increases) the reduced utility of the individual with attribute  $\check{x}$ . Note that, similarly to definition 4, definition 5 is not exclusive, allowing for further cardinal features.

**DEFINITION 6** (Bidirectional cardinal concerns for social status). *Reduced utility  $V^W$  is said to exhibit bidirectional concerns if it exhibits concerns for social status and there exists  $\check{x}$  for which  $V^W$  exhibits both downward and upward concerns.*

Definition 6 identifies the case where upward and downward concerns are jointly present. Note that, in order to satisfy definition 6, definition 4 and definition 5 must be satisfied for the same  $\check{x}$ .<sup>14</sup> This means that, to have bidirectional concerns, upward and downward concerns must jointly exist for the same individuals. Bidirectional cardinal concerns are a typical feature in economic models with concerns for status.

## B Appendix - Proofs

*Proof of Proposition 2.* Consider  $F$  and  $F'$  such that  $F' \succ_{fod} F$ , which implies that  $F(x) \geq F'(x)$  for all  $x \in X$ . Note that, because  $p$  is everywhere strictly positive,  $P$  is strictly

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<sup>14</sup>We note that the case where  $V^W$  exhibits upward concerns for some attribute levels and downward concerns for some different attribute levels does not satisfy definition 6. We want to remark that alternative definitions can be used. Here we insisted on the presence of at least one attribute level for which preferences exhibit both upward and downward concerns to stress the fact that the overall reduced utility exhibits bidirectional concerns.

increasing in its argument, so that  $F^{-1}(w) \leq F'^{-1}(w)$  for all  $w \in [0, 1]$  implies that  $P(x + \epsilon - F^{-1}(w)) \geq P(x + \epsilon - F'^{-1}(w))$  for all  $w \in [0, 1]$ . This in turn implies that  $\int_0^1 P(x + \epsilon - F^{-1}(s))ds \geq \int_0^1 P(x + \epsilon - F'^{-1}(s))ds$ . Hence, the integrand of (1) under  $F'$  is never greater than the integrand of (1) under  $F$ , and therefore that  $V^W(x, F) \geq V^W(x, F')$  for all  $x \in X$ , establishing point (i) of definition 1. To see why also point (ii) of definition 1 is satisfied consider  $F$  and  $F'$  such that  $F' \succ_{fod} F$ ,  $F'(x) < F(x)$  for all  $x \in (x_1, x_2) \subset X$ ,  $x_1 \neq x_2$ ,  $F'(x_1) = F(x_1)$  and  $F'(x_2) = F(x_2)$ . So, for every  $x \in X$ , we have that  $P(x + \epsilon - F^{-1}(w)) > P(x + \epsilon - F'^{-1}(w))$  for  $w \in (F(x_1), F(x_2))$ , which in turn implies, together with point (i) of definition 1, that  $\int_0^1 P(x + \epsilon - F^{-1}(s))ds > \int_0^1 P(x + \epsilon - F'^{-1}(s))ds$ , and hence that  $V^W(x, F) > V^W(x, F')$ .

Consider now  $F$ ,  $F'$ , and  $\check{x}$  such that  $F'(x) = F(x)$  for  $x \in [\underline{x}, \check{x}]$ , and  $F' \succ_{fod} F$  in  $[\check{x}, \bar{x}]$ . We observe that this is a particularization of the previous case where  $x_1 = \check{x}$ ,  $x_2 = \bar{x}$  and  $F'(x) = F(x)$  for all  $x \notin [x_1, x_2]$ . Hence,  $V^W(\check{x}, F) > V^W(\check{x}, F')$ . Therefore, function (1) does not satisfy definition 2 while it satisfies definition 3 and definition 5.

Finally, consider  $F$ ,  $F'$ , and  $\check{x}'$  such that  $F'(x) = F(x)$  for  $x \in [\check{x}', \bar{x}]$ , and  $F' \succ_{fod} F$  in  $[\underline{x}, \check{x}']$ . By an argument similar to the one applied above we can establish that  $V^W(\check{x}', F) > V^W(\check{x}', F')$ , implying that function (1) satisfies definition 4. Since we can set  $\check{x} = \check{x}'$ , we have that function (1) also satisfies definition 6.  $\square$

*Proof of Proposition 3.* Consider  $F$  and  $F'$  such that  $F' \succ_{fod} F$ , which implies that  $F(x) \geq F'(x)$  for all  $x \in X$  and  $F^{-1}(w) \leq F'^{-1}(w)$  for all  $w \in W$ . Consider also the following cases: (i) woman  $F(x)$  who chooses to signal  $\sigma^*(F(x))$  when the actual distribution is  $F$ , (ii) woman  $F(x)$  who chooses to signal  $\sigma^*(F'(x))$  when the actual distribution is  $F$ , and (iii) woman  $F'(x)$  who chooses to signal  $\sigma^*(F'(x))$  when the actual distribution is  $F'$ . The reduced utilities of women in the cases (i), (ii), and (iii) are, respectively:

$$U^W(x, G^{-1}(F(x))) - \int_0^{F(x)} \frac{\partial U^W(F^{-1}(s), G^{-1}(s))}{\partial y} \frac{1}{g(G^{-1}(s))} ds \quad (8)$$

$$U^W(x, G^{-1}(F'(x))) - \int_0^{F'(x)} \frac{\partial U^W(F^{-1}(s), G^{-1}(s))}{\partial y} \frac{1}{g(G^{-1}(s))} ds \quad (9)$$

$$U^W(x, G^{-1}(F'(x))) - \int_0^{F'(x)} \frac{\partial U^W(F'^{-1}(s), G^{-1}(s))}{\partial y} \frac{1}{g(G^{-1}(s))} ds \quad (10)$$

In case (i) woman  $F(x)$  acts optimally, so that utility (9) cannot be greater than utility (8). From  $F^{-1}(w) \leq F'^{-1}(w)$  for all  $w \in W$  and  $\partial^2 U^W / \partial x \partial y > 0$ , we have that  $\partial U(F^{-1}(w), G^{-1}(w)) / \partial y \leq \partial U(F'^{-1}(w), G^{-1}(w)) / \partial y$  for all  $w \in W$ . Therefore, the second



term in (10) is not greater than the second term in (9) for all  $x \in X$ . Since the first terms of (10) and (9) are identical for all  $x \in X$ , we have that utility (10) is not greater than utility (9) for all  $x \in X$ . This, together with the fact that utility (9) is never greater than utility (8), implies that utility (10) is never greater than utility (8), i.e.,  $V^W(x, F') \leq V^W(x, F)$  for all  $x \in X$ . Hence, function 4 satisfies point (i) of definition 1.

Consider now  $F$ ,  $F'$ , and  $\check{x}$  such that  $F' = F'$  in  $[\check{x}, \bar{x}]$ , and  $F' \succ_{fod} F$  in  $[\underline{x}, \check{x}]$ . Hence,  $F'(x) \leq F(x)$  for all  $x \in [\underline{x}, \check{x}]$ , with strict inequality holding for some interval  $(x_1, x_2) \subset [\underline{x}, \check{x}]$ ,  $x_1 \neq x_2$ . This and  $\partial^2 U^W / \partial x \partial y > 0$  allow us to conclude, by reasoning as we did above for the comparison between (10) and (9), that, for a woman with attribute  $\check{x}$ , the second term of (4) is strictly larger – i.e, smaller in absolute terms – under  $F$  than under  $F'$ . Moreover, since  $F'(\check{x}) = F(\check{x})$ , it follows that  $G^{-1}(F'(\check{x})) = G^{-1}(F(\check{x}))$  which in turn implies that, for a woman with attribute  $\check{x}$ , the first term of (4) is the same under both  $F$  and  $F'$ . From the latter two observations follows that  $V^W(\check{x}, F') < V^W(\check{x}, F)$ . This establishes that reduced utility (4) satisfies point (ii) of definition 1, and hence function (4) exhibits concerns for social status. Also, it shows that function (4) does not satisfy definition 2 while it satisfies definition 3 and definition 4.  $\square$

*Proof of Proposition 4.* Consider  $F$  and  $F'$  such that  $F' \succ_{fod} F$ , which implies that  $F'(x) \leq F(x)$  for all  $x \in X$  and, hence,  $F'^{-1}(w) \geq F^{-1}(w)$  for all  $w \in W$ . In particular,  $F'^{-1}(\hat{w}(F, G)) \geq F^{-1}(\hat{w}(F, G))$ . We now show that  $\hat{w}(F', G) \leq \hat{w}(F, G)$ . If  $\hat{w}(F, G) \in [0, 1)$ , then we have that:

$$\begin{aligned}
0 &\leq U^W(F^{-1}(\hat{w}(F, G)), G^{-1}(\hat{w})) - c - \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} U^W(F^{-1}(\hat{w}(F, G)), G^{-1}(m)) dm \\
&= \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} [U^W(F^{-1}(\hat{w}(F, G)), G^{-1}(\hat{w})) - U^W(F^{-1}(\hat{w}(F, G)), G^{-1}(m))] dm - c \\
&\leq \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} [U^W(F'^{-1}(\hat{w}(F, G)), G^{-1}(\hat{w})) - U^W(F'^{-1}(\hat{w}(F, G)), G^{-1}(m))] dm - c \\
&= U^W(F'^{-1}(\hat{w}(F, G)), G^{-1}(\hat{w})) - c - \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} U^W(F'^{-1}(\hat{w}(F, G)), G^{-1}(m)) dm \quad (11)
\end{aligned}$$

where the first inequality is implied by the fact that  $\hat{w}(F, G)$  is the cutoff woman in the equilibrium emerging under  $F$ , the first and last equalities follow from a rearrangement of terms, while the second inequality is implied by  $U^W$  satisfying increasing differences in  $x$  and  $y$ . Note that the last expression of (11) is expression (5) evaluated for  $\hat{w}(F, G)$  under  $F'$ . Since expression (5) is continuous and, for  $\hat{w} = 0$ , non-positive, we can conclude that (5) takes

value zero for some  $\hat{w} \in [0, \hat{w}(F, G))$  under  $F'$ . Therefore, the selected equilibrium under  $F'$  is such that  $\hat{w}(F', G) \leq \hat{w}(F, G)$ . Finally, if  $\hat{w}(F, G) = 1$ , then  $\hat{w}(F', G) \leq \hat{w}(F, G) = 1$  since 1 is the maximum of  $W$ .

We now prove that reduced utility (6) satisfies point (i) of definition 1. Since  $\hat{w}(F, G) \geq \hat{w}(F', G)$ , we can partition the set  $X$  in three subsets:  $X_{HH} = \{x \in X | F(x) \in W_H, F'(x) \in W_H\}$  collecting the attributes whose women do not reveal under both  $F$  and  $F'$ ,  $X_{HR} = \{x \in X | F(x) \in W_H, F'(x) \in W_R\}$  collecting the attributes whose women do not reveal under  $F$  but reveal under  $F'$ , and  $X_{RR} = \{x \in X | F(x) \in W_R, F'(x) \in W_R\}$  collecting the attributes whose women reveal under both  $F$  and  $F'$ . If  $x \in X_{RR}$ , then from  $F(x) \geq F'(x)$  follows that  $V^W(x, F') = U^W(x, G^{-1}(F'(x))) - c \leq U^W(x, G^{-1}(F(x))) - c = V^W(x, F)$ . If  $x \in X_{HH}$ , then it can be easily checked that the derivative of (6) with respect to  $\hat{w} \in (0, 1)$  is non-negative. This, together with the fact that  $\hat{w}(F, G) \geq \hat{w}(F', G)$ , implies that  $V^W(x, F) \geq V^W(x, F')$ . Finally, let  $x \in X_{HR}$ . Since woman  $F(x)$  finds it optimal not to reveal under  $F$ , we have a fortiori that  $V^W(x, F) \geq U^W(x, G^{-1}(F(x))) - c$ . Since  $F'(x) \leq F(x)$  for all  $x \in X$ , we have that  $U^W(x, G^{-1}(F'(x))) \leq U^W(x, G^{-1}(F(x)))$ , which implies that  $V^W(x, F') \leq U^W(x, G^{-1}(F(x))) - c = V^W(x, F)$ .

Consider now  $F$ ,  $F'$ , and  $\check{x}$  such that  $F' \succ_{fod} F$  for all  $x \in [\check{x}, \bar{x}]$ ,  $F'(x) = F(x)$  for all  $x \in [\underline{x}, \check{x}]$  and  $\check{x} < \hat{w}(F', G) < \hat{w}(F, G)$ . Note that for such  $F'$ ,  $F$ , and  $\check{x}$  to exist it is sufficient to have  $\hat{w}(F', G) \in (0, 1)$ , which in turn is guaranteed by  $c > 0$  being sufficiently small. In words, we are considering a woman with attribute  $\check{x}$  that does not reveal either under  $F$  or  $F'$  and who sees some women with attribute higher than her switching from non-revealing under  $F$  to revealing under  $F'$ . From  $\hat{w}(F, G) > \hat{w}(F', G)$  and the fact that the derivative of  $(1/\hat{w}) \int_0^{\hat{w}} U^W(\check{x}, G^{-1}(s)) ds$  with respect to  $\hat{w} \in (0, 1)$  is positive, we get:

$$V^W(F, \check{x}) = \frac{1}{\hat{w}(F, G)} \int_0^{\hat{w}(F, G)} U^W(x, G^{-1}(s)) ds > \frac{1}{\hat{w}(F', G)} \int_0^{\hat{w}(F', G)} U^W(x, G^{-1}(s)) ds = V^W(F', \check{x})$$

This establishes that reduced utility (6) satisfies point (ii) of definition 1, and hence function (6) exhibits concerns for social status. Moreover, it shows that function (6) does not satisfy definition 2 while it satisfies definition 3 and definition 5.  $\square$

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