Newtonian Forces Exerted by Electromagnetic Waves Traveling into Matter

Daniele Funaro

Dipartimento di Scienze Chimiche e Geologiche
Università di Modena e Reggio Emilia
Via Campi 103, 41125 Modena (Italy)
daniele.funaro@unimore.it

Abstract

Electromagnetic waves, developing in vacuum or into matter, produce dynamical alterations of the space-time metric. This is a consequence of Einstein's equation, that we are able to solve explicitly in some circumstances. Solutions are in fact obtained by plugging on the right-hand side of the equation some appropriate energy tensors. Hence, the passage of a wave generates both electrodynamics and 'gravitational' (local and temporary) modifications of the molecular lattice of a dielectric. If the wave or the dielectric body are asymmetric, we could theoretically obtain a distribution of Newtonian-like forces with nonzero resultant. This hypothesis suggested a laboratory experiment where an electromagnetic signal applied to a ring with a particular geometry imparts a directional thrust in apparent violation of the action-reaction principle. This test was recently realized with success. Therefore, the present theoretical approach, once appropriately refined, may constitute a crucial referring point for further developments.

Keywords: Electrodynamics, Stress-energy tensor, Einstein's equations, EM-Truster.

1 Introduction

A number of relatively recent papers analyze the role of general relativity in the description of phenomena happening inside matter. For instance, relations between phonons (or even sound waves) and space-time deformations have been theoretically studied in [1, 3, 2, 4]. The findings suggest possible applications in various fields, with the aim of taking advantage of the gravitational mass component. In the present paper we disclose the results of a preliminary study concerning the space-time deformations following the passage of an electromagnetic wave traveling into vacuum or a dielectric. This analysis, in part heuristic, has suggested the construction of a device able to generate asymmetric thrust [5]. The prototype has been successfully tested in laboratory, confirming the theoretical expectations. At this point, a further validation of both theory and practice follows parallel paths.

The device tested in [5] is an asymmetric ring supplied by radio-frequency electromagnetic signals, which undergoes a directional shift in violation of the laws of momentum conservation. The device, belonging to the family of the so called *EM-thrusters* (see for instance the EmDrive [6]), may represent a valid propulsion engine in particular in those applications where there is the need to generate unbalanced forces by only relying on electromagnetic sources, thus without the help of moving components, permanent magnets or some kind of fueling. Various explanations are possibly available. For example, momentum is transferred to the *quantum vacuum* [7, 8, 9, 10], a radiation of electromagnetic entity which pervades the universe. More sophisticated approaches rely upon the *Unruh effect* [11, 12, 13] or the concept of *warped space-time* [14, 15, 16]. As mentioned above, we are trying here to provide the reader with an alternative version of the facts, which is the one that actually inspired the construction of the new working device.

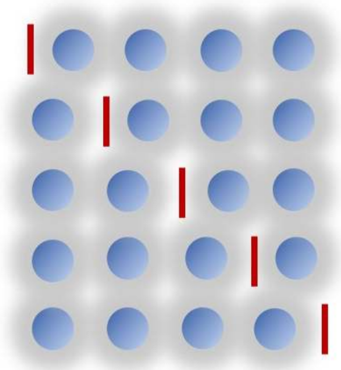

Figure 1: An electromagnetic pulse travels from left to right within the molecular lattice of a dielectric, causing a local deformation at its passage. The picture shows the modified displacements at different times. The signal has enough power and travels at a speed larger than that needed for the recovery reaction of the molecular links. At the end of the process, the initial framework turns out to be slightly shifted backwards. A continuous reiteration of the signal recalls the wakes shaking the body of a shuffling caterpillar.

Action-reaction principle can be put into discussion when taking into account the delay occurring between two events. For example, if our Sun suddenly disappears and we admit that the gravitational information travels at the speed of light, it takes more than 8 minutes before the change will be detected on Earth. During that time, our planet will continue its trajectory along an elliptic orbit, disregarding some basic classical assumptions. In the same way, an electromagnetic wave traveling inside a medium may produce local and temporary variations of the molecular displacement. The inertia of the molecules to regain their initial displacement plays a role where an asymmetry is encountered. Moreover, the uncertainty principle of quantum mechanics, in the form of momentum-position or energy-time, allows for the violation of classical conservation properties under certain restrictive conditions.

The question is how to induce effects as the one outlined in Fig. 1, where, hypothetically, a backward shift of the medium is realized after the passage of a pulse. More specifically, the problem is how to convert electromagnetic strength into Newtonian-like forces. Moreover, we would like to have a non vanishing resultant, able to confer a directional thrust. Therefore, the introduction of some asymmetry in the system becomes a primary concern. Note that, at frequencies of the order of GHz, the wavelengths are measured in centimeters. Thus, there are good margins to be able to work with tools having a manipulable size. Even if the results are negligible for a single pulse, a repetition at a rate of billions times per second may end up in significant outcomes.

The above mentioned laboratory experiment is still waiting to be replicated by other researchers. By the way, its realization was suggested by the same theoretical considerations that we would like here to report. In order to provide a background for a better understanding of the discovery, and suggest how the performances could be improved upon, in the following pages we try to give a formal description of the model equations ruling the phenomenon, as well as their main properties. Due to the technicality of the subject, we are not be able to explain all facts, but we shall limit our analysis to the development of the theoretical foundations and the discussion of some relatively simple examples.

2 The model equations

We denote by $\vec{E}$ the electric field, by $\vec{B}$ the magnetic one, and by $\vec{V}$ a velocity vector field. In particular, $\vec{V}$ follows the evolution of the electromagnetic information, which is not necessarily carried by real massive bodies, such as charged particles. For this reason, the equations will also have validity in pure vacuum. In the simplest cases, $\vec{V}$ maintains the direction of the Poynting's vector $\vec{E} \times \vec{B}$. A set of model equations coupling Maxwell's equations with Euler's equation for non viscous fluids can be obtained as follows:

$$\frac{\partial \vec{E}}{\partial t} = c^2 \mathrm{curl} \vec{B} - \rho \vec{V} \qquad \frac{\partial \vec{B}}{\partial t} = -\mathrm{curl} \vec{E} \qquad \mathrm{div} \vec{B} = 0 \tag{1}$$

$$\rho \left(\mu^{-1} \frac{D\vec{V}}{Dt} + \vec{E} + \vec{V} \times \vec{B} \right) = -\epsilon_0^{-1} \vec{\nabla} p \tag{2}$$

with $\rho = \text{div}\vec{E}$ and c is the speed of light. The above equations were introduced and discussed in [17, 18] and further analyzed in [19]. Hence, we refer to those publications for comments and explanations. The setting is not dissimilar from the ones reported in [20], chapter 10, within the framework of plasma physics. Analogous formulations are found in Magneto-HydroDynamics (see, e.g., [21, 22, 23]). The exception is that we do not introduce any mass density here. Indeed, the model corresponding to (1)-(2) represents, compatibly with the existing ones and in the respect of the physics canons, the most general way to handle electromagnetic phenomena without involving the presence of massive particles. The equations actually derive from a very general classical energy-stress tensor (see (9)). The model has been built to describe phenomena in vacuum, and it is naively applied to dielectrics. In truth, the description for the momentum of light in a medium is a serious issue that requires more attention, since it has been subject of different interpretations in the past [24].

We recall some useful facts. The first equation in (1) is the Ampère's law and the second one the Faraday's equation. As usual, the term $D\vec{V}/Dt$ denotes the substantial derivative and ϵ_0 is the dielectric constant in vacuum. The scalar p is a potential denoting pressure density per unit of surface, that, differently from fluid dynamics, can also attain negative values. The term $\vec{E} + \vec{V} \times \vec{B}$ recalls Lorentz's force. Finally, the constant μ is dimensionally equivalent to Coulomb/Kg. An estimation of μ is provided in [18], appendix H, in a specific circumstance. The most important difference is that density of mass does not appear in the equations and the link between electromagnetic and Newtonian-type forces is represented by the unknown p.

The above set of equations extends the classical Maxwell's model in vacuum. Indeed, by imposing $\rho = 0$, we get:

$$\frac{\partial \vec{E}}{\partial t} = c^2 \text{curl}\vec{B} \qquad \frac{\partial \vec{B}}{\partial t} = -\text{curl}\vec{E} \qquad \text{div}\vec{E} = 0 \qquad \text{div}\vec{B} = 0 \tag{3}$$

Another significant case is when $D\vec{V}/Dt = \vec{0}$ and $p = 0$, but $\rho \neq 0$. In such a circumstance, we get:

$$\frac{\partial \vec{E}}{\partial t} = c^2 \text{curl}\vec{B} - \rho\vec{V} \qquad \frac{\partial \vec{B}}{\partial t} = -\text{curl}\vec{E} \qquad \rho(\vec{E} + \vec{B} \times \vec{V}) = \vec{0} \qquad \text{div}\vec{B} = 0 \tag{4}$$

According to [17], the solutions of (4) (that include those of (3)) are named *free waves* and consist of electromagnetic phenomena that evolve according to the rules of geometrical optics. Indeed, as far as (4) is concerned, it turns out that the vector field $\vec{V}$ must be constant and orthogonal to both $\vec{E}$ and $\vec{B}$ (i.e., it is lined up with the Poynting's vector). The relation $|\vec{V}| = c$, stating that the velocity of propagation of the signal equals that of light, also becomes the *eikonal equation* [25], which actually rules the propagation of fronts in classical optics. As pointed out in [17, 18] there is plenty of these waves that do not satisfy (3), justifying the introduction of the extended model. By applying the divergence operator to the first equation in (4), the following continuity equation is automatically derived:

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}(\rho \vec{V}) \tag{5}$$

Note that the term $\rho\vec{V}$ is not necessarily a classical current, but is related to a kind of density charge accompanying the evolution of a wave. The reason for assuming that ρ can attain values different from zero also in vacuum is explained for instance in [19].

The equations (1)-(2) can be expressed in relativistic form, by using 4-vectors and introducing suitable tensors. As usual, in the system of coordinates (ct, x, y, z), we denote by $F_{\alpha\beta}$ the electromagnetic tensor (see, e.g., [26, 27] or other typical text on general relativity). Within the space-time metric denoted by $g_{\alpha\beta}$, we consider the electromagnetic stress tensor:

$$U_{\alpha\beta} = -g^{\gamma\delta} F_{\alpha\gamma} F_{\beta\delta} + \tfrac{1}{4}\, g_{\alpha\beta}\, F_{\gamma\delta} F^{\gamma\delta} \tag{6}$$

Afterwards, by introducing a velocity 4-vector $\vec{V}$, we define a mass tensor $M_{\alpha\beta}$ that recalls that of a dust of particles for a perfect fluid:

$$M_{\alpha\beta} = \mu^{-1} \rho\ V_\alpha V_\beta + \epsilon_0^{-1}\ g_{\alpha\beta} \Pi_{\alpha\beta} \tag{7}$$

with $\Pi_{\alpha\beta} = \mathrm{diag}(\mathcal{E}, -p, -p, -p)$, where the scalar $\mathcal{E}$ denotes an energy density per unit of volume and p our pressure density per unit of surface. The introduction of the constant μ is justified by the observation that ρ is a density of charge per unit of volume, and not a density of mass, as in standard mechanics. In this way, we remain within a pure electromagnetic context. Inside matter, the information will not be transported by massive particles (such as electrons) but develops by means of compression and rarefaction waves, as a kind of sound wave. Similar consideration may be applied when modeling the behavior of *phonons* [28].

By putting together the tensors so far examined, we arrive at a global stress-energy tensor:

$$T_{\alpha\beta} = \epsilon_0 (U_{\alpha\beta} - M_{\alpha\beta}) \tag{8}$$

The covariant version of the the model equations is obtained from requiring:

$$\nabla_\beta T^{\alpha\beta} = 0 \qquad\qquad \alpha = 0, 1, 2, 3 \tag{9}$$

By specializing the expression in a Minkowski metric, we finally get (1)-(2). In order to make this exposition more fluent, we omit the details of this computation. We also get the following equation:

$$\frac{\partial \mathcal{E}}{\partial t} = -\epsilon_0 \left(c\, \mathrm{div}(\vec{E} \times \vec{B}) + \rho\ \vec{E} \cdot \vec{V} \right) \tag{10}$$

where $\mathcal{E} = \frac{1}{2}\epsilon_0(|\vec{E}|^2 + c^2|\vec{B}|^2)$. This is akin to the Poynting's theorem and says that the local time variation of energy follows the evolution of the vector product $\vec{E} \times \vec{B}$ and also depends on the scalar product $\vec{E} \cdot \vec{V}$ (which is zero for a free wave).

3 Some interesting solutions

In Cartesian coordinates (x, y, z), the electric and magnetic fields of a standard (suitably polarized) plane wave traveling in the direction of the x-axis can be represented as follows:

$$\vec{E} = (E_1, E_2, E_3) = (0,\ f(\xi),\ 0) \qquad \vec{B} = (B_1, B_2, B_3) = (0,\ 0,\ f(\xi)/c) \quad (11)$$

where $\xi = x - ct$ and f is an arbitrary function. In this circumstance we have $\rho = 0$, so that we are in the usual Maxwell's setting in vacuum. More complex plane waves, where the function f also depends on y and z can be considered. In these cases it is necessary to use the modified model (4). The reader interested to this extension is addressed to [17, 18] for more clarifications. Here, for the sake of simplicity, we will not use such an option.

The full set of equations (1)-(2) also allows for solutions where a component of the electric field is lined up with $\vec{V}$ (that implies $\vec{E} \cdot \vec{V} \neq 0$). Electromagnetic waves having a component in the direction of motion actually exist when traveling within a dielectric. In this case, a viable solution is:

$$\vec{E} = (h(\xi),\ f(\xi),\ 0) \quad \vec{B} = (0,\ 0,\ f(\xi)/c) \quad \vec{V} = (V_1, V_2, V_3) = (c,\ 0,\ 0) \quad (12)$$

where h is another arbitrary function. We now have $\rho = h'(\xi)$, with the prime denoting derivation with respect to ξ. Moreover, we get:

$$p = -\frac{1}{2}\epsilon_0 h^2 \qquad \vec{\nabla} p = -\epsilon_0 (hh',\ 0,\ 0) \quad (13)$$

Of course, we return to the expression in (11) when $h = 0$. Note that the pressure has negative signature. It is worthwhile to observe that, when $f = 0$ (so that $\vec{B} = \vec{0}$), the constant c in (1)-(2) is multiplied by zero. This means that in principle a pure longitudinal wave is free to travel at any speed.

The last example teaches us that in order to create pressure we need two facts: there must be a divergence ρ different from zero (this also emerges from examining the left-hand side of (2)), there must be a component of $\vec{E}$ along the direction of propagation. We return to this issue in section 4.

We would like to plug the tensor (8) on the right-hand side of Einstein's equations. This means that we have to solve:

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R = \chi T_{\alpha\beta} \quad (14)$$

where $R_{\alpha\beta}$ is the Ricci's curvature tensor and $R = g^{\alpha\beta} R_{\alpha\beta}$ the scalar curvature. On the right-hand side a constant χ appears, that is not in relation with the gravitational constant. The dimension of χ is meters/joules. An estimate of such a constant in a very special circumstance has been given in [18], appendix E. The question of providing a value to χ is delicate. We expect that χ is many orders of magnitude greater than the corresponding constant related to gravitation. Indeed, inside matter, masses are concentrated in the nuclei whose volume size is 10^{-15} times smaller that of the corresponding atoms. The total mass is negligible from the classical gravitational viewpoint. The corresponding density of mass, taking into account the relevant spacing between nuclei, is also extremely small. Gravitational relativity refers to masses as a whole, without considering that the distribution is concentrated in tiny peaks. On the other hand, an enormous amount of free energy is circulating between atoms, with magnetic fields of the order of Teslas. The fields tend to cancel each other on the average, presenting a piece of matter like an almost neutral object, thus hiding the large energies that keep atoms together. This means that the role of χ in this context must be rethought. The constant will also depend on the material under consideration. We are unable in this short paper to provide a full theory, but the question is certainly of interest and should be further investigated, also in view of the observations that will be made in section 4.

Our analysis is conducted at a pure electromagnetic level. Hence, masses are not to be taken into account. When we talk about 'deformation of the space-time' we mean that this is due to some form of energy of electromagnetic nature which does not come from the presence of massive bodies or black holes. We are going to show that an electromagnetic wave will carry within itself a gravitational-type wave, i.e., a local time-dependent metric with nonzero curvature. Here the name 'gravity' should not be associated with the standard terminology. For instance, speaking about phonons, in [4] the authors observe that they actually carry mass, which is not due to the usual mass-energy conversion, since the effect survives in the nonrelativistic limit. Moreover, they remark that the effect is not of quantum type. Just to have an idea, the ionization energy of the Hydrogen atom is of the order of 10^{-18} J (more exactly 13.6 eV). The gravitational interaction would be only of order 10^{-56} J. Obviously, the description of the space-time metric inside matter cannot be a direct descendant of gravitational type phenomena at large scales.

Let us examine the first case, relative to the solution in (11). Since $E_1 = E_3 = B_1 = B_2 = 0$, our tensors are rather simple. We anticipate that only $U_{\alpha\beta}$ will be necessary in this case. We then look for a metric of the form:

$$(ds)^2 = (c\,dt)^2 - (dx)^2 - (\sigma(\xi)\,dy)^2 - (dz)^2 \tag{15}$$

where the function σ is the unknown to be determined. In other words, we have to find $g_{\alpha\beta} = \mathrm{diag}(c^2, -1, -\sigma^2, -1)$. In the special case we are examining, the non-vanishing entries of the electromagnetic tensor $F_{\alpha\beta}$ are: $F_{02} = E_2 = f$, $F_{20} = -E_2 = -f$, $F_{12} = -cB_3 = -f$, $F_{21} = cB_3 = f$. Correspondingly, the nonzero entries of the electromagnetic stress tensor $U_{\alpha\beta}$ are:

$$U_{00} = U_{11} = (f/\sigma)^2 \qquad\qquad U_{01} = U_{10} = -(f/\sigma)^2 \tag{16}$$

The next step is to evaluate the curvature tensor. Starting from (15), the Christoffel's symbols are:

$$\Gamma^0_{22} = \Gamma^1_{22} = -\sigma'\sigma \qquad \Gamma^2_{02} = \Gamma^2_{20} = -\sigma'/\sigma \qquad \Gamma^2_{12} = \Gamma^2_{21} = \sigma'/\sigma \tag{17}$$

Subsequently, we get:

$$G_{00} = G_{11} = -\sigma''/\sigma \qquad\qquad G_{01} = G_{10} = \sigma''/\sigma \qquad\qquad R = 0 \tag{18}$$

The above outputs have been evaluated with the help of an algebraic manipulation software. Finally, substituting into (14) we must have:

$$-\sigma\,\sigma'' = \chi\epsilon_0\, f^2 \tag{19}$$

which is a simple nonlinear second-order equation (see also [18], p. 248). Comments about its resolution will be given later in this section.

Let us now consider the electromagnetic displacement (12). For simplicity, we only treat the case when $f = 0$. This time we look for a metric of the following form:

$$(ds)^2 = (c\,dt)^2 - (\sigma(\xi)\,dx)^2 - (dy)^2 - (dz)^2 \tag{20}$$

corresponding to $g_{\alpha\beta} = \text{diag}(c^2, -\sigma^2, -1, -1)$. The nonzero entries of the electromagnetic tensors are (recall that now $\vec{B}$ is zero):

$$F_{10} = -h \qquad F_{01} = h \qquad U_{00} = U_{22} = U_{33} = \tfrac{1}{2}(h/\sigma)^2 \qquad U_{11} = -\tfrac{1}{2}h^2 \tag{21}$$

The Christoffel's symbols are:

$$\Gamma^0_{11} = -\sigma'\sigma \qquad \Gamma^1_{01} = \Gamma^1_{10} = -\sigma'/\sigma \qquad \Gamma^1_{11} = \sigma'/\sigma \tag{22}$$

As far as the curvature is concerned, we get:

$$G_{22} = G_{33} = -\sigma''/\sigma \qquad\qquad R = -2\sigma''/\sigma \tag{23}$$

Let us solve the Einstein's equation. It turns out that only the pressure part of the mass tensor $M_{\alpha\beta}$ in (7) is needed. We end up with the following relations:

$$-\sigma\,\sigma'' = \chi\epsilon_0\, h^2 \qquad\qquad \mathcal{E} = \tfrac{1}{2}\epsilon_0\, h^2/\sigma^2 \qquad\qquad p = -\tfrac{1}{2}\epsilon_0\, h^2/\sigma^2 \tag{24}$$

Again we get equation (19). The scalar curvature R is not zero and is proportional to p. This can be also checked by taking the trace of (14) and recalling that the trace of $G_{\alpha\beta}$ is equal to $-R$ and that of $U_{\alpha\beta}$ is zero. Thus, we obtain:

$$R = \chi(\mathcal{E} - 3p) = 2\chi\epsilon_0\, h^2/\sigma^2 = -4\chi p \tag{25}$$

Let us now approach the solution of the first equation in (24). If the right-hand side is $h(\xi) = \sin(\omega\xi)$, for some $\omega > 0$, it is easy to check that: $\sigma(\xi) = (\sqrt{\chi\epsilon_0}/\omega)\sin(\omega\xi)$. This immediately says that the magnitude of the gravitational deformation is inversely proportional to the frequency ω, which is quite an interesting result. Moreover, we get:

$$p = -\omega^2/2\chi \qquad\qquad R = 2\omega^2 \tag{26}$$

Thus, in presence of a longitudinal sinus wave, the scalar curvature is a constant only depending on the parameter ω.

Therefore, there might be conflicts with classical gravitational fields (the one relative to Earth, for instance), suggesting how to build tools for increasing or screening gravitational forces.

As far as different forcing terms h are concerned, the evaluation of σ must be done numerically. We show in Fig. 2 the plot of σ when h is a sawtooth signal. We imposed the boundary conditions $\sigma(0) = \sigma(1) = 0$. The corresponding σ is asymmetric, with σ' and σ'' growing at $\xi = 1$ as the slope of h on the right becomes steeper. This means that, if we prolong the plot periodically, the junctions are not smooth. The alteration of the space-time is continuous, but presents asperities.

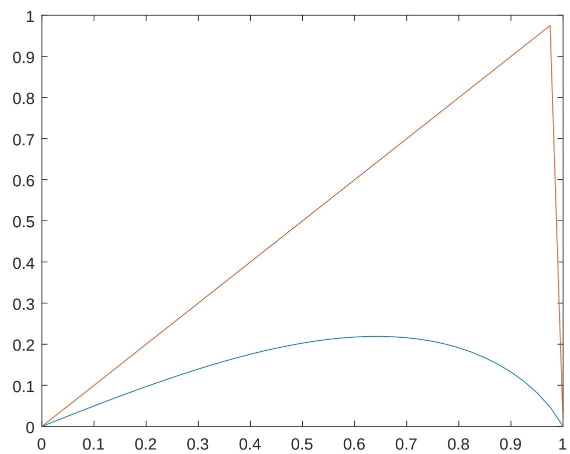

Figure 2: Plot of σ (blue line) solution of the first equation in (24), when h (orange line) is a sawtooth.

We recall that $\xi = x - ct$. An asymmetric signal shifting at speed c (not necessarily that of light) inside a molecular lattice (see Fig. 1) produces slight deformations at a local level. These are certainly electrodynamics effects acting on chemical bonds, but they could be also interpreted as microscopic deviations due to a flow of 'immaterial masses' as a consequence of a gravitational type wave traveling inside the body. This wave does not necessarily develop at the speed of light and may have a sensible intensity. In fact, for the reasons specified at the beginning of this section, the constant χ in (14) can be relevantly bigger in comparison to the classical gravitational case. Note that the pressure gradient (where p is given in (24)) is oriented along the longitudinal direction. Note instead that $p = 0$ in those cases when the electric field has no longitudinal component. We are reminded that $\vec{\nabla} p$ represents our link between forces of electrodynamical nature and forces of mechanical type. Here, we are not converting energy into matter in order to take advantage of gravitational forces. As it is known, this would give negligible outcomes. The idea is to convert electromagnetic energy directly into mass density, forgetting the intermediate passage. The trick is to stimulate asymmetrically the large electromagnetic energy hidden inside matter, having fields that tend to cancel each other, with the aim to extract a 'positive' resultant.

The phenomenon is certainly nonlinear and the reaction of the medium may be different if the signal imparted has a very slow growth followed by a fast damping. If this is true, we would entail that a periodic signal carrying an asymmetric time behavior could generate mechanical forces with nonzero resultant. Indeed, by sitting on a wheelchair without the feet touching the ground, we can impart neat acceleration in one direction by moving our body back and forth, each way with a different velocity law. In the case under discussion, we would take advantage of a sort of internal 'friction' hidden in the interatomic gaps. Heuristically, this phenomenon may be a consequence of the fact that the molecules take a finite fraction of time to regain their relative position after the passage of a pulse.

If the above argument is not convincing, and we want to stick to the classical rules on momentum conservation, we may say that the backward recoil shown in Fig. 1 is due to the forward ejection of a 'massive object' of electromagnetic type. Such an emission is totally different from waves generated by an antenna, that, with the exception of a brief transient near the source, carry a transversal signal satisfying the laws of geometrical optics (see the solutions in (11)). Waves of this last type have $p = 0$ (so they do not carry mass). They only exert very mild mechanical forces (radiation pressure) when interacting with matter (a survey on theory and applications is provided in [29]). For this reason, we do not expect a significant recoil from a classical antenna as a result of energy emission. As previously observed, the ejected mass is not comparable to that attributed to concrete objects (made of an enormous amount of molecules) and successively converted into gravitational forces of very small magnitude. We are dealing with a new interpretation of relativistic phenomena, where the constants involved are far more large. The electric energy supplied to the system is partly transformed in mass density (without masses) moving inside the material and following well-known rules of conservation. A power of 1 Watt is easily realized in an electric circuit and is a relevant quantity in classical mechanics. Even if such an energy conversion was highly inefficient there could still be margins for exploitation.

It is not straightforward however to build a device able to expel longitudinal signals. This is not trivial, at least, along rectilinear paths. The experiment becomes feasible if we opt for circular type paths. The discussion will follow in the next section.

4 Rotating electromagnetic waves

The set (1)-(2) also admits interesting solutions rotating about the axis of a cylinder. These are discussed in detail in [17, 18]. We report here a simplified version. We work in the system of coordinates (r, ϕ, z). For an integer $k \geq 1$

and arbitrary constants $\omega > 0$ we have for any z:

$$\vec{E} = \left(\frac{Qr}{\omega} + \frac{kJ_k(\omega r)}{\omega r} \cos(c\omega t - k\phi), \;\; J'_k(\omega r) \sin(c\omega t - k\phi), \;\; 0 \right) \tag{27}$$

$$\vec{B} \;=\; \frac{1}{c}\Big(0, \;\; 0, \;\; -\frac{Qr^2}{k} - \; J_k(\omega r)\cos(c\omega t - k\phi)\Big) \tag{28}$$

where Q is another constant and J_k denotes the k-th Bessel function of the first kind, that satisfies:

$$J''_k(\xi) \; + \; \frac{J'_k(\xi)}{\xi} \; - \; \frac{k^2 J_k \xi)}{\xi^2} \; + \; J_k(\xi) \; = \; 0 \tag{29}$$

We may take r between 0 and the first zero of J_k (denoted by δ_k), in order to actually obtain a solution defined on a disk (for any fixed z). The radial part of the stationary electric field, i.e. $(Qr/\omega, 0, 0)$, is the one corresponding to a uniformly charged dielectric. The stationary part of $\vec{B}$, i.e. $(0, 0, -Qr^2/ck)$, is the one generated inside a uniformly charged rotating cylinder (though there is no physical rotation of the body).

The displacement of the electric field for $Q = 0$ and $k = 1$ is shown in Fig. 3 at a fixed time t. The magnetic field is perpendicular to the page. As t evolves the picture rotates with constant angular velocity determined by the parameter ω. For $r = \delta_1$, we have $\vec{B} = \vec{0}$, whereas $\vec{E}$ is tangential to the cylinder surface ($E_1 = E_3 = 0$). Always for $Q = 0$, the electromagnetic fields satisfy Maxwell's equations (3), since we have $\rho = 0$ in this case.

As $Q \neq 0$, we need to use the full set of equations (1)-(2). In fact, we have that $\rho = \mathrm{div}\vec{E} = 2Q/\omega$ is constant and different from zero. We now define $\vec{V} = (0, \, c\omega r/k, \, 0)$. This velocity field describes a uniform angular rotation about the z-axis. As a consequence we get that $D\vec{V}/Dt = (-c^2\omega^2 r/k^2, \, 0, \, 0)$ is radial and points towards the center. Moreover, we start having $p \neq 0$. Indeed, we obtain the expression (up to an additive constant):

$$p \;= \epsilon_0 \left[\frac{c^2 \omega Q \, r^2}{\mu k^2} \; - \; \frac{Q^2 \, r^2}{\omega^2}\Big(1 - \frac{\omega^2 r^2}{2k^2}\Big) - \; \frac{2Q \, r}{k\omega} J'_k(\omega r)\cos(c\omega t - k\phi) \right] \tag{30}$$

Note that with this choice p is zero for $r = 0$. The check that (27)-(28)-(30) are actually solutions is straightforward. It is enough to compute partial derivatives and substitute into the corresponding equations. Note that (29) must be used several times. From (27) we observe that $\vec{E}$ has the component E_1 transversal to the direction of rotation (as in a classical plane wave in vacuum) and also a longitudinal one, i.e., E_2. The situation is similar to that illustrated in (12).

Unfortunately, the solution of Einstein's equations is rather complicated now, therefore we do not have results to show. Explicit solutions are extremely rare, and those proposed in the previous section are the result of an intensive study using algebraic manipulation. Numerical simulations aimed at approximating periodic solutions of Einstein's equation are rather challenging. The problem may be approached with techniques similar to those used for the study of rotating black-holes. Some scattered references in this field are for instance [30, 31, 32]. The case of a rotating cylinder with anisotropic fluid has been also treated [37] in the framework of general relativity. The metric in the case of the electromagnetic fields in (27)-(28) is not as simple as in (15) or (20). For sure, the term $g_{02} = g_{20}$, coupling time with the rotation angle ϕ, is activated. Our guess is that, due to the complicated structure of the right-hand side $T_{\alpha\beta}$, the term $g_{01} = g_{10}$ is also going to be different from zero, making the resolution of Einstein's equation very tough.

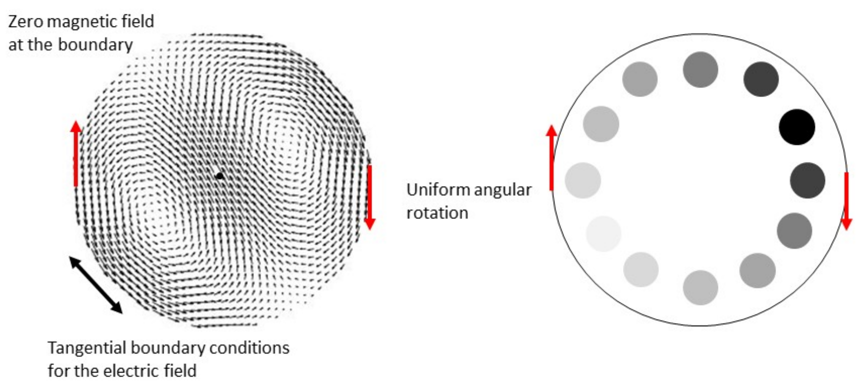

Figure 3: Electric field for $k = 1$ and $Q = 0$ on the disk of radius δ_1 (left). The magnetic field is orthogonal, so that the whole displacement is defined on a cylinder of infinite length. The system rotates with constant angular velocity. For $Q \neq 0$, according to (30), we have pressure waves developing circularly inside the body (right).

We cannot rely on exact (or approximated) solutions, but we can however come out with useful qualitative considerations. By putting together the existence of field displacements like in (27)-(28) together with the argumentation of the previous section, we entail that pressure waves, accompanied by suitable space-time deformations, rotate inside the cylinder. Some sort of non-homogeneous mass is then circulating around the axis. In the experiment in [5], the solicitations were obtained through a winding made of a conducting wire supplied with specific frequencies related to some resonance properties of the object. Thus, the rotating electromagnetic wave and the consequent gravitational one are expected to produce dynamical effects on the molecular bonds (Fig. 3, right). This should shake the entire body with oscillations that, of course, are supposed to be extremely small. On the other hand, as we mentioned in the introduction, these happen at a rate of a billion times per second, so that the final effect may be appreciable. We are however at a periodic regime inside a symmetric body. Therefore, we get a compensation of the various forces by integrating in a period of time. In other words, the resultant displays zero average.

The situation is different if the cylinder section is not a disk. In this occurrence, we may encounter dynamical displacements where the resultant of the forces at a fixed time is different from zero and, in addition, remains different from zero also after averaging over a period of time. This could generate a thrust towards a specific direction that depends on how the cylinder section has been designed.

As far as the determination of electromagnetic displacements is concerned, from the case of the cylinder it is relatively simple to pass to a ring geometry. The first computations were proposed in [33]. Numerical solutions of the full set of equations (1)-(2) circulating in rounded cavities (also including toroids), are also available in [17, 18, 34, 35, 36]. Their evolution is very similar to that of fluid dynamic vortex rings [38]. Periodic behaviors are obtained by requiring that the eigenfunctions associated with the vector Laplace operator on the domain have multiplicity at least four. From the viewpoint of general relativity, finding the metric generated by the corresponding stress tensors requires an effort that at the moment is out of our possibilities.

From the practical viewpoint, the adoption of the ring makes the realization more easy and forces the magnetic field to follow closed loops inside the body, conferring additional stability properties. The analysis of real-life situations becomes more intricate if we take into account that the supplied signal (the one carried by the conducting winding) does not travel at the speed of light and its velocity is not equal in general to the one propagating inside the dielectric. Thus, the correct imposition of boundary conditions should not be underestimated.

Together with a dynamical behavior, the fields in (27)-(28) also display a stationary component proportional to the constant Q. This option was also taken into account in the practical realization of the experimental device. In fact, the addition of a radial steady electric field emphasizes the properties of the thruster. We can provide a sort of justification by showing again some explicit results related to the solution of Einstein's equation. The cylinder and the ring turned out to be rather difficult to handle. We can say something regarding a charged dielectric sphere. The preparatory work was done in the recent paper [19].

We are in the spherical system of coordinates (ct, r, θ, ϕ). As in [19], we look for a metric of the form:

$$(ds)^2 = c^2 \ \tau^2(r)(dt)^2 - \sigma^2(r)(dr)^2 - r^2(d\theta)^2 - r^2 \sin^2\theta \ (d\phi)^2 \qquad (31)$$

and we consider (14) with the right-hand side (8), which is built on the radial electric field $\vec{E} = (Qr, 0, 0)$, whereas $\vec{B}$ and $\vec{V}$ are zero. Here Q is a dimensional constant proportional to charge and depending on the relative dielectric constant of the medium. By solving equation (28) in [19] with the term $\chi\epsilon_0 Q^2 r^2$ on the right-hand side, we discover that:

$$\tau(r) = \sqrt{1 + \chi\epsilon_0 Q^2 r^4/5} \qquad\qquad \sigma(r) = 1/\tau(r) \tag{32}$$

We also get:

$$\mathcal{E} = -p = \frac{3}{2}\epsilon_0 Q^2 r^2 \qquad\qquad R = 6\chi\epsilon_0 Q^2 r^2 \tag{33}$$

Moreover, we obtain $\rho = 3Q$ either in the flat or the curved space (interesting result, since ρ is not a relativistic invariant).

The above finding tells us that the sole fact of charging a body produces a deformation of the space-time metric. According to (32) the effect is minimal near the center ($r = 0$), where τ and σ are approximately equal to 1.

Quantitatively, the final effect depends on the magnitude of the parameter χ (see also the comments at the beginning of section 2), which is expected to depend on the atomic properties of the dielectric material. We can try to provide a rough estimate for χ as follows. A relativistic charged body can be described by the Reissner-Nordström metric. For the electric field external to a charged sphere $\vec{E} = (q/4\pi\epsilon_0 r^2, 0, 0)$, the metric is obtained by choosing an appropriate τ and σ in (31). In [19], working with the same tensor (8) for the study of the electric field external to a pulsating charged sphere, the following functions were deduced in the stationary case:

$$\tau(r) = \sqrt{1 - \frac{2mG}{c^2 r} + \frac{\chi q^2}{16\pi^2\epsilon_0 r^2}} \qquad\qquad \sigma(r) = 1/\tau(r) \tag{34}$$

where G is the gravitational constant and m is the mass of the body.

Let us suppose that we are in the situation in which the massive and the electric components are approximately of the same magnitude, i.e.:

$$\frac{2mG}{c^2 r} \approx \frac{\chi q^2}{16\pi^2\epsilon_0 r^2} \quad\Rightarrow\quad \chi \approx \frac{32\pi^2\epsilon_0 mG}{q^2 c^2}\, r \tag{35}$$

We now compare χ with the classical constant G/c^4 of Einstein's equation:

$$\chi = \gamma\,\frac{G}{c^4} \qquad \text{where} \qquad \gamma \approx 32\pi^2\,\frac{mc^2}{\dfrac{q^2}{\epsilon_0 r}} \tag{36}$$

If, for instance, our body is a proton, we can evaluate γ for r equal to its radius (about $.85 \times 10^{-15}$ meters), so obtaining the multiplicative factor:

$$\gamma \approx 13.93 \tag{37}$$

Therefore, thanks to (36), χ has a an order of magnitude not too far from the standard constant. Moreover, we can observe that γ defined in (36) is adimensional, being the ratio between two energies. The situation changes drastically when we consider an entire hydrogen atom. In this case, the diameter is of the order of 10^{-10} meters, but the mass is only concentrated on the nucleus and the corresponding gravitational field rapidly decays outside. On the other hand, the electric field mildly reduces its strenght. Hence, γ changes its meaning if we intepret it as a ratio of energy densities, so that the new G must be multiplied by a factor of order 10^{15}, at least.

Somebody certainly finds these considerations unconventional. Nevertheless, they help to better understand what actually happens inside matter. It is enough to think about the bending of light due to diffraction, where angles change within very small spatial ranges. The guess is that the rays are following geodesics of a modified geometry induced by the presence, not only of the nuclear lattice, but of the full electromagnetic internal environment. This approach supports the existence of solutions as those in Fig. 3 (see also (27)-(28)), pertaining to rotating electromagnetic waves. An interpretation in terms of general relativity can provide a more robust description of optics phenomena, beyond the abstract application of geometrical rules. In this framework, also the attraction or repulsion of charges due to Coulomb's law ceases to be a mere phenomenological rule and acquires a deeper meaning in terms of modifications of the space-time (note that the potential p may take either positive or negative values). The challenge would be to show that between two neutral object, the resultant of the nonlinear mutual interactions of their internal charges still amounts to a residual weak action, which is what we call gravitational force. If this was true, a theory unifying electromagnetic and gravitational forces would become a reality. A more thoroughgoing discussion on these issues has been put forth in [18], section 2.6, with more examples and remarks, though again at the level of an interrogative analysis .

Going back to the experiment, the role of the stationary electric field was proven to be significant Indeed, a second independent wiring was added to the primary one. The purpose of this circuitry was to apply a constant voltage difference of the order of KVolts between the secondary coil and the center of the dielectric (represented by a further wire running inside the ring). In this fashion, the ring itself becomes a charged capacitor.

With a mixture of rigorous and heuristic considerations, a ring with an asymmetric section was built as described in [5] (see Fig. 4). The design of its shape and the electric circuit benefit from the above theoretical hints. As a combination of stationary and dynamical signals is applied to the device, a sensible thrust is observed in the direction of the bottom part (the rounded one). In absence of the high frequency signal, the sole presence of the high voltage does not produce any measurable effect, and this is a confirmation of the theoretical predictions. In principle, performances may vary depending on many parameters, such as: the frequency injected in the system, the magnitude of the stationary electric field (see Q in (27)), the geometry of the ring, the properties of the dielectric, the conductivity and the number of spires in the wiring, etc. Quantitative information will be available as more tests will be conducted.

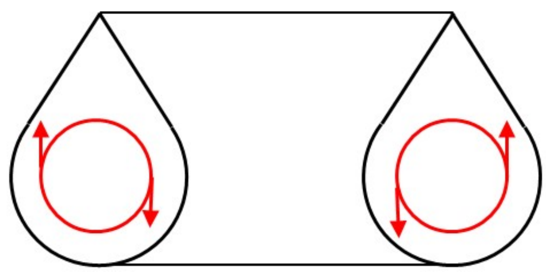

Figure 4: Section of the asymmetric ring used for the experiments in [5]. The geometry has been suggested by heuristic arguments. The idea is to allow for a smooth rotation of the signal in the lower part of the ring and emphasize instead the acceleration term $D\vec{V}/Dt$ on the upper side. This last is relatively large at the corner line on top and points downward. As the angular velocity of the signal is constant, by neglecting the term $\vec{E}+\vec{V}\times\vec{B}$ in (2), we expect that the integral of the pressure gradient $\vec{\nabla}p$ shows a neat resultant pointing downwards. The shape can be certainly ameliorated in order to increase performances. This should be done in conjunction with numerical experiments conducted on the model equations.

What really happens is difficult to explain without a good dose of numerical simulations that are expected to be carried out in the future. These will be confronted with the experimental outcomes and ameliorated when necessary.

According to the results of this paper, an 'immaterial' mass is circulating inside the ring, following trajectories as specified by the arrows in figure. There are two kinds of asymmetries. The first one is the section shape. The sharp form of the upper side was actually designed to emphasize diversity. Secondly, we note that the semi-path passing through the ring hole is different (in terms of surface density) from the one running outside. Our guess is that the orbits are not closed, but are spiraling towards the bottom. As we specified, this may be due to some nonlinear properties of the dielectric material, or because part of the signal is ejected from the top (or both the occurrences). In this framework, the violation of the action-reaction principle is not due to the interaction with the surrounding vacuum (that would be negligible). The asymmetric Newtonian forces should come instead from the induced repositioning of some 'virtual masses' partly already present inside matter due to the strong chemical bonds. The density of such 'alternative' mass is certainly small, but it is spread all over the volume of the body, contrary to what happens to standard masses that are only concentrated at the nuclei. The rotatory process is ignited by the electromagnetic component enforced externally. Unfortunately, we are not in the position to provide more details at the moment.

5 Conclusions

Electromagnetic waves traveling into matter bring along a space-time deformation that can be assimilated to a kind of mass flow, with the consequence of generating longitudinal mechanical pressure variations. Analogous theoretical results have been obtained by other authors with the purpose of finding feasible applications. The results discussed above actually suggested the construction of an asymmetric *thruster*, as experimented in [5]. Though the device still needs to be replicated and tested by independent research groups, the material here collected may represent a key point for improvements, as well as a source for further new ideas. In addition, the theoretical contributions here collected may open a new page for the study of relativistic phenomena inside matter.

References

[1] Sabín, C.; Bruschi, D. E.; Ahmadi, M.; Fuentes, I. Phonon creation by gravitational waves. *New J. Phys.*, 2014, *16*, 085003.

[2] Nicolis, A.; Penco, R. Mutual interactions of phonons, rotons, and gravity. *Phys. Rev. B*, 2018, *97*, 134516.

[3] Kornreich, P. Light induced gravity phonons. *J. Modern Phys.*, 2019, *10*, 1674-1695.

[4] Esposito, A.; Krichevsky, R.; Nicolis, A. Gravitational mass carried by sound waves. *Phys. Rev. Lett.*, 2019, *122*, 084501.

[5] Funaro, D.; Chiolerio, A. An efficient ring-shaped electromagnetic thruster. hal-03843874f. https://hal.archives-ouvertes.fr/hal-03843874, November 2022.

[6] Shawyer, R. J. Digitally controlled beam former for a spacecraft, USP 5543801, 2001.

[7] Milonni, P. W. *The Quantum Vacuum: An Introduction to Quantum Electrodynamics*, Academic Press, 1993.

[8] Cornille, P. *Advanced Electromagnetism and Vacuum Physics*, SCCP, v. 21, World Scientific, Singapore, 2004.

[9] Puthoff, H. E. Advanced space propulsion based on vacuum (spacetime metric) engineering. *JBIS*, 2010, *63*, 82-89.

[10] Meis, C. *Light and Vacuum*, 2nd ed., World Scientific, Singapore 2017.

[11] Cozzella, G.; Landulfo, A. G. S.; Matsas, G. E. A.; Vanzella, D. A. T. Proposal for observing the Unruh effect using classical electrodynamics. *Phys. Rev. Lett.*, 2017, *118*, 161102.

[12] Sudhir, V; Stritzelberger, N.; Kempf, A. Unruh effect of detectors with quantized center of mass. *Phys. Rev. D*, 2021, *103*, 105023.

[13] Šoda, B; Sudhir, V.; Kempf, A. Acceleration-induced effects in stimulated light-matter interactions. *Phys. Rev. Lett.*, 2022, *128*, 163603.

[14] Fischer, U. R.; Visser, M. Warped space-time for phonons moving in a perfect nonrelativistic fluid. *EPL* , 2003, *62*, 1, 1.

[15] Lobo, F.; Visser, M. Fundamental limitations on warp drive spacetimes. *Classical and Quantum Gravity*, 2004, *21*, 24, 5871.

[16] Lentz, E. W. Breaking the warp barrier: hyper-fast solitons in Einstein-Maxwell-plasma theory. *Classical and Quantum Gravity*, 2021, *38*, 7, 075015.

[17] Funaro, D. *Electromagnetism and the Structure of Matter*; World Scientific, Singapore, 2008.

[18] Funaro, D. *From Photons to Atoms, The Electromagnetic Nature of Matter*; World Scientific, Singapore, 2019.

[19] Funaro, D. The space-time outside a pulsating charged sphere. *Appl. Sci.*, 2022, *12*, 14, 7290.

[20] Jackson J. D.. *Classical Electrodynamics*, 2nd Edition, John Wiley & Sons, 1975.

[21] Jeffrey, A. *Magnetohydrodynamics*, Oliver & Boyd, Edinburgh, 1966.

[22] Davidson, P. A. *An Introduction to Magnetohydrodynamics*, Oxford Univ. Press, London, 2001.

[23] Molokov, S. S.; Moreau, R.; Moffatt, H. K. (Eds.) *Magnetohydrodynamics: Historical Evolution and Trends*, Springer, 2007.

[24] Barnett, S. M. Resolution of the Abraham-Minkowski dilemma. *Phys. Rev. Lett.*, 2010, *104*, 070401.

[25] Born, M.; Wolf, E. *Principles of Optics*, Pergamon Press, Oxford, 1987.

[26] Fock, V. *The Theory of Space, Time and Gravitation*, Pergamon Press, London, 1959.

[27] Misner, C. W.; Thorne, K. S.; Wheeler, J. A. *Gravitation*, W.H. Freeman & c., San Francisco, 1973.

[28] Brüesch, P. *Phonons: Theory and Experiments II, Experiments and Interpretation of Experimental Results*, SSSOL, v. 65, Springer, Heidelberg, 1986.

[29] Cohen-Tannoudji, C. N. Manipulating atoms with photons, Nobel Lecture. *Rev. Mod. Phys.*, 1998, *70*, 707.

[30] Price, R. H.; Pullin, J. Colliding black holes: The close limit. *Phys. Rev. Lett.*, 1994, *72*, 3297.

[31] Brügmann, B. Binary black hole mergers in 3d numerical relativity. *Int. J. Mod. Phys.*, 1999, *D8*, 85.

[32] Pretorius, F. Evolution of binary black hole spacetimes. *Phys. Rev. Lett.*, 2005, *95*, 121101.

[33] Chinosi, C.; Della Croce, L.; Funaro, D. Rotating electromagnetic waves in toroid shaped regions. *Int. J. Modern Phys. C*, 2010, *21*, 1, 11-32.

[34] Funaro, D. Trapping electromagnetic solitons in cylinders. *Math. Model. Anal.*, 2014, *19*, 1, 44-51.

[35] Funaro, D. High frequency electrical oscillations in cavities. *Math. Model. Anal.*, 2018, *23*, 3, 345-358.

[36] Funaro, D. Electromagnetic waves in annular regions. *Appl. Sci.*, 2020, *10*, 5, 1780.

[37] Bolokhov, S. V.; Bronnikov, K. A.; Skvortsova, M. V. Rotating cylinders with anisotropic fluids in general relativity. *Gravitation and Cosmology*, 2019, *25*, 122-130.

[38] Shariff, K.; Leonard, A. Vortex rings. *Annual Rev. Fluid Mech.*, 1992, *24*, 235.