On the Shear Models in Finite Elasticity

F. O. Falope[1,2], L. Lanzoni[1,2], A.M. Tarantino[1,2]

[1] *DIEF – Department of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia, 41125 Modena, Italy*
[2] *CRICT – Inter-departmental Research and Innovation Centre on Construction and Environmental Services, 41125 Modena, Italy*
E-mail: federicooyedeji.falope@unimo.it, luca.lanzoni@unimo.it, angelomarcello.tarantino@unimo.it

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Since the seminal work of Rivlin, shear deformations have been extensively investigated within the framework of finite elasticity. Most contributions focus on two classical deformation modes, commonly referred to as *simple shear* and *pure shear*. Despite their widespread use, both kinematic descriptions involve not only shear distortions but also extension and contraction of material fibers. As a consequence, the associated Cauchy stress tensor generally contains both tangential and normal stress components, irrespective of the adopted constitutive law, unless unrealistic restrictions are imposed on the material parameters.

The use of these classical deformation schemes for constitutive characterization may therefore lead to ambiguities and potentially contradictory interpretations. As an illustrative example, previous experimental studies have employed both simple and pure shear tests to identify the parameters of the same incompressible hyperelastic material, obtaining significantly different results despite the fact that the two deformation modes differ, for incompressible materials, only by a rigid-body rotation.

To overcome these limitations, an alternative kinematic framework, termed *purely angular shear*, is introduced. Inspired by the classical notion of angular variation associated with infinitesimal shear strains, the proposed homogeneous finite deformation is defined through finite rotations of material fibers along selected directions. This approach provides a more direct representation of shear deformation and has recently been employed to characterize the mechanical behavior of rubber-like materials through a dedicated experimental apparatus.

The formulation is further extended to the three-dimensional setting. Unlike the traditional simple and pure shear configurations, the resulting Green–St. Venant strain tensor is found to be traceless, thereby enabling the definition of a genuine state of pure shear deformation at finite strains. This property establishes a natural correspondence with the concept of pure shear commonly adopted in linearized elasticity and offers a consistent framework for both experimental characterization and constitutive modelling of hyperelastic materials under large deformations.

References

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