



WORKING PAPER SERIES

Disclosure of Information in Matching Markets with Non-Transferable Utility

Ennio Bilancini and Leonardo Boncinelli

Working Paper 94

March 2014

www.recent.unimore.it

Disclosure of Information in Matching Markets with Non-Transferable Utility

Ennio Bilancini^{*,a}, Leonardo Boncinelli^{**,b}

^a*Dipartimento di Economia “Marco Biagi”, Università degli Studi di Modena e Reggio Emilia, Viale Berengario 51, 43 ovest, 41121 Modena, Italia.*

^b*Dipartimento di Economia e Management, Università degli Studi di Pisa, Via Cosimo Ridolfi 10, 56124 Pisa, Italia.*

Abstract

We present a model of two-sided matching where utility is non-transferable and information about individuals' skills is private, utilities are strictly increasing in the partner's skill and satisfy increasing differences. Skills can be either revealed or kept hidden, but while agents on one side have verifiable skills, agents on the other side have skills that are unverifiable unless certified, and certification is costly. Agents who have revealed their skill enter a standard matching market, while others are matched randomly. We find that in equilibrium only agents with skills above a cutoff reveal, and then they match assortatively. We show that an equilibrium always exists, and we discuss multiplicity. Increasing differences play an important role to shape equilibria, and we remark that this is unusual in matching models with non-transferable utility. We close the paper with some comparative statics exercises where we show the existence of non-trivial externalities and welfare implications.

Key words: costly disclosure of information; matching markets; non-transferable utility; partial unraveling; positive assortative matching; increasing differences

JEL: C78, D82, L15

1. Introduction

Sorting patterns have been widely investigated in two-sided matching models where agents are ranked according to a trait (called skill or type) that is publicly visible. One important insight from the literature is that when utility is transferable (i.e., the surplus generated by a matched pair of agents can be freely distributed between them) the sorting of agents depends on comparative advantages. Indeed, if higher types gain relatively more than lower types to be matched with partners of a higher type, then their willingness to pay for such a match will be larger, and this determines positive assortative matching in equilibrium. By contrast, when utility is non-transferable (i.e., the surplus generated by a match is non-contractible) the sorting of agents depends on absolute advantages. This is so because agents cannot compete by offering larger shares of the total surplus, and therefore if utilities are increasing in the partner's type then in equilibrium high type agents match together; this is true even if the total surplus of a match does not exhibit the comparative advantage property.

*Tel.: +39 059 205 6843, fax: +39 059 205 6947.

**Tel.: +39 050 221 6219, fax: +39 050 221 0603.

Email addresses: ennio.bilancini@unimore.it (Ennio Bilancini), l.boncinelli@ec.unipi.it (Leonardo Boncinelli)

In this paper we show that, if we remove the assumption that types are publicly visible and we give agents the possibility to costly disclose their type, then comparative advantages become again important to understand the properties of the equilibrium matching even in a model where utility is non-transferable. More precisely, we assume that, prior to matching, agents face the following choice: either reveal one's own skill and participate in a standard two-sided matching market together with all agents who disclose their skill, or keep one's own skill hidden and go for a random match together with all agents who do not reveal their skill. We assume an asymmetry in skill verifiability between agents on the two sides. Agents on one side have skills that are verifiable, hence such skills become observable once revealed. Agents on the other side, instead, have skills that are unverifiable, and they must resort to costly certification in order to make their skills verifiable. It turns out that comparative advantages – that we model as the property of increasing differences for utilities – are crucial to assess whether certification costs are worth being paid; indeed, an agent must compare the option to be randomly matched within a pool of low type agents with the option to pay the cost of certification and end up matched with a higher type agent.

Our main modeling innovation with respect to the matching literature concerns the assumption that types are private information, and that agents on one side must rely on costly certification in order to credibly reveal such information. We believe that this assumption can help matching models to get closer to some real applications. The following sketched example illustrates. Consider the market for a given kind of job, with positions on one side and candidates on the other side. Positions are ranked in candidates' preferences according to some trait, like salary, job duties, working time. Such a trait can be costlessly revealed, and even reported in the job contract. Candidates, instead, are ranked in recruiters' preferences according to a skill level that is directly related to productivity in the kind of job that we are considering; however, candidates' skills are not verifiable, and hence cannot be simply revealed, since uncertified declarations would be non-credible. Therefore, candidates who want to certify their skills need to obtain – depending on the nature of job under consideration – educational or professional qualifications, or other kinds of certificates: for instance, the European Computer Driving Licence (ECDL) can be obtained at different levels and certifies knowledge in the field of information and communication technology, that can be relevant for a position of computer technician; Cambridge ESOL diplomas, like the First Certificate in English, or the Test Of English as a Foreign Language (TOEFL) prove one's adequacy in the English language, that can be useful for jobs that require to interact socially with English-speaking people; similarly, the Graduate Record Examinations (GRE) and the Graduate Management Admission Test (GMAT) measure skills related to verbal reasoning, quantitative reasoning and analytical writing, and are usually employed to discriminate the access to graduate positions; moreover, many other certificates and licences exist that guarantee one's ability as hairstylist, chef, musician, driver, etc. Needless to say, all these certifications are costly to be acquired.

In the paper, we start adapting the standard notion of stable matching – that requires two conditions to be satisfied, i.e., no blocking pair and individual rationality – to obtain a notion of equilibrium that is suited for our model. We provide a characterization of equilibria where matching is positive assortative between agents who reveal their skill, and cutoff types emerge in both populations, separating the agents who reveal information – that lie above the cutoff – from those who do not – that lie below the cutoff. Moreover, the cutoff type in the population whose skills are unverifiable unless certified must be indifferent between, on the one hand, certifying the skill and matching with the same-rank mate and, on the other hand, saving the cost of certification and relying on random matching in the set of mates who have kept their skill hidden.

Interestingly enough, multiple equilibria can emerge in our model. This is essentially due to a kind of “network effect” that is at play when considering the value of the outside option: since only higher types resort to certification, if the pool of uncertified agents gets larger, then the average type therein becomes higher and the value of a random match increases. More precisely, for a higher cutoff type certification

means to end up matched with a better mate, and hence it has a higher value; however, the outside option of a random match has a higher value as well, due to the network effect. Therefore, there may be multiple cutoff types that are indifferent to the choice of whether to certify or not, and this means that multiple equilibria exist. Due to equilibrium multiplicity, in the paper we proceed by distinguishing equilibria between stable and unstable – since that plays a role for comparative statics – and we then show that a stable equilibrium always exists provided that certification costs are positive but low enough.

In the final part of the paper we go through some comparative statics exercises and we comment on welfare implications. In particular, we consider the effects of a change in the cost of certification, and in the distribution of skills in the two populations. To have an idea of the kind of results that we obtain, consider the following case. Suppose that skills increase for agents who need certification. Because of increasing differences of their utilities, we obtain that the value of certification increases relatively to the value of a random match with agents having lower skills. We show that this leads to a reduction in the equilibrium cutoff level for stable equilibria, and hence the average value of a random match decreases in the new equilibrium. This nicely highlights the existence of a negative externality on the agents who still choose not to pay the certification cost and to go for a random match.

The paper is organized as follows. Section 2 places our contribution in the relevant literature. Section 3 presents the model, while Section 4 illustrates our main results: the characterizations of equilibria, the possibility of multiple equilibria with different cutoffs, the distinction between stable and unstable equilibria with the proof of existence for stable equilibria. Section 5 describes the comparative statics exercises and provides some comments on welfare implications. Section 6 briefly summarizes our contribution and outlines directions for future research. The Appendix reports a couple of examples which help to illustrate the variety of welfare outcomes that may arise.

2. Related literature

The present paper lies at the intersection of two streams of contributions: the literature on positive assortative matching and the literature on costly disclosure.

The first stream of literature studies under what conditions matched partners are sorted according to some ordered characteristics. In his seminal contribution [Becker \(1973\)](#) shows that, when utility is non-transferable, if payoffs are monotonic in the partner's skill then stable matchings are characterized by agents who are paired in a positive assortative way.¹ More recently, [Legros and Newman \(2010\)](#) show that monotonicity is not necessary for positive assortative matching. More precisely, they show that a weakening of Becker's condition, which they label co-ranking, is necessary and sufficient for having agents who match in positive assortative way. In our model too equilibrium configurations are characterized by positive assortative matching but, due to the presence of asymmetric information, this only holds for the pool of individuals who publicly disclose their skills – as the remaining individuals are matched randomly. In addition, for the individuals disclosing their skill we also have same-rank matchings, i.e., matches essentially take place between agents with the same rank in the distribution of skills over their own population. This is because in equilibrium agents in both populations adopt a cutoff entry rule to establish whether to disclose or not their skill. We also emphasize that, although in our model agents' preferences satisfy the conditions for a unique stable match when utility is non-transferable (see [Beckhout, 2000](#); [Clark, 2006](#)), there might be multiple equilibria with distinct cutoffs. This is due to the fact that the expected value of a random match is endogenous, depending on the skills of agents who decide not to disclose.

¹[Legros and Newman \(2007\)](#) provide sufficient conditions for positive assortative matching in the case of partial utility transferability.

The second stream of literature studies the strategic disclosure of quality information when it is both costly and credible to do so.² A central concept in this literature is “unraveling”, i.e., the process by which higher quality agents disclose their quality as a way to distinguish themselves from lower quality agents (Grossman, 1981; Milgrom, 1981). When disclosure is costless, then unraveling is total, i.e., all private information is made publicly available, but if disclosure is costly – which may be due to certification required to make credible a piece of information that otherwise would be unverifiable – then unraveling can be only partial (Grossman and Hart, 1980; Jovanovic, 1982). In particular, all and only the agents whose quality is above a certain threshold decide to disclose their quality.³ Note that, when disclosure is costly, it is not necessarily suboptimal to have partial disclosure instead of full disclosure.⁴ In our model we also obtain partial unraveling with positive certification costs, and our welfare analysis confirms that full disclosure can be far from social optimum. These findings suggest that the main features of partial unraveling due to costly disclosure of information also hold for matching markets.

To the best of our knowledge, there are only two other contributions that have so far provided results at the intersection of positive assortative matching and costly disclosure of information. The first paper is Bloch and Ryder (2000) that compares the case of a matchmaker charging a uniform fee with the case of a matchmaker charging a price proportional to joint value. A man and a woman with same rank can decide to go to the matchmaker and, if they pay their participation fee, they are matched together. If at least one of the two agents does not go to the matchmaker, no match occurs, and both agents end up in a decentralized market paying no cost. Under some regularity assumptions about the search technology and the skill distribution, it turns out that in the case of a uniform fee only agents in the upper tail pay the fee, i.e., there is a cutoff type in each population such that only those above it pay the fee, while in the case of a proportional price only agents in the lower tail pay, i.e., there is a cutoff type in each population such that only those below it pay the price. The second paper is Bergstrom and Bagnoli (1993) that applies an original OLG model to explain the empirical fact that the mean age at marriage of men exceeds that of women. Men types are private information while women types are observable. Each man can either match randomly with a woman in period one, or wait until period two (there is a fixed cost of waiting) when his type is revealed and then match assortatively. The only choice given to agents is whether to wait or not. In stationary equilibria a cutoff rule emerges such that the more desirable women marry successful older men while the less desirable women marry younger men with low prospects. Equilibrium uniqueness is attained if type distributions (in utility terms) are log-concave. We observe that the primary focus of both these papers is not about positive assortative matching and costly disclosure of information as a theoretical issue per se, but rather about the role of market intermediaries in one case and the empirically observed tendency of women to marry older men in the other case. Therefore, their results are derived under assumptions that are specific to the issues they address, while we provide a model that is clean from unnecessary hypotheses. In addition, in both models the positive assortativeness of matching is directly assumed, while we explicitly model the stability of matching and we obtain positive assortativeness as a result. Moreover, in both models utility is assumed to depend only on one’s partner type (implying constant differences in utilities) while we perform our analysis under the hypothesis of increasing differences of utilities for agents who need certification. Finally, we also explore how equilibrium cutoffs respond to exogenous changes in the cost of certification and the distribution of skills.

Furthermore, there are several recent papers which are related to ours but have a different focus. We list

²See Dranove and Jin (2010) for a complete survey on the theory and practice of quality disclosure.

³Total unraveling may also fail for other reasons, as shown for instance by Board (2009).

⁴This is true even if the cost is not associated with disclosure but with information acquisition in the first place (Mathews and Postlewaite, 1985; Shavell, 1994).

a few of them for each strand, with no claim of being exhaustive. Some papers are concerned with signaling in matching markets where information is asymmetric. [Hoppe et al. \(2009\)](#) study signaling of attributes in two-sided matching where populations are finite and agents on both sides can observe only their own skills. The focus is on signaling equilibria and on their limit properties as populations tend to infinity, while the assortativeness of matching is assumed. [Hopkins \(2011\)](#) studies the signaling of attributes in two-sided matching where, as in our model, populations are infinite and only agents on one side have private information on their own attributes. Positive assortativeness is derived in equilibrium under both transferable and non-transferable utility. [Booth and Coles \(2010\)](#) focus instead on comparing the economic implications of marriage markets based on positive assortative matching on education (which determines earnings) with those of markets based on random matching. The second case is interpreted as “romantic” marriage and is shown to potentially increase efficiency by inducing the most skilled women to invest in education and participate in the labor market. Other papers consider costly search of potential mates. For instance, [Atakan \(2006\)](#) shows that positive assortative matching emerges under additive search costs and complementarities in joint production. With an explicit focus on exchange, [Satterthwaite and Shneyerov \(2007\)](#) study decentralized trade via matching of buyers and sellers under incomplete information and costly search, showing that the economy tends to the Walrasian equilibrium as the search cost tends to zero. [Chade \(2006\)](#) explores matching with both search and information frictions, showing that there exists an equilibrium exhibiting a stochastic positive assorting of types and that being accepted reduces an agent’s estimate of a potential partner’s type. Finally, a series of papers considers matching under incomplete information about others’ preferences. [Chakraborty et al. \(2010\)](#) study two-sided matching with interdependent valuations and noisy signals received by one side, showing that the existence of a stable matching hinges on the diversity of students’ preferences and the transparency of the mechanism. [Chade et al. \(2007\)](#) study the behavior of students in the application process to colleges when they are uncertain about their own qualities and applications are costly, showing that a unique equilibrium with assortative matching exists provided that application costs are small and that the lower-ranked college has sufficiently high capacity. Finally, [Li and Rosen \(1998\)](#) and [Li and Suen \(2000\)](#) consider a multi-period setting and study issues of early contracting and strategic waiting, while [Ostrovsky and Schwarz \(2010\)](#) endogenize disclosure costs and consider third party certification that is strategic.

3. Model

3.1. Preliminaries

We consider two populations, with agents in each population differing by type (or skill). In one population, individual type is denoted with x and varies in $X = [\underline{x}, \bar{x}]$ according to cumulative distribution F . In the other population, individual type is denoted with y and varies in $Y = [\underline{y}, \bar{y}]$ according to cumulative distribution G . Both distributions are assumed to have bounded density functions that we denote with f and g respectively. We also assume that F and G are strictly increasing, so that every type is uniquely associated to a rank in the skill distribution.

Each agent in one population is interested in matching with one agent in the other population. We use $U(x, y)$ to denote the utility of an agent with skill x who matches with an agent having skill y , and we use $V(x, y)$ to denote the utility of the agent with skill y who matches with an agent having skill x . We assume that U and V are continuous in both arguments, U is strictly increasing in y and V is strictly increasing in x . We assume that U (but not V) satisfies *increasing differences* (ID): for all $x, x' \in X$, with $x < x'$, for all $y, y' \in Y$, with $y < y'$, we have that $U(x, y') - U(x, y) \leq U(x', y') - U(x', y)$. We speak about *strict increasing*

differences (SID) if the inequality holds strictly, and we speak about *constant differences* (CD) if we have an equality.⁵

Skill y is assumed to be hard information, that can either be disclosed or not by an agent, but if it is disclosed then it is recognized as true. Skill x is instead assumed to be soft information, that requires certification in order to be credible. Certification is costly, and the cost is denoted by $c > 0$. Let X^H be the set of types that are not certified and remain hidden, and $X^R = X \setminus X^H$ the set of types that are certified and revealed. Analogously, Y^H denotes the set of types that remain hidden, while $Y^R = Y \setminus Y^H$ is the set of types that are revealed. With a slight abuse of notation, we use $F(\widehat{X})$ to denote the mass of agents with skill in $\widehat{X} \subseteq X$, and, analogously, we use $G(\widehat{Y})$ for the mass of agents with skill in $\widehat{Y} \subseteq Y$. We restrict to consider X^H and Y^H that are finite unions of intervals, i.e., proper intervals (with or without extrema) and degenerate intervals (that are single points).⁶

Agents who reveal their skill then match together as in a standard matching market, while agents who do not reveal their skill are then randomly paired. A *matching* is a function $\mu : X^R \rightarrow Y^R$ that constitutes an isomorphism.⁷ When $X^R \neq \emptyset$, we say that a matching μ is a *positive assortative matching* (PAM) if $x, x' \in X^R$, $x < x'$ implies $\mu(x) < \mu(x')$.

The expected utility of an agent with skill y who is randomly paired with an agent in X^H , when $F(X^H) > 0$, is:

$$\mathbb{E}V(X^H, y) \equiv \int_{x \in X^H} V(x, y) \frac{f(x)}{F(X^H)} dx.$$

Analogously, the expected utility of an agent with skill y who is randomly paired with an agent in Y^H , when $G(Y^H) > 0$, is:

$$\mathbb{E}U(x, Y^H) \equiv \int_{y \in Y^H} U(x, y) \frac{g(y)}{G(Y^H)} dy.$$

To complete the definition of $\mathbb{E}V(X^H, y)$ and $\mathbb{E}U(x, Y^H)$ we need to deal with X^H and Y^H having measure zero. Since we have assumed that X^H and Y^H are made of a finite number of intervals, then they are either empty or made of a finite number of points. If X^H and Y^H are non-empty, then $\mathbb{E}V(X^H, y) \equiv (1/\|X^H\|) \sum_{x \in X^H} V(x, y)$, and $\mathbb{E}U(x, Y^H) \equiv (1/\|Y^H\|) \sum_{y \in Y^H} U(x, y)$. Finally, if X^H and Y^H are empty then we assign arbitrary values, with the only requirement that the assumptions we have made on utilities still hold if we consider \emptyset as the lowest type; more precisely: $\mathbb{E}V(\emptyset, y) \leq V(\underline{x}, y)$ for all $y \in Y$, and $\mathbb{E}U(x, \emptyset) \leq U(x, \underline{y})$ for all $x \in X$, and $U(x', y) - \mathbb{E}U(x', \emptyset) \geq U(x, y) - \mathbb{E}U(x, \emptyset)$ for all $x, x' \in X$, $x < x'$, and for all $y \in Y$.

3.2. Equilibrium definition

In this model an *equilibrium* is a triple (X^R, Y^R, μ) , such that two conditions hold, that are no blocking pair (NBP) and individual rationality (IR). These conditions are standard for matching models, but are here

⁵A natural alternative to increasing differences is the single crossing condition, that however turns out to be insufficient for our purpose. In fact, we want that, if type x finds it convenient to pay the certification cost, then any type $x' > x$ also finds it convenient to do the same, and this must hold for any level of the certification cost c . We could instead assume the single crossing condition on the utility net of the cost of certification, and for any level of the cost, i.e., $U(x, y') - c \geq (>) U(x, y)$ implies $U(x', y') - c \geq (>) U(x', y)$ for any $x' > x$, $y' > y$ and for all $c > 0$. We note that this last definition is equivalent to ID as we defined it.

⁶This restriction has some useful implications. First, it ensures that X^H , X^R , Y^H and Y^R are measurable sets, and moreover it allows us to work with Riemann integration when computing the expected value of a random match in X^H and Y^H . Second, we are able to complete the definition of $\mathbb{E}V(X^H, y)$ and $\mathbb{E}U(x, Y^H)$ in a natural way when X^H and Y^H are non-empty sets of measure zero. In our opinion, this assumption causes a negligible loss of generality – since neglected sets are inherently related to the continuum setup and have a poor interpretation – while it allows us a significant simplification in the exposition.

⁷This means that μ is an invertible map and both μ and its inverse μ^{-1} are measurable and measure preserving maps.

adapted⁸ to the current setup:

- NBP
- for $x \in X^R, y \in Y^R$, if $U(x, y) - c \geq (>) U(x, \mu(x)) - c$ then $V(x, y) \leq (<) V(\mu^{-1}(y), y)$;
 - for $x \in X^R, y \in Y^H$, if $U(x, y) - c \geq (>) U(x, \mu(x)) - c$ then $V(x, y) \leq (<) \mathbb{E}V(X^H, y)$;
 - for $x \in X^H, y \in Y^R$, if $U(x, y) - c \geq (>) \mathbb{E}U(x, Y^H)$ then $V(x, y) \leq (<) V(\mu^{-1}(y), y)$;
 - for $x \in X^H, y \in Y^H$, if $U(x, y) - c \geq (>) \mathbb{E}U(x, Y^H)$ then $V(x, y) \leq (<) \mathbb{E}V(X^H, y)$.
- IR
- for $x \in X^R, U(x, \mu(x)) - c \geq \mathbb{E}U(x, Y^H)$;
 - for $y \in Y^R, V(\mu^{-1}(y), y) \geq \mathbb{E}V(X^H, y)$.

An equilibrium is said to be of *full revelation* if $F(X^R) = 1$ and $G(Y^R) = 1$. An equilibrium is said to be of *no revelation* if $F(X^R) = 0$ and $G(Y^R) = 0$. An equilibrium is of *partial revelation* if it is of neither full or no revelation.

4. Results

In this section we first provide characterizations of equilibria, we then show that multiple equilibria can exist, we proceed by distinguishing between stable and unstable equilibria, and we finally provide an existence result for stable equilibria.

4.1. Equilibrium characterization

Proposition 1 gives a characterization of equilibria of partial revelation, and it is the fundamental result of the paper. The proof is rather long but not difficult, and it helps to understand the role played by ID of U for our results.

Proposition 1. (X^R, Y^R, μ) is an equilibrium of partial revelation if and only if:

- (i) μ is PAM;
- (ii) there exists $x_c \in (\underline{x}, \bar{x})$ such that:

- (a) $X^R = [x_c, \bar{x}]$, and $Y^R = [G^{-1}(F(x_c)), \bar{y}]$,
- (b) $U(x_c, G^{-1}(F(x_c))) - c = \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$.

Proof. We start showing that if (X^R, Y^R, μ) is an equilibrium of partial revelation then (i) holds. Suppose not, then there must exist $x, x' \in X^R$, such that $x < x'$ and $\mu(x) > \mu(x')$. But then $(x', \mu(x))$ would form a blocking pair, since $U(x', \mu(x)) > U(x', \mu(x'))$ and $V(x', \mu(x)) > V(x, \mu(x))$ due to the strict monotonicity of U in y and of V in x .

We now show that if (X^R, Y^R, μ) is an equilibrium of partial revelation then (ii) holds. We start from (a). We first prove that there exist cutoff types x_c and y_c that separate the types who do not reveal – that lie below – from those who do reveal – that lie above; then we show that $x_c \in X^R$ and $y_c \in Y^R$, and finally $y_c = G^{-1}(F(x_c))$ is established.

⁸In the no blocking pair condition we essentially require that no weak Pareto improvement is possible for any pair of agents, while strict Pareto improvement is usually required in analogous definitions. We remark that our choice has the only consequence to have equilibrium cutoffs x_c and y_c (see Proposition 1) that are necessarily matched together (since the match (x_c, y_c) represents a strict improvement for y_c , but not for x_c , who is indifferent due to the cost of revelation), while they might also remain unmatched if we required a strict improvement for both agents to form a blocking pair.

Suppose that a cutoff type x_c does not exist, therefore there must exist $x \in X^R$, with $x < \hat{x} \equiv \sup X^H$. There are two cases: case 1, if $\mu(x) \geq \hat{y} \equiv \sup Y^H$, and case 2, if $\mu(x) < \hat{y}$. Consider case 1. There must exist $x' \in X^H$ with $x < x'$. The following inequalities hold, the first is obtained by ID of U after taking expectations, and the second comes from $x \in X^R$.

$$U(x', \mu(x)) - \mathbb{E}U(x', Y^H) \geq U(x, \mu(x)) - \mathbb{E}U(x, Y^H) \geq c.$$

Moreover, $V(x', \mu(x)) > V(x, \mu(x))$ by V being strictly increasing in x . Hence, $(x', \mu(x))$ would form a blocking pair, against the definition of equilibrium.

Consider case 2. The following inequalities hold, the first is obtained by ID of U after taking expectations, the second is due to U being strictly increasing in y , and the third comes from $x \in X^R$.

$$U(\hat{x}, \hat{y}) - \mathbb{E}U(\hat{x}, Y^H) \geq U(x, \hat{y}) - \mathbb{E}U(x, Y^H) > U(x, \mu(x)) - \mathbb{E}U(x, Y^H) \geq c.$$

Moreover, $V(\hat{x}, \hat{y}) > \mathbb{E}V(X^H, \hat{y})$ by V being strictly increasing in x , after taking expectations. We observe that, by continuity of U and V , there must exist $x' \in X^H$ sufficiently close to \hat{x} , and $y' \in Y^H$ sufficiently close to \hat{y} , such that they would both strictly gain by matching together. Hence, (x', y') would form a blocking pair, against the definition of equilibrium.

Suppose now that a cutoff type y_c does not exist, therefore there must exist $y \in Y^R$, with $y < \hat{y} \equiv \sup Y^H$. There must exist $y' \in Y^H$ with $y < y'$. We have just shown that $\mu^{-1}(y) > x$ for any $x \in X^H$, and hence $V(\mu^{-1}(y), y') > \mathbb{E}V(X^H, y')$ due to strict monotonicity of V in x . Moreover, $U(\mu^{-1}(y), y') > U(\mu^{-1}(y), y)$, due to strict monotonicity of U in y . Hence, $(\mu^{-1}(y), y')$ would form a blocking pair, against the definition of equilibrium.

So far we have proven that X^R is either $[x_c, \bar{x}]$ or $(x_c, \bar{x}]$, and Y^R is either $[y_c, \bar{y}]$ or $(y_c, \bar{y}]$. We observe that we cannot have the case $[x_c, \bar{x}]$ and $(y_c, \bar{y}]$, since there would exist $y \in Y^R$ with $y < \mu(x_c)$, and necessarily we would have $\mu^{-1}(y) > x_c$, but this would violate μ being PAM. Analogously, we can reason against $(x_c, \bar{x}]$ and $[y_c, \bar{y}]$. Consider now the case in which $(x_c, \bar{x}]$ and $(y_c, \bar{y}]$. Since types that are slightly higher than x_c are matched with types that are slightly higher than y_c , and they must find convenient to do so, then $U(x_c, y_c) - \mathbb{E}U(x_c, Y^H) \geq c$ by continuity of U ; but we also have that $V(x_c, y_c) > \mathbb{E}V(X^H, y_c)$ by V being strictly increasing in x , and hence (x_c, y_c) would form a blocking pair. Finally, we observe that if $y_c \neq G^{-1}(F(x_c))$ then the mass of agents in X^R would differ from the one in Y^R , and hence no matching would be feasible between the two sets. Therefore, we are left with the only possibility that $X^R = [x_c, \bar{x}]$ and $Y^R = [G^{-1}(F(x_c)), \bar{y}]$, and so (a) is proven.

We now prove (b). Suppose that $U(x_c, G^{-1}(F(x_c))) - c < \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$. Since types that are slightly higher than x_c are matched with types that are slightly higher than $G^{-1}(F(x_c))$, by continuity of U we would have $U(x, \mu(x)) - c < \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$ for type x higher than x_c but sufficiently close to it, against IR. Suppose instead that $U(x_c, G^{-1}(F(x_c))) - c > \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$. Consider x and y that are lower than, respectively, x_c and $G^{-1}(F(x_c))$ but sufficiently close to them. By continuity of U we have that $U(x, y) - c > \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$, and by strict monotonicity of V in x we have that $V(x, y) > V([x, x_c], y)$; hence, (x, y) would form a blocking pair. We are left with the only possibility that $U(x_c, G^{-1}(F(x_c))) - c = \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$, and so (b) is proven.

Finally, we show that if (X^R, Y^R, μ) satisfies (i) and (ii) then it is an equilibrium of partial revelation. Consider $x \in [x_c, \bar{x}]$. The first of the following inequalities holds by strict monotonicity of U in x , the second inequality holds – after expectations are taken – by ID of U , and the final equality is condition (b) of (ii).

$$\begin{aligned} U(x, \mu(x)) - \mathbb{E}U(x, [y, G^{-1}(F(x_c))]) &\geq U(x, G^{-1}(F(x_c))) - \mathbb{E}U(x, [y, G^{-1}(F(x_c))]) \geq \\ &\geq U(x_c, G^{-1}(F(x_c))) - \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))]) = c. \end{aligned}$$

Therefore $U(x, \mu(x)) - c \geq \mathbb{E}U(x, [\underline{y}, G^{-1}(F(x_c))])$, and IR is established.

Consider $x \in X$ and suppose x strictly gains by matching with y . If $x \in X^R$, then $y > \mu(x)$ since U is strictly monotone in y , hence $y \in Y^R$, and $\mu^{-1}(y) > x$ by PAM. Therefore, y strictly loses by matching with x due to strict monotonicity of V in x . If $x \in X^H$, then we first show that $y \in Y^R$. Suppose not, and consider the following inequalities, which show that x would strictly lose by matching with such a y . The first inequality holds by strict monotonicity of U in y , the second one holds – after expectations are taken – by ID of U , and the final equality is condition (b) of (ii).

$$\begin{aligned} U(x, y) - \mathbb{E}U(x, [\underline{y}, G^{-1}(F(x_c))]) &< U(x, G^{-1}(F(x_c))) - \mathbb{E}U(x, [\underline{y}, G^{-1}(F(x_c))]) \leq \\ &\leq U(x_c, G^{-1}(F(x_c))) - \mathbb{E}U(x_c, [\underline{y}, G^{-1}(F(x_c))]) = c. \end{aligned}$$

Therefore, we must have $y \in Y^R$, and hence $\mu^{-1}(y) \geq x_c > x$, so that y strictly loses by matching with x due to strict monotonicity of V in x . We have established that NBP holds.

We have shown that (X^R, Y^R, μ) is an equilibrium, while to understand that is of partial revelation we can simply observe that from (a) of (ii) we have $F(X^R) = 1 - F(x_c) = 1 - G(y_c) = G(Y^R)$ and $F(x_c) \in (0, 1)$. \square

From the above proposition, we know that equilibria of partial revelation are characterized by a matching that is PAM between the agents revealing their types. This is a standard result in matching models with non-transferable utility, and it essentially depends on utilities being strictly increasing in the partner's skill. More interestingly, equilibria of partial revelation have a *cutoff* type x_c that separates the agents who do not certificate from those who do. This, together with matching being PAM, implies that the agents lying above the cutoff level are matched with same-rank mates; in particular, we have that type x_c is matched with type $G^{-1}(F(x_c))$. Basically, finding an equilibrium of partial revelation amounts to identifying a cutoff type that is indifferent between the option to pay the certification cost entering the matching market, and the alternative option to save such a cost relying on random matching outside the market, and then letting the agents above the cutoff level match assortatively. This cutoff feature of equilibria crucially depends on utilities satisfying ID for the agents who have to pay the certification cost.

Given its important role, we will refer to

$$U(x_c, G^{-1}(F(x_c))) - c - \mathbb{E}U(x_c, [\underline{y}, G^{-1}(F(x_c))]) \tag{1}$$

as *relative gain from certification for the cutoff type x_c* .

We now provide characterizations of equilibria of full revelation (Proposition 2) and of no revelation (Proposition 3). Proofs are omitted, since they are essentially contained in the proof of Proposition 1.^{9,10}

Proposition 2. (X^R, Y^R, μ) is an equilibrium of full revelation if and only if:

- (i) μ is PAM;
- (ii) $U(\underline{x}, \underline{y}) - c \geq \mathbb{E}U(\underline{x}, \emptyset)$.

⁹In Proposition 2, the only equilibrium of full revelation is with $X^R = X$ and $Y^R = Y$. This is so because if $X^H = \{\underline{x}\}$ and $Y^H = \{\underline{y}\}$, then \underline{x} strictly prefers being matched in Y^H than paying a positive certification cost and being matched with \underline{y} . By continuity, also types close to \underline{x} would strictly prefer random matching, and so a positive mass of agents would exit the certified matching market.

¹⁰In Proposition 3, two equilibria of no revelation are possible, one with $X^R = \emptyset = Y^R$, the other with $X^R = \{\bar{x}\}$ and $Y^R = \{\bar{y}\}$. In fact, if the inequality holds as an equality, then $X^R = \{\bar{x}\}$ and $Y^R = \{\bar{y}\}$, since \bar{x} is indifferent but \bar{y} strictly prefers to be matched with \bar{x} .

Proposition 3. (X^R, Y^R, μ) is an equilibrium of no revelation if and only if $U(\bar{x}, \bar{y}) - c \leq \mathbb{E}U(\bar{x}, Y)$.

According to Propositions 2 and 3, equilibria of full revelation and of no revelation can be interpreted as extrema of the partial revelation equilibria, where cutoff levels are \underline{x} and \bar{x} , and the indifference condition holds as inequality since we are at the boundary of X and, respectively, no further decrease and no further increase of the cutoff is feasible. We observe that in Proposition 3 there is no mention to matching being PAM since X^R is either empty, and hence PAM is undefined, or it contains only \bar{x} , and PAM is trivially satisfied in such a case.

4.2. Equilibrium multiplicity

A salient feature of our model is that the outside option – i.e., not to reveal one’s own skill and remaining out of the matching market – has an endogenous value. The larger the cutoffs x_c and y_c , the better is the option to randomly match with an agent in X^H and Y^H , respectively. This allows for equilibrium multiplicity, as can be intuitively understood by the following argument. In an equilibrium of partial revelation, the cutoff type x_c must be indifferent between revealing and not revealing the skill, i.e., $U(x_c, G^{-1}(F(x_c))) - c = \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$. If we start from an equilibrium and we consider an increase in x_c , we can observe that – neglecting the direct effect that x_c has on utility in both sides – there is a positive effect on the left-hand side due to a better mate $G^{-1}(F(x_c))$, and a positive effect on the right-hand side as well due to the higher average quality of a match in $[y, G^{-1}(F(x_c))]$. The possibility is left open that both sides have increased by the same amount so that a new equilibrium is reached, and the following example shows that multiple equilibria can actually exist.¹¹

Example 1.

Let us consider a case in which $X = [0, 1] = Y$, F and G are uniform cumulative distributions, i.e., $F(x) = x$ for every x and $G(y) = y$ for every y , and $U(x, y) = 1 + (y - 1)^3$. We do not need to specify a precise function for $V(x, y)$. Note that U satisfies CD. We set $c = 0.3$. We can write the following gain from certification as a function of the cutoff type x_c , where we use the fact that $y_c = x_c$ in this setup:

$$\begin{aligned} & U(x_c, G^{-1}(F(x_c))) - c - \mathbb{E}U(x_c, [0, G^{-1}(F(x_c))]) = \\ & = 1 + (x_c - 1)^3 - 0.3 - \frac{1}{x_c} \int_0^{x_c} 1 + (y - 1)^3 dy = (x_c - 1)^3 - 0.3 - \frac{(x_c - 1)^4}{4x_c} + \frac{1}{4x_c}. \end{aligned}$$

The graph of the above function is depicted in Figure 1, from which we can recognize that multiple equilibria exist. As the figure shows, there are two equilibria of partial revelation, and one equilibrium of no revelation.

4.3. Equilibrium stability

We find useful to distinguish between two types of equilibria of partial revelation, since they will behave differently in the comparative statics exercises of Section 5. Looking at Figure 1, function $U(x_c, G^{-1}(F(x_c))) - c - \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$ crosses the horizontal axis from below in the first equilibrium, and from above in the second equilibrium. This can be related to a difference in the stability of the two equilibria. Suppose that the cutoff x_c is slightly higher than the first equilibrium cutoff. Then, x_c finds more convenient to obtain certification and reveal the skill, and the same holds by continuity for close types. Hence, all such agents are likely to pay the certification cost and enter the matching market, thus lowering the cutoff towards its

¹¹We underline that, as intuitively understood from the argument used and better illustrated in Example 1, the assumption of U satisfying ID does not play any specific role in the possibility that multiple equilibria exist, which essentially relies only on the fact that the value of random matching increases as the cutoff gets larger.

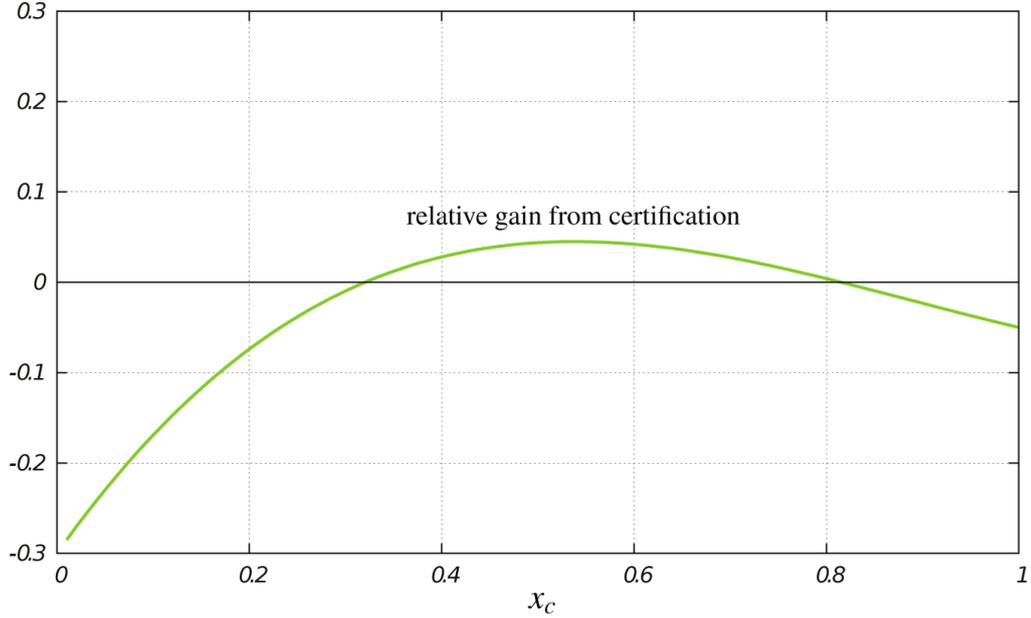


Figure 1: In the two equilibria of partial revelation the relative gain from certification for the cutoff type x_c is zero, while in the equilibrium of no revelation the relative gain from certification for the highest type \bar{x} is non-positive.

equilibrium level. If instead we start from a cutoff that is slightly lower than the equilibrium level, we can analogously argue in favor of an increase of the cutoff, thus showing the stability of the first equilibrium. Similar reasonings can be done for the second equilibrium, reaching the conclusion that it is unstable.

We will speak about *stable equilibrium* referring to an equilibrium of partial revelation such that function $U(x_c, G^{-1}(F(x_c))) - c - \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$ is strictly increasing in x_c at the equilibrium, while we will speak about *unstable equilibrium* for an equilibrium of partial revelation such that function $U(x_c, G^{-1}(F(x_c))) - c - \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$ is strictly decreasing in x_c at the equilibrium.¹²

4.4. Equilibrium existence

Our main focus is on stable equilibria of partial revelation. The following proposition provides an existence result for such a type of equilibria.

Proposition 4. *There exists \bar{c} such that if $0 < c < \bar{c}$, then there exists a stable equilibrium of partial revelation.*

¹²This distinction between stable and unstable equilibria is not exhaustive, since it does not cover with cases where the function $U(x_c, G^{-1}(F(x_c))) - c - \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))])$ is neither strictly increasing nor strictly decreasing in x_c at the equilibrium. Moreover, here we do not discuss equilibria of full revelation and of no revelation, for which a stability analysis might be done but would not be particularly interesting. What we do is because our focus in the comparative statics exercises of Section 5 will be mainly to understand how stable equilibria of partial revelation will react to exogenous changes, and unstable equilibria are useful since they provide a nice reference to compare the analysis.

Proof. If the certification cost is null, then the relative gain from certification for the cutoff type x_c is always positive for any $x_c \in (\underline{x}, \bar{x}]$, due to the strict monotonicity of U in y . Moreover, it tends to zero as x_c approaches zero, since $\lim_{x_c \rightarrow \underline{x}} \mathbb{E}U(x_c, [y, G^{-1}(F(x_c))]) = U(\underline{x}, y)$. This implies that there exists an interval (\underline{x}, x') where the relative gain from certification for the cutoff type is always strictly increasing. We set $\bar{c} = U(x', G^{-1}F(x')) - \mathbb{E}U(x', [y, G^{-1}F(x')])$, and the proof is completed. \square

The condition that guarantees existence of a stable equilibrium of partial revelation is on the level of certification costs. Intuitively, the relative gain from certification for the cutoff type is a function that is strictly increasing when x_c is very close to \underline{x} , and in the limit for x_c going to \underline{x} such a function goes to $-c$. Therefore, if the cost of certification is positive but low enough, then the relative gain from certification for the cutoff type will cross the zero level coming from below, and thus a stable equilibrium of partial revelation is found. Incidentally, we observe that if the cost of certification is instead high enough, so that the relative gain from certification for the cutoff type remains always negative, then an equilibrium of no revelation surely exists, while no equilibrium of partial revelation can exist.

5. Discussion

In this section we carry out some comparative statics exercises. In particular, we analyze the effects of changes in the cost of certification, and changes both in the distribution of skills that are unverifiable unless certified and in the distribution of skills that are verifiable. We conclude with some simple comments on welfare.

5.1. Comparative statics on the cost of certification cost and on the distribution of skills

We start by motivating a change of the unit of analysis from type – i.e., x and y – to rank – i.e., $F(x)$ and $G(y)$. The reason is that in Proposition 6 we will consider a general increase of x -skills, so that a generic agent with skill x will have a higher skill after the distribution has changed. Therefore, looking at the ex-ante and ex-post situation for the same type/skill is generally misleading, since the comparison will involve different agents. If we suppose that the generalized shift in skills does not cause changes in relative positions, then it is more reasonable to carry out the analysis taking the rank as unit of analysis. With this purpose, we preliminarily define the *relative gain from certification for the cutoff agent* r_c as

$$U(F^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)]), \quad (2)$$

where $\mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)]) \equiv \frac{1}{r_c} \int_0^{r_c} U(F^{-1}(r_c), G^{-1}(r)) dr$. We are now ready for some comparative statics results. We start considering a change in the cost of certification.

Proposition 5. *If the certification cost c increases, then the relative gain from certification for every cutoff agent r_c decreases.*

Proof. By looking at (2), it is immediate to recognize that an increase in c has a negative impact on the relative gain from certification for every cutoff agent r_c . \square

The next proposition analyzes the effects of a change in the distribution of x -skills. The result crucially relies on the ID property of the utility function U . We briefly remind that a distribution F' first-order stochastically dominates a distribution F if $F'(x) \leq F(x)$ for every x .

Proposition 6. *If F' first-order stochastically dominates F , then the relative gain from certification for every cutoff agent r_c does not decrease, and it increases if $F'(r_c) < F(r_c)$ and U satisfies SID.*

Proof. We observe that $U(F^{-1}(r_c), G^{-1}(r_c)) - U(F^{-1}(r_c), G^{-1}(r)) \leq U(F'^{-1}(r_c), G^{-1}(r_c)) - U(F'^{-1}(r_c), G^{-1}(r))$ for every $r \leq r_c$, due to ID of U and the fact that $F^{-1}(r_c) \leq F'^{-1}(r_c)$, that comes from F' first-order stochastically dominating F . Therefore, taking the expectations we obtain:

$$U(F^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)]) \leq U(F'^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F'^{-1}(r_c), [0, G^{-1}(r_c)]). \quad (3)$$

The initial inequality above becomes strict if U satisfies SID and $F^{-1}(r_c) < F'^{-1}(r_c)$, and the same holds when expectations are taken, thus completing the proof. \square

One might think that similar results hold for changes in the distribution of y -skills. The following proposition states that this is not the case, and Example 2 – which constitutes a proof of Proposition 7 – illustrates the variety of effects that can occur. Intuitively, a generalized increase in y -skills positively affects both the value of certification – since the skill of same-rank mate has increased – and the value of a random match – since the average skill in the pool of agents who do not certify has increased as well. In general, there is no way to decide which one has increased more.

Proposition 7. *If G' first-order stochastically dominates G , then the relative gain from certification for a cutoff agent r_c can either increase, or decrease, or remain unchanged.*

Example 2.

We provide three variants of an example, each variant illustrating a case that Proposition 7 states as a possibility. Let us assume that $X = [0, 1]$, $F^{-1}(r) = r$, (i) $Y = [0, k]$, $G^{-1}(r) = kr$ with $k > 0$, (ii) $Y = [k, 1]$, $G^{-1}(r) = k + (1 - k)r$ with $0 < k < 1$, (iii) $Y = [k, 1 + k]$, $G^{-1}(r) = k + r$. We also assume $U(x, y) = xy$, so that SID is satisfied, while we do not need to specify a precise function for $V(x, y)$. We can write the relative gain from certification as a function of the cutoff agent r_c in the three variants as follows:

$$\begin{aligned} & U(F^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)]) = \\ (i) \quad & = r_c - c - \frac{1}{r_c} \int_0^{r_c} r_c k r \, dr = \frac{kr_c^2}{2} - c \\ (ii) \quad & = r_c(k + (1 - k)r_c) - c - \frac{1}{r_c} \int_0^{r_c} r_c(k + (1 - k)r) \, dr = \frac{(1 - k)r_c^2}{2} - c \\ (iii) \quad & = r_c(k + r_c) - c - \frac{1}{r_c} \int_0^{r_c} r_c(k + r) \, dr = \frac{r_c^2}{2} - c \end{aligned}$$

From the above expression it is immediate to recognize that an increase in k has a (i) positive, (ii) negative, or (iii) null effect on the relative gain from certification for all $r_c \in (0, 1)$.

As a consequence of the displacement of the function measuring the relative gain from certification in response to changes in c , F and G , the set of equilibria changes in a rather straightforward way. We can facilitate intuition by means of a graphical example, like the one in Figure 2. It is easy to understand that equilibria react to a displacement of the relative gain from certification differently depending on whether they are stable or unstable. More precisely, an upward displacement of the function measuring the relative gain from certification causes stable equilibria to move leftwards, and unstable equilibrium to move rightwards.¹³ Similar arguments can be applied to draw conclusions on equilibrium cutoffs in many other comparative statics exercises.

¹³Clearly, a large enough increase of skills may cause equilibria to disappear (or, better, one stable equilibrium and one unstable equilibrium may collapse into a unique equilibrium and then vanish, similarly to what happens for solutions to nonlinear equations when translations are applied). So, previous statements are valid only for small enough displacements of function D .

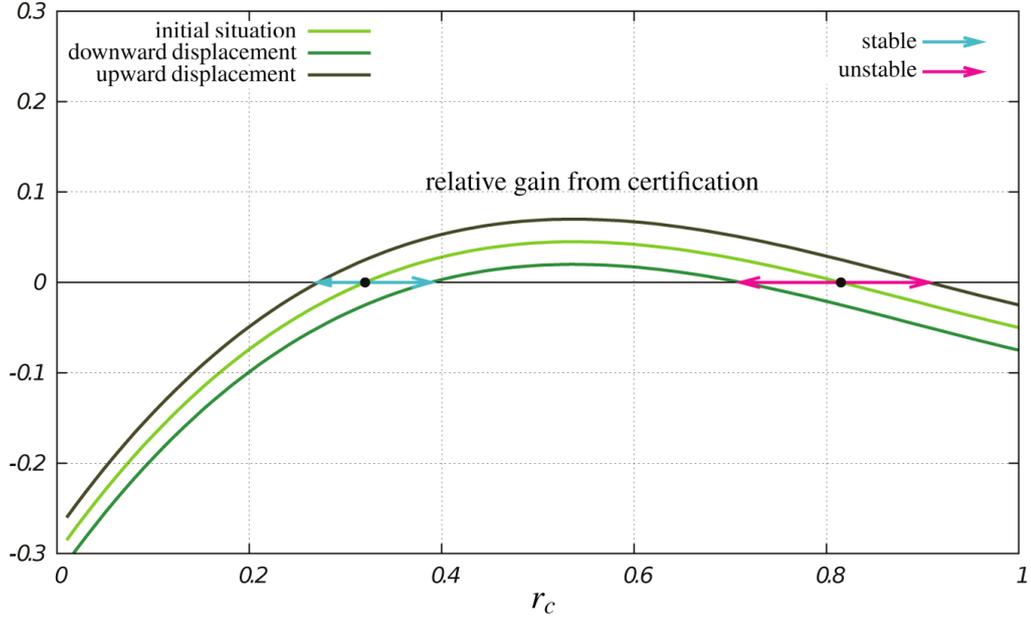


Figure 2: The two equilibria of partial revelation react differently to displacements of the function measuring the relative gain from certification depending on their stability properties.

5.2. Welfare

A first question that can be posed concerns the total welfare effect of imposing all agents to disclose credible information about their skill. This can be the case, for instance, of compulsory quality certification. To fix ideas, suppose that we want to compare a case in which all agents are forced to reveal their skill, so that $X^R = X$, to another case in which there is no imposition and an equilibrium of partial revelation occurs, so that $X^R = [x_c, \bar{x}]$ for some $x_c \in (\underline{x}, \bar{x})$. Two effects are involved in the passage from the first case to the second case. Let us first consider agents whose skills are unverifiable unless certified. On the one side, there are fewer people who pay the cost to certify the skill, and this reduces the total cost of information disclosure. On the other side, there are fewer pairs who are matched in a positive assortative way, and this is a social cost in the presence of strict increasing differences, while it is not in the case of constant differences. Let us now consider agents whose skills are verifiable. Obviously, the first effect is absent, since they do not pay any certification cost. The second effect instead is present, although again not in the case of constant differences. In general, no definite ranking in terms of welfare can be established between the two cases. Typically, full disclosure will not be the social optimum. In particular, if utilities of agents on both sides satisfy constant differences, then we can conclude that overall welfare decreases in the level of information disclosure.

A second question that can be posed regards the total welfare associated to different equilibria. Suppose that we have two distinct equilibria, that can be the ex-ante and ex-post state with respect to some exogenous change in c , F or G , or two different equilibria for the same c , F and G , if we are in the presence of multiple equilibria (like in Example 1). Whatever the case, we may be interested in a welfare comparison between such equilibria. We observe that the same two effects that we have discussed above apply here as well. Moreover, if the two equilibria that we compare are associated to changes in c , F or G , then further effects

are at play. Let us consider first agents whose skills are unverifiable unless certified. A third effect arises if c increases, since then agents who still pay to disclose their skill must sustain a higher cost. This effect combines with the former effect regarding the reduction of the set of agents who disclose their skill, leaving undetermined the net effect on total cost of disclosure. A fourth effect is triggered by the increase in one's own skill and it has an ambiguous effect on utility, depending on how $U(x, y)$ depends on x .¹⁴ A fifth effect is triggered by the increase in the partner's skill and it has a positive impact on utility, because $U(x, y)$ is strictly increasing in y . When we look at the utility of agents whose skills are verifiable, we have to consider that they pay no cost to reveal their skill, so that the third effect is absent. However, the fourth and the fifth effects are present and are similar to those we have just discussed. On the whole, we have argued that several contrasting effects are at play and, once combined together, the result of a welfare comparison remains ambiguous. Nevertheless, when the general model is applied to more specific setups, welfare assessments become easier, and often prove to be non-trivial and insightful. In the Appendix we provide a couple of examples to illustrate the variety of welfare analyses that may arise.

6. Conclusions

In this paper we have developed and studied a model which lies at the intersection of the research on positive assortative matching under non-transferable utility and the research on strategic revelation of information when disclosure is costly. We have obtained that when types are private information and disclosure is costly on one side of the market, only the best agents get certification and match with agents on the other side in a positive assortative way, while the remaining agents match randomly. Moreover, multiple equilibria may arise, as a consequence of the endogenous value of the outside option.

Typically, in models of matching with transferable utility the shape of a stable matching – and, in particular, whether agents match assortatively or not – is determined by comparative advantages. By contrast, if utility of a match is non-transferable, then what matters are absolute advantages. In this paper we have shown that under costly disclosure of types comparative advantages become again important to understand the properties of the equilibrium matching when utility is non-transferable.

A natural follow up of the paper would be to take into consideration the case in which agents on both sides have unverifiable skills that require certification in order to be trusted. In such a situation it is no longer enough to consider the relative gain of certification in one population only, since the corresponding cutoff mate might find it not convenient to reveal the skill, due to the cost of certification that must now be paid. Therefore, the choice of whether to certify or not must be jointly considered for agents on both sides. In particular, if the relative gain from certification is negative for the cutoff agent on one side, then no equilibrium can arise, since such agent would prefer to go for a random match. If, instead, the relative gain from certification is positive for the cutoff agent on one side, then we can have an equilibrium if the cutoff agent on the other side is indifferent whether to certify or not. Indeed, we would have additional agents on the first side who are willing to certify and form pairs with same-rank mates, but they would not find available partners on the other side. We observe that such a case is similar to what we have studied in the present paper, where agents with verifiable skill have null certification costs and hence their relative gain from certification is always positive.

¹⁴We remind that we have assumed that utilities strictly increase in the partner's skill, while no assumption is made concerning the effect of changes in one's own skill.

Acknowledgements

We are very much indebted to an anonymous referee, who has provided us with useful suggestions to make our main message clearer and to simplify the model accordingly. We also want to thank participants to the 7th Spain-Italy-Netherlands Meeting on Game Theory, Paris 2011, to the 4th World Congress of the Game Theory Society, Istanbul 2012, and to the 6th meeting of the GRASS workgroup, Rome 2012. In all these occasions we have benefitted from insightful discussions and invaluable comments. All mistakes remain ours.

Appendix - Examples on welfare

Example 3.

We assume $X = [0, 1] = Y$, $F^{-1}(r) = r$, $G^{-1}(r) = r$, $U(x, y) = y$ and $V(x, y) = x$, so that both U and V satisfy CD. The relative gain from certification for the cutoff agent r_c is:

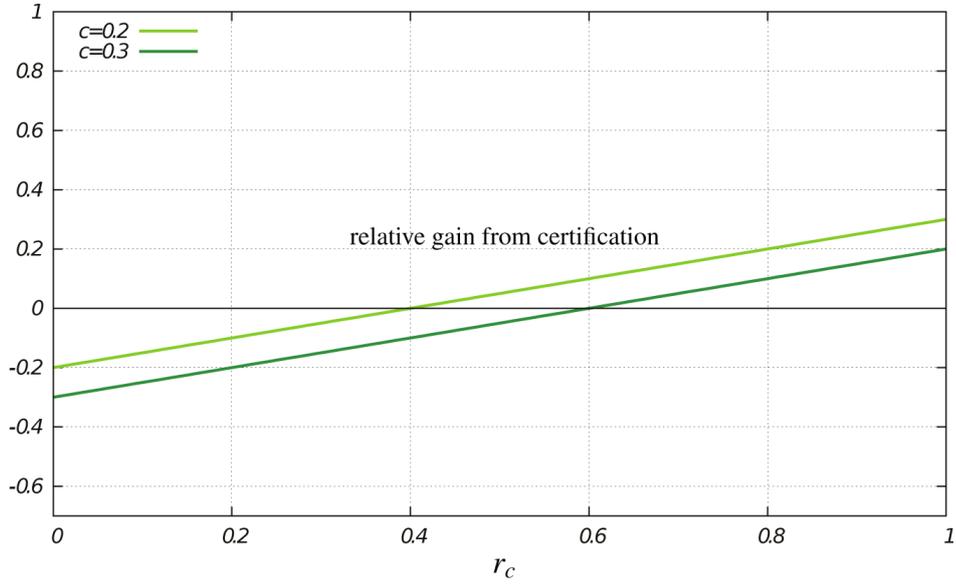
$$U(F^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)]) = r_c - c - \frac{1}{r_c} \int_0^{r_c} r \, dr = \frac{x_c}{2} - c.$$

Figure 3.A depicts the above expression as a function of r_c for $c = 0.2$ and for $c = 0.3$. The unique equilibrium of partial revelation is in the first case with $r_c = 0.4$, and in the second case with $r_c = 0.6$. Figure 3.B depicts the utility at equilibrium of agents whose skills are unverifiable unless certified for both levels of the certification cost. From it we can see that those who choose to hide the skill for both levels of cost – i.e., agents from 0 to 0.4 – are better off when $c = 0.3$, because of the increase in the average skill of a random match. Also some of the agents who exit the matching market in the passage from $c = 0.2$ to $c = 0.3$ – i.e., those from 0.4 to 0.5 – find themselves in better conditions when $c = 0.3$, while the others – i.e., those from 0.5 to 0.6 – find themselves in worse conditions. We think that it is an interesting theoretical possibility that some types may be induced to exit the market as a result of the increase in c , and find themselves better off in the ex-post situation. Finally, agents who remain in the matching market are clearly worse off when the cost increases, since they are matched with the same mate but have to pay a higher certification cost.

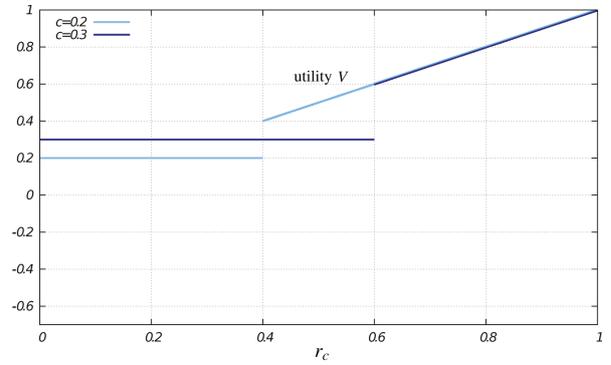
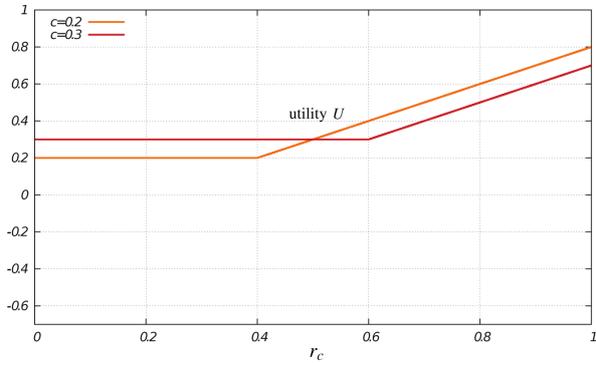
The analysis follows similar lines for agents whose skills are verifiable, with the only difference that no cost is paid. Figure 3.C illustrates. Those who are out of the matching market for both levels of costs – i.e., agents from 0 to 0.4 – are better off when the cost is higher, since the average skill of a random match has increased. Those who reveal their skill when $c = 0.2$ but that rely on a random match when $c = 0.3$ – i.e., agents from 0.4 to 0.6 – are all worse off when the cost is higher. This is so because the average skill of a random match for $c = 0.3$ is lower than the skill of their same-rank mate.¹⁵ Finally, agents from 0.6 to 1 are evidently not affected by the change in cost, since matched with the same partner in both cases.

We might be interested to compare the above two equilibria using an utilitarian welfare function. In the very simple setup of this example, this amounts to rank outcomes inversely on the basis of the total amount of costs, since both utility functions U and V exhibit constant differences. We observe that total expenditure is the same in the two equilibria, in particular it is equal to 0.12. Generalizing the question, we might look at the relationship between total cost (and, hence, utilitarian welfare) and the level of c . Using the fact that $2r_c = c$, we end up with $-c^2/(2+c)$ as the function describing such relationship in the cost interval $[0, 0.5]$. It is not surprising that the utilitarian welfare is maximized for $c = 0$ – when all agents enter the market for

¹⁵We note that this is not a general result. In some cases it may happen that the average skill of a random match for a higher cost is larger than the skill of the same-rank mate.



(a) $U(F^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)])$.



$$(b) \begin{cases} \mathbb{E}U(F^{-1}(r), [0, G^{-1}(r_c)]) & \text{for } r < r_c, \\ U(F^{-1}(r), G^{-1}(r)) - c & \text{for } r \geq r_c. \end{cases}$$

$$(c) \begin{cases} \mathbb{E}V([0, F^{-1}(r_c)], y) & \text{for } r < r_c, \\ V(G^{-1}(r), r) - c & \text{for } r \geq r_c. \end{cases}$$

Figure 3: (a) depicts the relative gain from certification for the cutoff agent r_c when $c = 0.2$ and $c = 0.3$. For the same cost values, (b) and (c) depict the utility at equilibrium (i.e., with $r_c = 0.4$ and $r_c = 0.6$) of agents with x -skills and y -skills, respectively. We note that in (c) the two graphs coincide when r_c is larger than 0.6.

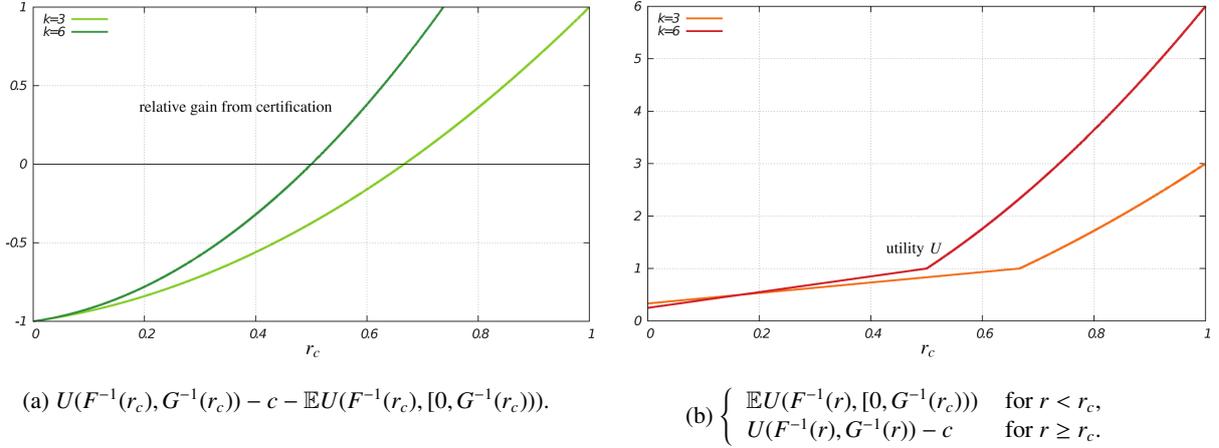


Figure 4: (a) and (b) illustrate the effects of changing k from 3 to 6 on, respectively, the relative gain from certification for the cutoff agent r_c and the utility of x -types at equilibrium (i.e., with $r_c = 2/3$ and $r_c = 0.5$).

free – and for $c \geq 0.5$ – when no certification occurs. The utilitarian welfare is minimized for $c = 0.25$, when total expenditure is maximized. Even in this extremely simple setup, we find of some interest that for a range of the cost level – i.e., $[0.25, 0.5]$ – the utilitarian welfare increases as the cost to enter the market becomes higher. The quality of this remark appears to be still valid in the presence of utility functions with moderate strictly increasing differences.

Example 4.

We assume that $X = [0, k]$, $Y = [0, 1]$, $F^{-1}(r) = kr$ with $k > 0$, $G^{-1}(r) = r$, $U(x, y) = (1 + x)y$ and $V(x, y) = (1 + y)x$, so that both U and V satisfy SID. The relative gain from certification for the cutoff agent r_c is:

$$U(F^{-1}(r_c), G^{-1}(r_c)) - c - \mathbb{E}U(F^{-1}(r_c), [0, G^{-1}(r_c)]) = (1 + kr_c)r_c - c - \frac{1}{r_c} \int_0^{r_c} (1 + kr_c)r \, dr = \frac{kr_c^2}{2} + \frac{r_c}{2} - c.$$

We set $c = 1$. Figure 4.A depicts the above expression as a function of r_c for $k = 3$ and $k = 6$. The unique equilibrium of partial revelation is in the first case with $r_c = 2/3$, and in the second case with $r_c = 0.5$. We limit ourselves to making a remark about the well-being of agents whose skills are unverifiable unless certified. Figure 4.B depicts the utility at equilibrium for agents whose skills are unverifiable unless certified when $k = 3$ and $k = 6$. From it we can observe that agents below $1/6$ are made worse off by the generalized increase of skills. This may appear surprising at a first glance, since the increase in x has a positive direct effect on utility. However, there is also an indirect effect, which is negative. In fact, the equilibrium cutoff gets lower as a consequence of the upward displacement of the relative gain from certification for the cutoff agent (as we know from Proposition 6). This means that the average skill of a random partner decreases. These two effects contrast with each other. Therefore, types for whom the latter effect dominates the former find themselves in worse conditions after the generalized increase of skills.

References

Atakan, A. (2006). Assortative matching with explicit search costs. *Econometrica* 74(3), 667–680.

- Becker, G. (1973). A theory of marriage: Part I. *Journal of Political Economy*, 813–846.
- Bergstrom, T. and M. Bagnoli (1993). Courtship as a waiting game. *Journal of Political Economy*, 185–202.
- Bloch, F. and H. Ryder (2000). Two-sided search, marriages, and matchmakers. *International Economic Review* 41(1), 93–116.
- Board, O. (2009). Competition and disclosure. *Journal of Industrial Economics* 57(1), 197–213.
- Booth, A. and M. Coles (2010). Education, matching, and the allocative value of romance. *Journal of the European Economic Association* 8(4), 744–775.
- Chade, H. (2006). Matching with noise and the acceptance curse. *Journal of Economic Theory* 129(1), 81–113.
- Chade, H., G. Lewis, and L. Smith (2007). The college admissions problem under uncertainty. *mimeo*.
- Chakraborty, A., A. Citanna, and M. Ostrovsky (2010). Two-sided matching with interdependent values. *Journal of Economic Theory* 145(1), 85–105.
- Clark, S. (2006). The uniqueness of stable matchings. *B.E. Journal of Theoretical Economics* 6(1), 8.
- Dranove, D. and G. Jin (2010). Quality disclosure: Theory and practice. *Journal of Economic Literature* 48(4), 935–63.
- Eckhout, J. (2000). On the uniqueness of stable marriage matchings. *Economics Letters* 69(1), 1–8.
- Grossman, S. (1981). The informational role of warranties and private disclosure about product quality. *Journal of Law & Economics* 24, 461.
- Grossman, S. and O. Hart (1980). Disclosure laws and takeover bids. *Journal of Finance* 35(2), 323–334.
- Hopkins, E. (2011). Job market signalling of relative position, or becker married to spence. *forthcoming Journal of European Economic Association* 134.
- Hoppe, H., B. Moldovanu, and A. Sela (2009). The theory of assortative matching based on costly signals. *Review of Economic Studies* 76(1), 253–281.
- Jovanovic, B. (1982). Truthful disclosure of information. *Bell Journal of Economics*, 36–44.
- Legros, P. and A. Newman (2007). Beauty is a Beast, frog is a prince: Assortative matching with nontransferabilities. *Econometrica* 75(4), 1073–1102.
- Legros, P. and A. Newman (2010). Co-ranking mates: Assortative matching in marriage markets. *Economics Letters* 106(3), 177–179.
- Li, H. and S. Rosen (1998). Unraveling in matching markets. *American Economic Review* 88(3), 371–387.
- Li, H. and W. Suen (2000). Risk sharing, sorting, and early contracting. *Journal of Political Economy* 108(5), 1058–1091.
- Matthews, S. and A. Postlewaite (1985). Quality testing and disclosure. *RAND Journal of Economics*, 328–340.
- Milgrom, P. (1981). Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 380–391.
- Ostrovsky, M. and M. Schwarz (2010). Information disclosure and unraveling in matching markets. *American Economic Journal: Microeconomics* 2(2), 34–63.
- Satterthwaite, M. and A. Shneyerov (2007). Dynamic matching, two-sided incomplete information, and participation costs: Existence and convergence to perfect competition. *Econometrica* 75(1), 155–200.
- Shavell, S. (1994). Acquisition and disclosure of information prior to sale. *RAND Journal of Economics*, 20–36.

RECent Working Papers Series

The 10 most RECent releases are:

- No. 94 DISCLOSURE OF INFORMATION IN MATCHING MARKETS WITH NON-TRANSFERABLE UTILITY (2014)
E. Bilancini and L. Boncinelli
- No. 93 BIBLIOMETRIC EVALUATION VS. INFORMED PEER REVIEW: EVIDENCE FROM ITALY (2013)
G. Bertocchi, A. Gambardella, T. Jappelli, C. A. Nappi and F. Peracchi
- No. 92 ITALY'S CURRENT ACCOUNT SUSTAINABILITY: A LONG RUN PERSPECTIVE, 1861-2000 (2013)
B. Pistoresi, A. Rinaldi
- No. 91 EDUCATION TIES AND INVESTMENTS ABROAD. EMPIRICAL EVIDENCE FROM THE US AND UK (2013)
M. Murat
- No. 90 INEFFICIENCY IN SURVEY EXCHANGE RATES FORECASTS (2013)
F. Pancotto, F. M. Pericoli and M. Pistagnesi
- No. 89 RETURNING HOME AT TIMES OF TROUBLE? RETURN MIGRATION OF EU ENLARGEMENT MIGRANTS DURING THE CRISIS (2013)
A. Zaiceva, K. F. Zimmermann
- No. 88 WOMEN, MEDIEVAL COMMERCE, AND THE EDUCATION GENDER GAP (2013)
G. Bertocchi, M. Bozzano
- No. 87 OUT OF SIGHT, NOT OUT OF MIND. EDUCATION NETWORKS AND INTERNATIONAL TRADE (2012)
M. Murat
- No. 86 THE FEDERAL FUNDS RATE AND THE CONDUCTION OF THE INTERNATIONAL ORCHESTRA (2012)
A. Ribba
- No. 85 ASSESSING GENDER INEQUALITY AMONG ITALIAN REGIONS: THE ITALIAN GENDER GAP INDEX (2012)
M. Bozzano

The full list of available working papers, together with their electronic versions, can be found on the RECent website: <http://www.recent.unimore.it/workingpapers.asp>