

CORES, JOINS AND THE FANO-FLOW CONJECTURES

LIGANG JIN, ECKHARD STEFFEN

Paderborn University
Institute of Mathematics and Paderborn Center for Advanced Studies
Warburger Str. 100, 33098 Paderborn, Germany

e-mail: ligang@mail.upb.de
es@upb.de

AND

GIUSEPPE MAZZUOCOLO

Università di Verona
Dipartimento di Informatica
Strada Le Grazie 15, Verona, Italy

e-mail: mazzuocolo.giuseppe@univr.it

Abstract

The Fan-Raspud Conjecture states that every bridgeless cubic graph has three 1-factors with empty intersection. A weaker one than this conjecture is that every bridgeless cubic graph has two 1-factors and one join with empty intersection. Both of these two conjectures can be related to conjectures on Fano-flows. In this paper, we show that these two conjectures are equivalent to some statements on cores and weak cores of a bridgeless cubic graph. In particular, we prove that the Fan-Raspud Conjecture is equivalent to a conjecture proposed in [E. Steffen, *1-factor and cycle covers of cubic graphs*, *J. Graph Theory* **78** (2015) 195–206]. Furthermore, we disprove a conjecture proposed in [G. Mazzuocolo, *New conjectures on perfect matchings in cubic graphs*, *Electron. Notes Discrete Math.* **40** (2013) 235–238] and we propose a new version of it under a stronger connectivity assumption. The weak oddness of a cubic graph G is the minimum number of odd components (i.e., with an odd number of vertices) in the complement of a join of G . We obtain an upper bound of weak oddness in terms of weak cores, and thus an upper bound of oddness in terms of cores as a by-product.

Keywords: cubic graphs, Fan-Raspud Conjecture, cores, weak-cores.

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