Green Design of Wireless Local Area Networks by Multiband Robust Optimization

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Abstract

We consider the problem of designing a wireless local area network according to a green paradigm, i.e. serving users with a telecommunication service while minimizing power consumption. To protect against fluctuations in data rate transmission that naturally affect the problem, because of unpredictable user mobility and wireless propagation conditions, we propose a new Multiband Robust Optimization model, and assess its performance on realistic network instances. The preliminary computational experience confirms the effectiveness of the new model in terms of power savings and resiliency against large variations of mobile user positions.

Keywords: Wireless Network Design, User Mobility Uncertainty, Binary Linear Programming, Robust Optimization

1 Introduction

We consider an optimization problem arising in the design of Green (or energy-saving) Wireless Local Area Networks (GWLAN). A WLAN is composed of a set of Access Points (APs) that provide wireless connectivity to a set of User Terminals (UTs). The problem of optimally designing a GWLAN, introduced in the studied form in [4], consists of minimizing the power consumption of a WLAN when the load is scarce, by powering-on just a subset of APs and associating UTs to powered-on APs, while taking into account the data rates between UTs and APs. To protect the GWLAN against natural fluctuations in the network performance that occur over short periods of time and lead to tricky reductions in data rates, we propose to adopt a Robust Optimization (RO) approach, based on a generalization of the classical Γ-Robustness (Γ-Rob) by Bertsimas and Sim (see [2]). The adoption of RO in GWLAN design aimed at tackling data rate fluctuation has been first investigated in the preliminary study [4], by considering the impact of both user movement and wireless propagation conditions on data rates. In fact, users can move around the service area, and this has a direct impact on the link data rates, which are a function of the distance between users and access points. Furthermore, the data rate of the links are sensible to the fluctuation in the signal propagation.

Here we propose an enhanced RO model for GWLAN design, which is based on Multiband Robust Optimization (MRO). MRO was originally proposed in [3] to refine Γ-Rob, while maintaining the computational tractability and accessibility of Γ-Rob. It is essentially based on the use of histogram-like uncertainty sets, which result particularly suitable to represent empirical distributions commonly available in real-world problems (e.g., [1,5]). Specifically, with respect to [4], we propose to use MRO to model the user mobility uncertainty, while we adopt Γ-Rob to model the channel fluctuation event. The rationale is that a more accurate model of the user mobility, via multi-

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ple deviation bands, can better represent the real link data rate variations, which are distance dependant, so allowing a finer allocation of the users to the access points, and therefore a stronger overall energy saving. We note that, with respect to the canonical MRO model proposed in [3], here we consider the presence of two distinct uncertain events that are mutually dependent, thus extending the theory of MRO.

The paper is organized as follows. The nominal GWLAN problem is presented in Section 2. The proposed MRO extension is described in Section 3, with a proof of integrality that allows a compact formulation. Section 4 reports the results of preliminary computational experiments, which compare the new robust approach with the one in [4].

2 The nominal GWLAN

Consider a GWLAN system constituted by a set $\mathcal{J}$ of deployed APs that can serve a set $\mathcal{I}$ of UTs. The traffic demand w_i of each UT i must be satisfied by exactly one AP. The power P_j consumed by the generic AP j can be essentially ascribed to two major components: 1) a fixed component b_j, which is bound to the mere fact that the device is powered-on; 2) a variable component a_j, which accounts for the so-called "airtime", i.e. the fraction of time the device is either transmitting or receiving frames . The component a_j is weighted by a constant "wireless" factor p^w, which accounts for the power drain of the radio frontend for the transmission and reception operations (see [4] for more details) and the overall power consumption P_j is then: $P_j = b_j + p^w a_j \ , \quad \forall j \in \mathcal{J}$.

The other parameters characterizing the GWLAN system are the r_{ij}, i.e. the data rate available between the UT i and the AP j, for $i \in \mathcal{I}$ and $j \in \mathcal{J}$. They depend on the physical properties of the system, such as the position of the UT i with respect to the AP j. To keep the notation simpler, we shall assume that the links are symmetric, i.e. $r_{ij} = r_{ji}$, $\forall i \in \mathcal{I}$, $j \in \mathcal{J}$.

The nominal version of the GWLAN problem consists in deciding which APs to power-on and to which powered-on AP assign each UT, so as to satisfy the demand of each UT and the capacity constraint of each AP. The goal is to minimise the overall power consumption of the WLAN. By introducing the following two sets of binary variables:

- x_{ij}, which is set to 1 if UT i is assigned to AP j, 0 otherwise, $i \in \mathcal{I}$, $j \in \mathcal{J}$,
- y_j, which is set to 1 if AP j is powered-on, 0 otherwise, $j \in \mathcal{J}$,

the considered optimization problem can be formulated as the following Binary

Linear Programming (BLP) presented below, initially proposed in [4]:

$$z = \min \sum_{j\in\mathcal{J}} P_j = \min \sum_{j\in\mathcal{J}} \left[b_j\, y_j + p^w \sum_{i\in\mathcal{I}} \frac{w_i}{r_{ij}}\, x_{ij} \right], \quad (1)$$

$$\sum_{j\in\mathcal{J}} x_{ij} = 1 \qquad i \in \mathcal{I} \quad (2)$$

$$\sum_{i\in\mathcal{I}} \frac{w_i}{r_{ij}}\, x_{ij} \leq y_j \qquad j \in \mathcal{J} \quad (3)$$

$$x_{ij} \in \{0,1\} \qquad i \in \mathcal{I}, j \in \mathcal{J} \quad (4)$$

$$y_j \in \{0,1\} \qquad j \in \mathcal{J}\ , \quad (5)$$

where the airtime a_j is expressed in terms of the variables x_{ij}: $a_j = \sum_{i\in\mathcal{I}} \frac{w_i}{r_{ij}}\, x_{ij}$. In this model, the constraints (2) express that each UT must be assigned to exactly one AP, whereas the constraints (3) express the capacity of the APs and also ensure that no UT is assigned to powered-off APs.

3 The multiband robust model for GWLAN design

Until now, we have assumed that the data rates r_{ij} are exactly known when the problem is solved. However, in practice this is not true, since r_{ij} depends on the mobility of the users and on the propagation conditions of the wireless channel, which are hard to exactly know a priori. The data rates r_{ij} are thus naturally subject to uncertainty.

Concerning the user mobility, we assume that the users can move from their original position in diverse annuli areas. Specifically, each user moves in an annulus whose ray is not exactly known but belongs to the range $[0, \rho^{\max}]$ and could be in any position of its annulus area. Following the MRO paradigm (see [3] and the consequent papers by Büsing and D'Andreagiovanni for an exhaustive description of theory and applications of MRO), the overall range $[0, \rho^{\max}]$ is partitioned into subranges that corresponds to a set B of so-called *bands*. Each band models a distinct range of deviation of the uncertain data and, in the case of GWLAN design, represents a different class of mobility of the users. Specifically, in our computational study we consider 4 classes of mobility corresponding to 4 deviations bands: 1) $b = 0$ - static: new position = old position, i.e. $\rho = 0$; 2) $b = 1$ - low mobility: the new position is in the annulus area defined by $\rho = (0, \rho_1]$; 3) $b = 2$ - medium mobility: the new position is in the annulus area defined by $\rho = (\rho_1, \rho_2]$; 4) $b = 3$ - high mobility: the new position is in the annulus area defined by $\rho = (\rho_2, \rho_3]$;

where $0 = \rho_0 < \rho_1 < \rho_2 < \rho_3 = \rho^{\max}$.

We assume that at most $H_b \geq 0$ UTs may move simultaneously according to the class of mobility b with $b \geq 1$. Additionally, we distinguish users who do not change their current position (mobility class $b = 0$) and whose wireless channel conditions vary and lead to a variation in the data rate and assume that at most H_0 UTs may belong to this category. Finally, we assume that at most K UTs are subject to any kind of uncertainty (due to mobility uncertainty or to wireless propagation uncertainty, discussed below).

The actual values r_{ij} are also influenced by variations in the wireless propagation conditions, due to fading phenomena that are really hard to precisely assess. For these phenomena, the Rayleigh model is widely used in the literature [6] and serves as the worst case for a broad class of fading distributions. We consider the case where the f-quantile of the Rayleigh fading varies within the interval, or monoband, $[f_L, \overline{f}]$, where $\overline{f} = 1 - e^{-\frac{\pi}{4}}$ is the nominal quantile, leading to the average value of the fading channel. The parameter f is equal to $\overline{f}$ in case of no fluctuations, while it is set to f_L in case of channel fluctuation, since f_L models the worst scenario under the considered uncertainty model.

According to what stated before, for each AP-UT couple (i, j), the data rate depends upon the distance and the propagation condition between i and j and thus depends upon the deviation band b and the fluctuation in the Rayleigh fading. We thus denote by $r_{ij}(b, f_L)$ the function representing the data rate for (i, j) when the UT i belongs to the mobility band b, $b \geq 1$, and the worst Rayleigh fading deviation occurs. Analogously, $r_{ij}(0, f_L)$ denotes the data rate for the no-mobility band $b = 0$ and worst fading case. We also introduce the notation $\overline{r}_{i,j} = r_{i,j}(0, \overline{f})$ to denote the nominal value of the data rate for (i, j), where no mobility and fading fluctuation occur.

In order to state the robust counterpart of the constraint (3) corresponding to the index $j \in \mathcal{J}$, let us associate *binary variables* q_{ij}^b with each UT i and band b: q_{ij}^b is set to 1 if either UT i moves according to the band $b \geq 1$, or i does not move ($b = 0$) but the fading channel is subject to fluctuation; it is set to 0 otherwise. The data rates of the links related to i thus vary according to $r_{ij}(b, f_L)$ with $b \in B$. The case of mobile UTs whose related data rates are not subject to channel fluctuation is not modelled here, since it is not significant in this context.

By using these additional variables, the robust version of each constraint (3) contains an inner BLP problem, which gives the maximum (i.e., the worst case) value that the left-hand-side may achieve under the considered robust framework. For each $j \in \mathcal{J}$, the (non-linear) robust capacity constraint that includes the maximization of the deviation through the variables q_{ij}^b writes as:

$$\sum_{i\in\mathcal{I}} \frac{w_i}{\bar{r}_{ij}}\, x_{ij} \;+\; \max \sum_{i\in\mathcal{I}}\sum_{b\in B} \left(\frac{w_i}{r_{ij}^{min-b}} - \frac{w_i}{\bar{r}_{ij}} \right) x_{ij}\; q_{ij}^b \;\le\; y_j\;, \tag{6}$$

where r_{ij}^{min-b} is the worst value assumed by the data rate function $r_{ij}(b, f_L)$ for UT i and band b. We note that when $\sum_{b\in B} q_{ij}^b = 0$ (i.e. UT i does not move and the related fading channel does not fluctuate), we obtain the fraction $\frac{w_i}{\bar{r}_{i,j}}$ including the nominal data rate. Also, we observe that the values in the denominators in (6) are *not* decision variables.

For each $j \in \mathcal{J}$, the feasible set of the inner maximisation problem is described by the following set of constraints, where (8) states that the capacities of at most K users may deviate simultaneously, assuming $K < \sum_{b\in B} H_b$:

$$\sum_{b\in B} q_{ij}^b \le 1 \qquad\qquad i\in\mathcal{I} \tag{7}$$

$$\sum_{i\in\mathcal{I}}\sum_{b\in B} q_{ij}^b \le K \tag{8}$$

$$\sum_{i\in\mathcal{I}} q_{ij}^b \le H_b \qquad\qquad b\in B \tag{9}$$

$$q_{ij}^b \in \{0,1\} \qquad\qquad i\in\mathcal{I}, b\in B \tag{10}$$

Proposition 3.1 *The polytope associated with the linear relaxation of* (7) – (10) *is integral.*

Proof. We consider the linear relaxation of (7) – (10) where we have dropped the constraints $q_{ij}^b \le 1$, $i\in\mathcal{I}, b\in B$, which are dominated by the constraints (7). To prove the integrality, we consider the constraints of this relaxation in matrix form $Aq \le b$, where the matrix A and the right-hand-side b are:

$$A = \left(\begin{array}{c|c|c} 1\cdots 1 & & \\ & \ddots & \\ & & 1\cdots 1 \\ \hline 1\cdots 1 & \cdots & 1\cdots 1 \\ \hline I & \cdots & I \end{array}\right) \qquad\qquad b = \left(\begin{array}{c} \vdots \\ 1 \\ \vdots \\ \hline K \\ \hline \vdots \\ H_b \\ \vdots \end{array}\right)$$

where I is the identity matrix of size equal to number of bands $|B|$. The matrix A is totally unimodular since: 1) each of its entries is in {+1, -1, 0} and 2) for each subset M of the rows, there exists a partition (M_1, M_2) of M

such that each column j satisfies: $|\sum_{i\in M_1} a_{ij} - \sum_{i\in M_2} a_{ij}| \leq 1$ (see [7], page 50). Since A is totally unimodular and the right-hand-side vector b is integral, it is well-known that the polytope defined by $Aq \leq b$ and $q \geq 0$ is integral. □

Proceeding in a similar way to [3], we can derive a linear and compact BLP formulation for the robust GWLAN problem under consideration by defining the dual problem of the linear relaxation of the maximum MRO deviation problem considered above. Since the relaxed problem associated with (7)−(10) is feasible and bounded also its dual is feasible and bounded and, by strong duality, the optimal values of the two problems coincide. We can then replace each inner maximisation problem of the original (non-linear) problem with the corresponding purely linear dual problem, obtaining the following robust (linear and compact) BLP model:

$$\min \sum_{j\in\mathcal{J}} \left[b_j y_j + p^w \left(\sum_{i\in\mathcal{I}} \frac{w_i}{\bar{r}_{ij}} x_{ij} + \sum_{i\in\mathcal{I}} \pi_i^j + K\,\delta^j + \sum_{b\in B} \mu_b^j\, H_b \right) \right] \tag{11}$$

$$\sum_{i\in\mathcal{I}} w_i \frac{x_{ij}}{\bar{r}_{ij}} + \sum_{i\in\mathcal{I}} \pi_i^j + K\,\delta^j + \sum_{b\in B} \mu_b^j\, H_b \leq y_j \qquad j\in\mathcal{J} \tag{12}$$

$$\pi_i^j + \delta^j + \mu_b^j \geq \left(\frac{w_i}{r_{ij}^{min-b}} - \frac{w_i}{\bar{r}_{ij}} \right) x_{ij} \qquad i\in\mathcal{I}, j\in\mathcal{J}, b\in B \tag{13}$$

$$\pi_i^j,\ \delta^j,\ \mu_b^j\ \geq 0 \qquad i\in\mathcal{I}, j\in\mathcal{J}, b\in B \tag{14}$$

s.t. (2), (4), (5)

which includes the robust version (11) and (12) of the objective function and of the capacity constraints, respectively. The additional constraints (13) and the variables (14) are those coming from the classical MRO dualization procedure.

4 Computational results

We preliminary assessed the computational effectiveness of the proposed multi-band robust model (hereafter MR) using 7 scenarios, each characterized by 100 instances. According to the description in Section 3, we set $|B| = 4$, $H_0 = H_1 = H_2 = H_3 = 0.25|I|$ and $K = 0.5|I|$. We assumed that the maximum speed of users moving according to band b, $b \geq 1$, is $b * v_m$, with $v_m = 0.5$ m/s. Hence we have $\rho_b = b * v_m * \Delta_t$ for $b \geq 1$, where Δ_t is the considered time horizon. Concerning the fading uncertainty, we experimented the lower end $f_L = 0.05$. The main goal of the computational experience has been to perform a comparison with the classical Γ-Robustness model by Bertsimas and Sim in [4] (hereafter BR), which assumes to have only one mobility class for the users, i.e. $(0, \rho_3]$, while the monoband of the fading model is the same as in the MR. Therefore, in testing BR we set the upper bound H on the total users who may move to $0.75|I|$ (i.e. the sum of the upper bounds of the mobility bands $H_1 + H_2 + H_3$), and K, with the same meaning, equal to $0.5|I|$, as for MR.

In the experiments each instance is considered twice, once in the present (time t_0) and once in the future (time $t_0 + \Delta_t$). At first, we solved the nominal (N hereafter) and the robust problems at time t_0. Then, we generated the future instance by moving the UTs, and recalculating the rates that are subject to fluctuations, at time $t_0 + \Delta_t$, by considering $\Delta_t = 1$s and $\Delta_t = 10$s. By this choice we wanted to investigate, with the former setting of Δ_t, scenarios of limited mobility, and with the latter one scenarios characterized by a greater movement distribution, where it could be more appropriate to model the user mobility with several bands. All models have been solved by IBM ILOG CPLEX IBM 12.5.1, with a time limit of 7200s, on a 64 bit Ubuntu OS, hosted by a virtual machine. The hosting operating system is Apple OS X (10.11.4) running on a 1.3 GHz Intel Core i5 processor with 8GB of memory.

Table 1
Performance results

	$\|I\|$	$\|J\|$	$\|I\|/\|J\|$	FF_M	FF_B	FF_N	BU	PR_{MP}	PR_{BP}	PR_{MF}	PR_{BF}
$\Delta_t = 1$ s	27	5	5.4	57	85	40	0	1.37	2.21	1.12	1.78
	38	7	5.42	35	68	23	10	1.21	2.11	0.97	1.68
	57	3	19	53	44	24	45	1.21	1.35	1.00	1.15
	83	4	20.75	56	67	43	2	1.23	1.41	1.29	1.42
	99	8	12.375	18	47	0	16	1.25	1.66	1.03	1.35
	95	7	13.57	31	54	0	14	1.27	1.72	1.11	1.44
	126	7	18	11	49	10	7	1.00	1.31	1.00	1.33
$\Delta_t = 10$ s	27	5	5.4	70	58	29	32	1.73	2.60	1.18	1.79
	38	7	5.42	34	49	19	30	1.21	2.47	1.02	2.03
	57	3	19	58	9	30	86	1.37	1.55	1.14	1.44
	83	4	20.75	67	44	46	39	1.37	1.52	1.38	1.50
	99	8	12.375	22	22	2	64	1.27	1.89	1.07	1.57
	95	7	13.57	25	17	0	62	1.32	1.89	1.17	1.67
	126	7	18	27	19	17	57	1.16	1.50	1.24	1.53

The results are reported in Table 1, where FF_M indicates the number of the solutions of MR, computed at time t_0, which are still feasible in the future. FF_B and FF_N report the same performance metric for BR and for

N, respectively. In the table, BU gives the number of instances which proved to be unfeasible for BR at time t_0. On the contrary, MR was always able to determine a feasible solution. The table also reports the average ratio between the power consumption induced by the MR/BR solutions and those induced by the nominal solutions, at the present time (PR_{MP} and PR_{BP}, respectively), and in the future (i.e., PR_{MF} and PR_{BF}, respectively).

For $\Delta_t = 1$s, i.e. for a small variation in the new positions of the UTs, the results suggest that the number of solutions of BR, still feasible in the future, overcomes the one of the MR counterpart. Thus, it seems to be not useful to use the multiband approach in case of limited mobility. On the contrary, MR guarantees a better resiliency to long movements, as shown by the results for $\Delta_t = 10$s. Also notice that the resiliency guaranteed by the two robust approaches is always greater than that achieved by the nominal approach.

An important advantage of MR is its tiny power consumption, which is always less than the one of its basic counterpart. In some cases, the solutions of MR lead to a power consumption which is about a half of the one required by the solutions of BR. The indicators PR_{MF} and PR_{BF}, in general smaller than PR_{MP} and PR_{BP}, respectively, testify this trend.

Table 2
CPU times

Scenario			$\Delta_t = 1$ s		$\Delta_t = 10$ s	
$\|I\|$	$\|J\|$	$\|I\|/\|J\|$	MR	BR	MR	BR
27	5	5.4	0.385	0.106	0.742	0.088
38	7	5.42	0.484	0.138	0.729	0.118
57	3	19	0.419	0.096	0.633	0.052
83	4	20.75	0.876	0.218	1.311	0.140
99	8	12.375	1.782	0.326	4.142	0.290
95	7	13.57	1.573	0.256	3.557	0.206
126	7	18	2.221	0.682	7.663	0.538

The average CPU times (in seconds) required by the tested approaches are reported in Table 2. We do not report the results for the nominal approach, since the computational times of N and BR are about the same. The table shows that BR usually needs less than one second, for all considered scenarios and both settings of Δ_t, whereas MR requires, on average, more seconds for solving the larger scenarios. The general conclusion that we can carry out from our computational campaign is therefore that (limited to the small number of tested scenarios): *i*) the multiband approach is always able to compute a feasible solution, and it is always very preferable in terms of power savings, at the expenses of a greater computational time for solving the scenarios of larger size; *ii*) the offered resiliency against the UTs mobility pends in favour of the multiband approach in the case of large variations of the mobile user positions (i.e., for $\Delta_t = 10$s in our experiments), while the vice-versa holds in the case of limited mobility (i.e., for $\Delta_t = 1$s in our tests).

Summing up, according to these preliminary tests, adopting an MRO approach is computationally efficient, grants important advantages with respect to Γ-Robustness and is thus worth of further investigations.

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