Research Article

Luigi Fenu, Eleonora Congiu*, Giuseppe Carlo Marano, and Bruno Briseghella

Shell-supported footbridges

https://doi.org/10.1515/cls-2020-0017
Received Sep 23, 2020; accepted Oct 13, 2020

Abstract: Architects and engineers have been always attracted by concrete shell structures due to their high efficiency and plastic shapes. In this paper the possibility to use concrete shells to support footbridges is explored. Starting from Musmeci's fundamental research and work in shell bridge design, the use of numerical form-finding methods is analysed. The form-finding of a shell-supported footbridge shaped following Musmeci's work is first introduced. Coupling Musmeci's and Nervi's experiences, an easy construction method using a stay-in-place ferrocement formwork is proposed. Moreover, the advantage of inserting holes in the shell through topology optimization to remove less exploited concrete has been considered. Curved shell-supported footbridges have been also studied, and the possibility of supporting the deck with the shell top edge, that is along a single curve only, has been investigated. The form-finding of curved shell-supported footbridges has been performed using a Particle-Spring System and Thrust Network Analysis. Finally, the form-finding of curved shell-supported footbridges subjected to both vertical and horizontal forces (*i.e.* earthquake action) has been implemented.

Keywords: concrete shells; footbridges; curved bridges; form-finding methods; topology optimization

***Corresponding Author: Eleonora Congiu:** Department of Civil and Environmental Engineering and Architecture, University of Cagliari, via Marengo 2, 09124 Cagliari, Italy; Email: e.congiu@studenti.unica.it
Luigi Fenu: Department of Civil and Environmental Engineering and Architecture, University of Cagliari, via Marengo 2, 09124 Cagliari, Italy
Giuseppe Carlo Marano: College of Civil Engineering, Fuzhou University, No. 2 Xue Yuan Road, University Town, Fuzhou 350108 – Fujian, China; Department of Structural, Geotechnical and Building Engineering, Technical University of Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy
Bruno Briseghella: College of Civil Engineering, Fuzhou University, No. 2 Xue Yuan Road, University Town, Fuzhou 350108 – Fujian, China

1 Introduction

Concrete shells have been always a special typology for engineers as, if well designed and shaped from the flow of forces, will be mostly subjected to axial compression forces. Following these promises, starting from the first half of the last century, several well-known engineers (*e.g.* Pier Luigi Nervi, Edoardo Torroja, Felix Candela, Heinz Isler, Jörg Schlaich and others) were involved in designing and building concrete shell structures. Nevertheless, this structural typology has not been used so much for bridges. Shell-supported footbridges are made of a shell supporting a deck, with the shell that can be continuous, usually made of concrete, or a grid-shell, for instance made of steel. The Basento Bridge, a concrete viaduct designed by the Italian engineer Sergio Musmeci at the end of the 1960s in Potenza (Italy), can be considered as the first modern shell-supported bridge. Musmeci achieved this goal after a deep study on the design and realization of shells in civil engineering and architecture, and in particular in designing shell-supported bridges. In the same period (1968-71) a concrete shell-supported road bridge designed by Alfred Pauser was built in Erdberger (Austria) [1]. At the beginning of this century, Jiri Strasky proposed two shell-supported footbridges in two international design competitions: the first held in Jersey (UK), and the second held in London for the Leamouth Footbridge [2, 3]. After about forty years from the last realized shell-supported bridges in Potenza and in Erdberger, in 2008 a steel shell-supported footbridge, with a curved concrete deck, designed by Laurent Ney for a design competition, was realized in Knokke-Heist, a resort town at the Belgian coast (*i.e.* the Lichtenlijn bridge [4]). In 2011 Hugo Corres Peiretti designed the Matadero Bridge in Madrid [5], a shell-supported footbridge with the deck suspended to a concrete canopy. More recently, SBP Consulting engineers designed the Trumpf Footbridge in Ditzingen (Germany, 2018), a pedestrian bridge whose deck is supported by a steel grid-shell [6]. Moreover, some studies on shell-supported footbridges have been carried out by Block's Research Group at ETH in Zurich [7] and by the Institute for Structural Engineering of the University of Vienna [8, 9]. The former prepared a prototype of a concrete shell bridge shaped with a pre-stressed hybrid knitted textile and a

Open Access. © 2020 L. Fenu *et al.*, published by De Gruyter. CC BY This work is licensed under the Creative Commons Attribution 4.0 License

bending-active structure that acted as a waste-free, stay-in-place, self-supporting formwork, successively thickened through spraying a thixotropic mortar and by then manual casting of the final concrete layer onto the stiffened fabric formwork. The latter group has proposed an efficient construction method of for concrete shells by pneumatic forming of hardened concrete. The method also allows the construction of concrete shell bridges by inflation. In the present work the studies on shell-supported bridges carried out in the last years by the Authors are introduced and compared with the most recent researches made in this field. The form finding methods usually used for concrete shells are briefly introduced in Section 2. The main steps and goals of the research developed by the Authors on shell-supported footbridges are explained in Section 3. The Musmeci's lesson on designing shell-supported bridges is discussed in Section 4. It has been applied to a case study of straight shell-supported footbridge in Section 5, and, following Nervi's lesson, in this section a shell construction method using a ferrocement stay-in-place formwork has been also proposed. The possibility to reduce tension stresses in shells using holes shaped by topology optimization is exanimated in Section 6. According to the most recent trends in footbridges design, curved shell-supported footbridges are analysed in Section 7 also considering the influence of possible horizontal forces (*e.g.* earthquake loads) on the bridge shape. Finally, the main conclusions are drawn.

2 Short summary of form-finding methods

In general, to shape the shell of shell-supported bridges, form-finding method are used. In this section a list and a brief summary of these methods are reported. Physical models were originally used in the form-finding of membrane structures. The most remarkable results were achieved by Heinz Isler [10], Sergio Musmeci [11, 12] and Frei Otto [13]. Unfortunately, physical models were not sufficiently reliable in designing tension structures, neither easy to be used, as well as very expensive. In the 1960s, the advent of computer inspired the earlier experimental shape finding techniques, and the first form-finding digital methods were implemented. The numerical form-finding methods are commonly grouped into three main families:

- Stiffness Matrix Methods
- Geometric Stiffness Methods
- Dynamic Equilibrium Methods.

Stiffness Matrix Methods are among the oldest numerical form-finding methods. Among these, the Natural Shape Finding (NSF) method, developed in 1974, is to be mentioned [14]. Contrary to the other form-finding digital methods, using the standard elastic and geometric stiffness matrices, they require material properties.

Geometric Stiffness Methods are material-independent since they only take into account geometric stiffness matrices of elements. The most popular among them is the Force Density Method (FDM), that was developed by Scheck in 1974 [15]. It does not need iterations or convergence criterion but just to solve an equation system. Defined the force densities as the ratios of axial forces over bar lengths, FDM obtains the position of the free nodes at the equilibrium state from the definition of a net topology, of the given coordinates of the boundary points and of the force densities. In the linear approach FDM just needs to solve a linear system of equations, thus providing a quick design tool for shaping tension structures and shells. The nonlinear approach is more adherent to the actual membrane behaviour and was introduced to optimize cable-nets under assigned constraints [16]. In 2007, Philippe Block developed a new geometric stiffness method, the Thrust Network Analysis (TNA), a three-dimensional version of Thrust-Line Analysis [17]. TNA is a new methodology based on graphic statics for generating compression-only vaulted surfaces and networks, subject to gravitational loading. The TNA method is based on the assumption (derived from descriptive geometry) that a three-dimensional network under vertical external loads is in compression when its projection on the horizontal plane is also in compression. The form-finding of the optimal shell shape by TNA is performed by the simultaneous manipulation of two diagrams: the form diagram Γ, which is the horizontal projection of the three-dimensional network G, and the force diagram Γ^*, which is constituted by the horizontal components of the forces that act on each bar of the compressed network. The primal grid Γ and dual grid Γ^* are related by a reciprocal relationship: *Corresponding lines in the two diagrams are parallel, and the equilibrium of a node in one of them is guaranteed by a closed polygon in the other and vice versa* (according to Maxwell's definition).

Dynamic Equilibrium Methods find static equilibrium states by iteratively solving dynamic equilibrium problems. The Dynamic Relaxation (DR) method [18] traces, step by step for small time increments, the motion of each interconnected node of an auxiliary grid, under applied loads, until the structure comes to rest in static equilibrium. This method is particularly suited to optimize the shape of grid-shells. The Particle-Spring System (PSS) aims to find structures in static equilibrium by defining the topology of a particle-spring network [19]. The auxiliary network consists of a system of lumped masses (particles) connected by linear elastic springs. The gravitational pull on a mass causes the displacement of the associated particle and subsequently the elongation of the attached springs. This elongation creates a counter force in the springs and stretching continues until the sum of the spring forces matches the downward force of the mass. The motion of the particle is governed by Newton's second law of motion, and the force in the spring by Hooke's law of elasticity. A static equilibrium is found (*i.e.* the dynamic simulation stops) once a balance between internal and external forces is iteratively achieved. Two different kinds of simulations can be performed:

- In a "hanging cloth simulation" the particles are allowed to "fall" under the influence of gravity and the rest lengths of springs along the boundary edges are set to be equal to their original lengths. This simulation is often often used to produce synclastic geometries.
- In a "stretched cloth simulation" gravity is usually turned off or set to a very low value and the rest-lengths of all the springs are very low or set to zero. This type of simulation is usually employed to produce anticlastic geometries.

Regarding applications to bridges, in the last decades several structural optimization techniques have been implemented and adopted to maximize the structural efficiency of arch bridges [20–22], as arches, like shells, owe their bearing capacity to their shape if optimally designed for certain boundary conditions. Among the above form-finding methods, in the following applications the Thrust Network Analysis and Particle Spring Systems are used.

3 Development of the research on shell supported footbridges

This research started at the beginning of 2000, accounting for Musmeci's work, who shaped his shell-supported bridges with equally-compressed shells with minimal surface through coupling physical models -even made of soap bubbles- and integration of the differential equation of isotropic membranes [11, 12]. While at Musmeci's time the only mathematical tool to optimise a shell shape was mathematical analysis, in 2004 evolutionary algorithms were available. Therefore, an optimization procedure was used to follow Musmeci's work in shaping his famous bridge over the Basento River in Potenza, Italy. To this aim, a "simulated annealing" algorithm was used [23]. In the '60s of the past century, when Musmeci designed the Basento Bridge, no reliable numerical methods to design tension structures were available. In fact, the first available form-finding method was the Force Density Method, also known as "the Stuttgart approach", that was defined and successfully used in designing vaults and tension structures only at the beginning of the '70s. Also the use of dynamic relaxation in the membrane form-finding dates about the same years [24], even if the method, that is an application of Newton's 2-nd law of motion and can be used in different fields of mechanics, was developed in the '60s [18]. Therefore, Musmeci developed a peculiar form-finding procedure to shape the shell supporting the deck of the Basento bridge as a compressed membrane. Using soap bubbles and integration of the differential equation of an isotropic membrane, Musmeci found an isotropic shell from which to obtain the actual form of the shell supporting the deck of the Basento Bridge. Since in a bridge the longitudinal direction is highly prevalent with respect to the transverse one, Musmeci reduced the transverse curvatures of the isotropic shell to find the final form of the anisotropic shell supporting the deck.

Following Musmeci's method, some other shell-supported footbridges were designed through imposing suitable boundary conditions to shape an isotropic membrane [25]. The form of the isotropic membrane was found using a "simulated annealing" algorithm to minimize the anticlastic surface, and then, the transverse curvatures were reduced with respect to the longitudinal ones to obtain the final form of the shell supporting the bridge deck. The shell was shown to be equally compressed, as expected by its anticlastic shape. Equally-compressed shells are particularly suitable for concrete shells, because crack opening is counteracted by uniform compressions throughout the shell. In 2006 it was also studied how to realize this type of footbridges following not only Musmeci's but also Nervi's lesson. In fact, Pier Luigi Nervi built his amazing concrete vaults using ferrocement, a cementitious composite made of steel nets and mortar matrix [25]. It was hence proposed to shape the nets in tension by imposing suitable boundary conditions to obtain the anticlastic form of the shell supporting the bridge and to then transform the nets in tension in a ferrocement membrane through filling the space between the net wires, thus obtaining a ferrocement formwork that can be thickened with thyssotropic micro-concrete to finally obtain the concrete shell. In recent years there has been a great development in the study and realization of cementitious composites, with improved mortars (highly ductile and

Table 1: Research steps on shell-supported footbridges with anticlastic equally-compressed shell

	Research Steps	Goals	Method/Algorithm
1	Basento's Bridge (first modern shell supported bridge)	Investigation of Basento's bridge using numerical form-finding methods	Musmeci's method coupled to an evolutionary algorithm
2	Straight shell-supported footbridges	Shaping straight shell-supported footbridges according to Form-finding methods	Equally-compressed anticlastic shell following Musmeci's method coupled to an evolutionary algorithm
3	Concrete Shell Construction Methods	Develop easy and economical construction techniques for concrete shells	Thickening with spaying concrete a stay-in-place ferrocement formwork following Nervi's lesson
4	Shell-supported footbridges with optimized shell topology	Minimize tensile stresses in concrete using holes in the shells	Topology optimization
5	Curved shell-supported footbridges	Shaping curved shell-supported footbridges according to Form-finding methods	Particle Spring System and Thrust Network Analysis
6	Curved shell-supported footbridges subjected to both vertical and horizontal forces	Seismic form-finding of curved shell-footbridges shape-resistant to earthquake forces	Particle-Spring method

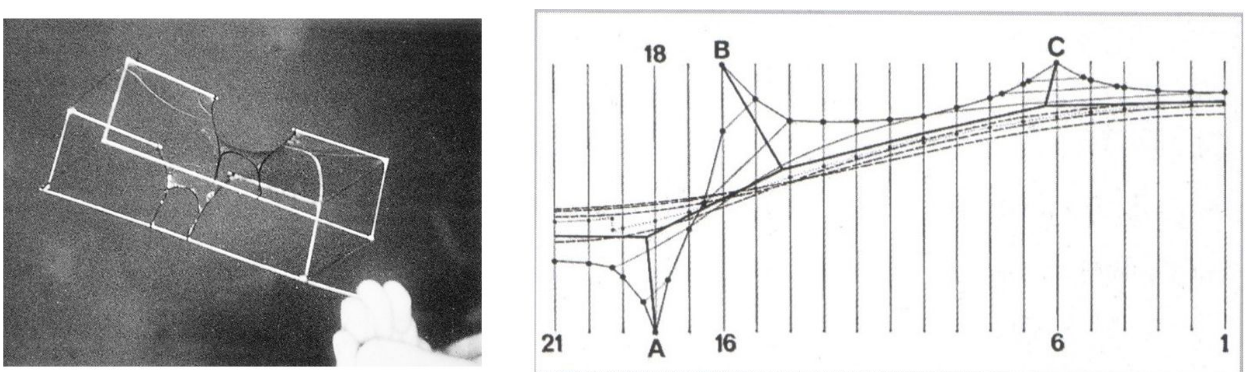
with high tensile strength) and different types of fibres, like basalt and carbon fibres. Therefore, this opens new perspectives in the construction of concrete shells [26, 27]. Although topology optimization is still rarely applied in civil engineering and in particular in shell construction, the research was successively extended to applying topology optimization for designing shell-supported bridges. This allowed to minimize the shell regions where second order moments arise and better understand the role of possible holes in eliminating tension stresses. Following not only Musmeci's and Nervi's lesson, but also Jorg Schlaich's work on curved bridges [28], the extension of shell-supported bridges to curved footbridges was then studied. Facing the problem of designing curved shell-supported footbridges, the philosophy of only considering equally-compressed shells was still followed. The problem of defining suitable boundary conditions to allow curvatures to be anticlastic of opposite sign in any shell region was to be solved for a shell supporting a curved deck. Although the problem is more difficult to solve than in straight shell-supported bridges, the fact that the deck can be considered as a ring girder helped to find a suitable shell form to solve such a design problem. To this aim, two form-finding methods were used, the Particle Spring method and Thrust Network Analysis, the former developed by Kilian and Ochsendorf in 2005 [19] and the latter by Block and Rippmann [17, 29]. Carrying on in following Schlaich's work, a shell-supported footbridge with S-curve and counter-curve along the deck was also studied [28]. The research on shell-supported bridges was finally turned to the form-finding of shells supporting a curved deck that allow the bridge to better resist horizontal forces. Seismic form-finding of shell-supported footbridges was thus studied, and reference to previous work on seismic resistant forms of shells was made [30–33]. The above described research steps are shortly listed in Table 1.

4 Musmeci's lesson on designing shell-supported bridges

Sergio Musmeci was an Italian structural designer, as well as a professor of the Faculty of Architecture of Rome University "La Sapienza". He cultivated interests and skills in different fields of culture, as mathematics, philosophy, astronomy, architecture and music, loving jazz and playing it as a pianist. Besides in Civil Engineering, he also graduated in Aeronautical Engineering, and applied the aerodynamics principles shaping a famous helicoidal skyscraper designed with his close friend and well-known architect Manfredi Nicoletti [11]. He started his career as structural engineer first in Riccardo Morandi's and then in Pier Luigi Nervi's design offices. For his entire professional life, he had fruitful collaborations with some of the most famous architects in Italy, as Carlo Mollino, Ludovico Quaroni and

a) b)

Figure 1: Basento Bridge in Potenza, Italy (Photos: Carlo Atzeni)



a) b)

Figure 2: Form-finding of the Basento Bridge, (a) using soap bubbles; (b) integrating the membrane equation by finite differences (b)

Bruno Zevi [11]. Similarly to Frei Otto's research but independently by him, an important part of his work was addressed in the study of shells with minimal surface. Unfortunately, some years after the inauguration of his masterpiece, the shell-supported bridge over the Basento River in Potenza, Italy (Figure 1), he prematurely died in 1981.

The Basento bridge is one of the very few shell-supported bridges realized in the world [11, 12], and it is by far the most important one by both a structural and an architectural point of view. In designing Basento bridge, the pioneering Musmeci's idea was to design a uniformly compressed concrete shell with minimal surface. His aim was to obtain a stiffer structure and reduce the volume of resistant material by better exploiting concrete characteristics, because concrete better resists compressions that in turn are able to counteract the opening of cracks unavoidably formed during concrete hardening. Nevertheless, his main aim was not to save concrete (that, incidentally, is advantageous, too), but to obtain a structure that was also an architectural object with aesthetical value. Similarly to what happens in the organic forms of living beings, including shells and skeletal structures of animals, this was in accordance with his quality statements, for which structural efficiency and resistant material saving are quality indexes of the intrinsic beauty of the form of his optimised structures.

The conceptual design of the bridge was firstly set up by Musmeci using a soap film model that, by suitably choosing the boundary conditions, allowed him to achieve qualitatively the bridge shape (Figure 2a). The soap film was a physical model in small scale of an isotropic surface in tension, that he also achieved with real longitudinal and elevation coordinates by integrating by finite differences the differential equation of isotropic membranes (Figure 2b). By then reducing the transverse dimensions of this isotropic membrane, he numerically achieved the actual coordinates of the points of the average surface of the shell that would have supported the bridge deck. This design procedure can be simulated with modern evolution-

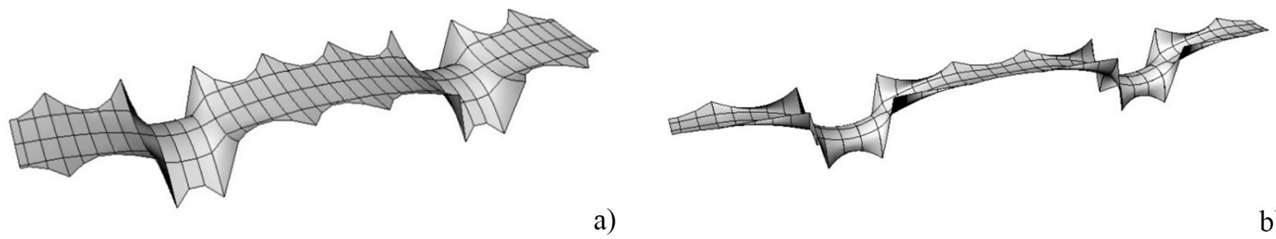


Figure 3: Form-finding of the Basento Bridge using simulated annealing: (a) with isotropic curvatures; (b) after reduction of transverse curvatures

ary algorithms. Marmo *et al.* [34] provided a comprehensive study on the form of the double-curved RC shell that bears the Basento bridge designed by Musmeci by adopting the actual geometry of the structure as a reference for experiencing a new iterative procedure based on the Force Density Method (FDM) and then compare the structural efficiency of the computed geometry with the designed and surveyed ones. The Authors, given the boundary conditions used by Musmeci to integrate the membrane equation, have obtained the shape of the shell of the Basento bridge by first calculating the isotropic membrane (Figure 3a), as that originally achieved by Musmeci, through minimising the distance between the coordinates of the nodes of a cable net structure and the points of a horizontal surface using a "simulated annealing" algorithm [23]. By then reducing the transverse coordinates to their actual values, the anisotropic curvatures of the shell of the Basento bridge were finally obtained (Figure 3b).

5 Designing shell-supported footbridges following Musmeci's work

The same form-finding procedure implemented to investigate Musmeci's work was then used to design a shell-supported footbridge to cross a deep canyon in

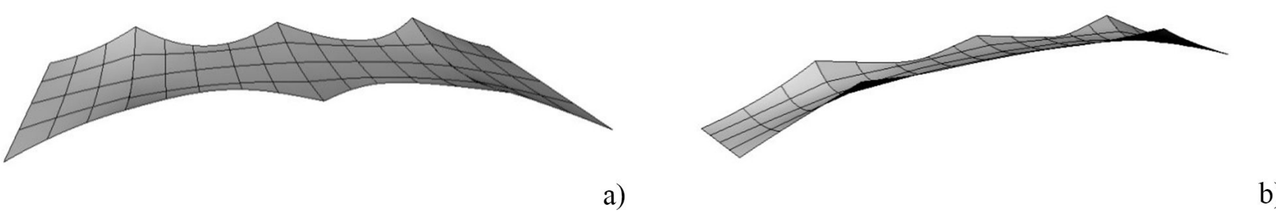


Figure 4: Form-finding of the shell-supported footbridge using simulated annealing: (a) with isotropic curvatures; (b) after reduction of transverse curvatures

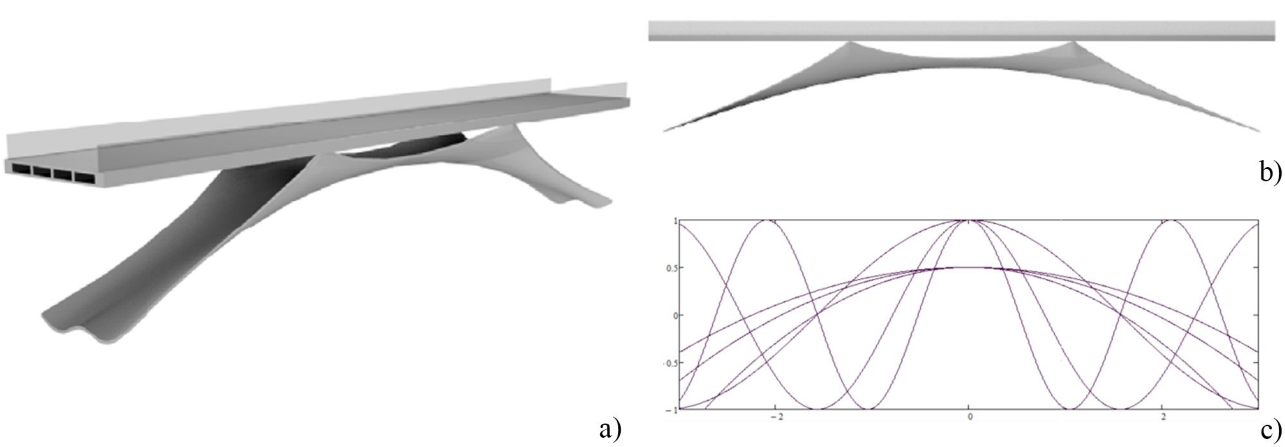


Figure 5: Shell-supported footbridge: (a) Perspective view; (b) lateral view; (c) parametric variation of the shell edge at the abutments to choose its optimum shape

Figure 6: Photomontage of the shell-supported footbridge over Tuvixeddu canyon in Cagliari, Italy

Cagliari [25], an Italian city in the South of Sardinia, the second biggest island of the Mediterranean Sea. This site was also adopted in the following researches developed by the same group to design innovative footbridges and compare the design results.

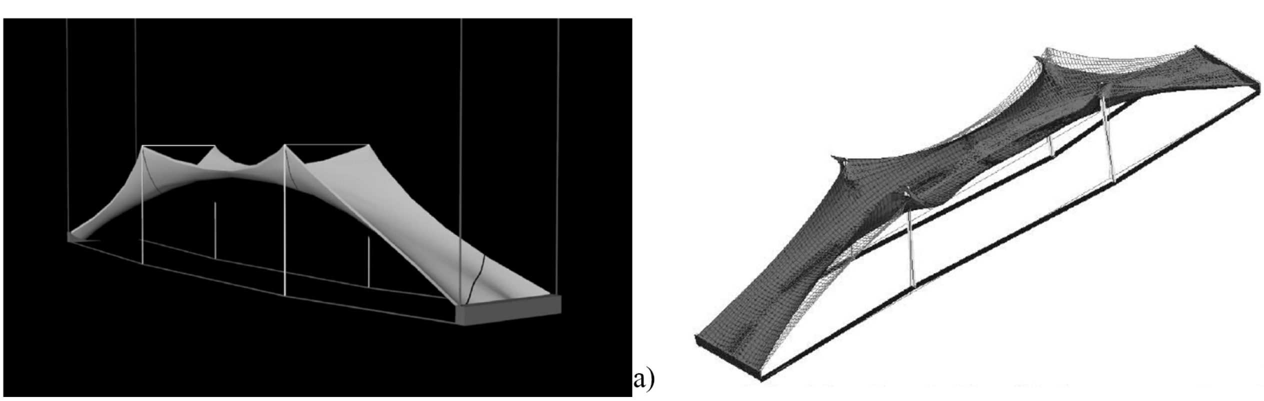
Following Musmeci, the shell-supported footbridge was initially shaped as a soap bubble by starting from an isotropic membrane obtained using "simulated annealing" to minimize its surface (Figure 4a), and by then reducing its transverse dimensions to meet the required width of the deck (Figure 4b). The bridge span was 40 m. Shell thickness was 15 cm. The deck was supported by the shell at six points (three by side), meaning that a four-span continuous deck was adopted.

A three-span shell supported footbridge with same span was also shaped (Figure 5a, 5b). The anticlastic shell was hinged at the abutments. Since a linear abutment leads to lose the opposite signs of the orthogonal curvatures in the longitudinal and transverse directions, a curved abutment was adopted to obtain a shell where unwished second order moments are minimised (Figure 5a).

The best shape of the curved abutment was chosen by making vary parabolic and sinusoidal curves defined parametrically (Figure 5c) [25]. A photomontage of the footbridge over Cagliari canyon is shown in Figure 6.

5.1 Construction method following Nervi's lesson

Although the construction of shell structures is still quite expensive and labour-consuming, nowadays a lot of research is addressed to find construction methods that simplify shell construction. When designing this footbridge following Musmeci's work in 2005, new construction methods for shells were not yet proposed, and therefore how to easier construct shell-supported bridges was a problem to solve. A construction method was then set up to realize shells with minimal surface following both Musmeci's and Nervi's lesson. Following Musmeci's experience in realizing his physical models, it was proposed to first shape a closely spaced net in tension made of thin cables and wires, and by then filling it with mortar. The cable net can be put in tension on yard, for instance, using masts and cable stays. A ferrocement thin shell (5 cm) is so obtained (Figure 7a), that is made of a cementitious composite following Nervi's lesson [25]. Of course, todays the net can be made not only of steel wires but also of other fibres, *i.e.* basalt e/or carbon fibres [7, 35, 36]. Such a thin shell is however sufficiently light to be easily set-in-place by modern cranes (Figure 7a) for a span length even quite higher than that of the footbridge under consideration. After setting-in-place the ferrocement thin shell, it can be used as the formwork to obtain the final concrete shell through placing a reinforcement grid over the ferrocement thin shell and by then thickening it through spraying a thixotropic micro-concrete.



a) b)

Figure 7: Stay-in-place, self-supporting formwork made of ferrocement. (a) setting-in-place by crane; (b) checking its buckling resistance

Of course, when setting-in-place the ferrocement thin shell, it could buckle, owing to its small thickness. A buckling analysis was then performed to check the ferrocement thin shell against buckling when lifted by crane (Figure 7b) [25]. Buckling analysis showed that the anticlastic shape of the ferrocement thin shell improves buckling resistance, thus allowing to reduce its thickness.

6 Topology optimization of the shell supporting the bridge deck

The insertion of holes in concrete shells can be favourable to their structural behaviour, because holes can highly lighten the shell, thus reducing internal forces caused by dead loads. It is well known how Romans could construct the big Pantheon vault in Rome through lightening it both using pumice aggregates but also inserting a big hole in the centre of the vault.

Therefore, inserting holes in a shell supporting the bridge deck can improve the behaviour of the bridge, and in particular of the shell. To suitably insert holes in a shell, topology optimization can be used. Two main methods have been proposed: the Evolutionary Structural Optimization (ESO) method, first introduced by Xie and Steven in 1992 [37] and successively updated as Bidirectional Evolutionary Structural Optimization (BESO) method at the end of the '90s [38], and the Solid Isotropic Material with Penalization (SIMP) method, introduced by Bendsoe and Kikuchi (1988) [39] and Rozvany and Zhou (1992) [40] that gave it the classical mathematical setting of structural optimization problems. For topology optimization of the shell supporting the bridge deck, the SIMP method was adopted for the above-mentioned mathematical reasons. It has been shown that suitably inserting holes in the shell reduces the shell regions where tensile stresses arise. A perforated shell where stresses were channelled between holes was obtained (Figure 8) [41, 42]. Moreover, some optimization indexes that allow to identify the best hole pattern were also defined [43].

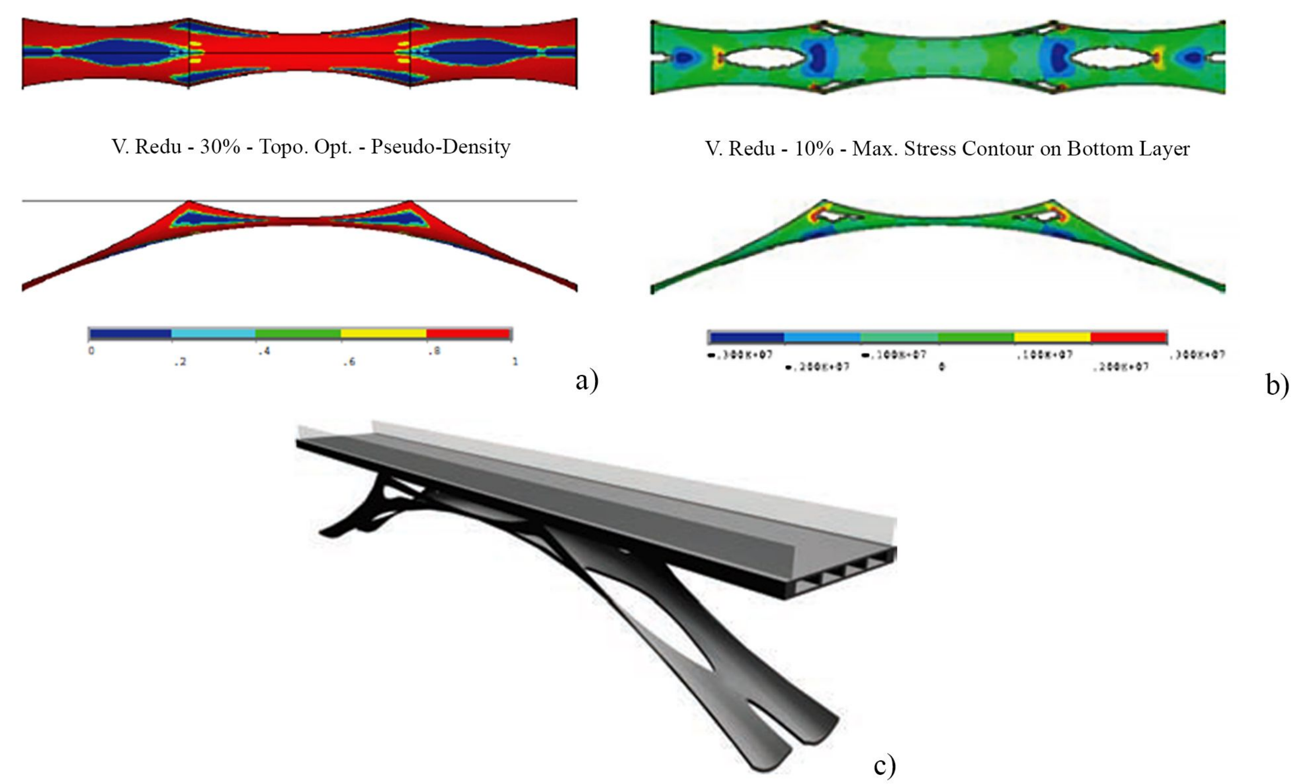


Figure 8: Topology optimization of the shell-supporting footbridge through the SIMP Method: (a) distribution of the pseudo-densities; (b) hole insertion; (c) rendering of the shell-supported footbridge after hole insertion

7 Curved shell-supported bridges

While Musmeci's Basento Bridge is the first of the few shell bridges built until now, after the first curved bridge designed by Maillart, the Schwandbach Bridge near Hinterfultigen in Switzerland (1933) [44], many other curved bridges (especially cable-stayed and suspended) have been built, in particular in the last thirty years, thanks to Jorg Schlaich's valuable work [28].

Therefore, the design problem that was asked to solve was: taking into account the form and the structural functioning of curved bridges and of shell-supported bridges, which is the form that a curved shell-supported bridge should have to better transfer deck loads to earth? To answer this question it is worth noting that while a straight deck supported along a single line by a straight girder is always unstable (Figure 9a), a curved deck supported by a curved girder along a curved line can be stable, thanks to the peculiar properties of ring girders [28, 45–48].

Therefore, while Musmeci needed to support the straight deck of Basento Bridge using point supports symmetrically aligned along two parallel lines (Figure 9b), Schlaich designed his suspended or cable-stayed curved bridges by hanging either the outer or the inner deck edge, that is hanging the deck along one curved line only. Therefore, to shape a curved shell-supported footbridge, Musmeci's and Schlaich's lessons had to meet each other's. This means that the curved deck (that can be considered as either a ring girder or made of a ring girder and transverse cantilevers, as in the solution herein adopted) could be supported by the concrete structure (the shell under consideration or, in general, even another ring girder, see Figure 10) along one curved line only, thus exploiting the peculiar properties of ring girders. A box girder with high torsional stiffness is usually used to absorb the torsional moments induced by the eccentric loads acting on the cantilever deck. In the recent years, an external pretensioning system applied to the upper flange of the girder has been used in some footbridges to equilibrate the torsional mo-

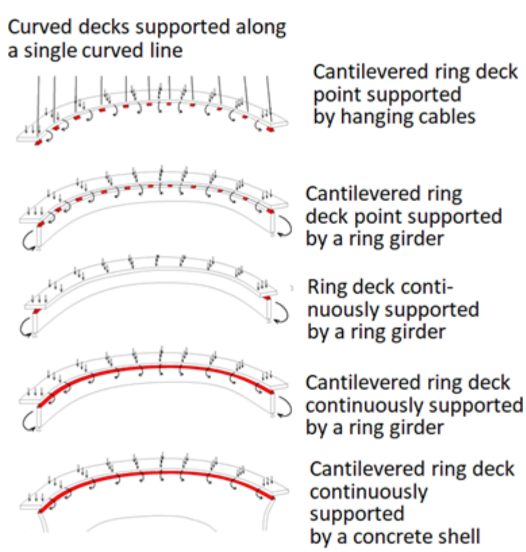


Figure 10: Curved cantilevered deck supported along a single curved line

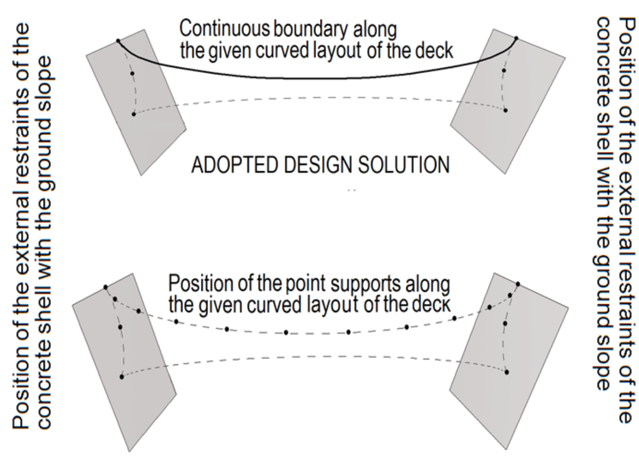


Figure 11: Assignment of the boundary conditions for the shell of a curved shell-supported bridge: (a) with the deck continuously supported by the shell top edge; (b) with the deck point-supported along the shell top edge

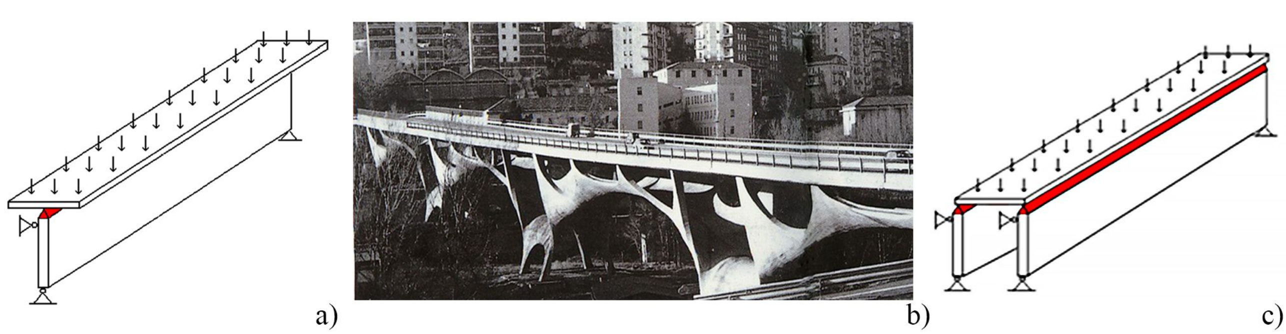


Figure 9: Unstable straight deck (a); Stable straight deck (b) and (c)

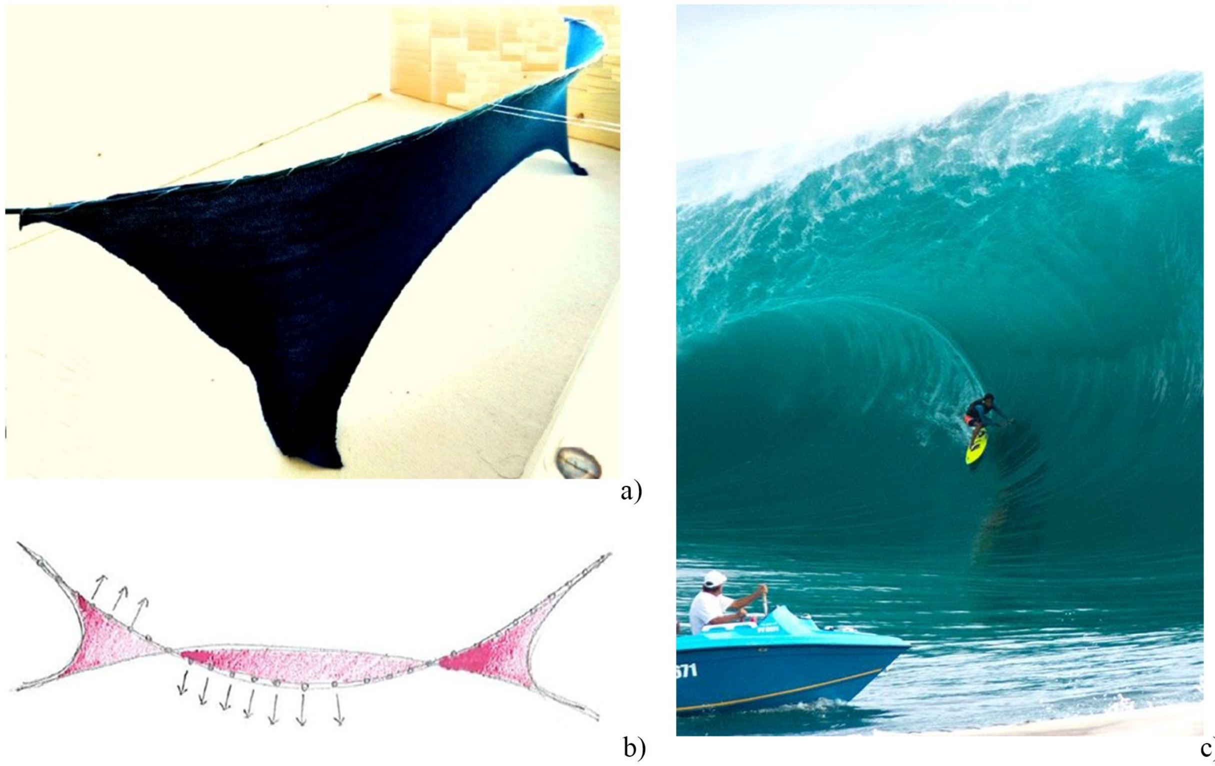


Figure 12: Initial form-finding of the curved shell-supported footbridge: (a) using a fabric physical model; (b) with a hand-made sketch; (c) referring to the shape of a sea wave

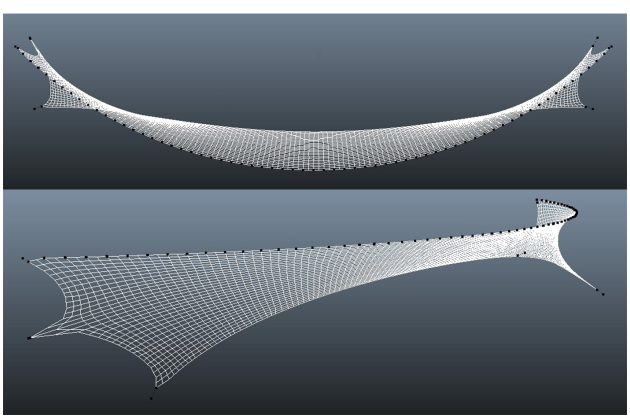


Figure 13: Form-finding of the shell of the curved shell-supported footbridge using a Particle Spring System

ments, as also done in the case study presented in this paper [28, 45–48].

Conversely, in the form-finding of the concrete shell one can impose a boundary condition for which the deck must be supported along the upper edge of the concrete shell only, that is along a curved line defined by the form of the curved deck (Figure 11). For instance, if the shell has to support the deck along one edge, the position of the deck supports is a boundary condition for the shell upper edge (Figure 11). Moreover, the fact that the curved deck defines a boundary condition for the shell upper edge means that the curvature of the shell meridians along the upper edge must be of the opposite sign with respect to the curvature of the upper edge. All these considerations favoured the conceptual design of the curved shell-supported footbridge, by also favouring the set-up of a simple physical model made of elastic nylon fabric whose form recalls that of a sea wave (Figure 12).

Some other boundary conditions are given along the abutments. For instance, at the abutments the shell can be restrained either at some points with concrete hinges [49] (three by side in the example under consideration) or along one curve, provided that the given boundary conditions allow the shell surface to be anticlastic. Contrary to the upper edge, the shell lower edge is not given, and the curve describing it is free to be found by the form-finding procedure [47, 48, 50]. For given boundary conditions, different form-finding methods can be used.

The Particle-Spring Method has shown to be suited for the form-finding of the shell supporting the curved deck of the bridge (see Figure 13). In a Particle-Spring System, the spring stiffness is in some way related to the material elastic properties. It is important to suitably choose the spring

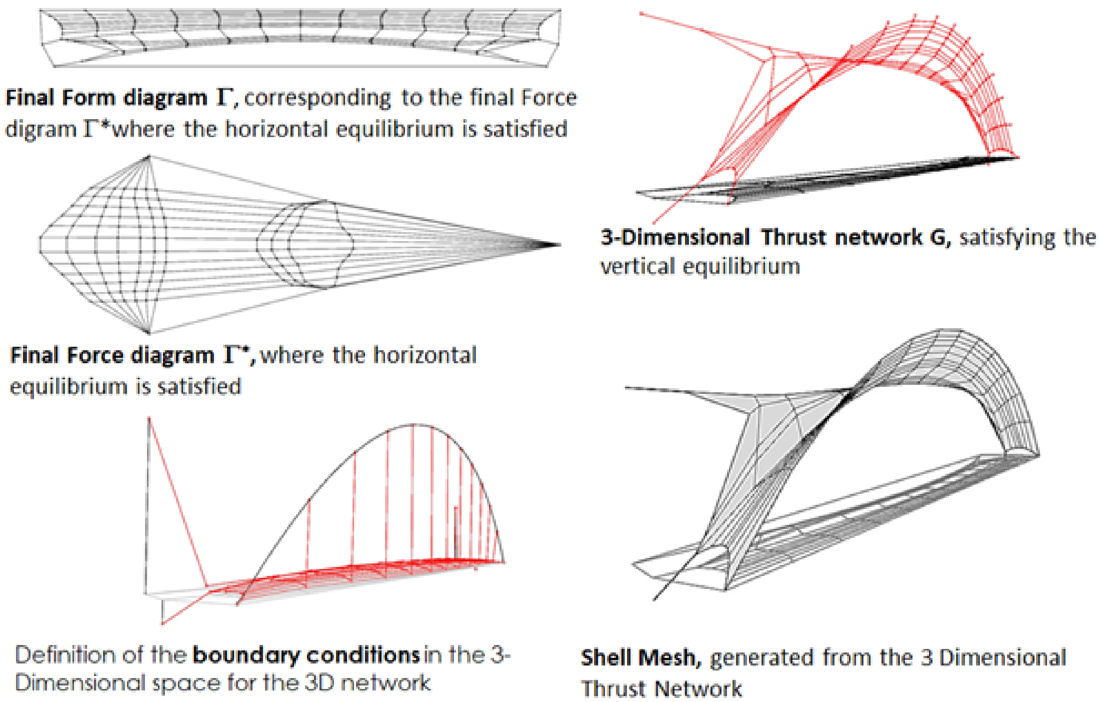


Figure 14: Shell form-finding of the curved shell-supported footbridge using Thrust Network Analysis

Figure 15: Photomontage of the curved shell-supported footbridge over Tuvixeddu canyon in Cagliari, Italy

stiffness because, besides the boundary conditions, they can affect the shell shape.

TNA was also used for the form-finding of the curved shell-supported footbridge. The original Block's method for TNA is not suited for such a problem, because the projection of different shell regions in the horizontal plane overlap, thus making impossible the construction of the form and force diagrams in accordance with the Block's method. This drawback could be overcome using the TNA version proposed by Marmo and Rosati [51, 52]. Moreover, it can be also overcome by using Block's method in a non-standard way as done in this research. In fact, it can be noted that, contrary to the vertical direction, the projections of different shell regions never overlap in the horizontal direction.

Therefore, the traditional TNA procedure can be carried out by constructing the form diagram in the horizontal direction (Figure 14). In doing so, vertical forces due to the shell dead loads are neglected, while the parallelism condition between form and force diagrams guarantees the equilibrium in the vertical direction for any magnitude of the vertical components of the shell internal forces.

The form-finding is then suitably completed through moving the net in the horizontal direction, that corresponds to have different horizontal components of the shell internal forces. It is worth noting that when using a Particle Spring System to shape anticlastic shells, dead loads are also neglected. The form-finding result is then checked both in terms of principal curvatures, that must be of the opposite sign throughout the shell, and in terms of tensile stresses, that are to be evaluated by finite elements, thus checking the actual effect of the vertical forces loading the shell. This means that on the one hand dead loads are neglected when shaping the shell, but their effect is in any case accounted for by checking the shell by finite elements; on the other hand, for their peculiar properties, in anticlastic shells any force increase in one direction results in a force increase also in the other directions. This means that while the form-finding of synclastic shells is highly affected by dead loads, they much less affect the shape of anticlastic shells.

Figure 15 shows a photomontage of the curved shell-supported bridge over the canyon in Cagliari.

7.1 Shell-supported footbridges with S-curve and counter-curve of the deck

Following Schlaich's work on curved arch bridges, the case of shell-supported footbridges with S-curve and counter-curve of the deck was also studied. This case is of particular importance because the designer wonders which is the form that the lower free edge of the shell will assume when the upper edge is constrained by a deck with S-curve and

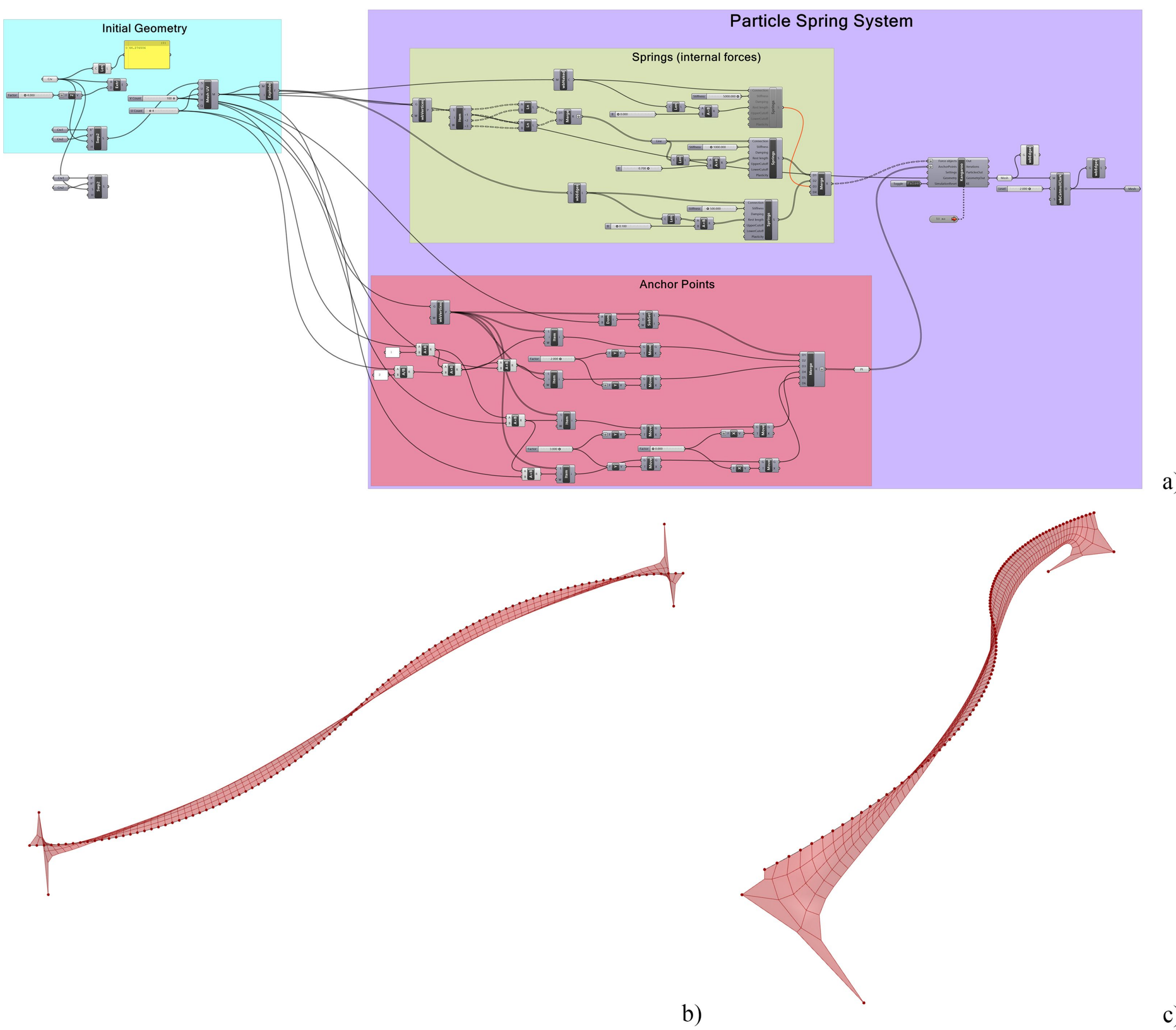


Figure 16: Form-finding by Particle-Spring System (PSS) of a S-curved curve and counter-curve shell: (a) Grasshopper/Kangaroo graphical form-finding algorithm; (b) top view of the optimal particle-spring network; (c) perspective view of the optimal particle-spring network

counter-curve. The form-finding was performed using the Particle-Spring method. Besides three given points by side at the abutments, the boundary condition for the shell upper edge was also imposed by constraining it to the deck. A peculiar shell shape was found (Figure 16). In particular, it can be noted that the lower free edge assumes the form of a three-dimensional twisted arch, similar to the form proposed by Schlaich for the arch of his arch bridge with S-curved and counter-curved deck.

7.2 Seismic form-finding of shell-supported footbridges

It has been already pointed out that the form of the anticlastic shell supporting the curved deck is suited to well bear both vertical and horizontal loads. Shaping the shell using non-standard TNA, it has been also shown that the anticlastic shell can well resist horizontal forces, the better the higher is the transverse vertical projection of the shell, that is the higher the nodes are moved from the vertical plane where the vertical equilibrium is imposed. The seismic form-finding of the shell-supported footbridge has been performed using the Particle-Spring Method. Since to increase the height of the transverse projection of the shell is favourable to seismic resistance, then a further design variable was added, that is the shape of the curved deck. Of course, in general any shape is not suitable for the curved deck, but a family of suitable curves can be defined among which to choose the most appropriate one for the deck. The chosen curve of the deck in turn becomes a boundary condition for the shell. Of course, this boundary condition affects the height of the transverse projection of the shell, as well as its resistance to horizontal forces. Therefore, similarly to the previous curved shell-supported footbridges, a

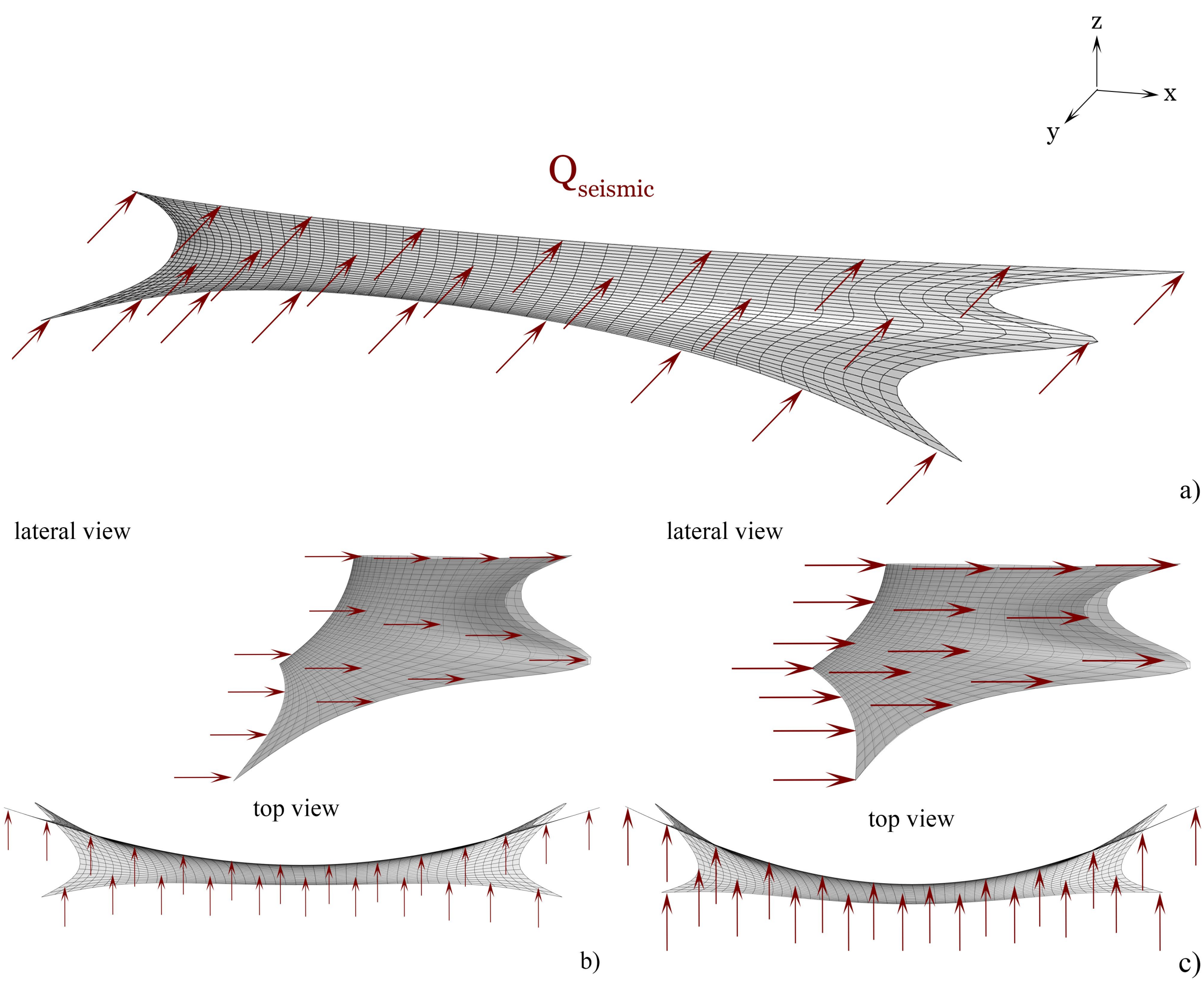


Figure 17: Shell form-finding of a curved shell-supported footbridge subjected to the seismic action: Perspective view of a seismic action acting orthogonally to the shell surface (a); lateral and top views on a shell form-found under a small seismic action (b); lateral and top views on a shell form-found under a greater seismic action (c)

Particle-Spring System has been defined but with a variable boundary condition for the top edge, whose form varies with the variation of the deck curve. For increasing seismic loads, higher curvature of the deck curve is needed to better resist the seismic loads, the higher the better (see Figure 17). The resulting shell shape is highly affected by this varying boundary condition.

8 Discussion on research findings and concluding remarks

The object of the paper is to investigate the research development on shell-supported bridges. The issue of designing shell-supported bridges with shells compressed in any direction has been faced. Moreover, a design methodology has been proposed for both straight and curved shell bridges. Finally, the problem has been extended to the case in which the form-finding is affected not only by vertical loads but also by a horizontal seismic action. This paper starts from studying and re-interpreting Musmeci's work following his peculiar way of designing shell-supported bridges. By following Musmeci's work in designing shell-supported footbridges, a way for realizing them meeting Musmeci's and Nervi's work was also proposed. The way of constructing the shell by first realizing a wire net in tension successively filled with mortar to obtain a light ferrocement formwork finally thickened to obtain the concrete shell is an important finding that has been successively proposed by also some other authors [7, 25, 35].

By instead designing a curved shell-supported footbridge, the main finding was in defining the conceptual design of the bridge, that is the intuition and definition of the shell form on the basis of the required boundary conditions. For this goal, the use of modern form-finding algorithms was necessary. Among them, the Particle Spring method and the Thrust Network Analysis (TNA) were used. The former appears to be more suitable for this aim, due to the advantage that spring stiffness can be related to shell material properties. TNA is instead independent of them, and in Block's formulation seems not suitable to be used for the form-finding of the curved shell supported footbridge due to the overlapping of the projections in the horizontal plane of different shell regions. In this research, the previous restriction was overcome with a non-standard use of Block's algorithm, as described in Section 5. Satisfying results were obtained in terms of both curvatures and minimization of unwished tensile stresses, thus allowing to extend the use of Block's TNA algorithm to anticlastic shells for which the projections in the horizontal plane of some shell regions overlap. Achieved the goal of defining the way to design curved shell footbridges with equally-compressed shell, the form-finding of curved shell-supported footbridges was extended to the case of curved shell footbridges subjected to the earthquake action. Since nowadays the seismic form-finding is an issue in form-finding research, and the anticlastic shape of the shell surface is favourable for bearing also horizontal forces, including the inertial forces due to the seismic action, this property of the anticlastic shell supporting the deck has naturally leaded to search for the best shell shape to resist seismic forces. Since the shell-shape is highly affected by the form of the curved deck, it is assumed as a design variable, of course subjected to give acceptable deck forms for pedestrian and bicycle traffic. The Particle-Spring method has been hence extended to shape the shell supporting the deck taking into account both vertical and horizontal forces (*i.e.* earthquake loads). Future research is needed to easier allow the realization of shell-supported footbridges. For this aim, the study of automatic construction methods as well as the use of robots in building appear to be a perspective development in this field.

Conflict of Interests: The authors declare no conflict of interest regarding the publication of this paper.

References

[1] Geier R., Monitoring of the Vienna Erdberger Bridge, In: Proceedings of IABSE Symp. - Large Struct. Infrastructures Environ. Constrained Urban. Areas, International Association for Bridge and Structural Engineering (IABSE), (22-24 September 2010, Venice, Italy), 389-90, doi:10.2749/222137810796025050.

[2] Strasky J., Bridges Utilizing High-strength concrete, In: Proceedings of 30th Conf. Slov. Struct. Eng., (2008, Bled, Slovenia), 15-34.

[3] Strasky J., Pedestrian bridges utilizing high strength concrete, Int. J. Sp. Struct., 2007, 22, 61-79, doi:10.1260/026635107781037275.

[4] Adriaenssens S., How and why Laurent Ney finds steel structural forms, J. Int. Assoc. Shell Spat. Struct., 2020, 61, 39-49, doi:10.20898/j.iass.2020.203.017.

[5] Corres-Peiretti H., Dieste S., León J., Pérez A., Sánchez J., Sanz C., New Materials and Construction Techniques in Bridge and Building Design, Innov. Mater. Tech. Concr. Constr., Dordrecht: Springer Netherlands, 2012, 17-41, doi:10.1007/978-94-007-1997-2_2.

[6] Kappelt H., Efficient Structure: Footbridge in Ditzingen, Structure, 2018, 4, 4-5.

[7] Popescu M., Reiter L., Liew A., Van Mele T., Flatt R.J., Block P., Building in Concrete with an Ultra-lightweight Knitted Stay-in-place Formwork: Prototype of a Concrete Shell Bridge, Structures 2018, 14, 322-32, doi:10.1016/j.istruc.2018.03.001.

[8] Kromoser B., Pachner T., Tang C., Kollegger J., Pottmann H., Form Finding of Shell Bridges Using the Pneumatic Forming of Hardened Concrete Construction Principle, Adv. Civ. Eng., 2018, 2018, 1-14, doi:10.1155/2018/6309460.

[9] Kromoser B., Kollegger J., Efficient construction of concrete shells by Pneumatic Forming of Hardened Concrete: Construction of a concrete shell bridge in Austria by inflation, Struct. Concr., 2020, 21, 4-14, doi:10.1002/suco.201900169.

[10] Isler H., Concrete Shells Derived from Experimental Shapes, Struct. Eng. Int., 1994, 4, 142-7. doi:10.2749/10168669478060 1935.

[11] Nicoletti M., Sergio Musmeci: organicità di forme e forze nello spazio, Testo & Immagine, Torino, 1999.

[12] Musmeci S., La statica e le strutture, Cremonese, Roma, 1971.

[13] Boller G., Schwartz J., Modelling the form. Heinz Isler, Frei Otto and their approaches to form-finding, In: Proceedings of Seventh Conf. Constr. Hist. Soc., (3-5 April 2020, Cambridge, UK), 565-76.

[14] Adriaenssens S., Block P., Veenendaal D., Williams C., Shell Structures for Architecture, Taylor & Francis - Routledge, New York, 2014, doi:10.4324/9781315849270.

[15] Schek H.-J., The force density method for form finding and computation of general networks, Comput. Methods Appl. Mech. Eng., 1974, 3, 115-34, doi:10.1016/0045-7825(74)90045-0.

[16] Aboul-Nasr G, Mourad SA. An extended force density method for form finding of constrained cable nets. Case Stud Struct Eng 2015;3:19-32. doi:10.1016/j.csse.2015.02.001.

[17] Block P., Thrust Network Analysis: exploring three-dimensional equilibrium, PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 2009.

[18] Day A.S., An Introduction to Dynamic Relaxation, Eng., 1965, 219, 218-21.

[19] Kilian A., Ochsendorf J., Particle-Spring Systems for structural form finding, J. Int. Assoc. Shell Spat. Struct., 2005, 46, 77-84.

[20] Trentadue F., Marano G.C., Vanzi I., Briseghella B., Optimal arches shape for single-point-supported deck bridges, Acta Mech., 2018, 229, 2291-7, doi:10.1007/s00707-017-2084-0.

[21] Fenu L., Marano G.C., Congiu E., Briseghella B., Optimum design of an arched truss under vertical and horizontal multi-load cases, In: Proceedings of IASS Annu. Symp. 2019 - Struct. Membr. 2019 Form Force, (7-10 October 2019, Barcelona, Spain), International Association for Shell and Spatial Structures (IASS), 2019, 1-8.

[22] Zordan T., Briseghella B., Mazzarolo E., Bridge structural optimization through step-by-step evolutionary process, Struct. Eng. Int., J. Int. Assoc. Bridg. Struct. Eng., 2010, 20, 72-8, doi: 10.2749/101686610791555586.

[23] Fenu L., Madama G., A method of shaping R/C shells with heuristic algorithms and with reference to Musmeci's work, Stud. e Ric. - Politec. Di Milano Sc. Di Spec. Costr. Cem. Armato, 2004, 199-238.

[24] Barnes M.R., Form finding and analysis of tension space structures by dynamic relaxation, PhD thesis, City University London, 1977.

[25] Fenu L., Madama G., Tattoni S., On the conceptual design of R/C footbridges with the deck supported by shells of minimal surface, Stud. e Ric. - Politec. Di Milano Sc. Di Spec. Costr. Cem. Armato, 2006, 26, 103-26.

[26] Fenu L., Forni D., Cadoni E., Dynamic behaviour of cement mortars reinforced with glass and basalt fibres, Compos, Part B Eng., 2016, 92, 142-50, doi:10.1016/j.compositesb.2016.02.035.

[27] Kruszka L., Moćko W., Fenu L., Cadoni E., Comparative experimental study of dynamic compressive strength of mortar with glass and basalt fibres, EPJ Web Conf., EDP Sciences, 94, 2015, doi:10.1051/epjconf/20159405008.

[28] Schlaich J., Bergermann R., Leicht weit/Light Structures, Prestel, Munchen, 2004.

[29] Rippmann M., Lachauer L., Block P., Interactive vault design, Int. J. Sp. Struct., 2012, 27, 219-30, doi:10.1260/0266-3511.27.4.219.

[30] Michiels T., Adriaenssens S., Dejong M., Form finding of corrugated shell structures for seismic design and validation using non-linear pushover analysis, Eng. Struct., 2019, 181, 362-73, doi:10.1016/j.engstruct.2018.12.043.

[31] Michiels T., Form finding of arches and shell structures subjected to seismic loading, PhD thesis, Princeton University, Princeton, NJ, USA, 2018, doi:10.13140/RG.2.2.18163.22562.

[32] Michiels T., Adriaenssens S., Jorquera-Lucerga J.J., Parametric Study of Masonry Shells Form-Found for Seismic Loading, J. Int. Assoc. Shell. Spat. Struct., 2017, 58, 267-75, doi:10.20898/j.iass.2017.194.892.

[33] Michiels T., Adriaenssens S., Identification of key design parameters for earthquake resistance of reinforced concrete shell structures, Eng. Struct., 2017, 153, 411-20, doi:10.1016/j.engstruct.2017.10.043.

[34] Marmo F., Demartino C., Candela G., Sulpizio C., Briseghella B., Spagnuolo R., *et al.* On the form of the Musmeci's bridge over the Basento river, Eng. Struct., 2019, 191, 658-73, doi:10.1016/j.engstruct.2019.04.069.

[35] Popescu M., Rippmann M., Liew A., Reiter L., Flatt R.J., Van Mele T., et al., Structural design, digital fabrication and construction of the cable-net and knitted formwork of the KnitCandela concrete shell, Structures, 2020, doi:10.1016/j.istruc.2020.02.013.

[36] Van Mele T., Pigram D., Liew A., Block P., A prototype of a thin, textile-reinforced concrete shell built using a novel, ultra-lightweight, flexible formwork system, DETAIL Struct., 2018, 1, 50-3.

[37] Xie Y.M., Steven G.P., A simple evolutionary procedure for structural optimization, Comput. Struct., 1993, 49, 885-96.

[38] Querin O.M., Steven G.P., Xie Y.M., Evolutionary structural optimisation (ESO) using a bidirectional algorithm, Eng. Comput. (Swansea, Wales), 1998, 15, 1031-48. doi:10.1108/02644 409810244129.

[39] Bendsøe M.P., Kikuchi N., Generating optimal topologies in structural design using a homogenization method, Comput. Methods Appl. Mech. Eng., 1988, 71, 197-224, doi:10.1016/0045-7825(88)90086-2.

[40] Rozvany G.I.N., Zhou M., Birker T., Generalized shape optimization without homogenization, Struct. Optim., 1992, 4, 250-2, doi:10.1007/bf01742754.

[41] Briseghella B., Fenu L., Feng Y., Mazzarolo E., Zordan T., Topology optimization of bridges supported by a concrete shell, Struct. Eng. Int., J. Int. Assoc. Bridg. Struct. Eng., 2013, 23, 285-94, doi:10.2749/101686613X13363929988214.

[42] Briseghella B., Fenu L., Lan C., Mazzarolo E., Zordan T., Application of Topological Optimization to Bridge Design, J. Bridg. Eng., 2013, 18, 790-800, doi:10.1061/(ASCE)BE.1943-5592.0000416.

[43] Briseghella B., Fenu L., Feng Y., Lan C., Mazzarolo E., Zordan T., Optimization Indexes to Identify the Optimal Design Solution of Shell-Supported Bridges, J. Bridg. Eng., 2016, 21, doi:10.1061/(ASCE)BE.1943-5592.0000838.

[44] Bill M., Robert Maillart. Bridges and constructions, Girsberger, Zurich, 1955.
[45] Fenu L., Congiu E., Lavorato D., Briseghella B., Marano G.C., Curved footbridges supported by a shell obtained through thrust network analysis, J. Traffic. Transp. Eng. (English Ed), 2019, 6, doi:10.1016/j.jtte.2018.10.007.
[46] Fenu L., Congiu E., Briseghella B., Curved deck arch bridges supported by an inclined arch, In: Elfgren, L. *et al.* (Ed.), Proceedings of 19th IABSE Congr. Stock. 2016 – Challenges Des. Constr. an Innov. Sustain. Built Environ., (21-23 September 2016, Stockholm, Sweden), International Association for Bridge and Structural Engineering (IABSE).
[47] Fenu L., Briseghella B., Congiu E., Curved footbridges supported by a shell obtained as an envelope of thrust-lines, In: Proceedings of ARCH'16 – 8th Int. Conf. Arch Bridg. (5-7 October 2016, Wroclaw, Poland), 921-32.
[48] Fenu L., Briseghella B., Zordan T., Curved shell-supported footbridges, In: Proceedings of IABSE Conf. 2015 – Struct. Eng.: Provid. Solut. to Glob. Challenges, (23-25 September 2015, Geneva, Switzerland), International Association for Bridge and Structural Engineering (IABSE), 2016, 105, 1-8.
[49] Lan C., Briseghella B., Fenu L., Xue J., Zordan T., The optimal shapes of piles in integral abutment bridges, J. Traffic. Transp. Eng. (English Ed), 2017, 4, 576-93, doi:10.1016/j.jtte.2017.11.001.
[50] Fenu L., Congiu E., Lavorato D., Briseghella B., Marano G.C., Curved footbridges supported by a shell obtained through thrust network analysis, J. Traffic. Transp. Eng. (English Ed.) 2019, 6, 65-75.
[51] Marmo F., Masi D., Rosati L., Thrust network analysis of masonry helical staircases, Int. J. Archit. Herit., 2018, 12, 828-48, doi:10.1080/15583058.2017.1419313.
[52] Marmo F., Rosati L., Reformulation and extension of the thrust network analysis, Comput. Struct., 2017, 182, 104-18, doi:10.1016/j.compstruc.2016.11.016.