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**Extrinsic Uncertainty and the  
Informal Role of Prices**

by

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**Abstract**

We study a financial economy with finitely many states of private information. There are two agents, informed and uninformed. We define individuals' expectations on future equilibrium prices as a probability distribution over  $K$  distinct values, i.e., we introduce  $K$  extrinsic events. We show that there are open sets of economies with nonrevealing (and also with partially revealing) equilibria: For an appropriate choice of self-fulfilling price expectations, observable equilibrium prices are signal invariant.

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## 1. Introduction

This paper analyzes how extrinsic uncertainty may affect the efficiency of prices in conveying information. We consider the simplest framework: There are finitely many states of information and two individuals, informed and uninformed. The economy extends over two periods: After the realization of the signal, individuals trade a given collection of real assets, (i.e., assets whose payoffs are denominated in terms of commodities) on an asset market. In the second period, one "fundamental" state of nature realizes and individuals trade in consumption goods.

As is well known, in competitive economies that do not satisfy the first welfare theorem at a competitive equilibrium extrinsic uncertainty may affect the equilibrium allocations (see Cass and Shell [7] and Shell [23]). In economies where intertemporal transactions take place through trade in an incomplete set of assets, sunspot equilibria are a pervasive phenomenon (see, for instance, Cass [6] and Mas-Colell [17]).

In economies with asymmetric information, a rational expectations equilibrium (REE) is a map from the states of the economy into the price domain. Individuals exploit the knowledge of the equilibrium map to refine their information through the information revealed by the observed equilibrium prices. Radner [21] proved that, when the number of states of private information is finite, typically there exists a REE and that, typically, all the REE are fully revealing, i.e., no difference of information can persist at equilibrium. Subsequent work has clarified the conditions under which, in economies with infinite signal space, observable equilibrium prices are a sufficient (or a nearly sufficient) statistics for the information available in the economy (see, Allen [1], Jordan [15] and Jordan and Radner [16]).

The existence of economies with noninformative (or partially revealing) REE is an important issue. For instance, as pointed out in Grossman [11] or Grossman and Stiglitz [12], revelation at equilibrium is incompatible with the costly acquisition of information. Furthermore, many important economic phenomena seem to be more satisfactorily explicable in a partial revelation framework (see, for instance, Ausubel [3]).

Examples of partially revealing (or nonrevealing) equilibria have been provided in the literature. In Grossman and Stiglitz [12], for instance, their existence follows from the introduction of ad hoc noise in the economy. Allen [2] provides a robust example where equilibrium prices are nonrevealing (but agents have full information) at the equilibrium. Ausubel [4] constructs a robust example of economies with partially revealing equilibria where asymmetric information persists at the equilibrium.

In economies with incomplete financial markets and nominal assets, Polemarchakis and Siconolfi [21] show the existence of nonrevealing equilibria, (see also Rahi [22]). Their result depends crucially upon the indeterminacy of equilibrium prices and allocations in economies with nominal assets and restricted participation.

We study the existence of nonrevealing (and partially revealing) equilibria in financial economies with real assets (hence, with - typically - locally unique equilibria). In our economy, there are two primitive sources of uncertainty: Finitely many states of private information (signals) and finitely many intrinsic events. Individuals, after the signal realization, observe equilibrium asset prices and form consistent expectations about future commodity prices for each possible realization of the uncertainty. Typically, there exists a fully revealing REE. Moreover, typically, all the equilibria are fully revealing. To construct noninformative equilibria (i.e., equilibria where the asset prices are signal-invariant) we add a third source of uncertainty. We assume that, for each possible signal, individuals' expectations on future commodity prices are defined as a probability distribution over  $K$  market clearing values, i.e., we introduce  $K$  *extrinsic events*. An equilibrium specifies, together with market clearing prices, a number  $K$  and a probability distribution (in general dependent on the realization of the signal) over these extrinsic events. Evidently, independently of  $K > 0$ , the equilibria of the primitive economy are still equilibria of the economy with extrinsic uncertainty. However, since the economy is now an intertemporal economy with incomplete financial markets, sunspots may matter. We exploit this feature to show that, for an open sets of economies, there are also nonrevealing equilibria.

Our construction is robust in the space of economies specified by the two primitive sources of uncertainty and the usual parameters. Though, it is not robust if we consider as part of the description of the economy the probability distribution over the extrinsic events. Typically, given the number  $K$ , the set of probability distributions that generate nonrevealing equilibria is negligible. This is hardly surprising. The economy with the state space augmented to include the extrinsic uncertainty is still an economy with real assets. Therefore, given an arbitrary probability distributions, equilibria are, typically, finite and continuously dependent on the economy parameters. These are the same properties shared by the equilibria of economies with a unique budget constraint, which, typically, do not have noninformative equilibria. The endogeneity over the probability law governing the extrinsic uncertainty generates the indeterminacy that we need to hide the information.

It is still an open issue if the set of nonrevealing equilibrium allocations is negligible in the set of equilibrium allocations associated with all the possible specifications of extrinsic uncertainty (i.e., of the number of extrinsic events and of their conditional probabilities).

We believe that this exercise is interesting for two main reasons. It provides a way to generate "noise" which allows for the existence of nonrevealing equilibria. This "noise" is of pure expectational nature and, contrary to the noisy REE tradition, it is compatible with individual optimization by all the agents and with complete (pooled) information. It points out a new and important way through which extrinsic uncertainty may affect the properties of competitive equilibria.

It can be shown that there are also *partially revealing* equilibria, where some (but not all) information is revealed by the asset prices. The analysis can be generalized to economies with many individuals and with more complex information structures. We consider economies where, in the first period, there is no consumption. Given that the analysis of the informational role of prices mostly refers to asset prices, we believe that this assumption is restrictive, but justified. We conjecture that open examples of this sort could be provided even for economies with period 1 consumption.

The equilibrium concept that we use in this paper is somewhat related to the notion of *Common Beliefs Equilibria* (CBE), previously introduced by Dutta and Morris [9] for the study of

economies with asymmetric information. At a CBE, the map from states to prices which defines a REE is weakened to be a correspondence, i.e., a non trivial distribution of equilibrium prices corresponds to a realization of the state of the economy. At a CBE, individuals share the same conditional probability distribution on equilibrium prices given signals. Conditional probabilities (of signals given prices) are not fixed parameters, but a part of the definition of the equilibrium. Example 2 in Dutta and Morris is particularly related to the issues considered in our paper. They consider an economy that, generically in the vectors of probability distributions over signals,  $\pi$ , has a unique fully revealing rational expectations equilibrium  $\hat{p}$ , while, for  $\pi = \bar{\pi}$ , the economy has a (unique) rational expectations noninformative equilibrium  $\bar{p}(\bar{\pi})$ . Dutta and Morris construct an equilibrium which is a lottery over  $\hat{p}$  and  $\bar{p}(\bar{\pi})$  and such that the actual conditional probabilities (of signals given  $\bar{p}(\bar{\pi})$ ) are  $\bar{\pi}$ . This equilibrium is a CBE and the authors refer to it as a "partially revealing" equilibrium. While the basic viewpoint adopted in our paper is obviously related to their result, there are some fundamental differences related to the basic structure of the model. In the economy considered by Dutta and Morris, individuals maximize subject to a unique budget constraint. Hence, given the information compatible with the observed equilibrium prices, sunspots do not matter. To the contrary, in our economy, given a noninformative equilibrium price, sunspot matters. It is precisely this feature that allows us to construct robust noninformative equilibria: By controlling the conditional probabilities of the extrinsic events (given a realization of the signal), we control the equilibrium asset prices and can "hide" the information. Finally, in our set up, we can construct fully nonrevealing equilibria (this seems to be impossible in the Dutta and Morris framework) and existence of fully nonrevealing equilibria allows a simple proof of the existence of partially revealing equilibria embedding all possible (asymmetric) information structures.

## 2. The model

We consider the simplest model of a general equilibrium economy with asymmetric information: There are two (types of identical) individuals: *Informed* (denoted by a subscript  $i$ ) and *uninformed* (denoted by  $u$ ). Let  $h$  denote a generic individual,  $h = i, u$ . There are three periods: At time 0 individual  $i$  observes a signal  $\sigma \in \Sigma \equiv \{1, \dots, \Sigma\}$ , which is not observed by individual  $u$ . Notice that we will use  $\Sigma$  to denote the last signal, the set of signals and their number. This should not induce any confusion. At time 1 asset trade takes place. In the last period the uninformed agent becomes informed of the realization  $\sigma$ , portfolio payoffs are paid and commodity trade takes place. There are two different sources of uncertainty: For each  $\sigma \in \Sigma$ , there are  $S$  *fundamental* (or *intrinsic*) events,  $s \in \{1, \dots, S\}$ , and  $K$  *extrinsic* events,  $k \in K = \{1, \dots, K\}$ , which realize before the intrinsic ones. Therefore, in all, there are  $\Gamma \equiv SK\Sigma$  states of nature. A state of nature in period 2 is identified by a triple  $(s, k, \sigma)$ . The time and uninformed agent's information structure of the economy is described in Figure 1. Evidently, for the informed agent, each node is an information set.

FIGURE 1 SHOULD BE HERE

Signal  $\sigma$  gives a complete specification of the realization of the economy in its intrinsic dimension. The map  $I: \Sigma \rightarrow \mathbf{E}'$  associates with each  $\sigma$  a realization of the fundamentals of the economy, i.e., it specifies a collection of utility functions, of endowments and of the coefficients determining asset payoffs  $((\dots, (\omega_h(\sigma), u_h^\sigma), \dots), \rho) \equiv \mathbf{E}' \in \mathbf{E}'$ . In this paper, the number of extrinsic events,  $K$ , and the vector of probabilities of the realization of the extrinsic events are always treated as an endogenous variable.

It is somewhat inappropriate to use the term "signal" to denote the random variable  $\sigma$ , because a  $\sigma$ -realization affects the endowment-utility profiles of the agents, not only the probability distribution over  $S$  (in fact, in our model, we do not restrict individuals to be expected utility maximizers over the set  $S$  and, hence, we do not even need to mention probability distribution over  $S$ ). Though, in the

Rational expectations literature, it is standard to treat  $\sigma$  in this way, e.g., Allen [1] and Jordan [15]. Moreover, at some notational cost, our argument can be applied as well to the case where  $\sigma$  only affects the probability distribution over the intrinsic events.

At each spot there are  $C$  physical commodities, denoted by a superscript  $c = 1, \dots, C$ . Individual  $h$ 's consumption vector at state  $(s, k, \sigma)$  is  $x_h(s, k, \sigma) = (\dots, x_h^c(s, k, \sigma), \dots)$ . Individual  $h$ 's consumption vector, associated with signal  $\sigma$ , is  $x_h(\sigma) = (\dots, x_h(s, k, \sigma), \dots)$ , while the consumption vector is  $x_h = (\dots, x_h(\sigma), \dots)$ . We use an analogous notation to refer to the endowment vectors  $(\omega_h)$ , to the excess demand vectors  $(z_h = (x_h - \omega_h))$  and to the commodity price vectors  $(p)$ .

There are  $A$  assets,  $A > 1$ . We do not place any upper bound on the relation between  $S$  and  $A$ . In particular, without extrinsic uncertainty, i.e.,  $K = 1$ , the asset market could be (potentially) complete. Given that the number of extrinsic events is endogenous, we can always obtain that (in the economy with extrinsic events) the asset market is incomplete. However, for technical reasons, our arguments goes through if  $C > 2(A + 1)$ .

Throughout the analysis of this paper we assume that the number of assets,  $A$ , satisfies  $SK > A > 1$ . The asset price vector, given signal  $\sigma$ , is  $q(\sigma) = (\dots, q^a(\sigma), \dots)$ , while  $q = (\dots, q(\sigma), \dots)$ . Assets are *real*, with typical yield  $r^a(s, k, \sigma) \equiv \rho^a(s)p(s, k, \sigma)$ , where  $\rho^a(s)$  is a  $C$ -dimensional vector specifying the quantities of the different commodities that a unit of asset  $a$  pays off if event  $s$  realizes. Let  $\rho \in \mathbb{R}^{ACS}$  be the collection of the coefficients  $(\rho^{ac}(s))$  defined above and let  $R(p(\sigma))$  be the matrix of the asset yields associated with signal  $\sigma$ . The vectors  $\rho^a$  are sunspot and signal invariant. Signal invariance simplifies notation. Sunspot invariance makes more difficult for sunspots to matter. Our main result holds (a fortiori) if we allow for sunspot and signal dependent asset payoffs.

Individual  $h$ 's portfolio, given  $\sigma$ , is  $y_h(\sigma) \in \mathbb{R}^A$ , while  $y_h = (\dots, y_h(\sigma), \dots)$ .

## 2.1 The space of the economies

For each agent  $h$ , the consumption space is  $\mathbb{R}_{++}^{CS\Sigma}$  and the endowment vector is  $\omega_h \in \mathbb{R}_{++}^{CS\Sigma}$ . For each signal  $\sigma$ , preferences are described by a utility function  $u_h^\sigma(x_h(\sigma)) \in C^\infty$ , strictly monotone and



differentiably strictly concave. The utility functions satisfy the boundary condition:  $\{x \in \mathbb{R}_{++}^{CS\Sigma} \mid \sum_{\sigma} \pi(\sigma)u_h^{\sigma}(x_h(\sigma)) \geq \sum_{\sigma} \pi(\sigma)u_h^{\sigma}(x_h^*(\sigma))\}$  is closed in  $\mathbb{R}_{++}^{CS\Sigma}$  for each  $x_h^* \gg 0$  and for each strictly positive probability distribution over  $\Sigma$ , i.e., for each  $(\dots, \pi(\sigma), \dots) \gg 0$ .

When we introduce the extrinsic events,  $k \in K = \{1, \dots, K\}$ , we assume that preferences can be represented by a Von Neumann-Morgenstern utility function. For given  $\sigma$ , let  $\pi(\cdot|\sigma) = (\dots, \pi(k|\sigma), \dots)$  be the conditional probability over the extrinsic states of uncertainty. In the sunspot economy, the utility function is  $V_h^{\sigma}(x_h(\sigma)) \equiv \sum_k \pi(k|\sigma)u_h^{\sigma}(x_h(k,\sigma))$ .

Let  $U^*$  be the space of functions satisfying the stated assumptions. Endow  $U^*$  with the  $C^2$  compact-open (weak) topology.

A realization of the random variable  $\sigma$  completely specifies the fundamentals of the sub-economy associated to  $\sigma$ ,  $E^{\sigma} \in \mathbf{E}$ . A sub-economy is a specification of an utility-endowment profiles  $((u_h^{\sigma}, \omega_h(\sigma))_{h,\sigma}) \in \prod_h (U^* \times \mathbb{R}_{++}^{CS})$  and of the ( $\sigma$ -invariant) asset payoff matrix  $\rho \in \mathbb{R}^{ACS}$ . Hence, a sub-economy is  $E^{\sigma} \equiv \{(u_h^{\sigma}, \omega_h(\sigma))_{h,\sigma}, \rho\}$ . The space of economies, endowed with the product topology, is  $\mathbf{E} \equiv \prod_{\sigma} [\prod_h (U^* \times \mathbb{R}_{++}^{CS})] \times \mathbb{R}^{ACS}$ . An economy is a collection  $\{(u_h^{\sigma}, \omega_h(\sigma))_{h,\sigma}, \rho\} \equiv E \in \mathbf{E}$ . Bear in mind that  $\mathbf{E}$  is a metric space.

To establish the properties of the equilibrium set, we use (locally) finite dimensional, linear perturbations of the utility functions and, therefore, we (locally) treat  $\mathbf{E}$  as a finite dimensional manifold. The parameterization of the space of the utility functions is the following: For each  $h$ , let  $B_h^j(\sigma)$ ,  $j = 1, \dots, J$ , be open (possibly empty) sets such that  $\text{cl}B_h^j(\sigma) \subset \mathbb{R}_{++}^{CS}$ , for each  $j$ . Also, assume that, for each  $j$  and given  $B_h^j(\sigma)$ , there is an open set  $B_h^{ej}(\sigma)$  which satisfies  $\text{cl}B_h^j(\sigma) \subset B_h^{ej}(\sigma) \subset \text{cl}B_h^{ej}(\sigma) \subset \mathbb{R}_{++}^{CS}$  and  $\text{cl}B_h^{ej}(\sigma) \cap \text{cl}B_h^{ej'}(\sigma) = \emptyset$ , for each  $j$  and  $j'$ .

Given  $B_h^{ej}(\sigma)$ , for each  $j$ , let  $\Theta_h^{j\sigma}(x_h(\sigma), B_h^{ej}(\sigma))$  be a smooth "bump" function:  $\Theta_h^{j\sigma}: \mathbb{R}_{++}^{CS} \rightarrow [0, 1]$ .  $\Theta_h^{j\sigma}(x_h(\sigma), B_h^{ej}(\sigma))$  takes the value 1 if  $x_h(\sigma) \in B_h^j(\sigma)$  and the value 0 if  $x_h(\sigma) \notin \text{cl}B_h^{ej}(\sigma)$  (see, Hirsch [7, p. 41]).

Given  $u_h^{\sigma} \in U^*$  and, for each  $h$  and each  $j$ , vectors  $\delta_h^j(\sigma) \in \mathbb{R}^{CS}$ , define  $u_h^{\delta\sigma}$  the economy where, for each  $h$ ,  $u_h^{\delta\sigma}(x_h(\sigma))$  is obtained replacing  $u_h^{\sigma}(x_h(\sigma))$  with the function

$$u_h^{\delta\sigma}(x_h(\sigma)) \equiv u_h^\sigma(x_h(\sigma)) + \sum_j \Theta_h^{j\sigma}(x_h(\sigma), B_h^{ej}(\sigma))[\delta_h^j(\sigma)x_h(\sigma)].$$

Evidently, if  $u_h^\sigma(x_h(\sigma)) \in U^*$ , given any collection of open balls  $B_h^j(\sigma)$  satisfying the properties defined above,  $u_h^{\delta\sigma}(x_h) \in U^*$  for  $\delta_h^j(\sigma)$  small enough for each  $j$ .

The parameterization adopted has two important properties:

- a. It allows independent perturbations of the utility function on the disjoint sets  $B_h^j(\sigma)$ ;
- b. Locally (i.e., on a given set  $B_h^j(\sigma)$ ), it allows us to modify the gradients of the utility functions by an (arbitrary) vector  $(\dots, \delta_h^j(\sigma), \dots)$  without altering the higher order derivatives.

## 2.2 Individual behavior and equilibrium

In our economy, individuals observe equilibrium asset prices and use them to refine their private information. Conditional on the realization of the signal  $\sigma$  and on the observation of asset prices  $q(\sigma)$ , individuals formulate a conjecture on the future realization of equilibrium spot commodity prices. A price conjecture is a probability distribution over a finite set of future market clearing commodity prices. We assume that, for each  $\sigma$ , there exists a set  $K = \{1, \dots, K\}$  of extrinsic events which serves as the support of the future price conjecture. All individuals in the economy share the same probability distribution over future prices, or, equivalently, over the extrinsic events,  $\pi(\cdot|\cdot) = (\dots, \pi(\cdot|\sigma), \dots)$ . More formally:

**Definition 1:** A *price conjecture* is a collection  $(p, \pi(\cdot|\cdot)) = (\dots, (p(k,\sigma), \pi(k|\sigma)), \dots) \in \mathbb{R}_{++}^{\Gamma(C+1)}$ . A *K-dimensional price conjecture* is a price conjecture  $(p, \pi(\cdot|\cdot))$  such that, for each  $s$  and  $\sigma$ , the commodity relative prices (i.e.,  $p(s,k,\sigma)/p^1(s,k,\sigma)$ ) are one-to-one in  $k$ .

At prices  $(p, q)$ , the partition on  $\Sigma$  induced by the vector of asset prices  $q$  is  $T(q) \equiv \{\dots, \theta, \dots\}$ ,  $\#T(q) \leq \#\Sigma$ , where  $\theta \equiv \{\sigma \mid q(\sigma) = q(\theta)\}$ . Having observed  $q(\theta)$ , the uninformed individual rules out signals that do not lie in the set  $\theta$ . Let  $\pi(\sigma|\theta)$  be the conditional probability of  $\sigma$  given  $\theta$ , i.e.,

$\pi(\sigma|\theta) = \pi(\sigma)/\sum_{j \in \theta} \pi(j)$ . Hence, given  $((p, \pi(\cdot)), q)$  and having observed  $q(\theta)$ , individual  $u$  solves

$$[u(\theta)] \quad \max \sum_{\sigma \in \theta} \pi(\sigma|\theta) V_u^\sigma(x_u(\sigma)) \quad \text{subject to} \quad q(\theta)y_u(\theta) = 0, \text{ for each } \sigma \in \theta;$$

$$\Psi(\sigma)z_u(\sigma) - R(p(\sigma))y_u(\sigma) = 0, \text{ for each } \sigma \in \theta.$$

where

$$\Psi(\sigma) \equiv \begin{bmatrix} p(1, 1, \sigma) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & p(S, K, \sigma) \end{bmatrix}$$

is a matrix of dimension  $SK \times CSK$ .

Evidently, the optimal solution to the collection of  $\#T(q)$  optimization problems above is identical to the optimal solution to the following maximization problem:

$$[u] \quad \max \sum_{\theta \in T(q)} \sum_{\sigma \in \theta} \pi(\sigma) V_u^\sigma(x_u(\sigma)) \quad \text{subject to} \quad q(\sigma)y_u(\sigma) = 0, \text{ for each } \sigma;$$

$$\Psi(\sigma)z_u(\sigma) - R(p(\sigma))y_u(\sigma) = 0, \text{ for each } \sigma;$$

$$y_u \text{ is measurable with respect to } T(q).$$

The specification of the behavior of the informed individual is trivial: Given  $((p, \pi(\cdot)), q)$  and after having observed a realization of the signal  $\sigma$  and of asset prices  $q(\sigma)$ , individual  $i$  solves:

$$[i(\sigma)] \quad \max V_i^\sigma(x_i(\sigma)) \quad \text{subject to} \quad q(\sigma)y_i(\sigma) = 0;$$

$$\Psi(\sigma)z_i(\sigma) - R(p(\sigma))y_i(\sigma) = 0.$$

Evidently, the optimal solution to the collection of  $\Sigma$  optimization problems above is identical to the optimal solution to the following optimization problem

$$[i] \quad \max \sum_{\sigma} V_i^{\sigma}(x_i(\sigma)) \quad \text{subject to} \quad q(\sigma)y_i(\sigma) = 0, \quad \text{for each } \sigma;$$

$$\Psi(\sigma)z_i(\sigma) - R(p(\sigma)y_i(\sigma) = 0, \quad \text{for each } \sigma.$$

**Definition 2:** A *financial equilibrium* is an asset price vector and a price conjecture  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , with associated portfolio and consumption allocation  $(\dots, (\hat{x}(\sigma), \hat{y}(\sigma)), \dots)$ , such that:

- i.  $\hat{p}^1(s, k, \sigma) = 1$ , for each  $(s, k, \sigma)$ ;
- ii. For each  $h$ ,  $(\hat{x}_h, \hat{y}_h)$  solves the ex-ante optimization problem [h] at  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ ;
- iii.  $\sum_h \hat{y}_h = 0$  and  $\sum_h (\hat{x}_h - \omega_h) = 0$ .

A *K-dimensional sunspot equilibrium* is a financial equilibrium  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , where  $(\hat{p}, \hat{\pi}(\cdot))$  is a K-dimensional price conjecture.

A *financial equilibrium* is *fully revealing* if  $\hat{q}(\sigma) \neq \hat{q}(\sigma')$  for each pair  $\sigma$  and  $\sigma'$ . It is *nonrevealing* if  $\hat{q}(\sigma) = \hat{q}(\sigma')$  for each pair  $\sigma, \sigma'$ . It is *partially revealing*, otherwise. Moreover, an equilibrium is a *full rank equilibrium* if the matrix  $R(p(\sigma))$  has full rank  $A$  for each  $\sigma$ .

Since assets are real, the normalization imposed by condition i in Definition 1 does not introduce any loss of generality. In the proofs of the theorems, we adopt several different asset price normalizations. Hence, we do not normalize asset prices in the definition above.

Bear in mind that both the number of extrinsic states,  $K$ , and the probability vector,  $\pi(\cdot)$ , are endogenous variables. Moreover, by the definition of K-dimensional price conjecture, at a K-dimensional sunspot equilibrium, the allocation  $\hat{x}_i(k, \sigma)$  is, for all  $\sigma$ , one-to-one in  $k$ .

Two final observations:

**Remark 1:** Given the structure of the two optimization problems, it suffices to analyze the two polar cases of fully revealing and nonrevealing equilibria. If there is partial revelation, the economy essentially decomposes into as many disjoint sub-economies as there are distinct asset price vectors. Each one of the sub-economies is characterized by nonrevelation.

**Remark 2:** The probability distribution  $\pi(\cdot|\sigma)$  may in principle depend on  $\sigma$ . Hence, since the random variable  $k$  is correlated with the random variable  $\sigma$  whose realization affects the fundamentals, strictly speaking,  $k$  is not a purely extrinsic event. On the other hand, at a fully revealing equilibrium, as already observed in Remark 1, the economy decomposes into  $\Sigma$  sub-economies. In that context, given a  $\sigma$ -realization,  $k$  is without doubt an extrinsic event. What it matter most, as the argument should clarify, at some heavy notational cost (and increasing the number of extrinsic events and of commodities), all the arguments of the paper can be rewritten restricting  $\pi(\cdot|\sigma)$  to be  $\sigma$ -invariant.

### 3. Main results

Our purpose is to show the existence of an open set of economies with nonrevealing equilibria. In our model, the existence of these equilibria depends entirely upon the role of extrinsic uncertainty. It does not depend upon any sort of "pathology" in the specification of the economy. In fact, equilibria where  $K$  is set equal to 1 are typically fully revealing. This is established in a companion paper, where we extend to our setup the classical generic revelation result. This is done exploiting only endowment perturbations. Hence, in [20] we define the space of economies in terms of endowments,  $\Omega$ . For completeness, we reproduce here the result.

**Theorem 1:** There is an open, dense set of full Lebesgue measure,  $\Omega'' \subset \Omega$ , such that, for each economy  $\omega \in \Omega''$ , there is a rational expectations equilibrium. Moreover, each financial equilibrium is a fully revealing rational expectations equilibrium.

**Proof of Theorem 1:** See Pietra and Siconolfi [20]. ■

Evidently, Theorem 1 implies that, even when  $K > 1$ , generically, there exists fully revealing equilibria where sunspot do not matter. We believe that Theorem 1 extends as well to economies with extrinsic uncertainty, where the probability distribution  $\pi(\cdot|.)$  is exogenously given, i.e., where  $K$  and  $\pi(\cdot|.)$  are not treated as equilibrium variable.

To properly formulate the main result of this paper we need two further pieces of notation.

Let

$$\text{diag}(\mathbf{E}) \equiv \{(E^1, \dots, E^S) \in \mathbf{E} \mid E^1 = \dots = E^S\}$$

be the set of economies such that the sub-economies associated with distinct realizations of the signal  $\sigma$  are identical (i.e., such that - from a substantive viewpoint - there is no asymmetry in information).

Moreover, define the set

$$\mathbf{E}(K) \equiv \{E \in \text{diag}(\mathbf{E}) \mid \text{the economy } E \text{ has a full rank, } K\text{-dimensional sunspot equilibrium}\}.$$

The main building block required to establish the existence of an open set of economies with nonrevealing equilibria is, of course, the analysis of the set of economies with  $K$ -dimensional sunspot equilibria, i.e. of economies with equilibria where sunspots matter. The following result, whose proof is postponed to section 3.1, is obtained by a straightforward modification of the argument proposed in Gottardi and Kajii [10], which is, in turn, based on Mas-Colell [17].

**Theorem 2:** Let  $C > K$  and  $1 < A < SK$ . The set  $\mathbf{E}(K)$  is open and non-empty.

Our main result, whose proof is postponed to section 3.2 is given by the following theorem:

**Theorem 3:** Let  $C > K > 2A+1$  and  $A > 1$ . There exists an open neighborhood  $B(\text{diag}(\mathbf{E}(K)))$  and an open, dense subset  $\mathbf{E}^{\text{NR}} \subset B(\text{diag}(\mathbf{E}(K)))$  such that, for each  $E \in \mathbf{E}^{\text{NR}}$ , there is a  $K$ -dimensional, nonrevealing sunspot equilibrium.

Hence, the set of economies with non-revealing equilibria contains a (relatively) open and

dense subset of the open set of economies which satisfy two properties:

- a. Each sub-economy has a full rank,  $K$ -dimensional sunspot equilibrium;
- b. The informational asymmetry is "small enough", i.e., the distance between the sub-economies associated with distinct realizations of the signal  $\sigma$  is sufficiently small.

As pointed out above, b constitutes a crucial difference between our result and previous examples of economies with non-revealing equilibria, where the essential role was played by restrictions on the parameters of the economy (e.g., on utility functions). In our setup, the set of economies with sunspot equilibria is open, but, unfortunately, it is not dense. Moreover, to establish the existence of nonrevealing equilibria, we need an additional restriction: For each  $\sigma$ , the minimal dimension of the sunspot equilibrium, i.e.,  $K$ , must be greater than  $2A+1$ . Once we restrict ourselves to the class of economies with sunspot equilibria of the right dimension, existence of nonrevealing equilibria is essentially related to the magnitude of the information asymmetries: Our approach to the construction of nonrevealing equilibria works when the difference across the sub-economies associated with the distinct signals is not "too large". This follows from purely technical aspects of our construction. However, in a competitive model, the result has the right flavor: If there is a large difference in the values of the parameters associated with distinct values of  $\sigma$ , this translates into very different behavior of the informed agents in the various sub-economies and, therefore, into large differences in equilibrium prices. To the contrary, if the difference in information is small, the effects of sunspot beliefs are strong enough to cover it, for some value of the probability distribution  $\pi(k|\sigma)$ .

### 3.1 The modified economy: Structure and basic properties

As is well known, competitive economies where individuals learn from the observation of prices generate discontinuous aggregate demand functions: Distinct, but arbitrarily close prices may reveal different information, thereby generating substantially different values of the aggregate excess demand. To avoid this problem, the classical argument for the existence of a REE (e.g., Radner [22]) analyzes a *fictitious* economy. In the fictitious economy, individuals do not learn from prices and they

are endowed with an exogenously given private information. An equilibrium price of the fictitious economy is a REE of the original economy only if it generates (were individuals able to learn from observed prices) the same information which is exogenously given in the fictitious economy.

We will prove Theorem 3 following, basically, the same strategy. However, we have to deal with an additional problem. Since we are looking for a nonrevealing equilibrium, the natural candidate for a fictitious economy is an economy defined by the maximization problems [i] and [u] where, in addition, we impose in [u] that, for all the asset prices  $q$ ,  $y_u(\sigma)$  is  $\sigma$ -invariant. Unfortunately, the excess demand functions of this economy is subject to two different sources of discontinuities. First, the matrix of asset yields may loose rank in some regions of the price domain. Second, the portfolio holdings of individual  $u$ ,  $y_u$ , must satisfy as many budget constraints as the number of linearly independent prices  $q(\sigma)$ ,  $\sigma \in \Sigma$ . Since assets are real, we have to live with the first problem, but we can get around the second.

Following Polemarchakis and Siconolfi [20], we introduce a *modified economy*. This is a standard economy with restricted participation on the asset market, which is quite easy to study and with the property that, when the asset prices are  $\sigma$ -invariant, its equilibria are nonrevealing equilibria of the original economy.

We define the *modified economy* as follows: Independently of the properties of the price system  $(p, q)$ , agent  $u$  solves the modified optimization problem

$$\begin{aligned}
 [u'] \max \quad & \sum_{\sigma} \pi(\sigma) V_u^{\sigma}(x_u(\sigma)) \equiv V_u(x_u) \quad \text{subject to} \quad \sum_{\sigma} (q(\sigma)/\Sigma) y_u(\sigma) = 0; \\
 & \Psi(\sigma) z_u(\sigma) - R(p(\sigma)) y_u(\sigma) = 0, \text{ for each } \sigma; \\
 & y_u \text{ is } \sigma\text{-invariant.}
 \end{aligned}$$

Bear in mind that, in [u'], we impose the condition  $y_u(\sigma) = y_u(\sigma')$ , for all  $\sigma$  and  $\sigma'$ , even when asset prices are fully (or partially) revealing. Moreover, if asset prices  $q(\sigma)$  are  $\sigma$ -invariant, the programming problems [u] and [u'] coincide.



Agent  $i$  solves the following modified optimization problem:

$$\begin{aligned}
 [i'] \quad \max \sum_{\sigma} V_i^{\sigma}(x_i(\sigma)) \equiv V_i(x_i) \quad \text{subject to} \quad & \sum_{\sigma} (q(\sigma)/\Sigma)y_i(\sigma) = 0; \\
 & \Psi(\sigma)z_i(\sigma) - R(p(\sigma))y_i(\sigma) = 0, \text{ for each } \sigma; \\
 & y_i(\sigma) \in \mathbb{R}^A, \text{ for each } \sigma.
 \end{aligned}$$

Observe that the budget constraint of the modified programming problem [i'] contains the budget constraint of the optimization problem [i].

The modified economy is an economy with  $A\Sigma$  assets, where individual  $u$  faces a linear constraint on portfolio holdings. Assets are indexed by the pair  $(a, \sigma)$ ,  $a = 1, \dots, A$  and  $\sigma \in \Sigma$ . We maintain the normalization  $p(s, k, \sigma) = 1$ , for each  $(s, k, \sigma)$ , and we set  $q^1(1) = 1$ .

Since assets are real, individual  $h$ 's excess demand function (for commodities and assets),  $(z_h, y_h)(p, \pi(\cdot), q, E)$ , can be discontinuous at commodity prices inducing a collapse of rank of the asset yield matrix. Otherwise,  $(z_h, y_h)(p, \pi(\cdot), q, E)$  is continuous and differentiable. Also, let  $\lambda_h^1$  be the Lagrange multiplier associated with the period 0 budget constraint and let  $\lambda_h^2$  be the vector of Lagrange multipliers associated with the spot market budget constraints. Finally, let  $\lambda_h \equiv (\lambda_h^1, \lambda_h^2)$ .

An equilibrium of the modified economy,  $E \in \mathbf{E}$ , is a price system  $((p, \pi(\cdot)), q)$  such that  $(z_u, y_u)(p, \pi(\cdot), q, E) + (z_h, y_h)(p, \pi(\cdot), q, E) = 0$ . An equilibrium is a *full rank equilibrium* if the matrix  $R(p(\sigma))$  has full rank  $A$ , for each  $\sigma$ .

Let  $\zeta_h(p, \pi(\cdot), q, E)$  be agent  $h$ 's excess demand for commodity  $c > 1$  at each spot and for the  $(A\Sigma - 1)$  assets,  $(a, \sigma) \neq (1, 1)$ . Let  $\zeta(p, \pi(\cdot), q, E)$  be the aggregate excess demand for the same commodities and assets. Let  $((p, \pi(\cdot)), q)$  be a price system such that  $R(p(\sigma))$  has full rank, for all  $\sigma$ . Then,  $((p, \pi(\cdot)), q)$  is an equilibrium if and only if  $\zeta(p, \pi(\cdot), q, E) = 0$ .

Our interest in the modified economy is entirely due to the relation between equilibria of the modified economy and nonrevealing equilibria of the original economy, which is summarized by the following Proposition.

**Proposition 1:** Let  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , with associated allocation  $(\hat{x}, \hat{y})$ , be an equilibrium of the modified economy  $E$ . Suppose that  $\hat{q}(\sigma) = \hat{q}(\sigma')$  for each  $\sigma$  and  $\sigma'$ . Then,  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , with associated allocation  $(\hat{x}, \hat{y})$ , is a nonrevealing equilibrium of the original economy  $E \in \mathbf{E}$ .

**Proof of Proposition 1:** See Polemarchakis and Siconolfi [21, Lemma 1]. ■

In Lemma 1, whose proof is postponed to the appendix, we establish the relevant properties of the map  $\zeta(p, \pi(\cdot), q, E)$ . The restriction  $\rho^1(s) = (1, 0, \dots, 0)$ , for each  $s$ , drastically simplifies its proof.

**Lemma 1:** Let  $((p, \pi(\cdot)), q)$  be a full rank equilibrium of the modified economy  $E$ . Then:

- i. The map  $\zeta(p, \pi(\cdot), q, E)$  is smooth.
- ii.  $\text{rank } D_{(p,q)} \zeta(\cdot) = \text{rank } (-\lambda_i^1 I + M(\lambda_i/\lambda_i^1, p, \pi(\cdot), q), D_x^2 V_i(x_i)) + D_{(p,q)} \zeta_u(\cdot))$ ,

where  $M(\cdot)$  is a square,  $((C-1)\Gamma + A\Sigma - 1) \times ((C-1)\Gamma + A\Sigma - 1)$ -dimensional matrix.  $M(\cdot)$  depends on the normalized Lagrange multipliers,  $\lambda_i/\lambda_i^1$ , the Hessian of individual  $i$  utility function,  $D_x^2 V_i(x_i)$ , and on  $((p, \pi(\cdot)), q)$ .  $M(\cdot)$  does not depend on  $\lambda_i^1$ .

Even though the probability distribution  $\pi(\cdot)$  enters the definition of equilibrium, we maintain the classical definition of regularity:  $((p, \pi(\cdot)), q)$  is a regular equilibrium of a modified economy  $E$  if  $D_{(p,q)} \zeta((p, \pi(\cdot)), q, E)$  is invertible.

Lemma 1 is instrumental in establishing that, generically, full rank equilibria of the modified economy are regular. Consider a critical full rank equilibrium. Roughly speaking, we restore the regularity of the equilibrium by performing an arbitrarily small perturbation of the economy. Lemma 1.ii suggests to look for a perturbation of the utility function that modifies the Lagrange multiplier  $\lambda_i^1$ , while keeping constant both  $\lambda_i/\lambda_i^1$  and the Hessian of the utility function. We introduce now a utility perturbation (local around  $x_i^*$ ) that, by modifying proportionally the gradient of the utility function, while keeping constant its Hessian, accomplishes this task. In the sequel, we will refer to this

perturbation as the  $\delta$ -perturbation.

States which are distinct just because of the index  $k$ ,  $k = 1, \dots, K$ , only differ with respect to the realization of purely extrinsic events. This poses severe limitations on the type of perturbations that can be used, because each perturbation must preserve the extrinsic nature of an event  $k$ . Since we are dealing with utility perturbations, this means that the "perturbed" utility function must be  $k$ -invariant.

By an inspection of the first order conditions of the programming problem [i], it is immediate to check that, for given  $\sigma$  and  $k \neq k'$ ,  $x_i^*(k, \sigma) \neq x_i^*(k', \sigma)$  if and only if  $p(k, \sigma) \neq p(k', \sigma)$ . Moreover, if  $x_i^*(k', \sigma) = x_i^*(k', \sigma)$  (and, hence,  $p(k, \sigma) = p(k', \sigma)$ ), then  $\lambda_i^2(k, \sigma)/\pi(k|\sigma) = \lambda_i^2(k', \sigma)/\pi(k'|\sigma)$ , for given  $\sigma$  and  $k \neq k'$ .

For given  $\sigma$ , define a collection of consumption vectors  $x_i^*(j, \sigma)$ ,  $j = 1, \dots, J(\sigma)$ , satisfying

- a.  $x_i^*(j, \sigma)$  is one-to-one in  $j$ ;
- b. For each  $j$ , there exists a  $k$  such that  $x_i^*(j, \sigma) = x_i^*(k, \sigma)$ .

Hence,  $x_i^*(j, \sigma)$ ,  $j = 1, \dots, J(\sigma)$ , are the distinct values taken up by the allocation  $x_i^*(k, \sigma)$ ,  $k = 1, \dots, K$ , and  $J(\sigma)$  is their number,  $J(\sigma) \leq K$ . Similarly, let  $\mu_i^2(j, \sigma)$  and  $p(j, \sigma)$ ,  $j = 1, \dots, J(\sigma)$ , be the  $J(\sigma)$  distinct values taken up by the ratios  $\lambda_i^2(k, \sigma)/\pi(k|\sigma)$  and by the commodity prices  $p(k, \sigma)$ ,  $k = 1, \dots, K$ .

For given  $\sigma$ , let  $B_i^{\sigma j}(x_i(j, \sigma))$ ,  $B_i^{\sigma j^e}(x_i(j, \sigma))$  and  $\Theta^{\sigma j}(x_i(j, \sigma), B_i^{\sigma j^e}(x_i(j, \sigma)))$ ,  $j = 1, \dots, J(\sigma)$ , be a collection of open neighborhoods of  $x_i^*(j, \sigma)$  and of bump functions satisfying the assumptions of section 2.1. For all  $\sigma$  and for  $\delta$  small enough,  $\delta$  independent of  $\sigma$ , consider the following perturbation of individual  $i$ 's Von Neumann-Morgenstern cardinality index,  $u_i^{\sigma}(\cdot): \mathbb{R}_{++}^{SC} \rightarrow \mathbb{R}$ :

$$u_i^{\sigma \delta}(x_i(\sigma)) \equiv u_i^{\sigma}(x_i(\sigma)) + \sum_j \Theta^{\sigma j}(x_i(j, \sigma), B_i^{\sigma j^e}(x_i(j, \sigma))) [\delta \sum_s \mu_i^2(s, j, \sigma) p(s, j, \sigma) x_i(s, j, \sigma)],$$

By construction,  $u_i^{\sigma \delta}$  is  $k$ -invariant, i.e., the extrinsic nature of the events indexed by  $k$  is preserved by the  $\delta$ -perturbation.

From the expression of  $u_i^{\sigma\delta}$  we immediately infer that the perturbation induced on the overall utility function  $V_i^{\sigma\delta}(x_i(\sigma))$  is given by :

$$V_i^{\sigma\delta}(x_i(\sigma)) \equiv V_i^\sigma(x_i(\sigma)) + \sum_{j=1}^{J_i(\sigma)} \Theta^{\sigma j}(x_i(j,\sigma), B_i^{\sigma j}(x_i(j,\sigma))) [\delta \sum_s \lambda_i^2(s,j,\sigma) p(s,j,\sigma) x_i(s,j,\sigma)].$$

Evidently, for  $\delta$  arbitrarily close to 0,  $V_i^\delta(x_i(\sigma))$  is arbitrarily close to  $V_i(x_i(\sigma))$ . Moreover, it is immediate to check, from an inspection of the first order conditions, that, at  $((p, \pi(\cdot)), q)$ , agent  $i$ 's optimal consumption-portfolio choice does not change if the utility function  $V_i$  is replaced with  $V_i^\delta$ . Though, this perturbation does change the Lagrange multipliers of the individual maximization problem from  $(\lambda_i^1, \lambda_i^2)$  to  $(\lambda_i^1(1+\delta), \lambda_i^2(1+\delta))$ , while keeping constant the Hessian of the utility function. This observation, together with Lemma 1.ii, is the key to the proof of the generic regularity of the modified economy equilibria. This is shown in Lemma 2.

**Lemma 2:** Let  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , with associated allocation  $(\hat{x}, \hat{y})$ , be a full rank equilibrium of the modified economy  $E$ . Let  $B(E)$  be any open neighborhood of  $E$ . Then, under the maintained assumptions, there is an economy  $E' \in B(E)$  such that  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , with associated allocation  $(\hat{x}, \hat{y})$ , is a regular equilibrium of the modified economy  $E'$ .

**Proof of Lemma 2:** By Lemma 1,

$$\text{rank } D_{(p,q)} \zeta(p,q,\pi(\cdot),E) = \text{rank } (-\lambda_i^1 I + M(\cdot) + D_{(p,q)} \zeta_u(p,q,\pi(\cdot),E)) \equiv \text{rank } (\lambda_i^1 I + M^{**}).$$

As already stated in Lemma 1, the entries of  $M(\cdot)$  and, therefore,  $M^{**}$  do not depend on  $\lambda_i^1$ , but only on  $\lambda_i^2/\lambda_i^1$ .

For given  $M^{**}$ , consider the matrix  $(-\lambda_i^1 I + M^{**})$  as a function of  $\lambda_i^1$ . If, for some values of  $\lambda_i^1$ ,  $\text{rank } (\lambda_i^1 I + M^{**}) < ((C - 1)S\Sigma K + A\Sigma - 1)$ ,  $\lambda_i^1$  is an eigenvalue of the matrix  $M^{**}$ . Given that the number of eigenvalues is finite, a small perturbation of  $\lambda_i^1$  restores full rank. Keeping this in mind,

suppose that the equilibrium  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  of the modified economy  $E$  is not regular. Select a value of  $\lambda_i^!$ , say  $\lambda_i^{!*}$ , such that the matrix  $(-\lambda_i^{!*}I + M^{**})$  has full rank. Let  $E'$  be the economy obtained with the  $\delta$ -perturbation of agent  $i$ 's utility function,  $\delta \equiv (\lambda_i^{!*} - \lambda_i^!)/\lambda_i^!$ . By construction,  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  is an equilibrium of  $E'$  and, by the choice of  $\delta$ , it is regular. ■

Two observations about Lemma 1 and 2 and about the  $\delta$ -perturbations will play an important role throughout the analysis:

- a. When  $\Sigma = 1$ , the modified economy reduces to a standard economy with an incomplete asset market and real assets. Since the proofs of the Lemma 1 and 2 do not depend on  $\#\Sigma$ , the stated results hold true for standard real asset incomplete market economies. Moreover, it is immediate to check that they extend as well to economies with nominal or real numeraire assets (not only this is shown in Pietra [18, 19], but actually the proof of Lemma 2 is just a minor modification of the argument in Pietra).
- b. Lemma 1 and 2 establish a result which is actually stronger than the generic regularity of equilibria. They establish that *any* full rank equilibrium price and allocation is regular up to a  $\delta$ -perturbation of the economy. Hence both equilibria where sunspots matter and equilibria where they do not matter are regular (up to a  $\delta$ -perturbation).

### 3.2 Proof of Theorem 2

Theorem 2 refers to economies where  $E^\sigma = E^{\sigma'}$  for each  $\sigma$  and  $\sigma'$ . To study their properties, it is convenient to take as a starting point the class of economies where  $\#\Sigma = 1$ . When  $\#\Sigma = 1$ , our economy reduces to a standard incomplete asset market economy with two individuals. Let us denote such an economy by  $E^\alpha$ . In the  $E^\alpha$ -economy, for both individuals, the individual programming problems are given by  $[i(\sigma)]$ . Evidently, to each economy  $(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E})$  it corresponds unambiguously an  $E^\alpha$ -economy.

The economy  $E^\alpha$  has  $S$  intrinsic and  $K$  extrinsic events, hence  $SK$  states of nature.

Most of the argument for the proof of Theorem 2 is based on the study of the  $E^\alpha$  economy. The link between the equilibria of the  $E^\alpha$ -economy and the economy  $(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E})$  is provided by the following Lemma, which refers to the *actual* economy. As a Corollary, the same result is shown to hold for the *modified* economy. Also, bear in mind that the equilibria of the economies in  $\mathbf{E}(\mathbf{K})$  need not to be regular.

**Lemma 3:** If there exists a regular, full rank,  $K$ -dimensional sunspot equilibrium of the  $E^\alpha$ -economy,  $((p^\alpha, (\dots, \pi(k), \dots)), q^\alpha)$ , then the set  $\mathbf{E}(\mathbf{K})$  is open (relatively to  $\text{diag}(\mathbf{E})$ ).

**Proof of Lemma 3:** Since  $((p^\alpha, (\dots, \pi(k), \dots)), q^\alpha)$  is a regular equilibrium of the  $E^\alpha$ -economy, there exists an open neighborhood of  $E^\alpha$ ,  $B(E^\alpha)$ , such that each economy  $E^{\alpha'} \in B(E^\alpha)$  has one regular, full rank,  $K$ -dimensional sunspot equilibrium,  $((p^{\alpha'}, (\dots, \pi(k), \dots)), q^{\alpha'})$ . Let  $(x_h^{\alpha'}, y_h^{\alpha'})$ ,  $h = i, u$ , be the equilibrium consumption and portfolio allocation associated with the equilibrium  $((p^{\alpha'}, (\dots, \pi(k), \dots)), q^{\alpha'})$  of an economy  $E^{\alpha'} \in B(E^\alpha)$ .

Associate with each economy  $E^{\alpha'} \in B(E^\alpha)$  an economy  $(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E})$  defined by  $E^\sigma = E^{\alpha'}$ , for each  $\sigma$ . Observe that the set  $\{(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E}) \mid E^\sigma = E^{\alpha'}, \text{ for each } \sigma, E^{\alpha'} \in B(E^\alpha)\}$  is an open subset of  $\text{diag}(\mathbf{E})$ .

For an arbitrary economy in  $B(E^\alpha)$ , say  $E^\alpha$  itself, define  $p(k, \sigma) = p^\alpha(k)$  and  $q(\sigma) = q^\alpha$ , for all  $\sigma$ ,  $\pi(k|\sigma) = \pi(k)$ , for all  $k$  and  $\sigma$ . Also let  $(x_h(\sigma), y_h(\sigma)) = (x_h^\alpha, y_h^\alpha)$ , for all  $\sigma$ ,  $h = i, u$ .

We want to show that  $((p, \pi(\cdot)), q)$  is a full rank,  $K$ -dimensional sunspot equilibrium of the economy  $(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E})$  and that  $(x, y) = (\dots, x(\sigma), y(\sigma), \dots)$  is the corresponding equilibrium allocation.

For individual  $i$ , decompose the programming problem  $[i]$  into the  $\Sigma$  problems  $[i(\sigma)]$ ,  $\sigma = 1, \dots, \Sigma$ . At prices  $((p, \pi(\cdot)), q)$ , each  $[i(\sigma)]$  problem coincides with the programming problem that individual  $i$  is solving in the  $E^\alpha$ -economy at prices  $(p^\alpha, q^\alpha)$ . Hence,  $(x_i, y_i)$  is the optimal solution.

For individual  $u$ , the budget set of the programming problem  $[u]$  is strictly contained in the budget set of the programming problem  $[i]$ . The  $\sigma$ -invariance of  $q(\sigma)$  implies that  $(x_u, y_u)$  is budget

feasible. Hence, at prices  $((p, \pi), q)$ ,  $(x_u, y_u)$  is the optimal solution to [u].

Therefore,  $((p, \pi(\cdot)), q)$  is an equilibrium of the economy  $(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E})$ .

The argument is concluded by observing that the set  $\{(\dots, E^\sigma, \dots) \in \text{diag}(\mathbf{E}) \mid E^\sigma = E^{\alpha'}\}$ , for each  $\sigma, E^{\alpha'} \in B(E^\alpha)$  is an open subset of  $\text{diag}(\mathbf{E})$  and that we can repeat for each  $E^{\alpha'} \in B(E^\alpha)$  the same argument used for the equilibrium of the economy  $E^\alpha$ . ■

As already mentioned, Theorem 3 will be proved by analyzing the modified economies. By Proposition 1, this is without loss of generality. Hence, we need to extend Theorem 2 to the modified economies. This is taken care of in the following corollary. Bear in mind that *actual* and *modified* economies are defined by the same fundamentals  $E \in \mathbf{E}$ , but by different programming problems.

**Corollary to Lemma 3:** If there exists a regular, full rank,  $K$ -dimensional sunspot equilibrium of the  $E^\alpha$ -economy,  $((p^\alpha, (\dots, \pi(k), \dots)), q^\alpha)$ , then the set  $\mathbf{E}(K)$  is open (relatively to  $\text{diag}(\mathbf{E})$ ) in the *modified* economy.

**Proof of the Corollary:** By Lemma 3, there exists a  $K$ -dimensional, full rank, sunspot equilibrium price and allocation,  $((p, \pi(\cdot)), q)$  and  $(x, y)$ , of an economy  $E \in \mathbf{E}(K)$  such that  $((p(\sigma), \pi(\cdot|\sigma)), q(\sigma))$  and  $(x(\sigma), y(\sigma))$  are  $\sigma$ -invariant. Since  $q(\sigma)$  is  $\sigma$ -invariant, for individual  $u$ , the budget constraints of the modified programming problems [u'] coincides, at prices  $((p, \pi(\cdot)), q)$ , with the budget constraints of the original economy [u]. Hence,  $(x_u, y_u)$  is the optimal solution to the modified programming problem [u] at prices  $((p, \pi(\cdot)), q)$ .

For individual  $i$ , the budget constraints of the modified programming problems [i'] contains, at  $((p, \pi(\cdot)), q)$  the budget constraints of the actual economy, i.e., of programming problem [i]. Hence, at  $((p, \pi(\cdot)), q)$ ,  $(x_i, y_i)$  is budget feasible. Let  $\lambda_i^1(\sigma)$  and  $\lambda_i^2(\sigma)$ ,  $\sigma = 1, \dots, \Sigma$ , be the Lagrange multipliers of the programming problem [i]. Since  $((p(\sigma), \pi(\cdot|\sigma)), q(\sigma))$  and  $(x_i(\sigma), y_i(\sigma))$  are  $\sigma$ -invariant,  $\lambda_i^1(\sigma) = \lambda_i^1$ , for all  $s$  and some scalar  $\lambda_i^1 > 0$ . Then, by an inspection of the first order conditions of the modified programming problem [i'], it is trivial to check that  $(x_i, y_i)$ ,  $\lambda_i^1$  and  $\lambda_i^2(\sigma)$ ,

$\sigma = 1, \dots, \Sigma$ , solve the first order conditions of the modified programming problem [i'] at prices  $((p, \pi(\cdot)), q)$ . ■

By Lemma 3, the proof of Theorem 2 reduces to show the existence of a regular, full rank,  $K$ -dimensional sunspot equilibrium of the economy  $E^\alpha$ . Moreover, by the Corollary to Lemma 3, Theorem 2 immediately extends to modified economies. Hereafter, fix  $K, K < C, SK > A$  and  $\pi(k) = 1/K$ , for all  $k$ . Bear in mind that the economy  $E^\alpha$  is completely defined by an utility-endowment profile,  $((V_h^\alpha, \omega_h)_{h=i,u}) \in \prod_h (U^* \times \mathbb{R}_{++}^{CS})$  and by a matrix of asset payoffs (in terms of commodities)  $\rho \in \mathbb{R}^{ACS}$ .

**Lemma 4:** Assume  $SK > A > 1$  and  $C > K$ . Then, there exists an open and dense set of utility-endowment profiles  $G \subset \prod_h (U^* \times \mathbb{R}_{++}^{CS})$ , such that, for each  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G$ , the economy  $E^\alpha = ((\omega_h, V_h^\alpha)_{h=i,u}, \rho)$ , has a regular, full rank,  $K$ -dimensional sunspot equilibrium, for some (relatively) open set of asset structures,  $\rho \in \mathbb{R}^{ACS}$  such that  $\rho(s) = (1, 0, \dots, 0)$  for each  $s$ .

Evidently, if  $A \leq 1$ , intertemporal autarky must prevail at an equilibrium. Hence, we assume  $A > 1$ . The motivation for the restriction  $C > K$  will become transparent later on. Also, regularity implies that we could take the set of asset structure to be open in  $\mathbb{R}^{ACS}$ , allowing asset 1 to be a generic real asset. Hence, the restriction on asset 1 payoff does not entail any loss of generality.

The proof of Lemma 4 is very long and will be divided in several steps. There are two main parts of the argument. First, we consider a *fictitious* economy defined by endowment-utility profiles  $((\omega_h, V_h^\alpha)_{h=i,u})$  and asset yield matrix  $R$  of dimension  $KS \times A, A < SK$ . The yield matrix  $R$  is independent of relative commodity spot prices (equivalently, it is defined in terms of numeraire commodities) and throughout the argument it has full column rank. To construct a regular, full rank,  $K$ -dimensional sunspot equilibrium of the fictitious economy, we use the asset yield matrix  $R$  as a free parameter. The last part of the argument shows that the sunspot equilibrium of the fictitious economy,



together with  $\pi(k) = 1/K$ , for all  $k$ , is an equilibrium of the  $E^\alpha$ -economy for some  $\rho$  (where asset 1 is a real numeraire asset). This line of proof follows very closely the argument in Gottardi and Kajji [10] and Mas-Colell [17].

The first step shows that, generically in endowment-utility profile, we can select two assets with yield vectors  $r^a$ ,  $a = 1, 2$ , such that the nonsunspot equilibrium of the resulting fictitious economy is regular. The second step uses the regularity of the nonsunspot equilibria to construct  $K$ -dimensional sunspot equilibria of the fictitious economy. Bear in mind that, throughout the proof of Lemma 4, we set  $\pi(k) \equiv 1/K$  and we treat this vector as a fixed parameter.

**Step 1:** There exists an open and dense set of utility-endowment profiles  $G^1 \subset \prod_h (U^* \times \mathbb{R}_{++}^{CS})$ , such that, for  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G^1$ , the fictitious economy  $((\omega_h, V_h^\alpha)_{h=i,u}, (r^1, r^2))$  has a regular nonsunspot equilibrium, for some  $(r^1, r^2) \in \mathbb{R}^{2SK}$ .

**Proof of Step 1:** Consider an utility-endowment profile  $((V_h^\alpha, \omega_h)_{h=i,u}) \in \prod_h (U^* \times \mathbb{R}_{++}^{CS})$ . Let  $p^* \equiv (\dots, p^*(s), \dots)$  be the competitive equilibrium price vector in the associated Walrasian economy *without* extrinsic uncertainty. Let  $\lambda_h$  be the associated Lagrange multiplier. Assume that  $p^*(s)z_h^*(s) \neq 0$ , for all  $s$ . As is well known, this property holds true for an open and dense set of endowment-utility profiles.

Let  $\hat{p}(s,k) = p^*(s)/p^*(s)$ , for each  $s$  and  $k$ . Introduce two assets. The first asset has safe yields  $\hat{r}^1(s,k) = 1$ , while the second asset yields  $\hat{r}^2(s,k) = \hat{p}(s,k)z_i^*(s) + 1$ , for all  $(s, k)$ . Evidently, since  $p^*(s)z_h^*(s) \neq 0$ , for all  $s$ , and  $\sum_s p^*(s)z_h^*(s) = 0$ , the matrix  $(\hat{r}^1, \hat{r}^2)$  has full column rank.

We want to show that  $(\hat{p}, \hat{q})$ , with  $\hat{q} = (1, 1)$ , is a nonsunspot equilibrium of the fictitious economy *with* extrinsic uncertainty and with  $\pi(k) = 1/K$ , for all  $k$ . To do that, we just have to show that the allocation  $\hat{x}_h(s,k) \equiv x_h^*(s)$ , for all  $k$  and  $s$ , is an optimal solution of the individual programming problem at prices  $(\hat{p}, \hat{q})$ .

Let  $\hat{y}_i = (-1, 1) = -\hat{y}_u$ . These portfolios are budget feasible and make the allocation  $\hat{x}_h(s,k)$

budget feasible, at spot prices  $\hat{p}$ . Define  $\hat{\lambda}_h^1 = \lambda_h^* \sum_s p^{*1}(s)/K$  and  $\hat{\lambda}_h^2(s,k) = \lambda_h^* p^{*1}(s)/K$ . Then, it is trivial to check that  $\hat{x}_h, \hat{y}_h, \hat{\lambda}_h^1$  and  $\hat{\lambda}_h^2$  solve the first order conditions of the individual programming problem at prices  $(\hat{p}, \hat{q})$ . Hence  $(\hat{p}, \hat{q})$  is a competitive equilibrium of the fictitious economy, where sunspots do not matter.

As already mentioned, Lemma 2 applies also to economies with numeraire assets (see Pietra [18]). Hence, up to an arbitrarily small  $\delta$ -*perturbation*, the nonsunspot equilibrium  $(\hat{p}, \hat{q})$  is a regular equilibrium for the economy with asset yield matrix  $(\hat{r}^1, \hat{r}^2)$  and endowment-utility profiles  $((\omega_h, V_h^\alpha)_{h=i,u})$ . ■

**Step 2:** For  $SK > A > 1$  and  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G^2$ , an open and dense subset of  $G^1$ , the fictitious economy  $((\omega_h, V_h^\alpha)_{h=i,u}, R)$  has a regular,  $K$ -dimensional sunspot equilibrium, for some  $(SK \times A)$ -dimensional yield matrix  $R$ .

**Proof of Step 2:** Pick  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G^1$  and yield vectors  $(\bar{r}^1, \bar{r}^2)$ , with  $\bar{r}^1 = \hat{r}^1$  and  $\bar{r}^2$  both arbitrarily close to  $\hat{r}^2$  and one-to-one in  $(s, k)$ . By regularity of the original nonsunspot equilibrium, for  $\bar{r}$  close enough to  $\hat{r}$ , there is an equilibrium  $(\bar{p}, \bar{q})$ , arbitrarily close to  $(\hat{p}, \hat{q})$ , such that  $\bar{y}_h \neq 0$ . Moreover, by construction of  $\bar{r}$ ,  $\bar{r}(s,k)\bar{y}_h = \bar{r}(s,k')\bar{y}_h$ , for some  $s, k, k', k' \neq k$ , and  $\bar{y}_h \neq 0$ , if and only if  $\bar{y}_h^2 = 0$ . By the first period budget constraint and by  $\bar{q} \gg 0$ ,  $\bar{y}_h^1 \neq 0$  if and only if  $\bar{y}_h^2 \neq 0$ . Hence, by continuity and because  $\hat{y}_h \neq 0$ , for  $\bar{r}$  close enough to  $\hat{r}$ ,  $\bar{r}(s,k)\bar{y}_h \neq \bar{r}(s,k')\bar{y}_h$  (therefore,  $\bar{x}_h(s,k) \neq \bar{x}_h(s,k')$ ), for each  $s, k$  and  $k', k \neq k'$ .

Let  $\lambda_h^1$  and  $\lambda_h^2$  be the Lagrange multipliers associated with the equilibrium allocation  $\bar{x}_h, h = i, u$ . Since sunspot matters,  $\bar{x}$  is Pareto inefficient. Hence the vectors  $\lambda_h^2/\lambda_h^1, h = i, u$ , are linearly independent. For each  $1 < A < KS$ , select a matrix  $R$  of dimension  $SK \times A$  such that

- a.  $R^1 = \bar{r}^1 = 1$  and  $R^2 = \bar{r}^2$ , where  $R^j$  is column  $j$  of the matrix  $R$ ;
- b.  $R$  has full column rank;
- c.  $\lambda_h^2/\lambda_h^1 R = \bar{q}'$ , for some  $\bar{q}'$  in  $\mathbb{R}^A$ .

Since the linear subspace in  $\mathbb{R}^{KS+1}$  orthogonal to the vectors  $(-1, \lambda_h^2/\lambda_h^1)$  has dimension  $KS-1$ ,

there always exist matrices  $R$  satisfying a-c. Then, it is trivial to show that  $(\bar{p}, \bar{q}')$  is an equilibrium of the fictitious economy  $((\omega_h, V_h^\alpha)_{h=i,u}, R)$ .

Observe that, even though  $(\bar{p}, \bar{q})$  is a regular equilibrium of the fictitious economy  $((\omega_h, V_h^\alpha)_{h=i,u}, (\bar{r}^1, \bar{r}^2))$ ,  $(\bar{p}, \bar{q}')$  needs not to be a regular equilibrium of the economy  $((\omega_h, V_h^\alpha)_{h=i,u}, R)$ . However, whenever  $(\bar{p}, \bar{q}')$  is not regular, by Lemma 2, an arbitrarily small  $\delta$ -*perturbation* makes  $(\bar{p}, \bar{q}')$  a regular sunspot equilibrium of the  $\delta$ -*perturbed* economy  $((\omega_h)_{h=i,u}, V_u^\alpha, V_i^{\delta\alpha}, R)$ . ■

We want to show that there exists an asset payoff matrix  $\rho$  such that  $((\bar{p}, (\dots, 1/K, \dots), \bar{q}')$  is a full rank,  $K$ -dimensional sunspot equilibrium, of the economy  $E^\alpha = ((\omega_h, V_h^\alpha)_{h=i,u}, \rho)$ . A sufficient condition is to find an asset payoff matrix  $\rho$  which generates, at commodity prices  $\bar{p}$ , the same asset yields of the matrix  $R$ . Equivalently, we have to show that, for each  $s$  and  $a$ ,  $a = 1, \dots, A$ , the system of  $K$  equations in  $C$  variables given by

$$(*) \quad \bar{p}(s,k)\rho^a(s) = \bar{r}^a(s,k), \quad \text{for all } k,$$

has a solution  $\bar{p}^a(s)$ . Since  $C > K$ , a sufficient condition for the existence of a solution to  $(*)$  is that, for each  $s$ , the  $K$  price vectors  $\bar{p}(s,k)$  are linearly independent. The next step shows that this is generically the case. Notice that, for asset 1, it suffices to set  $\rho^1(s) = (1, 0, \dots, 0)$  for each  $s$ .

**Step 3:** For each  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G^3$ , an open and dense subset of  $G^2$ , the regular,  $K$ -dimensional sunspot equilibrium of the fictitious economy  $((\omega_h, V_h^\alpha)_{h=i,u}, R)$  has the property that the  $K$  vectors  $\bar{p}(s,k)$ ,  $k = 1, \dots, K$ , are linearly independent.

**Proof of Step 3:** Consider a regular,  $K$ -dimensional sunspot equilibrium  $(\bar{p}, \bar{q})$  of the fictitious economy  $((\omega_h, V_h^\alpha)_{h=i,u}, R)$ , with  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G^2$ , and suppose that there exists at least one state  $s^*$  such that the  $K$  vectors  $\bar{p}(s^*, k)$ ,  $k = 1, \dots, K$ , are linearly dependent (otherwise there is nothing to prove).

Let  $p''$  be a vector of commodity prices arbitrarily close to  $\bar{p}$  and satisfying

- a.  $p''(s,k) = \bar{p}(s,k)$ , for each  $s$  and  $k$ ,  $s \neq s^*$ ;
- b.  $(p''(s^*,k) - \bar{p}(s^*,k))\bar{z}_h(s^*,k) = 0$ , for  $h = i, u$ ;
- c. The  $K$  vectors  $p''(s^*,k)$ ,  $k = 1, \dots, K$ , are linearly independent.

Remember that, since  $\bar{z}_i = -\bar{z}_u$ , condition b holds for individual  $i$  if and only if it holds for individual  $u$ . Given the normalization  $p''(s^*,k) = 1$ , the set of prices  $p''(s^*,k)$  satisfying b is an affine space of dimension  $C-2$ , for each  $k$ . This means that, for each  $k$ , there are  $C-2$  linearly independent vector  $p_j''(s^*,k)$ ,  $j = 1, \dots, C-2$ , solving  $(p''(s^*,k) - \bar{p}(s^*,k))\bar{z}_h(s^*,k) = 0$ . Bear in mind that to obtain property c we need to perturb at most  $K-1$  vectors  $p(s^*,k)$ . But then, since  $C-2 \geq K-1$ , the set of prices  $p''$  satisfying a-c is nonempty.

Let  $p''$  be a price vector satisfying a-c. We are going to show that, perturbing the utility functions, we can perturb the equilibrium commodity prices from  $\bar{p}$  to  $p''$ .

Let  $\bar{x}_h(k) = (\bar{x}_h(s,k))_{s \in S}$ ,  $k = 1, \dots, K$ . Since the equilibrium allocation  $\bar{x}$  is one-to one in  $k$ , we can construct a collection of  $K$  open neighborhoods of  $\bar{x}_h(k)$ ,  $B_h^k$ , for each  $k$  and  $h = i, u$ , such that, for some  $\varepsilon > 0$ , there are open balls  $B_h^k$  satisfying:

- d.  $B_h^k \cap B_h^{k'} = \emptyset$ , for each  $k \neq k'$ ;
- e. For each  $(s, k)$ ,  $B_h^k \subset \text{cl} B_h^k \subset B_h^k \subset \text{cl} B_h^k \subset \mathbb{R}_{++}^{CS}$ .

Then, the parameterization of the utility functions described in section 2.1 allows us to independently perturb the gradient of the utility functions on each ball  $B_h^k$ , for each  $k$ . Hence, to change equilibrium price from  $\bar{p}$  to  $p''$ , it suffices to replace each agent's utility function  $u_h^\alpha(x_h(\alpha))$  with the function

$$u_h^{\delta\alpha}(x_h) \equiv u_h^\alpha(x_h) + \sum_k \Theta_h^{\alpha k}(x_h, B_h^k)(\lambda_h^2(s^*,k)/\pi(k))[(p''(s^*,k) - \bar{p}(s^*,k))x_h(s^*,k)].$$

Since, for each  $h$  and each  $(s, k)$ ,  $(p''(s,k) - \bar{p}(s,k))\bar{z}_h(s,k) = 0$ , the perturbation of commodity prices has no effect on agents' behavior. Inspection of the first order conditions of the individual programming problem shows that if  $(\bar{x}_h, \bar{y}_h)$  with associated vector of Lagrange multipliers  $\lambda_h$  is an

optimal solution to [h] at  $((\bar{p}, \pi), \bar{q})$  given  $u_h^\alpha(x_h)$ , then  $(\bar{x}_h, \bar{y}_h)$  with associated vector of Lagrange multipliers  $\lambda_h$  is an optimal solution to [h] at  $((p'', \pi), \bar{q})$  given  $u_h^{\delta\alpha}(x_h)$ .

Bear in mind that, by construction of the balls  $B_h^k$ ,  $k = 1, \dots, K$ , the perturbations of agent h's utility function on the different balls are completely independent and do not affect the extrinsic nature of the uncertainty. Evidently, this argument can be repeated for each state  $s$  having linearly dependent commodity prices. Moreover, since the perturbed economy is arbitrarily close to the original one, the regularity property of the equilibrium is preserved. ■

Step 3 guarantees that the system of equations (\*) has a solution, i.e., it guarantees that for all endowment-utility profiles  $((\omega_h, V_h^\alpha)_{h=i,u}) \in G^3$ , there exists an asset payoff matrix  $\rho$  such that the  $E^\alpha$  - economy,  $E^\alpha = ((\omega_h, V_h^\alpha)_{h=i,u}, \rho)$ , has a full rank,  $K$ -dimensional sunspot equilibrium. Though, the fact that  $(\bar{p}, \bar{q})$  is regular for the fictitious economy, i.e., the one with given yield matrix  $R$ , does not imply necessarily that  $((\bar{p}, (\dots, 1/K, \dots)), \bar{q})$  is a regular equilibrium of the  $E^\alpha$ -economy. However, by Lemma 2, all equilibria are regular up to a small  $\delta$ -perturbation. ■

**Remark 4:** A natural question concerns the relationship between multiplicity of equilibria in the spot economies associated with event  $s = 1, \dots, S$ , and existence of sunspot equilibria. In economies with an incomplete asset market and real assets, there can be sunspot equilibria even if there is a *unique* equilibrium  $(p', q')$  of the intertemporal economy without extrinsic uncertainty. Existence of sunspot equilibria requires multiplicity of equilibria in the spot economy associated with the endowment vectors obtained taking into account the reallocation of endowments induced by the trade in the real assets, i.e., with the vectors  $\omega_h(\sigma) + \rho y_h(\sigma)$ . However, it does not require multiplicity of spot equilibria at the original endowments  $\omega_h(\sigma)$ . Notice that  $\omega_h(\sigma) + \rho y_h(\sigma)$  may not lie in the Edgeworth box and that the vector  $|\rho y_h(\sigma)|$  may be arbitrarily large. Finally, remember that, as pointed out, for instance, in Hens [13], unless all agents have homothetic and identical preferences (a nongeneric case for which, clearly, there can not be sunspot equilibria if assets are real), there is

always at least one reallocation of the endowments (non necessarily lying in the Edgeworth box) such that there are multiple spot equilibria.

### 3.3 Existence of nonrevealing sunspot equilibria

Given an equilibrium of the modified economy, we can change the equilibrium asset prices (without affecting equilibrium commodity prices and allocations) by varying appropriately  $\pi(\cdot)$ . The trick is to find conditions which are typically sufficient to show that, given equilibrium prices and allocation,  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  and  $(\hat{x}, \hat{y})$ , of the modified economy E, there exists a vector  $\pi(\cdot)$  such that  $((\hat{p}, \pi(\cdot)), (\dots, \sum_{\sigma} \hat{q}(\sigma)/\Sigma, \dots))$  and  $(\hat{x}, \hat{y})$  are equilibrium prices and allocation of the modified economy E, hence, by Proposition 1, of the actual economy. Such a set of sufficient conditions can be found studying the first order conditions of the optimization problems that the two agents face and is summarized in Lemma 5.

The first order conditions of optimization problem [i'] imply that  $\lambda_i^1 q(\sigma)^T/\Sigma = R(p(\sigma))^T \lambda_i^2(\sigma)^T$ , for all  $\sigma$ . Moreover, since, by normalization,  $p^1(s, k, \sigma) = 1$ , for each  $(s, k, \sigma)$ ,  $\lambda_i^2(s, k, \sigma) = \pi(k|\sigma) \partial u_i^\sigma(x_i(\sigma))/\partial x_i^1(s, k, \sigma)$ , for all  $(s, k, \sigma)$ .

Let  $R(p(k, \sigma))$  be the  $(S \times A)$ -dimensional sub-matrix associated with spots  $(1, k, \sigma), \dots, (S, k, \sigma)$  and let  $[\partial u_i^\sigma(x_i(\sigma))/\partial x_i^1(k, \sigma)]$  be the corresponding  $S$ -dimensional vector. Then, the first order conditions of optimization problem [i'] imply that

$$q(\sigma)^T/\Sigma = (\lambda_i^1)^{-1} \sum_k \pi(k|\sigma) R(p(k, \sigma))^T [\partial u_i^\sigma(x_i(\sigma))/\partial x_i^1(k, \sigma)]^T,$$

or, defining  $q_i(k, \sigma)^T \equiv R(p(k, \sigma))^T [\partial u_i^\sigma(x_i(\sigma))/\partial x_i^1(k, \sigma)]^T$ , a vector of dimension  $A \times 1$ ,

$$q(\sigma)^T/\Sigma = (\lambda_i^1)^{-1} \sum_k \pi(k|\sigma) q_i(k, \sigma)^T \equiv (\lambda_i^1)^{-1} Q_i(\sigma) \pi(\cdot|\sigma)^T,$$

with  $Q_i(\sigma)$  a matrix of dimension  $A \times K$ , with columns  $q_i(k, \sigma)^T$ .

Similarly, the first order conditions of optimization problem [u''] imply

$$\sum_{\sigma} q(\sigma)^T / \Sigma = (\lambda_u^1)^{-1} \sum_{\sigma, k} \pi(k|\sigma) q_u(k, \sigma)^T = (\lambda_u^1)^{-1} (Q_u(1), \dots, Q_u(\Sigma)) \pi(\cdot)^T,$$

where, this time,  $q_u(k, \sigma)^T \equiv (\lambda_u^1)^{-1} \pi(\sigma) R(p(k, \sigma))^T [\partial u_u^{\sigma}(x_u(\sigma)) / \partial x_u(k, \sigma)]^T$ , for all  $k$  and  $\sigma$ .

Define the system of equations

$$\begin{bmatrix} Q_i(1) / \lambda_i^1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & Q_i(\Sigma) / \lambda_i^1 \\ Q_u(1) [\pi(1) / \lambda_u^1] & \dots & \dots & Q_u(\Sigma) [\pi(\Sigma) / \lambda_u^1] \\ [1] & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & [1] \end{bmatrix} \begin{bmatrix} \pi(1|1) \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \pi(K|\Sigma) \end{bmatrix} = \begin{bmatrix} \sum_{\sigma} q(\sigma) / \Sigma \\ \dots \\ \dots \\ \sum_{\sigma} q(\sigma) / \Sigma \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

In compact form, let us rewrite the system of equations above as

$$[\mathbf{A}] \quad D(\hat{p}, E)[\dots, \pi(k|\sigma), \dots]^T = [(\dots, \sum_{\sigma} \hat{q}(\sigma) / \Sigma, \dots), [1]]^T.$$

where  $[1]$  is the  $K$ -dimensional vector with all the entries equal to 1.

Evidently, at a solution of  $[\mathbf{A}]$ ,  $\hat{q}^1(1) \equiv \sum_{\sigma} \hat{q}^1(\sigma) / \Sigma$  need not to be equal to 1. However, we can always renormalize asset prices (and adjust the period 1 Lagrange multipliers) so that this restriction is satisfied.

**Lemma 5:** Let  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  be an equilibrium of the modified economy  $E$  with associated allocation  $(\hat{x}, \hat{y})$ . Suppose that, at  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , there is a strictly positive solution  $\pi^*(\cdot)$  to the system of equations  $[\mathbf{A}]$ . Then,  $((\hat{p}, \pi^*(\cdot)), (\dots, \sum_{\sigma} \hat{q}(\sigma) / \Sigma, \dots))$  with associated allocation  $(\hat{x}, \hat{y})$  is an equilibrium of the modified economy  $E$ . By construction, at this equilibrium,  $q(\sigma)$  is  $\sigma$ -invariant.

**Proof of Lemma 5:** It suffices to show that, if  $(\hat{x}_h, \hat{y}_h)$  is an optimal solution to the optimization problems [u'] and [i'] at prices  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , then it is also an optimal solution at prices  $((\hat{p}, \pi^*(\cdot)), (\dots, \sum_{\sigma} \hat{q}(\sigma) / \Sigma, \dots))$ .

This is obviously true for individual  $u$ . For individual  $i$ , consider the first order conditions to problem [i] at  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ :

- a.  $\hat{\pi}(k|\sigma)D_{x(s,k,\sigma)}u_i^\sigma(\hat{x}_i(\sigma)) - \lambda_i^2(s,k,\sigma)\hat{p}(s,k,\sigma) = 0$ , for each  $(s, k, \sigma)$ ;
- b.  $-\lambda_i^1\hat{q}(\sigma)/\Sigma - \lambda_i^2(\sigma)R(\hat{p}(\sigma)) = 0$ , for each  $\sigma$ ;
- [FOC<sub>i</sub>] c.  $-\sum_\sigma (\hat{q}(\sigma)/\Sigma)\hat{y}_i(\sigma) = 0$ ;
- d.  $-\Psi(\sigma)\hat{z}_i(\sigma) - R(\hat{p}(\sigma))\hat{y}_i(\sigma) = 0$ , for each  $\sigma$ .

At prices  $((\hat{p}, \pi^*(\cdot)), (\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots))$ ,  $(\hat{x}_i, \hat{y}_i)$  obviously satisfies d. Moreover, the restriction " $\hat{y}_u$  is  $\sigma$ -invariant", together with the market clearing conditions,  $\hat{y}_u = -\hat{y}_i$ , implies that  $\hat{y}_i$  is also  $\sigma$ -invariant. Hence,  $\sum_\sigma (\hat{q}(\sigma)/\Sigma)\hat{y}_i(\sigma) = 0$  implies that  $\sum_\sigma (\sum_\sigma (\hat{q}(\sigma)/\Sigma))\hat{y}_i(\sigma) = 0$ . Hence,  $(\hat{x}_i, \hat{y}_i)$  satisfies c as well.

For each  $\sigma$ , define the vector of Lagrange multipliers associated with the equilibrium  $((\hat{p}, \pi^*(\cdot)), (\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots))$  as  $(\lambda_i^1, (\dots, \lambda_i^2(s,k,\sigma), \dots)) \equiv (\lambda_i^1, (\dots, \pi^*(k|\sigma)\partial u_i^\sigma(x_i(\sigma))/\partial x_i^1(s,k,\sigma), \dots))$ . Evidently,  $\hat{x}_i$  with associated Lagrange multiplier vector  $\lambda_i$  satisfies a, given the vector  $((\hat{p}, \pi^*(\cdot)), (\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots))$ .

Finally, by assumption,  $D(\hat{p}, E)[\dots, \pi^*(k|\sigma), \dots]^T = [(\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots), [1]]^T$ . Hence, by definition of the matrix  $D(\hat{p}, E)$ ,  $\lambda_i$  satisfies b at  $((\hat{p}, \pi^*(\cdot)), (\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots))$ . ■

Given Lemma 5, the properties of the matrix  $D(\hat{p}, E)$  are crucial. They are discussed in Lemma 6, whose proof is in appendix.

**Lemma 6:** Let  $K > 2A+1$ . Let  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , with associated allocation  $(\hat{x}, \hat{y})$ , be an equilibrium of the modified economy  $E$ . Suppose that  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  and  $(\hat{x}, \hat{y})$  satisfy

- i.  $\text{rank } R(\hat{p}(k,\sigma)) = A$ , for all  $(k, \sigma)$ ;
- ii.  $x_h(s,k,\sigma) \neq x_h(s,k',\sigma)$ , for each  $s, \sigma, k$  and  $k'$ , with  $k \neq k'$ .



Then, there exists a modified economy  $E'$ , arbitrarily close to  $E$ , with the same equilibrium prices and allocation and such that both  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  is a regular equilibrium and the matrix  $D(\hat{p}, E')$  has maximal rank.

**Proof of Theorem 3:** In view of Proposition 1, we can refer the analysis to the modified economy.

In Theorem 2 we have shown that the set of original economies  $E(K)$  is open and non-empty. In the Corollary to Lemma 3, we have shown that  $E(K)$  is a set of modified economies as well. By definition, for each  $E \in E(K)$ , there is a  $K$ -dimensional sunspot equilibrium,  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$ , such that

- a.  $x_h(s, k, \sigma) \neq x_h(s, k', \sigma)$ , for each  $s, \sigma, k$  and  $k'$ , with  $k \neq k'$ .

By the construction adopted in the proof of Theorem 2 (see Step 2):

- b.  $\text{rank } R(\hat{p}(k, \sigma)) = A$ , for all  $(k, \sigma)$ .

Moreover, by assumption,

- c.  $\hat{\pi}(k|\sigma) = 1/K$ , for each  $\sigma$ .

Hence, by Lemma 6, these equilibria are, generically, regular and the matrix  $D(\hat{p}, E)$  has maximal rank. Let  $E'(K)$  be the set of modified economies with  $K$ -dimensional equilibria which satisfy a-c and that are, in addition, regular and have a full rank matrix  $D(\hat{p}, E)$ . Evidently,  $E'(K)$  is an open and dense subset of  $E(K)$ .

Pick  $E \in E'(K)$  and an  $\varepsilon$ -ball  $B^\varepsilon(E)$ , for  $\varepsilon > 0$  and small enough. Evidently, in general, for  $E \in B_\varepsilon(E) \setminus \text{diag}(E)$ , it will be  $\hat{q}(\sigma) \neq \hat{q}(\sigma')$ , for  $\sigma \neq \sigma'$ . We need to show that, for some probability vector  $\pi''(\cdot)$ ,  $((\hat{p}, \pi''(\cdot)), (\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots))$  is also an equilibrium of  $E$ . By Lemma 6, this is true if the vector  $\pi''(\cdot)$  solves the system of equations **[A]**. Given that, by construction,  $D(\hat{p}, E)$  has full rank and that, by assumption,  $\Sigma K \geq (\Sigma+1)A + \Sigma$ , there is a solution to the system **[A]**. We only need to show that at least one solution  $\pi''(\cdot)$  is strictly positive.

Remember that, for each economy  $E \in E'(K)$  and for the equilibrium constructed in Theorem 2,  $(\dots, 1/K, \dots)$  solves the system  $D(\hat{p}, E)\pi(\cdot) = [(\dots, \sum_\sigma \hat{q}(\sigma)/\Sigma, \dots), [1]]$ , because  $\sum_\sigma \hat{q}(\sigma)/\Sigma = \bar{q}(\sigma)$  for

each  $\sigma$ . Hence, by (local) continuity of equilibrium prices, for  $E''$  close enough to  $E$ , the system of equations **[A]** has a strictly positive solution  $\pi''(\cdot, \cdot)$ . Hence, by Lemma 6,  $((p'', \pi''(\cdot, \cdot)), (\dots, \sum_{\sigma} q''(\sigma)/\Sigma, \dots))$  is a sunspot equilibrium of the economy  $E''$ . By construction, it is a nonrevealing equilibrium. Therefore, by Proposition 1, the asset price vector  $(\dots, \sum_{\sigma} \hat{q}(\sigma)/\Sigma, \dots)$ , together with the  $K$ -dimensional price conjecture  $(p'', \pi''(\cdot, \cdot))$ , is a nonrevealing equilibrium of the actual economy with fundamentals  $E''$ . Hence, for each  $E \in E'(K)$ , there exists  $\varepsilon > 0$  such that each  $E'' \in B^{\varepsilon}(E)$  has a full rank,  $K$ -dimensional, nonrevealing equilibrium. ■

**Remark 4:** **[A]** is a system of  $[(\Sigma+1)A+\Sigma]$  equations in  $K\Sigma$  variables. Hence, for  $K$  large enough, if there is a strictly positive solution and the matrix  $D(p, E)$  has maximal rank, the set of solutions is a manifold of dimension  $(\Sigma(K-1)-A(\Sigma+1))$ . Hence, typically, a particular nonrevealing equilibrium price vector can be supported by a continuum of probability vectors for the extrinsic states of nature. However, it is still true that the set of probability distributions solving **[A]** (hence supporting the nonrevealing equilibrium) is a zero measure subset of the set of all the possible probability distributions of dimension  $K$ . Hence, from this point of view, nonrevealing equilibria are exceptional. ■

### Appendix

**Proof of Lemma 1:** i. The proof is standard.

ii. The proof follows directly from Balasko and Cass [5] and Pietra [18, 19], hence, we just sketch the argument. When convenient, we drop the subscript  $i$ .

Let

$$R \equiv \begin{bmatrix} R(p(1)) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & R(p(\Sigma)) \end{bmatrix} \quad \text{and} \quad \Psi \equiv \begin{bmatrix} \Psi(1) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \Psi(\Sigma) \end{bmatrix}$$

Define the following matrices (the subscripts to the square brackets indicate the dimensions of the various sub-matrices).

$$D_{(x_i, y_i, \lambda_i)} FOC_i(p, q, \pi(\cdot, \cdot), V_i) =$$

$$\begin{bmatrix} \begin{bmatrix} [D_{x_i}^2 V_i(x_i)]_{(\mathcal{C}\Gamma \times \mathcal{C}\Gamma)} & [0]_{(\mathcal{C}\Gamma \times \Sigma A)} \\ [0]_{(\Sigma A \times \mathcal{C}\Gamma)} & [0]_{(\Sigma A \times \Sigma A)} \end{bmatrix} & \begin{bmatrix} [0]_{(\mathcal{C}\Gamma \times 1)} & [-\Psi^T]_{(\mathcal{C}\Gamma \times \Gamma)} \\ [-q^T/\Sigma]_{(\Sigma A \times 1)} & [R^T]_{(\Sigma A \times \Gamma)} \end{bmatrix} \\ \begin{bmatrix} [0]_{(1 \times \mathcal{C}\Gamma)} & [-q/\Sigma]_{(1 \times \Sigma A)} \\ [-\Psi]_{(\Gamma \times \mathcal{C}\Gamma)} & [R]_{\Gamma \times \Sigma A} \end{bmatrix} & [0]_{(\Gamma+1) \times (\Gamma+1)} \end{bmatrix}$$

$$\equiv \begin{bmatrix} [A^1]_{((\mathcal{C}\Gamma + \Sigma A) \times (\mathcal{C}\Gamma + \Sigma A))} & [A^{2T}]_{((\mathcal{C}\Gamma + \Sigma A) \times (\Gamma+1))} \\ [A^2]_{((\Gamma+1) \times (\mathcal{C}\Gamma + \Sigma A))} & [0]_{((\Gamma+1) \times (\Gamma+1))} \end{bmatrix}$$

The matrix  $D_{(x,y,\lambda)} FOC_i(p,q,\pi(\cdot),V_i)$  has full rank. Its - conformably partitioned - inverse is

$$B^* \equiv \begin{bmatrix} [B^{1*}]_{((\mathcal{C}\Gamma + \Sigma A) \times (\mathcal{C}\Gamma + \Sigma A))} & [B^{2*T}]_{((\mathcal{C}\Gamma + \Sigma A) \times (\Gamma+1))} \\ [B^{2*}]_{((\Gamma+1) \times (\mathcal{C}\Gamma + \Sigma A))} & [B^{3*}]_{((\Gamma+1) \times (\Gamma+1))} \end{bmatrix}$$

It is easy to check that the square,  $(C\Gamma+A\Sigma)$ -dimensional matrix  $A^1$  defined above is negative semi-definite. Furthermore, if  $A^2v = 0$  and  $v \neq 0$ , then  $v^T A^1 v < 0$ . Hence, the conditions required to apply Theorem 2 in Balasko and Cass [5] are satisfied and, therefore, the matrix  $B^{*1}$  is negative semidefinite, while the matrix  $B^{*1}$  obtained from  $B^{*1}$  by deleting the column and rows 1,  $(C+1)$ , ...,  $(C(\Gamma-1)+1)$ ,  $(C\Gamma+1)$  is negative definite, hence of full rank.

Let  $\rho$  be the  $(A \times (CSK))$ -dimensional matrix of the coefficients determining asset payoffs (in terms of commodity) given a signal (remember that this coefficients are  $k$ - and  $\sigma$ -invariant). Also, let  $\Lambda_i^2(\sigma) \circ \rho$  be the  $(A \times (CSK))$ -dimensional matrix obtained as a product of  $\rho$  and the  $(A \times (CSK))$ -dimensional diagonal matrix with diagonal  $[\lambda(1,1,\sigma), \dots, \lambda(1,1,\sigma), \dots, \lambda(S,K,\sigma), \dots, \lambda(S,K,\sigma)]$ . Define

$$D_{(p,q)} \text{FOC}_i(p,q,\pi(\cdot), V_i) \equiv$$

$$\begin{bmatrix} \begin{bmatrix} -\lambda_i^2(1,1,1) & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & -\lambda_i^2(1,1,1) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & -\lambda_i^2(S,K,\Sigma) \end{bmatrix} & \begin{bmatrix} 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 \\ -\lambda_i^2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & -\lambda_i^2 \end{bmatrix} \\ \begin{bmatrix} [\Lambda_i^2(1) \circ \rho] & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & [\Lambda_i^2(\Sigma) \circ \rho] \end{bmatrix} & \begin{bmatrix} 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & -\lambda_i^2 \end{bmatrix} \\ \begin{bmatrix} 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & -z_i(S,K,\Sigma)^T + [\dots, \rho^c(S) y_i(\Sigma), \dots] \end{bmatrix} & \begin{bmatrix} [-y_i(1)/\Sigma & \dots & \dots & -y_i(\Sigma)/\Sigma] \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 \end{bmatrix} \end{bmatrix}$$

$$\equiv \begin{bmatrix} [C^{1*}]_{((C\Gamma+A\Sigma) \times C\Gamma)} & [C^{2*}]_{((C\Gamma+A\Sigma) \times A\Sigma)} \\ [C^{3*}]_{((\Gamma+1) \times C\Gamma)} & [C^{4*}]_{((\Gamma+1) \times A\Sigma)} \end{bmatrix}$$

Evidently, the squared matrix  $[C^{1*}, C^{2*}]$  of dimension  $(\Gamma C + A\Sigma) \times (\Gamma C + A\Sigma)$  is invertible

Denote with  $(p, q)$  the  $(\Gamma C + A\Sigma)$ -dimensional vector of prices, with  $(p', q')$  the vector of the prices of all commodities  $c > 1$  and of all the assets, but asset  $(1, 1)$ .

By the implicit function theorem,

$$D_{(p,q)}(z_i, y_i)(p,q,\pi(\cdot), V_i) \equiv -[B^{1*} C^{1*} \mid B^{1*} C^{2*}] - [B^{2*T} C^{3*} \mid B^{2*T} C^{4*}] = -D^* + M^*$$

The matrix

$$D_{(p',q')} \zeta_i(p,q,\pi(\cdot), V_i) \equiv -\lambda_i^1 D - M'$$

is obtained from  $D_{(p,q)}(z_i, y_i)(p, q, \pi(\cdot), V_i)$  deleting rows and columns 1, (C+1), ..., (C(\Gamma-1)+1), (C\Gamma+1), i.e., deleting the given rows from the matrices  $B^{1*}$  and  $B^{2*T}$  and the given columns from the matrices  $C^{1*}$ , ...,  $C^{4*}$ . We need to show that:

- D and M' do not depend on  $\lambda_i^1$ ;
- D is a square,  $(\Gamma(C-1)+A\Sigma - 1)$ -dimensional matrix of full rank

Fact a follows immediately from the structure of the matrices  $B^*$  and  $C^*$ .

Let  $B^{11}$  be the matrix of dimension  $((C-1)\Gamma+(\Sigma A-1)) \times (\Gamma C+A\Sigma)$  obtained from  $B^{1*}$  deleting rows 1, (C+1), ..., (C(\Gamma-1)+1), (C\Gamma+1). Similarly, let  $[C^1, C^2]$  be the matrix of dimension  $(C\Gamma+\Sigma A) \times ((C-1)\Gamma+\Sigma A-1)$  obtained from  $[C^{1*}, C^{2*}]$  by deleting columns 1, (C+1), ..., (C(\Gamma-1)+1), (C\Gamma+1).

Evidently,  $D = B^{11}[C^1, C^2]$ .

By construction of the matrix  $[C^{1*}, C^{2*}]$ , the rows 1, (C+1), ..., (C(\Gamma-1)+1) of the matrix  $[C^1, C^2]$  are identically equal to zero. Denote by  $B^1$  the matrix of dimension  $((C-1)\Gamma+(\Sigma A-1)) \times (\Gamma(C-1) + A\Sigma)$  obtained by  $B^{11}$  deleting the columns 1, (C+1), ..., (C(\Gamma-1)+1). Similarly, denote by  $[C^1, C^2]$ , the matrix of dimension  $((C-1)\Gamma+\Sigma A) \times (\Gamma(C-1)+(A\Sigma-1))$  obtained from  $[C^1, C^2]$  deleting the rows 1, (C+1), ..., (C(\Gamma-1)+1). Then,  $D = B^1[C^1, C^2]$  and, moreover, since  $B^{11}$  is negative definite,  $B^1$  has full row rank, while  $[C^1, C^2]$  has full column rank. .

In addition, by an inspection of the matrix  $D_{(p,q)} \text{FOC}_i(p, q, \pi(\cdot), V_i)$ , it is easy to establish that, for any vector  $b = (b^1, b^2) \in \mathbb{R}^{\Gamma(C-1)+A\Sigma}$ ,  $b \neq 0$ , with  $b^1 \in \mathbb{R}^{\Gamma(C-1)}$  and  $b^2 \in \mathbb{R}^{A\Sigma}$ ,

$$B^1 b^T = 0 \text{ implies that } b^1 = 0.$$

But then, it is straightforward to show that D is invertible. Therefore, setting  $M = M'D^{-1}$ ,

$$\text{rank } D_{(p,q)} \zeta_i(p, q, \pi(\cdot), V_i) = \text{rank}[-\lambda_i^1 I - M]. \quad \blacksquare$$

### Proof of Lemma 6.

Regularity follows immediately from Lemma 1 and 2.

We need to show that the matrix  $D(\hat{p}, E')$  has maximal (row) rank. Given the structure of the

matrix and  $K > 2A+1$  (hence,  $K\Sigma > ((\Sigma+1)A+\Sigma)$ ), it suffices to show that, for each  $\sigma$ , the matrix  $[\hat{\delta}_i(\sigma)^T, \hat{\delta}_u(\sigma)^T]$  (of dimension  $K \times 2A$ ) has maximal rank  $2A$  and that the  $A$ -dimensional vector  $[1] \notin \text{span} [\hat{\delta}_i(\sigma)^T, \hat{\delta}_u(\sigma)^T]$ . Bear in mind that  $[1] \in \mathbb{R}^K$ , while the matrix  $[\hat{\delta}_i(\sigma)^T, \hat{\delta}_u(\sigma)^T]$  has at most maximal row rank  $2A < K$ . The condition  $[1] \notin \text{row span} [M]$  is generic, in the space of matrices  $M$  of dimension  $K \times 2A$ .

Consider an equilibrium  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  satisfying i and ii. Without loss of generality, assume that the first  $A$  rows of the matrix  $R(\hat{p}(k, \sigma))$  (say, the submatrix  $R_A(\hat{p}(k, \sigma))$ ) are linearly independent, for each  $k$  and  $\sigma$ . We want to show that we can perturb arbitrarily the first  $K-1$  vectors  $\hat{q}_h(k, \sigma)$ , for  $k = 1, \dots, K-1$  and for all  $\sigma$ .

Indeed, pick an arbitrary  $q'_h(k, \sigma)$  sufficiently close to  $\hat{q}_h(k, \sigma)$ , for  $k < K$ . To guarantee that the "personalized" asset prices  $q'_h(k, \sigma)$  are consistent with the market asset prices  $\hat{q}(\sigma)$ , at the given probabilities  $[\hat{\pi}(1|\sigma), \dots, \hat{\pi}(K|\sigma)]$ , set

$$q'_h(K, \sigma) \equiv (-\sum_{k < K} \hat{\pi}(k|\sigma) q'_h(k, \sigma) + \lambda_h^1 \hat{q}(\sigma)) / \hat{\pi}(K|\sigma),$$

so that  $\sum_k \hat{\pi}(k|\sigma) q'_h(k, \sigma) = \lambda_h^1 \hat{q}(\sigma)$ . Evidently, if  $q'_h(k, \sigma)$  is close enough to  $\hat{q}_h(k, \sigma)$ , for all  $k$ , then  $q'_h(K, \sigma)$  is close to  $\hat{q}_h(K, \sigma)$ .

For each  $k$ , let  $\delta_{hA}(k, \sigma) \in \mathbb{R}^A$  be given by a solution to

$$(q'_h(k, \sigma) - \hat{q}_h(k, \sigma))^T = \pi(k|\sigma) R_A(\hat{p}(k, \sigma))^T [\delta_{hA}(k, \sigma)]^T,$$

and define  $\delta_h(k, \sigma) \equiv (\delta_{hA}(k, \sigma), 0)$ , where  $0 \in \mathbb{R}^{K-A}$ .

For given  $\sigma$ , let  $B_h^{\sigma k}(x_h(k, \sigma))$ ,  $B_h^{\sigma k \epsilon}(x_h(k, \sigma))$  and  $\Theta^{\sigma k}(x_i(k, \sigma), B_h^{\sigma j \epsilon}(x_i(k, \sigma)))$  be a collection of open neighborhoods of  $x_h(k, \sigma)$  and bump functions satisfying the assumptions stated in section 2.1. There exists such a collection since  $x_h(k, \sigma)$  is one-to-one in  $k$ , for all  $\sigma$ . Locally replace  $u_h^\sigma(x_h(k, \sigma))$  with the function  $u_h^{\delta \sigma}(x_h(k, \sigma))$ , defined as

$$u_h^{\delta\sigma}(x_h(k,\sigma)) \equiv u_h^\sigma(x_h(k,\sigma)) + \sum_k \Theta_h^{\sigma k}(x_h(k,\sigma), B_h^{\sigma k}(x_h(k,\sigma))) [\delta_h(k,\sigma) \hat{p}(k,\sigma) x_h(k,\sigma)],$$

Consider the economy  $E'$  obtained iterating over  $\sigma$  the perturbation described above. It is easy to check that, if  $(\hat{x}_h, \hat{y}_h)$  is an optimal solution to problem [h] at  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  given the utility function  $u_h(x_h)$ , then  $(\hat{x}_h, \hat{y}_h)$  is an optimal solution to problem [h] at  $((\hat{p}, \hat{\pi}(\cdot)), \hat{q})$  given the utility function  $u_h^\delta(x_h)$ .

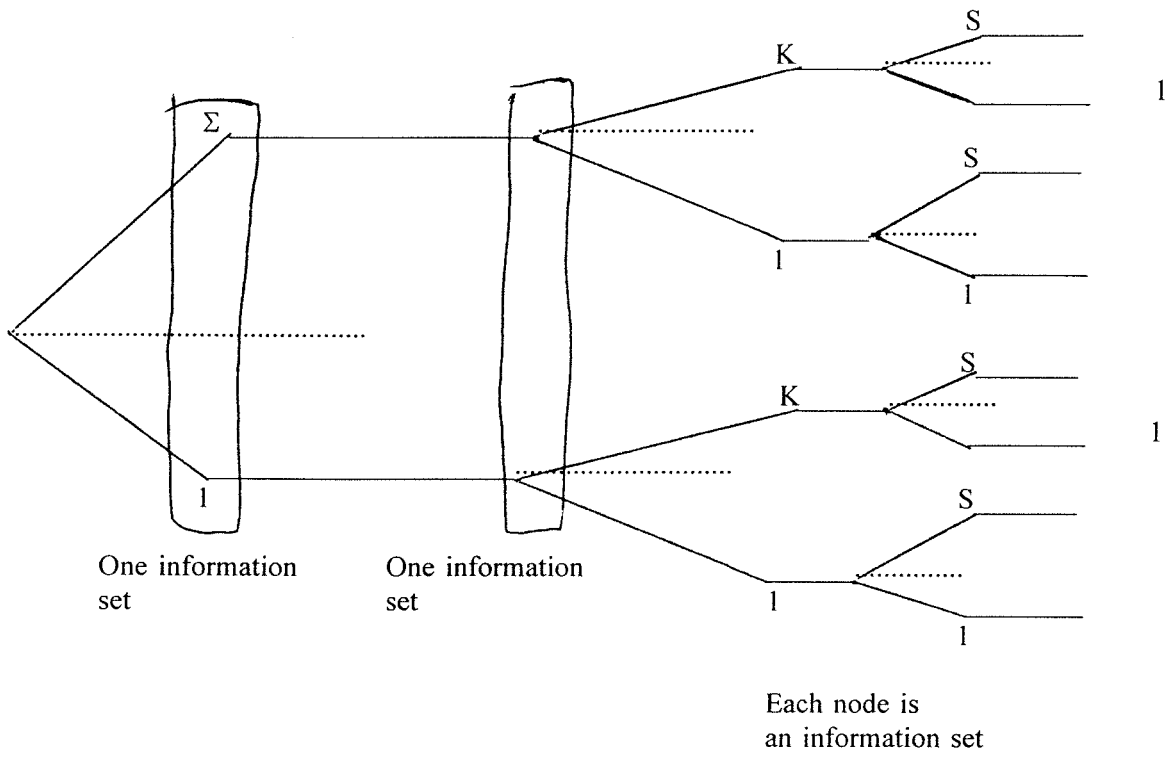
Since, for each  $\sigma$ , we can perturb arbitrarily the first  $(K-1)$  columns of the matrices  $[\hat{q}_i(k,\sigma)]^T$ ,  $[\hat{q}_u(k,\sigma)]^T$  (without affecting equilibrium prices and allocation) and since,  $(K-1) > 2A$ , we can choose a perturbation such that both each matrix  $[\hat{q}_i(k,\sigma)]^T$ ,  $[\hat{q}_u(k,\sigma)]^T$  has maximal rank  $2A$  and its span does not contain the vector  $[1]$ , as required above. ■

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**Figure 1**

The information structure of the uninformed agent

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