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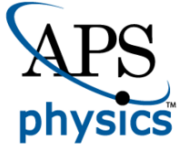
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# Constraints from Triple Gauge Couplings on Vectorlike Leptons

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**ABSTRACT:** We study the contributions of colorless vectorlike fermions to the triple gauge couplings  $W^+W^-\gamma$  and  $W^+W^-Z^0$ . We consider models in which their coupling to the Standard Model Higgs boson is allowed or forbidden by quantum numbers. We assess the sensitivity of the future accelerators FCC-ee, ILC and CLIC to the parameters of these models, assuming they will be able to constrain the anomalous triple gauge couplings with a precision  $\delta\kappa_V \sim \mathcal{O}(10^{-4})$ ,  $V = \gamma, Z^0$ . We show that the combination of measurements at different center-of-mass energies helps to improve the sensitivity to the contribution of vectorlike fermions, in particular when they couple to the Higgs. In fact, the measurements at the FCC-ee and, especially, the ILC and the CLIC, may turn the triple gauge couplings into a new set of precision parameters able to constrain the models better than the oblique parameters or the  $H \rightarrow \gamma\gamma$  decay, even assuming the considerable improvement of the latter measurements achievable at the new machines.

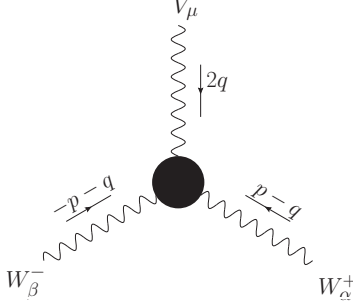
## 1 Introduction

All experimental data collected so far have confirmed the Standard Model (SM) predictions, including the existence of a scalar particle that seems to have the right properties to match those of a Higgs boson. The SM cannot however be the final theory of Particle Physics, since it does not explain neutrino masses, the baryon asymmetry of the universe and it does not contain a Dark Matter (DM) candidate. Moreover, if the naturalness principle applies, New Physics (NP) is expected.

The nature of the NP models that are supposed to complete the SM is elusive and unknown. Taking a bottom up approach, however, we can suppose that, exactly as the SM particles are vectorlike from the low energy QED/QCD point of view, the first particles to be discovered (if any) will be vectorlike from the SM point of view [1]. In addition, vectorlike fermions arise in many well motivated SM extensions such as models with extra dimensions [2–5], composite Higgs [6–8], two Higgs doublet model extensions [9], low-scale supersymmetry [10, 11] and, more recently, in new solutions of the hierarchy problem [12, 13]. Vectorlike fermions are much less constrained than extra chiral families, which in fact are now pretty much ruled out by data after the observation of the 125 GeV boson at the LHC [14, 15]. Vectorlike quarks masses are typically bounded from ATLAS and CMS Run 1 data to be  $\gtrsim$  (800-1000) GeV [16–23], while direct constraints on vectorlike leptons come only from the LEP experiments and are constrained to be  $\gtrsim$  100 GeV [24]. Bounds from electric and magnetic dipole moments and electroweak precision measurements have been also considered [25, 26].

As no new particles have been discovered so far, there is a growing interest of the community in future  $e^+ e^-$  colliders that could pursue the electroweak precision tests started by LEP and the SLC profiting of higher energies and luminosities. This moves from the observation that, for heavy enough particles, NP may first show up through loop effects, and as such be bounded by electroweak precision measurements, modifications of  $H \rightarrow \gamma\gamma$  or anomalous triple gauge couplings (TGC). In particular, the new machines can probe the anomalous TGC's  $W^+W^-\gamma$ ,  $W^+W^-Z^0$  and  $Z^0Z^0\gamma$  to unprecedented levels. Since the structure of the TGC's is a direct manifestation of the non abelian nature of the SM gauge group, they are sensitive to the presence of NP with  $SU(2)_L \times U(1)_Y$  representation and, in particular, to the presence of vectorlike fermions.

The purpose of this paper is to estimate the sensitivity of future  $e^+ e^-$  machines to vectorlike leptons, in many possible realizations, via the measurements of triple gauge couplings which will putatively reach a  $\mathcal{O}(10^{-4})$  precision. The paper is organized as follows. In Sec. 2 we start by defining the TGC's form factors that can be modified by SM loop corrections and new physics. Next, in Sec. 3 we describe the vectorlike lepton models that we will study in this paper and how they can contribute to the TGC's form factors. In Sec. 4 we estimate the constraints on these models that can be achieved by TGC's measurements at three proposed future accelerator facilities: the Future Circular Collider (FCC-ee) [27], International Linear



**Figure 1.** Feynman diagram of the  $WWV$  vertex in momentum space.

Collider (ILC) [28] and the Compact Linear Collider (CLIC) [29]. Finally, in Sec.5 we discuss our conclusions.

## 2 Triple Gauge Couplings

The typical structure of the charged TGC's that we will consider in this paper is shown in Fig. 1, where  $V$  can be either the  $Z^0$  boson or the photon. The complete one-loop SM contribution to the charged TGC's  $W^+W^-\gamma$  and  $W^+W^-Z^0$  have been computed some time ago [30–32], while the contribution to the neutral TGC  $Z^0Z^0\gamma$  have been studied in [30, 33]. The charged couplings can be directly studied in future  $e^+e^-$  colliders, through  $e^+e^- \rightarrow W^+W^-$ . The neutral couplings, on the other hand, can be studied using the processes  $e^+e^- \rightarrow Z^0\gamma$  or  $e^+e^- \rightarrow Z^0Z^0$ , with subsequent decays  $Z^0 \rightarrow \bar{\nu}\nu$  and  $Z^0 \rightarrow \ell^+\ell^-$  [34–36]. Let us note that only fermions with an axial coupling to the  $Z^0$  boson can generate non vanishing corrections to the neutral TGC's [33]. As such, since our focus are vectorlike fermions, we will just consider the effects on the charged vertexes.

The generic charged TGC vertex  $WWV$ , with  $V = \gamma, Z^0$ , can be parametrized using the following effective lagrangian [37]

$$\begin{aligned} \mathcal{L}_{WWV} = & -ig_V[(W_{\mu\nu}^\dagger W^{\mu\nu} V^\nu - W_{\mu\nu} W^{\mu\dagger} V^\nu) + \kappa_V W_\mu^\dagger W_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\mu\tau}^\dagger W_\nu^\tau V^{\nu\mu} \\ & + \mathcal{L}_{WWV}^{\text{nCP}}, \end{aligned} \quad (2.1)$$

where  $\mathcal{L}_{WWV}^{\text{nCP}}$  contains  $P$  or  $C$  odd terms,  $\kappa_V$  and  $\lambda_V$  are form factors, the field strengths are defined as  $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ ,<sup>1</sup>  $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ , and the coupling  $g_V$  is given by

$$g_V = \begin{cases} e & \text{for } V = \gamma, \\ e \cot \theta_W & \text{for } V = Z^0. \end{cases} \quad (2.2)$$

<sup>1</sup>Notice that with this definition, the  $W$  field strength is not  $U(1)_{em}$  invariant. New quadrilinear terms must be introduced in  $\mathcal{L}$  to make the whole Lagrangian gauge invariant.

In the SM at tree level,  $\kappa_V = 1$  and  $\lambda_V = 0$ . We will focus only on the  $C$  and  $P$  conserving terms, discarding  $\mathcal{L}_{WWV}^{\text{nCP}}$  in the following. In the photon case, the form factors are related to the static properties of the  $W$  boson (namely the magnetic dipole  $\mu_W$  and the electric quadrupole moment  $Q_W$ ) through the relations [37]

$$\begin{aligned}\mu_W &= \frac{e}{2M_W}(1 + \kappa_\gamma + \lambda_\gamma), \\ Q_W &= -\frac{e}{M_W^2}(\kappa_\gamma - \lambda_\gamma).\end{aligned}\tag{2.3}$$

Following a notation analogous to the one used in [30] (see Figure 1 for the definition of the momenta), the  $WWV$  vertex in momentum space can be written as

$$\begin{aligned}\Gamma_{\mu\alpha\beta}^V &= -ig_V \left\{ f(q^2) [2g_{\alpha\beta}p_\mu + 4(g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha)] + 2\Delta\kappa_V(q^2)(g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha) \right. \\ &\quad \left. + 4\frac{\Delta Q(q^2)}{M_W^2} \left( p_\mu q_\alpha q_\beta - \frac{1}{2}q^2 g_{\alpha\beta} p_\mu \right) \right\},\end{aligned}\tag{2.4}$$

with the  $f(q^2)$  form factor connected to the renormalization of the charge, while  $\Delta\kappa_V(q^2)$  and  $\Delta Q_V(q^2)$ , related to  $\kappa_V$  and  $\lambda_V$  through the expressions

$$\begin{aligned}\Delta\kappa_V &= \kappa_V + \lambda_V - 1 \equiv \Delta\kappa_V^{SM} + \Delta\kappa_V^{NP}, \\ \Delta Q_V &= -2\lambda_V \equiv \Delta Q_V^{SM} + \Delta Q_V^{NP},\end{aligned}\tag{2.5}$$

are designed to be zero at tree level in the SM. The SM 1-loop contributions can be found in Refs. [30–32], while the explicit calculation of  $\Delta\kappa_V^{NP}$  and  $\Delta Q_V^{NP}$  in the case of vectorlike fermions is presented in Appendix A.

The quantity used by the experimental collaborations to show their results is the deviation from the SM value of  $\kappa_V$  a tree level,  $\delta\kappa_V = \kappa_V - 1$ , which will correspond to a linear combination of  $\Delta\kappa_V$  and  $\Delta Q_V$ , namely

$$\delta\kappa_V = \Delta\kappa_V + \frac{1}{2}\Delta Q_V,\tag{2.6}$$

and this is the quantity we will be using throughout the paper.

### 3 Models of Colorless Vectorlike Fermions

For our study, we will consider two classes of colorless vectorlike fermions: *(i)* a set of fermions in a unique  $\text{SU}(2)_L$  representation, with no couplings to the Higgs boson allowed, and *(ii)* a set of at least two extra fermions in representations such that a Yukawa term with the Higgs boson is allowed. In both cases we will assume that, due to some unspecified symmetry  $\mathcal{G}$ , all the mixing between the vectorlike and the SM fermions are forbidden.

### 3.1 Unmixed Colorless Vectorlike Fermions

As already mentioned, we start adding to the SM particle content one vectorlike fermion  $\Psi$ , transforming under  $SU(2)_L \times U(1)_Y$  as  $\Psi \sim (\mathbf{2}_j + \mathbf{1}, Y)$  and with mass  $m_\Psi$ . The lagrangian is given by

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu(\partial_\mu - igW_\mu^a T^a - ig'Y B_\mu)\Psi - m_\Psi\bar{\Psi}\Psi, \quad (3.1)$$

where  $T^a$  are the  $2j + 1$  dimensional generators of the  $SU(2)_L$  Lie algebra. An important consequence of considering a unique  $SU(2)_L$  representation for all the  $N_F$  vectorlike fermions is that the  $\delta\kappa_V^\Psi$  form factor just depends on the hypercharge and on the dimension  $j$  of the  $SU(2)_L$  representation, and not on the eigenvalues of the  $T^3$  operator. This is shown explicitly in Appendix B, from which we see that we can write

$$\delta\kappa_V^\Psi \propto F_j I(m_\Psi), \quad F_j \equiv N_F Y \frac{2}{3} j(j+1)(2j+1), \quad (3.2)$$

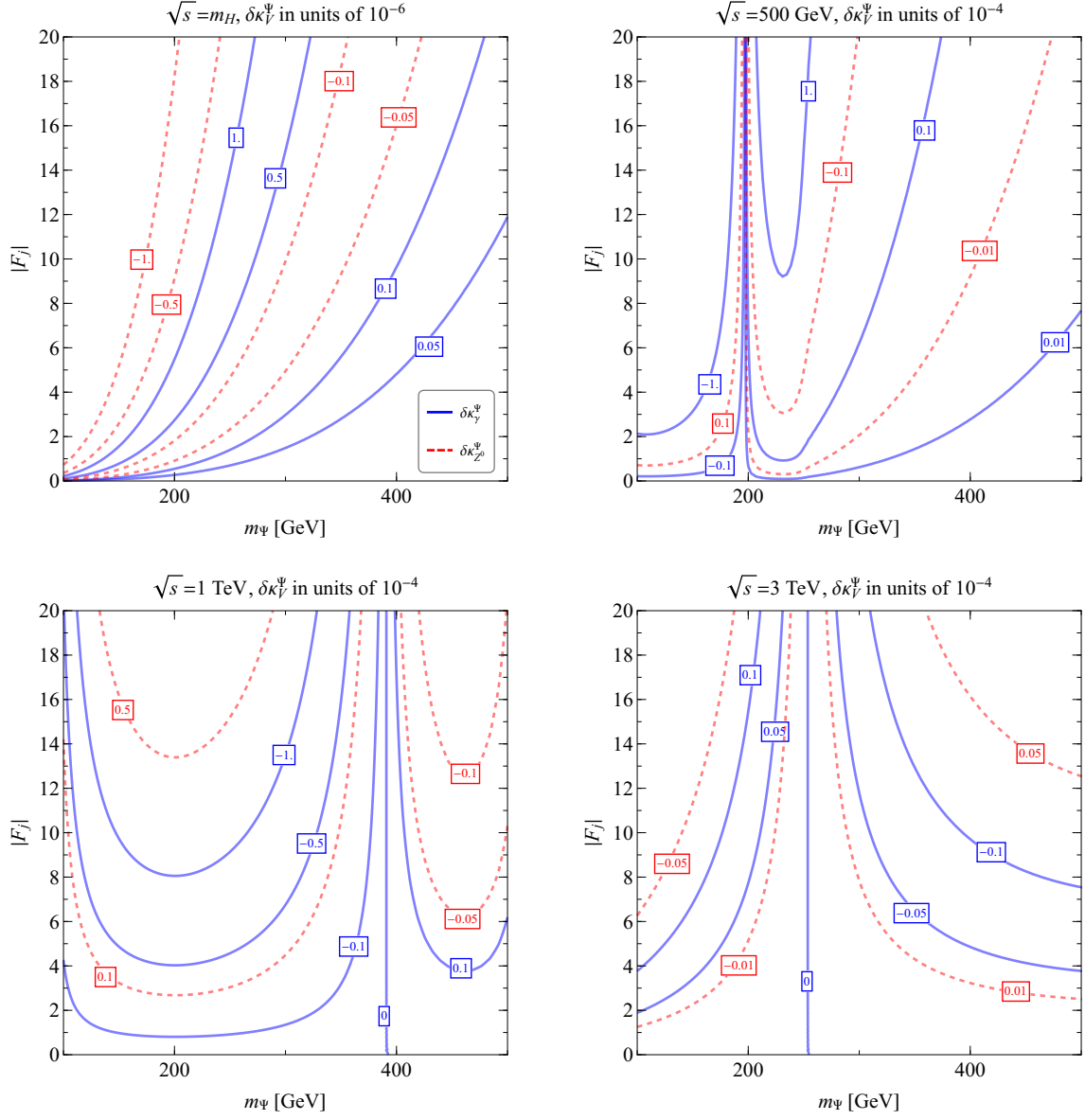
where  $I(m_\Psi)$  is a loop factor that only depends on the vectorlike lepton mass  $m_\Psi$ . An equivalent statement is that all the contributions to the  $W^+W^-W^3$  TGC cancel out, leaving only  $W^+W^-B$  (with  $B$  the hypercharge gauge boson). Integrating numerically over the Feynman parameters of Eq. (A.1) we obtain  $\Delta\kappa_V^\Psi$  and  $\Delta\kappa_Z^\Psi$  as a function of  $\sqrt{s} = \sqrt{(2q)^2}$  (see Appendix A for details).

In Fig. 2 we show the contour lines for  $\delta\kappa_V^\Psi$  in the  $(m_\Psi, |F_j|)$  plane for the four different center-of-mass energies  $\sqrt{s} = m_H, 500 \text{ GeV}, 1 \text{ TeV}$  and  $3 \text{ TeV}$ . We observe that  $|\delta\kappa_\gamma^\Psi| < |\delta\kappa_{Z^0}^\Psi|$  and they have opposite sign (see Eq. (B.12) in Appendix B). The typical values of  $|\delta\kappa_V^\Psi|$  are smaller than a few  $10^{-4}$ .

For fixed  $\sqrt{s}$ , the loop factor in Eq. (3.2) vanishes for  $m_\Psi = m_{\Psi_1}$  and  $m_\Psi = m_{\Psi_2}$ , where  $m_{\Psi_1, \Psi_2}$  are complicated functions of  $\sqrt{s}$ . The general behavior of  $\delta\kappa_\gamma^\Psi$  as a function of  $m_\Psi$  is the following: it starts positive, it vanishes for  $m_\Psi = m_{\Psi_1}$ , goes through a minimum (negative) value, it increases again until it reaches zero for  $m_\Psi = m_{\Psi_2}$ , goes through a maximum (positive) value and then decreases again until it goes back to zero. Because of the flip in sign,  $\delta\kappa_{Z^0}^\Psi$  has the opposite behavior. For  $\sqrt{s} = m_H$  both cancellations occur for  $m_\Psi < 100 \text{ GeV}$  so they do not appear in the plot. For  $\sqrt{s} = 500 \text{ GeV}$  and  $1 \text{ TeV}$ , we can only see in Fig. 2 the second cancellation at  $m_{\Psi_2} \approx 200 \text{ GeV}$  and  $400 \text{ GeV}$ , respectively, while for  $\sqrt{s} = 3 \text{ TeV}$  we can see the first cancellation at  $m_{\Psi_1} \approx 250 \text{ GeV}$ . Note that after the second cancellation the loop integral gets suppressed ( $m_\Psi$  becomes too off-shell for that specific center-of-mass energy) so to reach the same  $|\delta\kappa_V^\Psi|$  one has to increase the effective coupling, *i.e.* go to higher values of  $|F_j|$ .

### 3.2 Mixed Colorless Vectorlike Fermions

Let us now consider the case in which the colorless vectorlike fermions transform in different  $SU(2)_L \times U(1)_Y$  representations, such that an invariant Yukawa coupling with the Higgs boson



**Figure 2.** Contour lines of  $\delta\kappa_V^\Psi$  (see Eq. (2.6)) in the plane  $(m_\Psi, |F_j|)$  for the models with unmixed vectorlike colorless fermions (vectorlike leptons) at four different center-of-mass energies:  $\sqrt{s} = m_H$ , 500 GeV, 1 TeV and 3 TeV. For the definition of  $F_j$  see Eq. (3.2). The full blue (dashed red) lines correspond to  $V = \gamma$  ( $Z^0$ ).

is allowed. Since a general discussion would be quite involved, we will consider two examples to illustrate the impact of the future experiments measuring the TGC's. Specifically, we will examine the two models studied in [38], corresponding to the addition of a singlet and a doublet, and a doublet plus a triplet of fermions.



**Doublet-singlet model.** We introduce a singlet Dirac fermion  $N = N_L + N_R$  with hypercharge  $Y$  and a doublet Dirac fermion  $L = L_L + L_R$  with hypercharge  $Y - \frac{1}{2}$ .<sup>2</sup> We will write explicitly the components of the  $L$  doublet as  $L = (N_0, E)^T$  for the two chiralities. The lagrangian is given by

$$\mathcal{L}_{2+1} = i\bar{L}\not{D}L + i\bar{N}\not{D}N - M_N\bar{N}_R N_L - M_L\bar{L}_R L_L - c\bar{N}_R H L_L - c'\bar{N}_L H L_R + h.c. \quad (3.3)$$

With the hypercharge assignment we are considering, the electric charges of the various components are

$$\begin{aligned} E &\rightarrow q_\chi \equiv Y - 1, \\ N, N_0 &\rightarrow q_\omega \equiv Y, \end{aligned} \quad (3.4)$$

so that after electroweak symmetry breaking the Higgs introduces a mixing between  $N_0$  and  $N$ , while  $E$  does not mix.

The three mass eigenstates  $\omega_{1,2}$  and  $\chi$  are defined as

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = U_L^\dagger \begin{pmatrix} N \\ N_0 \end{pmatrix}_L + U_R^\dagger \begin{pmatrix} N \\ N_0 \end{pmatrix}_R, \quad \chi = E_L + E_R, \quad (3.5)$$

with  $U_{L/R}$  the unitary matrices that diagonalize the mass matrix obtained from Eq. (3.3) after electroweak symmetry breaking.

In terms of the mass eigenstates the gauge lagrangian can be written as

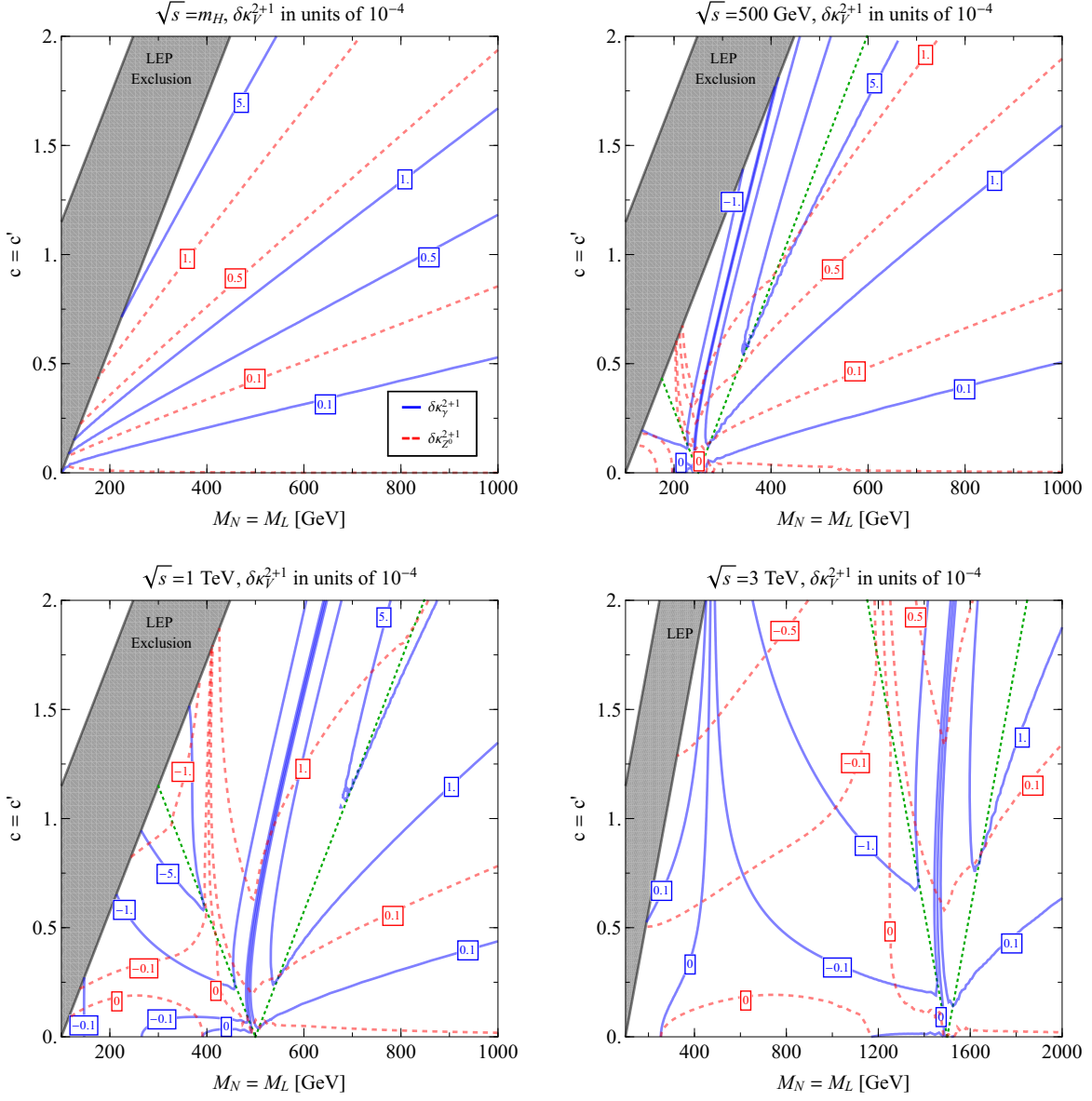
$$\begin{aligned} \mathcal{L}_{\text{gauge}}^{2+1} = & e q_\chi \bar{\chi} \gamma^\mu \chi A_\mu + e q_\omega \bar{\omega} \gamma^\mu \omega A_\mu - \frac{1}{2} \left( (2Y - 1) g' s_W + g c_W \right) \bar{\chi} \gamma^\mu \chi Z_\mu \\ & + \bar{\omega} \left[ U_L^\dagger \begin{pmatrix} -Y g' s_W & 0 \\ 0 & \frac{1}{2} (g c_W - (2Y - 1) g' s_W) \end{pmatrix} U_L P_L + (L \rightarrow R) \right] \gamma^\mu \omega Z_\mu, \\ & + \frac{g}{\sqrt{2}} \bar{\omega} \gamma^\mu [U_L^\dagger P_L + U_R^\dagger P_R] (0 \quad W_\mu^+)^T \chi, \end{aligned} \quad (3.6)$$

where  $g$  and  $g'$  are the usual SM gauge couplings,  $s_W = \sin \theta_W$  and  $c_W = \cos \theta_W$ .

Having established our model, we proceed to compute the 1-loop contributions of the new vectorlike fermions to the TGCs. Using the general result for the 1-loop contribution, given in Appendix A, we computed the  $\Delta\kappa_V^{2+1}$  and  $\Delta Q_V^{2+1}$  form factors for this model. Note that the  $W^+W^-Z^0$  vertex gets an additional correction with respect to the  $W^+W^-\gamma$  one, due to the mixing between the doublet and the singlet.

In Fig. 3 we show the contour lines for  $\delta\kappa_V^{2+1}$  in the  $(M, c)$  plane, where  $M = M_L = M_N$  and  $c' = c$ , for the same four center-of-mass energies as before. Assuming  $c$  real, the mass spectrum is  $m_\chi = M$ ,  $m_{\omega_1, \omega_2} = |M \pm 2cv|$ , where  $v = 175$  GeV is the SM Higgs vacuum

<sup>2</sup>Notice that although we use a notation suggesting heavier copies of a lepton doublet and right handed neutrinos, we leave the hypercharge  $Y$  of  $N$  unspecified. The case  $Y = 0$  corresponds, for example, to the situation studied in [13].



**Figure 3.** Iso-contour lines of the deviations  $\delta\kappa_V^{2+1}$  from the SM couplings in the plane ( $M_N = M_L, c = c'$ ) for the vectorlike colorless fermion doublet-singlet model at four different center-of-mass energies:  $\sqrt{s} = m_H, 500 \text{ GeV}, 1 \text{ TeV}$  and  $3 \text{ TeV}$ . We have chosen  $Y = 1$ , so  $\omega_1$  and  $\omega_2$  are charged whereas  $\chi$  is neutral. The full blue (dashed red) lines correspond to  $V = \gamma$  ( $Z^0$ ). The dotted green lines correspond to the physical masses  $m_{\omega_1}$  and  $m_{\omega_2}$ , for  $M_N = M_L = \sqrt{s}/2$ .

expectation value. As an illustration we have chosen the case  $Y = 1$ , so  $\omega_1$  and  $\omega_2$  are particles with charge 1 that participate in both  $\delta\kappa_\gamma^{2+1}$  and  $\delta\kappa_{Z^0}^{2+1}$ , whereas  $\chi$  is a neutral fermion and so it only contributes to the latter. For a fixed coupling  $c = c'$ ,  $\delta\kappa_\gamma^{2+1}$  has the following behavior as a function of  $M = M_L = M_N$ . It starts positive when, for a given center-of-mass energy,

all vectorlike fermion masses are irrelevant for the loop function. Then, decreases as the lowest fermion mass starts to play a role, until it reaches a minimum at  $m_{\omega_1} = |\sqrt{s}/2 - 2cv|$ ; next, increases when the next massive vectorlike fermion starts to contribute, passes again through zero before reaching a maximum at  $m_{\omega_2} = \sqrt{s}/2 + 2cv$ . As  $M_N$  continues to increase  $\delta\kappa_\gamma^{2+1} \rightarrow 0$  as we approach the decoupling limit. The behavior of  $\delta\kappa_{Z^0}^{2+1}$  is somewhat similar but a bit more involved at lower values of  $M_N$  due to the mixing between  $\omega_{1,2}$ . Also as  $M_N$  increases, the contribution of the neutral vectorlike fermion,  $\chi$ , appears giving rise to the maximum value for  $\delta\kappa_{Z^0}^{2+1}$  at  $M_N = m_\chi$ . Here again the typical values of  $|\delta\kappa_V^{2+1}|$  are smaller than a few  $10^{-4}$ . The green dotted lines that can be seen on the  $\sqrt{s} = 500$  GeV and 1 TeV panels correspond to the values of  $m_{\omega_1}$  and  $m_{\omega_2}$  computed with  $M = \sqrt{s}/2$ . At the other center-of-mass energies these masses lie outside of the plot range.

**Triplet-doublet model.** We will now add to the SM particle content a Dirac  $SU(2)_L$  doublet  $L = L_L + L_R$ , and a Dirac triplet  $T = T_L + T_R$ , with hypercharges  $Y$  and  $Y - \frac{1}{2}$ , respectively. The total lagrangian is given by

$$\mathcal{L}_{3+2} = i\bar{L}\not{D}L + i\bar{T}\not{D}T - M_L\bar{L}_L L_R - M_T\bar{T}_L T_R - c\bar{L}_L T_R H - c'\bar{L}_R T_L H + h.c., \quad (3.7)$$

where the doublet and triplet fermions are written as

$$L = \begin{pmatrix} N_0 \\ E \end{pmatrix}, \quad T = \begin{pmatrix} \frac{T_a}{\sqrt{2}} & T_b \\ T_c & -\frac{T_a}{\sqrt{2}} \end{pmatrix}. \quad (3.8)$$

With the hypercharge assignment we are considering, the electric charge of the various components read

$$\begin{aligned} T_c &\rightarrow q_\chi \equiv Y - \frac{3}{2}, \\ T_a, E &\rightarrow q_\xi \equiv Y - \frac{1}{2}, \\ T_b, N_0 &\rightarrow q_\omega \equiv Y + \frac{1}{2}, \end{aligned} \quad (3.9)$$

in such a way that, after electroweak symmetry breaking, there is a mixing between  $T_a$  and  $E$ , as well as between  $T_b$  and  $N_0$ . Defining the mass eigenstates as

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = U_L^\dagger \begin{pmatrix} N_0 \\ T_b \end{pmatrix}_L + U_R^\dagger \begin{pmatrix} N_0 \\ T_b \end{pmatrix}_R, \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = V_L^\dagger \begin{pmatrix} E \\ T_a \end{pmatrix}_L + V_R^\dagger \begin{pmatrix} E \\ T_a \end{pmatrix}_R, \quad (3.10)$$

$$\chi = T_{cL} + T_{cR},$$

the gauge lagrangian can be written as

$$\begin{aligned}
\mathcal{L}_{3+2} = & e q_\chi \bar{\chi} \gamma^\mu \chi A_\mu + e q_\omega \bar{\omega} \gamma^\mu \omega A_\mu + e q_\xi \bar{\xi} \gamma^\mu \xi A_\mu - (q_\xi g' s_W + g c_W) \bar{\chi} \gamma^\mu \chi Z_\mu \\
& + \bar{\omega} \left[ U_L^\dagger \begin{pmatrix} \frac{g}{2} c_W - Y g' s_W & 0 \\ 0 & g c_W - q_\xi g' s_W \end{pmatrix} U_L P_L + (L \rightarrow R) \right] \gamma^\mu \omega Z_\mu \\
& + \bar{\xi} \left[ V_L^\dagger \begin{pmatrix} -\frac{g}{2} c_W - Y g' s_W & 0 \\ 0 & -q_\xi g' s_W \end{pmatrix} V_L P_L + (L \rightarrow R) \right] \gamma^\mu \xi Z_\mu \\
& + g (\bar{\omega} \bar{\xi} \bar{\chi}) \gamma^\mu \left[ \begin{pmatrix} 0_{2 \times 2} & W_\mu^+ U_L^\dagger V_L' & 0_{2 \times 1} \\ W_\mu^- V_L'^\dagger U_L & 0_{2 \times 2} & V_L^\dagger \widetilde{W}_\mu^{+T} \\ 0_{1 \times 2} & \widetilde{W}_\mu^{-T} V_L & 0 \end{pmatrix} P_L + (L \rightarrow R) \right] \begin{pmatrix} \omega \\ \xi \\ \chi \end{pmatrix}, \quad (3.11)
\end{aligned}$$

where  $\widetilde{W}_\mu^\pm = (0 \ W_\mu^\pm)$  and

$$V_L' = \frac{1}{\sqrt{2}} \begin{pmatrix} V_{L11} & V_{L12} \\ \sqrt{2} V_{L21} & \sqrt{2} V_{L22} \end{pmatrix}.$$

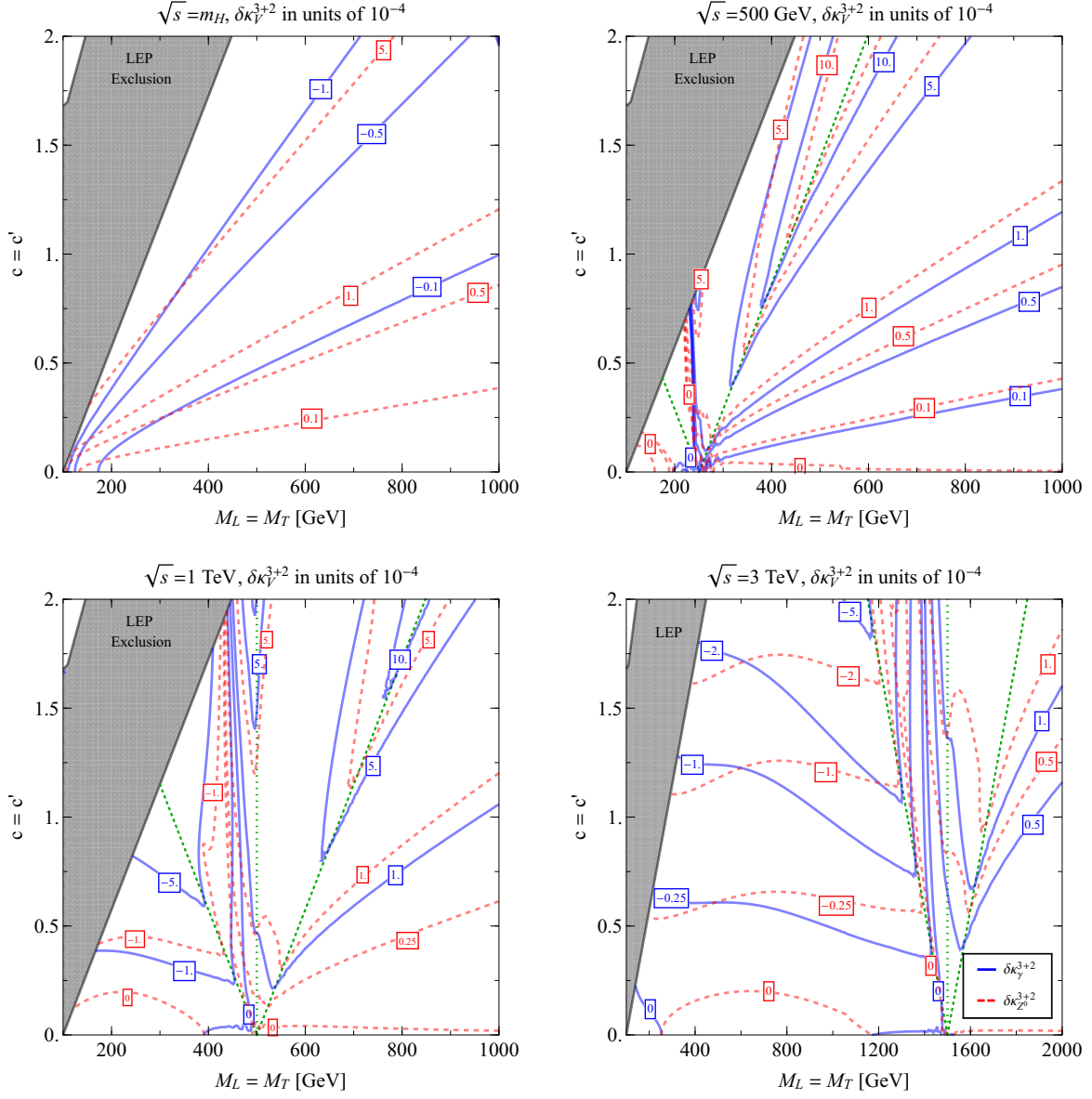
In Fig. 4 we show the iso-contour lines for the  $\delta\kappa_V^{3+2}$  combinations for this model in the plane  $M_L = M_T$  versus  $c = c'$  for the same four different center-of-mass energies as before. In this case the physical mass spectrum is:  $m_\chi = \sqrt{s}/2$ ,  $m_{\omega_1} = |\sqrt{s}/2 - 2cv|$ ,  $m_{\omega_2} = \sqrt{s}/2 + 2cv$ ,  $m_{\xi_1} = |\sqrt{s}/2 - \sqrt{2}cv|$  and  $m_{\xi_2} = \sqrt{s}/2 + \sqrt{2}cv$ . The green dotted lines that can be seen on the  $\sqrt{s} = 500$  GeV and 1 TeV panels correspond to the values of the charged particle masses  $m_{\omega_1}$ ,  $m_{\omega_2}$  and  $m_\chi$ . At the other center-of-mass energies these masses lie outside of the plot range.

Here we show the case  $Y = 1/2$ , so  $\chi$ ,  $\omega_1$  and  $\omega_2$  are charged particles that participate of both  $\delta\kappa_\gamma^{3+2}$  and  $\delta\kappa_{Z^0}^{3+2}$ , whereas  $\xi_1$  and  $\xi_2$  are neutral fermions and only contribute to the latter. Here the typical values of  $|\delta\kappa_V^{3+2}|$  can get about an order of magnitude larger than in the previous models, but always smaller than a few  $10^{-3}$ .

For a fixed coupling  $c$ ,  $\delta\kappa_\gamma^{3+2}$  as a function of  $M_L$  has the same general behavior as for the doublet-singlet model. It goes through a minimum at  $m_{\omega_1}$ , and through a maximum at  $m_\chi$  and  $m_{\omega_2}$ . This can be best seen on the panel for  $\sqrt{s} = 1$  TeV. The behavior of  $\delta\kappa_{Z^0}^{3+2}$  is somewhat similar but even more involved than the previous mixed case because now we have five particles coupling to the  $Z^0$  so in addition to the charged particle peaks, we also have peaks for the neutral particles. We note that in this case  $|\delta\kappa_{Z^0}^{3+2}| \sim |\delta\kappa_\gamma^{3+2}|$  and sometimes even a bit larger.

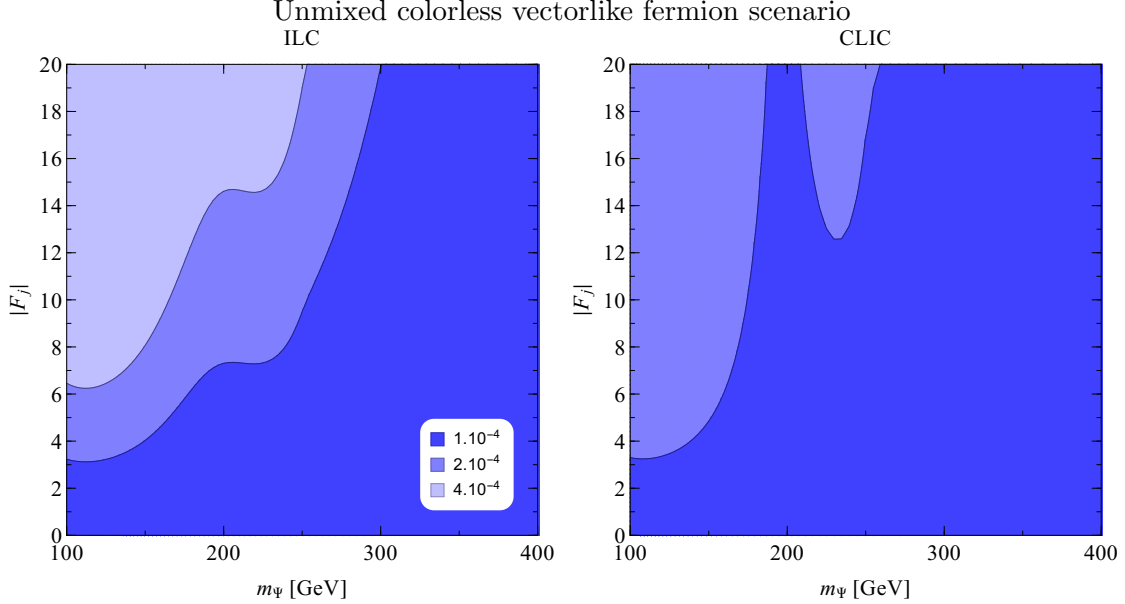
#### 4 TGC Constraints on Vectorlike Colorless Fermion Models

We move now to estimate the possible future constraints that can be imposed on vectorlike colorless fermion models by TGC measurements at future  $e^+e^-$  accelerator facilities such as



**Figure 4.** Iso-contour lines of the deviations  $\delta\kappa_V^{3+2}$  from the SM couplings in the plane ( $M_L = M_T, c = c'$ ) for the vectorlike colorless fermion triplet-doublet model at four different center-of-mass energies:  $\sqrt{s} = m_H, 500 \text{ GeV}, 1 \text{ TeV}$  and  $3 \text{ TeV}$ . We have chosen  $Y = 1/2$ , so there are three charged states and two neutral ones. The full blue (dashed red) lines correspond to  $V = \gamma (Z^0)$  and the dotted green lines correspond to the physical masses  $m_{\omega_1}, m_{\omega_2}$ , and  $m_\chi$ .

the proposed Future Circular Collider (FCC-ee) [27], International Linear Collider (ILC) [28] and the Compact Linear Collider (CLIC) [29]. For the FCC-ee experiment we considered the following center-of-mass energies:  $\sqrt{s} = m_Z, m_H, 2m_Z$  and  $2m_t$  [27], for the ILC:  $\sqrt{s} = 500, 800$  and  $1000 \text{ GeV}$  [28] and for the CLIC (in the so-called scenario A):  $\sqrt{s} = 500, 1400$



**Figure 5.** Possible TGC reach to probe the parameters of the unmixed vectorlike colorless fermion models by combining different center-of-mass energies at the ILC ( $\sqrt{s} = 500, 800, 1000$  GeV) and the CLIC ( $\sqrt{s} = 500, 1400, 3000$  GeV) facilities. We assume the same three different sensitivities for  $\delta\kappa_\gamma$  and  $\delta\kappa_{Z^0}$  at all center-of-mass energies considered:  $4 \times 10^{-4}$ ,  $2 \times 10^{-4}$  and  $1 \times 10^{-4}$ . The regions of accessibility were computed at 95.45% CL. See text for more details.

and 3000 GeV [29].

We do this for each of the models addressed in this paper by minimizing a combined  $\chi^2(\delta\kappa_Z, \delta\kappa_\gamma; \sqrt{s_i})$  assuming the following three different benchmark sensitivities for both TGCs:  $4 \times 10^{-4}$ ,  $2 \times 10^{-4}$  and  $1 \times 10^{-4}$  [39, 40]. We assume the same benchmarks for all facilities at all center-of-mass energies.

In Fig. 5 we show the regions on the plane  $(m_\Psi, |F_j|)$  of the unmixed vectorlike model that can be probed at  $2\sigma$  CL by combining the various center-of-mass energies at these accelerators. Because of the relatively low center-of-mass energies proposed for the FCC-ee, it can only probe a very limited range of  $m_\Psi \lesssim 200$  GeV for  $|F_j| \gtrsim (1 - 4)$  at  $2\sigma$  CL if the sensitivity is at least  $1 \times 10^{-4}$ . This is why we do not show this case on Fig. 5. The ILC will be able to test  $m_\Psi \lesssim 250$  GeV ( $m_\Psi \lesssim 300$  GeV) for  $|F_j| \gtrsim 16$  if a sensitivity of  $2 \times 10^{-4}$  ( $1 \times 10^{-4}$ ) can be achieved. At the CLIC the reach is somewhat reduced, as, for instance, no region is accessible at  $2\sigma$  CL even for a sensitivity of  $2 \times 10^{-4}$  for  $|F_j| < 20$ . Note that CLIC is less sensitive to the unmixed colorless vectorlike scenario than ILC due to its higher center-of-mass energies as explained by the following reasoning. As can be seen in Fig. 2, the contribution to TGCs is higher when  $\sqrt{s}$  is close to the vectorlike fermions mass threshold, but the heavier are the fermions, the smaller is the TGC deviation in general. Deviations at the  $\mathcal{O}(10^{-4})$  level are typically caused by particles below the TeV scale, and thus having a lower center-of-mass

energy leads to better sensitivity.

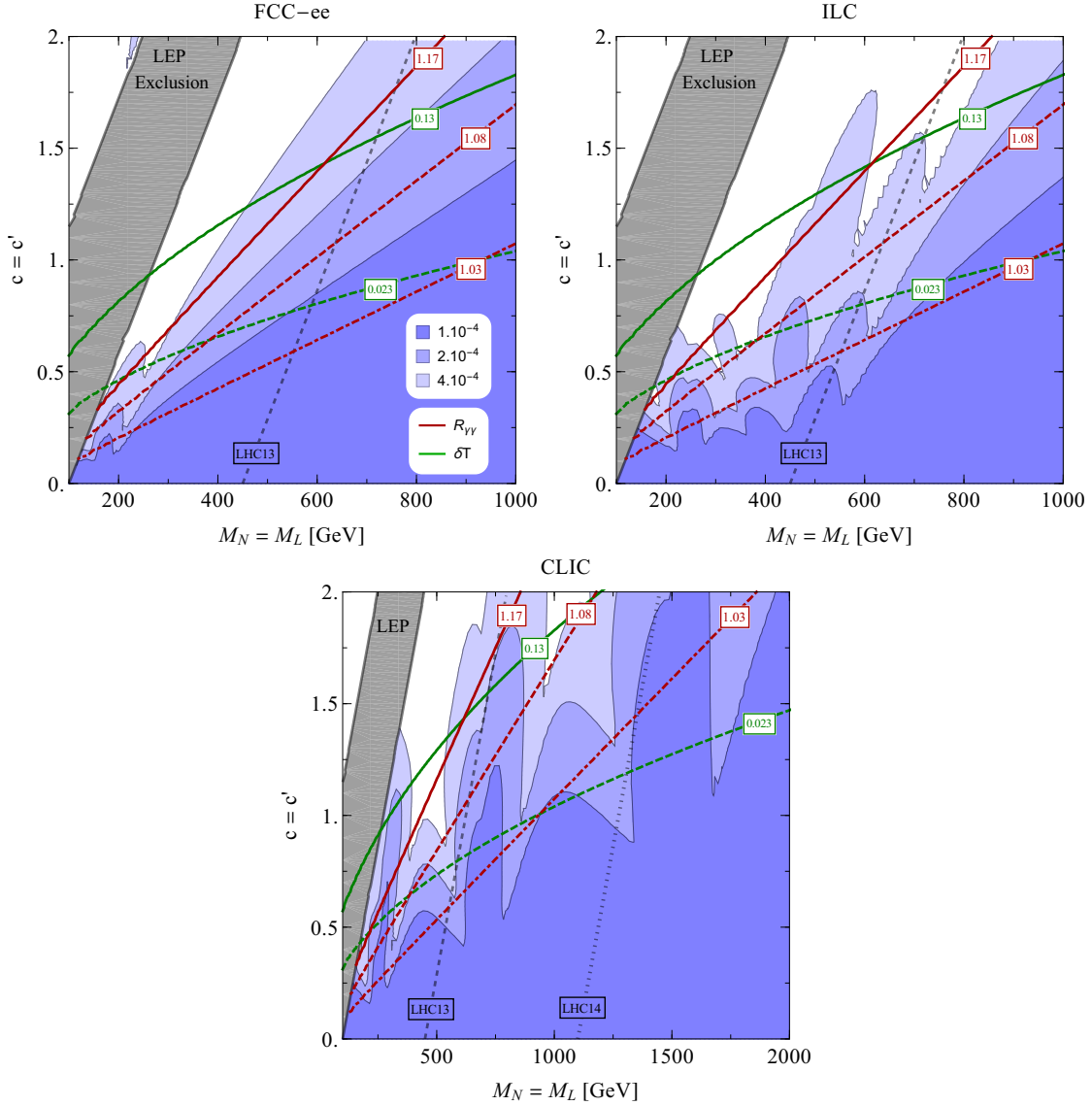
In Fig. 6 we show the regions on the plane ( $M_N = M_L$ ,  $c = c'$ ) of the doublet-singlet model with  $Y = 1$  than can be explored  $2\sigma$  CL by the FCC-ee, ILC and CLIC by combining the same center-of-mass energies as before. For comparison we also show the current limits one can obtain from  $H \rightarrow \gamma\gamma$  ( $R_{\gamma\gamma}$ , full red line; see e.g. Ref. [41]) and electroweak precision measurements ( $\delta T$ , full dark green line), as well as the effect of a future possible improvement on the uncertainty on  $R_{\gamma\gamma}$  to 8% (dashed red line) or 3% (dotted-dashed red line) and on the uncertainty on  $\delta T$  (dashed dark green line). These future prospects on the uncertainties were taken from [28, 42]; for comparison we show the same  $\delta T$  and  $R_{\gamma\gamma}$  sensitivities for all proposed facilities. The region in gray was excluded by LEP searches for neutral and charged leptons [24].

At present  $R_{\gamma\gamma}$  excludes more of the parameter space of the doublet-singlet model than  $\delta T$  if  $M_N \lesssim 600$  GeV, but for larger values of  $M_N$ ,  $\delta T$  is more restrictive. We see that at the FCC-ee one can have the sensitivity to probe and exclude a larger region of the parameter space, that can only be comparable to a future sensitivity on  $R_{\gamma\gamma}$  of 8% or better, if one can reach a sensitivity of  $\sim 1.5 \times 10^{-4}$  on the TGCs. Here since the center-of-mass energies that we have combined are comparatively low, the peak structure only appears around  $M_N \sim 180$  GeV, the rest of the exclusion region being quite smooth. At the ILC, because the center-of-mass energies are higher, the exclusion region is more complicated due to the maxima and minima that appear for the different masses of the vectorlike fermions that run in the loop functions at different  $\sqrt{s}$ . In general, the ILC can exclude the same regions probed by the FCC-ee but, for the most part of the parameter space, requiring a less challenging sensitivity to the TGCs.

The CLIC, involving even higher center-of-mass energies, in spite of the fact that, because of the peak structure, loses some sensitivity for  $M_N \sim 700$  GeV, can test  $800 \lesssim M_N/\text{GeV} \lesssim 1400$  and  $1600 \lesssim M_N/\text{GeV} \lesssim 1900$  for a TGC sensitivity of  $1 \times 10^{-4}$ , a region that could only be otherwise inspected by a  $R_{\gamma\gamma}$  or a  $\delta T$  measurement with 2-3% uncertainty.

Finally, in Fig. 7 we show the regions on the plane ( $M_L = M_T$ ,  $c = c'$ ) of the triplet-doublet model with  $Y = 1/2$  than can be explored at  $2\sigma$  CL by the FCC-ee, ILC and CLIC again combining the same center-of-mass energies as before. In this case, the FCC-ee can explore a region than can only be attainable by measuring  $R_{\gamma\gamma}$  with an uncertainty of at least 3% if the TGC sensitivity is  $2 \times 10^{-4}$ , while the ILC is a bit better except for  $M_L \lesssim 250$  GeV. As before CLIC is, in general, less sensitive for  $M_L \lesssim 700$  GeV because of the peak structure but becomes more sensitive for higher masses, probing the model down to regions where even a very aggressive measurement of  $R_{\gamma\gamma}$  would not reach.

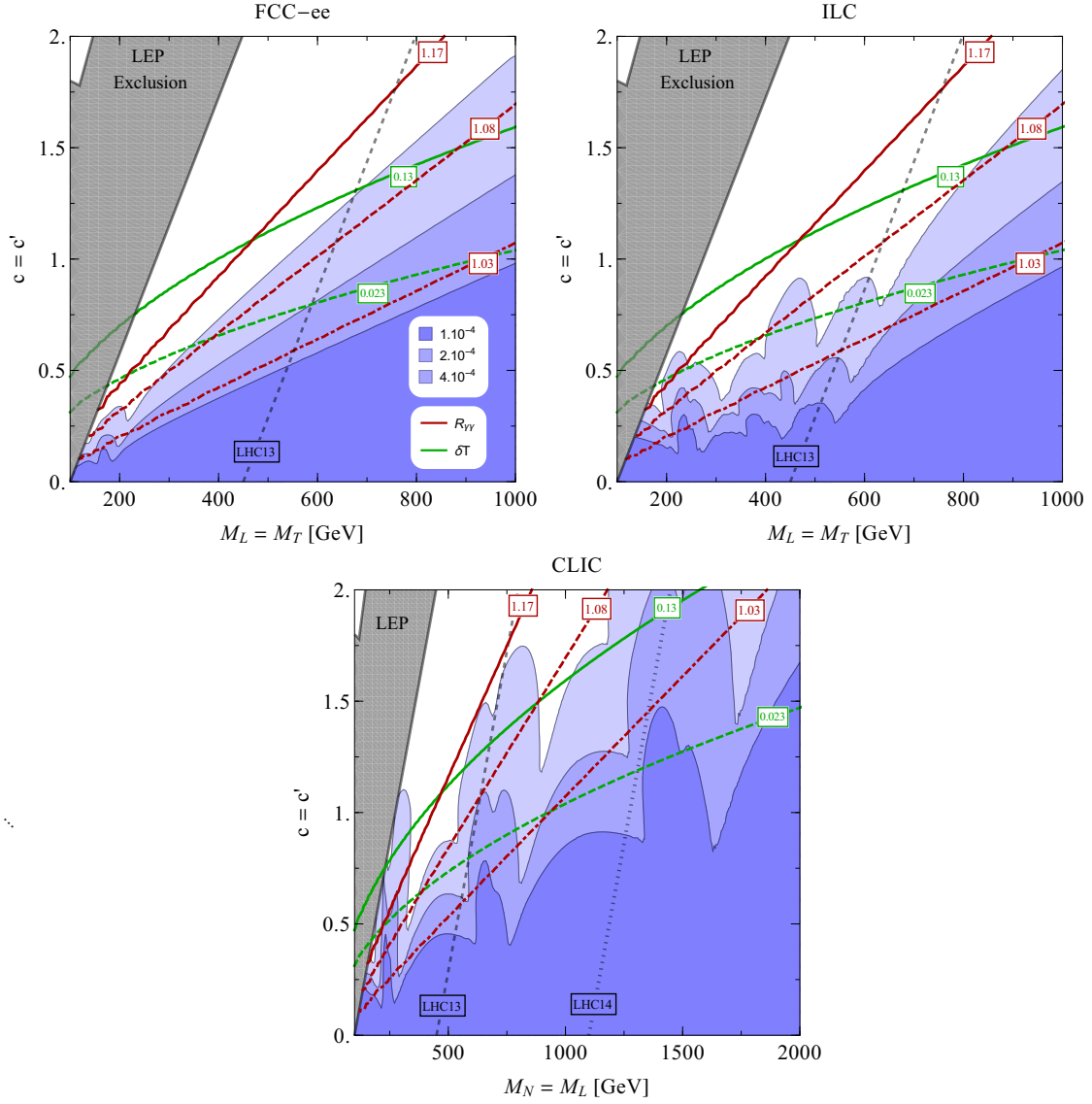
Let us conclude with some remarks about the limits from direct searches at the LHC. As shown for instance in [13, 43], the collider signatures of the doublet-singlet model are very similar to those of electroweakinos in minimal SUSY models. Moreover, we expect the limits



**Figure 6.** Possible TGC reach to probe the parameters of the doublet-singlet vectorlike colorless fermion model with  $Y = 1$ , by combining different center-of-mass energies at the FCC-ee, the ILC and the CLIC facilities at  $2\sigma$  CL. We also show the current limits from  $H \rightarrow \gamma\gamma$  ( $R_{\gamma\gamma}$ , full red line) and electroweak precision measurements ( $\delta T$ , full dark green line), as well as the possible future sensitivities of  $R_{\gamma\gamma}$  assuming an uncertainty of 8% (dashed red line) or 3% (dotted-dashed red line) and of  $\delta T$  (dashed dark green line). The gray region has been excluded by LEP [24] while the black dashed (dotted) lines correspond to the LHC current limit (future sensitivity).

for the other representations not to be too different. Current lower bounds can be found in [44], and are of order 150 GeV for the lightest neutral state and of order 450 GeV for the heavier states. Future sensitivities have been estimated in [45]; with a luminosity of 3000





**Figure 7.** Same as Fig. 6 but for the triplet-doublet vectorlike colorless fermion model with  $Y = 1/2$ .

$\text{fb}^{-1}$  (at  $\sqrt{s} = 14$  TeV), the lower bound on the lightest neutral mass becomes 400 GeV, while the lower bound on the heavier states becomes 1.1 TeV. We included the current limit (dashed black line) and future sensitivity (dotted black line) in figures 6 and 7. As can be seen, even considering the future LHC reach there are regions not probed by the LHC that will be probed by TGC's searches.

## 5 Conclusions

We have studied vectorlike colorless fermions contributions to the triple gauge couplings  $W^+W^-\gamma$  and  $W^+W^-Z^0$  in the context of two classes of models. First we consider the unmixed case, where an arbitrary set of fermions in a given representation of  $SU(2)_L$  cannot couple to the SM Higgs boson. Second we consider the mixed case, where the vectorlike fermion fields transform as different representations of  $SU(2)_L$  allowing for invariant Yukawa couplings with the Higgs boson. In the latter case we study two concrete situations: the doublet-singlet model, where three new vectorlike physical particles are introduced, and the triplet-doublet model, where five new vectorlike physical particles appear.

We established that the contributions of the above vectorlike fermion models to the combination of the form factors,  $\delta\kappa_V$ ,  $V = \gamma, Z^0$ , used by the experimental collaborations, have several minima and maxima as a function of the mass parameters of the model. Since to go from a negative minimum to a positive maximum one has to cross zero, this also implies that there are values of the mass parameter for which  $\delta\kappa_V \rightarrow 0$ . These maxima and minima will depend on the center-of-mass energy considered, and how close one is to a physical particle which contributes to the TGC loop function being on the mass-shell.

In the case of the unmixed vectorlike colorless fermion model, we have assumed that all fermions, independent of how many multiplets of a given representation, are degenerate in mass ( $m_\Psi$ ). Since  $|\delta\kappa_\gamma|$  starts large when  $m_\Psi \ll \sqrt{s}/2$ , and we expect a maximum at  $m_\Psi \sim \sqrt{s}/2$ , there are, in general, two values of  $m_\Psi$ , for a given  $\sqrt{s}$ , where  $\delta\kappa_V \rightarrow 0$ .

For the doublet-singlet and triplet-doublet model the minima and maxima for  $\delta\kappa_\gamma$  ( $\delta\kappa_{Z^0}$ ) as a function of  $M_L$ , the mass parameter, correspond to the values of the charged (all) physical particles of the model, which clearly depend on  $\sqrt{s}$  and the hypercharge  $Y$ , which defines the charges of the particles.

We made an assessment of the sensitivity of the proposed future precision test accelerators FCC-ee, ILC and CLIC to the parameters of these models assuming they will be able to constrain  $\delta\kappa_V \sim \mathcal{O}(10^{-4})$  at different  $\sqrt{s}$ . Using the same benchmark sensitivities for all accelerators allow us to clearly see the effect of the different center-of-mass energy combinations. For the FCC-ee experiment we considered the following center-of-mass energies:  $\sqrt{s} = m_Z, m_H, 2m_Z$  and  $2m_t$ . For the ILC:  $\sqrt{s} = 500, 800$  and  $1000$  GeV and for the CLIC (in the so-called scenario A):  $\sqrt{s} = 500, 1400$  and  $3000$  GeV.

Only for the unmixed vectorlike colorless fermion case the FCC-ee is definitely not as capable to probe the model as the ILC or the CLIC. However, for both mixed vectorlike models we have examined, the ILC is generally better than the FCC-ee, but not as powerful as CLIC at larger values of the mass parameters  $M_N$  or  $M_L$ . This is because the  $\sqrt{s}$  used by FCC-ee are all quite low, making the exclusion region basically insensitive to the maxima and minima caused by the physical particle masses. For the ILC the gaps between the center-of-mass

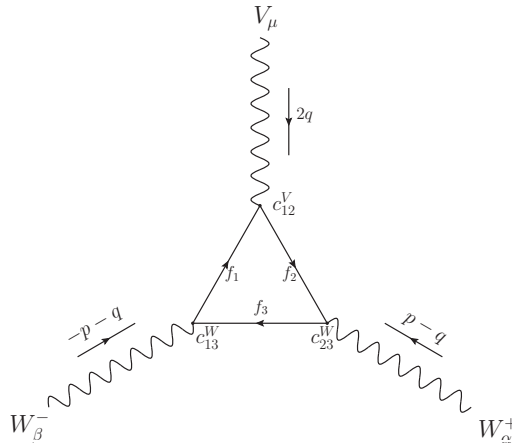
energies and their high values exhibit some synergy that helps to improve the sensitivity in a large region of the parameter. This also happens for the CLIC, but since the center-of-mass energies are more spread out there is an overall decrease in sensitivity to the model parameters for  $M_N, M_L \lesssim 700$  GeV, with respect to the ILC. However, for higher masses (due to the 3000 GeV center-of-mass energy contribution) we have again an increase of sensitivity because heavier vectorlike fermion physical masses come into play.

It is also important to note that if one is able to achieve  $\mathcal{O}(10^{-4})$  sensitivity on TGC's with the FCC-ee ILC or CLIC, one will be able to use them to do precision measurements that surpass the sensitivities of the oblique parameters or  $H \rightarrow \gamma\gamma$  even assuming a considerable improvement of the latter measurements in these new machines.

## Acknowledgments

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## A Vectorlike Fermion Contribution to Triple Gauge Couplings



**Figure 8.** Vectorlike fermions contribution to TGC.

The 1-loop correction to the TGCs coming from a set of  $N_F$  vectorlike fermions can be obtained from the diagram in Fig. 8. Here, we will keep as general as possible, by supposing

that three different fermions run into the loop,  $f_i$ ,  $i = \{1, 2, 3\}$ , with masses  $m_i$  and generic couplings between them and the gauge bosons,  $c_{ij}^B$ , where  $i, j = \{1, 2, 3\}$  and  $B = \{\gamma, W, Z\}$ . Proceeding in a standard way, we find the  $\Delta\kappa_V^{NP}$  and  $\Delta Q_V^{NP}$  form factors,

$$\begin{aligned} \Delta\kappa_V^{NP} = & -N_F \frac{c_{12}^V c_{23}^W c_{31}^W}{8\pi^2 g_V} \int_0^1 dx \int_0^1 dy \frac{x}{\tilde{\Lambda}} \left\{ \frac{4q^2}{M_W^2} x^2 (3x-2)y(1-y) + x^2(x-1) \right. \\ & + (R_1 - R_2)xy(x-1) + (R_3 - R_1)x(x-1) + \sqrt{R_1 R_2}x \\ & \left. + \sqrt{R_2 R_3}(1-x-2xy) + \sqrt{R_1 R_3}(1-3x+2xy) \right\}, \end{aligned} \quad (\text{A.1a})$$

$$\Delta Q_V^{NP} = -N_F \frac{c_{12}^V c_{23}^W c_{31}^W}{\pi^2 g_V} \int_0^1 dx \int_0^1 dy \frac{x^3(1-x)y(1-y)}{\tilde{\Lambda}}, \quad (\text{A.1b})$$

where

$$\tilde{\Lambda} = -\frac{4q^2}{M_W^2} x^2 y(1-y) + x^2 - x(1+R_3-R_1) - (R_1-R_2)xy + R_3, \quad (\text{A.2})$$

and  $R_i = \frac{m_i^2}{M_W^2}$ .

## B Dependence on the Hypercharge in the Unmixed Case

The proof that the one-loop contributions to the TGC are independent of the eigenvalues of the  $T^3$  operator is as follows; for simplicity in the notation, we consider here just one copy of the multiplet. Writing the multiplet in terms of its  $2j+1$  states,  $j$  the principal quantum number, as

$$\Psi = \{\psi_{j,m}\} = \begin{pmatrix} \psi_{j,j} \\ \psi_{j,j-1} \\ \vdots \\ \psi_{j,-j+1} \\ \psi_{j,-j} \end{pmatrix}, \quad (\text{B.1})$$

where  $m = j, j-1, \dots, 0$  (or  $\frac{1}{2}, -\frac{1}{2}, \dots, -j+1, -j$ ) is the *magnetic* quantum number, we first rotate to the physical gauge boson states,  $W^\pm, Z^0, \gamma$ . Introducing the ladder operators as usual,

$$T^\pm = T^1 \pm iT^2, \quad (\text{B.2})$$

together with the  $T^3$  operator, we write the covariant derivative acting on the multiplet as

$$\mathcal{L}_G = i\bar{\Psi}\gamma^\mu \left( \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{c_W}(c_W^2 T^3 - s_W^2 Y)Z_\mu - ie(T^3 + Y)A_\mu \right) \Psi, \quad (\text{B.3})$$

where  $c_W = \cos \theta_W$  and  $s_W = \sin \theta_W$ ,  $\theta_W$  is the weak angle. In terms of the function multiplet of  $\Psi$ , eq (B.1), we get

$$i\bar{\Psi}\gamma^\mu D_\mu\Psi = \sum_{m=-j}^j \left[ i\bar{\psi}_m\gamma^\mu \left( \partial_\mu - i\frac{g}{c_W}(c_W^2 m - s_W^2 Y)Z_\mu - ie(m+Y)A_\mu \right) \psi_m + \frac{g}{\sqrt{2}}\sqrt{(j+1-m)(j+m)}W_\mu^- \bar{\psi}_{m-1}\gamma^\mu\psi_m + \text{h.c.} \right], \quad (\text{B.4})$$

here we used the action of the ladder operators on the multiplet.

Now, we have to compute the 1-loop correction to the charged TGCs coming from the new fermions. We have to add all the possible diagrams,

$$\Gamma_{\mu\alpha\beta}^V = \frac{1}{2} \sum_{m=-j}^j \left( \text{Diagram 1} + \text{Diagram 2} \right)$$

to determine the form factors  $\Delta\kappa_V^\Psi$  and  $\Delta Q_V^\Psi$ . Each diagram can be written as a product of the couplings of the fermions with the gauge bosons times a loop integral,  $I_{\mu\alpha\beta}(m_m, m_m, m_{m\pm 1})$ . Therefore, the amplitude will be

$$\Gamma_{\mu\alpha\beta}^V = \frac{g^2}{4} \sum_{m=-j}^j g_V^m [(j+1-m)(j+m)I_{\mu\alpha\beta}(m_m, m_m, m_{m-1}) + (j-m)(j+m+1)I_{\mu\alpha\beta}(m_m, m_m, m_{m+1})], \quad (\text{B.5})$$

where

$$g_V^m = \begin{cases} e(m+Y) & \text{for } \gamma, \\ \frac{g}{c_W}(c_W^2 m - s_W^2 Y) & \text{for } Z^0. \end{cases} \quad (\text{B.6})$$

Since the mass of the components of the multiplet is the same, we have that the loop integral will depend only in the mass  $m_\Psi$ ,

$$I_{\mu\alpha\beta}(m_m, m_m, m_{m\pm 1}) = I_{\mu\alpha\beta}(m_\Psi),$$

then, the amplitude will take a simpler form,

$$\Gamma_{\mu\alpha\beta}^V = \frac{g^2}{2} I_{\mu\alpha\beta}(m_\Psi) \sum_{m=-j}^j g_V^m [j(j+1) - m^2]. \quad (\text{B.7})$$

Summing over the *magnetic* quantum number  $m$ ,

$$\sum_{m=-j}^j [j(j+1) - m^2] = \frac{2}{3}j(j+1)(2j+1), \quad (\text{B.8a})$$

$$\sum_{m=-j}^j m[j(j+1) - m^2] = 0, \quad (\text{B.8b})$$

we see here that the amplitude of the 1-loop correction will be proportional to the hypercharge,

$$\Gamma_{\mu\alpha\beta}^V = \frac{g^2 c_\Psi^V Y}{3} j(j+1)(2j+1) I_{\mu\alpha\beta}(m_\Psi), \quad (\text{B.9})$$

being

$$c_\Psi^V = \begin{cases} e & \text{for } \gamma, \\ -e t_W & \text{for } Z^0, \end{cases} \quad (\text{B.10})$$

with  $t_W = \tan \theta_W$ . Finally, the form factors will be computed in a standard manner. The expressions for  $\Delta\kappa_V^\Psi$  and  $\Delta Q_V^\Psi$  can be obtained from the general expressions in the Appendix A by taking all the masses as identical and

$$c_{23}^W = c_{13}^W = \frac{g}{\sqrt{2}} G_j, \quad (\text{B.11})$$

$$c_{12}^V = c_\Psi^V Y, \quad (\text{B.12})$$

where  $G_j$  is the square root of the multiplet factor,

$$G_j = \sqrt{\frac{2}{3}j(j+1)(2j+1)}. \quad (\text{B.13})$$

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