Modeling the Parallel Drone Scheduling Vehicle Routing Problem as a Heterogeneous Vehicle Routing Problem

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ABSTRACT

Drones are currently seen as a viable way for improving the distribution of parcels in urban and rural environments, especially while working in coordination with traditional vehicles like trucks. In this paper, we consider the parallel drone scheduling vehicle routing problem, where the service of a set of customers requiring a visit is accomplished by a fleet of trucks performing tours and a set drones moving back and forth from a single depot. In this work we propose a simple transformation that allows mapping the problem into a heterogeneous vehicle routing problem, which is a classic problem in the literature and for which an abondant collection of solving methods is available. We finally experimentally demonstrate how freely available softwares for the heterogeneous vehicle routing problem, such as Google OR-tools, can be used to obtained results comparable to those found by more complex methods especially tailored to the parallel drone vehicle routing problem.

CCS CONCEPTS

• **Mathematics of computing → Combinatorial optimization**; • **Applied computing → Transportation**.

KEYWORDS

Parallel Drone Scheduling Vehicle Routing Problems, Vehicle Routing Problem, OR Tools, Optimization

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1 INTRODUCTION

According to Forbes [13], the "Drone Explosion" of the last years has seen consistent substantial investments, due to its potential in many fields. Drones can enhance the current situation in many sectors, including logistics, surveillance, and disaster relief [22]. The potential impact on E-commerce, which has experienced an exponential growth in the last decades has raised a particularly strong interest [8], [5]. In [28], the authors forecast that autonomous vehicles will deliver about 80% of parcels in the following ten years. An obvious advantages associated with the use of aerial drones can be identified: they do not stick to the existing road network, but they can instead fly approximately straight line, and they are not affected by road traffic congestions. Therefore, the adoption of drones for delivering parcels can lead to innovative solutions of interest for the companies (operational costs reduction), for the customers (faster deliveries) and for the whole society (decarbonized traffic). In this work, we will analyze a transition scenario, where operational delivery strategies are optimized for a mixed fleet using both traditional trucks and drones.

The seminal work of Murray and Chu [20] pioneered a new routing problem in which a truck and a drone collaborate to make deliveries. From an operations research perspective, the authors present two new prototypical variants expanding from the traditional Traveling Salesman Problem (TSP) called the Flying Sidekick Traveling Salesman Problem (FSTSP) and the Parallel Drone Scheduling Traveling Salesman Problem (PDSTSP). In both cases a truck and drones collaborate to deliver parcels, the difference being however that in the former model drones can be launched and collected from the truck during its tour, while in the latter one drones are operated directly from the central depot, while the truck executes a traditional delivery tour. In the remainder of the paper we will focus on the latter problem, addressing the interested reader, for example, to [7] for full details and some solution strategies for the FSTSP. In the PDSTSP there is a truck that can leave the depot, serve a set of customers, and returns to the depot. In parallel, a set of drones moving back and forth from the depot to serve a single customer at a time. Not all the customer can be served by the drones, either due to their location or the characteristic of their parcel. The objective of the problem is to minimize the completion time of the last vehicle returning to the depot, while serving all the customers.

To tackle this problem, many approaches have been proposed. A first Mixed Integer Linear Programming (MILP) model and some simple heuristics were proposed in [20]. A more refined MILP model along with a two-step metaheuristic embedding dynamic programming components were discussed in [16]. A similar two-step metaheuristic approach, although based on matheuristics concepts, was presented in [6]. A hybrid ant colony metaheuristic was discussed in [9], and an improved variable neighborhood search procedure was outlined in [14].A constraint programming approach able to optimally solved all the benchmark instances adopted in the literature up to that time, was presented in [18]. In [21] another exact approach based on branch-and-cut was finally proposed, bringing also some new benchmark problems.

Several PDSTSP variants are also introduced and studied in the literature. We refer the interested reader to [22] and [23] for a complete survey. In the following we review only the extensions of the original problem relevant to the present study, where multiple trucks are employed out of a same depot. The recent work [17] discussed the *Parallel Drone Scheduling Vehicle Routing Problem* (PDSVRP), which is the topic of the present study and can be seen as a straightforward extension of the PDSTSP where multiple trucks are employed and the target is to minimize the time required to complete the delivery to the last customer serviced and go back to the depot. The authors proposed a hybrid metaheuristic, a mixed integer linear model and a branch-and-cut approach working on such a model. The same problem was also introduced at the same time in [25], where the authors propose three mixed integer linear programming models, one of which is arc-based and the other two are set covering-based, together with a branch-and-price approach based on one of the set covering-based models. A heuristic version of the branch-and-cut method is also discussed, targeting larger instances. Constraint programming models for the PDSVRP were finally discussed in [19].

Heterogeneous Vehicle Routing Problems (HVRPs) are extensions of the traditional VRP where vehicles have different characteristics and this impacts on their travel times or other factors less of interest for the present study such as capacity or range. The literature on the HVRP is vast, both in terms of variations of the problem and solving approaches. We refer the interested reader to [2] and [12] for comprehensive surveys, while in the remainder of the section we will only mention the works relevant to our study. Most of the papers focus on the so-called *min-sum* version of the problem, where the target is to minimize the sum of the travel times of all the vehicles. A review on exact methods for this problem can be found in [3]. Heuristic approaches were instead discussed in [4] and [15]. Concerning the so-called *min-max* HVRP, which is the version of the problem of interest for the present study, it is possible to track a few exact approaches. A branch-and-cut approach was proposed in [1]. In [10], two exact search schemes were proposed together with a simple tabu search algorithm. Concerning works purely on heuristic approaches, [11] proposed another tabu search metaheuristic with an adaptive memory. A hybrid genetic proposal for the HVRPD with min-max objective can finally be found in [27].

The present work shows how the PDSVRP can be easily and efficiently transformed in a HVRP and solved with the algorithms proposed for the latter in the past decades. The rest of the paper is organized as follows. After having formally described the PDSVRP and we HVRP, we will show how an instance of the former can be modelled as an instance of the latter. Then we describe some experimental results obtained with the *Routing* solver of Google OR-Tools [24], with the aim of validating the problem reduction we propose.

2 THE PARALLEL DRONE SCHEDULING VEHICLE ROUTING PROBLEM

The PDSVRP can be represented on a complete directed graph $G = (V, A)$, where the node set $V = \{0, 1, \ldots, n\}$ represents the depot (node 0) and a set of customers $C = \{1, \ldots, n\}$ to be serviced. A set T of homogeneous trucks and a set D of homogeneous drones are available to deliver parcels to the customers. Each truck starts from the depot 0, visits a subset of the customers, and returns back to the depot, operating a single route. The drones operate back and forth from the depot, delivering one parcel per trip and operating multiple routes if necessary. Not all the customers can be served by a drone, due to the weight of the parcel, an excessive distance of the customer location from the depot, or eventual terrain obstacles such as hills or areas with high-rise buildings. Let $C^D \subseteq C$ denotes the set of customers that can be served by drones. These customers are referred to as *drone-eligible* in the remainder of the paper. The travel time incurred by a truck to go from node i to node j is denoted as t^T_{ij}, while the time required by a drone to serve a customer i (back and forth from the depot) is denoted as t^D_i. The trucks and the drones start from the depot at time 0, and the objective of the PDSVRP is to minimize the time required to complete all the deliveries and to have all the trucks and all the drones back at the depot. Note that since truck and drones work in parallel, the objective function translates into minimizing the time required by the vehicle of the fleet with the longest total operational time.

An example of a PDSVRP instance and an associated solution is provided in Figure 1.

3 THE PARALLEL DRONE VEHICLE ROUTING PROBLEM AS A HETEROGENEOUS VEHICLE ROUTING PROBLEM

In this section we formally define the min-max HVRP and we show how a PDSVRP can be mapped into a HVRP instance.

3.1 The Min-Max Heterogeneous Vehicle Routing Problem

We here consider an uncapacitated version of the min-max Heterogeneous Vehicle Routing Problem (HVRP), that can be formally described as follows. A directed graph $\overline{G} = (\overline{V}, \overline{A})$ is given, where $\overline{V} = \{0, 1, \ldots, n\}$ is the set of nodes and $\overline{A}$ is the set of arcs. Node 0 represents the depot, while the remaining nodes correspond to the n customers. Each customer $i \in V \backslash \{0\}$ requires a delivery. The set M represents a heterogeneous fleet of vehicles that is stationed at the depot and is used to supply the customers. For each vehicle $k \in M$ and for each arc $(i, j) \in A$ a non-negative traveling cost c^k_{ij} is given. The route followed by vehicle k is a solution is denoted as R^k, where $R^k = \{i^k_1, i^k_2, \ldots, i^k_{|R^k|}\}$, with $i^k_1 = i^k_{|R^k|} = 0$

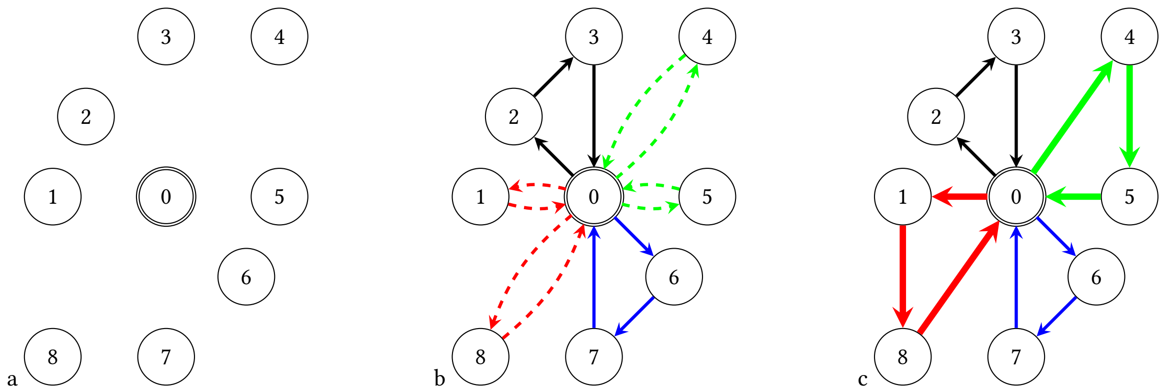

Figure 1: An example of a PDSVRP instance is provided in part a; we assume that two trucks and two drones are available (travel times are omitted for the sake of simplicity). A solution of the PDSVRP is provided in part b; the black and blue arcs represent the tour of the two truck. The first visits nodes 2 and 3 before going back to the depot, the second visits nodes 6 and 7 and goes back to the depot. The dashed red arcs depict the missions of the first drone (that serves nodes 1 and 8), while the dashed green arcs depict the missions of the second drone (that serves nodes 4 and 5). The same solution with the problem modelled as a HVRP is provided in part c; the green and red tours are now truck tours carried out by vehicles with different characteristics (thicker lines).

and $\{i_2^k, i_3^k, \ldots, i_{|R^k|-1}^k\} \subseteq V\backslash\{0\}$, is a simple circuit in G containing the depot. A feasible solution is a set of routes such that every customer is visited exactly once. The target is to minimize the makespan of the solution, i.e. the length of the longest route: $\min \max_{k \in M} \left\{ \sum_{s=2}^{|R^k|} c^k_{i^k_{s-1} i^k_s} \right\}$.

3.2 A Reduction from PDSVRP to HVRP

In order to model and solver a PDSVRP instance as a min-max HVRP one, the following reduction can be applied. We define $C_0^D = C^D \cup \{0\}$ to ease the notation.

- $\overline{V} = V$;
- $M = \{1, 2, \ldots, |T| + |D|\}$. The first $|T|$ vehicles of the HVRP represent the trucks of the PDSVRP, the remaining ones represent the drones;
- $c_{ij}^k = \begin{cases} t_{ij}^T & \text{if } (k \leq |T|) \\ t_j^D & \text{if } (k > |T|) \wedge (i < j) \wedge (i \in C_0^D) \wedge (j \in C^D) \\ 0 & \text{if } (k > |T|) \wedge (i \in C_0^D) \wedge (j = 0) \\ +\infty & \text{if } (k > |T|) \wedge \neg((i < j) \wedge (i \in C_0^D) \wedge (j \in C_0^D)) \end{cases}$
  $\forall i, j \in \overline{V}, k \in M$.

We create a truck of the HVRP for each vehicle of the PDSVRP. The traveling costs are such that if a truck k if the HVRP represents a truck of the PDSVRP ($k \leq |T|$), the cost in transferred directly, while if k models a drone ($k > |T|$), the cost of an arc (i, j) with $i < j$ is given by the time required to visit customer j by a drone in case $i \in C_0^D$ and $j \in C^D$; 0 when $i = 0$ or $i \in C^D$) and $j = 0$; $+\infty$ in case either i or j are not drone-eligible, or when $i \geq j$. The latter condition imposes that truck of the HVRP representing a drone of the PDSVRP will visit the customer assigned to it in increasing order of index. This condition would is not necessary, but it substantially reduces the search space and makes the HVRP easier to solve.

An example of representation of a same solution first as PDSVRP and then as HVRP is provided in Figure 1.

4 COMPUTATIONAL EXPERIMENTS

The HVRP instances corresponding to the PDSVRP benchmarks investigated have been solved via the *Routing* solver of Google OR-Tools 9.7 [24], a widely used and freely available solver. The settings used for the experiments are *FirstSolutionStrategy=PATH_CHEAPEST_ARC* and *LocalSearchMetaheuristic=GUIDED_LOCAL_SEARCH*. These values have been chosen according to some preliminary tests that indicated a clear dominance with respect to the other possible alternatives.

The experiments have been run on a single core of a computer equipped with 32 GB of RAM and an Intel Core i7 12700F CPU running at 4.90 GHz, with a maximum computation time of 600 seconds for each instance.

The outcome of the experimental champaign is discussed in the remainder of this section, and is organized according to the different benchmark sets attacked. All the tables present first the information of the instances, both in terns of names and number of trucks and drones. Note that the number of nodes of each instance can be inferred by its name (or it is written explicitly in brackets if this is not the case. The tables also report the cost of the best heuristic solutions reported in the previous literature.

4.1 Instances from Mbiadou Saleu et al. [17]

A first set of benchmarks for the PDSVRP was proposed in [17] starting from classic instances for the capacitated vehicle routing problem. A total of 20 instances with a number of customers ranging between 50 and 199 was obtained. Note that for each instance considered, the number of nodes of each instance can be inferred by its name (or it is written explicitly if this is not the case). We refer the interested reader to [17] for full details about the elements of the instances. The outcome of the experiments is summarized in

Table 1: Results on the instances from Mbiadou Saleu et al. [17].

Instances			BC	HM	CP1	CP2	OR-Tools
Name	$\|T\|$	$\|D\|$	[17]	[17]	[19]	[19]	Routing
CMT1 ($\|V\| = 51$)	3	2	188.00	166.00	166.00	174.00	204.00
CMT2 ($\|V\| = 76$)	5	5	3630.86	130.23	158.00	144.00	146.00
CMT3 ($\|V\| = 101$)	4	4	4537.11	184.00	196.00	210.00	208.00
CMT4($\|V\| = 151$)	6	6	-	160.38	282.00	180.00	168.00
CMT5 ($\|V\| = 200$)	9	8	-	138.00	312.00	158.00	140.00
E-n51-k5	3	2	188.00	168.00	166.00	174.00	204.00
E-n76-k8	4	4	2975.51	154.00	180.00	164.00	156.00
E-n101-k8	4	4	4537.11	184.00	244.00	224.00	208.00
M-n151-k12	6	6	-	154.00	270.00	188.00	168.00
M-n200-k16	8	8	-	144.00	302.00	166.00	152.00
P-n51-k10	5	5	230.00	111.07	118.00	124.00	124.00
P-n55-k7	4	3	308.00	126.00	136.00	130.00	148.00
P-n60-k10	5	5	246.00	114.00	124.00	120.00	124.00
P-n65-k10	5	5	580.00	126.00	144.00	138.00	140.57
P-n70-k10	5	5	3166.25	128.00	164.00	150.00	135.05
P-n76-k5	3	2	280.00	200.00	206.00	214.00	224.91
P-n101-k4	2	2	4725.47	342.00	340.00	362.00	406.69
X-n110-k13	7	6	-	1864.00	2702.00	2048.00	1896.00
X-n115-k10	5	5	-	2258.00	2920.00	2488.00	2602.00
X-n139-k10	5	5	-	2492.00	-	2744.00	2796.45

Table 1. The algorithms considered in the table are those discussed in [17], namely a branch and cut (BC) approach and nine variations of a hybrid metaheuristc approach (the best result is reported in column HM). Two different constraint programming models (CP1 and CP2), both presented in [19], are also considered.

The results in Table 1 suggest that even a general purpose HVRP solver like the one adopted can consistently provide good results on PDSVRP instances reduced to HVRP instances: the quality of the solutions is comparable with those of the PDSVRP-tailored and complex state-of-the-art solvers published in very recent times.

4.2 Instances from Raj et al. [25]

A second set of instances was built starting from TSPLIB [26] in [25]. The number of customers range between 48 and 229. Full details about the instances can be found in [25]. The results are summarized in Table 2. The methods considered in the table are: an arc-based model (MILP); a branch and price (BP) method based on a set covering MILP model, and a math-heuristic method (MH), all from [25] and two constraint programming models (CP1 and CP2) proposed in [19].

Table 2 confirms the observation of Section 4.1 about the functionality of solving PDSVRP instances as HVRP ones. The only remarkable difference is represented by the results on the last instances of the second table, for which the results obtained by the OR-Tools are clearly suboptimal. This depends on the limitations of the HVRP solver, which is designed to work efficiently on instances with up to 150 nodes. A more elaborated HVRP solver could potentially provide results more competitive with the state-of-the-art on these instances.

5 CONCLUSIONS

It has been shown that the parallel drone scheduling vehicle routing problem, a problem arising when parcels are distributed by a combined fleet of drones and trucks, can be reduced to a classic min-max heterogeneous vehicle routing problem. The conversione brings the advantage of using for the former problem the abondant set of methods originally developed in decades of research for the latter problem. Some experimental results are also presented to demonstrate that the reduction proposed is effective.

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Table 2: Results on the instances from Raj et al. [25].

Instances			MILP	BP	MH	CP1	CP2	OR-Tools
Name	$\|T\|$	$\|D\|$	[25]	[25]	[25]	[19]	[19]	Routing
att48_0_80	1	2	29048.00	28796.00	28894.00	28686.00	28686.00	29098.00
att48_0_80	1	4	28784.00	28610.00	31008.00	28610.00	28610.00	28610.00
att48_0_80	1	6	28610.00	28784.00	30708.00	28610.00	28610.00	28610.00
att48_0_80	2	2	17032.00	17056.00	16940.18	16940.18	17288.00	17538.00
att48_0_80	2	4	16500.00	16500.00	16500.00	16500.00	16564.00	16500.00
att48_0_80	2	6	16500.00	16500.00	16500.00	16500.00	16500.00	16500.00
att48_0_80	3	2	15218.00	14062.00	13452.00	13452.00	14652.00	13768.00
att48_0_80	3	4	13394.00	14652.00	10605.21	13394.00	13394.00	13394.00
att48_0_80	3	6	13756.00	13394.00	10369.78	13394.00	13394.00	13394.00
berlin52_0_80	1	2	5290.65	5291.10	5600.00	5290.65	5290.65	5595.00
berlin52_0_80	1	4	5190.00	5190.00	6055.00	5190.00	5190.00	5190.00
berlin52_0_80	1	6	5190.00	5190.00	5685.00	5190.00	5190.00	5190.00
berlin52_0_80	2	2	3415.00	3415.00	3285.00	3285.00	3480.00	3542.58
berlin52_0_80	2	4	2995.00	2995.00	2995.00	2995.00	2995.00	2995.00
berlin52_0_80	2	6	2995.00	2995.00	2995.00	2995.00	2995.00	2995.00
berlin52_0_80	3	2	3403.10	2995.00	2471.35	2935.00	2995.00	2995.00
berlin52_0_80	3	4	2635.00	2625.00	2013.82	2625.00	2925.00	2785.00
berlin52_0_80	3	6	2625.00	2625.00	2132.20	2625.00	2925.00	2675.00
eil101_0_80	1	2	458.80	498.00	496.00	456.00	456.00	539.00
eil101_0_80	1	4	360.00	358.00	354.00	346.00	346.00	414.00
eil101_0_80	1	6	314.00	320.00	365.00	314.00	314.00	392.93
eil101_0_80	2	2	316.00	305.40	293.00	293.00	331.00	336.77
eil101_0_80	2	4	266.80	253.70	232.84	248.00	283.00	296.53
eil101_0_80	2	6	219.00	215.30	198.00	212.00	272.00	263.00
eil101_0_80	3	2	260.00	235.00	209.00	252.00	278.00	255.00
eil101_0_80	3	4	221.00	200.10	177.00	198.00	227.00	232.00
eil101_0_80	3	6	228.00	181.00	155.20	218.00	204.00	247.00
gr120_0_80	1	2	1202.20	1263.10	1246.10	1186.00	1186.00	1320.00
gr120_0_80	1	4	949.00	1005.00	1003.00	943.00	943.00	1374.39
gr120_0_80	1	6	851.40	889.00	851.50	820.00	820.00	1101.37
gr120_0_80	2	2	806.30	764.00	703.50	735.00	866.00	800.00
gr120_0_80	2	4	676.00	646.00	590.54	676.00	777.00	732.00
gr120_0_80	2	6	769.00	581.20	518.00	549.00	688.00	690.00
gr120_0_80	3	2	611.00	597.00	509.50	688.00	734.00	610.00
gr120_0_80	3	4	536.00	528.00	444.00	688.00	601.00	589.00
gr120_0_80	3	6	590.90	527.20	397.50	526.00	547.00	567.00
pr152_0_80	1	2	97630.00	79686.00	79686.00	70148.00	70148.00	197466.33
pr152_0_80	1	4	66228.80	63990.00	63990.00	59756.00	59756.00	124163.46
pr152_0_80	1	6	65140.00	57794.00	57794.00	53478.00	53478.00	91438.54
pr152_0_80	2	2	55477.00	48967.00	38873.80	41371.00	46897.00	80326.62
pr152_0_80	2	4	54980.00	52353.00	33593.10	41295.58	44059.00	50905.61
pr152_0_80	2	6	50855.40	50854.40	31020.74	37413.00	43573.00	50905.61
pr152_0_80	3	2	48135.00	48135.00	25154.41	38395.00	36443.00	36131.00
pr152_0_80	3	4	43024.00	43024.00	21913.95	36940.00	37183.00	54073.19
pr152_0_80	3	6	46403.00	46403.00	20584.69	36940.00	36281.00	51291.68
gr229_0_80	1	2	1851.50	2020.90	2020.90	1664.77	1664.77	8123.55
gr229_0_80	1	4	1899.90	1935.00	1935.00	1511.74	1511.74	4680.14
gr229_0_80	1	6	1664.60	1883.80	1883.80	1415.12	1415.12	2750.83
gr229_0_80	2	2	1403.70	1403.70	881.91	1064.97	-	1196.85
gr229_0_80	2	4	1346.20	1346.20	818.33	1064.97	-	1196.85
gr229_0_80	2	6	1327.60	1327.60	776.98	1056.10	-	1196.85
gr229_0_80	3	2	1145.40	1145.40	579.62	981.59	-	1144.18
gr229_0_80	3	4	1132.30	1132.30	561.47	969.92	-	1144.18
gr229_0_80	3	6	1130.00	1130.00	517.37	1004.89	-	930.35

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