Journal of Physics: Conference Series

Pescara benchmark: overview of modelling, testing and identification

To cite this article: A Bellino *et al* 2011 *J. Phys.: Conf. Ser.* **305** 012131

Pescara benchmark: overview of modelling, testing and identification

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Abstract. The "Pescara benchmark" is part of the national research project "BriViDi" (BRIdge VIbrations and DIagnosis) supported by the Italian Ministero dell'Università e Ricerca. The project is aimed at developing an integrated methodology for the structural health evaluation of railway r/c, p/c bridges. The methodology should provide for applicability in operating conditions, easy data acquisition through common industrial instrumentation, robustness and reliability against structural and environmental uncertainties. The Pescara benchmark consisted in lab tests to get a consistent and large experimental data base and subsequent data processing. Special tests were devised to simulate the train transit effects in actual field conditions. Prestressed concrete beams of current industrial production both sound and damaged at various severity corrosion levels were tested. The results were collected either in a deterministic setting and in a form suitable to deal with experimental uncertainties. Damage identification was split in two approaches: with or without a reference model. In the first case f.e. models were used in conjunction with non conventional updating techniques. In the second case, specialized output-only identification techniques capable to deal with time-variant and possibly non linear systems were developed. The lab tests allowed validating the above approaches and the performances of classical modal based damage indicators.

1. Introduction

The paper is aimed at presenting the general framework of the "Pescara benchmark" and the related research activities. The detailed results pertaining to each activity are discussed in a number of separate companion papers. The Pescara benchmark is part of a larger national research project co-funded by the Italian Ministero dell'Università e Ricerca named "BriViDi" (BRIdge VIbrations and

DIagnosis). The project involved the joint cooperation among specialised research teams of the Torino, Bologna, Roma, Trieste and Chieti-Pescara Universities providing expertise in the different field of testing, signal processing and f.e. modeling and updating. The final target of the BriViDi project is the development of an integrated methodology for the structural health evaluation of reinforced concrete (r/c) or prestressed concrete (p/c) railway bridges subjected to damage caused by natural factors such as corrosion of the reinforcement. Corrosion damage, moving loads and prestress force effects are the three keywords that make innovative the character of the research. In fact, in the literature, only recently has been given attention to these topics and the results are often scarcely coherent and sometimes contradictories. A brief account of the main literature results is given below. Apart from theoretical aspects, the methodology should also satisfy three further important practical requirements that are: suitability for applications in operating conditions; easy data acquisition through common industrial instrumentation; robustness and reliability against structural and environmental uncertainties. Such a challenging research needs validation of the procedures and of the theoretical / numerical results against controlled laboratory data. In order to get a consistent and large data base of experimental data to work with, special tests have been devised to simulate actual field conditions and carried out at the laboratory of Pescara University. This constitute the so called Pescara benchmark. The tests involved prestressed concrete beams of mass industrial production. The beams were either sound or damaged at various severity corrosion levels. Static and dynamic characterization tests accompanied simulations of train transits. Special mechanical devices running on the beams were realised on purpose. Three beams were arranged in sequence to capture the dynamics of the incoming, crossing and outgoing train on the central damaged and monitored span. The results are provided either in a deterministic setting or in a form suitable to deal with experimental uncertainties. Two different processing were followed to analyse the experimental data. The first makes use of f.e. reference models and exploits concepts of model updating to identify damage in different corrosion states of the beam. Non conventional methods were used and developed: one based on interval analysis formulation to account for modeling and experimental uncertainties; the other operating in a deterministic setting and based on concepts of high order, adjusting shape functions. The second adopts output only identification techniques of various sophistication level ranging from standard Fourier based processing mounted on common industrial instrumentation to time-frequency or short-time stochastic subspace identification capable to deal with non linear, time-variant systems. Stationary and evolutionary modal parameters are estimated and damage estimated through the analysis of their changes. The paper is organised as follows. Initially, the relevance of corrosion, prestress and moving loads on the beam dynamics are briefly discussed according to the main findings of the literature. Then, the activities of the different research units are introduced and discussed. They involve the experimental campaign (Pescara University PE) and the pretest phase (Trieste University TS); the signal processing for (quasi)modal parameters identification: stationary (Pescara University) and non stationary (Torino Politecnico TO); the f.e. model parameters updating with deterministic FRF (Bologna University BO) and experimental uncertainties (Roma Tre University RM3).

2. Factors affecting the p.c. beam dynamics

The three main causes that influence the dynamic response of a prestressed concrete bridge beam are: the loss of stiffness and strength due to ageing effects such as corrosion, the possible changes of the prestressing force, the traffic loads that are not negligible for railway bridges. A brief account of the state of the art concerning these topics is given in the following.

2.1. Corrosion damage effects

Experimental modal analysis is often used as a tool in health monitoring strategies to assess the system state. Safe results can be obtained depending on how much the modal parameters are actually sensitive to damage. This problem is still an open question for civil structures prone to corrosion. In [1] the result of dynamic tests on a p.c. beam are studied. Preliminary static bending tests exceeding the cracking load were performed. Then, damage due to corrosion was simulated by the progressive cut of the prestressing strands until 50% reduction. Changes in natural frequencies were detected when the reinforcement reduction was over 30%. The first bending frequency suffered the highest decay. In [2] three r.c. beams subjected to cycles of accelerated corrosion in the compressive zone are tested. One beam was kept undamaged for reference. Neither the corrosion entity nor its effect on the concrete are quantified. It is reported that the first and third frequencies decays respectively of about 4% and 3%. In [3] two r.c. beams were corroded until a 7% loss of steel area in the tensile zone and one was kept sound as control beam. Concrete cracking ranged from very slight spalling to moderate cracking. The drop of the first seven frequencies was quite different between the two beams: 4% to 11% and 1 to 4%. The higher frequencies suffered the higher decay. Changes of the modal damping ratios were also measured for the first three modes. However the changes were not consistent with those observed for the frequencies. In [4] 26 r.c. beams, with the same geometry and reinforcement scheme, were subjected to different chemical attacks (sulfate, carbonation, chloride). All the beams were subjected to a preliminary loading process to induce cracks and favour the chemical attack. After one year of chemical attack variations in natural frequencies were not clearly defined. Actually, a slight increment of the stiffness was observed. The proper account of the supports stiffness was considered important for correct identification. In [5] the dependence of the first three natural frequencies on the level of reinforcement corrosion is examined by the help of a non linear f.e. model calibrated on experimental results. Corrosion damage up to 20% of steel area loss is considered. It is found that higher order frequencies are more sensitive to corrosion damage. In view of the above, no clear and univocal conclusions can be drawn.

2.2. Prestress force effects

The problem of the evaluation of the influence of the prestress force P_f on the dynamic behaviour of p.c. beams has received increasing attention. This is particularly true in the present case where the corrosion attack reduces the net area of the steel strands and causes fracturing of the surrounding concrete. First consideration was given in [6] where P_f was thought as an external force. Using this assumption it was found that frequencies are lowered by P_f. In [7] hypotheses similar to [6] were used to compared theoretical and laboratory results. A mismatch between the two was observed. It was found that the experimentally evaluated frequencies show an opposite trend: they decrease as P_f decreases. The motivation is that the coupling between the compressed beam and the tensioned strands is not accounted for. Following [7] a debate was opened. Various authors arrive at similar conclusions: in a linear model the influence of P_f is negligible [8], the influence of P_f does not modify the beam frequencies in any case [9], P_f cannot cause changes of stiffness since it should be considered an internal force [10]. Subsequent works faced the problem, but remained defective in the theoretical assumptions [11], [12] and [13]. A general and rigorous model was proposed by [14]. The mathematical formulation considers both cases of bounded and un bounded cables in an Euler beam subjected to large displacements. The coupling between the compressed concrete beam and the tensioned steel strands is accounted by proper equilibrium and compatibility conditions. The outcome of the work is that P_f does not affect in any case the natural frequencies of the beam. In conclusion, as concerns the present work, this parameter will not be further considered in the interpretation of results.

2.3. Moving loads effects

The effects of moving loads on the beam dynamic response received considerable attention in the literature since several decades [15]. Many works deal with the evaluation of the amplification factor as in [16] or the identification of the moving forces [17]. Damage evaluation is carried out through equivalent loss of stiffness either concentrated (sided cracks) [18] or distributed [19]. Time varying structural matrices, nonlinearities and spatial incompleteness of measured data are also accounted for [19]. Theoretical or numerical formulations are usually followed. Tests and experimental validation are only occasionally employed. Also in more recent works [20], mechanical damage and concrete beams are almost exclusively addressed. Natural damage such as corrosion and prestressed concrete beams seem to be a relatively novel subject. Moreover, the study of simultaneous nonlinear and time-varying systems is usually treated separately, mainly due to the articulated approaches needed to face these topics. Few researchers have treated the joint problem. For example, in [21] the authors resort to a mechanical device including these two effects. In [22], instead, the proper orthogonal decomposition is applied to both nonlinear and time-varying systems, in order to detect the different characteristics. In general, however, the nonlinear systems are identified with specific tools, as in [23], [24] and [25]. On the other side, the time-varying systems are usually analyzed with parametric methods [26] or techniques extended from the linear time-invariant case, such as the “frozen technique” [27] and [28].

3. Experimental setup and testing

The execution of the experimental tests was preceded by a design phase targeted to define all the main variables in order to get results as controlled as possible. The design phase involved the optimal sensors placement, the type of p.c. beams, the corrosion pattern and entity, the device providing for the moving load. These topics are briefly discussed in the following.

3.1. Optimal sensors placement

Pretest techniques help to select the best measurement stations where to locate the sensors during the design phase of dynamic tests. For each structural point and direction, the selection makes use of a merit index derived from the numerical modal shapes of the structure under consideration. In the present case, where the fraction of moving mass (train) cannot be neglected respect to the beam mass (bridge deck) the classical indices are scarcely valuable since they do not account for the changes of the modal shapes during the train transit. To this end a new merit index has been formulated where the classical modal shapes are substituted by modal envelopes. These latter are defined as a proper combination of the different modal shapes displayed by the beam for a discrete time set of the moving load positions. The new merit index considers simultaneously all the relevant modal shapes and in this sense it characterises globally the measurement stations. The combination of the modal shapes relevant to different load positions should be done carefully since the modal shapes can result differently scaled. The effectiveness of three different methods MSSP (Mode shape summation plot), NMD (Normalised modal displacements) and NKE (Nodal kinetic energy) were analysed. The results showed satisfactorily coherent, figure 1.

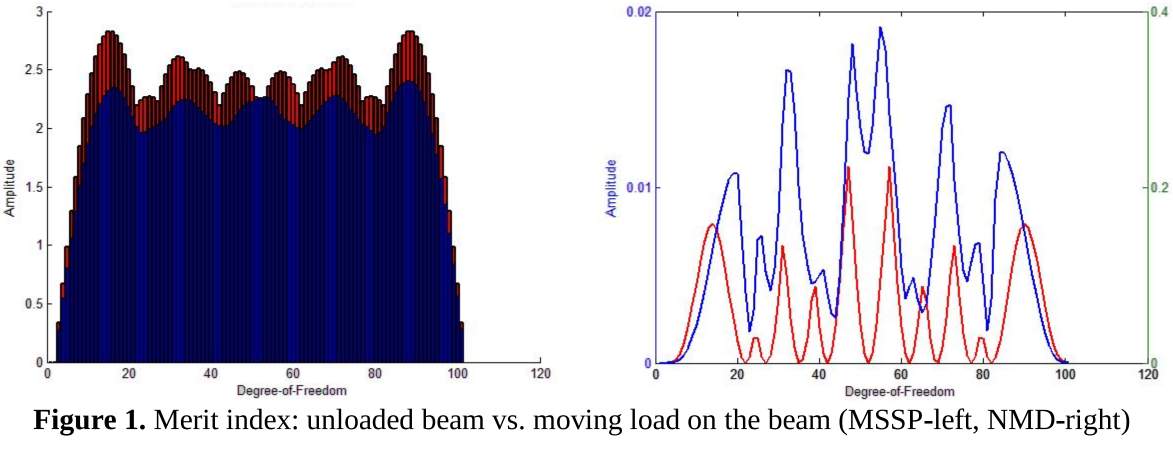
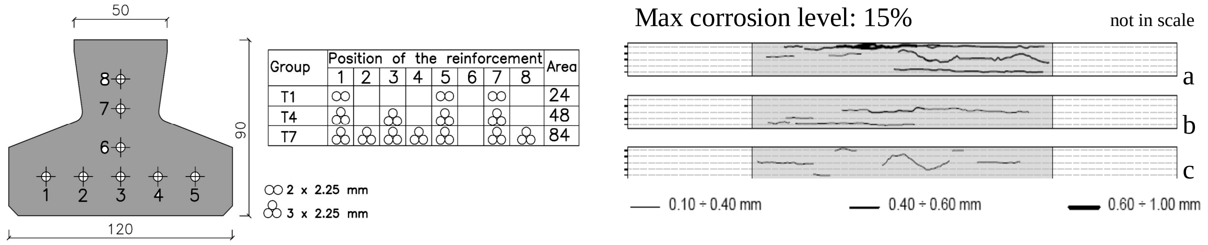


Figure 1. Merit index: unloaded beam vs. moving load on the beam (MSSP-left, NMD-right)

3.2. P.c. beams

Three different groups of beams derived from mass production were considered: T1, T4 and T7 composed by 5 joists each. One beam per group was preserved sound, the others corroded. The beams have nominal identical geometry, but differ by the amount and arrangement of the prestressing steel strands, figure 2. In view of the corrosion attack, the beams are representative of small (T1) to high

(T7) reinforcing steel percentage and the number and position of the steel strands is such to favour different cracking mechanisms: from spalling or localised cracking (T) to delamination or diffused cracking (T7).



Group	Position of the reinforcement								Area
	1	2	3	4	5	6	7	8	
T1	○○				○○		○○		24
T4	○○○		○○○		○○○		○○○		48
T7	○○○	○○○	○○○	○○○	○○○		○○○	○○○	84

○○ 2 x 2.25 mm
○○○ 3 x 2.25 mm

Figure 2. Beam cross section geometry and typical cracking patterns.

3.3. Corrosion induced damage

The beams were corroded only in the central part equal to 60% of the total length = 320 cm. Both end were preserved by corrosion to prevent loss of bond. The corrosion entity is measured by the percent loss of the effective steel area since this parameter has direct relevance in terms of the beam strength. Two corrosion levels were considered for each beam: one up to 5% and the second up to 15%. The corrosion process was artificial and accelerated. The central part of the beams was immersed in a electrolitic water solution at 5% NaCl and the current provided and kept constant by a dc power supply. The time duration of the process to achieve predefined corrosion levels was estimated using the Faraday's law previously validated. At the end of each corrosion step the map of the concrete cracking was taken. The cracking maps of group T7 are depicted in figure 2 where it is evident that the damage suffered (concrete cracking) revealed quite different even if the beams experienced the same corrosion process and level.

3.4. Moving loads

Two different trolleys have been designed and realized to perform dynamic tests with moving mass (MV) figure 3. The shape of the cross section of the beam acts as a natural rail. Because of the rugged surface of the beams, it was necessary to add some guides (external discs or internal tongues) to stabilize the run. The MV_S trolley was designed to provide a point force in order to have data comparable with classical theoretical assumptions. The MV_D trolley was designed with the purpose of getting changes of the moving mass. For this reason trolley MV_S is made by a single axle with two heavy metal wheels, whereas trolley MV_D is double axles C-shaped with hard rubber coated wheels and houses a top box to host variable masses. The mass ratios between the trolleys and the beam are 1:5 for MV_S and 1:15, 1:10; 1:5 for MV_D.

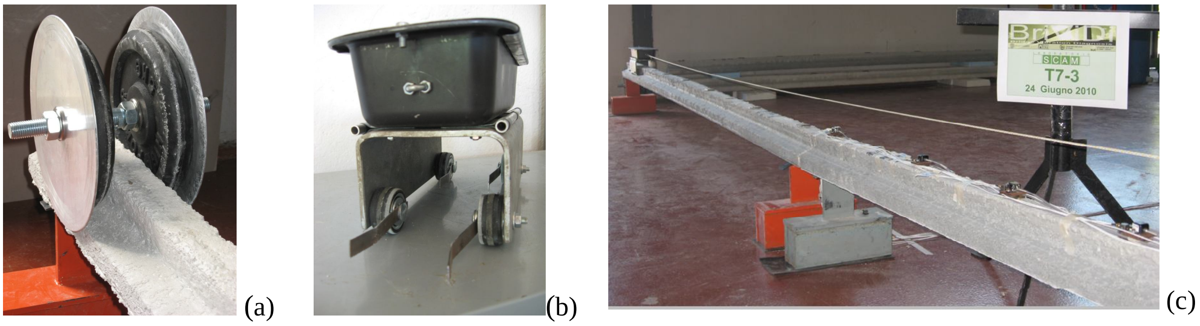


Figure 3. Moving loads: (a) trolley MV_S, (b) trolley MV_D, (c) trolley MV_D at launch

3.5. Experimental tests

Due to some unexpected results during dynamic testing of group T1 and T4, it was decided to perform a static characterization of the remaining group T7 before starting the dynamic tests. The static tests were conceived to give a preliminary and independent evaluation of the damage suffered by the beams. Damage is herein intended as the equivalent loss of stiffness uniformly distributed along the corroded beam length. Two types of dynamic tests and their combination were performed: impulse (I) tests and moving load (MV) tests according to the static scheme of a simply supported beam resting on heavy steel supports (span length L = 300 cm). Three different I tests were considered: (M) hammer, (R) sudden release of a point load, (C) sudden settlement at one of the supports. The impulse was not measured. In the MV case two further spans with the same static scheme were added before and after the instrumented beam to account for the effects of the incoming and exiting moving trolley as for a real bridge. The trolley speed was not measured. Two different type of transit were considered: constant velocity driven by an engine and constant acceleration obtained by the pulling force exerted by a calibrated falling body. The dynamic response was recorded using 9 accelerometers with f_{sampl} = 2000 Hz. The accelerometer placement was driven by the results of the pretest phase. Each type of test was repeated a number of times to estimate the experimental uncertainties involved.

4. Identification methods

Two separate processing of the acquired data were performed by the PE and TO research units. The PE unit analysed only the impulse tests in order to get a preliminary characterisation and qualification of the recorded data either deterministic or not. The average modal parameters (frequency, damping and modal shapes) and the associated uncertainties were identified. The TO unit analysed both impulse and moving load tests with innovative signal processing techniques capable to deal with nonlinearity and time-variance simultaneously and to keep trace of the evolutionary behaviour of frequency and damping.

4.1. Identification of equivalent modal parameters

The impulse tests were processed using three different identification methods according to as many analysis domain: frequency, time and joint time-frequency domain. The three methods are all of output-only type and in this sense the operational modal parameters were identified. The frequency domain method was considered as a first level processing extended to all measurements. For that reason the peak picking method based on the Fourier transform (FT) was used [29]. It is in essence a sdof type method that is repeatedly applied for all the interested frequencies. All the modal parameters can be identified yet with some approximation. To get a more robust and accurate identification of the frequency and damping the time domain Hilbert transform method (HT) [30] was applied to a selected subset of tests. This is also a sdof type method and was used to process only the fundamental vibration mode. The HT was used also to qualify the type of damping and possible non linearity of the beam response. The same subset of tests was analysed using the Gabor transform method (GT). This is a considerably general method of mdof type [31] with intermediate accuracy between FT and HT.

4.2. Identification of nonlinearity and moving loads

The moving load tests were processed using a modified version of the SSI (Stochastic Subspace Identification) algorithm capable to account for the “instantaneous” evaluation of the modal parameters of the beam and hence termed ST-SSI (Short Time SSI). The capability to estimate the modal parameters in successive signal time windows allows to find the relationship between frequency and time. In this respect the algorithm, pertaining to the class of the so called “frozen techniques” [28], can be applied to non linear systems as well. When non linear time-variant structures are the main concern, as in the present case where a moving load excites a corrosion damaged beam, then the frequency changes come from two different sources and should be separated for a proper damage detection and quantification. The identification procedure estimates first the possible nonlinearity presence, then quantifies the nonlinearity and finally “subtracts” the nonlinearity contribution from the beam dynamic properties. At the end of the process the “purified” natural frequencies will depend only on the load properties (average velocity and mass). These latter are estimated by fitting the theoretical expression of the frequency of a simply supported beam traversed by a constant velocity point mass to the experimental “purified” frequencies. Two sample identifications are given in figure 4. The first refers to free decaying vibrations and shows a softening behaviour of the vibrating beam. The second refers to the moving load forced vibrations and show the frequency after the elimination of the nonlinear effects (i.e. constant frequency value after the MV passage, t > 12 s).

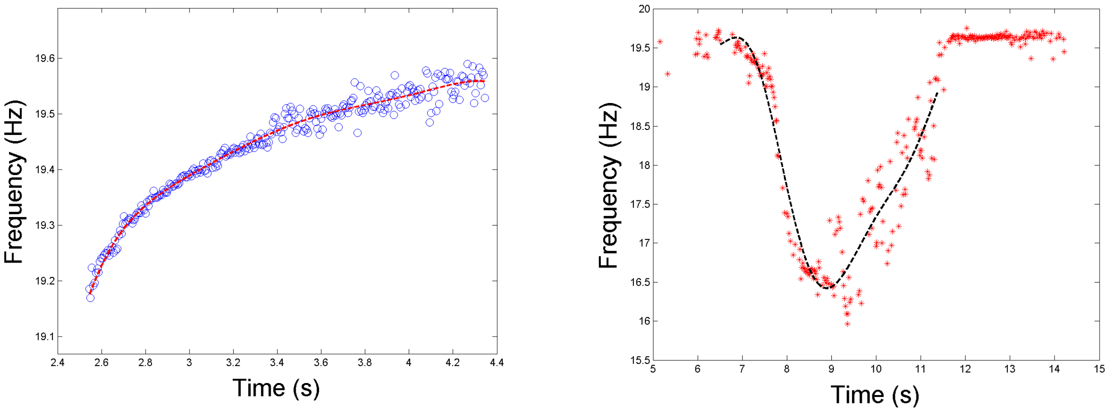


Figure 4. First natural frequency. Test type C (left); test type MV_D (right)

5. Model updating

The identified modal data were used by the BO and RM3 research units within model updating procedures. The BO unit worked with frequency domain data (FRF) in a deterministic setting using non conventional f.e. formulations. The RM3 unit worked with modal parameters in the context of interval analysis to account for experimental and modelling uncertainties. In both cases a twofold objective was pursued: to set up accurate f.e. models of the sound beams and to detect damage through changes of the beam mechanical properties between the corroded and sound state.

5.1. FRF based B-spline f.e. model updating

A large variety of f.e. updating methods can be found in the literature [32]. Iterative techniques allow to update a large number of parameters and to obtain parameters corrections usually having physical meanings, on the contrary they reveal computationally demanding. This drawback can be relieved by the use of B-spline shape functions that show superior accuracy as compared with classical polynomial f.e. models, especially when dealing with vibration problems [33]. This result may be useful in application such as f.e. updating to reduce the number of dofs [34]. In view of the above, an updating procedure of a B-spline based f.e. beam model was developed. The f.e. beam was formulated according to the Euler-Bernoulli kinematic assumptions in terms of only transversal displacement dofs with condensed rotational inertia. The beam model coefficients (e.g. flexural stiffness, mass density, damping ratio, joint stiffness) and the displacement field are described as continuous parametric functions by means of B-spline shape functions. The updating is carried out through the least squares minimization of an objective function of the residues, defined as the difference between the model response and the experimental FRF response, at the same frequency and excitation. The minimization is solved by using a truncated linear Taylor series of the objective function and an iterative formulation. The use of FRFs provides a large amount of input data that mitigate the general ill conditioned problem especially when the number of updating parameters is increased. The parameters to be identified are the coefficients of a distributed constraint stiffness model and the damping ratios, both modelled by means of B-spline functions.

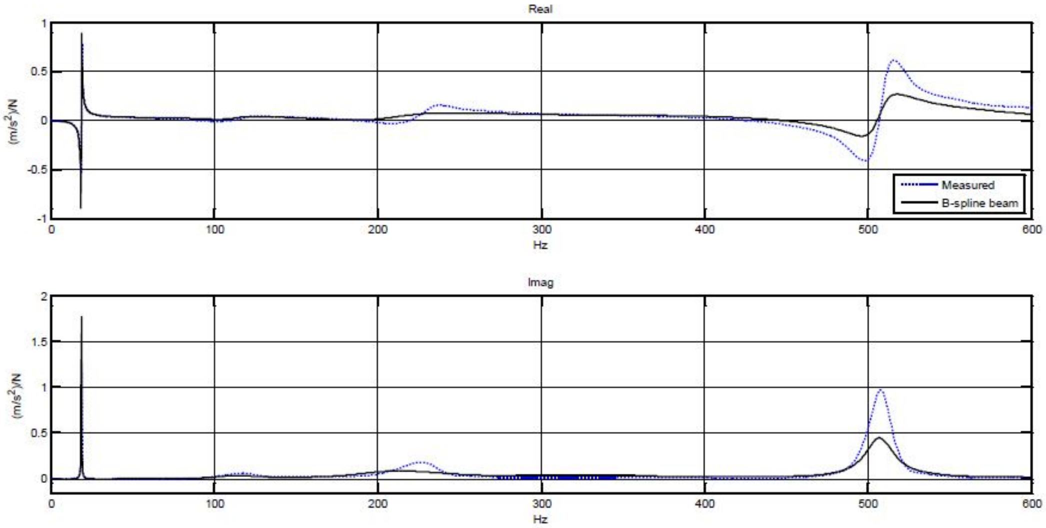
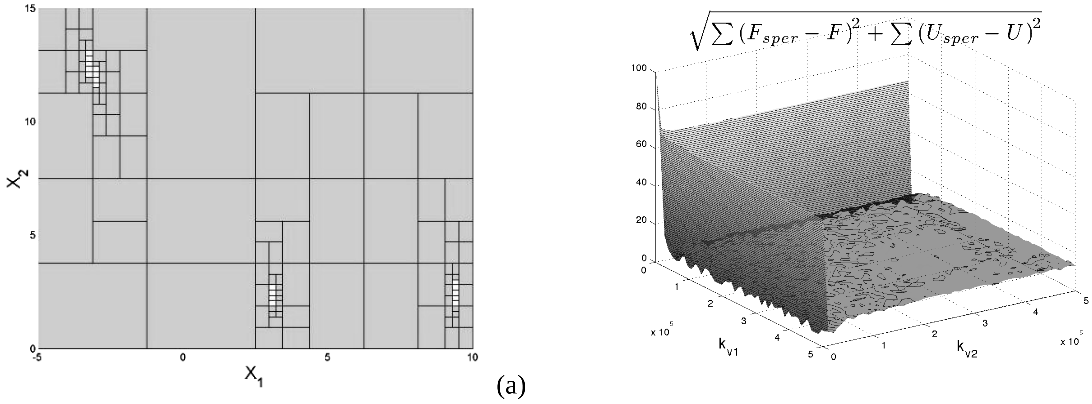


Figure 5. FRF comparison after updating: input data (blue dotted) vs. model (black continuous)

5.2. F.e. model updating based on interval analysis

Some difficulties of standard f.e. model updating techniques and mainly the capability to embody the modelling and experimental uncertainties in a natural setting can be overcome if the problem is recast in the framework of interval analysis [35]. A recently developed method called Interval Intersection Method (INTIM) was used to treat model updating with uncertainties [36]. In this framework, uncertain quantities as assumed as interval numbers and the inclusion theorem of interval analysis is used to select between actual and fictitious solutions. This distinction allows also to judge on the admissibility of f.e. models, i.e. their intrinsic capability to model the physical problem under study. The method implements global optimisation strategies [37] based on modal parameters: frequencies and modal shapes. The capability of the method to detect all the global minima in the search domain is highlighted in figure 6a where the interval discretized map of the Branin test function is projected in the parameters space.



(a) (b)

Figure 6. Interval bounding of the Branin test function (a);
Error function of beam on elastic supports (b).

The apparently simple problem under consideration of a vibrating beam resting on finite stiffness supports reveals hard to deal with if the beam bending stiffness and the support stiffness are chosen as updating parameters. The section of the objective function in the subspace of the support stiffness is plotted in figure 6b. It is computed using frequencies (*F*) and modal shapes (*U*) and turns out to be constituted by a flat rugged surface endowed with a large number of local minima and two very narrow valleys bounded by steep and almost vertical planes.

6. Conclusions

The rational of the Pescara benchmark activities has been presented. The benchmark is devoted to the study of the dynamic behaviour of prestressed concrete beams damaged by corrosion and excited by moving loads. The purpose is to contribute to the structural health evaluation of new and deteriorated railway bridges. Five specialised research teams of as many Italian Universities were involved in the project each providing expertise in a particular field. A large amount of experimental data was collected. Innovative identification techniques and model updating strategies were developed and validated. Ranges for robust damage identification were defined.

Acknowledgement

The work is part of the 2007 Italian national PRIN project BriViDi: Bridge Vibration and Diagnosis. The authors wish to thank Mr. M. Colantuono and Mr. C. Di Cintio for their valuable support in the design, realization and execution of the experimental tests.

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