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600 Oil & Gas future returns

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Model-free moments: predictability of *STOXX Europe 600 Oil & Gas* future returns.

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Abstract

The relationship between prices and volatility of energy assets (primarily oil and gas) is of paramount importance for investors and policy makers. We construct a volatility index for the European oil and gas market based on a model-free approach to obtain a European counterpart of US volatility indices for the energy market, such as the CBOE Crude Oil Volatility Index (OVX). Given that investors are averse to volatility of losses, but appreciate volatility of gains, we also derive risk measures that focus on positive and negative returns and their imbalance. We assess whether the constructed indices have predictive power on future returns. We show that in the medium term all the risk indices behave as market greed indicators, whereas in the short term they behave as fear indicators since rises in risk indices are linked with negative returns. The implications for investors and policy-makers are outlined.

Keywords: Corridor implied volatility, Energy market, Model-free implied volatility, Return predictability, Risk-asymmetry index, Risk measures

JEL Codes: C02, C53, G13, G15, G17

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1 Introduction

Financial market risks (e.g. credit risk, liquidity risk or market risk) are important in investments assessment. Volatility is a crucial indicator for quantification of risk. The Chicago Board Options Exchange (CBOE) introduced the VIX Index in 1993, as a 30-day expectation indicator for the volatility of the S&P 100. The methodology was later updated in 2003, adopting a model-free approach based on extracting volatility from a basket of option prices related to the S&P 500. Since then, it has become the reference benchmark for volatility in the US stock market. However, this measure does explicitly account for tail risk of the distribution, leaving ground for other skewness measures. It is worth noting that, for the computation of model-free implied volatility the risk-neutral distribution of returns is used, which may differ from the realized one. In fact, the risk-neutral distribution is often more left-skewed than the realized distribution, as traders assign a higher probability to extreme events in the left tail of the distribution. Put out-of-the-money (OTM) prices consistently exceed call OTM prices, indicating traders' willingness to hedge against extremely negative events. Therefore, asymmetry in the distribution is necessary to accurately predict future realized returns (see Foresi & Wu (2005)). In 2011, the CBOE introduced the CBOE SKEW Index to measure the perceived tail risk by financial operators, capturing the probability they associate with extremely negative returns (Gambarelli & Muzzioli (2019)). However, the CBOE SKEW Index did not achieve the same success as the VIX Index because the combined use of the two indicators led to results that are difficult to interpret. In fact, while the VIX tends to assume high values during periods of market instability, the asymmetry of the distribution can also increase during calm periods. To address this discrepancy, new risk measures have been proposed, as Corridor Implied Volatility (CIV) of Andersen & Bondarenko (2007) and the Risk Asymmetry Index (RAX) of Elyasiani et al. (2018).

In this paper we construct for the first time a volatility index for the European oil and gas market based on a model-free approach to obtain a European counterpart of US volatility indices for the energy market, such as the CBOE Crude Oil Volatility Index (OVX). More-

over, we derive risk measures that focus on positive and negative returns and their imbalance (Corridor implied volatility and Risk asymmetry index). We assess whether the constructed indices have predictive power on future returns of Oil & Gas market. Both markets are subject to strong pressures, including energy transition and geopolitical tensions, influencing energy asset prices and inflation. Additionally, oil and gas prices are linked to significant cost items in many sectors of the economy, affecting the inflation rate dynamics. According to Ciner (2013), oil prices affect stock prices in two ways: by altering expectations about future cash flows of companies and influencing inflation expectations, which in turn affect the discount rate used for securities valuation.

The structure of the paper is as follows. In Section 2 we provide a literature review. In Section 3 we illustrate the data used in the empirical analysis. In Section 4 we investigate the capability of CIV and RAX to predict realized returns. The last section concludes.

2 Literature review

In this section, we describe the extensive literature regarding the relationship between volatility and prices in the energy market, focusing on oil and gas markets in Subsections 2.1 and 2.2 respectively. In Subsection 2.3 we present the main challenges in the estimation of volatility and skewness.

2.1 Relationship between oil prices and stock market

The literature regarding the relationship between prices and volatility of energy assets (primarily oil) and stock markets is extensive. Urom et al. (2021) examine the relationship between economic activity and stock market volatility, energy assets and gold in the United States, both in the short and long term, using data from March 12, 2011 to May 2, 2020. They implement an asymmetric AutoRegressive Distributed Lag (ARDL) model and a Granger causality test showing an asymmetric interaction between the variables, both in the short

and long term. Specifically, in the long term, there is an increase in stock market volatility in response to both positive and negative shocks from the energy asset market and conversely. However, energy asset volatility decreases in response to both positive and negative shocks to economic activity and in relation to negative shocks from the gold market. In all cases, they use OVX as implicit volatility.

Choi & Hong (2020) focus on the causal relationship between implicit volatility indices related to crude oil (OVX), the S&P 500 (VIX) and the KOSPI 200 (VKOSPI), representing the South Korean stock market, using a methodology similar to Urom et al. (2021). Unlike Urom et al. (2021), they split the sample in two sub-periods to examine the relationship between the measures considered before and after the *shale gas revolution*. The results show that during the period of the shale gas revolution, there is a bi-directional causal relationship between OVX and VIX. Furthermore, OVX appears to influence the dynamics of VKOSPI. However, these relationships seem to diminish in the period that does not include the shale gas revolution. Instead, in both sub-periods, there is a unidirectional causality from VIX to VKOSPI.

Xiao et al. (2019) examine how changes in the values of OVX impact the implicit volatility index of the Chinese market (VXFXI). They implement a quantile regression model that, unlike the standard linear regression model, provides a comprehensive frame of the distribution of the dependent variable under different market conditions and it produces estimates of the dependent variable robust to outliers, heteroscedasticity, and skewness. The results of the standard linear regression model show a statistically significant and positive relationship between OVX changes and VXFXI changes. A more in-depth analysis through quantile regression, however, reveals that this relationship persists in all quantiles but intensifies from the seventieth percentile. It is therefore inferred that the effect of OVX changes on VXFXI is greater in extreme market conditions. The analysis is then repeated, including VIX in the model, as a control variable. As suggested by the Adjusted R^2 , the inclusion of this variable provides a better description of the relationship between OVX changes and VXFXI

changes, which is still significant and positive. However, it is a less pronounced relationship overall, but one that still has a greater intensity in the upper quantiles. Controlling for VIX also suggests that changes in this index have a significantly positive impact on VXFXI changes, meaning that higher volatility in the US stock market results in higher volatility in the Chinese stock market.

Creti et al. (2013) analyse the relationship between the returns of 25 commodities and the US stock market from January 2001 to November 2011 using a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The results show that the relationship between commodities and the stock market evolves over time and exhibits high volatility, especially following the 2007-2008 crisis, confirming previous studies' findings that this relationship is rather unstable. It is highlighted that, in the short term, the stock market crash following the sub-prime crisis led to a loosening of this relationship. This is mainly due to a flight-to-quality phenomenon, whereby commodities are seen as safe-haven assets in portfolio diversification. However, in the long term, the relationship between the stock market and commodities is stronger, and its dynamics are mainly driven by industrialization and financialization phenomena. This latter aspect is particularly evident when considering the oil market. This energy asset is highly correlated with the stock market because an increase in oil prices results in increased production costs, potential profit erosion, and therefore, loss of value for shareholders. Creti et al. (2013) also highlight how the relationship between stock prices and oil prices even reverses in crisis period.

Dutta (2017) uses an ARDL model, a Bound Test and a Granger Causality Test in the Toda-Yamamoto version to verify the relationship, over different time horizons, between global oil market implicit volatility indices and the US energy stock market. They introduce the US VIX as control variable for potential effects resulting from global uncertainty. The results reveal both a long and short term relationship between oil market volatility indices. Among implicit volatilities in the oil market and the US energy asset market, the Toda-Yamamoto test highlights, as expected, a bidirectional causal relationship between the energy sector's

VIX and OVX. However, no significance is found regarding the mutual influence between VIX and OVX, contradicting previous studies. This discrepancy can be explained by use of different models and/or periods in the analysis. In any case, the results indicate that changes in implicit volatility of crude oil prices are indicative of future changes in implicit volatility of energy sector stock prices and viceversa, given the bidirectional relationship.

2.2 Relationship between natural gas prices and stock market

Natural gas plays a crucial role in the economy, although the literature on this topic is rather limited. The majority of the papers focuses on volatility spillovers between crude oil and natural gas.

Perifanis & Dagoumas (2020) examine the mechanism of transmission of price and volatility between the main European natural gas markets (National Balancing Point, NBP, and Title Transfer Facility, TTF), the Japan-Korean Marker (JKM), and the Brent crude oil market, used as a benchmark in Europe and Asia. They use a Vector autoregression (VAR) model with Granger causality tests and Wald tests to verify the connection between prices and a Dynamic Conditional Covariance (DCC) GARCH model to test the transmission of information between markets through volatility transmission. The results show that, regarding the NBP, oil and natural gas do not mutually influence price formation, which is rather based on fundamental factors (i.e. supply and demand) and that only gas, for some brief periods, acts as an information transmitter to the Brent oil market. The results are the same for the TTF. Perifanis & Dagoumas (2020) find that, in the European market, there is independence between the two commodities. Opposite results are obtained when considering the JKM. In this case, each commodity has an explanatory power over the other, although the influence exerted by Brent oil on natural gas is greater. However, the cause of this connection is to be found in exogenous factors. Indeed, the period in which this connection was most evident corresponds to the period when Japan shut down some of its nuclear power plants following the 2011 tsunami, thus increasing the need for alternative sources for electricity production.

Brent oil, unsurprisingly, led price formation in the natural gas market only until Japan resumed nuclear energy production. Even considering potential connections between volatilities, the result is the same, except for the period characterized by the first gas crisis in Europe. Therefore, the authors conclude that the two markets are not connected, and price formation follows supply and demand dynamics.

Geng et al. (2021) on the other hand, analyse the level of connection between the natural gas market, uncertainty, and the stock market in Europe and the USA. They observe an higher level of connection in the United States due to a greater maturity of the markets, although this difference is rather limited. The authors also assess how the level of connection varies over time. They highlight that a higher level of connection is associated with periods of turmoil in financial markets, confirming the hypothesis of previous studies that there is greater interaction between the commodity market and the stock market in times of stress. Geng et al. (2021) also observe that much of the spillover effect between markets occurs mainly in the short term. They also study in which direction spillovers move. In Europe, after 2014 and until mid-2016, the natural gas market acted as an information transmitter to other markets, probably due to the collapse of crude oil prices. In the long term, however, the intensity of the connection between markets in both regions is rather limited. Focusing on the European case, they show that, in recent years, the impact of the natural gas market on overall uncertainty has increased: it acts as an information transmitter for energy market uncertainty and the stock market, while it is a net receiver regarding uncertainty arising from economic policy. Both in the long and short term, in both regions, the natural gas market is mainly influenced by uncertainty from financial markets, indicating an increasingly financial asset.

Acaravci et al. (2012) assess the long-term relationship between natural gas prices and stock market prices in 15 European Union countries, for the period from 1990 to 2008. Empirical evidence shows a long-term relationship between natural gas prices, industrial production, and stock market prices in Austria, Denmark, Finland, Germany, and Luxembourg, con-

trary to what happens for other EU countries. However, this causal relationship is not direct; rather, gas prices affect the stock market through economic activity. The authors evidence a certain heterogeneity among EU countries regarding the effects of natural gas price changes on stock prices. Given the economic importance and considering that natural gas is currently among the most volatile commodities, it is essential to have tools to accurately predict future realized volatility.

Ding (2021) propose the Natural Gas Volatility Index (NGVX), built using a state-preference approach based on 30-day expectations of future realized volatility. This measure falls among those based on a forward-looking approach, as it is derived from option prices that incorporate market participants' expectations. According to Poon & Granger (2005), volatility estimates extracted from option prices have better predictive performance than those derived from historical data. This is because option prices incorporate all information and volatility expectations in response to market events such as monetary policy announcements or the publication of gas and oil inventory reports. The results show that NGVX is an unbiased estimator of future realized volatility and it is able to capture the strong seasonality of the natural gas market, showing peaks when gas demand increases. The study also analyses the characteristics of this market to justify daily price volatility movements. The authors highlight that the main source of information is the weekly gas inventory reports, which is why volatility tends to increase until the day before the publication of these reports (Thursday) and then decrease once the data is released.

2.3 Challenges in the estimation of volatility and skewness

Volatility estimation is crucial for pricing and risk management: a gain and a loss of the same absolute value do not have the same effect on decision-making, but a loss has proportionally a greater impact (see the prospect theory of Kahneman & Tversky (1979)).

We can distinguish two approaches: a backward-looking approach (BLa) and a forward-looking approach (FLa). BLa is based on the Random Walk model, which assumes that

it is possible to estimate volatility based on the previous period, using historical average, moving average, exponential weighted moving average or Exponential Smoothing methods Poon & Granger (2003). On the other hand, FLa involve extracting volatility from option prices, which incorporate traders' expectations. The volatility estimation following FLa can be based on specific theoretical model or model-free. In the former case, volatility is extracted from the price of a single option through the inversion of a formula, such as Black & Scholes (1973). Specifically, at-the-money (ATM) calls are used because they have the most informative content (see Christensen & Prabhala (1998)) and the resulting volatility is considered as the market's expectation of future realized volatility until the option's expiry date. However, this estimate should be less accurate than the model-free estimate proposed by Britten-Jones & Neuberger (2000), because the latter, derived from prices of options with different strikes, maturities and types, has more information content.

RAX belongs to FLa and is based on the concept of CIV introduced by Carr et al. (1998) and Andersen & Bondarenko (2007). In order to compute corridor implied volatility, option prices are used, but the integration domain is truncated at the forward price to obtain the two corridor implied volatilities: one for the upper part of the distribution (CIV_{up}) and one for the lower part (CIV_{down}). The two corridor implied volatilities CIV_{up} and CIV_{down} are used to calculate the RAX. This index is useful because it combines the information contained in both the VIX, which is an indicator of overall volatility, and the skewness index, which is expected to express perceived tail risk.

Elyasiani et al. (2018) show that an asymmetric distribution does not necessarily imply thick tails, and conversely, a symmetric distribution may have them. The ability to synthesize the various information is crucial for those making investment decisions, as the combined use of VIX and SKEW may lead to misleading results. The usefulness of the skewness index and whether it is an indicator of market fear or market greed is a widely debated topic in the literature. The direction of the relationship between the skewness index, constructed according to CBOE guidelines, with overall volatility index and market returns is decidedly

counterintuitive. Elyasiani et al. (2018) conduct a study on the properties of the skewness index (ITSKEW) and its relationship with volatility (ITVIX) and stock market returns in Italy. The empirical evidence is consistent with studies on different markets. The first result concerns the relationship between ITVIX and ITSKEW with FTSE Mib returns, respectively. The volatility index is known to be an indicator of market fear: an increase in its value is associated with negative returns. This occurs because when there are expectations of increasing volatility, investors demand higher returns, and thus stock prices decrease. What is rather surprising is the positive relationship between the skewness index and stock returns. This index is constructed such that values above 100 indicate left skewness (negative skewness) of the distribution, and viceversa. In economic terms, a value of the skewness index above 100 indicates that purchasing protection against market downturns, i.e., buying put options, is more expensive. The results show that increasing values of ITSKEW are associated with higher returns, suggesting that ITSKEW is more an indicator of market greed than an indicator of market fear. The authors indicate a possible explanation in the fact that investors, with a view to protecting their profits, tend to assign a relatively higher probability to negative events located in the left tail of the distribution.

Another factor that creates confusion in interpreting SKEW values arises from the controversial relationship between this index and VIX. The relationship between skewness and volatility has been studied through a regression model for 12 European countries (see Gambarelli & Muzzioli (2019)). The results show that the sign of the relationship depends on the country considered and the maturity, highlighting an unclear relationship between the variables. Daily variations in RAX are, instead, positively correlated with those of the overall volatility index, for almost all countries, thus showing a clearer relationship. Therefore, the authors conclude that there is no clear relationship between the variables, at least regarding SKEW and VIX, and that this depends on the characteristics of each market. A further analysis considers market returns regressed against the overall volatility index and skewness index simultaneously. The results show that the main contribution to explain market move-

ments comes from daily variations in VIX, and only marginally from variations in skewness indices. Furthermore, while VIX is an indicator of market fear, being negatively correlated with market returns, the contribution of the skewness index shows a positive sign, confirming that, at least for the countries considered, it acts as an indicator of market greed. However, considering additional studies, this relationship is not always respected, generating confusion among those operating in financial markets.

3 Data and methodology for constructing the implied volatility indices

For the calculation of implied volatility indices on the STOXX Europe 600 Oil & Gas, we use data on options for the period from January 1, 2005, to December 31, 2020. These data include the issuance date, option price, underlying price, implied volatility, volume, strike and expiry date. Dividend yield for the STOXX Europe 600 Oil & Gas is also utilized to adjust its price, along with risk-free rates and futures prices of natural gas and Brent oil. All data were extracted from the *OptionMetrics IvyDB Europe Database*. STOXX Sector indices categorize companies based on their primary source of earnings using the Industry Classification Benchmark (ICB) market standard. According to this standard, 11 industries, 20 supersectors, 45 sectors, and 173 subsectors are defined. The STOXX Europe 600 Oil & Gas belongs to the STOXX Europe 600 family and pertains to the energy supersector (Oil and Gas). Specifically, it comprises companies operating in 10 European countries. The most represented country is the United Kingdom (45.7%). It is followed by France (16.2%), Italy (9.6%), Norway (9.4%), Denmark (5.3%), Spain (4.6%), Finland (3.6%), Germany (1.6%), Poland (1.5%), and Austria (1.3%). The index encompasses 600 companies with high, medium, and low capitalization. Table 1 contains the top 10 companies by market capitalization included in the index (as of December 30, 2022).

We follow Elyasiani et al. (2018) in constructing of VIX, CIV and RAX indices for STOXX

Europe 600 Oil & Gas in order to assess whether these risk measures can explain the returns of the index and whether they add information content compared to other variables such as futures prices on the European market for Brent oil and natural gas.

3.1 Methodology

Option data are filtered according to the following criteria. First, we eliminate options with time-to-maturity of less than eight days, as these may be subject to pricing anomalies close to expiration. Second, only ATM and OTM options are retained, following the criteria of Aït-Sahalia & Lo (1998), which means put options with moneyness lower than 1.03 and call options with moneyness higher than 0.97. In-the-money (ITM) options are excluded. Last, we eliminate the option prices violating the standard no-arbitrage constraints and those with positive prices for butterfly spreads, as outlined by Carr et al. (1998). To handle volatility-strike knots, cubic splines are used for interpolation, and a constant extrapolation scheme is employed outside the existing domain of strike prices, as suggested by Muzzioli (2013a). This approach ensures a minimal truncation and discretization error (for further details, refer to Muzzioli (2013a) and Muzzioli (2013b)).

We construct VIX as follow (see Appendix A):

$$\text{VIX} = \frac{2e^{rT}}{T} \int_0^\infty \frac{M(K, T)}{K^2} dK \quad (1)$$

where r is the risk-free rate, T is the time to maturity, $M(K, T)$ is the minimum between the call and put price, with strike price K and maturity T . B_1 and B_2 represent the cut-off points in which the variance is accumulated.

CIV is calculated likewise implied volatility but the integration domain is not $(0, \infty)$. In fact, it is truncated as follow:

$$\text{CIV} = \frac{2e^{rT}}{T} \int_{B_1}^{B_2} \frac{M(K, T)}{K^2} dK \quad (2)$$

The RAX is calculated as follow:

$$\text{RAX} = \frac{\sigma_{up}(0, T) - \sigma_{down}(0, T)}{\sigma_{overall}(0, T)} \quad (3)$$

where σ_{up} and σ_{down} represent, respectively, the volatility of the right tail (CIV_{up}) and the left tail (CIV_{down}) of the distribution and $\sigma_{overall}$ the overall volatility, with integration domain $(0, \infty)$. Indeed:

$$\sigma_{up}(0, T) = \sqrt{\frac{2e^{rT}}{T} \int_{F_t}^{\infty} \frac{M(K, T)}{K^2} dK} \quad (4)$$

$$\sigma_{down}(0, T) = \sqrt{\frac{2e^{rT}}{T} \int_0^{F_t} \frac{M(K, T)}{K^2} dK} \quad (5)$$

where F_t is the forward index price, which is the value based on which the integration domain is divided, equal to:

$$F_t = K^* e^{rT} \Delta \quad (6)$$

where K^* is the *reference strike price*, which is the strike at which the absolute difference (Δ) in price between the call and the put ATM is minimized. Given that strike prices are spaced by a finite interval and are available on a limited domain, we overcome truncation and discretization errors by interpolating the implied volatility function by cubic splines.

For each trading day, two maturities are reported: one for the near-term options and the other for the next-term options. For each of these, we calculate the model-free implied volatility and the two CIVs. However, since the goal is to obtain risk measures for a 30-day period, interpolation is performed between the two values corresponding to different expirations. Thus, for each trading day, implied volatility, CIV_{up}, CIV_{down}, for 30 days are obtained. These values are then inserted into (3) to obtain the 30-day RAX.

Alternatively, a value of RAX can be calculated for near-term options and one for next-term options. There will be two values for the forward index price: one for the near-term options (with expiry less than 30 days) and one for the next-term options (with expiry more than

30 days). Consequently, there will also be two values for the RAX. Elyasiani et al. (2018) construct a single 30-day skewness measure, proceeding to calculate a weighted average of the two RAX values based on their maturities, using the following formula:

$$\text{RAX}_{30} = w\text{RAX}_{near} + (1 - w)\text{RAX}_{next} \quad (7)$$

where

$$w = \frac{T_{next} - 30}{T_{next} - T_{near}}$$

Through interpolation (Equation 7), we obtain the respective 30-day RAX measure. Since interpolation is performed on different levels, the values obtained using the two calculation methods are not identical, but the difference is negligible. The correlation between the two series is indeed 99.95%.

In order to have a positive value for the RAX index, we use the following formula:

$$\text{RAX} = 100 - 10\text{RAX}_{30} \quad (8)$$

Values of RAX greater than 100 indicate a higher volatility of the left tail compared to the right tail of the distribution, and viceversa ¹. We use *Matlab R2021b* to perform the calculation of indices.

3.2 Descriptive analysis

Table 2 present presents descriptive statistics for the log-returns (contemporaneous and 30-day future) of the STOXX Europe 600 Oil & Gas, for the volatility index, the Corridor Implied Volatility (CIV_{up} and CIV_{down}), and the RAX, covering the period from January 1, 2005, to December 31, 2020. These statistics include mean, median, maximum, minimum,

¹The same considerations apply to the other risk indices: the goal is to obtain a single 30-day risk measure by interpolating the near-term and next-term measures. In this case, the values obtained are simply multiplied by 100.

standard deviation, skewness, kurtosis, and the p-value for the Jarque-Bera test. The distributions of STOXX Europe 600 Oil & Gas returns do not adhere to the assumptions of normality: there is a noticeable negative skewness and high kurtosis for both 30-day future returns and contemporaneous returns. The normality assumption is rejected for all variables considered. In the case of the implied volatility index, the asymmetry of the distribution is already apparent when considering the means of the two CIV, which are its components. The mean of CIV_{down} is higher than that of CIV_{up} (17.63 and 14.7, respectively). Since these are risk-neutral distributions, based on the expectations of option traders, this discrepancy highlights how traders in this market assign a higher probability to price movements located in the left tail of the distribution. Consequently, volatility spikes are also more frequent in the left tail of the distribution. This is also evident from the fact that the mean of the Risk Asymmetry Index is greater than 100 (101.2), indicating a negative asymmetry of the risk-neutral distribution. Additionally, it is observed that the distributions of the implied volatility index and the Corridor Implied Volatilities are characterized by positive skewness. This implies that relatively high values for VOL, CIV_{up} , and CIV_{down} are more likely to occur.

In Table 3 we present the correlation matrix of the variables. A negative relationship is observed between contemporaneous daily returns and all risk indices, indicating that higher perceived risk is associated with lower daily returns on average. However, in the case of 30-day future returns, the relationship with risk indices differs: increasing perceived risk is associated with positive average returns. Therefore, it is expected that the implied volatility index, CIV, and Risk Asymmetry Index are not suitable, in the medium term, as indicators of market fear. It is also interesting to observe the correlation between the various indices, especially between the Risk Asymmetry Index and the implied volatility index. The high (positive) correlation between the implied volatility index and the two CIV is quite intuitive since CIVs are obtained by model-free implied volatility. There is a positive relationship between the RAX and the implied volatility index, indicating that its value tends to increase

in the presence of tensions in the European energy market. This result is consistent with expectations. In times of turbulence in financial markets, it is expected that the volatility of negative returns will be higher. Consequently, the spread between CIV_{down} and CIV_{up} increases, leading to higher values of the RAX. The increase in the spread between the two CIV is evident in Figure 1, which presents STOXX Europe 600 Oil & Gas prices (left axis) and the values of the implied volatility index and CIVs (right axis).

4 Econometric analysis and results

The following analysis consists of five subsections. Subsection 4.1 concerns the evaluation of the risk indices' ability to predict the medium-term returns of the STOXX Europe 600 Oil & Gas. Subsection 4.2 focuses on a one-day time horizon to assess the information content available in the indices for explaining the Oil & Gas index daily returns. Subsection 4.3 investigates the potential asymmetry in the relationship between risk indices and one-day returns. Finally, we assess in Subsection 4.4 whether past information also influences price formation and in Subsection 4.5 we verify whether risk indices provide additional information after introducing Brent oil futures prices into the models. We implement several linear regression models in order to assess the information content and predictive power of model-free volatility and asymmetry measures on the returns of the STOXX Europe 600 Oil & Gas, both in the medium term and short term (30 and 1 days, respectively).

4.1 The predictive power of risk indices on returns in the medium term

We use the following model to evaluate the predictive power of volatility and asymmetry measures in the medium term.

$$R_{t,t+30} = \alpha + \beta I_t + \varepsilon_t \tag{9}$$

where $R_{t,t+30}$ identifies the 30-day log-returns of the STOXX Europe 600 Oil & Gas and I_t represent, at time t , alternatively, the implied volatility (VOL_t), CIV_{up_t} , CIV_{down_t} and RAX_t . If the estimate of the β is statistically significant and negative (positive), it means that the i -th index is an indicator of market fear (market greed) given an inverse (direct) relationship between returns and indices.

The result is presented in Table 4. On a 30-day horizon, all four risk measures are statistically significant at the 1% level in explaining medium-term future returns of the STOXX Europe 600 Oil & Gas. Moreover, the implied volatility index and the two Corridor Implied Volatility measures have quite similar information content: they explain approximately 1.8% of the total variance of returns, unlike the model containing RAX, whose Adjusted R-squared is significantly lower. Such modest information content of the indices was foreseeable over a 30-day horizon. Indeed, other factors come into play in determining prices over the medium term, such as monetary policy interventions or geopolitical events. The sign of the β estimates suggests that, over the medium term, high index values are generally associated with positive returns. For example, in the case of CIV_{down} , a 1% change in the index value is associated with a 0.031 change in 30-day future log-returns. Regarding VOL and CIV_{up} , the beta estimates are almost identical to that of CIV_{down} (slightly above 0.03). Even in the model containing RAX, the β estimate is positive, but compared to other models, the intercept value is higher. This, along with a relatively lower Adjusted R-squared, suggests that RAX has lower content compared to the implied volatility index and the two CIVs.

Contrary to Elyasiani et al. (2018), in the European energy market, options-derived risk indices cannot be considered indicators of market fear. Instead, given their association with positive future returns, market participants' expectations may incorporate the idea of an oversold market that may return to "normal" levels in the medium term. For this reason, we perform the same regression taking into account two distinct market conditions: high volatility and low volatility. Additionally, it is suggested that during periods of high volatility, the indices carry a higher level of information, as demonstrated in previous research

(i.e. Gambarelli & Muzzioli (2019)). The sample characterized by high risk includes the subprime crisis, the Euro crisis, the Ukraine gas crisis that began in 2014, and the COVID-19 pandemic crisis, totaling 2312 observations compared to the remaining set of 1681 observations, defined residually. Table 5 present the results of the estimates made for Regression 9 under different volatility scenarios. Even in periods characterized by high volatility, the sign of the estimates remains unchanged, reinforcing the idea that the energy market tends to exhibit positive returns following even pronounced turbulence. Furthermore, in such circumstances, the implicit volatility index, CIV_{up} , and CIV_{down} respectively explain 5.2%, 4.8%, and 5.3% of the total variance of returns, thus possessing greater information content compared to normal market conditions. Moreover, double beta estimates are observed compared to those of the entire sample, indicating a more pronounced relationship: a one percent change in the volatility index and Corridor Implied Volatility is associated with future returns being approximately 0.06 higher on average. Even the RAX has improved its performance, although it still has significantly lower information content compared to the other indices, explaining approximately 1.3% of the total variance of returns. The intercept level also assumes a rather high value, suggesting a potential omitted variable problem within the model. However, even in this case, an increase in risk is associated with higher average returns.

Conversely, in periods of calm, the indices have less relevance in explaining future returns of the STOXX Europe 600 Oil & Gas. The levels of Adjusted R-squared are indeed less than 1% for all indices. However, even in low volatility conditions, increases in risk are associated with positive returns, although the relationship is less pronounced overall. An increase in the volatility of the left tail more than proportional to that of the right tail is associated with lower returns, but overall the model is only marginally significant, so it cannot be concluded that there is a reversal of the relationship in calm market periods. Despite differences in performance among the various indices, it is quite evident that, regardless of market conditions, an increase in volatility is indicative of future positive returns in the medium term.

4.2 The information content of risk indices in the short term

In order to investigate the information content of risk indices in the short term, we consider first differences of the indices (ΔI) in the following regression:

$$R_{t,t+1} = \alpha + \beta \Delta I_{t+1} + \varepsilon_t \quad (10)$$

where $R_{t,t+1}$ represents the contemporaneous daily log return. From Table 6, it can be observed that in the short term, volatility risk is a predominant factor in explaining the variance of contemporaneous returns. However, considering the entire time span, the RAX does not have sufficient information content to explain the dynamics of returns as it is not statistically significant. As for the volatility measures, adjusted R-squared values of approximately 23% are observed for the two CIVs measures, and 29.6% for the VOL. In the short term, an increase in perceived risk, whether related to the left tail or the right tail of the distribution, is generally associated with a decline in prices of the STOXX Europe 600 Oil & Gas.

The estimation is replicated in the two different volatility conditions (Table 7). In the period characterized by high volatility, the coefficient estimates are essentially unchanged compared to Table 6. However, the statistical significance of the models has increased. Indeed, higher levels of Adjusted R^2 are recorded (above 33% for implied volatility and around 25% for both Corridor Implied Volatility measures). Additionally, the economic significance has slightly increased with larger standardized coefficients (in absolute value) compared to those estimated over the entire sample. It's evident that the strongest relationship, in economic terms, is between contemporaneous returns and ΔVOL . As for the CIV, the standardized coefficients are quite similar among them.

4.3 Is the relationship between returns and risk indices symmetrical?

Given the relevance of risk in explaining the dynamics of contemporaneous returns, the analysis was deepened to identify any specificities in the risk-return relationship. Specifically, this assessment aims to identify differences in the mean returns depending on whether traders' expectations are for increasing or decreasing volatility (thus considering the first differences of volatility indices). In order to evaluate the potential asymmetry in the relationship between the analysed measures and daily returns, we create a variable (ΔIP_t) to account for any additional effect resulting from positive differentials (referring to the difference in index value between time t and time $t - 1$). This variable takes a value of zero if the differential is negative; otherwise, it takes on the value of the observation. We estimate the Regression 11 in the case of high-volatility sample and on the low-volatility sample. Results are reported in Table 8.

$$R_{t,t+1} = \alpha + \beta\Delta I_{t+1} + \gamma\Delta IP_{t+1} + \varepsilon_t \quad (11)$$

In calm market conditions, the coefficient of the constructed variable is not statistically significant. No conclusion can be drawn regarding a potential asymmetric relationship between risk and return. However, conducting the estimates on the sample characterized by high volatility, we highlight a possible asymmetry in the case of ΔVOL and ΔCIV_{down} . The estimated coefficients for these measures are statistically significant at the 1% level, suggesting that daily returns differ on average depending on whether volatility expectations are increasing or decreasing. Given the positive estimation of coefficients for these variables, on average, lower returns are observed when volatility expectations are decreasing. Consequently, for positive values of VOL and ΔCIV_{down} , the relationship with returns is less pronounced. For example, for positive differentials of the volatility index (CIV_{down}), a unitary increase is associated with average contemporaneous returns lower by 0.00567 (0.00577); conversely, in the case of negative spreads, a unitary decrease is associated with higher returns by approxi-

mately 0.00756 (0.00814) on average. Therefore, it is evident that the risk-return relationship tends to change under different market conditions, at least concerning the implicit volatility index and CIV_{down} . While the risk-return relationship appears symmetrical for all indices in calm periods (except for RAX, which is not significant), periods characterized by financial market turbulence show an asymmetric relationship between volatility and returns, except for volatility related to the right tail of the distribution (i.e., "good" volatility).

4.4 Is past information relevant?

We assess whether the contemporaneous first differences related to the risk indices (ΔI_{t+1}) contain all the available information or if, in price formation, past information is crucial. We estimate the following regression:

$$R_{t,t+1} = \alpha + \beta \Delta I_{t+1} + \gamma \Delta I_t + \varepsilon_{t+1} \quad (12)$$

We regress contemporaneous daily log returns on contemporaneous first difference of indices (ΔI_{t+1}) and lagged first difference of indices ΔI_t . We present in Table 9 the result of the Regression 12 for high and low volatility sub-samples. Either way, the lagged variable (ΔI_t) is statistically significant only in the models containing the two Corridor Implied Volatility indices. The RAX is not statistically significant. Therefore, concerning the implied volatility index only, the lagged variable (ΔVOL_t) does not add any significant information beyond that contained in the contemporaneous variable (ΔVOL_{t+1}). In the case of the two Corridor Implied Volatility indices, it was observed that the coefficients related to the lagged variable are statistically significant in both market conditions, although there are differences between different volatility regimes. In the low volatility period the coefficient related to the lagged variable (ΔCIV_{upt}) is statistically significant at the 1% level, the information content is lower compared to that of the model with a single regressor (Regression 10). A lower value of Adjusted R^2 is observed.

Conversely, in the high volatility period, the inclusion of the lagged variable among the regressors leads to a better descriptive ability of the models (Table 3.2.8). For both CIV_{up_t} and CIV_{down_t} , the coefficients related to the lagged variable are statistically significant (at the 5% and 1% levels, respectively), and the increase in Adjusted R^2 values confirms that, for the Corridor Implied Volatility indices, past information is relevant in explaining the dynamics of STOXX Europe 600 Oil & Gas returns. Economically, the effect of the lagged variable is quite similar to the contemporaneous variable, but the relationship with returns is less pronounced. It is approximately 14% of the effect exerted of the contemporaneous variable. Indeed, while variations of one standard deviation of $\Delta CIV_{up_{t+1}}$ are on average associated with movements in returns of about 0.54 standard deviations, the standardized effect of the lagged variable (ΔCIV_{up_t}) is approximately 0.07 standard deviations of returns, in the opposite direction. This holds true in the presence of market turbulence. In conditions of low volatility, however, this ratio remains almost unchanged, but the overall economic significance is reduced. For instance, in such a context, variations of $\Delta CIV_{up_{t+1}}$ of one standard deviation are associated with changes in returns averaging 0.39 standard deviations.

4.5 Using oil futures prices as control variable

In this subsection, we investigate whether the risk indicators considered add information content and remain significant in explaining both contemporaneous and medium-term future returns, even after including futures prices of natural gas and Brent oil among the regressors (Brent oil, like crude oil in the USA, is the benchmark reference in the European market). It is expected that the prices of these two commodities are significant in explaining the returns of the European sector index. To verify their actual significance, we estimate the following models:

$$R_{t,t+30} = \alpha + \beta CO1_t + \gamma NG1_t + \varepsilon_t \quad (13)$$

$$R_{t,t+1} = \alpha + \beta CO_{t,t+1} + \gamma NG_{t,t+1} + \varepsilon_{t+1} \quad (14)$$

where $CO1_t$ and $NG1_t$ are, respectively, future prices of Brent oil and natural gas and $CO_{t,t+1}$ ($NG_{t,t+1}$) represent the logarithmic difference of $CO1_t$ ($NG1_t$). The results are presented in Table 10. We find that only the variables related to Brent oil have significant information content in explaining both contemporaneous and medium-term returns of the STOXX Europe 600 Oil & Gas. For this reason, we decide to exclude the future prices of natural gas as control variable. We estimate the following regressions, for both different sub-sample of volatility scenario:

$$R_{t,t+30} = \alpha + \beta I_t + \gamma CO1_t + \varepsilon_t \quad (15)$$

$$R_{t,t+1} = \alpha + \beta \Delta I_{t+1} + \gamma CO_{t+1} + \varepsilon_{t+1} \quad (16)$$

We present in Table 11 the results of Regression 15 for different volatility scenarios. We show that in periods characterized by market turbulence, VOL and CIV_{down} have information content such that Brent oil futures prices, which are statistically insignificant, do not add any additional information about the dynamics of future returns over 30 days for the STOXX Europe 600 Oil & Gas. In periods of calm, however, the situation is reversed. In such a context, the implied volatility index and CIV_{down} are not significant at all, while Brent oil futures prices alone explain over 3.5% of the overall variance of future returns for the STOXX Europe 600 Oil & Gas. CIV_{up} and RAX, on the other hand, are statistically significant regardless of market conditions, thus adding information compared to that contained in Brent oil futures prices. However, the information content they possess is quite different. From the estimates made on models with a single regressor (Regression 9 and Regression 10), it emerged that RAX explains a very small percentage of the overall variance of returns. The same considerations apply to CIV_{up} : in periods of calm, this indicator adds information content compared to prices, but most of the variance in returns is explained by the latter. Instead, CIV_{up} has much of the information contained in the control variable in periods characterized by high volatility. In such cases, indeed, Brent oil futures prices are

scarcely significant, while CIV_{up} explains about 4.8% of the variance in returns.

Finally, we present the result of Regression 16 in Table 12. Unlike what happens over a medium-term horizon, the daily variation in Brent oil futures prices is always significant, both in periods of high volatility and in calm periods. However, the relationship between risk indicators (except for RAX) and daily returns remains very strong: the estimated coefficients for the first differences of the volatility index and the two Corridor Implied Volatility measures are statistically significant at 1%, regardless of market conditions. Despite the significant increase in Adjusted R^2 of the models after the inclusion of the control variable, risk remains a determining factor in price formation.

There is a different relevance of the risk factor in describing the dynamics of returns depending on the time horizon and the index considered. In the medium term, the risk indicators are always significant even after controlling for Brent Oil future prices as control variable but the Adjusted R^2 is lower than the short term analysis. In the short term, we find that the lagged RAX is no significant.

5 Conclusions

Given the strong relationship between risk and financial asset returns, quantifying the volatility of a security plays a crucial role in defining appropriate investment strategies. Currently, all trading platforms provide investors with a multitude of risk indicators to support their investment decisions. Among these, the VIX Index is certainly the most widely used, becoming the benchmark for overall market volatility in the US financial market. However, investors distinguish between "good" volatility and "bad" volatility, as a security with high risk can offer excellent returns but also significant losses. Therefore, it is necessary to consider the asymmetry of returns distribution and consequently evaluate which tail this volatility falls into. Thus, our work focus on evaluating the information content and predictive power of both volatility and skewness indices on returns. In particular, the analysis was conducted with reference to the short and medium-term returns of the STOXX Europe 600 Oil & Gas, a sector index of the European energy market that includes 600 companies with high, medium, and low market capitalization exposed to the price risk of oil and natural gas.

We construct, following a forward looking approach (FLa) based on model-free implied volatility (i.e. Britten-Jones & Neuberger (2000)), the risk indices (VOL, CIVs and RAX) for STOXX Europe 600 Oil & Gas. Specifically, the implied volatility index (VOL), the Upside Corridor Implied Volatility (CIV_{up}), and the Downside Corridor Implied Volatility (CIV_{down}) were considered as volatility indices while the Risk Asymmetry Index (RAX) was considered as a skewness index.

Compared to previous studies Elyasani et al. (2016), the analysis was conducted over a longer observation period (January 1, 2005, to December 31, 2020) and focused on a specific sector of the European financial market. The choice of using RAX as a skewness index stems from the fact that the joint use of the SKEW index Bakshi et al. (2003) and the implied volatility index leads to misleading results for those making investment decisions. Indeed, if volatility increases in periods characterized by market tensions, skewness of the distribution can also increase in periods of calm. RAX, on the other hand, being positively correlated with the

implied volatility index, shows increases in value with increases in volatility. Moreover, since RAX contains the information available in both the volatility index and the skewness index, investors can use only this measure to obtain unbiased indications regarding future returns Elyasani et al. (2016). In particular, to assess the information content of the risk measures in explaining sector returns, we estimated a linear regression where the returns were regressed on the implied volatility indices. 30-day returns were regressed on the levels of the risk measures (as they are indicative of 30-day volatility by construction), while 1-day returns were regressed on the first differences of the same to preserve temporal coherence within the analysis.

Unlike previous studies (Elyasani et al. (2016)), on a medium-term horizon (30 days), risk measures cannot be considered indicators of market fear. Rather, given the positive sign of the coefficient estimates, expectations of increasing volatility are indicative of investors' fear of missing investment opportunities (indicators of market greed). However, the results of the analysis conducted over the short term (1 day), are consistent with expectations that increases in risk measures are reflected in negative returns. Over this time horizon, the RAX is statistically non-significant: it is unable to explain the returns of the STOXX Europe 600 Oil & Gas.

Given the crucial role of volatility in determining short-term returns, the analysis was deepened by studying the relationship between volatility indices and returns. It emerged that, in periods characterized by low volatility, the relationship between the indices under consideration and 1-day returns of the STOXX Europe 600 Oil & Gas is symmetric. However, in the presence of market turmoil, the existence of an asymmetric relationship between the first differences of the VOL and the CIV_{down} and the 1-day returns of the STOXX Europe 600 Oil & Gas cannot be excluded. In fact, although an overall inverse relationship between the first differences of the two indices and 1-day returns is observed, for positive values of the two volatility measures, negative returns are still observed, albeit lower in absolute value. In the case of the Upside Corridor Implied Volatility, however, the relationship with returns

remains symmetric even in periods of market tension.

In order to check the robustness of the results, 30-day futures prices of Brent oil, the benchmark for oil prices in the European market, were introduced into the analysis. Also in this case, for the very short-term analysis, the daily logarithmic differences of the futures prices were considered, as the 30-day prices were unable to capture the dynamics of returns over this time horizon. It emerged that the risk measures under study provide additional information even after the introduction of Brent oil futures prices, which are significant in explaining the returns of the STOXX Europe 600 Oil & Gas, both in the medium and short term. This has led to a redefinition of the economic significance of volatility measures on returns. In fact, the implied volatility index and the two CIVs measures are statistically significant in explaining Brent oil futures prices as well.

The study has thus confirmed the importance of model-free volatility and skewness measures in describing returns in the European energy sector (with the exception of skewness index in the short term). However, it highlights the different information content of these measures in explaining sector returns over different time horizons. If in the medium term risk measures are to be considered indicators of market greed, the same cannot be said for short term, where increases in volatility risk are associated with negative returns. Future research will extend the analysis over longer time horizons to verify the information contained in these measures and include the recent period characterized by the war between Russia and Ukraine, during which natural gas prices showed marked volatility.

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Conflict of interest

The authors declare that they have no conflict of interest.

Availability of data and materials

Not applicable.

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Not applicable.

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Tables

Table 1: Top 10 companies for market capitalization.

| Companies | Supersector | Country | Wage (%) |
|---------------------|-------------|----------------|----------|
| SHELL | Energy | United Kingdom | 29.84% |
| TOTALENERGIES | Energy | France | 15.61% |
| BP | Energy | United Kingdom | 15.05% |
| EQUINOR | Energy | Norway | 6.76% |
| ENI | Energy | Italy | 6.35% |
| VESTAS WIND SYSTEMS | Energy | Denmark | 5.29% |
| REPSOL | Energy | Spain | 3.94% |
| NESTE | Energy | Finland | 3.56% |
| SNAM RETE GAS | Energy | Italy | 1.79% |
| AKER BP | Energy | Norway | 1.71% |

Table 2: Descriptive statistics

| | VOL | CIV_{up} | CIV_{down} | RAX | $R_{t,t+30}$ | $R_{t,t+1}$ |
|------------|-------|------------|--------------|--------|--------------|-------------|
| mean | 23.38 | 15.36 | 18.04 | 101.20 | 0.00 | 0.00 |
| median | 20.30 | 13.06 | 15.45 | 101.22 | 0.00 | 0.00 |
| max | 93.20 | 54.35 | 76.11 | 103.58 | 0.35 | 0.14 |
| min | 10.49 | 7.03 | 8.17 | 99.51 | -1.13 | -0.18 |
| std. Dev. | 9.56 | 6.22 | 8.04 | 0.55 | 0.08 | 0.02 |
| skewness | 2.29 | 2.23 | 2.34 | 0.07 | -1.26 | -0.39 |
| kurtosis | 10.50 | 10.06 | 11.38 | 3.24 | 11.35 | 15.11 |
| p-value JB | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Note: We show the descriptive statistics for the implied volatility index (VOL), the Upside Corridor Implied Volatility (CIV_{up}), the Downside Corridor Implied Volatility (CIV_{down}), the Risk Asymmetry Index (RAX), the 30-day future returns ($R_{t,t+30}$), and contemporaneous daily returns ($R_{t,t+1}$) of the STOXX Europe 600 Oil & Gas. The p-value corresponds to the Jarque-Bera normality test (the null hypothesis being that skewness and kurtosis are both zero).

Table 3: Correlation matrix of variables

| | VOL | CIV _{up} | CIV _{down} | RAX | R _{t,t+30} | R _{t,t+1} |
|---------------------|-------|-------------------|---------------------|-------|---------------------|--------------------|
| VOL | 1.00 | | | | | |
| CIV _{up} | 16.32 | 1.00 | | | | |
| CIV _{down} | 16.37 | 16.18 | 1.00 | | | |
| RAX | 5.31 | 3.42 | 6.38 | 1.00 | | |
| R _{t,t+30} | 2.28 | 2.25 | 2.28 | 0.38 | 1.00 | |
| R _{t,t+1} | 22.56 | 22.56 | 22.58 | 23.37 | -1.04 | 1.00 |

Note: Table 3 represents correlation matrix between the implied volatility index (VOL), the Upside Corridor Implied Volatility (CIV_{up}), the Downside Corridor Implied Volatility (CIV_{down}), the Risk Asymmetry Index (RAX), the 30-day future returns (R_{t,t+30}) and the contemporaneous daily returns (R_{t,t+1}) of the STOXX Europe 600 Oil & Gas.

Table 4: Results of Regression 9 based on the total observations

| | R _{t,t+30} | R _{t,t+30} | R _{t,t+30} | R _{t,t+30} |
|-------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| VOL _t | 0.03228*** (0.00374) | | | |
| CIV _{up,t} | | 0.03300*** (0.00389) | | |
| CIV _{down,t} | | | 0.03094*** (0.00361) | |
| RAX _t | | | | 0.00701*** (0.00235) |
| Constant | -0.10193*** (0.01155) | -0.08955*** (0.01030) | -0.08948*** (0.01019) | -0.71189*** (0.23818) |
| Adjusted R ² | 1.83% | 1.77% | 1.81% | 0.22% |

Note: Table 4 presents the estimated output for the following regression: $R_{t,t+30} = \alpha + \beta I_t + \varepsilon_t$ where I_t is represented by VOL, CIV_{up}, CIV_{down} and RAX. The t-statistics are reported in parentheses. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 5: Results of Regression 9 based on different volatility scenarios.

| High volatility | | | | |
|-----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ |
| VOL_t | 0.06384*** (0.00565) | | | |
| $CIV_{up,t}$ | | 0.06378*** (0.00588) | | |
| $CIV_{down,t}$ | | | 0.06237*** (0.00546) | |
| RAX_t | | | | 0.02059*** (0.00364) |
| Constant | -0.21330*** (0.01833) | -0.18448*** (0.01645) | -0.19204*** (0.01629) | -2.09262*** (0.36898) |
| Adjusted R^2 | 5.20% | 4.81% | 5.31% | 1.32% |
| Low volatility | | | | |
| | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ |
| VOL_t | 0.02209*** (0.00703) | | | |
| $CIV_{up,t}$ | | 0.02930*** (0.00728) | | |
| $CIV_{down,t}$ | | | 0.01630** (0.00665) | |
| RAX_t | | | | 0.00802*** (0.00252) |
| Constant | -0.05977*** (0.02011) | -0.06767*** (0.01769) | -0.03875** (0.01720) | 0.81384*** (0.25468) |
| Adjusted R^2 | 0.53% | 0.90% | 0.30% | 0.54% |

Note: Table 5 presents the estimated output for the following regression: $R_{t,t+30} = \alpha + \beta I_t + \varepsilon_t$ where I_t is represented by VOL , CIV_{up} , CIV_{down} and RAX . The t-statistics are reported in parentheses.

p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 6: Results of Regression 10 based on the total observations

| | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ |
|-------------------------------|--------------------------|--------------------------|--------------------------|----------------------|
| ΔVOL_{t+1} | -0.00608*** (0.00015) | | | |
| $\Delta\text{CIV}_{up,t+1}$ | | -0.00731*** (0.00021) | | |
| $\Delta\text{CIV}_{down,t+1}$ | | | -0.00637*** (0.00018) | |
| ΔRAX_{t+1} | | | | 0.00033 (0.00074) |
| Constant | -0.00004 (0.00022) | -0.00006 (0.00023) | -0.00005 (0.00023) | -0.0001 (0.00026) |
| Adjusted R^2 | 29.62% | 22.95% | 23.24% | 0% |

Note: Table 6 presents the estimated output for the following regression: $R_{t,t+30} = \alpha + \beta\Delta I_t + \varepsilon_t$ where ΔI_t is represented by ΔVOL , ΔCIV_{up} , ΔCIV_{down} and ΔRAX . The t-statistics are reported in parentheses. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 7: Results of Regression 10 for different volatility scenarios

| High volatility | | | | |
|-------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ |
| ΔVOL_{t+1} | -0.00637*** (0.00019) | | | |
| $\Delta CIV_{up,t+1}$ | | -0.00756*** (0.00027) | | |
| $\Delta CIV_{down,t+1}$ | | | -0.00666*** (0.00023) | |
| ΔRAX_{t+1} | | | | 0.00063 (0.00114) |
| Constant | -0.00018 (0.00033) | -0.00019 (0.00035) | -0.00019 (0.00035) | -0.00024 (0.0004) |
| Adjusted R^2 | 33.23% | 25.51% | 25.92% | 0% |
| Low volatility | | | | |
| ΔVOL_{t+1} | -0.00465*** (0.00026) | | | |
| $\Delta CIV_{up,t+1}$ | | -0.00593*** (0.00038) | | |
| $\Delta CIV_{down,t+1}$ | | | -0.00494*** (0.00032) | |
| ΔRAX_{t+1} | | | | -0.00012 (0.00078) |
| Constant | 0.00014 (0.00024) | 0.00012 (0.00025) | 0.00013 (0.00025) | 0.00009 (0.00026) |
| Adjusted R^2 | 15.50% | 12.43% | 12.67% | 0% |

Note: Table 7 presents the estimated output for the following regression: $R_{t,t+30} = \alpha + \beta \Delta I_t + \varepsilon_t$ where ΔI_t is represented by ΔVOL , ΔCIV_{up} , ΔCIV_{down} and ΔRAX . The t-statistics are reported in parentheses. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 8: Results of Regression 11 based on different volatility scenarios

| High volatility | | | | |
|---------------------------|--------------------------|--------------------------|--------------------------|-------------------------|
| | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ |
| ΔVOL_{t+1} | -0.00756*** (0.00034) | | | |
| $\Delta VOL P_{t+1}$ | 0.00189*** (0.00045) | | | |
| $\Delta CIV_{up,t+1}$ | | -0.00742*** (0.00044) | | |
| $\Delta CIV P_{up,t+1}$ | | -0.00024 (0.00063) | | |
| $\Delta CIV_{down,t+1}$ | | | -0.00814*** (0.00043) | |
| $\Delta CIV P_{down,t+1}$ | | | 0.00237*** (0.00057) | |
| ΔRAX_{t+1} | | | | -0.00324* (0.00197) |
| $\Delta RAX P_{t+1}$ | | | | 0.00747** (0.00311) |
| Constant | -0.00099*** (0.00038) | -0.0001 (0.00041) | -0.00110*** (0.00041) | -0.00114** (0.00055) |
| Adjusted R2 | 33.69% | 25.47% | 26.42% | 0.18% |
| Low volatility | | | | |
| ΔVOL_{t+1} | -0.00454*** (0.00044) | | | |
| $\Delta VOL P_{t+1}$ | -0.0002 (0.00063) | | | |
| $\Delta CIV_{up,t+1}$ | | -0.00515*** (0.00062) | | |
| $\Delta CIV P_{up,t+1}$ | | -0.0016 (0.00098) | | |
| $\Delta CIV_{down,t+1}$ | | | -0.00539*** (0.00057) | |
| $\Delta CIV P_{down,t+1}$ | | | 0.00076 (0.00079) | |
| ΔRAX_{t+1} | | | | -0.00007 (0.0014) |
| $\Delta RAX P_{t+1}$ | | | | -0.00006 (0.00226) |
| Constant | 0.00018 (0.00029) | 0.0004 (0.00032) | -0.00004 (0.0003) | 0.00009 (0.00038) |
| Adjusted R^2 | 15.50% | 12.56% | 12.68% | 0.00% |

Note: Table 8 shows the estimated output for the following regression: $R_{t,t+1} = \alpha + \beta \Delta I_{t+1} + \gamma \Delta IP_{t+1} + \varepsilon_t$, where ΔI_{t+1} is represented by ΔVOL , ΔCIV_{up} , ΔCIV_{down} and ΔRAX and ΔIP_{t+1} is the asymmetry variable for each measure, taking a value of zero for negative differentials, otherwise the value of the observation. T-statistics are reported in parentheses. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 9: Results of Regression 12 based on different volatility scenarios

| | High volatility | | | |
|-------------------------------|--------------------------|--------------------------|--------------------------|-------------------------|
| | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ |
| ΔVOL_{t+1} | -0.00668*** (0.00019) | | | |
| ΔVOL_t | -0.00027 (0.00019) | | | |
| $\Delta\text{CIV}_{up,t+1}$ | | -0.00876*** (0.0003) | | |
| $\Delta\text{CIV}_{up,t}$ | | -0.00059** (0.0003) | | |
| $\Delta\text{CIV}_{down,t+1}$ | | | -0.00729*** (0.00024) | |
| $\Delta\text{CIV}_{down,t}$ | | | -0.00100*** (0.00024) | |
| ΔRAX_{t+1} | | | | -0.00229* (0.00136) |
| ΔRAX_t | | | | -0.00283** (0.00136) |
| Constant | -0.0002 (0.00033) | -0.00022 (0.00034) | -0.00021 (0.00034) | 0.00009 (0.00038) |
| Adjusted R2 | 33.86% | 27.30% | 29.13% | 0.14% |
| | Low volatility | | | |
| ΔVOL_{t+1} | -0.00473*** (0.00026) | | | |
| ΔVOL_t | -0.00045* (0.00026) | | | |
| $\Delta\text{CIV}_{up,t+1}$ | | -0.00593*** (0.00039) | | |
| $\Delta\text{CIV}_{up,t}$ | | -0.00106*** (0.0004) | | |
| $\Delta\text{CIV}_{down,t+1}$ | | | -0.00532*** (0.00031) | |
| $\Delta\text{CIV}_{down,t}$ | | | -0.00072** (0.00031) | |
| ΔRAX_{t+1} | | | | -0.00199** (0.00087) |
| ΔRAX_t | | | | -0.00079 (0.0087) |
| Constant | 0.00021 (0.00023) | 0.00021 (0.00024) | 0.00021 (0.00023) | 0.00021 (0.00025) |
| Adjusted R^2 | 16.33% | 11.79% | 15.03% | 0.19% |

Note: Table 9 shows the estimated output for the following regression: $R_{t,t+1} = \alpha + \beta\Delta I_{t+1} + \gamma\Delta I_t + \varepsilon_{t+1}$, where ΔI_{t+1} is represented by ΔVOL , ΔCIV_{up} , ΔCIV_{down} and ΔRAX and ΔI_t represents the differences in value of the same risk measures between time t and time $t - 1$. T-statistics are reported in parentheses. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 10: Results of Regressions 13 and Regression 14 based on the total observations

| | Regression13 | Regression 14 |
|----------------|--------------------------|-------------------------|
| | $R_{t,t+30}$ | $R_{t,t+1}$ |
| $NG1_t$ | 0.00026 (0.00054) | |
| $CO1_t$ | -0.00050*** (0.00005) | |
| $NG_{t,t+1}$ | | -0.00003 (0.00702) |
| $CO_{t,t+1}$ | | 0.34740*** (0.00974) |
| Constant | 0.03443*** (0.00022) | -0.00017 (0.00438) |
| Adjusted R^2 | 2.32% | 24.67% |

Note: We report the results of Regression 13 and Regression 14, $R_{t,t+30} = \alpha + \beta CO1_t + \gamma NG1_t + \varepsilon_t$ and $R_{t,t+1} = \alpha + \beta CO_{t,t+1} + \gamma NG_{t,t+1} + \varepsilon_{t+1}$, respectively. T-statistics are reported in parentheses. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Table 11: Results of Regression 15 based on different volatility scenarios

| High volatility | | | | |
|-----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ | $R_{t,t+30}$ |
| VOL_t | 0.06126*** (0.00669) | | | |
| $CIV_{up,t}$ | | 0.05966*** (0.00685) | | |
| $CIV_{down,t}$ | | | 0.06051*** (0.00652) | |
| RAX_t | | | | 0.01524*** (0.00399) |
| $CO1_t$ | -0.00008 (0.00008) | -0.00012 (0.00008) | -0.00006 (0.00008) | -0.00035*** (0.00007) |
| Constant | -0.19864*** (0.02544) | -0.16330*** (0.02274) | -0.18169*** (0.02334) | -1.52279*** (0.40575) |
| Adjusted R^2 | 5.16% | 4.84% | 5.25% | 2.36% |
| Low volatility | | | | |
| VOL_t | 0.01350* (0.00752) | | | |
| $CIV_{up,t}$ | | 0.02230*** (0.00792) | | |
| $CIV_{down,t}$ | | | 0.00741 (0.00701) | |
| RAX_t | | | | 0.00863*** (0.00399) |
| $CO1_t$ | -0.00054*** -0.00008 | -0.00051*** -0.00008 | -0.00056*** -0.00008 | -0.00054*** -0.00007 |
| Constant | 0.0038 -0.02351 | -0.01383 -0.02152 | 0.02449 -0.02002 | 0.91515*** -0.2479 |
| Adjusted R^2 | 3.62% | 3.90% | 3.50% | 4.14% |

Note: In Table 11, we report the estimated outputs for $R_{t,t+30} = \alpha + \beta I_t + \gamma CO1_t + \varepsilon_t$, where $R_{t,t+30}$ represents the log-returns of 30 days future of the STOXX Europe 600 Oil & Gas, I_t is represented by the implicit volatility index VOL_t , Upside Corridor Implied Volatility $CIV_{up,t}$, Downside Corridor Implied Volatility $CIV_{down,t}$ and Risk Asymmetry Index RAX_t . $CO1_t$ is the Brent oil futures price at time t . In parentheses, t-statistics are provided. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

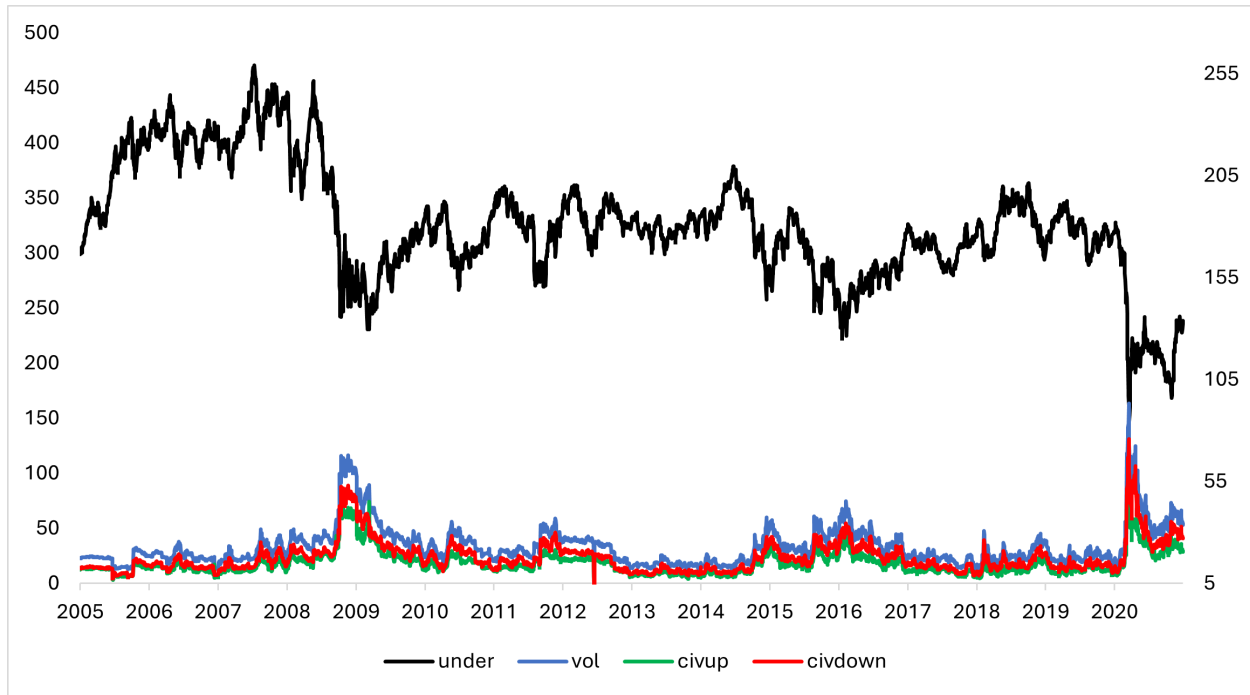
Table 12: Results of Regression 16 based on different volatility scenarios

| High volatility | | | | |
|-------------------------|-------------|-------------|-------------|-------------|
| | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ | $R_{t,t+1}$ |
| ΔVOL_{t+1} | -0.00538*** | | | |
| | -0.00018 | | | |
| $\Delta CIV_{up,t+1}$ | | -0.00699*** | | |
| | | -0.00027 | | |
| $\Delta CIV_{down,t+1}$ | | | -0.00568*** | |
| | | | -0.00022 | |
| ΔRAX_{t+1} | | | | 0.00047 |
| | | | | -0.00106 |
| $CO_{t,t+1}$ | 0.27066*** | 0.29746*** | 0.28439*** | 0.37352*** |
| | -0.00008 | -0.00008 | -0.01179 | -0.01278 |
| Constant | -0.00022 | -0.00023 | -0.00023 | -0.00026 |
| | -0.00029 | -0.0003 | -0.0003 | -0.00034 |
| Adjusted R^2 | 46.77% | 43.28% | 42.90% | 26.81% |
| Low volatility | | | | |
| ΔVOL_{t+1} | -0.00395*** | | | |
| | -0.00025 | | | |
| $\Delta CIV_{up,t+1}$ | | -0.00490*** | | |
| | | -0.00037 | | |
| $\Delta CIV_{down,t+1}$ | | | -0.00434*** | |
| | | | -0.00029 | |
| ΔRAX_{t+1} | | | | -0.00102 |
| | | | | -0.0073 |
| $CO_{t,t+1}$ | 0.21702*** | 0.23143*** | 0.21908*** | 0.25635*** |
| | -0.01333 | -0.0135 | -0.01345 | -0.01407 |
| Constant | 0.00007 | 0.00006 | 0.00007 | 0.00004 |
| | -0.00021 | -0.00022 | -0.00022 | -0.00023 |
| Adjusted R^2 | 27.62% | 24.61% | 26.40% | 16.64% |

Note: In Table 12, we report the estimated outputs for Regression 16, $R_{t,t+1} = \alpha + \beta \Delta I_{t+1} + \gamma CO_{t+1} + \varepsilon_{t+1}$. In parentheses, t-statistics are provided. *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01.

Figures

Figure 1: STOXX Europe 600 Oil & Gas prices, VOL, CIV_{up} and CIV_{down}



Note: The figure displays STOXX Europe 600 Oil & Gas prices (left axis) and the values of the implied volatility index (vol), Upside Corridor Implied Volatility (civup) and Downside Corridor Implied Volatility (civdown) (right axis) for the period from January 3, 2005, to December 31, 2020.

Appendix A. VIX methodology

We present the methodology of VIX index, based on CBOE (2021). Starting from May 2007, the same methodology is applied for calculating the CBOE crude oil ETF volatility index (OVX), whose underlying options refer to the United States Oil Fund (USO), an ETF created to replicate the returns of WTI (see Chen et al. (2018)). Given the following formula:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \quad (17)$$

VIX is equal to

$$\text{VIX} = 100 \cdot \sigma \quad (18)$$

The Equation 17 include:

- T is the time to expiration (in years);
- F is option-implied forward price;
- K_0 is first strike equal to or otherwise immediately below the forward index level, F ;
- K_i is the strike price of the i^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$;
- ΔK_i is the interval between strike prices- half the difference between the strike on either side of K_i ;

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

- R is the risk-free interest rate to expiration;
- $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i .

The VIX consists of put and call options (near-term and next-term) with more than 23 days and less than 37 days to expiry. Among these are the standard options that expire on the third Friday of the month and the "weekly" options that expire every Friday except the third. The time to expiration is calculated according to the following formula:

$$T = \frac{M_{cd} + M_{sd} + M_{od}}{m} \quad (19)$$

where M_{cd} represent the minutes remaining until midnight of the current day, M_{sd} represent the minutes from midnight to 9:30 a.m. for standard options and until 4:00 p.m. for weekly

options, M_{od} is the total minutes of the days between the current day and the expiration day and m is the minutes in a year. The risk-free interest rate consistent with the option's time to expiration is derived from the U.S. Yield curve, to which approximations are applied using cubic spline.

The first step in calculating the index is to select the options to be used. Firstly, it is necessary to determine F for both the near-term and the next-term options according to the formula:

$$F = S + e^{RT}(C - P) \quad (20)$$

where S is the strike price at which the absolute difference between the call price and the put price is minimal and $(C - P)$ represents the minimum difference. Then, K_0 is determined, which is the strike that equals F or, if not, the immediately lower strike. Finally, out-of-the-money (OTM) puts with strike $K < K_0$ and OTM calls with strike $K > K_0$ are selected until two consecutive options with a bid price of 0 are found. For options with a strike equal to K_0 , both the put and the call are selected. For each selected option, reference is made to the midpoint between the quoted bid price and ask price. Then, Equation 17 is used to calculate σ^2 for both the selected near-term and next-term options. The third and final step involves calculating the VIX using the two values of σ^2 :

$$\text{VIX} = 100 \cdot \sqrt{\left[T_1 \sigma_1^2 \left(\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right) + T_2 \sigma_2^2 \left(\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right) \right]} \cdot \frac{N_{365}}{N_{30}} \quad (21)$$