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Luca Gambarelli[1], Silvia Muzzioli[2], Bernard De Baets[3]

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[1]University of Modena and Reggio Emilia, Department of Economics Marco Biagi
Email: luca.gambarelli@unimore.it
2University of Modena and Reggio Emilia, Department of Economics Marco Biagi
Address: Viale Berengario 51, 41121, Modena, Italy
Email: silvia.muzzioli@unimore.it
[3] Ghent University, Department of Data Analysis and Mathematical Modelling and KERMIT
Address: Viale Berengario 51, 41121, Modena, Italy
Email: bernard.debaets@ughent.be

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Luca Gambarelli[a], Silvia Muzzioli[b*], Bernard De Baets[c]

[a] *Marco Biagi Foundation, Department of Economics, University of Modena and Reggio Emilia, Viale Berengario 51, 41121, Modena, Italy, luca.gambarelli@unimore.it*

[b] *Department of Economics, University of Modena and Reggio Emilia, Viale Berengario 51, 41121, Modena, Italy, silvia.muzzioli@unimore.it*

[c] *KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University, Coupure links 653, B-9000, Ghent, Belgium, bernard.debaets@ugent.be*

Abstract

In this paper, we analyze the information value of the VSTOXX (volatility) index as a measure of risk for the EU stock market. Employing daily data from 2007 to 2017, we inspect and contrast the properties of the VSTOXX index under various market conditions and in high- and low-volatility periods. Moreover, to evaluate the contribution of each country-specific index to the VSTOXX index, we employ the Ordered Weighted Averaging (OWA) operator, which provides a flexible aggregation procedure ranging between the minimum and the maximum of the input values. We obtain a number of useful insights. The correlation between the VSTOXX index and the volatility indices is high during the entire period only for France and Germany. Moreover, the VSTOXX index acts more like an OR-like measure than as an AND-like measure of volatility for the EU stock markets and acts as an average only during periods of extreme volatility.

Keywords: volatility indices; OWA aggregation; VSTOXX; EU market.

* Corresponding author: Department of Economics and CEFIN, University of Modena and Reggio Emilia, Viale Berengario 51, 41121 Modena (I), Tel. +390592056771 Fax +390592056947,ORCID iD: 0000-0003-0738-6690.

1. Introduction

Recent studies highlight the importance of using option-implied measures in asset pricing and portfolio management (see, e.g., Elyasiani et al., 2020a for a literature review). Option-implied volatilities are commonly used to capture market volatility as a proxy for market-wide risk (Leppin and Reitz, 2016). In fact, option prices reflect the risk of rare economic events, such as consumption disasters (Seo and Wachter, 2019), and can convey informed investors' negative news through their trades. They also become very useful in highly uncertain situations such as build-ups of potential systemic risk (Bevilacqua et al., 2020), since they represent natural financial instruments for hedging purposes.

Nowadays, the only option-implied index intended to measure the aggregate volatility of the EU markets is the VSTOXX. The VSTOXX, officially EURO STOXX 50 Volatility Index, is referred to as the "European VIX" since it represents the equivalent of the VIX index for the European markets (Kalyvas et al., 2020), and it is the most widely used measure of expected volatility in Europe (Peterburgsky, 2021). The VSTOXX has been designed to reflect the investor sentiment and overall EU economic uncertainty by measuring the 30-day implied volatility of the EURO STOXX 50 Index,[1] using near-term EURO STOXX 50 option prices. The EURO STOXX 50 Index is the most widely followed benchmark to track equity market performance and development in the Eurozone[2] and comprises fifty of the largest and most liquid stocks covering Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain (Li, 2013).[3] Its composition is annually reviewed in September.

Although the VSTOXX is generally accepted as the leading market indicator of risk sentiment in the Eurozone (see, e.g., Zhang et al., 2017; Forte and Lovreta, 2019), it has received some criticism in the literature. Indeed, there is mixed evidence on its representativeness for the EU market. For instance, Peterburgsky (2021) points out that EURO STOXX 50 companies account for less than 35% of the European stock market value. Moreover, important EU financial markets such as the UK and Switzerland are not considered in the EURO STOXX 50 index, even if many studies find significant interactions between these markets, especially before the campaign for the EU referendum started in January 2016 (see e.g., Li, 2020). Since there is a lack of studies investigating the behavior of the VSTOXX as a measure of risk for all the EU markets, it is our aim to fill this void.

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[1] https://www.eurexchange.com/resource/blob/146798/29a67e60c49811de80c7252b627ee738/data/factsheet_eurex_ vstoxx .pdf

[2] www.stoxx.com

[3] A stock index of Eurozone stocks developed by STOXX, an index provider owned by Deutsche Börse Group, introduced on 26 February 1998. According to STOXX, its goal is "to provide a blue-chip representation of Supersector leaders in the Eurozone".

Among existing studies, Peterburgsky (2021) finds that during the 2002-2016 period, investors do not care about aggregate volatility measured by the VSTOXX. Other criticisms in the literature concern the fact that the VSTOXX is characterized by a slow response to shocks in non-equity markets, such as the bond market; moreover, the VSTOXX cannot be used to hedge specific sectors of the stock market (Zghal et al., 2018). Furthermore, López and Esparcia (2021) provide evidence that the VSTOXX reacts to the German unemployment rate and ESI (Economic sentiment indicator) release, but not the release of the corresponding economic indicators for the Eurozone. This result is affected by the fact that German indicators are announced before those of the Eurozone. Thus, part of the information content of the latter has already been discounted in option prices. However, the reaction of the VSTOXX to the German economic indicators may be at least partly due to the importance (in terms of relative weight) that German companies have in the EURO STOXX 50 index computation.

In an international setting, Clements et al. (2019) find evidence of declining informational dominance of the VIX as global volatility leader, replaced by the VSTOXX since the European debt crisis. The importance of the VSTOXX is also supported by Shu and Chang (2019), who investigate the interaction between the VIX (the US volatility index), VSTOXX (the EU volatility index), VKOSPI (the Korean volatility index) and international stock market indices over the 2004-2014 period. Even if the VIX is the most influential volatility index in terms of effects on the stock market returns, they find that the VSTOXX also has a significant impact on stock returns in the U.S., European and Asian markets, and has been an originator of spillovers during the European fiscal crisis.

We contribute to the literature in several respects. First, we introduce model-free implied volatility indices for nine index options markets in the EU during the 2007-2017 period. The index options markets under investigation include AEX (the Netherlands), BEL (Belgium), CAC (France), DAX (Germany), FTSE (the United Kingdom), IBEX (Spain), MIB (Italy), OMX (Sweden), and SMI (Switzerland). This sample period is a suitable framework to investigate the behavior of implied volatility measures because it is characterized by the occurrence of both the subprime crisis (2007-2009) and the European debt crisis (2010-2012). Second, the presence of high-volatility periods in the sample period allows us to inspect and contrast the properties of the VSTOXX under various market conditions and economies under stress, such as EU peripheral countries. Third, we provide for the first time a deep analysis of the relationships between the VSTOXX and the country-specific volatility indices computed from major EU economies and their behavior over time. To investigate the information value of each country-specific index for the VSTOXX, we exploit the Ordered Weighted Averaging (OWA) operator, which provides a flexible aggregation procedure ranging between the minimum and the maximum of the input values. Most studies rely on expert opinions to select the OWA weights, whereas only a few researchers determine these weights from data (Dominguez-Catena et al., 2021). We adopt the latter approach with the aim to analyze the properties and the information on single countries embedded in the VSTOXX index. We find three main results that are of interest for investors and policymakers.

First, the VSTOXX index recorded an average volatility above the mean of the country-specific volatility indices during the sample period. Peripheral countries such as Spain and Italy, which suffered the most from the European debt crisis, recorded a very high average volatility. On the other hand, other countries that do not take part in the EUROSTOXX, such as Switzerland and the UK, recorded a very low volatility, acting as a safe haven during the EU debt crisis. Second, the VSTOXX index is strongly related to the French and German volatility indices during the entire sample period, proving to be a good measure of volatility for these countries. In contrast, the relationship between the VSTOXX index and the volatility measures in other countries highly depends on the specific period under investigation (especially for the peripheral ones), thus casting doubt on the ability of the VSTOXX index to measure risk for these countries correctly. Third, the results of the fitting exercise show that the VSTOXX index acts more like an OR-like measure than as an AND-like measure of volatility for the EU stock markets. More specifically, the VSTOXX index is higher than the average of the volatility indices; in fact, it is sufficient that a few volatility indices reach a high value for the index to spike. On the other hand, we find that the VSTOXX index acts more as an average (*orness* around 0.5) during periods of extreme volatility.

The remainder of this paper is structured as follows. Section 2 introduces the dataset and the methodology adopted in our study. Section 3 investigates the properties of the VSTOXX index and the volatility indices obtained for the nine countries. In Section 4, we exploit the OWA operator to assess how the information content of the nine volatility indices is embedded into the VSTOXX index. Finally, Section 5 draws some conclusions and provides policy implications.

2. Data and Methodology

This section introduces our dataset and the methodology used to obtain the volatility indices for the nine EU countries. The data set consists of daily closing prices of index options of nine different countries,[4] recorded from 2 January 2007 to 29 December 2017. The options data set, the dividend yield, and the Euribor rates are obtained from OptionMetrics (IvyDB Europe). The underlying assets, the time series of the underlying assets, and the daily closing values of the VSTOXX index are

[4] The choice of the countries is motivated by the availability of option prices in the IvyDB Europe dataset, which includes the most liquid and traded options market in Europe.

obtained from Bloomberg. Following Muzzioli et al. (2018, 2020), if the underlying of the option series (S_t) is an asset that pays a dividend yield δ_t, we compute its adjusted value ($\hat{S}_t$) as:

$$\hat{S}_t = S_t e^{-\delta_t \Delta t} \quad (1)$$

where Δt is the time to maturity of the option. Euribor rates with maturities of one week, one month, two months, and three months are used as a proxy for the risk-free rate. The appropriate yield to maturity is computed by linear interpolation.

In line with previous studies (see, e.g., Elyasiani et al., 2018), we applied several filters to the options dataset to remove arbitrage opportunities and other irregularities in the prices and to be consistent with the computational method of other listed indices. More specifically, we eliminate options with a time to maturity of less than eight days that may suffer from pricing anomalies that might occur close to expiration. Also, in-the-money options are removed because they are infrequently traded compared to the other options and can be affected by illiquidity problems. In particular, we retain only out-, and at-the-money option contracts, i.e., put options with moneyness values lower than 1.03 (K/S < 1.03), and call options with moneyness values higher than 0.97 (K/S > 0.97), where K is the strike price and S is the index value. Moreover, we eliminate option prices violating the standard no-arbitrage constraints. Finally, to have a one-to-one correspondence between strikes and implied volatilities, we average the implied volatilities of options that correspond to the same strike price.

The standard approach used to compute an option-implied volatility index is the one introduced by the Chicago Board Options Exchange (CBOE) for the VIX index, the measure of 30-day volatility of the S&P 500 index. Many market volatility indices have been quoted in European markets based on the same formula, such as the VSTOXX index, VDAX, and the Italian volatility index (IVI MIB), among others.[5] Given the market prices of at- and out-of-the-money options for a single option series, the volatility index can be computed as the square root of the model-free implied variance, which is estimated by using the following equation by Britten-Jones and Neuberger (2000):

$$E^Q\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C_0\left(T, Ke^{rT}\right) - \max\left(S_0 - K, 0\right)}{K^2} dK \quad (2)$$

where Q represents the expectation under the risk-neutral probability, S_t is the underlying asset price at time $t = 0, ..., T$, and $C_0\left(T, K\right)$ is a call option price at t = 0, with maturity T and strike price K; r is the risk-free rate.

[5] Although the VIX is calculated from 2014 using weekly options, we excluded these options due to their lower liquidity in the EU market.

Since Eq. (2) requires as input a continuum of strike prices ranging from zero to infinity, and in the market only a discrete and limited number of strike prices is available, the CBOE computes the VIX index using a subset of quoted option prices (see, e.g., Muzzioli et al., 2018, for a detailed discussion). Consequently, truncation and discretization errors could occur due to a finite range of strike prices and a discrete summation instead of the integral in Eq. (2). While this assumption is mitigated for the US market by the high number of option prices traded (more than 100 per day), truncation and discretization errors can impact the estimation of volatility for peripheral European markets, which are characterized by a limited number of strike prices traded (Elyasiani et al., 2021).

To mitigate both truncation and discretization errors, Jiang and Tian (2005) propose an interpolation-extrapolation method based on an interpolation among implied volatilities of available option prices with cubic splines and an extrapolation procedure outside the domain of quoted option prices using a constant volatility function. To cope with different numbers of strike prices traded, we adopt a country-specific procedure to make truncation and discretization errors negligible and significantly improve the precision of the volatility estimate. For each country, the procedure takes the following steps. First, we create a table of available strike prices and implied volatilities, which serves as our initial input. Second, following Jiang and Tian (2005), implied volatilities are interpolated between two adjacent knots using cubic splines to keep the function smooth at the knots. Volatility is assumed to be constant for strike prices higher (resp. lower) than the maximum (resp. minimum) strike price available. More specifically, for strikes below (resp. above) the minimum (resp. maximum) value, implied volatility is equal to the volatility of the minimum (resp. maximum) strike price available to avoid negative implied volatilities (Muzzioli et al., 2018). To mitigate the occurrence of truncation errors, a fixed parameter value of u equal to 2 for all countries is used to extend the integration domain by computing a matrix of strike prices and implied volatility in the interval $S/(1+u) \leq K \leq S(1+u)$, where S is the underlying asset value. Finally, a country-specific spacing between strike prices (details are reported in Table 1) is adopted to ensure insignificant discretization errors and compute missing implied volatility and strike prices from the interpolated-extrapolated smile.

The implied volatilities obtained are finally converted into option prices and used to compute model-free variance through the approximated Britten-Jones and Neuberger (2000) formula:

$$2\int_{0}^{\infty} \frac{C_0\left(T, Ke^{rT}\right) - \max\left(S_0 - K, 0\right)}{K^2} dK \approx \sum_{i=1}^{m} \left[g\left(T, K_i\right) + g\left(T, K_{i-1}\right) \right] \Delta K \qquad (3)$$

where $g\left(T, K_i\right) = \left[C_0\left(T, K_i\right) - \max\left(0, F_0 - K_i\right) \right] / K_i^2$, $C_0\left(T, K_i\right)$ is the price of a call option with strike price K_i and time to maturity T, $\Delta K = \left(K_{max} - K_{min}\right)/m$, m is the number of abscissas;

$K_i = K_{min} + i\Delta K$, $0 < i < m$, K_{min} and $K_{\max}$ are the minimum and the maximum strike prices, respectively.[6]

Moreover, to obtain constant 30-day measures of implied volatility that can be directly compared with the VSTOXX index, the daily estimate of volatility is computed by linear interpolation, using the same formula adopted for the VIX index. In particular, two values of risk-neutral variance obtained from Eq. (3) with different time-to-maturity (i.e., one for each of the two-option series considered, given a first option series with a maturity of less than 30 days and a second one with time to maturity greater than 30 days) are used:

$$\mathrm{VIX} = 100 \times \sqrt{\left[T_1 \sigma_1^2 \left(\frac{N_{T_2} - 30}{N_{T_2} - N_{T_1}} \right) + T_2 \sigma_2^2 \left(\frac{30 - N_{T_1}}{N_{T_2} - N_{T_1}} \right) \right] \times \left(\frac{365}{30} \right)} \qquad (4)$$

where T_1 and T_2 are the time-to-maturity of the first and the second option series used, respectively, and σ_1^2 and σ_2^2 the estimated variances. N_{T_1} and N_{T_2} are the time-to-maturity of the near-term and next-term options, respectively. All computations were done in Matlab R2021a.

3. Properties of the European volatility indices

This section presents the basic statistics and the properties of the nine country-specific volatility indices obtained by applying our approach to the dataset described in Section 2. As a result, we obtain 2869 daily closing values for each of the nine volatility indices, spanning from January 2, 2007, to December 28, 2017.

3.1 Basic descriptive statistics

The descriptive statistics of the country-specific volatility indices and the VSTOXX index are reported in Table 2. Several considerations are in order. First, the average volatility for most of the countries is around 20. Higher values are recorded for the VSTOXX index and for volatility indices measured in peripheral countries (Spain, Italy), which suffered the most from the European debt crisis. On the other hand, volatility is relatively low for Switzerland and the UK, which act as a safe haven during the European debt crisis. Second, the volatility range is the largest for Switzerland (SMI) and the UK (FTSE), while it is the smallest for highly volatile countries such as Italy and Spain. The different behavior of volatility indices could also be affected by different institutional features in the European markets in this study, such as dissimilarity in sectoral diversification and market depth. Third, all the volatility indices show a positive skewness. The right tail is long compared to the left

[6] In order to obtain the risk-neutral variance in Eq. (3), the trapezoidal rule for numerical integration is used.

tail since extreme volatility values are recorded for limited periods during crises. Consequently, the normal distribution hypothesis is strongly rejected for all the series.

3.2 Correlation between the volatility indices

In Table 3, which reports the Pearson correlation coefficients between the series under investigation, shows a high degree of association between all the variables. The correlation between the volatility indices of France (CAC) and Germany (DAX) and the VSTOXX index are close to 1. On the other hand, the Italian volatility index (MIB) shows the lowest correlation with the other variables, followed by the Spanish volatility index (IBEX). The correlation between these two indices (MIB and IBEX) is very high, suggesting that the European debt crisis may have influenced their behavior to the same extent.

In Figure 1 (resp. Figure 2) we present the scatterplots of the correlation between the VSTOXX index and each country-specific index depending on the level of the VSTOXX index (resp. sample period). The dashed grey line represents the case of a volatility index that is perfectly correlated with the VSTOXX index in terms of daily levels: the more the observations deviate from the grey line, the less the volatility index under investigation is correlated with the VSTOXX index. From Figure 1, we can see that the relation between the VSTOXX index and the other volatility indices is strong for very low values of the VSTOXX index (in red) or very high levels of the VSTOXX index (in blue). On the other hand, the relationship tends to weaken when the VSTOXX index ranges between 20 and 40, especially for peripheral EU countries in our dataset (Italy and Spain). A similar pattern could also be discerned for BEL, FTSE, and SMI.

Similarly, the relationship between the VSTOXX index and the country-specific volatility indices varies across different sample periods, as highlighted in Figure 2. The association is strong at the beginning and the end of the sample period (represented in green and in yellow, respectively), while it changes in shape during the 2010-2012 European debt crisis (in red), especially for the countries that are most affected by the latter. We can conclude that the VSTOXX index has not fully captured the different levels of risks during the 2010-2012 period. In particular, while the VSTOXX index was experiencing intermediate levels of volatility, consistent with the market conditions of Germany and France, some other countries experienced very high levels of volatility and stressful market conditions.

3.3 Ranking of the volatility indices

The analysis of volatility indices in terms of levels and their evolution over time can provide further insights into the EU markets' uncertainty. The value of a volatility index is closely linked to the possible of return of the underlying index in the next time period. More specifically, claiming that a volatility index is equal to 10 means that there is about a 68% chance that the underlying market's return is expected to stay within a $\pm$10% range over 1 year, or $\pm$ 2.89% over the next 30 days.[7] Therefore, accounting for the level of volatility is important and the ranking of volatility indices over time can provide insightful information about each market's relative risk and uncertainty compared to the others. To investigate the ranking of the volatility indices and its evolution during the sample period (2007-2017), each day, we rank the nine volatility indices plus the VSTOXX index from the highest (1) to the lowest (10). Since the ranking over time is highly volatile, we compute for each volatility index its 5-day moving average to enhance the readability of the plots, and we display the results in Figure 3. Several observations are in order. First, the ranking of the country-specific volatility indices is highly volatile and changes significantly over the sample period. Changes in the ranking are observed particularly in crises and market turbulence, such as the 2007-2009 financial crisis and the European debt crisis in 2010-2012. Second, both SMI and FTSE, which show high ranks during the 2007-2009 financial crises (probably attributable to the central role of these financial markets in the transmission of the financial crisis and to large banking groups listed on the Zurich and London stock exchanges), show low average ranks in the last part of the sample period. This result is probably due to the non-belonging of these two countries to the Euro area, thus allowing them to act as a safe haven for investors during the European debt crisis. A rare exception is the peak at the beginning of 2015 for the Swiss market due to the unexpected end of peg between the Swiss franc and the Euro.[8] Third, the IBEX and MIB volatility indices, characterized by low ranks at the beginning of the sample period, change significantly after the global financial crisis and during the EU sovereign debt crisis, remaining among the highest until the end of the sample period. Fourth, the remaining indices are characterized by a fairly volatile ranking, with the AEX and BEL showing a slightly lower ranking than CAC and DAX. Last, despite the high correlation level with CAC and DAX shown in Table 2, the VSTOXX index has maintained one of the top ranks (around the third position) during most of the sample period, being in many occurrences higher than both the CAC and the DAX volatility indices. In particular, the average VSTOXX index is higher than the average of the nine volatility indices. The composition of the VSTOXX in terms of the country-specific volatility indices will be closely investigated in the next section.

[7] The value of 2.89% is obtained as $10/\sqrt{12}$, where 10 is the index value and $\sqrt{12}$ is the factor that allows us to move from the annualized index (obtained using Eq. (3)) to that on a monthly basis. Further details are available at: https://www.cboe.com/tradable_products/vix/faqs/

[8] https://www.cnbc.com/2015/01/15/swiss-franc-sours-stocks-tank-as-euro-peg-scrapped.html

4. Fitting exercise

The results obtained in Section 3 reveal that the VSTOXX index is in general higher than the average volatility of the nine EU markets in our dataset. In this section, we propose to use the OWA operator in order to investigate how the country-specific volatility indices are represented in the VSTOXX index. In Subsection 4.1, we theoretically introduce the analysis of the composition of the VSTOXX based on the OWA operator, while in Subsection 4.2 we detail the application of the OWA operator to our dataset.

4.1 The analysis of the composition of the VSTOXX based on the OWA operator

Investigating the behavior of the VSTOXX index is important for investors who monitor this index as a measure of volatility for all European markets. Moreover, we aim at understanding whether its behavior has been fairly homogeneous over time, or whether on the contrary, it has been determined by the market phase. As far we know, there are currently no studies in the literature evaluating the effectiveness of the VSTOXX index for representing risk and uncertainty in different EU markets. To fill this gap, we propose an approach based on the Ordered Weighted Averaging aggregation operator (hereafter, OWA operator), introduced in Yager (1988), and successfully adopted in many fields, including volatility forecasting (Flores-Sosa et al., 2021), multi-criteria and group decision making (Wang and Parkan, 2005; Wang, 2021), multi-attribute decision making (Reimann et al., 2017), forecasting (Yager, 2008), data mining and data smoothing (Torra, 2004), financial decision making (Merigó and Casanovas, 2011). The OWA operator covers aggregation procedures ranging between the minimum and the maximum. An advantage of the OWA operator compared to standard aggregation procedures adopted in financial market applications (mainly the average and weighted average) is the possibility of weighting the values relying on their ordering. In this way, if we order a set of indicators from the highest to the lowest, we can give more importance to a subset of the input values in this ordering than to another subset. This characteristic is at the basis of the choice of the OWA operator for our analysis. Exploiting the OWA operator, we can better understand how the VSTOXX index reacts to high and low values in the country-specific volatility indices, independently of the country in which they originated.

Given **w**, a weighting vector of dimension N, Yager (1988) and Yager and Kacprzyk (1997) define a mapping $\mathrm{OWA}_{\mathbf{w}}:\mathbb{R}^N \to \mathbb{R}$ as an Ordered Weighting Averaging (OWA) operator of dimension N as follows:

$$\mathrm{OWA}_{\mathbf{w}}(a_1,\dots,a_N) = \sum_{i=1}^{N} w_i a_{\sigma(i)}, \tag{5}$$

where $\left(\sigma(1),...,\sigma(N)\right)$ is a permutation of $\left(1,...,N\right)$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ for all $i = 2,...,N$, i.e. $a_{\sigma(i)}$ is the *i*-th largest element in the input vector **a**, and the weights satisfy the properties $w_i \in \left[0,1\right]$ and $\sum_i w_i = 1$.

According to Xu (2005), the aggregation by means of an OWA operator can be synthesized according to the following three steps:

1) Ordering the input arguments in descending order;
2) Determine the weight associated with the OWA operator by using a proper method;
3) Use the OWA weights to aggregate the reordered input arguments.

The choice of the OWA weights is of crucial importance, and has thus attracted a broad strand of literature, and several methods have been proposed for determining the weights (see, e.g., Xu, 2005 for a detailed discussion). The possible range of the OWA outcome varies from the minimum to the maximum value. For instance, the minimum, arithmetic average, and maximum operations can be generated using the following three weighting vectors:

$$\begin{aligned}
&\mathbf{w} = (0,0,0,\, ...,0,1)^T,\ \mathrm{OWA}_{\mathbf{w}}(a_1,a_2,\, ...,a_n) = \min_j a_j\,, \\
&\mathbf{w} = \left(\frac{1}{n},\frac{1}{n},\frac{1}{n},\, ...,\frac{1}{n},\frac{1}{n}\right)^T,\ \mathrm{OWA}_{\mathbf{w}}(a_1,a_2,\, ...,a_n) = \frac{1}{n}\sum_n a_j\,, \\
&\mathbf{w} = (1,0,0,\, ...,0,0)^T,\ \mathrm{OWA}_{\mathbf{w}}(a_1,a_2,\, ...,a_n) = \max_j a_j\,.
\end{aligned} \tag{6}$$

Therefore, the OWA operator is similar to the weighted mean, while departing from the latter in the ordering step, thus producing a different interpretation. While in the weighted mean, the weights are attached to the information sources, in the OWA operator, the weights are attached to the data regarding their relative position. In this way, the decision-maker can give more importance to a subset of the input values in this ordering than to another subset (i.e., weights allow the decision-maker to attribute more importance to, e.g., low values, central values, or high values), allowing for a degree of compensation. The degree of compensation in the OWA operator is measured with the *orness* degree. *Orness* is a measure that gauges to what extent the outcome of the aggregation is near to the maximum of the data being aggregated, i.e., it indicates the position of the OWA operator on a continuum between the AND (i.e. min) and OR (i.e. max) operations. The larger the outcome, the larger the *orness* and the larger the compensation, i.e., the *orness* measures to what extent the outcome of an operator tends to be similar to the OR. The *orness* measure for the OWA operator introduced by Yager (1988) is defined as:

$$orness(\mathbf{w}) = \frac{1}{n-1}\sum_{i=1}^{N}(n-i)w_i\ . \tag{7}$$

The OWA operator allow us to model any desired degree of *orness* between 0 (corresponding to the

AND operator) and 1 (corresponding to the OR operator), by means of an appropriate selection of parameters, the so-called OWA weights. Some notable examples are the following:

$$\begin{aligned} &orness(0,0,0,\ldots,0,1)=0 \\ &orness\left(\frac{1}{n},\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n},\frac{1}{n}\right)=0.5 \\ &orness(1,0,0,\ldots,0,0)=1\,. \end{aligned} \tag{8}$$

Symmetrically, the measure of *andness* can be defined as $andness(\mathbf{w}) = 1 - orness(\mathbf{w})$.

Another important quantity associated with the weighting vector is its dispersion, also known as *entropy*. For a given weighting vector **w**, Yager (1988) defined its measure of dispersion (*entropy*) as:

$$\text{disp}(\mathbf{w}) = -\sum_{i=1}^{N} w_i \log w_i\,, \tag{9}$$

with the convention $0 \times \log 0 = 0$. The measure of dispersion (*entropy*) aims to represent the degree to which an aggregation operator considers all inputs. In particular, weights are often required to have a maximum dispersion (given a set of constraints, e.g., an appropriate level of *orness*). Such maximum dispersion is desirable because it is inappropriate to assign too much weight or importance to a single source of information. Also, a normalized measure of dispersion could be obtained as:

$$\text{ndisp}(\mathbf{w}) = -\frac{1}{\log n}\sum_{i=1}^{N} w_i \log w_i\,. \tag{10}$$

4.2 Application of the OWA operator to the VSTOXX index

Several ways of determining the weighting vector of an OWA operator have been proposed in the literature (Xu, 2005). Most of these methods rely on supervised approaches translating expert opinions into a weighting vector (Dominguez-Catena et al., 2021). On the contrary, few works have explored ways to determine the weighting vector from data (see, e.g., Beliakov, 2003). We adopt the latter approach and derive the weighting vector from the information embedded in the VSTOXX index.

To better understand the properties of the VSTOXX index and compute its *orness* over the sample period, we take the following steps. First, we collect the daily values of the nine country-specific volatility indices and the VSTOXX index. Second, we group them over different time windows (one-, three-, and six-month horizons). Therefore, for each time window, we have *M* daily realizations that depend on how many trading days are present in the one-month, three-month and six-month time period (approximately 21, 63, and 121 trading days, respectively). Third, for each time window, the series of daily values of the nine country-specific volatility indices sorted in descending order are used as input for the OWA operator. We estimate the weighting vector for the OWA operator by solving the following optimization problem:

$$\underset{\mathbf{w}}{\arg\min} \ \sum_{j=1}^{M} (\mathrm{OWA}_{\mathbf{w}}(a_1^j, ..., a_N^j) - b^j)^2$$

$$\text{subject to: } \sum_{i=1}^{N} w_i = 1, \qquad 0 \le w_i \le 1, \qquad i = 1, 2, ..., N \tag{11}$$

where $\left(a_1^j, ..., a_N^j\right)$ is the vector of volatility indices at day *j*, and b^j is the target of the optimization problem, represented by the VSTOXX index at the same date. The weights $\mathbf{w} = (w_1, ..., w_N)$ are the ones that minimize the sum of squared differences between the OWA operator values and the VSTOXX index values for the different time windows, or equivalently, the ones that minimize the Root Mean Square Error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{j=1}^{M} (\mathrm{OWA}_{\mathbf{w}}(a_1^j, ..., a_N^j) - b^j)^2}{M}} \ . \tag{12}$$

As the last step, we move the estimation window forward to the next period.

To evaluate the robustness of the window choice, we perform the proposed methodology using both overlapping and non-overlapping windows. In the first case (with overlapping windows), our estimation window considers 21, 63, and 121 trading days as a proxy for 1-month, 3-month, and 6-month time horizons. At each step, we move the window one week forward. One drawback of the proposed approach is that important market events such as the annual rebalancing of the EURO STOXX 50 index[9] can occur within the estimation window. Therefore, to evaluate the robustness of the results with respect to the choice of the estimation windows, we use non-overlapping windows based on calendar months with lengths equal to one, three, and six months. Shorter periods are not taken into account due to the existence of differences in the number of trading days on a weekly basis, especially during the Easter and Christmas periods. Moreover, for the three- and six-month estimation windows, we start the estimation from March 2007 instead of January to correctly match the index rebalancing date in September. The analysis was performed using Matlab R2021a. The results for the overlapping (resp. non-overlapping) exercise are reported in Table 4, Panel A (resp. Panel B), where we display for each estimation window the average weights, *orness*, and *entropy* (computed according to Eq. (10)), and the average RMSE.

Several observations can be made. First, the choice of the estimation window has a limited effect on the weighting vector. More specifically, the weights are focused mainly on the third (around 30%), the second and the fourth inputs (both around 20%), followed by the first one (usually between

[9] The EURO STOXX 50 composition is reviewed annually in September.

10% and 20%). Therefore, the VSTOXX index acts more like an OR-like measure than as an AND-like measure of volatility for the EU stock markets during the 2007-2017 period. This result is also confirmed by the average *orness*, which is slightly higher than 0.7. An average *orness* value of 0.7 indicates that the VSTOXX index signals high risk when four out of the nine country-specific volatility indices are high. Therefore, the VSTOXX index can detect a risky situation when a group of countries experiences a high level of volatility, but could fail to capture risky situations related to a single or a small group of countries such as peripheral ones.

Second, the sum of the weights associated with the indices ranging from the fifth to the ninth position is always lower than 20% and their relative weight tends to decrease as the length of the time window considered increases.

Third, the average RMSE increases with an increasing time window size, suggesting that optimal weights frequently vary over time. This intuition is supported by Figures 4-6, where we depict the evolution of weights estimated in Table 4, Panel A, along with the evolution of RMSE, *orness*, and *entropy*. Although the estimated weights are highly time-varying, the changes occur mainly among the first five indices, confirming that the VSTOXX index acted more like the maximum operation than as the minimum operation. In some cases, we can see non-zero weights on the right side of the figure (focused mainly on the seventh input), especially in the period characterized by the 2007-2009 financial crisis. The graphical representation of *orness* over time provides further insights into the VSTOXX index behavior. In particular, the index has always been above the 0.5 threshold except for one or two drops (depending on the estimation window used) during the 2007-2009 financial crisis. This means that the VSTOXX index is higher than the average of the volatility indices, except than during periods of extreme volatility where it acts like an average (*orness* = 0.5).

On the other hand, looking at the 1-month estimation window, the VSTOXX index can be obtained as the maximum three times during our sample period. However, this pattern occurs during periods of low (in the first part of 2007), medium (January 2010), and high (autumn 2015) values of the VSTOXX index, thus suggesting the absence of a clear relationship between the VSTOXX index behavior and volatility levels. We empirically checked this hypothesis by computing the correlation coefficients between the VSTOXX index level and the *orness* estimates, which turned out to be very close to zero. Therefore, the VSTOXX index changed its behavior during the sample period, and the changes were not related to an increased or decreased volatility risk. This result casts doubt on the suitability of the VSTOXX index as a measure of market volatility for all the EU countries.

Last, the *entropy* is also highly time-varying, indicating that depending on the time window considered, it was necessary to account for a different number of inputs to obtain the optimal fit. We do not detect a strong relationship between *entropy* and volatility, except during the 2007-2009 subprime crisis in which higher levels of the VSTOXX index are associated with a higher value of *entropy*.

5. Conclusions

The VSTOXX index is the only option-implied index intended to monitor the risk of the EU financial market as a whole. While it represents the equivalent of the VIX index for the European markets (Kalyvas et al., 2020), the VSTOXX index has not gained the same outstanding reputation as the VIX, and has received some criticism in the literature regarding its representativeness of the EU market (see, e.g., López and Esparcia, 2020; Peterburgsky, 2021). Despite the crucial role of the VSTOXX index as a measure of risk for the EU stock market, there are no studies investigating its behavior and the relationships between the VSTOXX index and the country-specific volatility indices. To fill this gap, we computed model-free implied volatility indices for nine index options markets in the EU during the 2007-2017 period, and we inspected and contrasted the properties of the VSTOXX index under various market conditions.

We found several results. First, the VSTOXX index attains an average value higher than the average of the country-specific volatility indices. The volatility indices of Spain and Italy were high during the sample period, while the volatility indices of Switzerland and the UK were the lowest on average, indicating that these markets acted as a safe haven during the European debt crises. Second, the VSTOXX index is strongly related to the French and German volatility indices during the entire sample period, given the high weight of these countries in the EURO STOXX 50 index. On the other hand, the relationship between the VSTOXX index and volatility measures in other countries depends on the specific period under investigation. Moreover, peripheral country volatility indices in our dataset (Italy, Spain) are the least correlated with the VSTOXX index, especially in the 2010-2012 period, thus casting doubt on the ability of the VSTOXX index to measure risk for these countries correctly.

Third, the OWA analysis shows that the VSTOXX index acts more like an OR-like measure than as an AND-like measure of volatility for the EU stock markets during the 2007-2017 period. since the average *orness* is slightly higher than 0.7. In particular, the index has always been above the 0.5 threshold except for one or two peaks (depending on the estimation window used) during the 2007-2009 financial crisis. This means that the VSTOXX index is higher than the average of the volatility indices, except than during periods of extreme volatility where it acts like an average (*orness* = 0.5). The fact that the behavior of the VSTOXX index changes during the sample period casts further doubts on the suitability of the VSTOXX index as a measure of market volatility for all the

EU countries. Moreover, an average *orness* level of about 0.7 indicates that the VSTOXX index signals high risk when 4 out of the 9 country-specific volatility indices are high. Therefore, the VSTOXX index can detect a risky situation when a group of countries experiences a high level of volatility. On the other hand, the VSTOXX index could fail to capture risky situations related to a single or a small group of countries such as peripheral ones. This result calls for new measures of risk that can complement the information provided by the VSTOXX index and capture the complexity and heterogeneity of the EU markets.

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Table 1 – Options market characteristics in the nine European countries in our dataset

Index (Country)	(i)	(ii)	(iii)	(iv)
AEX (Netherlands)	72	37	0.25	4500
BEL (Belgium)	37	23	2.00	4300
CAC (France)	71	22	2.50	4800
DAX (Germany)	172	53	5.00	4500
FTSE (UK)	119	37	2.50	6600
IBEX (Spain)	93	23	5.00	5500
MIB (Italy)	60	27	1.00	6000
OMX (Sweden)	64	38	0.50	6600
SMI (Switzerland)	145	81	5.00	4200

The table reports:

i) The average number of strike prices available before filtering;
ii) The average number of strike prices available after filtering;
iii) The country-specific parameter ΔK chosen to make discretization errors negligible;
iv) The average number of strike prices (after the interpolation-extrapolation procedure) used to plug-in formulas in Eq. (3).

Table 2 - Descriptive statistics

Index	Average	Median	Min	Max	Std. dev.	Skewness	Kurtosis	Jarque-Bera
AEX	22.12	19.23	8.97	93.20	10.38	2.29	10.44	9126.54***
BEL	20.77	18.30	7.14	88.71	8.77	2.40	12.02	12479.42***
CAC	23.33	21.24	5.73	86.76	9.18	2.04	9.42	6915.80***
DAX	22.99	20.61	10.20	93.82	9.24	2.35	11.35	10969.81***
FTSE	19.82	17.39	8.67	95.91	9.18	2.41	12.06	12608.51***
IBEX	26.05	24.16	8.11	81.21	9.24	1.69	7.48	3767.30***
MIB	26.70	24.57	10.99	81.20	9.29	1.49	6.34	2396.32***
OMX	21.37	19.22	7.05	88.75	8.86	2.32	11.23	10661.36***
SMI	18.99	16.59	6.94	97.66	8.45	2.86	15.12	21463.75***
VSTOXX	24.28	22.23	10.68	87.51	9.14	1.95	8.78	5820.29***

The table reports, for each EU country under investigation plus the VSTOXX, the descriptive statistics of the risk-neutral volatility measure. The abbreviations for the nine countries (the most left-hand side column) are described in Table 1.

Table 3 –Pearson correlation coefficients

	AEX	BEL	CAC	DAX	FTSE	IBEX	MIB	OMX	SMI	VSTOXX
AEX	1.000									
BEL	0.969	1.000								
CAC	0.970	0.962	1.000							
DAX	0.969	0.951	0.979	1.000						
FTSE	0.972	0.969	0.961	0.953	1.000					
IBEX	0.867	0.886	0.921	0.881	0.865	1.000				
MIB	0.845	0.843	0.906	0.884	0.813	0.918	1.000			
OMX	0.960	0.978	0.968	0.960	0.966	0.899	0.863	1.000		
SMI	0.958	0.953	0.939	0.946	0.969	0.823	0.788	0.946	1.000	
VSTOXX	0.969	0.958	0.992	0.980	0.957	0.932	0.916	0.964	0.931	1.000

Table 4 – Fitting exercise: OWA weight results and statistics

Window length:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	RMSE	*orness*	*entropy*
Panel A: overlapping estimation												
1-month	16.65%	18.90%	30.42%	17.25%	7.50%	2.66%	3.01%	1.93%	1.68%	0.481	0.725	0.436
3-month	12.36%	20.41%	30.93%	21.96%	6.82%	1.92%	2.50%	1.16%	1.94%	0.649	0.720	0.485
6-month	10.54%	22.84%	29.82%	23.59%	5.87%	1.83%	2.44%	1.56%	1.52%	0.739	0.721	0.501
Panel B: non-overlapping estimation												
1-month	15.76%	18.99%	31.71%	16.71%	8.46%	2.46%	2.80%	1.97%	1.14%	0.482	0.727	0.430
3-month	13.50%	20.51%	30.60%	22.16%	5.96%	1.37%	2.90%	1.36%	1.65%	0.652	0.726	0.463
6-month	11.20%	21.73%	30.78%	23.06%	6.62%	1.48%	1.42%	1.88%	1.84%	0.751	0.722	0.482

We report in the table the results for the fitting exercise proposed in Section 4. For each time window reported in the first column, the series of daily values of the nine country-specific indices (sorted in descending order) are used as input for the OWA operator, whose weights are computed by solving the optimization problem described by Eq. (11). The results for the overlapping (resp. non-overlapping) exercise are reported in Panel A (resp. Panel B), where we display for each estimation window the average values of weights, root mean square error (RMSE), orness, and entropy. For a definition of the measures, see Section 4.

Figure 1 – Relation between the VSTOXX index and country-specific volatility indices for different levels of the VSTOXX index.

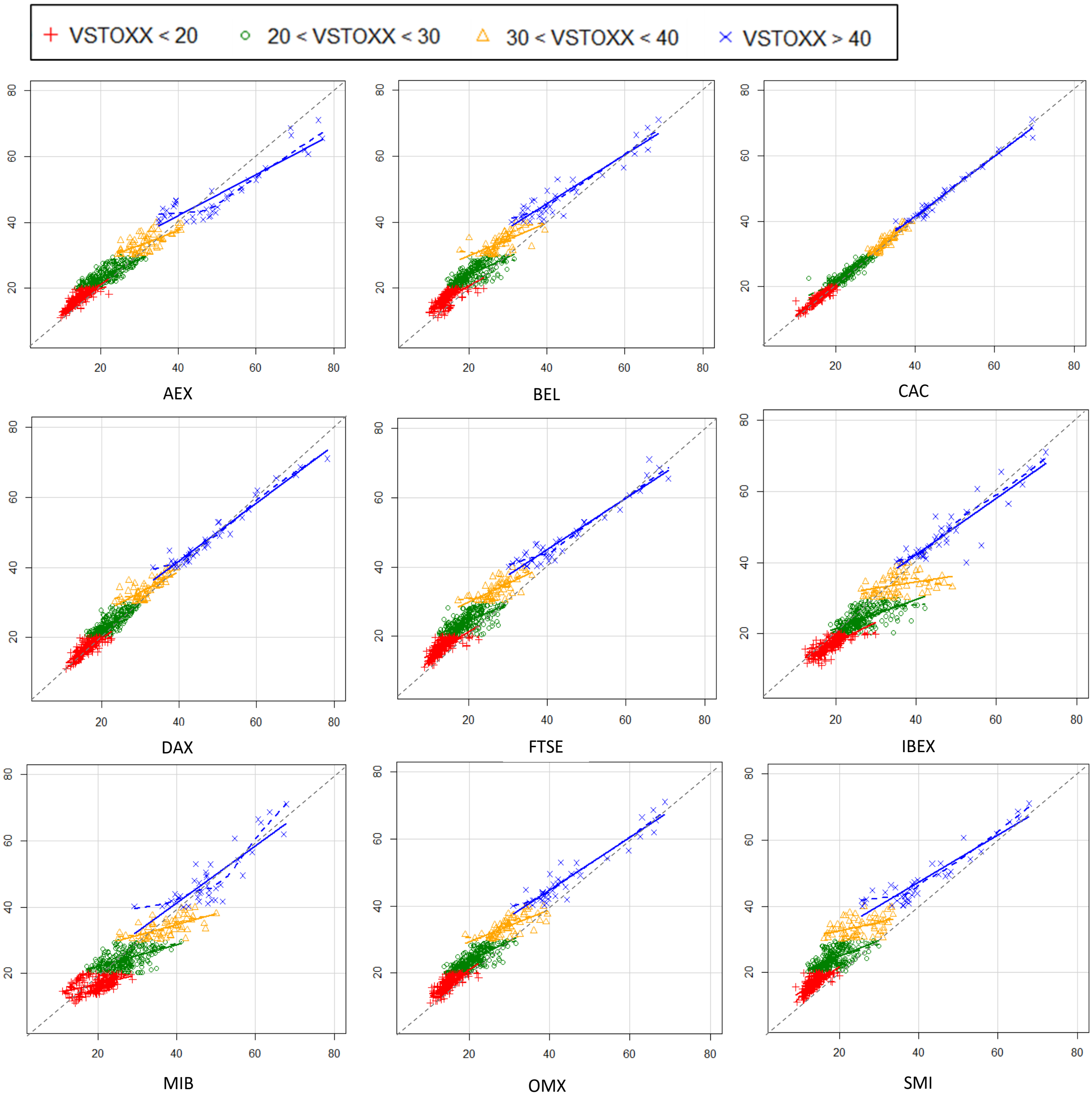


Figure 2 – Relation between the VSTOXX index and country-specific volatility indices during different sample periods.

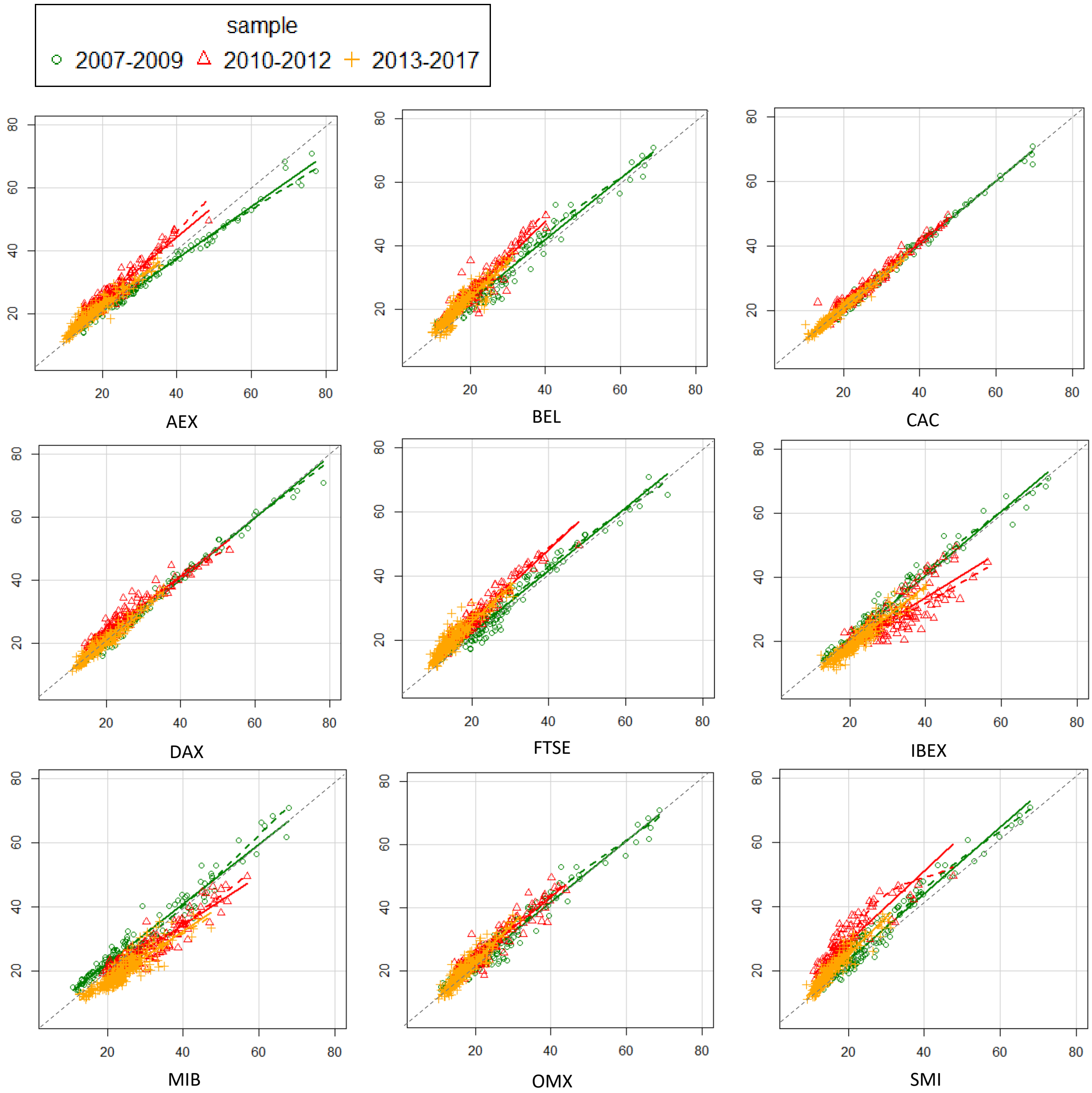
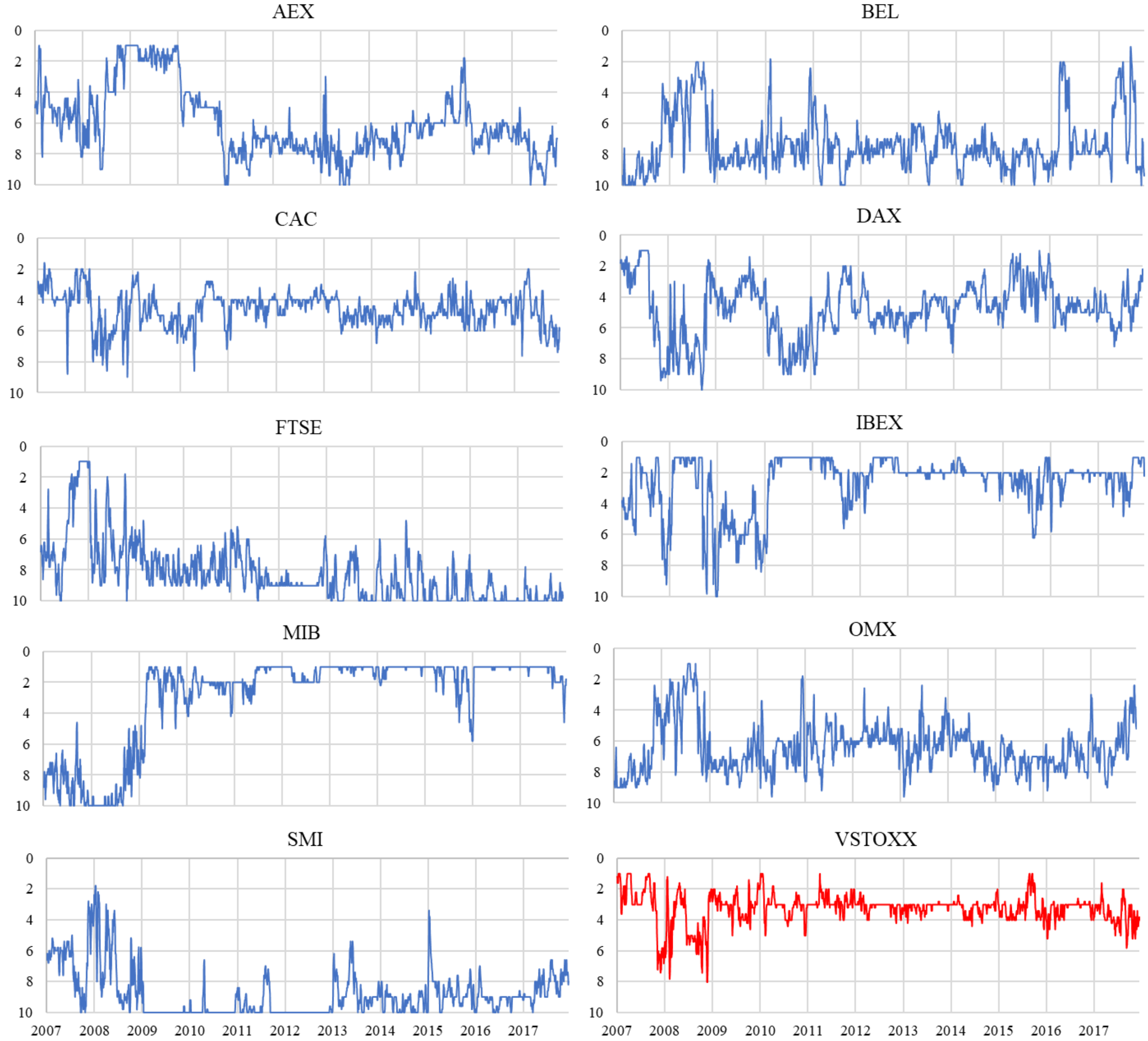
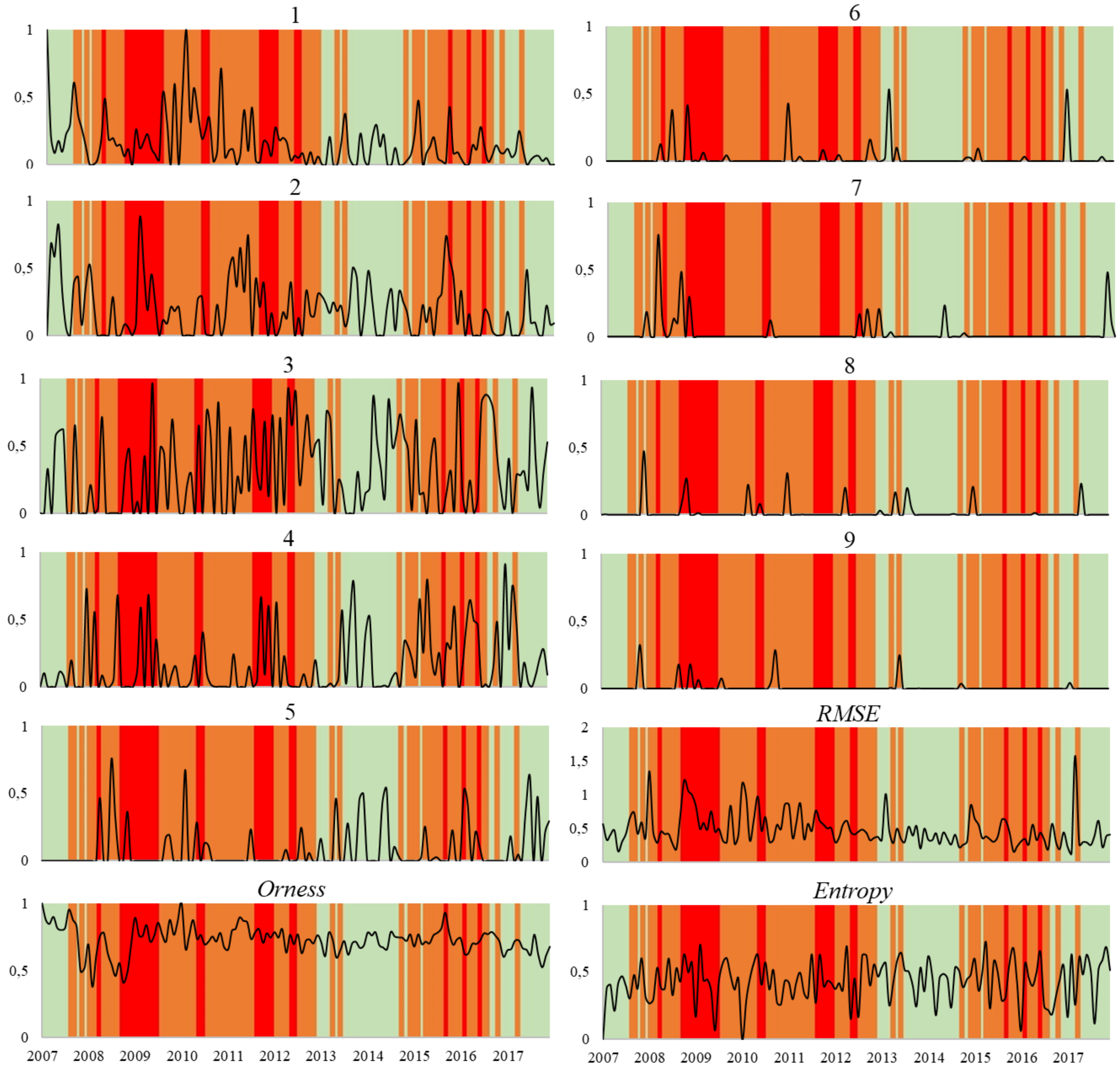


Figure 3 – Ranking of the volatility indices over the sample period (2007-2017)



We rank each of the nine volatility indices plus the VSTOXX index from the highest (1) to the lowest (10) and we report the evolution over time of the 5-day average of the ranking.

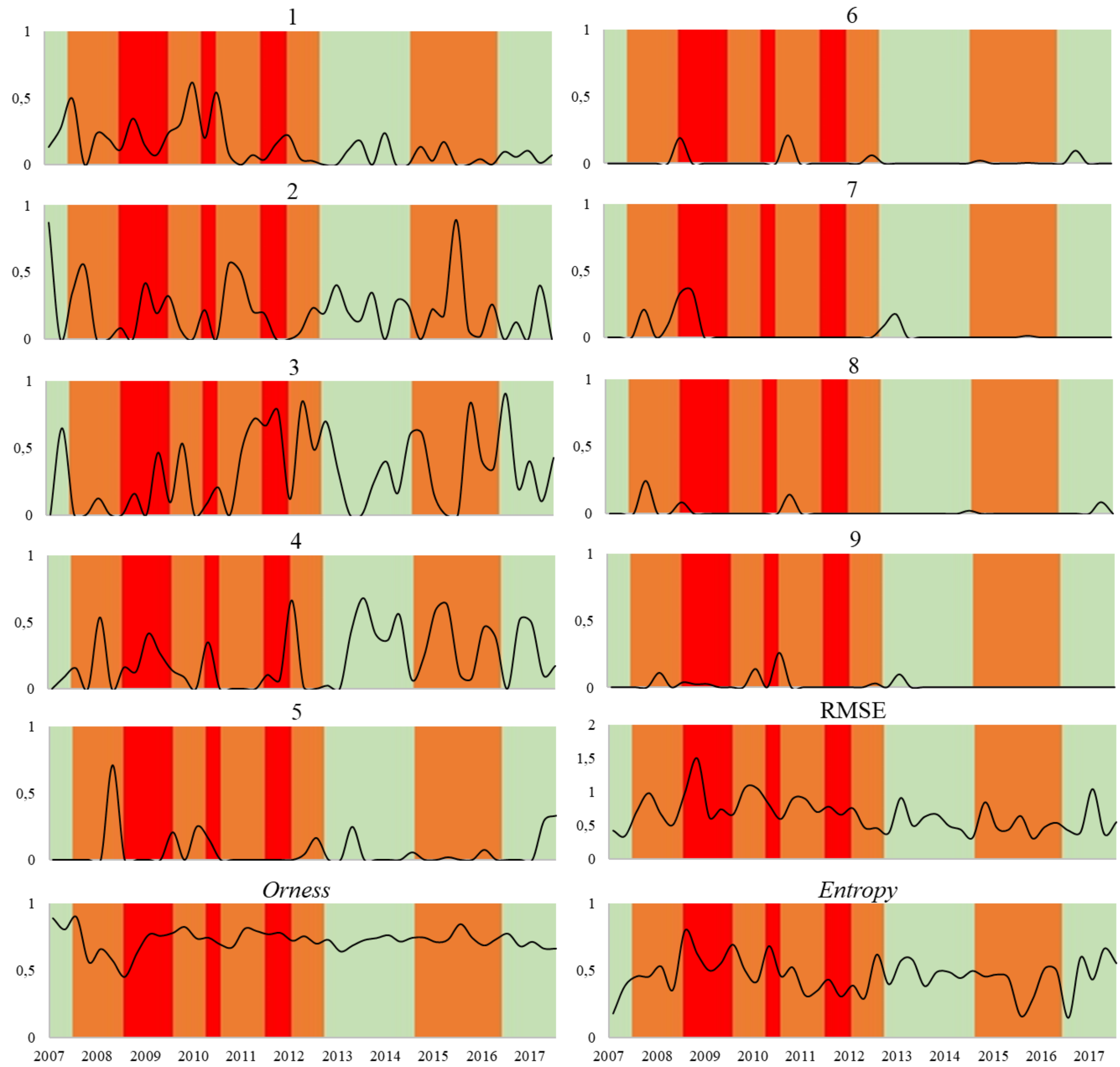
Figure 4 – OWA estimation results for the 1-month estimation window (non-overlapping)



We report the evolution over time of the estimated weights, root mean square error (RMSE), *orness*, and entropy measures obtained for the fitting exercise proposed in Section 4. Shaded areas refer to different levels of volatility measured by the VSTOXX and used to contrast the results of the optimization exercise during different market conditions:

- green: VSTOXX < 20;
- orange: 20 < VSTOXX < 30;
- red: VSTOXX > 30.

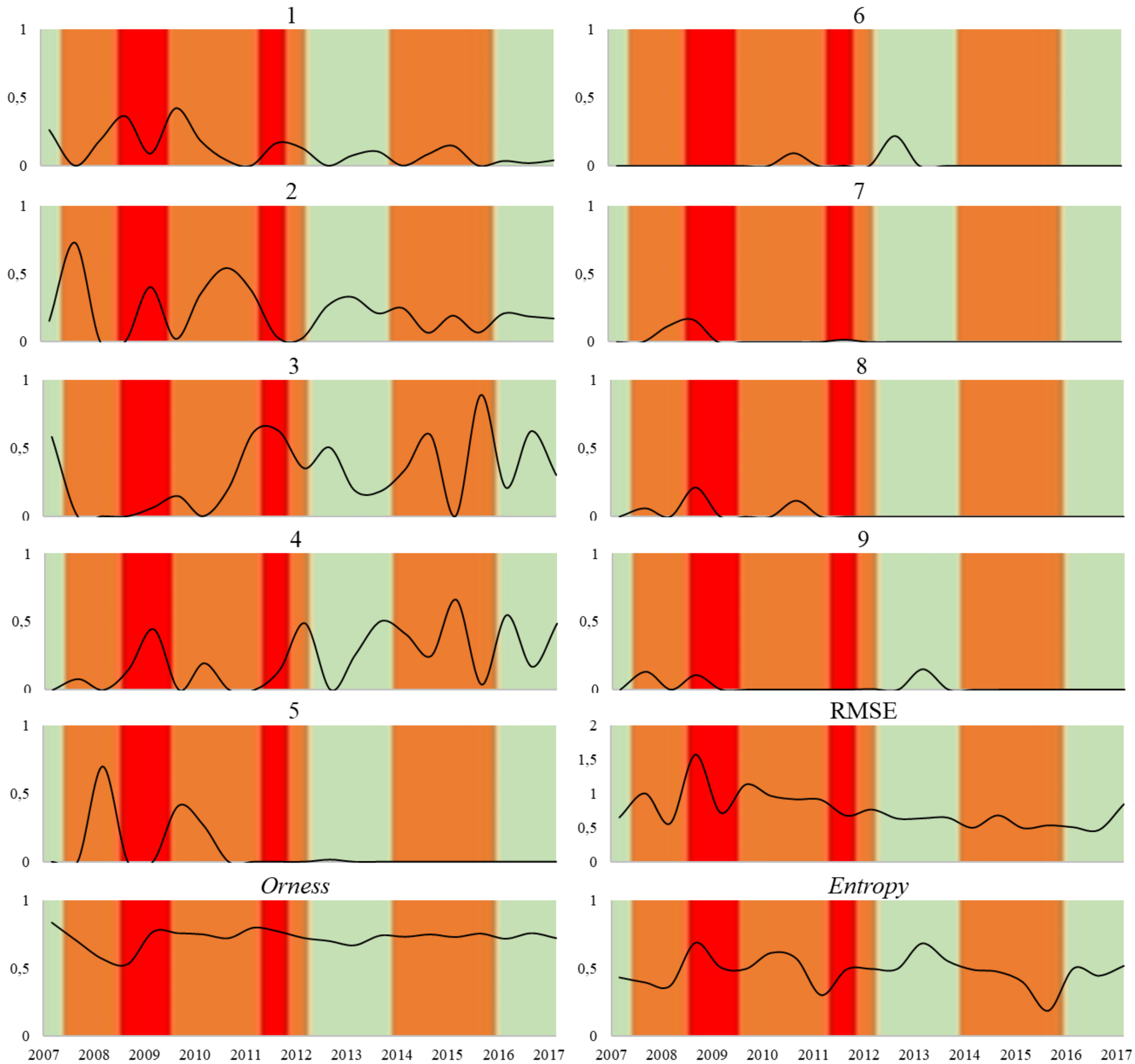
Figure 5 - OWA estimation results for the 3-month estimation window (non-overlapping)



We report the evolution over time of the estimated weights, root mean square error (RMSE), *orness*, and entropy measures obtained for the fitting exercise proposed in Section 4. Shaded areas refer to different levels of volatility measured by the VSTOXX index and used to contrast the results of the optimization exercise during different market conditions:

- green: VSTOXX < 20;
- orange: 20 < VSTOXX < 30;
- red: VSTOXX > 30.

Figure 6 - OWA estimation results for the 6-month estimation window (non-overlapping)



We report the evolution over time of the estimated weights, root mean square error (RMSE), *orness*, and entropy measures obtained for the fitting exercise proposed in Section 4. Shaded areas refer to different levels of volatility measured by the VSTOXX index and used to contrast the results of the optimization exercise procedure during different market conditions:

- green: VSTOXX < 20;
- orange: 20 < VSTOXX < 30;
- red: VSTOXX > 30.