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EFFECT OF MARKOV AND NON-MARKOV CLASSICAL NOISE ON ENTANGLEMENT DYNAMICS

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We analyze the effect of a classical noise into the entanglement dynamics between two particles, initially entangled, subject to continuous time quantum walks in a one-dimensional lattice. The noise is modeled by randomizing the transition amplitudes from one site to another. Both Markovian and non-Markovian environments are considered. For the Markov regime an exponential decay of the initial quantum correlation is found, while the loss of coherence of the quantum state increases monotonically with time up to a saturation value depending upon the degrees of freedom of the system. For the non-Markov regime the presence or absence of entanglement revival and entanglement sudden death phenomena is found or deduced depending on the peculiar characteristics of the noise. Our results indicate that the entanglement dynamics in the non-Markovian regime is affected by the persistence of the memory effects of the environment and by its intrinsic features.

1. Introduction

Quantum entanglement represents undoubtedly one of the most peculiar aspects of quantum mechanics as it can be viewed as the furthest departure of the quantum world from the classical one [1]. Furthermore, in the last years, entanglement has been recognized as the fundamental resource for quantum information processing and communication, such as quantum cryptography, teleportation and exponential speed-up of specific computational tasks [2, 3]. The single-system decoherence,

due to the environment, can make the entanglement disappear on the short-range time scale, making the quantum parallelism, essential for quantum computation, ineffective [4]. On the other hand, environment can even resume entanglement or preserve it. Thus, it is still very important to analyze the effect of the various kinds of environmental noise on the entanglement dynamics in realistic quantum systems which can exhibit peculiar phenomena, such as entanglement sudden death (ESD), and entanglement revival (ER) [5].

ESD, namely the disappearance of the quantum correlations between two quantum systems at finite time in spite of an exponential decay of the system coherence, appears to be peculiar of the Markov environments, namely environments with short, or rather instantaneous, self-correlations [6]. Instead, ER, that is the rise up of quantum correlations in a bipartite system after a finite time period when they completely disappear, occurs in presence of non-Markovian noise [7], which represents the dominant source of decoherence in solid-state systems. The presence of environments with memory [7] is crucial for the non-monotonic time dependence of the entanglement which cannot be ascribed to interaction between the two sub-systems. Indeed, ER is found both for couples of quantum systems interacting directly or indirectly in a common quantum reservoir and for noninteracting quantum systems in independent non-Markovian quantum environments.

The effect of the quantum noise on the entanglement dynamics between two quantum systems has been interpreted in terms of the transfer of the correlations back and forth from the systems themselves to the environment. This is due to the back-action of the systems on the environment. However, recent works showed that the non-monotonic time dependence of the amount of quantum correlations may occur in two-qubit systems under the local action of a system-unaffected environments, such as classical random external potentials [8,9]. Due to the classical nature of the noise, in this case no back-action-induced correlations can be transferred from the system to the environment. Thus, the occurrence of ER in a non-Markovian classical environment is in contrast to the well-established interpretation of revivals in terms of system-environment quantum back action and rises the fundamental question of how one could explain the effect of classical noise on entanglement dynamics in bipartite systems.

In this work, we intend to analyze the role played by a classical noisy environment into the entanglement dynamics between two quantum particles in a simple model, which not only allows us to relate the intrinsic features of the two-particle dynamics to the revival of the correlations, but can also be of great relevance in various physical phenomena. The diffusion of two quantum particles along a one-dimensional lattice in the presence of static and dynamic disorder, mimicked by a noisy potential, represents a good candidate for our theoretical investigation. In fact, the spreading of a quantum-mechanical wave packet in a tight-binding model with a stochastic potential is able to describe electronic transport in solid-state systems, particle diffusion in molecular crystals, diffusion of excitations, photon propagation in coupled waveguide lattices. Furthermore, it gives the possibility to

connect the transition from quantum to classical behavior of the two particles to the features of the environmental noise (correlation time), as suggested by investigations on the diffusion of single-particle wavepackets [10]. This connection could be viewed as a valid guideline to study in details, and, possibly, to clarify how Markov and non-Markov classical noise affect the occurrence of ESD and ER.

In our approach, the dynamics of the two-particles subject to a random potential will be faced numerically. The effect of noise will be simulated both by assuming random and time-independent amplitudes for the transitions from one site to another and by using a sort of dynamic disorder, expressed by a potential term describing a random telegraph noise.

The paper is organized as follows. The physical model adopted is described in Sec. 2. In Sec. 3 are presented the results for the various cases considered: static and dynamic noise, Markovian and non-Markovian environments. Conclusions, comments and future perspectives are given in Sec. 4.

2. Physical Model

A model of two distinguishable particles continuous-time quantum walks (CTQW) in 1D graphs is solved by means of numerical techniques and the time evolution of the wavefunction describing the system is then used to estimate the degree of quantum correlations.

We study CTQW on a 1D ring lattices of N sites (with N even) with periodic boundary conditions. In agreement with previous single-particle investigations [11], the topology of the graphs here considered is simple, in the sense that each node is connected to its two first neighbors, still it is good enough to describe experimental implementations of CTQW, such as two-photon transport in an array of waveguide lattices where non-classical correlations appear. Therefore our model represents a valid tool to analyze the amount of quantum correlations appearing in two-particle CTQW.

The two-particle Hamiltonian describing the dynamical evolution of the system is given by

$$\mathcal{H} = \mathcal{H}_A^0 + \mathcal{H}_B^0, \quad (1)$$

where $\mathcal{H}_{A(B)}^0$ is the single-particle Hamiltonian acting on the particle $A(B)$:

$$\mathcal{H}_{A(B)}^0 = \sum_{j=1}^N \left\{ \beta_j^{A(B)} |j\rangle_{A(B)} \langle j| - c_j^{A(B)}(t) (|j+1\rangle_{A(B)} \langle j| + |j\rangle_{A(B)} \langle j+1|) \right\}. \quad (2)$$

The kets $|j\rangle_{A(B)}$ indicate the quantum states describing the particle $A(B)$ localized in the j node and form a complete, orthonormalized basis set, which span the whole accessible Hilbert space. $\beta_j^{A(B)}$ is the on-site energy, and $c_j^{A(B)}(t)$ is the tunneling amplitude between nearest neighbors [12]. Due to the periodic boundary conditions, here we assume that the site $N+1$ coincides with site 1.

To evaluate the dynamics of the entanglement between the two particles and of the decoherence induced by the noisy environment on the two-particle state, we need to calculate the density matrix of the system as a function of time. To this aim we need to estimate the evolution operator of the system which is a function of the eigenvectors and eigenvalues of the Hamiltonian which, in turns, depends on the parameters $\beta_j^{A(B)}$ and $c_j^{A(B)}(t)$. Therefore, once a suitable choice of such parameters is done, we diagonalize numerically the Hamiltonian for specific times and get the corresponding density matrix values.

In order to describe the effect of the environment on the two-particle quantum state, one can introduce noise either through the coupling constant $\beta_j^{A(B)}$ or through $c_j^{A(B)}(t)$. In agreement with previous investigations [12, 13], we adopt the latter. Once noise has been included in the model, the state of the system is represented by a density matrix $\langle \rho(t) \rangle$ which is the result of an average over a number of density matrices each one obtained with a specific choice of the $c_j^{A(B)}(t)$'s. In particular, to estimate $\langle \rho(t) \rangle$, first we evaluate a single numerical *run* by generating a sequence of transitions with random amplitudes and then by solving exactly the two-particle Schrödinger equation for that given sequence. The final numerical simulation is found by producing a large number of *runs*, each with a different sequence, and then calculating the average density matrix over all the *runs*.

Decoherence, which is an evaluator of the degree of entanglement between the two-particle system and the noisy environment, is here estimated by means of the von Neumann entropy [14] of $\langle \rho(t) \rangle$:

$$\epsilon_{vN}(t) = -\text{Tr} \{ \langle \rho(t) \rangle \ln \langle \rho(t) \rangle \} . \quad (3)$$

Note that such calculation implies the numerical diagonalization of the $N \times N$ matrix $\langle \rho \rangle$, that becomes more and more demanding with increasing the number of sites of the graph.

The particle-particle entanglement is quantified in terms of the negativity [15]:

$$\mathcal{N} = \sum_i |\lambda_i (\langle \rho^{T_{A(B)}} \rangle)| - 1 , \quad (4)$$

with $\lambda_i (\langle \rho^{T_{A(B)}} \rangle)$ the eigenvalues of $\langle \rho^{T_{A(B)}} \rangle$ which is the partial transpose related to the subsystems $A(B)$ of the total density matrix $\langle \rho \rangle$. Negativity has been shown to be a valid measure of the degree of quantum correlations only between two quantum sub-systems each having two degrees of freedom, namely qubits. In fact no measure discriminating separable from entangled states for a mixed state with more than two degrees of freedom is known. As a matter of fact, non-zero negativity, for the general case of N degrees of freedom, is just a sufficient condition for entanglement [16].

3. Numerical results

In this section we present the numerical results obtained for the case of $N = 4, 6, 8$ sites for two different ways of modeling classical noise, namely through static and

dynamic disorder. In particular the former is simulated by assuming random and time-independent values for each $c_j^{A(B)}$, while the latter is modeled by selecting the $c_j^{A(B)}(t)$ according to a random telegraph signal.

3.1. Static noise

In the static disorder case, the parameters $c_j^{A(B)}$'s are assumed to be random variables following the flat probability distribution $P(c) = \frac{1}{\Delta}$ for $|c - c_0| \leq \frac{\Delta}{2}$ and 0 otherwise [13]. c_0 is the mean value of the distribution and Δ is a measure of the disorder of the environment, in fact, when Δ goes to zero, the noise effect vanishes. The autocorrelation function of the $c_j^{A(B)}$'s, given by $\langle \delta c(t) \delta c(0) \rangle = \frac{\Delta^2}{12}$, is time independent and this implies that its power spectrum is proportional to a δ -function centered on zero frequency. As a consequence, this kind of noise has a characteristic time which is always much longer than the characteristic time of the system-environment coupling. Therefore, the static disorder can be considered as representative of a non-Markovian noise.

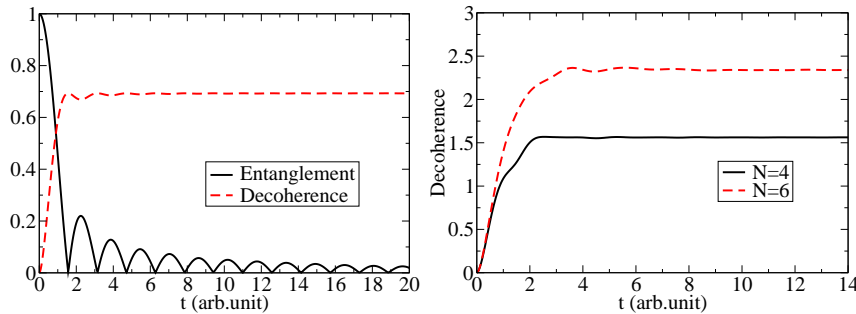


Fig. 1. Left panel: Entanglement (solid line) and decoherence (dashed line) as a function of time for $N = 2$. Right panel: decoherence time evolution for $N = 4$ (solid line) and $N = 6$ (dashed line).

In Fig.1 the time evolution of decoherence and entanglement are shown at $N = 2, 4, 6$, for the input maximally entangled state $|\psi(t=0)\rangle = \frac{1}{\sqrt{N}} \sum_i^N |i_A i_B\rangle$. The entanglement, here expressed in terms of negativity, as above explained is calculated for the $N = 2$ case only. From the left panel of Fig.1 we see that the entanglement exhibits ESD and ER phenomena [8], corresponding to an oscillating behavior of the decoherence curve (here oscillations are much less pronounced than for the entanglement case, because of the logarithmic dependence displayed in Eq.(3)). By increasing the number of sites (right panel), the saturation value of the decoherence increases because, as a consequence of the environment action, the system is described by a statistical mixture involving a larger number of terms, and, for this kind of noise, we observe that these oscillations tend to disappear.

3.2. Dynamic noise

In the dynamic disorder case the parameters $c_j^{A(B)}$'s are set equal to $\nu Q_j^{A(B)}(t)$ where $Q(t)$ is described by a stochastic process of the telegraph noise type, which flips randomly between the two values 1 and -1 at a rate γ , and ν is the particle-environment coupling constant. The autocorrelation function of $Q(t)$, as well known, is given by $\langle \delta Q(t) \delta Q(0) \rangle = e^{-2\gamma t}$ with the Lorentzian power spectrum $\langle \delta Q \delta Q \rangle_\omega = 4\gamma / (\omega^2 + 4\gamma^2)$. The relative length of the two time scales $\tau_e \sim 1/\gamma$ (characteristic time of the environment) and $\tau_c \sim 1/\nu$ (characteristic time of the particle-environment coupling) are critical for decoherence and entanglement. Indeed, $\tau_e < \tau_c$ is identified as the Markovian regime, while $\tau_e > \tau_c$ is identified as the non-Markovian regime [8].

3.2.1. Markov case

In Fig.2 the results for the case $\tau_e/\tau_c = 1/5$ are presented, for the same initial state $|\psi(0)\rangle$ used in the previous section. Unlike the case of static disorder, because of the short time correlation of the environmental noise, the degree of entanglement between the two-particles decays continuously and no ESD and ER phenomena are observed. Decoherence is a monotonic function of time, and reaches asymptotic values larger and larger as the number of nodes increase.

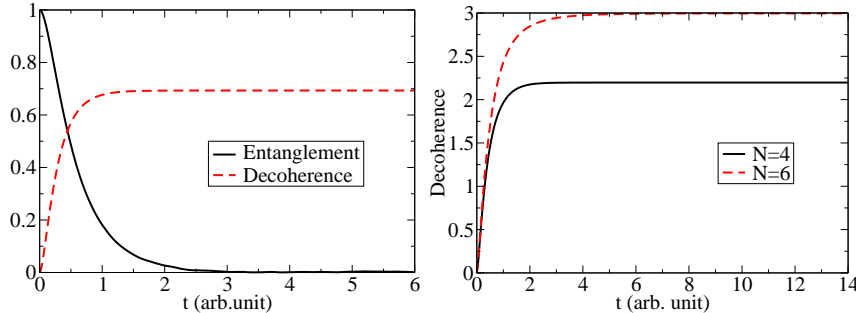


Fig. 2. Left panel: Entanglement (solid line) and decoherence (dashed line) as a function of time for $N = 2$. Right panel: decoherence time evolution for $N = 4$ (solid line) and $N = 6$ (dashed line). Here the Markov regime is obtained by setting $\tau_e/\tau_c = 1/5$

3.2.2. Non-Markov case

The non-Markovian behavior has been simulated by setting the ratio $\tau_e/\tau_c = 10$. The results are displayed in Fig.3. For $N = 2$ (left panel), the entanglement is a damped oscillating function of time, thus showing ESD and ER phenomena. Since the envelope is exponential we can assume that revival happens an infinite number of times, in agreement with previous findings [8]. Decoherence, unlike the Markovian

case, presents evident oscillations before reaching the saturation value. In particular we note that the local minima of decoherence correspond to the local maxima of entanglement, thus suggesting a strict connection between the two quantities. In fact, in our model, we can attribute the disentanglement of the two particles to the movement of the initial state $|\psi(0)\rangle$ towards mixed state. Such a relation can reasonably be thought to hold also for graphs with $N > 2$, where no rigorous evaluations of the entanglement can be performed. Therefore, the oscillating behavior of decoherence observed for $N = 4, 6$ (right panel of Fig.3) can be considered as an indicator of the presence of ER.

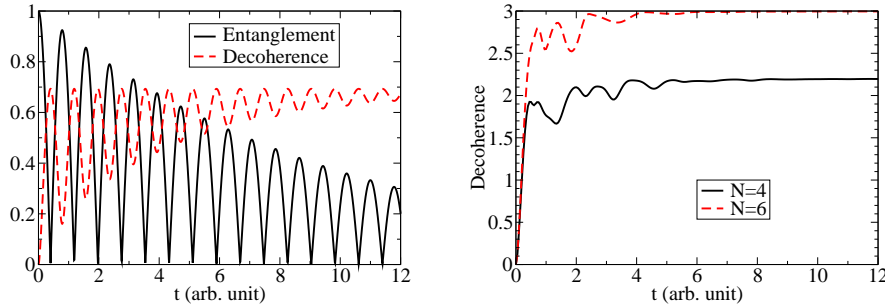


Fig. 3. Left panel: Entanglement (solid line) and decoherence (dashed line) as a function of time for $N = 2$. Right panel: decoherence time evolution for $N = 4$ (solid line) and $N = 6$ (dashed line). Here the non-Markov regime is obtained by setting $\tau_e/\tau_c = 10$.

4. Conclusions

We have investigated the effect of a noisy environment on the time evolution of the entanglement between two-quantum particles and of the loss of coherence of the two-particle system for CTQW in a one-dimensional ring lattice with periodic boundary conditions. The two-particle initial state has always been chosen as a maximally entangled one. The environmental noise has been simulated introducing both static and dynamic disorder in the lattice. To this aim the transition amplitudes from one site to another have been assumed random and time independent (static case) or subject to a random telegraph signal (dynamic case).

In the dynamic case, the persistence of the memory effect of the environment is related to the switching rate of the random telegraph signal. Therefore two distinct regimes can be identified according to the ratio between the correlation time of the environment and the one characteristic of the system-environment interaction. If such a ratio is smaller than 1, Markovian behavior can be assumed, on the contrary, values larger than 1 identify non-Markovian regime. The static disorder represents an environment whose memory effect can not be neglected at any finite

time, therefore the characteristic time of its correlations is always larger than the one of the environment-system coupling. Thus we classify this kind of noise as non-Markovian. It should be noticed that such model is different from the one adopted in the literature [17–19] where the non-Markovian noise is commonly represented only in terms of random telegraph signal.

For the Markovian case we found an exponential decay of the entanglement, while decoherence increases monotonically with time up to a saturation value, depending upon the number of lattice sites. No peculiar phenomena, like ESD or ER, are observed, in agreement with previous findings [8].

For the non-Markovian behavior we find that the dynamics of entanglement and decoherence exhibits non-monotonic time dependence. In particular ESD and ER are detected [7–9] for $N = 2$, and can even be supposed to be present for $N > 2$ under some specific conditions (dynamic noise). Indeed our approach suggests that different kinds of noise leads to somehow different results. For static disorder both entanglement and decoherence show less pronounced oscillations with respect to the non-Markovian random telegraph noise for $N = 2$, while for $N > 2$ decoherence increase practically in monotonic way, thus suggesting that ESD and ER phenomena are not present, unlike what is found for the dynamic noise case.

In conclusion, our analysis points out that the entanglement of a quantum system coupled to a classical source of noise is affected by the persistence of the memory effect of the environment and by the intrinsic features of noise itself. Therefore we believe that the extension of the present investigation to the ubiquitous case of $1/f$ noise could be of great interest to get a better insight into the effect of a classical noise on a quantum system [20]. Furthermore, in order to get a deeper understanding of how a classical environment affects the appearance of quantum correlations, the study of more complex systems, including many-body effects and a higher number of degrees of freedom, is required. In this view, the extension of the approach here adopted to model many particles interacting with each other is not straightforward and will need the development of more sophisticated and efficient numerical tools.

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