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Abstract:

This paper presents several proposals for creating an aggregate sentiment index for the European stock market. We achieve this objective by using the OWA and WOWA operators, which have been successful in finance and have a strong financial interpretation. We compute ten different aggregate sentiment indices for the 2007-2021 period and evaluate their ability to provide information about current and future market volatility and returns. We find several results of interest for both investors and policymakers. Sentiment indices have a strong negative relationship with market volatility. Extreme values of sentiment can predict future market returns, with low values indicating positive returns and high values suggesting negative returns. Finally, using stock market capitalisation as an input of the WOWA operator enhances explanatory power of the indices on future market returns compared to the OWA operator.

*Keywords**:** sentiment indices, OWA aggregation, WOWA aggregation, volatility, returns.*

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1. Introduction

Investor sentiment refers to the expectations and perceptions of future cash flow and investment risks that may not necessarily be based on available facts. It can also be described as a tendency towards speculation and an overall sense of positivity or negativity towards a particular asset (Baker and Wurgler, 2006; 2007). Understanding sentiment is crucial because it provides useful insights into the underlying market dynamics allowing investors and regulators to better assess market behaviour and foresee probable changes in demand and asset prices. Additionally, sentiment offers a nuanced perspective for understanding market returns and guiding prudent investing choices. It plays a crucial role in explaining both short-term market swings and long-term trends, being able to affect financial decisions with implications for asset prices (Benhabib et al., 2016; Piccione and Spiegler, 2014). In addition, sentiment has been able to explain financial market movements in periods of irrational panic or unjustified optimism (Reis and Pinho, 2020a). Furthermore, investor sentiment seems to be the main driving force behind the co-movements of stock returns, being able to explain the level, variance, and covariance of the non-fundamental component of returns (Frijns et al. 2017). As a result, studying sentiment has become essential in navigating the complexity of financial markets and making well-informed decisions that coincide with investor sentiment.

Since investor sentiment is not a directly measurable variable, researchers have historically relied on proxy variables like trading volume, option-implied volatility, and surveys. Recently, sentiment measures and indicators have been extracted from news, microblogging sites, and web searches. One example is the EPU index proposed by Baker et al. (2016), which uses newspaper coverage to capture uncertainty around economic policy decisions. However, the index is not able to distinguish between good and bad news, making it less useful for gauging market fear or greed. To address this limitation, some researchers have turned to text mining to classify comments as positive or negative (see e.g., Gao et al., 2020). However, there is no consensus on how to construct sentiment indicators (Chan et al., 2017), leaving investors and fund managers without a single reliable measure. To fill this gap, companies like Thomson Reuters and Bloomberg have developed their own investor sentiment indices at the individual stock level, using data from news and social media content. Thomson Reuters and Bloomberg both use comprehensive finance-specific data from news and social media to generate sentiment indicators. Bloomberg has developed the Investor Sentiment Index, which measures investors' reactions to news related to companies. This index considers news and tweets relevant to a specific company and assigns a numerical valuation of investor sentiment. Bloomberg categorises the sentiment as positive, negative, or neutral, depending on how it would affect investors with a long position. This information is then used in automatic learning models to create the sentiment index (see Bloomberg, 2016, for a detailed description).

A crucial issue that requires consideration in the European market is the paucity of sentiment indicators (Gambarelli and Muzzioli, 2023). These indicators are helpful in determining investor confidence, market expectations, and risk appetite, which are essential in making informed decisions. Unfortunately, most existing sentiment measures are geared towards the US market (such as the S&P 500 Twitter Sentiment Index), leaving European investors with limited options for accurately assessing sentiment. Moreover, the relationship between sentiment and future returns is debated in the literature. Many studies find evidence of a relationship between investor sentiment and market prices (see, e.g., Uhl, 2014), while others (see, e.g., González-Sánchez and Morales de Vega, 2021) conclude that there is no significant relationship between investor sentiment and market returns. In addition, there are also studies arguing that the explanatory power is in the opposite direction (Das et al., 2005), suggesting that sentiment is affected by variations in returns and volatility, not the other way around.

To fill the existing gap, in this paper we pursue a threefold objective. First, we develop several proposals for an aggregate sentiment index for the European stock market by combining sentiment at the individual stock level. Second, we investigate the explanatory power of the obtained sentiment indices on contemporaneous and future realised volatility, to evaluate their potential value for risk assessment. Third, we shed light on the debated relationship between the sentiment indices and both contemporaneous and future market returns. For constructing the sentiment indices, we rely on the Bloomberg sentiment index score at the individual stock data. While existing aggregate sentiment indices are commonly constructed by aggregating several economic and financial variables (see e.g., Reis and Pinho, 2020a; 2020b), or indicators based on surveys (for the European markets, see e.g., the European Commission’s monthly consumer confidence indicator and the economic sentiment indicator (ESI)), we believe that aggregating sentiment from individual stocks could provide more insights into future market outcomes to investors and policymakers.

Regarding the choice of the aggregation method, we resort to the Ordered Weighted Averaging (OWA) (Yager, 1988) operator. The OWA aggregates input values from the largest to the smallest and then takes a weighted average of the ordered inputs according to a vector of weights, which regulates the relative importance of each value. The choice of the OWA operator is motivated by its successful applications in many fields, including finance. In particular, in the financial fields, several studies adopt the OWA operator and its extensions for decision-making (Merigó and Casanovas, 2011), forecasting (Fonseca-Cifuentes et al., 2021), and volatility measuring (Leon-Castro et al., 2021; Flores-Sosa et al., 2021). In addition, the OWA operator has been successfully adopted to estimate variance and covariance (Merigó et al., 2015; Blanco-Mesa et al., 2019, 2020) and has been applied in the mean-variance model (Laengle et al., 2016). In fact, one of the key advantages of the OWA operator, when applied to the financial field, is the ability to represent complex scenarios using the degree of optimism and pessimism of the decision maker (Laengle et al., 2016). More specifically, the OWA operator is able to consider the decision-makers' attitude in the weight selection process in terms of different degrees of optimism and pessimism (measured by the degree of orness) (Laengle et al., 2016). Moreover, given the widespread use of weighted averages in finance (particularly for computing market indices), for the aggregation of the individual sentiment indices, we will also consider the weighted OWA operator (WOWA) introduced by Torra (1996, 1997). The WOWA operator is a generalisation of both weighted mean and OWA, which aims at encompassing in a single operator the advantages of the weighted mean and OWA operator (Yager et al. 2011) and is particularly suitable for applications in which it is important to assign a weight to information sources and the compensation degree (or the relative importance of the input values). The WOWA is an aggregation operator that permits the aggregation of a set of numerical data with respect to two weighting vectors: one of the weighting vectors has the same interpretation of the weighted mean vector of weights, while the other has the same interpretation of the one of the OWA.

Exploiting the OWA and the WOWA operators to aggregate sentiment data obtained at the individual stock level, we develop ten different aggregate sentiment indices related to different values of the compensation degree (degree of orness), and we evaluate their information content about contemporaneous and future market volatility and returns. The majority of sentiment indices is strongly and negatively related to market realised volatility: an increase in sentiment is associated to a decrease in realised volatility for the current and next three months. On the other hand, while the aggregate sentiment indices do not reveal explanatory power on returns when considering the entire distribution of the variables, we find notable results when investigating their extreme values. In particular, we find that aggregate sentiment indices act as contrarian predictors of future market returns: low (high) values of the indices signal positive (negative) returns over the medium term. Therefore, very low and very high values of aggregate sentiment indices suggest the possibility of an oversold and an overly complacent market, respectively, and hence a point of possible reversal for the market trend. The results also show that accounting for the stock market capitalisation as an input of the WOWA operators enhances the explanatory power on market returns compared to the OWA operator. Finally, regarding the compensation degree, a degree of orness between 0.6 and 0.8 is found to be the best in explaining contemporaneous and one-month ahead realised volatility. On the other hand, a degree of orness between 0.2 and 0.4 seems optimal for increasing the predictive power of the aggregate index on future market reversal when it reaches very high values. Finally, in order to detect an upward reversal in the stock market trend over the medium term when the sentiment index is very low, the weighted average is found to be appropriate.

The remainder of the paper is structured as follows. In Section 2, we review the OWA and the WOWA operators and identify an appropriate method to conduct the sentiment indices aggregation. In Section 3, we introduce the dataset and the methodology adopted to aggregate sentiment scores at the individual stock level in an aggregate sentiment index. Section 4 investigates the relationship between the developed sentiment indices and contemporaneous and future market volatility and returns. Finally, Section 5 concludes and provides policy implications.

2. The OWA the Ordered Weighted Averaging operators

In this section, we review the Ordered Weighted Averaging aggregation operator (hereafter, OWA operator), introduced in Yager (1988) and successfully adopted in many fields, including volatility forecasting (Flores-Sosa et al., 2021), multi-criteria and group decision making (Wang and Parkan, 2005; Wang, 2021), multi-attribute decision making (Reimann et al., 2017), forecasting (Yager, 2008), data mining and data smoothing (Torra, 2004), financial decision making (Merigó and Casanovas, 2011). Section 2.1 provides a detailed discussion of the OWA operator and its various features. Moving on to Section 2.2, we explore the weighted OWA (WOWA) operator, which is a generalisation of the OWA operator introduced by Torra (1996, 1997). Finally, in Section 2.3 we have identified a proper method to derive the weights for both the OWA and the WOWA operators.

2.1 The OWA operator

Given **w**, a weighting vector of dimension *N*, Yager (1988) and Yager and Kacprzyk (1997) define a mapping $\mathrm{OWA}_{\mathbf{w}}:\mathbb{R}^N \to \mathbb{R}$ as an Ordered Weighting Averaging (OWA) operator of dimension *N* as follows:

$$\mathrm{OWA}_{\mathbf{w}}(a_1,...,a_N)=\sum\nolimits_{i=1}^{N} w_i a_{\sigma(i)}, \tag{1}$$

where $(\sigma(1),...,\sigma(N))$ is a permutation of $(1,...,N)$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ for all $i=2,...,N$, i.e. $a_{\sigma(i)}$ is the *i*-th largest element in the input vector **a**, and the weights satisfy the properties $w_i \in [0,1]$ and $\sum_i w_i = 1$.

The OWA operator covers aggregation procedures ranging between the minimum and the maximum. An advantage of the OWA operator compared to standard aggregation procedures adopted in financial market applications (mainly the average and weighted average) is the possibility of weighting the values relying on their ordering. In this way, if we order a set of indicators from the highest to the lowest, we can give more importance to a subset of the input values in this ordering than to another subset. According to Xu (2005), the aggregation through an OWA operator can be synthesised according to the following three steps:

1) Ordering the input arguments in descending order;
2) Determine the weight associated with the OWA operator by using a proper method;
3) Use the OWA weights to aggregate the reordered input arguments.

The choice of the OWA weights is of crucial importance and has thus attracted a broad strand of literature, and several methods have been proposed for determining the weights (see, e.g., Xu, 2005 for a detailed discussion). The possible range of the OWA outcome varies from the minimum to the maximum value. For instance, the minimum, arithmetic average, and maximum operations can be generated using the following three weighting vectors:

$$\begin{aligned}
&\mathbf{w} = (0,0,0,\ ...,0,1)^T,\ \mathrm{OWA}_{\mathbf{w}}(a_1, a_2,\ ..., a_n) = \min_j a_j\,, \\
&\mathbf{w} = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n},\ ..., \frac{1}{n}, \frac{1}{n}\right)^T,\ \mathrm{OWA}_{\mathbf{w}}(a_1, a_2,\ ..., a_n) = \frac{1}{n}\sum_n a_j\,, \qquad (2) \\
&\mathbf{w} = (1,0,0,\ ...,0,0)^T,\ \mathrm{OWA}_{\mathbf{w}}(a_1, a_2,\ ..., a_n) = \max_j a_j\,.
\end{aligned}$$

Therefore, the OWA operator is similar to the weighted mean while departing from the latter in the ordering step, thus producing a different interpretation. While in the weighted mean, the weights are attached to the information sources, in the OWA operator, the weights are attached to the data regarding their relative position. In this way, the decision-maker can give more importance to a subset of the input values in this ordering than to another subset (i.e., weights allow the decision-maker to attribute more importance to, e.g., low values, central values, or high values), allowing for a degree of compensation. The degree of compensation in the OWA operator is measured with the degree of orness. The degree of orness is a measure that gauges to what extent the outcome of the aggregation is near to the maximum of the data being aggregated, i.e., it indicates the position of the OWA operator on a continuum between the AND (i.e. min) and OR (i.e. max) operations. The larger the outcome, the larger the degree of orness and the larger the compensation, i.e., the degree of orness measures to what extent the outcome of an operator tends to be similar to the OR. The degree of orness for the OWA operator introduced by Yager (1988) is defined as:

$$\mathrm{orness}(\mathbf{w}) = \frac{1}{n-1}\sum_{i=1}^{N}(n-i)w_i\,. \qquad (3)$$

The OWA operator allows us to model any desired degree of orness between 0 (corresponding to the AND operator) and 1 (corresponding to the OR operator), by means of an appropriate selection of parameters, the so-called OWA weights. Some notable examples are the following:

$$\mathrm{orness}(0,0,0,...,0,1) = 0$$

$$\text{orness}\left(\frac{1}{n},\frac{1}{n},\frac{1}{n},...,\frac{1}{n},\frac{1}{n}\right)=0.5 \tag{4}$$

$$\text{orness}(1,0,0,...,0,0)=1.$$

Symmetrically, the measure of andness can be defined as $\text{andness}(\mathbf{w})=1-\text{orness}(\mathbf{w})$.

The orness represents one of the key advantages of the OWA operator for financial applications since it makes it possible to represent complex scenarios using the degree of optimism and pessimism of the decision maker. Another important quantity associated with the weighting vector is its dispersion, also known as entropy. For a given weighting vector **w**, Yager (1988) defined its measure of dispersion (entropy) as:

$$\text{disp}(\mathbf{w})=-\sum_{i=1}^{N} w_i \log w_i , \tag{5}$$

with the convention $0\times\log 0=0$. The measure of dispersion (entropy) aims to represent the degree to which an aggregation operator considers all inputs. In particular, weights are often required to have a maximum dispersion (given a set of constraints, e.g., an appropriate level of orness). Such maximum dispersion is desirable because it is inappropriate to assign too much weight or importance to a single source of information. Also, a normalised measure of dispersion could be obtained as:

$$\text{ndisp}(\mathbf{w})=-\frac{1}{\log n}\sum_{i=1}^{N} w_i \log w_i . \tag{6}$$

2.2 The weighted OWA (WOWA) operator

Given the different interpretations of the weights between the weighted average and the OWA operator, it is straightforward that they meet very different needs. However, for some applications, it is important to assign a weight to information sources and the compensation degree (or the relative importance of the input values).

For this purpose, Torra (1996, 1997) introduces the WOWA operator as a generalisation of both weighted mean and OWA, which aims at encompassing the advantages of the weighted mean and the OWA in a single operator (Yager et al. 2011). In particular, the WOWA is an aggregation operator that permits the aggregation of a set of numerical data with respect to two weighting vectors: one of the weighting vectors has the interpretation of the ones in the weighted mean (***p***), and the other has the interpretation of the ones in the OWA (***w***) (Torra 2011). Note that both weighting vectors have the same mathematical properties.

Given ***p*** and ***w***, two weighting vectors of dimension *N*, Torra (1996, 1997) define a mapping WOWA: $\mathbb{R}^N \rightarrow \mathbb{R}$ as a Weighted Ordered Weighted Averaging (WOWA) operator of dimension *N* if:

$$WOWA_{p,w}(a_1,...,a_N)=\sum_{i=1}^{N}\omega_i a_{\sigma(i)}, \qquad (7)$$

where as in the case of the OWA, $\{\sigma(1),...,\sigma(N)\}$ is a permutation of $\{1,...,N\}$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ for all $i=\{2,...,N\}$, i.e. $a_{\sigma(i)}$ is the ith largest element in the input vector a. The weight ω_i is defined as:

$$\omega_i = w^*\left(\sum_{j\leq i} p_{\sigma(j)}\right) - w^*\left(\sum_{j<i} p_{\sigma(j)}\right) \qquad (8)$$

where w^* is a nondecreasing function that interpolates the points:

$$\left\{\left(i/N, \sum_{j\leq i} w_j\right)\right\}_{i=1,...,N} \cup \{(0,0)\}.$$

Note that the function w^* is required to be a straight line when the points can be interpolated in this way.

Given that the WOWA operator generalises the weighted mean and the OWA operator, it is possible to achieve the latter as particular cases of WOWA. In particular, when $w=(1/N,...,1/N)$ we have:

$$WOWA_{p,w}(a_1,...,a_N)=WM(a_1,...,a_N).$$

Also, when $p=(1/N,...,1/N)$ we get:

$$WOWA_{p,w}(a_1,...,a_N)=OWA_w(a_1,...,a_N).$$

2.3 The interpolation function w^*

Since the choice of OWA weights is of paramount importance to model any desired degree of orness between 0 (corresponding to a pure and) and 1 (corresponding to a pure or), several methods have been proposed in the literature (see, Xu, 2005, for a literature review). Similarly, the set of WOWA-consistent interpolation methods is relatively large, and several interpolation approaches have been used in the literature (see Torra and Lv, 2009). Consequently, the freedom in selecting the interpolation method might lead to different functions w* when different interpolation methods are exploited. However, not all the methodologies and the functions that have been proposed are able to achieve a very high level of flexibility in weight determination. Consider, for example, the case in which the DM wants to assign a specific weight to the highest value and gradually halve or reduce it to the following elements, thus obtaining an exponential shape for the cumulative weights. Consider also the opposite case, in which the DM wants to start from a sufficiently small weight and then increase it to a specific amount for the subsequent elements. To address this issue, we resort to a flexible weighting function reviewed by Liu and Han (2008) that allows us to generate a multiplicity of weighting vectors by simply changing one parameter. The function is defined as:

$$Q(x)=\begin{cases}\dfrac{a^x-1}{a-1} & \text{if } a>0 \text{ and } a\neq 1\\ x & \text{if } a=1\end{cases}. \tag{9}$$

The function has many desirable properties. First, it monotonically decreases with parameter *a*. Second, its degree of orness can be obtained in terms of *a* as:

$$Q(x)=\begin{cases}\dfrac{a-1-\ln(a)}{(a-1)\ln(a)} & \text{if } a>0 \text{ and } a\neq 1\\ \dfrac{1}{2} & \text{otherwise}\end{cases} \tag{10}$$

with $\lim_{a\to 0} orness(Q)=1$ and $\lim_{x\to +\infty} orness(Q)=0$, thus being a consisted parameterised RIM quantifier with degree of orness ranging between 0 and 1 (Feng and Dillon, 2003). Third, it can also be verified that *Q(x)* is the optimal solution in the sense of maximum entropy (Liu, 2008).

For the benefit of the reader, the membership function of Q(x) for different levels of *a* is depicted in Figure 1. For low values of *a*, more weight is attributed to larger input values (we recall that in both the OWA and the WOWA operators, inputs are sorted in decreasing order). On the other hand, as the value of *a* increases and departs from the threshold level of 1, more weight is attributed to smaller input values.

Further details on how the function will be applied to operators to obtain the aggregate sentiment index are given in Section 3

3. Data and methodology

This study is intended to assess the importance of aggregating individual stock data of the Bloomberg sentiment index to explain the behaviour of current and future stock market returns in the European market. In Section 3.1, we describe the dataset adopted to estimate sentiment at the individual stock level, while in Section 3.2, we will detail how sentiment estimates are aggregated in a unique sentiment index for the European market by exploiting the aggregation methods outlined in Section 2.

3.1 Data preparation

Daily stock market capitalisation in the Eurozone and the Bloomberg sentiment index for each stock in the Euro-denominated area are obtained from Bloomberg for the period between January 2007 and March 2021. The Bloomberg sentiment index differs from the Thompson Reuters News Analytics index (TRNA) as it aggregates all the news published daily for a company (González-Sánchez and Morales de Vega, 2021). The initial input is a matrix of around 3500 daily observations for 3474 stocks. The dataset includes at least one observation of the Bloomberg sentiment index for 2409 stocks: these stocks will compose the filtered dataset.

The number of companies for which the sentiment indicator is available is relatively low at the beginning of the sample but grows fast from 2010 and then remains stable, with on average between 100 and 200 stocks covered per day. To avoid excessive volatility of the final sentiment index, we compute, for each stock, its monthly sentiment by averaging the daily values of the Bloomberg sentiment index over the month[1]. Consequently, we obtained 170 monthly sentiment estimates for each stock in our filtered dataset.

Finally, we obtained the monthly returns of the Eurostoxx 50 index from Bloomberg, which is considered the primary benchmark for Eurozone stocks (see Gambarelli et al., 2023), to determine the overall stock market return.

3.2 The aggregation procedure

Starting from the prepared dataset consisting of monthly estimates of the sentiment index for a high number of stocks, we aim to aggregate the information from all the stocks in a unique sentiment index. Each month, we select stocks with available sentiment data and collect their market capitalisation. Then, given the possibility of exploiting the OWA and the WOWA operators to model investors' and regulators' risk preferences through their degree of orness, we apply both operators to estimate the aggregate sentiment index by taking the following steps.

First, in order to consider different levels of risk preference for the OWA and the WOWA operators, we adopt the same interpolation function described in Section 2.3, and we derive the values for the parameter a corresponding to the following degrees of orness: 0.2, 0.4, 0.5, 0.6, 0.8. Second, since formulas presented in Section 2.1 do not consider the use of an interpolation function, we perform the OWA aggregation by exploiting equation (8) with $p = (1/N,...,1/N)$. Moreover, when the degree of orness is set equal to 0.5, we obtain the simple average as a particular case of the OWA aggregation. Third, to understand whether the aggregate sentiment index obtained using the WOWA operator, which combines the advantages of the weighted mean and OWA operator, provides more insightful information about future market returns, we adopt the same interpolation function and the same level of a selected for the OWA aggregation. Unlike the OWA operator, the weight vector p in equation (8) is filled using the relative market capitalisation of the stocks whose sentiment is used as input of the OWA. Similarly to the OWA operator, when the degree of orness is set equal to 0.5, we obtain the weighted average of sentiment as a particular case of the WOWA aggregation. In this way,

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[1] In line with previous studies (e.g. Elyasiani et al., 2020), we consider 21 trading days as a monthly period.

we are able to compare and contrast the two aggregation operators with the commonly used operators in financial applications.

The procedure described above is performed each month for the entire sample period, thus obtaining a total of 166 observations for ten aggregate indices (five based on OWA and five based on WOWA). The descriptive statistics of the aggregate indices are reported in Table 1. Several considerations are in order. First, only the aggregate indices obtained with a degree of orness set equal to 0.2 (close to the minimum) are, on average, negative during the sample period, regardless of the operator used for the aggregation (OWA or WOWA). Second, both the average and the weighted average aggregate indices (owa_{05} and $wowa_{05}$, respectively) are on average positive, indicating that sentiment scores at the individual stock level are more frequently positive than negative, or positive scores are larger in magnitude compared to negative ones. Third, aggregate indices based on WOWA are, on average, lower in absolute value than the corresponding indices (i.e. the indices with the same parameter a) obtained using the OWA operator. Based on the results, it appears that stocks with extremely positive or negative sentiment scores tend to have lower market capitalisation. In fact, indices based on WOWA attribute more weight to stocks with large market capitalisation, since they are expected to affect the stock market to a larger extent compared to small capitalisation stocks.

Similar conclusions can be drawn from Figure 2, which represents the evolution over time of the aggregate sentiment indices obtained using OWA (upper panel) and WOWA (lower panel). Moreover, from the figure, it is possible to note another interesting pattern. The difference between indices with a high and low degree of orness is quite stable until 2016 and then increases, indicating a greater dispersion in the individual stock scores. Once again, the aggregate indices obtained using the WOWA operator show a lower dispersion. One possible explanation is that the most extreme scores (both positive and negative) are attributed to small- and mid-capitalisation companies.

4. Empirical results: the relationship between aggregate sentiment and market returns

In this section, we focus on the relationship between sentiment on the one hand and market volatility and returns on the other, which are still highly debated in the literature. There is a growing number of studies exploring the impact of investor sentiment on realised stock market volatility (see e.g., Lang et al. 2023; Xie et al. 2023), mainly focused on the Chinese stock market. However, there is no consensus in the literature about the relationship between sentiment indicators and both contemporaneous and future stock prices. Some studies find evidence of a relationship between investor sentiment and market prices (see, e.g., Uhl, 2014), whereas others (see, e.g., González-Sánchez and Morales de Vega, 2021) conclude that there is no significant relationship between investor sentiment and stock returns. In addition, there are studies showing that the explanatory power is in the opposite direction (Das et al., 2005), in the sense that returns and volatility variations affect sentiment rather than the other way around. In Section 4.1, we investigate the relationship between sentiment indices on the one hand and contemporaneous and future market returns on the other. In Section 4.2, we shed light on the information content of extreme values of the sentiment indices.

4.1 The level of sentiment and volatility

To investigate the relationship between aggregate sentiment indices and realised volatility for different horizons, we take the following steps. First, following Muzzioli et al. (2018), we compute realised volatility from daily Eurostoxx 50 log-returns using rolling windows of 30, 60, and 90 calendar days, as an estimate of future realised volatility. Moreover, we estimate contemporaneous market volatility using daily Eurostoxx 50 log-returns over the same period covered by the sentiment indices measurement (described in section 3.2). Second, for each of the ten obtained sentiment indices, we estimate the following regression model:

$$Vol_{t,t+n} = \alpha + \beta_1 index_t + \varepsilon_t \quad , \tag{11}$$

where $Vol_{t,t+n}$ is proxied by the contemporaneous realised volatility and by the 30-, 60-, and 90-day future realised volatility.

The results, reported in Table 3, show a strong relationship between the majority of the sentiment index and both contemporaneous and future realised volatility. The negative sign of the relationship indicates that as investor sentiment rises (declines), stock market volatility decreases (increases), indicating a safer (riskier) market environment. The relationship over the short term (contemporaneous and one-month returns) is stronger when the weighted OWA operator is used, particularly for levels of the degree of orness between 0.6 and 0.8. Therefore, an OR-like aggregation (degree of orness higher than 0.5) seems more appropriate to track the behaviour of contemporaneous and future market volatility behaviour. On the other hand, for explaining the two- and three-month realised volatility, the simple average operator (owa05) is well-suited.

4.2 The level of sentiment and market returns

To establish whether the index values are associated with positive or negative contemporaneous and future market returns, thus being able to capture prevailing fear or greed in the market, we estimate the following regression model for each of the ten aggregate indices (five based on OWA, five based on WOWA) under investigation:

$$R_{t,t+n} = \alpha + \beta_1 index_t + \varepsilon_t \quad , \tag{12}$$

where $index_t$ is alternatively proxied by levels of the sentiment indices and R_t is proxied alternatively by contemporaneous and future market returns over the next one-, two-, and three-month horizon. The results, reported in Table 4, do not reveal any significant relationship between aggregate sentiment indices and contemporaneous stock market returns, indicating that the aggregate sentiment indices are not well suited to measure fear and greed in the market. Similarly, that none of the indices is able to explain the future returns of the European stock market over the next 30, 60, and 90 days. One possible explanation is that the eventual predictive power of aggregate sentiment on returns is not constant along the variable distributions. Since aggregate sentiment indices exhibit several peaks and dips during the sample period, it will be interesting to assess whether such extreme movements may be early signs of positive or negative returns. Both hypotheses are investigated in the following section.

4.3 The information content of extreme values of the asymmetry indices

An alternative perspective for investigating the relationship between sentiment indices and future market returns is proposed by Rubbaniy et al. (2014), who estimate the relation between different levels of implied volatility indices (index values higher than 90%, 95%, and 99% percentiles or lower than the 1%, 5%, and 10% percentiles) and the corresponding future index returns. The underlying rationale is that high or very high implied volatility levels may indicate an oversold market and, as a result, possible positive future returns for long positions in the underlying market (Giot, 2005). Conversely, very high levels for the sentiment indices may indicate an overly complacent market, and hence a point of possible reversal for the market trend.

The procedure proposed by Rubbaniy et al. (2014) that analyses the relationship between indices and returns in the sub-samples at extreme levels suffers from a critical drawback. Specifically, when the model is estimated on the sub-samples that consider only extreme index values, the number of observations in each sample could be very low. As a result, the regression model's assumptions may not be satisfied. To avoid this issue, following Gambarelli and Muzzioli (2019), we adopt an alternative test based on a trading strategy that takes a long position in the underlying asset (the Eurostoxx 50 index) when the sentiment index level is lower than its 10% percentile or is higher than its 90% percentile. For each of the ten sentiment indices, we investigate the profitability of the strategy for both short (5-, and 10-day, corresponding to one and two trading weeks, respectively) and medium-term (21-, 42-, and 63-day, corresponding to one, two, and three months) holding periods and report the results in Table 4 (Table 5) for index levels lower than its 10% (90%) percentiles.

The results reported in Table 4 (indices level lower than their 10% percentile) reveal that when aggregate sentiment indices based on OWA are very low, there is no clear pattern about future market returns. An exception is represented by the owa_{02}, owa_{04}, and owa_{05} indices. In particular, when owa_{02} is very low, market return is positive and marginally significant over the next month. On the other hand, when owa_{04}, and owa_{05} attain very low values, positive and marginally significant returns are detected over the following three months. The relationship between low values of the sentiment indices and future market returns is stronger when we consider aggregate indices based on the WOWA operator. In particular, when $wowa_{05}$ and $wowa_{06}$ are lower than their 10% percentiles, future market returns over the next 1-,2-, and 3-month horizons are positive and statistically significant at the 1% level. Two main observations arise from these results. First, aggregate sentiment indices act as a contrarian predictor of future market returns over the medium term. Second, using stock market capitalisation as an input of the WOWA operator improves the usefulness of the aggregate sentiment indices in explaining future market returns since stocks with high market capitalisation are expected to affect the stock market to a larger extent compared to small capitalisation ones.

The main findings obtained in Table 4 are confirmed in Table 5, where we investigate the information content of very high values of the sentiment indices on future market returns. More specifically, aggregate indices based on WOWA outperform those based on OWA in explaining future market returns when attaining very high levels. Moreover, the sign of future market returns is negative and statistically significant over the next 2-, and 3-month horizons when $wowa_{02}$ and $wowa_{04}$ are higher than their 90% percentile. The negative sign of future returns indicates the role of aggregate sentiment indices as a contrarian predictor of future market returns: when the sentiment is at the highest levels, the indices may signal an overly complacent market and hence a point of possible reversal for the market trend. It is worth noting that also the aggregate index, based on the weighted average operator ($wowa_{05}$) when reaching very high values, points to a negative return over the next three months.

To sum up, our analysis suggests that aggregate indices of sentiment are contrarian predictors of future market returns when reaching very low or very high values. In contrast, the relationship between aggregate sentiment and returns is not straightforward for the whole distribution of the variables. Moreover, accounting for stock market capitalisation as an input of the WOWA operator is important in enhancing the predictive ability of such indices on market returns. Finally, a degree of orness between 0.2 and 0.4 is optimal for increasing the predictive power of the aggregate index on future market reversal when it reaches very high values. On the other hand, to detect an upward turn in the stock market trend over the medium term, an aggregate sentiment index based on a weighted average appears appropriate.

5. Conclusion

Investors sentiment indicators are of paramount importance in determining investor confidence, market expectations, and risk appetite, which are essential in making informed decisions. Empirically, market sentiment has been able to explain financial market movements, in particular in periods of irrational panic or unjustified optimism (Reis and Pinho, 2020a). Unfortunately, most existing sentiment measures are geared towards the US market (such as the S&P 500 Twitter Sentiment Index), leaving European investors with limited options for accurately assessing sentiment.

In this paper, we develop several proposals for constructing an aggregate sentiment index for the European stock market based on the OWA and the weighted OWA (WOWA) operators. The choice of the operator is motivated by its successful applications in finance and its strong financial interpretation. We develop ten different aggregate sentiment indices related to different values of orness, and we evaluate their information content about contemporaneous and future market returns.

Several results are obtained. First, when we evaluate the explanatory power of aggregate sentiment indices on contemporaneous and future market volatility, we find evidence of a strong and negative relationship between the variables. The aggregate sentiment indices act as contrarian predictors of contemporaneous and future market volatility since increases (decreases) in sentiment are associated to decreases (increases) in market volatility. Second, when investigating the association between aggregate sentiment indices on contemporaneous and future market returns, we fail to find empirical evidence supporting the relationship along the entire distribution of variables. Third, we find interesting results when we evaluate the information content of very low and very high values of the aggregate sentiment indices. The aggregate sentiment indices act as contrarian predictors of future market returns since low (high) values of the indices signal positive (negative) returns over the medium term. This result suggests that a very low value in an aggregate sentiment index may signal an oversold market and, as a result, possible positive future returns for long positions in the underlying market. Conversely, very high levels for the sentiment index may indicate an overly complacent market and hence a point of possible reversal for the market trend. Fourth, we find that accounting for the stock market capitalisation as an input of the WOWA operators enhances the explanatory power on contemporaneous volatility and future market returns compared to the OWA operator. More specifically, stocks with large market capitalisation are expected to affect the stock market to a larger extent with respect to small capitalisation stocks. Fifth, a degree of orness between 0.6 and 0.8 seems more appropriate for explaining contemporaneous and one-month ahead market realised volatility. On the other hand, a degree of orness between 0.2 and 0.4 seems optimal for increasing the predictive power of the aggregate index on future market reversal when it reaches very high values, indicating that an AND-like aggregate index is preferable to an OR-like index. On the other hand, in order to detect an upward reversal in the stock market trend over the medium term when the sentiment index is very low, the weighted average is found to be appropriate. Finally, the simple average is more appropriate to explain future realised volatility for forecasting horizons longer than one month.

The results of the present study are important for investors and policymakers, who can exploit the benefit of combining the advantages of the weighted mean and of the OWA operator in a single aggregate index, aiming to detect the condition of oversold or overly complacent stock markets. Since different indexes seem to be more suitable for different purposes (volatility or return forecasting), one possible solution for investors and regulators is to combine non-compensatory measures with compensatory ones. As suggested by Gibari et al. (2021), decision-makers can benefit from combining a measure of the overall performance (compensatory indicator) with alert signs signalling bad performances in certain indices that might remain unnoticed otherwise (noncompensatory indicators).

The present study represents only a small step in discovering how sentiment information at the individual firm level can be aggregated. Future research can further explore whether an AND-like aggregate sentiment index is preferable to an OR-like sentiment index in different market conditions and for different types of stocks (e.g., small capitalisation versus big capitalisation). In addition, it would be interesting to study whether aggregating sentiment at the sector or geographic level can provide additional information about future market returns.

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Table 1. Descriptive statistics for the sentiment aggregate indices and the market returns.

Index	*a*	Avg.	Median	Min	Max	Std. dev.	Skew	Kurt
owa_{02}	0.008	-0.137	-0.125	-0.378	0.008	0.084	-0.939	0.400
owa_{04}	0.292	0.055	0.063	-0.141	0.169	0.050	-0.593	0.727
owa_{05}	1.000	0.232	0.224	0.074	0.412	0.072	0.288	-0.268
owa_{06}	3.421	0.416	0.378	0.225	0.684	0.115	0.565	-0.820
owa_{08}	121.632	0.144	0.142	-0.014	0.281	0.055	-0.059	0.201
$wowa_{02}$	0.008	-0.076	-0.062	-0.285	0.411	0.084	0.605	6.126
$wowa_{04}$	0.292	0.053	0.060	-0.078	0.412	0.062	0.898	5.964
$wowa_{05}$	1.000	0.176	0.181	0.057	0.413	0.052	0.184	2.118
$wowa_{06}$	3.421	0.314	0.310	0.146	0.482	0.067	-0.049	0.147
$wowa_{08}$	121.632	0.114	0.123	-0.003	0.413	0.055	0.705	4.457

Note: The table reports the descriptive statistics for the monthly aggregate sentiment measures obtained using the OWA ($owa_{02,}$ $owa_{04,}$ $owa_{05,}$ $owa_{06,}$ owa_{08}) and the WOWA operators ($wowa_{02,}$ $wowa_{04,}$ $wowa_{05,}$ $wowa_{06,}$ $wowa_{08}$) for different degrees of orness (0.2, 0.4, 0.5, 0.6, and 0.8). The parameter *a* represents the input used to plug in equation (9) in order to obtain the corresponding degree of orness.

Table 2. Regression output for the relationship between sentiment indices and realised volatility

	index	ret_t	$ret_{t,\ t+1}$	$ret_{t,\ t+2}$	$ret_{t,\ t+3}$
Coef.		-0.137	-0.001	-0.030	-0.030
t-stat	owa_{02}	(-0.775)	(-0.004)	(-0.180)	(-0.172)
R^2		0.74%	0.00%	0.04%	0.04%
Coef.		-1.016***	-0.720**	-0.729***	-0.725***
t-stat	owa_{04}	(-3.094)	(-2.348)	(-2.641)	(-2.855)
R^2		14.55%	7.25%	8.44%	9.21%
Coef.		-1.064***	-0.844***	-0.824***	-0.814***
t-stat	owa_{05}	(-3.685)	(-2.776)	(-2.978)	(-3.207)
R^2		19.41%	12.11%	13.11%	14.10%
Coef.		-0.756***	-0.637**	-0.610**	-0.598***
t-stat	owa_{06}	(-3.367)	(-2.507)	(-2.518)	(-2.605)
R^2		16.51%	11.62%	12.11%	12.86%
Coef.		-0.371**	-0.333*	-0.313*	-0.304*
t-stat	owa_{08}	(-2.415)	(-1.928)	(-1.831)	(-1.841)
R^2		10.28%	8.20%	8.27%	8.59%
Coef.		-0.310*	-0.226**	-0.218**	-0.200**
t-stat	$wowa_{02}$	(-1.698)	(-1.360)	(-1.339)	(-1.251)
R^2		3.80%	2.01%	2.12%	1.97%
Coef.		-0.665**	-0.507**	-0.459**	-0.427**
t-stat	$wowa_{04}$	(-2.434)	(-2.112)	(-2.102)	(-2.050)
R^2		9.43%	5.44%	5.07%	4.83%
Coef.		-0.928***	-0.719***	-0.639***	-0.596***
t-stat	$wowa_{05}$	(-2.893)	(-2.586)	(-2.623)	(-2.637)
R^2		14.45%	8.59%	7.72%	7.40%
Coef.		-1.146***	-0.901***	-0.792***	-0.738***
t-stat	$wowa_{06}$	(-3.373)	(-3.041)	(-3.059)	(-3.155)
R^2		19.77%	12.12%	10.64%	10.21%
Coef.		-0.882***	-0.724***	-0.632***	-0.586***
t-stat	$wowa_{08}$	(-3.601)	(-2.957)	(-2.660)	(-2.592)
R^2		19.40%	12.95%	11.21%	10.62%

Note: The table presents the estimated output of the following regressions:

$$Vol_{t,t+n} = \alpha + \beta_1 index_t + \varepsilon_t ,$$

where *index* is proxied alternatively by monthly levels of the sentiment indices, and $Vol_{t,t+n}$ is proxied alternatively by the contemporaneus realised volatility and by the 30-, 60-, and 90-day future realised volatility. All the regressions are run by using Ordinary Least Squares (OLS), with the Newey-West (1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 3. Regression output for the relationship between sentiment indices and market returns

	index	ret_t	$ret_{t,\,t+1}$	$ret_{t,\,t+2}$	$ret_{t,\,t+3}$
Coef.		-0.027	-0.054	-0.080	-0.134
t-stat	owa_{02}	(-0.566)	(-1.264)	(-0.925)	(-1.058)
R^2		0.12%	0.50%	0.57%	1.23%
Coef.		-0.037	-0.085	-0.053	-0.131
t-stat	owa_{04}	(-0.309)	(-0.992)	(-0.382)	(-0.705)
R^2		0.08%	0.44%	0.09%	0.42%
Coef.		-0.013	-0.048	0.011	-0.030
t-stat	owa_{05}	(-0.117)	(-0.485)	(0.073)	(-0.156)
R^2		0.01%	0.17%	0.00%	0.03%
Coef.		0.002	-0.016	0.036	0.024
t-stat	owa_{06}	(0.022)	(-0.202)	(0.289)	(0.144)
R^2		0.00%	0.03%	0.08%	0.03%
Coef.		0.007	0.003	0.037	0.039
t-stat	owa_{08}	(0.156)	(0.056)	(0.459)	(0.369)
R^2		0.02%	0.00%	0.23%	0.19%
Coef.		0.025	-0.045	-0.085	-0.092
t-stat	$wowa_{02}$	(0.462)	(-1.069)	(-1.293)	(-0.905)
R^2		0.11%	0.35%	0.64%	0.59%
Coef.		0.062	-0.075	-0.122	-0.133
t-stat	$wowa_{04}$	(0.757)	(-1.188)	(-1.288)	(-0.899)
R^2		0.34%	0.53%	0.72%	0.66%
Coef.		0.081	-0.092	-0.136	-0.150
t-stat	$wowa_{05}$	(0.825)	(-1.157)	(-1.141)	(-0.830)
R^2		0.46%	0.61%	0.70%	0.65%
Coef.		0.090	-0.095	-0.127	-0.144
t-stat	$wowa_{06}$	(0.825)	(-0.997)	(-0.866)	(-0.686)
R^2		0.51%	0.59%	0.55%	0.54%
Coef.		0.050	-0.035	-0.032	-0.049
t-stat	$wowa_{08}$	(0.638)	(-0.414)	(-0.229)	(-0.267)
R^2		0.26%	0.14%	0.06%	0.10%

Note: The table presents the estimated output of the following regression model:

$$R_{t,t+n} = \alpha + \beta_1 index_t + \varepsilon_t ,$$

where *index* is proxied alternatively by monthly levels of the sentiment indices, and $R_{t,t+n}$ is proxied alternatively by contemporaneous and future market returns (Eurostoxx 50 return continuously compounded) over the next one-, two-, and three-month horizon. All the regressions are run by using Ordinary Least Squares (OLS), with the Newey-West (1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 5. Average returns for a long position in the Eurostoxx50 index when the aggregate sentiment index is higher than its 90% percentile.

	$R_{t,\ t+5}$	$R_{t,\ t+10}$	$R_{t,\ t+21}$	$R_{t,\ t+42}$	$R_{t,\ t+63}$
owa_{02}	-0.006 (-2.009)	-0.004 (-0.873)	0.000 (-0.056)	-0.001 (-0.098)	0.005 (0.303)
owa_{04}	0.001 (0.329)	0.003 (0.592)	0.005 (0.647)	0.011 (0.909)	0.009 (0.601)
owa_{05}	0.000 (0.112)	0.002 (0.574)	0.007 (1.056)	0.015 (1.380)	0.014 (1.327)
owa_{06}	-0.007 (-1.501)	-0.008 (-1.041)	-0.019 (-0.682)	0.001 (0.031)	0.001 (0.035)
owa_{08}	0.006 (0.576)	0.005 (0.487)	-0.003 (-0.129)	0.014 (0.690)	-0.004 (-0.114)
$wowa_{02}$	-0.005 (-0.832)	-0.008 (-1.173)	-0.014 (-1.285)	-0.036** (-2.567)	-0.038** (-2.264)
$wowa_{04}$	-0.006 (-1.210)	-0.003 (-0.345)	-0.005 (-0.551)	-0.026** (-2.135)	-0.032** (-2.085)
$wowa_{05}$	-0.007 (-1.266)	-0.004 (-0.478)	-0.009 (-0.677)	-0.027 (-1.398)	-0.035** (-2.389)
$wowa_{06}$	-0.002 (-0.533)	0.001 (0.147)	0.002 (0.161)	0.000 (-0.001)	0.001 (0.060)
$wowa_{08}$	-0.007 (-1.329)	-0.002 (-0.253)	-0.012 (-0.431)	-0.028 (-0.656)	-0.023 (-0.730)

Note: All the series are tested for significance by using the Newey-West (1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Figure 1. The interpolation functions of *Q(x)* for different levels of *a*.

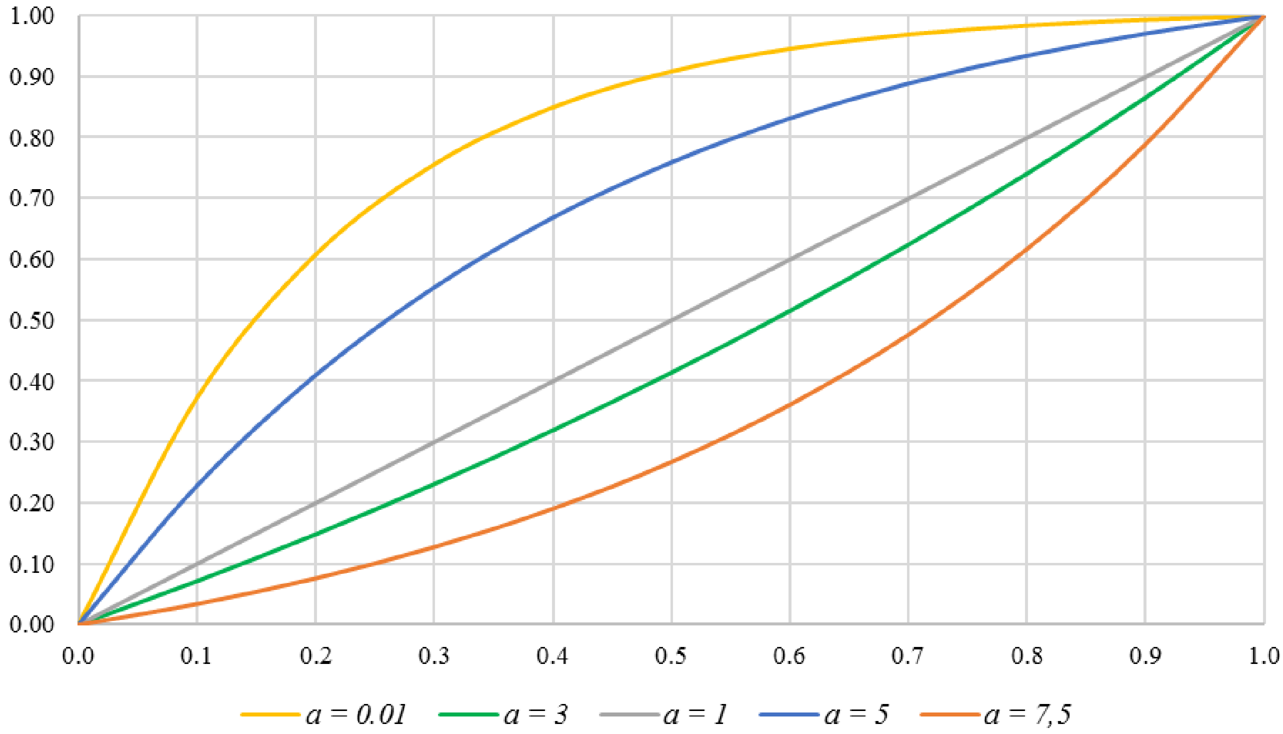
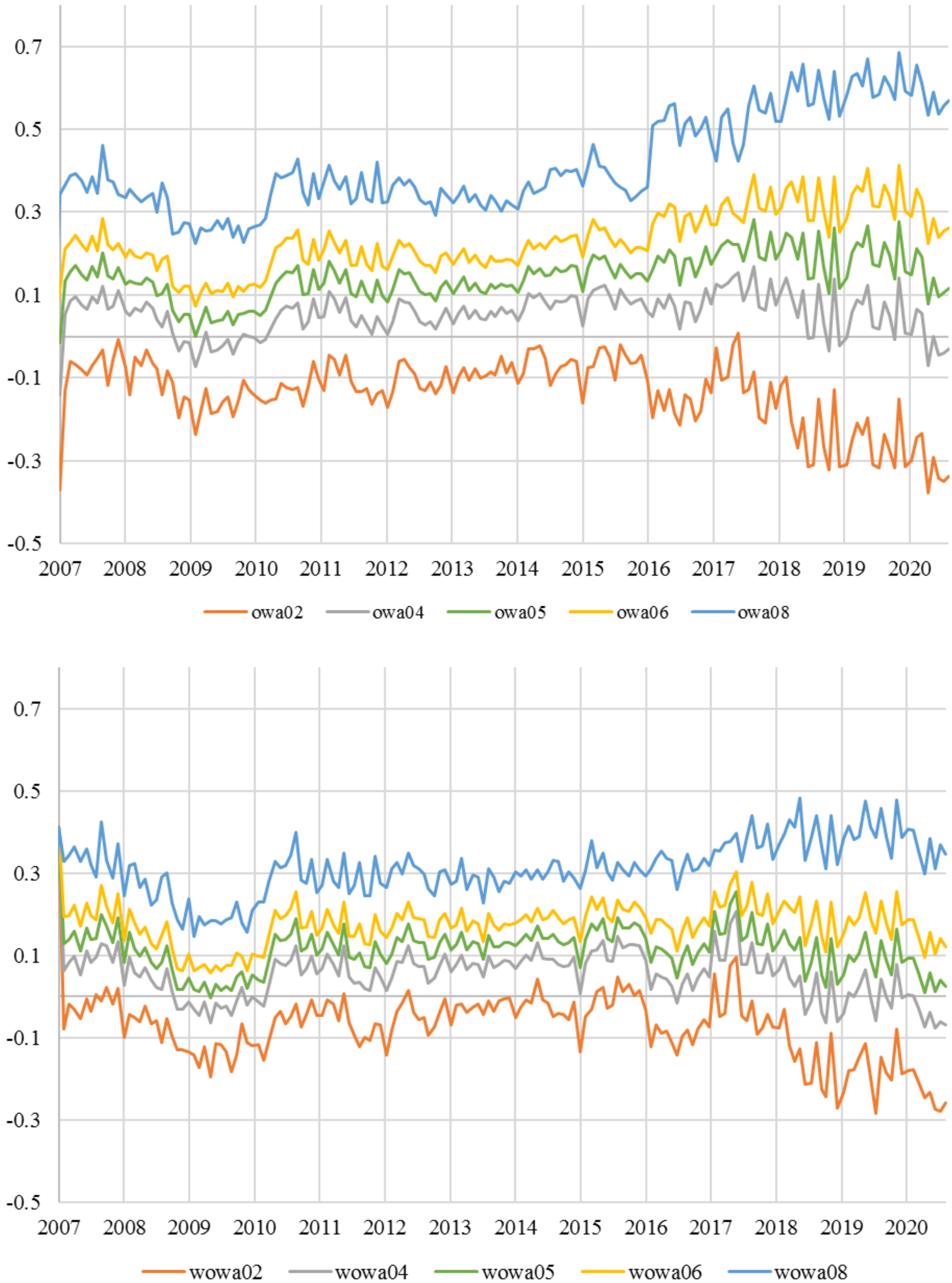


Figure 2. The aggregate indices obtained through the OWA and the WOWA operators.



Note: The figure depicts the evolution over the sample period (2007-2021) of the sentiment indices obtained using the OWA (upper panel) and the WOWA (lower panel) operators for different degrees of orness.