Solution of a Practical Vehicle Routing Problem for Monitoring Water Distribution Networks
SUPPLEMENTAL ONLINE MATERIAL

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Appendix

In this supplemental online material we present additional mathematical models and computational results. In particular, in Section A.1 two alternative MILP models for the VRPWDN are described. The comparison among the mathematical models is reported in Section A.2.

A.1. Alternative Mathematical Models

In this section, we report two alternative mathematical models that describe the VRPWDN and are derived from the literature. The first model is based on a time representation of the problem and is inspired by the formulation proposed by Desaulniers et al. (2014) for the VRP with time windows. The second is a node-based model that we derive from the classical Miller, Tucker and Zemlin formulation (see, e.g., Bektaş and Gouveia 2014).

Time-based Model

Let y_{ik} be a binary variable taking value 1 if node i is visited by vehicle k and 0 otherwise, x_{ijk} be another binary variable taking value 1 if arc (i, j) is covered by vehicle k and 0 otherwise, and T_{ik} be a continuous variable corresponding to the time at which vehicle k arrives at node i. The time-based model for the VRPWDN can be formulated as follows:

$$(\text{VRPWDN}_{\text{tb}}) \quad \min z_{(\text{VRPWDN}_{\text{tb}})} = \sum_{k\in K} \sum_{i\in V\setminus\{n+1\}} \sum_{j\in V\setminus\{0\}} (t_{ij} + v_i)x_{ijk} \tag{1}$$

subject to

$$\sum_{k\in K} \sum_{j\in V\setminus\{0\}} x_{ijk} = 1 \qquad i \in V_1 \cup V_2 \tag{2}$$

$$\sum_{j\in V\setminus\{0\}} x_{0jk} = 1 \qquad k \in K \tag{3}$$

$$y_{ik} = \sum_{j\in V\setminus\{0\}} x_{ijk} = \sum_{j\in V\setminus\{n+1\}} x_{jik} \qquad i \in V \setminus \{0, n+1\}, k \in K \tag{4}$$

$$\sum_{i \in V \setminus \{n+1\}} x_{i,n+1,k} = 1 \qquad k \in K \quad (5)$$

$$\sum_{k \in K} T_{0k} = 0 \qquad (6)$$

$$0 \leq T_{ik} \leq L y_{ik} \qquad i \in V, k \in K \quad (7)$$

$$T_{jk} \geq T_{ik} + v_i + t_{ij} - M_{ij}(1 - x_{ijk}) \qquad i \in V \setminus \{n+1\}, j \in V \setminus \{0\}, k \in K \quad (8)$$

$$y_{p_i k} + y_{d_i k} \geq 2 y_{ik} \qquad i \in V_2, k \in K \quad (9)$$

$$T_{p_i k} + v_{p_i} + t_{p_i i} - M'_i(1 - y_{ik}) \leq T_{ik} \leq T_{d_i k} - (t_{i d_i} + v_i) y_{ik} \qquad i \in V_2, k \in K \quad (10)$$

$$x_{ijk} \in \{0, 1\} \qquad i, j \in V, k \in K \quad (11)$$

$$y_{ik} \in \{0, 1\} \qquad i \in V, k \in K \quad (12)$$

Objective function (1) is to minimise the total duration of the routes. Constraints (2) impose that each node $i \in V_1 \cup V_2$ has exactly one outgoing arc. Each vehicle starts its route from the depot and such condition is imposed by means of constraints (3). Constraints (4) and (5) ensure that each node i has exactly one incoming and one outgoing arc and that each vehicle k end its route at the depot. Constraints (6) impose that all routes start at time 0. Constraints (7) impose that arrival times are non-negative and limit the duration of each route to be at most L. The time at which vehicle k arrives at node j is modelled by means of constraints (8), in which we set $M_{ij} = L + v_i + t_{ij} - t_{j,n+1}$. Constraints (9) impose that if vehicle k visits node i, then it also visits nodes p_i and d_i. Since p_i may contain keys not only for i but for other nodes, vehicle k may visit p_i but not i, and the same holds for d_i. For this reason, the equation cannot be an equality. Constraints (10), in which we set $M'_i = L + v_{p_i} + t_{p_i,i} - t_{i,n+1}$, guarantee the respect of precedence constraints, by forcing time dependency between visits to p_i, i and d_i. Note that if y_{ik} is equal to 0, constraints (10) become redundant with respect to constraints (7). Constraints (11) and (12) define the domain of the x_{ijk} and y_{ik} variables.

Furthermore, the aforementioned model can be enhanced with the addition of the following valid inequalities

$$(t_{0p_i} + v_{p_i} + t_{p_i i}) y_{ik} \leq T_{ik} \qquad i \in V_2, k \in K \quad (13)$$

$$(t_{0p_i} + v_{p_i} + t_{p_i i} + v_i + t_{i d_i}) y_{ik} \leq T_{d_i k} \qquad i \in V_2, k \in K \quad (14)$$

$$T_{jk} \geq (t_{0i} + v_i + t_{ij}) x_{ijk} \qquad i \in V \setminus \{n+1\}, j \in V \setminus \{0\}, k \in K \quad (15)$$

which strengthen the values taken by the arrival time variables.

Node-based Model

Let u_{ik} be a variable representing the load on vehicle k after leaving node i. With respect to the model presented in Section 4, this implies setting $u_{ik} = \sum_{j \in V} f_{ijk}$. We can model the VRPWDN as follows:

$$(\text{VRPWDN}_{\text{nb}}) \quad \min z_{(\text{VRPWDN}_{\text{nb}})} = \sum_{k \in K} \sum_{i \in V \setminus \{n+1\}} \sum_{j \in V \setminus \{0\}} (t_{ij} + v_i) x_{ijk} \qquad (16)$$

subject to (2), (3), (11) and

$$\sum_{j \in V \setminus \{0\}} x_{ijk} = \sum_{j \in V \setminus \{n+1\}} x_{jik} \qquad i \in V \setminus \{0, n+1\}, k \in K \quad (17)$$

$$\sum_{i \in V \setminus \{n+1\}} \sum_{j \in V \setminus \{0\}} (t_{ij} + v_i) x_{ijk} \leq L \qquad k \in K \quad (18)$$

$$u_{0k} = 0 \qquad k \in K \quad (19)$$

$$u_{n+1,k} = \sum_{i \in V \setminus \{n+1\}} \sum_{j \in V \setminus \{0\}} x_{ijk} \qquad k \in K \quad (20)$$

$$u_{ik} - u_{jk} + n x_{ijk} \leq (n-1) \qquad i \in V \setminus \{n+1\}, j \in V \setminus \{0\}, k \in K \quad (21)$$

$$u_{p_i k} - u_{ik} + \sum_{j \in V \setminus \{0\}} x_{ijk} \leq n(1 - \sum_{j \in V \setminus \{0\}} x_{ijk}) \qquad i \in V_2, k \in K \quad (22)$$

$$u_{p_i k} \geq \sum_{j \in \{0\}} x_{ijk} \qquad i \in V_2, k \in K \quad (23)$$

$$u_{d_i k} - u_{ik} \geq \sum_{j \in V \setminus \{0\}} x_{ijk} \qquad i \in V_2, k \in K \quad (24)$$

$$0 \leq u_{ik} \leq n \sum_{j \in V \setminus \{0\}} x_{ijk} \qquad i \in V, k \in K \quad (25)$$

Constraints (17) correspond to the previous constraints (4) except for the y_{ik} term. The maximum duration of each route is bounded by means of constraints (18). For each vehicle k, constraints (19) set the load after leaving node 0, while constraints (20) define the load when arriving at node $n+1$. Constraints (21) impose the load conservation when travelling from node i to node j. Constraints (22)–(24) guarantee the respect of precedence constraints. Constraints (25) impose both the non-negativity of the u_{ik} variables and their relation with the x_{ijk} variables.

The model can be improved by the addition of the following valid inequalities

$$\sum_{j \in V \setminus \{n+1\}} x_{j p_i k} + \sum_{j \in V \setminus \{0\}} x_{d_i j k} \geq 2 \sum_{j \in V \setminus \{n+1\}} x_{jik} \qquad i \in V_2, k \in K \quad (26)$$

$$u_{d_i k} - u_{p_i k} \geq \sum_{l \in V \setminus \{n+1\}: p_i = p_l} \sum_{j \in V \setminus \{0\}} x_{ljk} \qquad i \in V_2, k \in K \quad (27)$$

$$u_{ik} - u_{jk} + n x_{ijk} + (n-2) x_{jik} \leq (n-1) \qquad i, j \in V \setminus \{0, n+1\}, k \in K \quad (28)$$

A.2. Comparison among the Mathematical Models

In this section, the performance of the three mathematical models (i.e., time-based, node-based, and flow-based) is investigated. The mathematical models were coded in C++ and the computational tests were executed on a PC equipped with an Intel Core i7 CPU processor @ 2.70 GHz and 6 GB of RAM, using CPLEX 12.3 as MILP solver. A time limit of 3,600 CPU seconds was imposed on each execution. The detailed results that we obtained with the addition of the valid inequalities are reported in Table 1. For each instance, columns "z_{lb}" and "z_{ub}" give the lower and upper bound values, respectively, column "%gap" gives the percentage gap and column "Sec." the run time. An entry "tlim" indicates that the time limit was reached for that instance. Column "opt" gives value 1 if the instance was solved to proven optimality and 0 otherwise.

From Table 1, we can observe that just on a few large-size instances the time-based model and the node-based model find better results in terms of upper bound values. Overall, the flow-based model outperforms the other two models in terms of lower bound values, percentage gap, run time, and number of optimal solutions obtained. Note that CPLEX could not find any feasible solution for the last six large-size instances when running the time-based model and for the last three large-size instances when running the node-based model.

Table 1. Comparison of mathematical models. Best average lower and upper bound values in **boldface**

					time-based					node-based					flow-based				
$\|V_1 \cup V_2\|$	$\|V_2\|$	$\|V_3\|$	$\|K\|$	Instance No.	z_{lb}	z_{ub}	%gap	Sec.	opt	z_{lb}	z_{ub}	%gap	Sec.	opt	z_{lb}	z_{ub}	%gap	Sec.	opt
10	1	1	1	1	**597.11**	**597.11**	0.00	0.70	1	**597.11**	**597.11**	0.00	0.82	1	**597.11**	**597.11**	0.00	0.35	1
10	1	1	1	2	**781.27**	**781.27**	0.00	0.83	1	**781.27**	**781.27**	0.00	0.70	1	**781.27**	**781.27**	0.00	1.07	1
10	1	1	1	3	**740.78**	**740.78**	0.00	1.09	1	**740.78**	**740.78**	0.00	2.73	1	**740.78**	**740.78**	0.00	1.96	1
10	1	1	2	1	**636.49**	**636.49**	0.00	6.35	1	**636.49**	**636.49**	0.00	6.73	1	**636.49**	**636.49**	0.00	2.32	1
10	1	1	2	2	**787.26**	**787.26**	0.00	1.60	1	**787.26**	**787.26**	0.00	2.30	1	**787.26**	**787.26**	0.00	2.12	1
10	1	1	2	3	**768.98**	**768.98**	0.00	14.42	1	**768.98**	**768.98**	0.00	27.59	1	**768.98**	**768.98**	0.00	5.34	1
10	2	1	1	1	**605.66**	**605.66**	0.00	0.98	1	**605.66**	**605.66**	0.00	0.25	1	**605.66**	**605.66**	0.00	1.17	1
10	2	1	1	2	**870.93**	**870.93**	0.00	15.15	1	**870.93**	**870.93**	0.00	1.65	1	**870.93**	**870.93**	0.00	7.95	1
10	2	1	1	3	**753.17**	**753.17**	0.00	3.49	1	**753.17**	**753.17**	0.00	0.07	1	**753.17**	**753.17**	0.00	0.27	1
10	2	1	2	1	**645.04**	**645.04**	0.00	2.16	1	**645.04**	**645.04**	0.00	4.39	1	**645.04**	**645.04**	0.00	4.42	1
10	2	1	2	2	**940.39**	**940.39**	0.00	54.97	1	**940.39**	**940.39**	0.00	27.71	1	**940.39**	**940.39**	0.00	22.99	1
10	2	1	2	3	**795.83**	**795.83**	0.00	12.24	1	**795.83**	**795.83**	0.00	0.76	1	**795.83**	**795.83**	0.00	0.93	1
10	2	2	1	1	**685.38**	**685.38**	0.00	14.87	1	**685.38**	**685.38**	0.00	20.16	1	**685.38**	**685.38**	0.00	3.04	1
10	2	2	1	2	**858.32**	**858.32**	0.00	11.03	1	**858.32**	**858.32**	0.00	9.12	1	**858.32**	**858.32**	0.00	27.03	1
10	2	2	1	3	**868.18**	**868.18**	0.00	506.87	1	**868.18**	**868.18**	0.00	105.13	1	**868.18**	**868.18**	0.00	20.13	1
10	2	2	2	1	**729.06**	**729.06**	0.00	115.14	1	**729.06**	**729.06**	0.00	501.49	1	**729.06**	**729.06**	0.00	17.66	1
10	2	2	2	2	**920.64**	**920.64**	0.00	110.92	1	**920.64**	**920.64**	0.00	214.14	1	**920.64**	**920.64**	0.00	153.78	1
10	2	2	2	3	**851.63**	**851.63**	0.00	105.97	1	**851.63**	**851.63**	0.00	594.37	1	**851.63**	**851.63**	0.00	53.59	1
10	4	2	2	1	**847.30**	**847.30**	0.00	144.32	1	**847.30**	**847.30**	0.00	39.61	1	**847.30**	**847.30**	0.00	12.04	1
10	4	2	2	2	**878.02**	**878.02**	0.00	26.02	1	**878.02**	**878.02**	0.00	20.00	1	**878.02**	**878.02**	0.00	22.93	1
10	4	2	2	3	999.51	**1081.15**	7.55	tlim	0	**1081.15**	**1081.15**	0.00	677.11	1	**1081.15**	**1081.15**	0.00	187.62	1
10	4	3	2	1	**949.71**	**949.71**	0.00	1380.84	1	**949.71**	**949.71**	0.00	155.02	1	**949.71**	**949.71**	0.00	164.94	1
10	4	3	2	2	888.91	**1094.38**	18.78	tlim	0	**1094.38**	**1094.38**	0.00	1087.61	1	**1094.38**	**1094.38**	0.00	127.09	1
10	4	3	2	3	**1131.74**	**1131.74**	0.00	2721.66	1	**1131.74**	**1131.74**	0.00	176.10	1	**1131.74**	**1131.74**	0.00	530.96	1
sum/avg (10)					813.80	**825.77**	1.10	518.82	22	**825.77**	**825.77**	0.00	153.15	24	**825.77**	**825.77**	0.00	57.16	24
15	3	2	2	1	**895.86**	**895.86**	0.00	1252.06	1	**895.86**	**895.86**	0.00	417.06	1	**895.86**	**895.86**	0.00	710.38	1
15	3	2	2	2	**1035.29**	**1035.29**	0.00	1876.44	1	**1035.29**	**1035.29**	0.00	1789.68	1	**1035.29**	**1035.29**	0.00	693.32	1
15	3	2	2	3	**1178.69**	**1178.69**	0.00	2306.41	1	**1178.69**	**1178.69**	0.00	263.48	1	**1178.69**	**1178.69**	0.00	125.43	1
15	3	2	3	1	**918.44**	**918.44**	0.00	2497.55	1	902.71	**918.44**	1.71	tlim	0	**918.44**	**918.44**	0.00	1694.28	1
15	3	2	3	2	**999.23**	**999.23**	0.00	65.50	1	**999.23**	**999.23**	0.00	59.29	1	**999.23**	**999.23**	0.00	6.56	1
15	3	2	3	3	1124.45	**1230.67**	8.63	tlim	0	1162.82	**1230.67**	5.51	tlim	0	**1230.67**	**1230.67**	0.00	2276.20	1
15	3	3	2	1	1040.98	**1103.95**	5.70	tlim	0	1047.85	**1103.95**	5.08	tlim	0	**1103.95**	**1103.95**	0.00	394.63	1
15	3	3	2	2	1051.15	**1112.54**	5.52	tlim	0	1026.18	**1112.54**	7.76	tlim	0	**1112.54**	**1112.54**	0.00	2866.55	1
15	3	3	2	3	1105.75	**1127.54**	1.93	tlim	0	1127.54	**1127.54**	0.00	1719.49	1	**1127.54**	**1127.54**	0.00	51.12	1
15	3	3	3	1	1074.75	**1150.01**	6.54	tlim	0	1066.37	**1150.01**	7.27	tlim	0	**1150.01**	**1150.01**	0.00	347.12	1
15	3	3	3	2	1101.89	**1167.69**	5.64	tlim	0	1077.29	**1167.69**	7.74	tlim	0	**1146.03**	1168.95	1.96	tlim	0
15	3	3	3	3	1128.80	**1163.25**	2.96	tlim	0	1146.43	**1163.25**	1.45	tlim	0	**1163.25**	**1163.25**	0.00	128.67	1
15	4	2	2	1	**895.86**	**895.86**	0.00	1260.85	1	**895.86**	**895.86**	0.00	418.23	1	**895.86**	**895.86**	0.00	711.73	1
15	4	2	2	2	**1035.29**	**1035.29**	0.00	1889.53	1	**1035.29**	**1035.29**	0.00	1796.38	1	**1035.29**	**1035.29**	0.00	694.41	1
15	4	2	2	3	**1178.69**	**1178.69**	0.00	2315.62	1	**1178.69**	**1178.69**	0.00	261.99	1	**1178.69**	**1178.69**	0.00	126.28	1
15	4	2	3	1	939.54	**947.47**	0.84	tlim	0	**947.47**	**947.47**	0.00	2023.54	1	**947.47**	**947.47**	0.00	2696.66	1
15	4	2	3	2	**1090.48**	**1090.48**	0.00	1300.56	1	**1090.48**	**1090.48**	0.00	2484.80	1	**1090.48**	**1090.48**	0.00	548.97	1
15	4	2	3	3	**1214.40**	**1214.40**	0.00	2418.55	1	**1214.40**	**1214.40**	0.00	404.71	1	**1214.40**	**1214.40**	0.00	80.05	1
15	4	3	2	1	1085.39	1166.00	6.91	tlim	0	1123.33	1166.00	3.66	tlim	0	**1162.99**	**1162.99**	0.00	504.43	1
15	4	3	2	2	973.51	**1090.24**	10.71	tlim	0	974.24	**1090.24**	10.64	tlim	0	**1067.56**	**1090.24**	2.08	tlim	0
15	4	3	2	3	1162.54	**1196.18**	2.81	tlim	0	1150.67	**1196.18**	3.80	tlim	0	**1196.18**	**1196.18**	0.00	158.51	1
15	4	3	3	1	1140.73	**1217.11**	6.28	tlim	0	1166.71	1224.46	4.72	tlim	0	**1217.11**	**1217.11**	0.00	313.20	1
15	4	3	3	2	1003.83	1141.12	12.03	tlim	0	995.76	**1138.66**	12.55	tlim	0	**1103.37**	**1138.66**	3.10	tlim	0
15	4	3	3	3	1190.87	**1251.44**	4.84	tlim	0	1177.41	**1251.44**	5.92	tlim	0	**1251.44**	**1251.44**	0.00	378.33	1
15	7	3	3	1	1262.65	**1419.20**	11.03	tlim	0	**1419.20**	**1419.20**	0.00	1626.99	1	**1419.20**	**1419.20**	0.00	241.07	1
15	7	3	3	2	1150.47	**1250.24**	7.98	tlim	0	**1250.24**	**1250.24**	0.00	1159.81	1	**1250.24**	**1250.24**	0.00	388.52	1
15	7	3	3	3	1189.33	**1358.36**	12.44	tlim	0	1154.97	1374.99	16.00	tlim	0	**1358.36**	**1358.36**	0.00	2096.81	1
15	7	4	3	1	1230.71	1607.67	23.45	tlim	0	1483.07	1673.69	11.39	tlim	0	**1544.48**	**1595.43**	3.19	tlim	0
15	7	4	3	2	1154.95	1417.28	18.51	tlim	0	1312.13	1474.45	11.01	tlim	0	**1398.20**	**1399.92**	0.12	tlim	0
15	7	4	3	3	954.49	1239.21	22.98	tlim	0	1147.24	1279.08	10.31	tlim	0	**1174.37**	**1224.00**	4.05	tlim	0
sum/avg (15)					1083.63	1159.98	5.92	2972.77	10	1112.78	1166.13	4.22	2521.41	13	**1152.24**	**1158.35**	0.48	1327.77	24
20	2	2	2	1	1166.26	**1298.79**	10.20	tlim	0	1157.90	**1298.79**	10.85	tlim	0	**1203.65**	1305.93	7.83	tlim	0
20	2	2	2	2	1214.34	**1231.59**	1.40	tlim	0	**1231.59**	**1231.59**	0.00	2754.73	1	**1231.59**	**1231.59**	0.00	410.53	1
20	2	2	2	3	**1295.94**	**1295.94**	0.00	2624.94	1	**1295.94**	**1295.94**	0.00	443.14	1	**1295.94**	**1295.94**	0.00	54.06	1
20	2	2	3	1	1188.42	1341.16	11.39	tlim	0	1150.38	1347.29	14.62	tlim	0	**1233.42**	**1335.93**	7.67	tlim	0
20	2	2	3	2	1230.27	**1262.13**	2.52	tlim	0	1224.26	**1262.13**	3.00	tlim	0	**1262.13**	**1262.13**	0.00	628.18	1
20	2	2	3	3	1368.42	**1389.31**	1.50	tlim	0	1380.15	**1389.31**	0.66	tlim	0	**1389.31**	**1389.31**	0.00	147.40	1
20	3	2	2	1	1179.95	**1298.79**	9.15	tlim	0	1198.30	**1298.79**	7.74	tlim	0	**1220.22**	**1298.79**	6.05	tlim	0
20	3	2	2	2	1195.60	**1214.02**	1.52	tlim	0	**1214.02**	**1214.02**	0.00	334.14	1	**1214.02**	**1214.02**	0.00	101.36	1
20	3	2	2	3	1216.76	**1296.31**	6.14	tlim	0	**1296.31**	**1296.31**	0.00	659.52	1	**1296.31**	**1296.31**	0.00	111.86	1
20	3	2	3	1	1194.78	**1338.93**	10.77	tlim	0	1195.33	**1338.93**	10.73	tlim	0	**1338.93**	**1338.93**	0.00	2716.13	1
20	3	2	3	2	1221.39	**1240.01**	1.50	tlim	0	**1240.01**	**1240.01**	0.00	608.59	1	**1240.01**	**1240.01**	0.00	290.72	1
20	3	2	3	3	1244.87	**1326.25**	6.14	tlim	0	**1326.25**	**1326.25**	0.00	3323.27	1	**1326.25**	**1326.25**	0.00	376.04	1
20	3	3	2	1	1259.09	**1373.81**	8.35	tlim	0	1247.42	1393.76	10.50	tlim	0	**1301.73**	1396.60	6.79	tlim	0
20	3	3	2	2	1187.22	**1194.67**	0.62	tlim	0	1175.59	**1194.67**	1.60	tlim	0	**1194.67**	**1194.67**	0.00	784.93	1
20	3	3	2	3	1183.73	**1298.57**	8.84	tlim	0	1223.93	**1298.57**	5.75	tlim	0	**1285.68**	**1298.57**	0.99	tlim	0
20	3	3	3	1	1269.61	**1406.51**	9.73	tlim	0	1229.09	1443.90	14.88	tlim	0	**1331.97**	**1406.51**	5.30	tlim	0
20	3	3	3	2	1191.36	**1230.49**	3.18	tlim	0	1159.99	1238.69	6.35	tlim	0	**1218.56**	**1230.49**	0.97	tlim	0
20	3	3	3	3	1210.59	**1328.51**	8.88	tlim	0	1224.45	**1328.51**	7.83	tlim	0	**1293.39**	**1328.51**	2.64	tlim	0
20	5	3	2	1	1204.32	1352.44	10.95	tlim	0	1252.35	**1347.49**	7.06	tlim	0	**1270.20**	1353.84	6.18	tlim	0
20	5	3	2	2	1209.31	1248.41	3.13	tlim	0	**1247.84**	**1247.84**	0.00	1018.41	1	**1247.84**	**1247.84**	0.00	1760.38	1
20	5	3	2	3	1186.65	**1239.62**	4.27	tlim	0	1206.60	**1239.62**	2.66	tlim	0	**1207.89**	**1239.62**	2.56	tlim	0
20	5	3	3	1	1243.05	**1392.58**	10.74	tlim	0	1244.23	1652.73	24.72	tlim	0	**1300.39**	1413.05	7.97	tlim	0
20	5	3	3	2	1241.81	**1282.29**	3.16	tlim	0	**1282.29**	**1282.29**	0.00	1475.60	1	**1282.29**	**1282.29**	0.00	1390.70	1
20	5	3	3	3	1200.87	1297.62	7.46	tlim	0	1227.18	**1272.76**	3.58	tlim	0	**1227.66**	**1272.76**	3.54	tlim	0
20	10	4	4	1	1239.52	–	–	tlim	0	1512.39	1896.32	20.25	tlim	0	**1704.05**	**1723.12**	1.11	tlim	0
20	10	4	4	2	1377.55	–	–	tlim	0	1729.04	2796.96	38.18	tlim	0	**1983.38**	**2004.87**	1.07	tlim	0
20	10	4	4	3	1381.21	–	–	tlim	0	**1592.77**	2389.81	33.35	tlim	0	1577.30	**1813.64**	13.03	tlim	0
20	10	6	4	1	1474.55	–	–	tlim	0	1309.62	–	–	tlim	0	**1815.94**	**1902.57**	4.55	tlim	0
20	10	6	4	2	1272.48	–	–	tlim	0	1133.22	–	–	tlim	0	**1717.81**	**1728.85**	0.64	tlim	0
20	10	6	4	3	1168.19	–	–	tlim	0	1164.49	–	–	tlim	0	**2034.31**	**2075.43**	1.98	tlim	0
sum/avg (20)					1240.60	1299.11	5.90	3567.50	1	1269.10	1428.27	8.31	2994.07	8	**1374.89**	1414.95	2.70	2452.41	12
overall sum/avg					1062.60	1099.96	4.43	2484.04	33	1086.60	1152.66	4.33	2013.57	45	**1138.48**	1154.97	1.14	1366.40	60

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