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Fundamentalness, Granger Causality and Aggregation

A comment on Canova and Sahneh, JEEA, 2018

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Abstract

The testing procedure suggested in Canova and Sahneh (2018) is essentially the same as the one proposed in Forni and Gambetti (2014), the only one difference being the use of Geweke, Meese and Dent (1983) version of Sims (1972) test in place of a standard Granger causality test. The two procedures produce similar results, both for small and large samples, and perform remarkably well in detecting non-fundamentalness. Neither methods have anything to do with the problem of aggregation. A "structural aggregate model" does not exist.

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1 Introduction

Starting with the seminal contributions of Lippi and Reichlin (1993, 1994a, 1994b) non-fundamentalness, or non-invertibility, of structural MA representations of macroeconomic variables has been widely discussed in both the business cycle and time series literature. A partial list of recent contributions includes Giannone, Reichlin and Sala (2006), Giannone and Reichlin (2006), Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007), Ravenna (2007), Yang (2008), Forni, Giannone, Lippi and Reichlin (2009), Sims (2012), Leeper, Walker and Yang (2013), Giacomini (2013), Forni, Gambetti and Sala (2014, 2017), Forni and Gambetti (2014), Beaudry, Fève, Guay and Portier (2015), Chen, Choi and Escanciano (2015), Forni, Gambetti, Lippi and Sala (2017a, 2017b), Canova and Sahneh (2018).

The problem can be shortly summarized as follows. Structural VAR methods are aimed at estimating the impulse-response functions and the shocks of a structural, Moving Average representation of the macro economy. If such representation is non-fundamental, then the variables in the VAR do not convey enough information to recover the structural shocks and the related impulse-response functions, so that the empirical results obtained from the VAR may be misleading. Non-fundamentalness is likely to occur with small VARs, particularly in presence of news shocks, announcement effects of policy and fiscal foresight, and is an intrinsic feature of models with noise shocks.

Fundamentalness has long been assumed without testing in applied macroeconomic work, since fundamentalness tests were not available. Recently, a few papers have suggested methods to test for fundamentalness (Forni and Gambetti, 2014, Chen, Choi and Escanciano, 2017, Canova and Sahneh, 2018).

Forni and Gambetti (2014, FG henceforth) show that, if the macroeconomic variables follow a factor structure, then, under appropriate conditions, the structural representation of the VAR variables is non-fundamental if and only if the factors Granger-cause the VAR variables. The proposed testing procedure (also called FG from now on) is then to take the principal components of a large data set of macroeconomic time series (which are consistent estimators of the factors) and test whether such principal components Granger-cause the VAR variables.

Canova and Sahneh (2018, CS henceforth) claim that FG reject fundamentalness too often, particularly when there are problems related to aggregation of sectoral shocks or omitted variables. Results from Monte Carlo exercises in CS show that the size distortion may be very large. As a consequence, by applying the FG method, small VAR specifications might be rejected even if they are not informationally deficient. The authors then suggest a procedure (also called CS from now on), which, according to the reported Monte Carlo results, is not over-sized (indeed, it is under-sized).

In this paper, we argue that CS is asymptotically equivalent to FG, the only one difference being the use of a version of Sims' (1972) test proposed by Geweke, Meese and Dent (1983) in place of a standard Granger causality test. It is therefore puzzling that the two methods in CS behave so much differently even for large samples.

We then do by ourselves the Monte Carlo exercises of CS and find different results. According to our results, FG and CS perform similarly (and fairly well) for both Data Generating Processes considered in CS.

CS studies a DGP where there are three shocks: a technology shock and two sectoral tax shocks. The econometrician however observes just two variables, i.e. capital and an aggregate tax variable. The structural representation of the two variables, let us say the "disaggregate" representation, is non-fundamental, since two variables cannot provide enough information to recover three shocks. In fact, both testing procedures reject fundamentalness with similar power.

The two variables have also representations with just two shocks, let us say "aggregate" representations. Now, CS interprets the results of the two competing testing procedures as if the null hypothesis were fundamentalness of what they call "aggregate structural representation". Here, we argue that this interpretation is untenable, for two reasons.

First, structural moving average models do not aggregate, i.e. in general we do not have any aggregate representation which can be regarded as "structural" (Forni and Lippi, 1997, 1999). The case studied in CS is not an exception: we have both fundamental and non-fundamental aggregate representations, but all of them are merely statistical representations, devoid of any economic interpretation.

Second, when the Data Generating Process is the disaggregate one, all aggregate representations are observationally equivalent, so that there is no way to discriminate between them by using the data.

2 Fundamentalness and Granger causality

Let us consider the impulse-response function representation

$$x_t = A(L)\zeta_t, \tag{1}$$

where x_t is an n-dimensional vector of macro variables, $A(L)$ is an $n \times q$ matrix of rational IRFs and ζ_t is a q-dimensional vector of structural shocks, which we assume serially uncorrelated and mutually orthogonal at all leads and lags.

This representation is fundamental if ζ_t lies in the information space spanned by the present and past of x_t. In this case, the variables deliver exactly the same information as the shocks. By contrast, the representation is non-fundamental if the history of the x's until time t does not provide enough information to recover ζ_t. In this case, the variables contain less information than the shocks.

This simple and general definition of fundamentalness is valid for both "square" systems, i.e. representations with as many shocks as variables, and rectangular systems. "Short" systems, i.e. models with $n < q$, are always non-fundamental, since n variables cannot provide the same information as $q > n$ shocks.[1] For square systems, fundamentalness is characterized by a well-known condition on the roots of the determinant of the IRF matrix: all such roots must be in modulus greater than 1, or equal to 1.[2]

Fundamentalness is important for structural VAR analysis, since if we can find the structural shocks by using present and past values of the observables, standard structural VAR techniques are valid. By contrast, if representation (1) is non-fundamental, a VAR specification including only the x's is informationally deficient and VAR analysis might fail.

Fundamentalness is related to Granger causality (Forni and Reichlin, 1996, Giannone and Reichlin, 2006, FG, 2014). FG considers a vector of variables $y_t = (y_{1,t} \ \cdots \ y_{N,t})'$, not included in the VAR, which are also driven by the structural shocks in ζ_t and, possibly, the idiosyncratic measurement errors $\xi_t = (\xi_{1,t} \ \cdots \ \xi_{N,t})'$:

$$y_t = B(L)\zeta_t + \xi_t. \tag{2}$$

Clearly, by using the variables in y_t we can get independent information on ζ_t. Such information can be efficiently extracted by taking the principal components of the y's;

[1] By contrast, "tall" systems are fundamental, except for very special cases (Forni et al., 2009).

[2] Invertibility requires that all roots are greater than 1. Hence invertibility implies fundamentalness.

let us call such principal components $f_t = (f_{1,t} \ \cdots \ f_{s,t})'$. Indeed, the above model is a factor model; under suitable conditions, the principal components are consistent estimators of the factors (Stock and Watson, 2002) and the factors deliver the same information as the shocks (Forni, Giannone, Lippi and Reichlin, 2009).

Since f_t provides information on the shocks, it can in principle help predicting x_t. But of course this may happen only in the non-fundamentalness case, since in the fundamentalness case the x's already contain all the information provided by the shocks. Hence, if f_t Granger-causes x_t, the structural representation (1) is non-fundamental.

The converse implication does not hold in general: it may be the case that the shocks are more informative than the x's, but the additional information provided by the principal components does not help predicting the variables. FG shows that this special case cannot occur when the system is square, i.e. when we have as many variables as shocks (see FG, Proposition 3). For square systems, absence of Granger causation implies fundamentalness, so that fundamentalness and Granger causation are equivalent.

3 The competing testing procedures

Based on the above analysis, FG proposes the following testing procedure.

FG.1. Compute the first s ordinary principal components of the auxiliary data set, $f_t = (f_{1,t} \ \ldots \ f_{s,t})'$.

FG.2. Run the regression[3]

$$x_t = \mu + \sum_{j=1}^{p_1} \alpha_j x_{t-j} + \sum_{j=1}^{p_2} \beta_j f_{t-j} + v_t$$

and test if the coefficients β_j are equal to zero using an F-test.[4]

CS argues that FG does not have good properties, particularly for models like the aggregation model below, and suggests to use instead the following method.

CS.1. Procede as in FG.1.

[3]Indeed, FG use the out-of-sample Granger causality test proposed by Gelper and Croux, 2007. This detail however is not of special interest for the question we focus on in the present note, so that we prefer to develop our arguments by using a standard Granger causality test.

[4]The asymptotic distribution of the quadratic form λ_W of the Wald test is χ^2 with degrees of freedom equal to the number of restrictions, N. In order to have a better small sample distribution, we use below an F-test, where the statistic $\lambda_F = \lambda_W / N$ is approximately distributed as $F(N, \#\text{obs} - \#\text{parameters})$, see Lütkepohl, 2005.

CS.2. Run a VAR on

$$x_t = \lambda + \sum_{j=1}^{r} \rho_j x_{t-j} + u_t$$

and take the residual u_t.

CS.3. Run the regression:

$$f_t = \nu + \sum_{j=1}^{p_3} \varphi_j f_{t-j} + \sum_{j=0}^{p_4} \eta_j u_{t-j} + \sum_{j=1}^{p_5} \psi_j u_{t+j} + e_t \tag{3}$$

and test if the coefficients ψ_j are equal to zero using an F-test.

Step CS.3 is presented in CS as something different from a Granger causality test. However, regression (3) is essentially equivalent to a version of Sims (1972) test proposed in Geweke, Meese and Dent (1983). The only one difference is that we have u_t in place of x_t, i.e. the variables are pre-whitened before testing. This is not a major difference, since of course f_t Granger-causes u_t if and only if it Granger-causes x_t.

Hence the test proposed by CS is a Granger causality test and the proposed procedure is essentially the same as FG. The two procedures should be asymptotically equivalent, even if of course results may differ for small samples.

According to CS simulations, FG presents a disproportionately large size distortion (it almost always rejects the null hypothesis even if it is correct). By contrast, CS does not suffer from this problem (indeed, it is under-sized). This is true not only for a sample size T equal to 200, but also for $T = 2000$. This is surprising, since as argued above, the two methods should deliver similar results, at least for large samples.

4 The reference model

CS discuss their procedure and results using a simple DSGE model taken from Leeper, Walker and Yang (2013). A representative agent maximizes an infinite stream of discounted utilities over consumption:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

subject to

$$C_t + (1 - \tau_{t,k})K_t + T_t = (1 - \tau_{t,y})Y_t$$

where $Y_t = A_t K_{t-1}^{\alpha}$ denotes production. The government sets tax rates over income and capital and adjusts transfers to satisfy: $T_t = \tau_{t,y} Y_t + \tau_{t,k} K_t$. There are three i.i.d shocks hitting the economy: the first, $\zeta_{t,a}$, is a technology shock affecting the production function, the second, $\zeta_{t,a}$, is a shock to the tax rate on capital and the third, $\zeta_{t,y}$, affects the tax rate on income.

In the log-linearized version of the model, $\hat{A}_t = \zeta_{t,a}$, $\hat{\tau}_{t,k} = \zeta_{t,k}$ and $\hat{\tau}_{t,y} = \zeta_{t,y} + b\zeta_{t-1,y}$. The log-linearized solution for capital is: $\hat{K}_t = \alpha \hat{K}_{t-1} + \zeta_{t,a} - \kappa_k \zeta_{t,k} - \kappa_y b \zeta_{t,y}$.[5]

The econometrician observes $\hat{K}_t$ and an aggregate tax variable $\hat{\tau}_t = \hat{\tau}_{t,k} + \omega \hat{\tau}_{t,y} = \zeta_{t,k} + \omega(\zeta_{t,y} + b\zeta_{t-1,y})$.[6]

The Data Generating Process is therefore:

$$\begin{bmatrix} (1-\alpha L)\hat{K}_t \\ \hat{\tau}_t \end{bmatrix} = \begin{bmatrix} 1 & -\kappa_k & -\kappa_y b \\ 0 & 1 & 1+bL \end{bmatrix} \begin{bmatrix} \zeta_{t,a} \\ \zeta_{t,k} \\ \zeta_{t,y} \end{bmatrix}. \tag{4}$$

This model is labeled by CS as the "aggregation" model, since the econometrician observes the aggregated tax variables in place of the two tax variables.

The data generating process for the auxiliary variables $y_{i,t}$, $i = 1, \ldots, N$ is the factor model

$$(1-\delta L) y_{i,t} = \zeta_{t,a} + \gamma_i \zeta_{t,y} + (1-\gamma_i)\zeta_{t,k} + \xi_{i,t}, \tag{5}$$

where the $\xi_{i,t}$'s are serially uncorrelated, mutually orthogonal and orthogonal to the common factors at all leads and lags. The common factors are the structural shocks $\zeta_{t,a}$, $\zeta_{t,k}$ and $\zeta_{t,y}$. Note that representation (4) is "short". Hence, it is non-fundamental irrespective of the parameter values, since we cannot recover three shocks with just two variables.

CS considers also a simplified version of the model, named the "no aggregation" case. In this version of the model, the tax rate on capital is set to zero and therefore we have just two structural shocks, the technology shock and the shock affecting the tax rate on income:

$$\begin{bmatrix} (1-\alpha L)\hat{K}_t \\ \hat{\tau}_t \end{bmatrix} = \begin{bmatrix} 1 & -\kappa_y b \\ 0 & 1+bL \end{bmatrix} \begin{bmatrix} \zeta_{t,a} \\ \zeta_{t,y} \end{bmatrix}, \tag{6}$$

[5]Note that: $\kappa_k = \frac{\tau_k(1-\theta)}{1-\tau_k}$, $\kappa_y = \frac{\tau_y(1-\theta)}{1-\tau_y}$ where $\theta = \alpha\beta\frac{1-\tau_y}{1-\tau_k}$ and τ_y and τ_k are respectively, the steady-state income and capital taxes.

[6]As in CS, ω is set to 1 in the simulations.

For this model, the auxiliary variables $y_{i,t}$ are assumed to follow the two-factor model

$$(1-\delta L)y_{i,t} = \zeta_{t,a} + \gamma_i \zeta_{t,y} + \xi_{i,t},$$

where the idiosyncratic terms $\xi_{i,t}$ are as above.

The square system (6) may be fundamental or not, depending on the parameter values. As we have already seen, we have fundamentalness if and only if the determinant of the MA matrix on the right-hand side has no roots smaller than 1 in modulus. Since in the present case the determinant is $1 + bL$, the root is $L = -1/b$ and the condition is fulfilled if and only if $|b| \leq 1$.

5 Simulation results

For the simulation, we use the same parameters and distributions as in CS, i.e. $\zeta_{t,a}$, $\zeta_{t,k}$ and $\zeta_{t,y}$ i.i.d. $N(0,1)$; $\alpha = 0.36$, $\beta = 0.99$, $\tau_y = 0.25$, $\tau_k = 0.1$. We let b take on several values shown in the tables below. For the auxiliary variables, following CS we set $\delta = 0.9$, $N = 30$, $\xi_{i,t}$ i.i.d. $N(0,1)$, γ_i Bernoulli taking value 1 with probability 0.5. As in CS, we use two values for T, i.e. $T = 200$ and $T = 2,000$. The number of replications is 1,000.

Let us consider first the "no aggregation" model. The null hypothesis is fundamentalness, i.e. $b \leq 1$. We report results for $b = 0.6, 0.8, 0.9, 0.95, 1.05, 1.1$. Since we have values of b very close to 1, a large number of lags is needed in order to get a good approximation to the VAR(∞) representation of x_t. Hence for FG we set $p_1 = 16$, $p_2 = 2$. As for CS we try two dynamic specification: in the former one (specification [i]) we set $r = 8$, $p_3 = 2$, $p_4 = 8$, $p_5 = 8$; in the latter one (specification [ii]) we set $r = 0$, $p_3 = 2$, $p_4 = 8$, $p_5 = 8$. As for the number of principal components we set $s = 2$, i.e. the true number of factors. Notice that with specification [ii] the CS test reduces to Geweke, Meese and Dent's version of Sims' test, since with $r = 0$ we do not have pre-whitening. We do not consider the orthogonality test, which is inappropriate to detect global fundamentalness.

	b	0.6	0.8	0.9	0.95	1.05	1.1
	10%	0.146	0.163	0.147	0.186	0.802	0.993
CS [i]	5%	0.077	0.087	0.080	0.109	0.722	0.988
	1%	0.019	0.030	0.020	0.039	0.547	0.968
	10%	0.122	0.134	0.155	0.222	0.930	1.000
CS [ii]	5%	0.058	0.073	0.097	0.126	0.886	1.000
	1%	0.021	0.021	0.029	0.047	0.757	1.000
	10%	0.110	0.106	0.137	0.207	0.976	1.000
FG	5%	0.058	0.064	0.077	0.132	0.920	1.000
	1%	0.016	0.018	0.024	0.040	0.679	0.994

Table 1: No aggregation model: number of rejections / number of replications. T = 2,000 number of replications = 1,000. Dynamic specification: for the FG test $s = 2$, $p_1 = 16$, $p_2 = 2$; for the CS[i] test $s = 2$, $r = 8$, $p_3 = 2$, $p_4 = 8$, $p_5 = 8$; for the CS[ii] test $s = 2$, $r = 0$, $p_3 = 2$, $p_4 = 8$, $p_5 = 8$.

Table 1 reports the results. For values smaller than 1 the null is true. FG does not exhibits the large size distortions reported in CS for $b = 0.6$ and 0.8 (indeed, for these values of b, the size distortion of FG is very small). With $b = 0.9$ and $b = 0.95$ there is an upward size distortion similar for all procedures and dynamic specifications.

For $b = 1.05$ and $b = 1.1$ the null is false, so that the table reports the empirical power of the tests. The power of CS, specification [i], is considerably less than CS, specification [ii] and FG. Overall, CS[ii] and FG perform similarly (and reasonably well) in discriminating between the region of fundamentalness and that of non-fundamentalness.

Table 1 shows results for $T = 2000$. With $T = 2,000$ we need an even richer dynamic specification to discriminate correctly between fundamentalness and non-fundamentalness around $b = 1$ ($p_1 = 40$, $p_2 = 2$, $r = 0$, $p_3 = 4$, $p_4 = 20$, $p_5 = 20$). The performance of both procedures is now almost perfect.

	b	0.6	0.8	0.9	0.95	1.05	1.1
	10%	0.120	0.122	0.092	0.133	1.000	1.000
CS[ii]	5%	0.055	0.057	0.047	0.072	1.000	1.000
	1%	0.015	0.012	0.014	0.011	1.000	1.000
	10%	0.097	0.109	0.092	0.103	1.000	1.000
FG	5%	0.040	0.056	0.040	0.054	1.000	1.000
	1%	0.008	0.012	0.011	0.011	1.000	1.000

Table 2: No aggregation model: number of rejections / number of replications. T = 2,000, number of replications = 1,000. Dynamic specification: for the FG test $s = 2$, $p_1 = 40$, $p_2 = 2$; for the CS test $s = 2$, $r = 0$, $p_3 = 4$, $p_4 = 20$, $p_5 = 20$.

Coming to the "aggregation" case, we have non-fundamentalness irrespective of the parameters values. However, as observed above, when the model is "short" non-fundamentalness does not imply Granger causality. Hence in order to establish for which values of the parameters the null is false we directly investigate Granger causality.

Let us consider first $(1-\alpha L)\hat{K}_t$. Its theoretical projection onto the information space spanned by the past of both variables and the past of the factors is zero, and the residual is the variable itself, which is white noise and orthogonal to the projection space. Hence the factors do not Granger-cause $(1-\alpha L)\hat{K}_t$ and therefore $\hat{K}_t$.

Coming to $\hat{\tau}_t$, its projection onto the past of both variables and the past of the three structural shocks is $b\zeta_{t-1,y}$.[7] Hence the factors Granger-cause $\hat{\tau}_t$, whenever $b \neq 0$.

Summing up, the factors do not Granger-cause the first variable in x_t, i.e. $\hat{K}_t$, but Granger-cause the second variable $\hat{\tau}_t$ (so that they Granger-cause x_t as a vector) if and only if $b \neq 0$. It follows that, if $b = 0$, the null is true, whereas if $b \neq 0$ the null is false.

For this simulation, we set $s = 3$, which is the true number of factors in the present case. As for the dynamic specification, we use two sets of values. In the former one (A) we follow CS and set $p_1 = 4$, $p_2 = 2$ for the FG test; $r = 4$, $p_3 = 4$, $p_4 = 0$, $p_5 = 2$ for the CS test. In the latter one (B) we set $p_1 = 4$, $p_2 = 1$ for the FG test; $r = 0$, $p_3 = 1$, $p_4 = 3$, $p_5 = 1$ for the CS test. The rationale for dynamic specification B is that it is the one maximizing the power of the two procedures for $b = 0.4$ over a wide range of

[7] This is easily seen if we consider that $b\zeta_{t-1,y}$ belongs to the projection space, whereas the difference $\hat{\tau}_t - b\zeta_{t-1,y} = \zeta_{t,k} + \zeta_{t,y}$ is orthogonal to the past of all variables and shocks. Uniqueness of the orthogonal decomposition ensures the result.

possible specifications.

	b	0	0.2	0.4	0.6	0.8	1.5
	10%	0.093	0.138	0.425	0.821	0.986	1.000
CS	5%	0.046	0.072	0.306	0.744	0.976	1.000
	1%	0.013	0.019	0.131	0.543	0.917	1.000
	10%	0.099	0.185	0.480	0.863	0.993	1.000
FG	5%	0.054	0.107	0.370	0.802	0.986	1.000
	1%	0.014	0.033	0.174	0.603	0.938	1.000

Table 3: Aggregation model: number of rejections / number of replications. T = 200, number of replications = 1,000. Dynamic specification A: for the FG test $s = 3$, $p_1 = 4$, $p_2 = 2$; for the CS test $s = 3$, $r = 4$, $p_3 = 4$, $p_4 = 0$, $p_5 = 2$.

Table 3 reports the results for the dynamic specification A and $T = 200$. Looking at the results for $b = 0$ we see the empirical size of the test, since in this case the null is true. The size distortions are negligible.

By looking at the remaining columns, wee see the empirical power of the test, since in this case the null is false. Both procedures clearly reject the null for large values of b. FG is slightly more powerful than CS for all values of b between 0.2 and 0.8.

With specification B, reported in Table 4, both procedures have enhanced power. The empirical size of the test is close to the theoretical one for both procedures. The power performance is also very similar.

	b	0	0.2	0.4	0.6	0.8	1.5
	10%	0.114	0.223	0.603	0.941	0.998	1.000
CS	5%	0.049	0.127	0.481	0.886	0.995	1.000
	1%	0.013	0.045	0.249	0.725	0.972	1.000
	10%	0.106	0.211	0.609	0.942	0.999	1.000
FG	5%	0.051	0.119	0.482	0.891	0.997	1.000
	1%	0.010	0.041	0.256	0.736	0.974	1.000

Table 4: Aggregation model: number of rejections / number of replications. T = 200, number of replications = 1,000. Dynamic specification B: for the FG test $s = 3$, $p_1 = 4$, $p_2 = 1$; for the CS test $s = 3$, $r = 0$, $p_3 = 1$, $p_4 = 3$, $p_5 = 1$.

	b	0	0.05	0.1	0.2	0.3	0.4
	10%	0.103	0.163	0.430	0.957	1.000	1.000
CS	5%	0.048	0.091	0.318	0.923	1.000	1.000
	1%	0.011	0.028	0.149	0.821	0.999	1.000
	10%	0.103	0.168	0.431	0.958	1.000	1.000
FG	5%	0.049	0.090	0.314	0.923	1.000	1.000
	1%	0.011	0.028	0.154	0.820	0.999	1.000

Table 5: Aggregation model: number of rejections / number of replications. T = 2000, number of replications = 1000. Dynamic specification B: for the FG test $s = 3$, $p_1 = 4$, $p_2 = 1$; for the CS test $s = 3$, $r = 0$, $p_3 = 1$, $p_4 = 3$, $p_5 = 1$.

Table 5 shows the results for specification B, $T = 2000$. In this case, we do not need a rich dynamic specification to get good results. Of course, for a large T both tests have more power. For instance, with $b = 0.2$, the rejection rate of CS at the 5% level is 92.3%, as against 12.7% for the sample size $T = 200$. Size distortions are not there. The power performances of CS and FG are now almost identical, confirming that the methods are asymptotically equivalent.

Fundamentalness is clearly rejected for $b \geq 0.2$. For very small values of b non-fundamentalness is still there, but the test is not able to detect it, since Granger causation is too modest.

Summing up, according to our Monte Carlo exercises, the large differences reported by CS are not there. The two procedures perform similarly both for small and for large samples.

6 Aggregation

CS interpret the results of the two competing testing procedures as if the null hypothesis was fundamentalness of what they call the "aggregate structural representation".

In this subsection we argue that this interpretation is untenable. We show that for model (4) we have both fundamental and non-fundamental aggregate representations. None of them can be regarded as "structural". All aggregate representations are observationally equivalent, so that there are no testing procedures which can discriminate between them.

An aggregate representation of the vector x_t is simply a square representation, driven by just two "aggregate" white noise shocks. Existence of at least one such representation is guaranteed by the Wold theorem. An aggregate representation for model (4) can be derived as follows. The auto- and cross-covariances implied by model (4) tell us that the variables have a bivariate aggregate MA(1) representation. For convenience we normalize such representation by imposing that: (a) the shocks are orthogonal; (b) the second shock does not affect the first variable on impact; (c) the impact effects of both shocks on the corresponding variable is equal to 1. The remaining parameters can be found by matching the moments implied by the disaggregate model (4) with those implied by the aggregate model itself.[8]

We have:

$$\begin{bmatrix} (1-\alpha L)\hat{K}_t \\ \hat{\tau}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ d+gL & 1+cL \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix}. \tag{7}$$

In the first line we do not have lagged effects since the lagged covariance of $(1-\alpha L)\hat{K}_t$ with both variables are zero.

Let σ_1^2 and σ_2^2 be the variances of $v_{1,t}$ and $v_{2,t}$, respectively. By equating the variance of $(1-\alpha L)\hat{K}_t$ implied by the disaggregate and the aggregate systems —equations (4) and (7)— we get

$$\sigma_1^2 = 1 + \kappa_k^2 + \kappa_y^2 b^2.$$

By equating the contemporaneous covariance of $(1-\alpha L)\hat{K}_t$ and $\hat{\tau}_t$ we get

$$d = -(\kappa_k + \kappa_y b)/\sigma_1^2.$$

By equating $\text{cov}(\hat{\tau}_t, (1-\alpha L)\hat{K}_{t-1})$ we get

$$g = -\kappa_y b^2/\sigma_1^2.$$

By equating $\text{cov}(\hat{\tau}_t, \hat{\tau}_{t-1})$, if $b = 0$ we get $c = 0$; if $b \neq 0$ we get σ_2^2 as a function of c:

$$\sigma_2^2 = (b - dg\sigma_1^2)/c.$$

Finally, by equating the variance of $\hat{\tau}_t$, if $b = 0$ we get $\sigma_2^2 = (2+\kappa_k^2)/(1+\kappa_k^2)$; if $b \neq 0$, using the above displayed equation we get

$$c^2 + \frac{(d^2+g^2)\sigma_1^2 - 2 - b^2}{b - dg\sigma_1^2} c + 1 = 0.$$

[8]The aggregate representation derived in CS is incorrect, in that it does not match the covariance of $\hat{\tau}_t$ with $(1-\alpha L)\hat{K}_{t-1}$. The four parameters ρ, c, σ_1^2 and σ_2^2 cannot match the five nonzero moments of x_t.

This equation has two real reciprocal solutions, say $\bar{c}$, $|\bar{c}| < 1$, and $1/\bar{c}$, implying two alternative values for σ_2^2 (and two different solutions for $v_{2,t}$). For $c = \bar{c}$ the aggregate representation is fundamental, since the determinant $1 + cL$ vanishes only for $L = -1/\bar{c}$, which is greater than 1 in modulus. For $c = 1/\bar{c}$ the representation is non-fundamental, since the determinant vanishes within the open unit disc.

CS claim that, by varying the parameter b of the disaggregate model we can move the parameter c from the fundamentalness to the non-fundamentalness region. But this is not the case. By varying b we can change $\bar{c}$ and $1/\bar{c}$, but for the same b we always have two solutions for c, one in the fundamentalness region and the other one in the non-fundamentalness region. Such representations are observationally equivalent, since there is nothing in the data (either the x's or the y's) that depends on one or the other. Neither FG, nor CS can discriminate between them.

The matrix in (7) can be inverted toward the past for $c = \bar{c}$ or toward the future for for $c = 1/\bar{c}$ in order to express the aggregate shocks in terms of the variables and, using (4), in terms of the structural shocks. Doing this, we get

$$\begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa_k & -\kappa_y b \\ -\dfrac{d+gL}{1+cL} & \dfrac{h-pL}{1+cL} & \dfrac{q+wL}{1+cL} \end{bmatrix} \begin{bmatrix} \zeta_{t,a} \\ \zeta_{t,k} \\ \zeta_{t,y} \end{bmatrix}, \tag{8}$$

where

$$\begin{aligned} h &= (1+\kappa_k^2 - \kappa_k\kappa_y b)/\sigma_1^2 \\ p &= \kappa_k\kappa_y b^2/\sigma_1^2 \\ q &= (1+\kappa_k^2 b^2 - \kappa_k\kappa_y b)/\sigma_1^2 \\ w &= b(1+\kappa_k^2)/\sigma_1^2. \end{aligned}$$

From (8) it is seen that both aggregate shocks are mixtures of the technology shocks and the two tax shocks and there are no orthogonal transformations allowing us to distinguish an aggregate technology shock, depending only on $\zeta_{t,a}$ from an aggregate tax shock, depending only on $\zeta_{t,k}$ and $\zeta_{t,y}$.

In conclusion, the shocks of the aggregate representation cannot be given a meaningful economic interpretation. The aggregate representations (both the fundamental and the non-fundamental ones) are merely statistical representations. This finding is just an example of a general negative result about aggregation in structural VAR models which is already known in the literature (Forni and Lippi 1997, 1999). The only one structural representation here is the disaggregate representation (4). Unfortunately, a structural aggregate representation does not exists.

7 Empirics

Beaudry and Portier (2006) use a bivariate VAR specification, including TFP and stock prices, to assess the relevance of news technology shocks as a source of business cycle fluctuations. Forni, Gambetti and Sala (2014) use FG to test for the adequacy of such specification and find that it is severely deficient.[9] When amending the information set by including the principal components in the VAR, results change substantially and the role of news shocks is reduced, in line with the findings of Barsky and Sims (2011).

CS argues that, when using the CS test, fundamentalness is no longer rejected, thus validating the estimates by Beaudry and Portier (2006). We applied the CS procedure to the data used in Forni, Gambetti and Sala (2014), and found the results reported in Table 7. We used $r = 4$ lags in the VAR and the following six dynamic specifications:

Sims1	$p_3 = 0$,	$p_4 = 2$,	$p_5 = 2$
Sims2	$p_3 = 0$,	$p_4 = 4$,	$p_5 = 4$
Geweke1	$p_3 = 1$,	$p_4 = 2$,	$p_5 = 2$
Geweke2	$p_3 = 1$,	$p_4 = 4$,	$p_5 = 4$
Geweke3	$p_3 = 2$,	$p_4 = 2$,	$p_5 = 2$
Geweke4	$p_3 = 2$,	$p_4 = 4$,	$p_5 = 4$

Table 6: Dynamic specifications for the CS test.

[9] Indeed, Forni, Gambetti and Sala (2014) use the orthogonality test, which is a variant of FG designed to detect partial fundamentalness, i.e. fundamentalness of a single shock, rather than global fundamentalness. On the concept of partial fundamentalness, see Forni, Gambetti and Sala (2017).

Dyn. spec.	$s=3$	$s=4$	$s=5$	$s=6$	$s=7$	$s=8$	$s=9$	$s=10$
Sims1	0.094	0.022	0.036	0.024	0.014	0.018	0.039	0.022
Sims2	0.020	0.011	0.013	0.001	0.003	0.001	0.003	0.002
Geweke1	0.017	0.005	0.010	0.004	0.003	0.001	0.005	0.002
Geweke2	0.006	0.004	0.009	0.002	0.005	0.002	0.003	0.001
Geweke3	0.086	0.066	0.091	0.038	0.023	0.016	0.045	0.006
Geweke4	0.004	0.003	0.015	0.007	0.008	0.002	0.008	0.009

Table 7: Real data from Forni et al. (2014). p-values of the CS testing procedure. Dynamic specifications: see Table 6.

Contrary to CS results, we find a strong rejection of the null of fundamentalness for most parameter configurations, confirming the results in Forni, Gambetti and Sala (2014).

8 Conclusions

Canova and Sahneh (2018) propose a fundamentalness testing procedure which is a variant of the one proposed in Forni and Gambetti (2014), using a different Granger causality test. According to our Monte Carlo exercises, the two procedures perform similarly for both small and large samples.

When the underlying theoretical model includes disaggregate shocks, unfortunately, the two procedures cannot be used to infer whether the “structural aggregate representation” is fundamental or not. There are two reasons. First, there are no aggregate representations that can be regarded as structural. Second, fundamental and non-fundamental aggregate representations are observationally equivalent.

The bivariate news shock specification of Beaudry and Portier (2006) which was found to be fundamental by Canova and Sahneh (2018) is on the contrary non-fundamental as found in Forni, Gambetti and Sala (2014).

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