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A VALUE-BASED DECISION FRAMEWORK FOR EVALUATING THE INTEGRATED EFFICIENCY OF SMART GRIDS

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Abstract

The evaluation of Smart Grid (SG) efficiency requires a value-based perspective that incorporates decision-maker preferences alongside technical metrics. This study develops an integrated framework combining Data Envelopment Analysis (DEA) with the Flexible and Interactive Tradeoff (FITradeoff) method to assess SG efficiency. The approach begins with standardized DEA to identify an initial set of efficient DMUs, followed by an iterative preference elicitation process in which decision-makers provide pairwise comparisons of weight ratios. These preferences are translated into linear constraints that progressively contract the weight space through a robust optimization procedure with baseline weight restrictions. The algorithm systematically reduces the efficient set, terminating when a unique DMU is identified or the candidate set becomes sufficiently narrow. This framework offers a structured, transparent, and mathematically rigorous tool for identifying SG alternatives that align with strategic priorities and support preference-driven decision making.

Keywords: Smart Grids; Data Envelopment Analysis (DEA); Flexible and Interactive Tradeoff (FITradeoff)

1. Introduction

The global energy sector is undergoing a paradigm shift driven by the increasing penetration of renewable energy, digitalization, and the need for enhanced efficiency in energy management [1], [2], [3], [4]. Renewable energy sources, such as wind and solar, introduce variability and complexity that challenge traditional centralized grids, requiring sophisticated evaluation and optimization strategies to ensure reliability and sustainability [1], [2].

In this context, Smart Grids (SGs) have emerged as a cornerstone of modern energy infrastructure, integrating distributed energy resources, enabling self-healing capabilities, and optimizing large-scale grid operations [5], [6]. However, assessing the performance of these complex systems remains challenging, as it involves balancing multiple, often conflicting criteria spanning economic, technical, and environmental dimensions [7]. This multi-faceted nature creates a classic multi-criteria decision-making (MCDM) problem [8], necessitating frameworks that can holistically capture the integrated efficiency of SGs to guide strategic investment and policy decisions. To address the need for multi-dimensional benchmarking, Data Envelopment Analysis (DEA) [9] has been extensively applied in energy and environmental studies. As a non-parametric frontier analysis technique, DEA excels at evaluating the relative efficiency of homogeneous Decision-Making Units (DMUs), such as regional power grids or electricity distribution companies, without requiring a priori weight assignments. Its application in the power sector is well-established, ranging from assessing the sustainability performance of electric utilities in the United States [10] to benchmarking the operational efficiency of thermal power plants in China [11]. Nevertheless, the core strength of conventional DEA allowing each DMU to select its most favourable weights to maximize its own efficiency score, introduces a critical drawback for decision-making. This flexibility often leads to the "multiple-winner dilemma," where a large number of DMUs are identified as fully efficient [12]. The output is a set of efficient utilities or regional grids, leaving planners and investors without a definitive, justifiable answer to the fundamental question: "Which one is the best choice"? This indecisiveness severely limits DEA's utility in contexts requiring a unique selection, such as competitive funding allocation for large-scale grid modernization or strategic national infrastructure planning. Integrating preferences into DEA: The quest to incorporate decision-maker preferences into DEA to overcome its limitations has followed two primary streams, each with significant and well-documented shortcomings: (1) Direct weight restrictions, including approaches such as assurance regions [12] and cone-ratios, impose explicit bounds on the DEA weight space. By constraining the set of feasible weights, these methods can reduce the number of units classified as efficient, as overly extreme or unrealistic weight combinations are eliminated [13]. However, such restrictions have been widely criticized for their reliance on externally imposed and often subjective limits, which may not accurately capture the nuanced preferences or value system of the decision maker [14]. Consequently, while mathematically convenient, these constraints may lack a robust decision-theoretic foundation and do not necessarily produce solutions that are truly value-optimal [13]. (2) Integration with MCDM Methods. A more popular approach hybrids DEA with MCDM techniques. The most common is the AHP-DEA model, frequently applied in energy project selection, such as in the efficiency assessment of thermal power plants [15]. While decisive, this method inherits AHP's well-documented flaws: it requires many precise pairwise comparisons at the ratio scale upfront, creating a high cognitive burden [16]. Furthermore, it suffers from potential rank reversal, and its theoretical foundation is sometimes questioned for being disconnected from the axioms of multi-attribute value theory (MAVT) [17]. Other integrations, like those with TOPSIS [18], often determine a static preference structure ex-ante, failing to interactively refine the solution based on the evolving understanding of the efficient frontier.

In parallel, the field of MCDM has seen advances in more robust preference elicitation methods grounded in MAVT. The FITradeoff method [19] stands out as a state-of-the-art approach. Its core mechanism involves eliciting holistic pairwise comparisons of consequences from the DM, from which it infers linear constraints on the ratios of the scale constants in an additive utility function. Through an iterative process, it progressively reduces the feasible weight space to identify the most preferred alternative. Owing to its theoretical foundation in MAVT, which relies on more intuitive trade-off judgments rather than precise ratio-scale comparisons, a growing body of literature positions FITradeoff as a cognitively more efficient alternative to AHP, typically requiring fewer and less demanding pairwise inputs from the decision maker [20], and is theoretically sounder due to its direct foundation in MAVT axioms. Its applications have grown, including in supply chain selection and maintenance planning [21]. However, a critical analysis of the literature reveals a stark limitation: its powerful engine for iteratively constraining the weight space has been exclusively applied in traditional MCDM problems with a pre-defined, limited set of alternatives. Its potential has never been harnessed to address the fundamental weight flexibility problem in DEA, where the alternatives are the virtually infinite weight vectors within the DEA feasible region, and the goal is to rank a set of DMUs on an efficiency frontier.

The literature reveals a clear and compelling schism. DEA provides an objective efficiency benchmarking tool but fails to deliver a unique solution [12], while existing preference-integration methods are either theoretically and cognitively flawed (AHP) or have not been adapted to the specific mathematical and philosophical context of frontier-based efficiency analysis. The shortcomings of existing hybrid models highlight the need for a new synthesis that is both methodologically rigorous and practically decisive. This study bridges this gap by proposing a novel Value-Based Decision Framework that integrates DEA [9] with the FITradeoff method. Our work is designed to directly counteract the identified limitations and makes the following contributions. (1) To overcome the indecisiveness of DEA and the arbitrariness of weight restrictions [14], we develop a model where the weight space is not subjectively bounded but is systematically and rationally reduced by the DM's own value judgments, elicited through FITradeoff 's trade-off questions. This ensures the final unique solution is both Pareto-efficient on the frontier and value-aligned, directly solving the "multiple-winner dilemma" [12]. (2) To overcome the cognitive burden and theoretical issues of AHP-DEA integrations [16], [17], we replace AHP with FITradeoff. Our framework leverages FITradeoff 's cognitively efficient, holistic trade-off questions, significantly easing the DM's burden and providing a theoretically rigorous foundation for preference incorporation based on the axioms of MAVT. (3) To unlock the potential of FITradeoff for a new class of problems, we pioneer its application to frontier analysis. We create a novel, dynamic feedback loop where DEA identifies the efficient frontier and FITradeoff interactively refines the weights, converging to a single, preference-optimal SG.

This provides a defensible, decision-centric tool for energy investment and policy, effectively translating the theoretical power of trade-off-based elicitation into a practical solution for complex efficiency evaluation problems. The remainder of this paper is organized as follows. Section 2 reviews the theoretical foundations of DEA and the FITradeoff method. Section 3 presents the proposed preference-guided DEA framework integrating the FITradeoff approach. In Section 4, the applicability and effectiveness of the proposed method are demonstrated through an empirical evaluation of SG efficiency. Finally, Section 5 concludes the paper and discusses potential directions for future research.

2. Theoretical background: DEA and FITradeoff methods

This section delineates the theoretical underpinnings of the integrated evaluation framework proposed in this study. It commences with a critical exposition of DEA, establishing its role as the foundational engine for efficiency benchmarking. Subsequently, the discussion pivots to the imperatives for incorporating decision-maker preferences, thereby introducing the FITradeoff method as the mechanism for value-driven refinement. A nuanced comprehension of these constituent methodologies is paramount for appreciating their synthesis in the subsequent model development.

2.1 Data envelopment analysis

DEA is a non-parametric performance evaluation technique that utilizes linear programming to assess the relative efficiency of a set of homogeneous DMUs. Its core premise is to construct a piecewise linear efficiency frontier from the best-performing DMUs, against which all other units are benchmarked. The seminal CCR model establishes the foundational linear programming formulation under the assumption of constant returns to scale [9]. For a target DMU_0, the input-oriented model is specified as follows:

$$\text{Max } \theta_0 = \sum_{r=1}^{s} u_r y_{r0} \tag{1}$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1 \tag{2}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \forall j = 1,2,\dots,n \tag{3}$$

$$u_r \geq \varepsilon, v_i \geq \varepsilon, \forall\, r, i \tag{4}$$

where x_i: Input indicator i; y_r: Output indicator r; v_i, u_r: Virtual weights (multipliers) for inputs and outputs, respectively; θ_0: Efficiency score of the target DMU_0

- Equation (1) maximizes the virtual output of DMU_0.
- Equation (2) normalizes the virtual input of DMU_0 to 1, establishing a common benchmark for comparison.
- Equation (3) ensures that no DMU can achieve an efficiency score greater than 1 under the same set of weights, thereby defining the efficiency frontier.
- Equation (4) with a non-Archimedean infinitesimal (ε) prevent the model from assigning zero weights to any factor, ensuring all inputs and outputs are considered.

The efficiency score θ_0 for each DMU is obtained by solving this linear program. A DMU is considered Pareto-efficient (lying on the frontier) if $\theta_0 = 1$. A score of $\theta_0 < 1$ indicates that the DMU is inefficient, meaning there exists a combination of other DMUs that can produce at least the same level of outputs with fewer inputs. The principal strength of DEA lies in its ability to identify an objective efficiency frontier without requiring prior parametric assumptions about production technology. However, a key limitation is the potential for multiple optimal solutions, the "multiple-winner dilemma" as each DMU is evaluated using the set of weights most favourable to itself.

2.2 Flexible and Interactive Tradeoff method

The FITradeoff method is an advanced MCDM technique grounded in MAVT [19]. Its core strength lies in its adaptive preference elicitation process, which progressively refines the decision-maker's preferences through intuitive, holistic trade-off questions, significantly reducing the cognitive burden associated with methods like AHP that require precise ratio-scale comparisons. FITradeoff does not require the decision-maker to specify precise criterion weights upfront. Instead, it employs an iterative questioning procedure where the decision-maker compares pairs of consequence profiles. These preference statements are translated into linear constraints on the feasible space of criterion weights, systematically narrowing down the set of possible optimal solutions. At the heart of FITradeoff's iterative process is a linear programming model designed to test the potential optimality of an alternative a_k given the current set of preference constraints. The linear programming model for testing alternative a_k is formulated as follows:

$$\text{Max} \sum_{i=1}^{n} w_i \cdot v_i(a_{ki}) \tag{5}$$

$$\sum_{i=1}^{n} w_i \cdot v_i(a_{ki}) \geq \sum_{i=1}^{n} w_i \cdot v_i(a_{ji}), \forall j \neq k \tag{6}$$

$$w_{i+1} \leq w_i \cdot k^1 - \varepsilon, i = 1,2, \dots, n-1 \tag{7}$$

$$w_{i+1} \geq w_i \cdot k^2 + \varepsilon, i = 1,2, \dots, n-1 \tag{8}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{9}$$

$$w_i > 0, i = 1,2, \dots, n-1 \tag{10}$$

where n: Number of criteria. w_i: Weight (scale constant) for criterion i. $v_i(a_k)$: Value score of alternatives a_k under criterion i (standardized). a_j: Other alternatives being compared.

- Equation (5) checks whether there exists *any* set of weights within the current feasible region that would make alternative a_k have the highest total value.
- Equation (6) ensures that under the same set of weights, the total value of a_k is at least as high as that of every other alternative.
- Equation (7) or (8) is the core of the interaction, progressively shrinking the feasible weight space to reflect the decision maker's nuanced value system.
- Equations (9) and (10) ensure the weights form a valid, positive, and normalized set.

If the LP is feasible, alternative a_k remains a candidate for the most preferred solution. If it is infeasible, a_k is eliminated. The method then intelligently selects the next most informative trade-off question based on the updated constraints. This process iterates, creating a dynamic feedback loop that rapidly converges to a single, value-optimal alternative. The power of FITradeoff lies in transforming the search for the best alternative from a static, all-information-required-a-priori judgment into a dynamic, value-driven, and highly efficient search process, making it uniquely suited for complex decision scenarios.

3. Preference-guided DEA with FITradeoff method

In this section, we propose a novel hybrid framework, the FITradeoff-DEA model, which integrates interactive preference information from MCDM. By iteratively incorporating these preferences to constrain the feasible weight space, the model converges multiple efficient DMUs to a single, preference-driven optimal solution, thereby shifting the evaluation focus from purely technical efficiency to value-based decision-making. The model operates on an iterative principle of gradually narrowing the DEA weight space based on decision-maker preferences. Initially, all DMUs are evaluated without preference constraints, potentially resulting in multiple efficient units. Through an interactive process, preference information is elicited and converted into linear constraints, progressively reducing the feasible region. The iteration continues until only one DMU achieves the maximum efficiency score under the given preferences, representing the unique preference-driven optimal solution. The following notations and definitions are introduced for clarity and consistency throughout the subsequent model formulation.

- DMUs: Consider a set of n homogeneous DMUs, denoted as DMU_j $(j = 1,2, \dots, n)$.
- Input and Output Indicators: x_{ij} $(i = 1,2, \dots, m)$ represents the amount of input i for DMU_j . y_{rj} $(r = 1,2, \dots, s)$ represent the amount of output r for DMU_j .
- Weight Variables: v_i is the weight assigned to input i; ϑ_r is the weight assigned to output r.
- Efficiency Score: For a given weight combination (v, ϑ), the efficiency of DMU_j is $\theta_j = \sum_{r=1}^{s} \vartheta_r y_{rj}$.

The following Linear Programming (LP) model is formulated for evaluating a target unit DMU_k :

$$\text{Max } \theta_k = \sum_{r=1}^{s} u_r y_{rk} \tag{11}$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1 \tag{12}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 , \forall j = 1,2, \dots, n \tag{13}$$

$$\frac{u_r}{u_{r+1}} \in [l, p], \frac{v_i}{v_{i+1}} \in [l, p] \tag{14}$$

$$u_r \geq k_1' \cdot u_{r+1} \text{ or } u_r \leq k_1'' \cdot u_{r+1} \tag{15}$$

$$v_i \geq k_2' \cdot v_{i+1} \text{ or } v_i \leq k_2'' \cdot v_{i+1} \tag{16}$$

$$u_r \geq \varepsilon, v_i \geq \varepsilon, \forall\, r, i \tag{17}$$

where (12) is the DEA normalization constraint. (13) is the DEA envelopment constraints. (14) are the weight ratio bounds. (15) and (16) are preference constraint set, a set of linear inequalities incrementally incorporated during the interactive process, C= {all preference information from decision maker}: output vs. output: $u_r \geq k^1 \cdot u_{r+1}$ implies output r is deemed more important than k_1' times output $r+1$ or $u_r \leq k_1'' \cdot u_{r+1}$ implies output r is deemed less important than k_1'' times output $r+1$; input vs. input: $v_i \geq k_2'' \cdot v_{i+1}$ implies input i is deemed more important than k_2' times input $i+1$ or $v_i \leq k_2'' \cdot v_{i+1}$ implies input i is deemed less important than k_2'' times input $i+1$. (17) is the non-negativity constraints, a minimal positive number used to enforce strict positivity of weights. The model executes through a dynamic, interactive, and iterative procedure, as shown in Algorithm 1.

Step 1: Problem initialization and data standardization

Step 1.1 Load input-output data

Inputs*: x_{ij} $(i = 1,2, \dots, m)$ represents the amount of input i for DMU_j .*

Outputs*: y_{rj} $(r = 1,2, \dots, s)$ represent the amount of output r for DMU_j*

Step 1.2 Data standardization process

For each indicator (input x_{ij}or output y_{rj}), the standardized value is calculated as:

$$z' = 0.1 + 0.9\,\frac{z - \min(z)}{\max(z) - \min(z)} \tag{18}$$

[0.1,1] represents the original value of any input or output variable.

Step 1.3 Set baseline weight constraints

Minimum weight: $\mathrm{v_i}, \mathrm{u_r} \geq \varepsilon$

Weight ratio bounds:

Input weights $\frac{v_i}{v_{i+1}} \in [L_{bound}, U_{bound}]$ for $i = 1,2 \dots, m-1$ (19)

Input weights $\frac{u_r}{u_{r+1}} \in [L_{bound}, U_{bound}]$ for $r = 1,2 \dots, s-1$ (20)

Step 2: Initial DEA efficiency calculation

Step 2.1 Solve multiplier DEA model for each DMU j by (11)-(17).

Step 2.2 Identify initial efficiency frontier.

$E^1 = \{DMU_j \mid \theta_j^* \geq 0.999\}$

Record initial efficiency scores θ_j^1 for all DMUs on the frontier.

Step 3: FITradeoff interactive process

While** $|\ E^k\ |> 1$ **and iteration < max iteration do:

Step 3.1 Select weight comparison pair.

In each iteration, the algorithm selects one weight ratio from the set of candidate pairs P =$\{v_i/v_{i+1} | i = 1,2 \dots, m-1\} \cup \{u_r/u_{r+1} | r = 1,2 \dots, s-1\}$ according to a two-level criterion:

Primary criterion*: the pair with the smallest number of prior elicitation questions is preferred.*

Secondary criterion*: among pairs with equal numbers of prior questions, the pair with the largest current range ($U_{bound} - L_{bound}$) is selected, as it represents the greatest remaining uncertainty.*

Only pairs with a current range exceeding a predefined threshold (e.g., 0.05) are considered for selection.

Step 3.2 Generate comparison question.

$$\text{midpoint} = \frac{L_{bound}+U_{bound}}{2} \tag{21}$$

is the ration for the selected paired of weights.

Step 3.3 Simulate DM response.

Computing true ratio from preset weights

Response:

$$response = \begin{cases} I & if|true_{ration} - midpoint| < 0.001 \\ B & if\ true_{ration} < midpoint \\ A & if\ true_{ration} > midpoint \end{cases} \tag{22}$$

Step 3.4 Update weight boundaries

$$Case\ response = \begin{cases} I & L_{bound} = U_{bound} = midpoint \\ B & U_{bound} = midpoint \\ A & L_{bound} = midpoint \end{cases} \tag{23}$$

Step 3.5 Add linear constraints to constraint set C.

If the response is not I, add corresponding linear inequality:

For v_i/v_{i+1}: $v_i - midpoint \cdot v_{i+1} \geq 0$ *if (A) or* ≤ 0 *if (B)*

For u_r/u_{r+1}: $u_r - midpoint \cdot u_{r+1} \geq 0$ *if (A) or* ≤ 0 *if (B)*

This inequality is then incorporated into the set of preference constraints C.

Step 4: Constrained re-evaluation of Efficiency

Step 4.1 Solve constrained DEA for each DMU in E^k ***with constraints*** C.

Step 4.2 Update efficiency frontier

$$E^{k+1} = \{\text{DMU}_j \in E^k \mid \theta_j^* \geq 0.999\} \tag{24}$$

Step 4.3 Handle degenerate cases

If $|E^{k+1}| = 0$, *revert last constraint, restore previous weight boundaries, set* $E^{k+1} = E^k$

Step 5: Convergence check and termination

Step 5.1 Termination conditions

If $|E^{k+1}| = 1$:

Final optimal DMU = unique member of E^{k+1}

Output: Found preference-optimal solution→ terminate

Else if $|E^{k+1}| > 1$ *and maximum of iterations reached:*

Output: Multiple candidate DMUs remain

Conduct cross-efficiency analysis → terminate

Else:

$k = k + 1$→ *return to Step 3*

Step 6: Result analysis and recommendation

Step 6.1 Calculate final efficiency rankings

Compute θ_j *for all DMUs under final constraint set* C

Step 6.2 Output recommendation

If unique optimal DMU, final recommendation DMU can be considered as the optimal choice under preferences.

Algorithm 1. FITradeoff -DEA preference elicitation

4. Case study: application to power company efficiency evaluation

To empirically evaluate the effectiveness of the proposed FITradeoff -DEA model, we perform numerical analysis using a widely recognized dataset in SG efficiency studies. Specifically, we employ the input and output indicators of 15 power companies originally reported by Yu et al. [22]. This dataset is well suited to our analysis as it is constructed under the EPRI framework and sourced from the Transportation Monitoring Centre of the State Grid Corporation of China-an authoritative and reliable data provider. Building upon this established dataset allows for a direct and consistent comparison with prior research outcomes, thereby underscoring the distinctive contributions of our proposed approach.

	Input					Output		
Power company	**Power grid investment (10^4 yuan)**	**Infrastructure investment (10^4 yuan)**	**Technological investment (10^4 yuan)**	**Marketing investment (10^4 yuan)**	**Information technology (10^4 yuan)**	**Total profit (10^4 yuan)**	**Electricity sales (10^6 kwh)**	**Purchase price (Yuan / 10^3 kwh)**
Fujian	1260422	1158031	69000	153615	17139	239868	1397.80	214.16
Tianjin	620911	547966	22845	111471	12491	95404	6078.01	237.29
Hebei	912164	850828	40977	118861	13541	89700	13908.7	210.08
Jiangsu	2977447	2765963	121570	388372	17518	686197	38488.3	152.91
Shandong	2804030	2615571	107399	283868	18150	595476	32978.29	123.09
Shanghai	1079683	628907	128095	153731	29877	110202	11174.96	224.62
Shanxi	926989	807855	46466	185648	13989	88613	16295.99	126.32
Zhejiang	2290509	2134974	99994	239306	18355	588559	28026.15	217.98
Anhui	1125446	961436	107345	126241	15914	75821	1250.51	191.20
Beijing	643251	594054	28141	132705	14041	152093	7920.60	206.48
Hubei	1136556	1008999	53240	171112	17442	71546	1190.21	196.62
Hunan	891914	776652	45236	155489	16123	71723	961.89	200.39
Henan	1387564	1258219	87000	122140	18701	126149	2399.22	112.26
Jiangxi	877297	800046	22470	102954	16176	65327	730.40	252.21
Sichuan	2838232	2592478	157049	181748	19244	99523	180.40	159.64

Table 1. The dataset for SG efficiency evaluation

The FITradeoff-DEA methodology was implemented following the algorithmic procedure described in Section 3, with parameters configured to ensure both robustness and practical applicability. Key settings included: weight ratio bounds of 0.01–100 for all input (v_i/v_{i+1}) and output (u_r/u_{r+1}) comparisons, providing reasonable proportionality while allowing flexibility; a minimum weight threshold of 10^{-6} to prevent trivial solutions; a convergence threshold of 0.05 for weight ranges, excluding sufficiently determined weight pairs from further elicitation; an efficiency frontier threshold of $\theta \geq 0.999$ to account for numerical precision; and a maximum of 25 iterations to balance solution quality and computational effort. These settings were informed by preliminary sensitivity analyses and are consistent with established practices in DEA and MCDM literature. All input and output indicators were first standardized using min-max normalization to map values onto a [0.1, 1] scale, ensuring comparability across heterogeneous units. Initial DEA evaluation identified multiple efficient DMUs along the production frontier, confirming the presence of the "multiple-winner dilemma" that motivates the preference-based approach. The subsequent FITradeoff interactive preference elicitation involved six weight ratio comparisons (v_1/v_2, v_2/v_3, v_3/v_4, v_4/v_5 for inputs u_1/u_2, u_2/u_3 for outputs). Simulated decision-maker responses, derived from preset preference weights, were used iteratively to refine the feasible weight space by adding linear constraints until convergence criteria were met.

4.1 Iteration Process and Results

The FITradeoff-DEA methodology produced a systematic convergence pattern that effectively resolved the initial multiple-winner dilemma. Table 2 summarizes the iterative DMU elimination process. Initially, seven DMUs: Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi were identified on the production frontier with perfect efficiency scores ($\theta = 1.000$). As iterative weight ratio comparisons were applied, the candidate set was progressively narrowed. Early iterations exerted limited impact on the overall candidate set, whereas later iterations particularly from iteration 10 onwards resulted in a marked reduction of remaining DMUs. The majority of decision-maker responses were "B" (indicating the true ratio was below the midpoint), guiding the refinement of the feasible weight space and the elimination of less preferred DMUs. Through 16 iterations, 16 linear constraints were incorporated, progressively narrowing the candidate set and ultimately identifying Tianjin as the unique preference-optimal DMU. The corresponding output weights were $u_1 = 0.0110$, $u_2 = 0.0110$, and $u_3 = 1.1015$, and input weights were $v_1 = 0.0593$, $v_2 = 0.0593$, $v_3 = 0.0593$, $v_4 = 5.9271$, and $v_5 = 2.3018$, reflecting the relative importance of each indicator according to the decision-maker's preferences. Table 3 shows the evolution of weight ratio boundaries and the number of elicitation queries for each weight pair. The first three input ratios (v_1/v_2,v_2/v_3, v_3/v_4) converged to narrow bounds ([0.100, 1.337]) and required the maximum number of queries (three each), reflecting higher sensitivity to decision-maker preferences. In contrast, the fourth input ratio (v_4/v_5) and the two output ratios (u_1/u_2, u_2/u_3) maintained wider bounds ([1.337, 2.575] for v_4/v_5; [0.100, 2.575] for u_1/u_2 and u_2/u_3) and required fewer queries (two or fewer), indicating greater flexibility or less decisive preference information.

Iteration	Selected Weight Pair	midpoint	answer	Remaining Efficient DMUs
0 (Initial)	-	-	-	Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi
1	v_1/v_2	5.050	B	Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi
2	v_2/v_3	5.050	B	Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi
3	v_3/v_4	5.050	B	Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi
4	v_4/v_5	5.050	B	Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi
5	u_1/u_2	5.050	B	Tianjin, Hebei, Jiangsu, Shanxi, Zhejiang, Beijing, and Jiangxi
6	u_2/u_3	5.050	B	Tianjin, Hebei, Jiangsu, Zhejiang, Beijing, Jiangxi
7	v_1/v_2	2.575	B	Tianjin, Hebei, Jiangsu, Zhejiang, Beijing, Jiangxi
8	v_2/v_3	2.575	B	Tianjin, Hebei, Jiangsu, Zhejiang, Beijing, Jiangxi
9	v_3/v_4	2.575	B	Tianjin, Hebei, Jiangsu, Zhejiang, Beijing, Jiangxi
10	v_4/v_5	2.575	B	Tianjin, Hebei, Jiangsu, Zhejiang, Beijing
11	u_1/u_2	2.575	B	Tianjin, Hebei, Jiangsu, Beijing
12	u_2/u_3	2.575	B	Tianjin, Jiangsu, Beijing
13	v_1/v_2	1.337	B	Tianjin, Jiangsu
14	v_2/v_3	1.337	B	Tianjin, Jiangsu
15	v_3/v_4	1.337	B	Tianjin, Jiangsu
16	v_4/v_5	1.337	A	Tianjin

Table 2. FITradeoff iteration process and DMU elimination sequence

Weight ratio	Weight ratio bound	Question count
v_1/v_2	[0.100, 1.337]	3
v_2/v_3	[0.100, 1.337]	3
v_3/v_4	[0.100, 1.337]	3
v_4/v_5	[1.337, 2.575]	3
u_1/u_2	[0.100, 2.575]	2
u_2/u_3	[0.100, 2.575]	2

Table 3. Evolution of weight ratio bounds and question counts during iterations.

4.2 Analysis and discussion

The iterative process systematically reduced the candidate set of efficient DMUs while preserving both DEA structural constraints and decision-maker preferences. The results confirm the FITradeoff-DEA model's capability to resolve multiple-winner scenarios, yielding refined efficiency rankings and identifying a unique preference-optimal DMU, Tianjin. Notably, the first three input weight ratios exhibited pronounced convergence, reflected in narrower bounds and a higher number of elicitation queries, indicating strong sensitivity to decision-maker preferences. In contrast, the remaining input and output ratios maintained wider bounds with fewer queries, suggesting either greater flexibility or less decisive preference information. These findings demonstrate how linear constraints derived from preference elicitation can be systematically integrated into DEA, allowing simultaneous consideration of technical efficiency and subjective priorities. The differential convergence of weight ratios further highlights the algorithm's ability to prioritize under-explored or uncertain regions of the weight space, thereby reducing ambiguity among candidate DMUs and refining efficiency assessments. Overall, the results underscore the robustness and practical applicability of the FITradeoff-DEA framework in incorporating decision-maker preferences, particularly in scenarios characterized by multiple winners.

5. Conclusion

This study proposes a novel FITradeoff-DEA framework that effectively resolves the "multiple-winner dilemma" in traditional DEA by integrating decision-maker preferences through an iterative, weight-space contraction mechanism. The model successfully bridges the gap between technical efficiency analysis and value-based decision-making, converging multiple efficient DMUs to a single, preference-optimal solution, as validated in our SG case study. Notwithstanding its contributions, this study has limitations. The model's effectiveness partially relies on the consistency of decision-makers' responses during the interactive elicitation. Moreover, the current framework assumes a deterministic environment and does not account for uncertain data. Future research will focus on extending the model to handle data uncertainty through fuzzy or stochastic formulations. Exploring its application to other sectors, such as healthcare and finance, and integrating it with other advanced preference modelling techniques present promising avenues for further development. This study develops an innovative FITradeoff-DEA methodology that systematically addresses the issue of multiple efficient DMUs in conventional DEA analysis. By incorporating decision maker preferences through an interactive weight constraint mechanism, the framework progressively narrows the feasible weight space to identify the preferred efficient unit. The proposed approach effectively bridges technical efficiency measurement with value judgment, transforming the DEA frontier from a set of technically optimal DMUs to a single preference-optimal solution. The case study on Chinese power companies demonstrates the practical applicability of the method, showing how iterative preference elicitation on weight ratios leads to convergence toward a consensus solution. The algorithm's dynamic selection of comparison pairs ensures balanced preference exploration across all criteria while maintaining computational feasibility. While methodology offers significant advantages, certain limitations warrant attention. The solution quality depends on the consistency of preference statements during the interactive process. Additionally, the current implementation assumes deterministic data and may benefit from extensions to handle uncertain environments. Future research directions include developing robust versions for stochastic data, exploring applications in other regulatory contexts, and integrating machine learning techniques to reduce the cognitive burden on decision makers through preference prediction.

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