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**Analysis of human-induced vibrations of
footbridges based on experimental tests and
numerical simulations**

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Abstract

Over the last decades, footbridges have become more and more lively and slender structures, due to the increasing strength of new structural materials accompanied with aesthetic requirements. However, these kinds of structures are typically characterized by natural frequencies within the range of the pedestrian dynamic actions and thus they can be very sensitive to dynamic excitation. For this reason, measures to prevent the functionality of the structure to be compromised by excessive vibrations are often required, whether they are structural modifications or vibration serviceability assessment.

This work characterizes the main stages of the procedure for the evaluation of human-induced vibrations of footbridges, assessing and comparing the reliability of pedestrian load models proposed in literature. The procedure includes experimental investigations to identify the footbridge dynamic behaviour and characterize human-induced actions, calibration of structural numerical models and adoption of suitable mathematical models to simulate pedestrian loads. Finally, the reliability of the different pedestrian load models is assessed comparing numerical and experimental accelerations and the influence of the different load model hypothesis is evaluated.

Full-scale field-tests are performed on two real footbridges to experimentally identify their dynamic behaviour as well as evaluate the structural response due to pedestrian actions, namely different-sized groups of pedestrians crossing the footbridge at different velocities. The reliability of the predicted response depends on both the adoption of adequate models to simulate pedestrian loads and the capability of representing the actual footbridge behaviour. Hence, the characterization of the structural dynamic behaviour is an important stage in the prediction of the structural response. When finite element models are adopted to represent the structural behaviour, model updating techniques are applied to calibrate these models in order to achieve a reliable description of the dynamic behaviour.

Finally, the performances of different mathematical models to simulate human-induced loads are tested and compared, with a specific focus on the vertical components. At first, the classic periodic load model is considered, which is based on the assumption that both human feet produce the same force and that the force is periodic. In this approach, updated FE model is adopted to perform dynamic analyses simulating different pedestrian loading conditions. Then, to take into account that main parameters of the walking force are characterized by a certain level of randomness, a probabilistic simulation model is considered. As the statistical characterization of the dynamic structural response relies on the numerical simulations of many samples of pedestrian traffic conditions, it can be time consuming. An interesting alternative is the modelling of pedestrian loading as a stationary random process through the analytical definition of an equivalent spectral model of pedestrian-induced forces. The model can represent the pedestrian-induced loading even

in crowded conditions thanks to the definition of a physically based function describing the correlation among pedestrians. To allow for a more accurate prediction of the maximum structural response in crowded conditions, the present study in addition accounts for the vertical mechanical interaction between pedestrians and the supporting structure. In the last two models, the structural behaviour is defined by the experimentally identified dynamic parameters, with no need to develop FE models, which implies, on one hand, an approximation in the description of the footbridge behaviour and, on the other hand, avoiding uncertainties associated with the modelling.

Abstract

Negli ultimi decenni, le passerelle pedonali sono diventate strutture sempre più snelle e leggere, grazie allo sviluppo di materiali da costruzione ad alte prestazioni e nuove esigenze estetiche. Tuttavia, queste strutture possono essere sensibili a sollecitazioni dinamiche essendo solitamente caratterizzate da frequenze naturali prossime a quelle tipiche dell'azione dinamica dei pedoni. Pertanto, si rendono spesso necessarie misure atte a evitare che la funzionalità di queste strutture sia compromessa da eccessive vibrazioni, siano esse modifiche strutturali o previsioni delle vibrazioni indotte.

Il presente studio caratterizza le principali fasi della procedura di valutazione delle vibrazioni indotte dai pedoni su passerelle pedonali, valutando e confrontando le prestazioni di modelli di simulazione dell'azione dei pedoni proposti in letteratura. Tale procedura include campagne sperimentali atte a caratterizzare i parametri modali della struttura e valutare le sollecitazioni indotte dai pedoni, la calibrazione di modelli numerici strutturali e l'adozione di appropriati modelli matematici per simulare l'azione dei pedoni. Le prestazioni dei modelli di simulazione sono infine valutate attraverso un confronto tra le accelerazioni sperimentali e numeriche.

Test dinamici su scala reale sono effettuati su due passerelle pedonali al fine di caratterizzare il loro comportamento dinamico e valutare la risposta a diverse condizioni di carico dei pedoni. Poiché l'affidabilità della risposta simulata dipende sia dall'adozione di adeguati modelli di carico del pedone sia dalla capacità di rappresentare il reale comportamento strutturale, la caratterizzazione del comportamento dinamico è una fase chiave nella procedura di previsione della risposta. Nel caso in cui il esso sia rappresentato attraverso modelli a elementi finiti, l'impegno di tecniche di calibrazione numerica si rende necessaria per ottenere una descrizione accurata del reale comportamento dinamico.

Infine, si testano e confrontano le prestazioni dei diversi modelli matematici per la simulazione del carico dei pedoni, ponendo l'attenzione sulla componente verticale. Il primo modello analizzato è il classico modello di carico periodico, basato sull'ipotesi che la forza prodotta da ciascun piede sia la stessa e che essa sia periodica. In questo approccio, il modello FE della struttura è utilizzato per effettuare analisi dinamiche simulando diverse condizioni di carico. In seguito, per tenere in conto che i principali parametri che definiscono le forze indotte dai pedoni sono caratterizzati da un certo livello di variabilità, si considera un modello di simulazione probabilistico. La caratterizzazione statistica della risposta dinamica è basata sulla simulazione di numerose condizioni di traffico pedonale e perciò può essere molto onerosa in termini di tempi di calcolo. Un'interessante alternativa riguarda la modellazione dell'azione dei pedoni come un processo stazionario, basata su una definizione analitica della densità di potenza spettrale delle forze indotte. Questo modello può rappresentare il carico indotto dai pedoni anche in condizioni di intenso traffico pe-

donale, grazie alla definizione di una funzione che descrive la correlazione tra i pedoni. Inoltre, allo scopo di fornire una più accurata predizione della risposta strutturale in condizioni di traffico intenso, nel presente studio si tiene in conto dell'interazione meccanica tra la folla e la struttura. Negli ultimi due modelli, il comportamento strutturale è caratterizzato attraverso i parametri modali sperimentali, senza la necessità di sviluppare modelli FE; ciò implica sia un'approssimazione nella descrizione del comportamento strutturale ma anche l'eliminazione delle incertezze legate alla modellazione.

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Chapter 1

Introduction

Over the last decades, footbridges have become more and more lively and slender structures, due to the improved mechanical characteristics of new structural materials accompanied with aesthetic requirements for greater slenderness. As a result of these construction trends, many footbridges have become more sensitive to vibrations when subjected to dynamic loads. In the past, many cases in which dynamic excitation caused excessive vibrations of this type of structures have been reported [4, 5, 6, 50]. Nevertheless, this problem attracted considerably attention after the infamous swaying of the Millennium Bridge in London during its opening day on 10 June 2000 [35]. After that, the bridge was closed and a damping system to control these vibrations was installed. Finally, it was reopened to the public in February 2002 [3]. The excessive vibrations were caused by near resonance of one or more modes of vibration. Excessive vibration problems are indeed usually due to the fact that the dominant frequencies of the human-induced action often fall within the range of natural frequencies of these kinds of structures [72].

Measures to prevent the functionality of the structure to be compromised by excessive vibrations are often required. Vibration serviceability requires a characterization of the vibration source and the vibrating system. The most important source of vibration for footbridges is the human action, that appears in different types of human-induced loading. The most prominent are walking people, who can appear as individuals, in groups or even in crowd [32]. Other types of human action involve running and vandal loading, i.e. intentional actions in the form of tuned rhythmical body movements [30]. Modelling of human-induced dynamic forces is a complicated task because they depend on many parameters and are influenced by the number of persons as well as their degree of correlation, which is difficult to generalize [114]. Moreover, during footbridge vibration under crowd load, phenomena of human-structure interaction may occur. These can result in the influence of walking people on the footbridge vibration properties, such as the natural frequency and damping [104, 105, 114] as well as in the potential change in the walking pattern of an individual due to the movement of the structure [64]. The latter is more likely to occur in the horizontal direction, due to the nature of human walking and desire to maintain the body balance on a laterally moving surface [114]. Human-human interaction phenomenon may also occur, causing changes in the walking pattern of an individual due to the surrounding people [64].

To assess the vibration serviceability of footbridges, there are some design procedures

which are proposed by different authors and not formally codified as well as design checking methods which are formally codified. However, the latter do not cover all aspects of the problem comprehensively. The main design codes and recommendations have been developed in the past few years. In particular, the first international guide was published in 2005 by *fib* (fédération internationale du béton), dealing with the general design of footbridges [49]. Some more recent documents, specifically dealing with the dynamic behaviour of footbridges, include the widely used guideline from the French road authority Sétra [99], the European research project SYNPEX [29, 60, 89] and the work commissioned by the UK Highway Agency [8, 9, 10, 74], which was later adopted in the UK National Annex to Eurocode 1991-2:2003 [21]. International standards for buildings and walkways are also provided by ISO 10137:2007 [67].

During walking, a pedestrian produces a dynamic time varying force which has components in all three directions: vertical, horizontal-lateral and horizontal-longitudinal [6]. Over the years, there have been and still are many attempts to characterize the human-induced dynamic forces and model them analytically. The problem can be tackled in two ways: (i) by directly characterizing the pedestrian forces in the time domain or (ii) by analysing the frequency content of the walking forces. Although time domain models are more common than frequency domain models, the latter are of great interest due to their capability of representing the human force as a narrow band process. In fact, time domain models for walking and running are based on the assumption that both human feet produce exactly the same force and that the force is periodic. Starting from the analysis of real walking forces obtained from instrumented treadmills or force plates, some authors have demonstrated that the walking force is not a perfectly periodic sinusoidal force, but rather a considerably more complex narrow band random process [23, 103]. This results in a ‘leak’ of energy into adjacent frequencies on either side of the fundamental pacing rate in the amplitude spectrum of a series of consecutive steps [86], reflecting a slight variation of the pacing rate during the walking path. Brownjohn et al. [23] analysis real continuous walking forces obtained from an instrumented treadmill and the effect of their random imperfections through time simulations of structural response. Based on these analyses, they show that there are significant differences between responses due to the imperfect real walking forces and the equivalent perfectly periodic simulation. Similar results are obtained from the study performed by Van Nimmen et al. [103]. They show that in contrast to perfectly periodic forces that are exclusively composed of the harmonics of the step frequency, imperfect real walking results in a distribution of forces around the dominant harmonics [65, 85]. These effects are due to the naturally present intra-subject variability in the walking force, which means that a single pedestrian induces a walking force that differs with each step. This is due to the fact that some parameters such as step length and forward speed may vary along the path. While the perfectly periodic model, which is expanded in Fourier coefficients with a precise frequency, leads itself to deterministic time domain analysis, the random nature of human walking is more suited to the techniques of stochastic analysis in frequency domain, where the walking forces are represented as power-spectral densities (PSDs) [23]. Moreover, walking forces induced by pedestrians are also affected by a certain level of inter-subject variability, which implies that, as each pedestrian has its own characteristics, such as weight, step frequency and walking speed, different pedestrians can generate different dynamic forces. This can be addressed, both in time and frequency domain, via probability-based modelling, [112], that is defining variables that describe the

human-induced forces via their probability density functions.

During the measurements of pedestrian-induced forces conducted in the past, the attention was mainly paid to the vertical component of the force. It was regarded as the most important of the three forces (vertical, horizontal-lateral and horizontal-longitudinal) because it has the highest magnitude [114]. Nevertheless, even if the horizontal-component of the force is generally very small, an additional non-negligible force component is potentially generated when the walking surface moves laterally as well. This is due to the interaction between the movement of the pedestrian's centre of gravity and that of the structure [64]. The fundamental nature of human-structure interaction and the importance of synchronization is still somewhat disputed [63]. Heinemeyer and Feldmann [60] motivate this phenomenon by the fact that when a person walks on a laterally vibrating footbridge, he is inclined to compensate the additional movement of his centre of gravity by swaying with the bridge displacement for lateral stability. An adjustment of the movement of the centre of gravity is caused even by small and not perceptible vibrations. The movement of the centre of gravity goes together with an adaptation of the walking frequency and a widening of the gait [35]. Hence, the lateral ground reaction forces are applied in resonance and the widening of the gait causes and increase in the lateral ground reaction forces. The human-structure synchronization effect is such that a low-damped footbridge can be excited to large vibrations [60]. Other authors confirm this hypothesis (such as Fujino et al. [50]) and attribute excessive lateral vibrations to human-structure synchronization, as pedestrians seem to modify their gait to match the frequency and phase to that of the footbridge. On the contrary, Ingólfsson and Georgakis [63] state that the assumption of synchronization among pedestrians and the laterally moving surface is not verified by the several full scale tests that have been carried out. Moreover, Ingólfsson et al. [66] have recently shown that synchronization among pedestrians is not a necessary condition for the development of excessive vibrations. However, experimental tests have verified the existence of a form of instability for which a transition between limited and excessive lateral vibration occurs for a small increase in the number of people occupying the bridge [63].

1.1 Objectives of the thesis

The assessment of the vibration serviceability of footbridges has gained significant importance, not only in the research area but also with regard to design. This entails the need for design procedures which should be simple and as accurate as possible at the same time. Two kinds of design checking methods exist: codified procedures and methods proposed by different authors which are not formally codified. Design codes are usually based on the simplified representation of a pedestrian as a pulsating point load moving across the footbridge at a constant speed. The speed is evaluated depending on the pacing frequency, which is in turn assumed equal to the natural frequency of the footbridge (hypothesis of resonance). Moreover, the effect of groups of random pedestrians is evaluated defining an equivalent number of synchronized pedestrians. Alternatively, a spectral design method is provided, which describes in a simplified way the stochastic loading and system response. Design codes do not consider that the footbridge can respond to human-induced excitation in more vibration modes simultaneously and the pedestrian force is a stochastic and narrow band process. Furthermore, the fact that dynamic forces induced by pedestrian groups

or crowd are significantly influenced by the number of pedestrian as well as their degree of correlation, is addressed in a simplified way, defining equivalent load representative of crowd-induced loading. Finally, the fact that pedestrians interact with the supporting structure and that this can lead to changes in the structural behaviour is not accounted for.

Within the framework of the methods proposed in literature, there are methods which are able to account for some of those features. However, most of these methods still need to be improved and validated in order to be adopted at the design stage.

The aim of the present work is twofold. On one hand, the characterization of the main stages of the procedure for the evaluation of human-induced vibrations of footbridges, assessing and comparing the performance of accurate pedestrian load models proposed in literature. The procedure includes (i) experimental investigations to identify the footbridge dynamic behaviour and characterize human-induced actions, (ii) calibration of structural numerical models and (iii) adoption of suitable mathematical models to simulate pedestrian loads. For the purpose of modelling vertical human-induced forces, two time domain load models are adopted. The first is the classic load model, which is based on the assumption that both feet produce exactly the same force and that force is periodic. The second one is a probabilistic load model which can take into account that the main parameters of the walking force are characterized by a certain level of randomness and provides a statistical characterization of the response. In the second model, the structural behaviour is defined by the experimentally identified dynamic parameters, with no need to develop FE models. This implies, on one hand, an approximation in the description of the footbridge behaviour and, on the other hand, avoiding uncertainties associated with the modelling.

Full-scale field-tests are performed on two real footbridges to experimentally identify their dynamic behaviour as well as evaluate the structural response due to pedestrian actions, namely different-sized groups of pedestrians crossing the footbridge at different velocities. The reliability of the predicted response depends on both the adoption of adequate models to simulate pedestrian loads and the capability of representing the actual footbridge behaviour. Hence, the characterization of the structural dynamic behaviour is an important stage in the prediction of the structural response. Dynamic identification procedures are applied to evaluate natural frequencies, damping ratios and mode shapes of the structure starting from experimental measurements. When finite element models are adopted to represent the structural behaviour, model updating techniques are applied to calibrate those models in order to achieve a reliable description of the dynamic behaviour.

The second objective concerns the analysis and development of a spectral model for vertical pedestrian-induced loading proposed in literature. Being a spectral model, it allows a simple description of the stochastic load and structural response with no need of time analysis and takes into account that the pedestrian force is a narrow band process. In addition, the model can represent the pedestrian-induced loading in unrestricted and crowded pedestrian traffic conditions, thanks to the definition of a physically-based function describing the correlation among pedestrians. Furthermore, the presence of a crowd of pedestrians can change the dynamic properties of the system composed of the structure and the system. Hence, in the present study, to take into account the effects of the human-structure interaction (HSI) on the dynamic response, an improvement of the spectral load model is proposed. The HSI effects are accounted for by characterizing the dynamic behaviour of the coupled system composed of the crowd and the footbridge.

1.2 Outline of the thesis

The thesis is organized as follows. First, chapter 2 provides a description of the human-induced forces and a brief overview of some of the main mathematical models proposed in literature. Next, two load models are analysed in detail: the single step force model proposed by Li et al. [73] and the multi-harmonic force model proposed by Živanović et al. [115].

Chapter 3 focuses on the dynamic behaviour of the footbridges and the simulation of their dynamic response to human-induced loads. The analysed footbridges are the Pasternak and the Correggio footbridge. The first one is a curved cable-stayed footbridge located in Modena and about 270 m long. The Correggio footbridge is a steel structure composed of five simple-supported spans with a total length of 170 m. Their dynamic behaviour is characterized thanks to experimental investigations. Starting from experimental measurements, the structural dynamic properties are evaluated by means of identification techniques. A description of the algorithm adopted in the present study, the Enhanced Frequency Domain Decomposition method, is provided. Thanks to the identified modal parameters, FE models of the footbridges are calibrated so that their dynamic behaviour matches as close as possible the experimental one. Furthermore, accelerations caused by pedestrian-induced loads are analysed. The structural response is simulated employing the pedestrian load models described in chapter 2. Finally, predicted accelerations are compared with those recorded experimentally for the same loading conditions.

In chapter 4, the spectral model proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] for the modal force induced by pedestrian groups modelled as a stationary random process is thoroughly analysed. The spectral load model allows a simple description of the stochastic load and structural response. In addition, it can represent the pedestrian-induced loading in unrestricted and crowded pedestrian traffic conditions thanks to the definition of the physically-based function representing the correlation among pedestrians.

Chapter 5 presents and discusses the proposed developments of the spectral load model. First, the maximum structural response is evaluated considering different relevant parameters for the footbridge and the pedestrian action. Moreover, the dynamic behaviour of the coupled crowd-structure system is characterized and the structural response in presence of HSI is investigated.

Finally, a summary of the main conclusions from the research work as well as recommendations for future research are presented in chapter 6.

Chapter 2

Human-induced loading

A pedestrian walking on a structure causes a ground reaction force (GRF), which is due to acceleration (and deceleration) of the centre of mass of the body. The GRF is a three-dimensional vector (in the vertical, lateral and longitudinal direction) which varies in time and space due to the forward movement of the person [64, 87]. In the past, many studies have been performed to characterize this single pedestrian walking force, some of which are summarized in the following. Other types of human-induced forces involve running and vandal loading, i.e. internal action in the form of tuned rhythmical body movements. Some types of human-induced forces have also been investigated for small groups of people, but generally large group of pedestrians have seldom been investigated [114].

A brief summary of the vertical contact forces experimentally identified during the years is reported in the following. Afterwards, some mathematical load models proposed by various authors as an idealization of the experimentally identified contact forces are presented.

2.1 Force measurements

Harper et al. [59] and Harper [58] conducted one of the first measurements of pedestrian-induced forces with the aim to investigate the friction and slipperiness of a floor surface. They measured horizontal and vertical force from a single footstep using a force plate [34] and recorded the single footstep vertical force characterized by the shape with two peaks and a trough. In the years that followed, many researchers confirmed this general shape of the single-step force time history, among which Galbraith and Barton [51], Blanchard et al. [17], Ohlsson [81], Kerr [70] and many others. A lot of studies into walking forces have been done in the field of biomechanics. These include the one performed by Andriacchi et al. [2], based on the measurements of single step walking forces in the three directions by means of a force plate. They demonstrated the complex nature of human-induced dynamic forces and their dependence on many parameters, showing how the dynamic effect of walking forces changes with the walking speed. In particular, they reported that increasing walking velocity leads to increasing step length and peak force magnitude [114]. Galbraith and Barton [51] measured the single step vertical force caused by a pedestrian moving at different velocities, ranging from slow walking to running. They identified the running single step vertical force characterized by only one peak, instead of two as for the

Author(s)	Walking		Running		Jumping	
	f_m [Hz]	σ_{f_m} [Hz]	f_m [Hz]	σ_{f_m} [Hz]	f_m [Hz]	σ_{f_m} [Hz]
Matsumoto et al. [76]	2.0	0.173	-	-	-	-
Kerr and Bishop [71]	1.9	-	-	-	-	-
Leonard [72]	1.7-2.3	-	-	-	-	-
Bachmann et al. [7]	1.6-2.4	-	2.0-3.5	-	1.8-3.4	-

Table 2.1: Typical frequencies and standard deviations for different human activities

single step walking force. Moreover, they showed that during running there are periods when both feet are off the ground leading to zero force recorded. On the contrary, during walking there are some short time periods when both feet are on the ground which gives an overlapping between the left and right foot in the walking time history. In 1980, Wheeler [117, 118] summarized and systematized the work of other researchers related to different models of human moving from slow walking to running, providing a very comprehensive research into human forces relevant to footbridge dynamic excitation. He also highlighted the dependence of many walking parameters, such as step length, moving velocity, peak force and contact time (the time while one foot is in the contact with the ground) on the pacing frequency. Although all these parameters are different for different pedestrians, walking forces generated by different pedestrians have in common that the peak amplitude, stride length and velocity increase with increasing step frequency while contact time decreases.

The introduction of more advanced and informative measurement techniques has allowed the measurements of continuous walking time histories comprising several steps. The measured time-histories were near periodic with the average period equal to the reciprocal value of the average step frequency. For instance, Ebrahimpour [39, 40] measured continuous walking force using a platform instrumented with several force plates while Rainer et al. [88] used a floor strip.

Matsumoto et al. [76, 77] were the first to provide a reliable statistical description of normal walking frequencies. They investigated a sample of 505 people and concluded that the frequencies followed a normal distribution with a mean pacing rate of 2.0 Hz and standard deviation of 0.173 Hz. Other researchers found mean frequency of the normal walking in broad agreement with what Matsumoto et al. [76] and successive researchers have found, as those reported in Table 2.1. Similar comprehensive statistically based investigations, such as the one given by Matsumoto et al. [76] for walking, do not exist for other types of human induced forces [114]. However, there are some proposals for the typical frequency ranges for different human activities (running, jumping, bouncing, etc.), as those proposed by Bachmann et al. (Table 2.1).

However, it has to be stressed that practically all mentioned measurements of the walking forces were conducted on rigid surfaces, such as a force plate on stiff ground, gait machine or high-frequency structure. This leaves the possibility that the reported forces could be different from the ones that actually occur on low-frequency footbridges that vibrate perceptibly [114].

2.2 Force modelling

For the purpose of applying the measured dynamic forces in design, they have to be modelled analytically. Modelling of human-induced dynamic force is not straightforward, as there are many different types of human-induced forces which can change not only in time but also in space (e.g. walking and running) and are dependent on many parameters. Moreover, the dynamic force generated by a single person is essentially a narrow band process which is not well understood as well as the influence of the number of persons and their degree of synchronization.

Any mathematical model for the definition of pedestrian-induced loading, relies on the characterization of spatial and temporal parameters that describe the gait cycle of the periodic locomotion (such as running and walking). Typical temporal parameters are:

- **Step frequency or pacing rate** f_s [Hz] The fundamental loading frequency of the periodical activities, denoting the number of steps taken in a given time and commonly expressed in steps per seconds;
- **Gait cycle** T_s [s] Period of time between any two nominally identical events, defined as:

$$T_s = f_s^{-1} \quad (2.1)$$

According to Eq.(2.1), the gait cycle T_s is the period of time from the initial contact of the left foot with the ground to the initial contact of the right foot immediately thereafter. However, in literature the gait cycle is also defined as the period of time from the initial contact of the foot with the ground (also referred to as the heel strike) to the next point of initial contact for that same limb [3, 115]. The latter definition implies that the period of the walking force is given by the placement of the right and left foot. The former means that one period is given by the placement of a single foot, whether it is the right or the left one. In any case, the pacing rate is the inverse of the time from the initial contact of the left foot with the ground to the initial contact of the right foot immediately thereafter.

- **Walking speed** c_s [m/s] The walking speed can be related to the step frequency through the step length l_s :

$$c_s = l_s f_s \quad (2.2)$$

The principal spatial parameters are:

- **Step length** l_s [m] Distance in the direction of progression between the placement of the right and left foot. Walking frequency f_s and step length l_s can be considered as two independent [71, 112] or linked [55] modelling parameters. Kerr and Bishop [71] propose the following formulation for the step length dependent only on the pedestrian's height h [m]:

$$l_s = 0.45h \quad (2.3)$$

Živanović [112] describes the step length as an independent parameter that follows the normal distributions:

$$l_s = \mathcal{N}(0.71, 0.071) \text{ [m]} \quad (2.4)$$

On the contrary, Grundmann and Schneider [55] propose a formulation for the step length depending on the step frequency:

$$l_s = 0.24s \, h f_s \quad (2.5)$$

- **Stride length** $[m]$ Distance in the direction of progression between two successive placements of the same foot, enclosing two step lengths (left and right) [107];
- **Step width** $w_s \text{ [m]}$ Side-to-side distance between the paths taken by the two feet, usually measured at the mid-point of the heels [107]. Based on measurements of the lateral foot placement during walking conducted by some authors, a coupling between step length and step width has been proposed. This can be expressed as:

$$w_s = C l_s \quad (2.6)$$

where the constant C is assumed equal to 0.13 and 0.12 according to the studies performed by Bauby and Kuo [14] and Donelan et al. [38], respectively.

Time domain force models should be distinguished in deterministic and probabilistic models. The first establish one general force model for each type of human activity. The others take into account the fact that some parameters which influence human force, such as the step frequency and person's weight, are random variables whose statistical nature should be considered in terms of their probability distribution functions. In any case, time-domain models for walking and running are based on the assumption that both human feet produce exactly the same force and that the force is periodic [114].

The most common model of perfectly periodic human-induced force is the continuous walking load model, which describes the sum of the forces exerted by both feet [101]. Assuming a perfectly periodic walking cycle and that both feet produce exactly the same force, the single pedestrian vertical walking load is represented by a sum of Fourier harmonic components [1, 28, 49, 99]:

$$f(t) = G + \sum_{k=1}^n G \alpha_k(f_s) \sin(2\pi k f_s t + \Psi_k) \quad (2.7)$$

where $G \text{ [N]}$ is the pedestrian's weight, k the number of the harmonic, $\Psi_k \text{ [rad]}$ the phase shift of the k th harmonic, $\alpha_k \text{ [-]}$ the Fourier's coefficient (called Dynamic Loading Factor) of the k th harmonic normalized to the pedestrian's weight and n the total number of contributing harmonics.

According to Eq.(2.7), the walking force is composed of n harmonics characterized by frequencies equal to the step frequency and its integer multiples. However, from the analysis of the force spectrum of measured walking force, some authors (such as [6, 23, 91, 115]) show that also some subharmonics appear at frequencies between the main harmonics. This is explained as a consequence of more pronounced footfall on one side, meaning that

the walking process for two legs can be described by slightly different parameters [6]. In fact, the walking force of Eq.(2.7) is periodic with fundamental frequency equal to f_s , i.e. with a fundamental period equal to the time required to make one step (Eq(2.1)). On the contrary, considering that forces exerted by the right and left foot can slightly differ, the fundamental period of the walking force becomes the time required to make two consecutive steps, as had been widely accepted in literature [114]. Hence, being the fundamental period approximately two times higher than when analysing one step only, the fundamental frequency of the force becomes approximately two times lower than the one for a single step [91].

Eq.(2.7) shows that the amplitude of the harmonic component depends on the step frequency through the DLF $\alpha_k(f_s)$, that has to be quantified based on the Fourier spectrum of the experimentally registered ground reaction forces [101]. Over the years, many studies have been conducted in order to quantify DLFs. In the late 1980s, Rainer et al. [88] measured continuous single-person force from running and jumping and confirmed that DLFs strongly depended on the frequency of the activity. In his Ph.D. thesis, Kerr [70] presented about 1000 force records produced by 40 subjects walking from 1 Hz up to 3 Hz. A large scatter in the DLF values was observed, even if for the first harmonic an increasing trend with increasing pacing frequency was clearly identified, similarly to the results reported by Rainer et al. [88]. As DLFs of the higher harmonics obtained from Kerr were very scattered, they have been characterized statistically by mean values and coefficient of variations. Based on the work of Kerr and other researchers, Young [120] proposed DLFs for the first four harmonics as a function of the walking frequency in the range from 1 to 2.8 Hz. As the DLFs are obtained by direct or indirect force measurements on rigid surfaces, real DLFs can differ from those reported in literature. With this aim, Pimentel [84] investigated two full-scale footbridges both analytically and experimentally, and found that DLFs for resonant vertical harmonics (the first and second harmonics) were considerably lower than those reported in literature. The human-induced force differs from that measured on a rigid surface probably due to the interaction between humans and low-frequency structures like footbridges [114].

More recently, single step load models defining the ground reaction forces of single footsteps have been derived. As during the walking cycle a single foot is in discontinuous contact the supporting structure, the load applied by a single step is defined only for the duration of the contact (T_c). When assuming a perfectly periodic walking cycle, the corresponding continuous walking load is found as the sum of the single step load with a time shift of $T_s = f_s^{-1}$ [101]. Among different single step models reported in literature (e.g. [27, 93]), the model proposed by Li et al. [73] is described in detail in section 2.3.

Table 2.2 shows the DLF values for the single pedestrian force models proposed by Kerr [70], Young [120] and Li et al. [73].

The model in Eq.(2.7) is a deterministic model; however, many authors have highlighted that all the parameters in Eq.(2.7) should be modelled from a probabilistic point of view to account for the variability in walking forces induced by different pedestrians (e.g. [115]). This is the so-called *inter-subject variability* which represents that a different pedestrians will never produce exactly the same walking force. For a single person force, which is still assumed to be periodic, randomness can be taken into account by defining the probability distributions of the parameters defining the walking force. In particular, according to

Author(s)	DLFs for considered harmonics	Validity range
Kerr [70]	$\alpha_1 = -0.2649f_s^3 + 1.3206f_s^2 - 1.7597f_s + 0.7613$	$1 \leq f_s \leq 2.65 \text{ Hz}$
	$\alpha_2 = 0.07$	
	$\alpha_3 = 0.06$	
Young [120]	$\alpha_1 = 0.41(f_s - 0.95) \leq 0.56$	$1 \leq f_s \leq 2.8 \text{ Hz}$
	$\alpha_2 = 0.069 + 0.0112f_s$	
	$\alpha_3 = 0.033 + 0.0192f_s$	
	$\alpha_4 = 0.013 + 0.0260f_s$	
Li et al. [73]	$\alpha_1 = -0.0698f_s + 1.211$	$1.6 \leq f_s \leq 2.32 \text{ Hz}$
	$\alpha_2 = 0.1052f_s - 0.1284$	
	$\alpha_3 = 0.3002f_s - 0.1534$	
	$\alpha_4 = 0.0416f_s - 0.0288$	
	$\alpha_5 = -0.0275f_s + 0.0608$	$2.32 \leq f_s \leq 2.4 \text{ Hz}$
	$\alpha_1 = -0.1784f_s + 1.463$	
	$\alpha_2 = -0.4716f_s + 1.210$	
	$\alpha_3 = -0.0118f_s + 0.5703$	
	$\alpha_4 = -0.2600f_s + 0.6711$	
	$\alpha_5 = 0.0906f_s - 0.2132$	

Table 2.2: DLF values for single pedestrian force model proposed by different authors

experimental measurements and different models available in literature, the step frequency f_s , the pedestrian weight G , the DLF α_k and the walking velocity c_s can be considered as Gaussian random variables. As regards the phase angle Ψ , few measurements are available in literature and it is usually assumed as a random variable uniformly distributed in the interval $[0, 2\pi]$, as a consequence of the assumed random arrival of pedestrians (see, e.g., [113]).

In addition, the walking force caused by a single pedestrian is affected by the *intra-subject variability*, which means that even a single pedestrian induces a walking force that differs with each step, as some parameters such as step length and walking speed may vary along the path. The naturally present intra-subject variability makes the walking force not a perfectly periodic sinusoidal force, but rather a considerably more complex narrow band random process, as demonstrated by Brownjohn et al. [23]. This results in a ‘leak’ of energy into adjacent frequencies on either side of the fundamental pacing rate in the amplitude spectrum of a series of consecutive steps [86], reflecting a slight variation of the pacing rate during the walking path. The effect of the associated slightly varying pacing rate is even more pronounced for higher harmonics [23].

Another sensitive aspect of the force modelling regards the effect of people appearing in groups or crowd. The increase in load, in comparison with a single pedestrian force, is not directly proportional to the number of people involved. In fact, the load is directly proportional to the number of pedestrians only when they are perfectly synchronized. Neither only the static weight of the crowd should be taken into account (i.e. neglecting the dynamic effects), rather the actual behaviour is somewhere between the two scenarios

[114]. One of the first proposals to characterize the load induced by several pedestrians was given by Matsumoto et al. [76], who defined it in terms of multiplication of the load induced by a single pedestrian. They assumed that pedestrians arrive on the bridge following a Poisson distribution and then stochastically superimposed individual responses. On that basis, they found that the total response can be obtained by multiplying a single pedestrian response by the multiplication factor equal to $\sqrt{\lambda T_{cr}}$. T_{cr} represents the time required to cross over the footbridge while λ is the mean arrival rate, expressed as the number of pedestrians per second per width of the bridge. Hence, $\sqrt{\lambda T_{cr}}$ is equal to $\sqrt{n}$, where n is the number of pedestrians on the bridge at any time instant. Random vibration theory [80] states that when the response due to n equal and randomly distributed inputs is $\sqrt{n}$ times higher than the response due to a single input, these inputs (in this case pedestrians) are absolutely uncorrelated (unsynchronized) [114]. To sum up, the effect of a group of pedestrians can be evaluated by multiplying the response due to a single pedestrian by n if pedestrians are perfectly synchronized, or by $\sqrt{n}$ if they are totally unsynchronized. The use of the synchronization factor does not allow to account for the effective synchronization among pedestrians in case of nonrandom walking, which is likely to be somewhere between the perfect and unsynchronized case. Other researchers simulated the vertical forces from walking groups, among which Wheeler [118] and Mouring [78] who obtained results in agreement with Matsumoto et al.'s findings. However, Matsumoto et al.'s proposal does not consider the possibility of synchronization among people in a dense crowd, phenomenon which is likely to occur due to the limited space on the bridge deck [114].

Mathematical modelling of the pedestrian-induced dynamic force is a complicated task. However, design codes propose some force models for vibration serviceability assessment of footbridges, which are based on some simplifying assumptions. Existing walking models are typically based on measurements of single footfalls replicated at precise intervals [23]. This assumed perfect periodicity allows walking force to be modelled as a Fourier series based on the walking pace and its multiple integer [23], whether it is adopting continuous load models or single step models. Moreover, the force induced by a single person walking is usually modelled as a harmonic force having a frequency that matches one of the footbridge natural frequencies. This amounts to assume that, among the infinite number of harmonics a walking force is composed of, only a single harmonic is important for the vibration serviceability check [115]. Another usual assumption is that the footbridge can be modelled as a SDOF system, implying that only vibration in a single mode is of interest. However, there are footbridges, even with simple configurations, that respond to pedestrian-induced excitation in several vibration modes simultaneously, with more than one of them being important. These modes often can be excited by energy around different force harmonics. Therefore, it is necessary to take into account all relevant harmonics of the walking force. Moreover, as pedestrians usually appear in groups or crowd, the corresponding load should account for inter- and intra-subject-variability, which are not considered in the deterministic approaches usually adopted by the guidelines [115].

To address the shortcomings found in the existing design codes, Živanović et al. [115] propose a probability-based model which takes into account all the relevant harmonics of the walking forces. The model allows the calculation of the multi-mode response of the structure via the mode superposition principle [33]. Moreover, the model relies on a description of the pedestrian force in the time domain that can account for (i) the intra-

subject, variability thanks to the characterization of the induced force in the frequency domain, and (ii) the inter-subject variability via probability-based modelling of the walking parameters.

In the following, the single-step load model proposed by Li et al. [73] and the multi-harmonic force model proposed by Živanović et al. [115] are presented.

2.3 Periodic load model proposed by Li et al.

The dynamic walking force induced by a single pedestrian can be modeled as a series of moving single footsteps, for which the time-history of the vertical component can be described by a Fourier series. Li et al. [73] proposed a step-by-step load model to simulate the vertical ground reaction forces, whose model parameters are determined based on the registered body motion [103]. The step frequency f_s , along with the static weight G of the pedestrian, determines the vertical single foot force f_1 , defined by Li et al. [73] as:

$$f_1(t) = G \sum_{k=1}^5 \alpha_k \sin\left(\frac{\pi k}{T_c} t\right), \quad 0 \leq t \leq T_c \quad (2.8)$$

where α_k are the Fourier coefficients, normalized to the weight of the pedestrian, and T_c is the duration of the contact between the foot and the ground, both defined as a function of the step frequency f_s . It is assumed that the relevant contribution to the walking force is given by the first five harmonics. According to the statistical results of Ebrahimpour et al. [39], the ratio of the single foot force duration to the period during which both feet have contact with ground Δt (see Figure 2.1(a)) is basically insensitive to different walking speeds, and this ratio is 4.165 on average:

$$\frac{T_c}{\Delta t} = 4.165 \rightarrow \Delta t = 0.24 T_c \quad (2.9)$$

Thus, the cycle of the continuous walking force T_s , which is the inverse of the stride rate f_s , can be written as:

$$T_s = T_c - \Delta t = 0.76 T_c \quad (2.10)$$

Li et al. [73] propose the expressions of Fourier coefficients reported in Table 2.2.

To define the walking force, the step frequency is first assigned. From that, the cycle of the continuous walking force is obtained as $T_s = f_s^{-1}$ and in turn the contact duration is calculated by inverting Eq.(2.10), namely $T_c = T_s/0.76$. Then, consecutive footsteps are considered and their time-histories added together, obtaining the continuous walking force, as the one reported in Figure 2.1(b). If the pacing rate decreases, i.e. the walking speed decreases, the single foot contact duration increases and the overlapping Δt increases as well. This is shown in Figure 2.2(a), where the waking force obtained considering $f_s = 1.6$ Hz is reported. On the contrary, increasing pacing rate leads to decreasing contact duration and overlapping length, as obtained for the case $f_s = 2.0$ Hz presented in Figure 2.2(b). Values of T_s , T_c and Δt for the presented pacing rates are reported in Table 2.3.

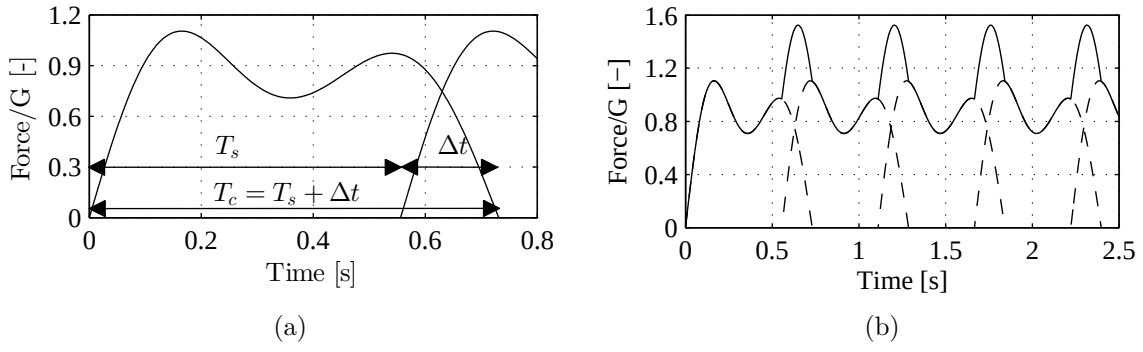


Figure 2.1: Normalized (a) single and (b) continuous foot force obtained for $f_s = 1.8$ Hz

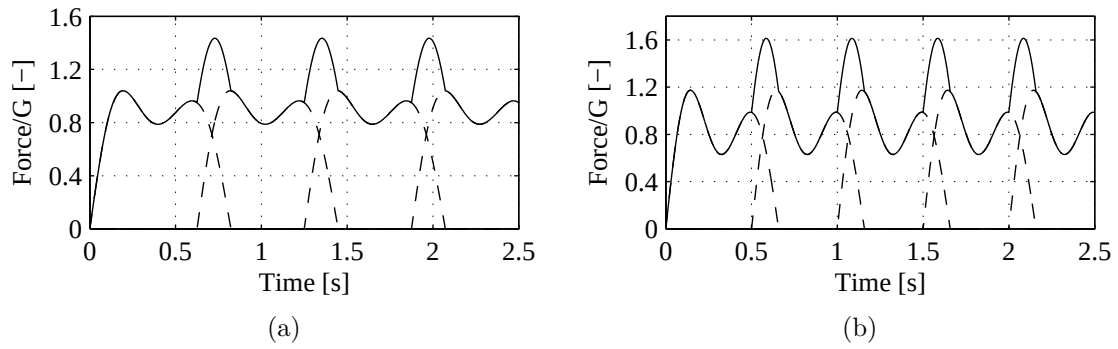


Figure 2.2: Normalized continuous foot force obtained for (a) $f_s = 1.6$ Hz and (b) $f_s = 2.0$ Hz

For the single foot force model no statistical characteristics are provided to assess the variability of the dynamic walking loads. This can be overcome by varying all the relevant parameters with each footstep in the simulation model, such as the step length and the step frequency. However, the intra-subject variability of the walking force along the path is more suited to be represented in the frequency domain. Finally, a stream of pedestrians can be created by defining a set of arrival times at the beginning of the bridge [100].

f_s [Hz]	T_s [s]	T_c [s]	Δt [s]
1.6	0.625	0.822	0.292
1.8	0.556	0.731	0.175
2.0	0.500	0.658	0.158

Table 2.3: Values of gait cycle T_s , single foot contact duration T_c and overlapping length Δt for different pacing rates

2.4 Multi-harmonic force model proposed by Živanović et al.

Vibration serviceability checks are usually based on the assumption that the pedestrian step frequency matches one of the structural natural frequencies and the footbridge responds in that single vibration mode. Moreover, the human-induced force is assumed to be a perfectly periodic sinusoidal force [6, 31, 20, 22, 61, 62]. By doing so, the inter and intra-subject variability in the walking force induced are neglected, and thus the footbridge vibration response can be significantly overestimated [23, 39, 70]. To account for the inter- and intra-subject variability of the walking force, in 2006 Živanović [112] developed a preliminary probability-based model for the case when a single force harmonic and the corresponding single mode response are sufficient for the vibration serviceability check. Then, to take into account that there are footbridges that respond to pedestrian-induced excitation in several vibration modes simultaneously, Živanović et al. [115] improved the probability-based model proposed in 2006. The improved model is to calculate the multi-mode structural response to a single person walking across a footbridge considering all relevant harmonics of the walking force.

The model contains not only the main harmonics usually dealt with in the literature, but also subharmonics that appear between main harmonics in the force spectrum [91]. In fact, as a consequence of the more pronounced footfall on one side, the force spectrum is characterized by main harmonics present at frequencies equal to the average walking frequency and its integer multiples as well as subharmonics appearing at frequencies between the main harmonics.

The inter-subject variability of the walking force is accounted for via probability-based modelling, i.e. by defining variables that describe the human-induced force via their probability density functions [112]. Parameters that describe the variability in walking forces induced by different pedestrians are the walking frequency, the step length and the magnitude of the walking force. On the contrary, the pedestrian's weight is assumed as a deterministic variable, equal to its mean value. According to statistics provided by the Department of Health of the United Kingdom, a mean pedestrian's weight of 750 N can be adopted.

Živanović [112] proposes to describe the walking frequency and the step length as two independent modelling parameters that follow the normal distributions:

$$f_s = \mathcal{N}(1.87, 0.186) \text{ [Hz]} \quad (2.11)$$

$$l_s = \mathcal{N}(0.71, 0.071) \text{ [m]} \quad (2.12)$$

The step frequency f_s , together with the step length l_s , defines the time T_{cr} required for footbridge crossing, i.e. the duration of the walking force. It is obtained as:

$$T_{cr} = \frac{L}{f_s l_s} \quad (2.13)$$

where L is the footbridge length.

To describe the amplitude of the walking force, it is necessary to define the amplitude of harmonics and subharmonics that make it up. Moreover, also the spreading of energy

Harmonic n.	μ	σ
1	$-0.2649f_s^3 + 1.3206f_s^2 - 1.7597f_s + 0.7613$ [70]	0.16μ [112]
2	0.07 [70]	0.03 [70]
3	0.06 [70]	0.02 [70]
4	0.05 [70]	0.02 [70]
5	0.03 [23, 115]	0.015 [23, 115]

Table 2.4: Parameters describing distribution of DLFs for the main harmonics

around the main harmonics and subharmonics has to be considered, which is related to the intra-subject variability. The amplitude of each (sub)harmonics is characterized through the dynamic loading factors and their probability distribution. Živanović et al. [115] deal with the first five harmonics of the walking force, assuming that higher harmonics do not induce perceptible vibrations in footbridges. The mean values μ and standard deviations σ describing the normal distributions of the first five harmonics are listed in Table 2.4. As regards the DLFs for the subharmonics, these data are missing in the literature available. Hence, Živanović et al. [115] propose a relationship between DLFs for the subharmonics and the DLF of the first walking harmonic α_1 . These are based on the analysis in the frequency domain of the time histories measured by Brownjohn et al. [23]. Proposed relations are the following:

$$\begin{aligned}
\alpha_1^s &= 0.026 \alpha_1 + 0.0031 \\
\alpha_2^s &= 0.074 \alpha_1 + 0.01 \\
\alpha_3^s &= 0.012 \alpha_1 + 0.016 \\
\alpha_4^s &= 0.013 \alpha_1 + 0.0093 \\
\alpha_5^s &= 0.015 \alpha_1 + 0.0072
\end{aligned} \tag{2.14}$$

Imperfections in the human walking force, i.e. the intra-subject variability, can be described via slight changes of the walking frequency and phase lag in each step. These changes are taken into account through the investigation of the force in the frequency domain described in section 2.4.1.

2.4.1 Force modelling

The model proposed by Živanović et al. [115] relies on a definition of the pedestrian force in the time domain, which is also able to represent the intra-subject variability of the walking force. For this purpose, it is necessary to characterize the spreading of energy around the (sub)harmonics of the walking force in the force spectrum. This is achieved by analysing and modelling of the frequency content of the walking force measured by Brownjohn et al. [23]. Starting from its frequency content, the force is finally reconstructed in the time domain.

The force induced by walking can be presented in the frequency domain via amplitudes and phases characterizing each frequency line in the force spectrum. The power of each (sub)harmonics is calculated in the range $\pm 0.25f_s$ and $\pm 0.25(0.5f_s)$ around each harmonic

k	1	2	3	4	5
$a_{k,1}$	0.785200	0.513000	0.390800	0.325500	0.280600
$b_{k,1}$	0.999900	2.000000	3.000000	4.000000	4.999000
$c_{k,1}$	0.008314	0.011050	0.009560	0.008797	0.007939
$a_{k,2}$	0.020600	0.133000	0.156700	0.164700	0.158400
$b_{k,2}$	1.034000	1.957000	3.000000	4.001000	5.004000
$c_{k,2}$	0.252400	0.263200	0.055250	0.066410	0.078250
$a_{k,3}$	0.107400	-0.049840	0.068660	0.068880	0.072890
$b_{k,3}$	1.001000	1.882000	2.957000	3.991000	4.987000
$c_{k,3}$	0.036530	0.058070	0.560700	0.375000	0.450100

Table 2.5: Fitting parameters for the first five harmonics

and subharmonic, respectively. Further, as the force model is related to the first five harmonics and subharmonics, it covers the frequency range $[0.25f_s, 5.25f_s]$. The normalized amplitude $\bar{\alpha}_k(\bar{f}_j)$ for the k th harmonic is given by [115]:

$$\bar{\alpha}_k(\bar{f}_j) = a_{k,1}e^{-\left(\frac{\bar{f}_j - b_{k,1}}{c_{k,1}}\right)^2} + a_{k,2}e^{-\left(\frac{\bar{f}_j - b_{k,2}}{c_{k,2}}\right)^2} + a_{k,3}e^{-\left(\frac{\bar{f}_j - b_{k,3}}{c_{k,3}}\right)^2} \quad (2.15)$$

where $a_{k,i}$, $b_{k,i}$ and $c_{k,i}$ ($i = 1, 2, 3$) are nine fitting parameters for the k th harmonic, listed in Table 2.5. $\bar{f}_j$ is the frequency ratio between the current frequency line and the step frequency f_s , belonging to the interval $[k - 0.25, k + 0.25[$. Hence, the variable $\bar{f}_j$ is in the range $[0.75, 1.25]$ for the first harmonic, $[1.75, 2.25]$ for the second harmonic and so on. Therefore, the spectrum width of $0.5f_s$ around each harmonic is taken into account when defining the normalized DLF for that harmonic. The normalized amplitudes for subharmonics $\overline{DLF}_k^s$ are fitted by the function [115]:

$$\bar{\alpha}_k^s(\bar{f}_j^s) = a_{k,1}^se^{-\left(\frac{\bar{f}_j^s - b_{k,1}^s}{c_{k,1}^s}\right)^2} + a_{k,2}^se^{-\left(\frac{\bar{f}_j^s - b_{k,2}^s}{c_{k,2}^s}\right)^2} \quad (2.16)$$

where $a_{k,i}^s$, $b_{k,i}^s$ and $c_{k,i}$ ($i = 1, 2$) are six fitting parameters for the k th subharmonic, listed in Table 2.6. $\bar{f}_j^s$ is the frequency ratio between the current frequency line and the step frequency f_s , belonging to the interval $[k - 0.75, k - 0.25]$. Hence, the variable $\bar{f}_j^s$ is in the range $[0.25, 0.75]$ for the first subharmonic, $[1.25, 2.75]$ for the second subharmonic and so on. Once the amplitudes in the spectrum of walking force is defined, information about the phase for each frequency line is needed to represent the force in the time domain. Based on the analysis of the phase spectra of measured forces, Živanović et al. [115] adopt a uniformly distributed random phase in the range $[-\pi, +\pi]$.

Finally, from the definition of amplitudes and phases in the spectrum of the walking force, the force in the time domain can be reconstructed. For the k th harmonic, occurring at frequency kf_s , the force in the time domain can be reconstructed as:

$$f_k(t) = G \alpha_k \sum_{\bar{f}_j=k-0.25}^{k+0.25} \bar{\alpha}_k(\bar{f}_j) \cos(2\pi \bar{f}_j f_s t + \Psi(\bar{f}_j)) \quad (2.17)$$

k	1	2	3	4	5
$a_{k,1}^s$	0.340600	0.302400	0.262700	0.234400	0.264500
$b_{k,1}^s$	0.498800	1.500000	2.500000	3.501000	4.499000
$c_{k,1}^s$	0.008337	0.008735	0.009748	0.009898	0.010190
$a_{k,2}^s$	0.280300	0.134500	0.245600	0.235500	0.238900
$b_{k,2}^s$	1.133000	1.532000	0.231200	-1.576000	1.153000
$c_{k,2}^s$	0.638800	0.723300	2.932000	7.050000	4.561000

Table 2.6: Fitting parameters for the first five subharmonics

while for the k th subharmonic it would be:

$$f_k^s(t) = G \alpha_k^s \sum_{\bar{f}_j^s=k-0.75}^{k-0.25} \bar{\alpha}_k^s(\bar{f}_j^s) \cos(2\pi \bar{f}_j^s f_s t + \Psi(\bar{f}_j^s)) \quad (2.18)$$

where, k is the (sub)harmonic considered, $\bar{f}_j f_s$ is a frequency line in the energy range of the analysed harmonic and $\Psi(\bar{f}_j)$ is the phase assigned to the spectrum line, based on the uniform distribution of phases in the range $[-\pi, +\pi]$, α_k is the DLF for the harmonic analysed, $\bar{\alpha}_k(\bar{f}_j)$ is the normalized amplitude for the same harmonic for each line and G is the average weight of a pedestrian. Superscript s is related to subharmonics.

Finally, the total force can be obtained adding the contribution of each (sub)harmonics:

$$f(t) = \sum_{k=1}^5 f_k(t) + \sum_{k=1}^5 f_k^s(t) \quad (2.19)$$

The force has to be reconstructed in the time domain for time equal to the crossing time T_{cr} for the person analysed. Thanks to the force in the time domain (Eq.(2.19)), is it possible to weight the human-induced force by mode shape. In section 2.4.2, this weighted modal force is adopted to calculate the structural modal response in a particular mode of vibration [115].

2.4.2 Evaluation of the structural response

The equation of motion of a supported beam travelled by a moving load is expressed in the following differential form:

$$m\ddot{q}(x, t) + c\dot{q}(x, t) + kq(x, t) = P(x, t) \quad (2.20)$$

where m , c and k are the mass, damping and stiffness per unit length and $P(x, t)$ is the pedestrian force. In the case the pedestrian is moving at a constant speed v_p along the centreline of the bridge deck, the force of the pedestrian can be represented as the product of the time component $f(t)$, i.e. the walking force of Eq.(2.19), and a component describing its time dependent position $\delta(x - v_p t)$, obtaining:

$$m\ddot{q}(x, t) + c\dot{q}(x, t) + kq(x, t) = f(t)\delta(x - v_p t) \quad (2.21)$$

where δ is the Dirac function and x the position of the pedestrian along the bridge centreline [102]. Based on the modal superposition principle [33], the vertical displacement $q(x, t)$ is obtained from the solution of the differential equation for the decoupled vibration modes. Each mode in the structure is modelled as a SDOF system with a load representing the pedestrian walking along the deck. Then, the contribution to the total displacement of the j th structural mode $q_j(x, t)$ is given by:

$$q_j(x, t) = p_j(t)\varphi_j(x) \quad (2.22)$$

where $p_j(t)$ is the structural response in the principal coordinates and $\varphi_j(x)$ is the j th mode shape. Substituting Eq.(2.22) into Eq.(2.21), the decoupled equation of motion for the j th mode is obtained:

$$m\ddot{p}_j(t)\varphi_j(x) + c\dot{p}_j(t)\varphi_j(x) + kp_j(t)\varphi_j(x) = f(t)\delta(x - v_pt) \quad (2.23)$$

where $\ddot{p}(t)$, $\dot{p}(t)$ and $p(t)$ are the modal acceleration, velocity and displacement respectively. Multiplying both sides of Eq(2.23) by the mode shape $\varphi_j(x)$, it becomes:

$$m_j\ddot{p}_j(t) + c_j\dot{p}_j(t) + k_jp_j(t) = f(t)\delta(x - v_pt)\varphi_j(x) \quad (2.24)$$

where $m_j = \varphi_j m \varphi_j$, $k_j = \varphi_j k \varphi_j$ and $c_j = \varphi_j c \varphi_j$ are the generalized mass, stiffness and damping, respectively. Dividing by the generalized mass m_j and integrating over the structure length x , the final expression of the equation of motion for the j th decoupled system is obtained:

$$\ddot{p}_j(t) + 2\xi_j\omega_j\dot{p}_j(t) + \omega_j^2p_j(t) = F_j(t) \quad (2.25)$$

with:

$$F_j(t) = \int_0^L \frac{f(t)}{m_j} \varphi_j(x) \delta(x - v_pt) dx = \frac{f(t)}{m_j} \varphi_j(x) \quad (2.26)$$

The modal damping ξ_j is defined as:

$$\xi_j = \frac{c_j}{c_{cj}} = \frac{c_j}{2\omega_j m_j} \quad (2.27)$$

where ω_j is the modal frequency in rad/s and c_{cj} is the critical damping coefficient. Thanks to the relation $x = v_pt$ that gives the position of the load as a function of time, the mode shape $\varphi_j(x)$ can be transform from the space to the time domain obtaining:

$$\ddot{p}_j(t) + 2\xi_j\omega_j\dot{p}_j(t) + \omega_j^2p_j(t) = \frac{f(t)}{m_j} \varphi_j(t) \quad (2.28)$$

The right hand side of the equation represents the modal force acting on the SDOF system, expressed as the product of the walking force $f(t)$ and the mode shape $\varphi_j(t)$. The modal force is used to calculate the footbridge modal response based on the time integration of the equation formulated for the modal coordinates. Then, the modal responses obtained for individual vibration modes have to be added according to the mode superposition principle [33].

Chapter 3

Experimental investigations and simulation of the structural response

The full assessment of the footbridge dynamic response under human-induced loading starts from the characterization of the structural dynamic behaviour. The reliability of the predicted structural response depends on the capability of representing the structural dynamic behaviour as well as the adoption of adequate models to simulate the pedestrian action. The present chapter is devoted to the characterization of the dynamic behaviour of the Pasternak and Correggio footbridges and the simulation of their structural response to human-induced loading. The pedestrian load models described in chapter 2 are adopted.

Structural dynamic identification is considered as the applied methodology to characterize the behaviour of a structure on the basis of measurements obtained from an array of sensors, usually accelerometers. The monitoring process involves the observation of a system over time by means of periodically sampled dynamic response measurements from the array of sensors with a suitable sampling rate. Thanks to experimental data, the real response of the structure to external and internal excitations (e.g. wind, vehicle and pedestrian traffic, seismic events, mass movements, mechanical shakers...) can be investigated also considering the influence of different environmental conditions, such as the temperature shifts.

Starting from experimental measurements, the structural dynamic properties are identified by the application of identification techniques. In the present study, the modal properties are identified through the classic Enhanced Frequency Domain Decomposition (EFDD) method, which is based on the diagonalization of the spectral density matrix of the acquired accelerations. The algorithm is described in section 3.2.

To simulate the dynamic response of footbridges under human-induced loads, FE models can be required. Due to the large number of uncertainties and the unavoidable simplifications performed during the modelling process, the real structural behaviour is different from the description obtained with structural models [56]. FE models can be improved using experimental data from the identification process [26]. In this respect, model updating approaches based on surrogate-assisted evolutionary strategies have shown advantages in reducing the computational effort in optimization problems [69]. Here, the so called DE-Q algorithm [52, 111] is adopted to develop reliable models that can accurately simulate the dynamic behaviour of real structures. A brief overview of the updating procedure is pre-

sented in section 3.3.

Furthermore, thanks to experimental investigations, the actual response to human-induced loads can be characterized and employed to evaluate the reliability of the simulated response.

The chapter is organized as follows. First, the acquisition system adopted for the dynamic tests is presented in section 3.1. Second, the identification algorithm and the calibration procedure are described. Experimental tests, identification results and response simulations of the Pasternak and Correggio footbridges are presented in sections 3.4 and 3.5, respectively.

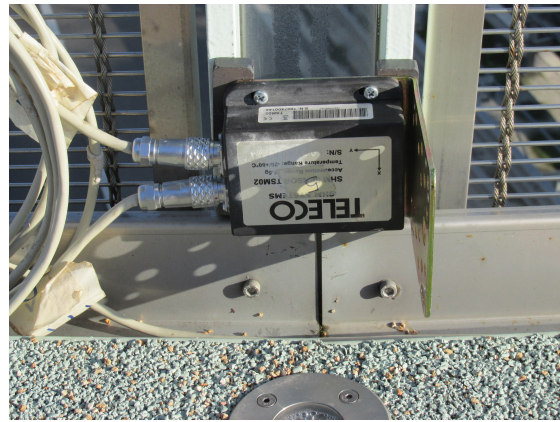
3.1 The acquisition system

The main components of traditional Structural Health Monitoring (SHM) systems include several analog sensors, such as strain gauges, accelerometers and temperature sensors connected to multichannel data loggers through signal conditioning units. Over years, new advanced tools based on microelectronics and microprocessor technologies and wireless communications have been developed and increasingly integrated in monitoring systems. Current SHM systems more and more frequently combine advanced sensor technologies and distributed computational power as well as efficient implementations of identification methodologies. These advanced systems rely on intelligent sensors that elaborate local models and exchange data, information and models on a local network under the supervision of a control and storage unit accessible also on line. Particularly, the introduction of MEMS sensors in SHM systems has increasingly allowed the design of systems that conjugate a reduced cost with adequate performances suitable for structural applications.

Recently, an advanced MEMS-based SHM system, named SHM602, has been developed by the University of Bologna, the University of Modena and Reggio Emilia and engineered by Teleco S.p.a. [56]. It was designed compliant with the recommendations reported in [48]. The SHM602 system is based on an advanced architecture composed of digital bus and MEMS sensors.



(a)



(b)

Figure 3.1: Acquisition system: (a) the main unit and (b) a biaxial sensor

Experimental investigations performed on the Pasternak and Correggio footbridges have been one of the first chance to test the SHM602 system on full-scale measurements. Previously, the system was employed to characterize the dynamic behaviour of the Tower of the School of Engineering of the Bologna University [56] and of a masonry bell tower located in Ficarolo (Italy) [13].

The main components of the SHM602 system consist in a controller and storage unit (Figure 3.1(a)) and several intelligent sensing units (Figure 3.1(b)) connected to the main unit by means of a serial bus. The sensor bus connection assures a high degree of reliability and prevention against electromagnetic interferences. Each sensing unit can record the accelerations along two orthogonal axes as well as the temperature. The sampling frequency can be selected by the user in the range $[20, 80]$ Hz. Thanks to local digital filtering techniques and oversampling rates implemented, these units based on MEMS sensors can exhibit a noise floor in the range $[0.3, 0.5]$ mg [56]. In literature, higher noise values referring to other MEMS-based systems are reported (e.g. [92]). The main features of the SHM602 system are the transmission of data in digital form and the possibility of performing some system analyses directly on-board of the sensors, transmitting the processed synthetic data to the main computer. Moreover, due to its high signal-to-noise ratio even in the low and medium-low frequency range and its adaptability, the SHM602 system is a very suitable tool for the monitoring of flexible structures as footbridges and tall towers as well as for more stiff structures, such as historical buildings and short-span bridges.

3.2 The identification algorithm

Modal parameter identification is a procedure to identify dynamic properties of a vibrating structure starting from experimental tests. Modal parameters can be estimated from measurements in time and frequency domain. These measurements can include free-decays, forced responses, frequency response functions or impulse response functions. The structural response can be the dynamic result of a known excitation (force vibration test) or unknown excitation (impact test or ambient test).

The estimation of the modal parameters starting from output measurements only is known as stochastic system identification, where the structure is excited by an unmeasurable input force. In this case, deterministic knowledge of inputs is replaced by the assumption that the input is a realization of a stochastic process, i.e. a white noise [109].

The Frequency Domain Decomposition (FDD) method was introduced by Brincker et al. [19] and subsequently enhanced [18] in order to extract modal damping factor estimate. The Enhanced Frequency Domain Decomposition (EFDD) method is an extension of the frequency domain approach referred to as the Peak Picking technique. The latter relies on simple signal processing using the Discrete Fourier Transform (DFT) and the estimation of the modes directly from the PSD matrix at the peak [15]. However, this technique gives reliable estimates of natural frequencies and mode shapes only if the modes are well separated. On the contrary, the EFDD method allows the identification of close spaced modes as well.

The EFDD method is for the modal identification from ambient response, i.e. when the loads are unknown and the identification has to be carried out based on the response only. Hence, the assumption of white noise input is made.

The EFDD methods relies on the singular value decomposition (SVD) of the PSD matrix of the acquired accelerations. As such, the spectral matrix is decomposed into a set of auto spectral density functions, each corresponding to a single degree of freedom (SDOF) system. The decomposition into SDOF systems allows the close spaced modes to be clearly detected. Considering a structural system characterized by input $x_i(t)$, with $i = 1, \dots, N_i$, and output $y_o(t)$, with $o = 1, \dots, N_o$, signals, the DFT of all the input and output functions is first performed. The Fourier transforms of the input and output signals are referred to as $X_i(i\omega)$ and $Y_o(i\omega)$, respectively. Then, the PSD or auto spectral density of the signal is calculated as:

$$S_{oo}(i\omega) = \overline{Y}_o(i\omega) Y_o(i\omega) = |Y_o(i\omega)|^2 \quad (3.1)$$

where “-” denotes the complex conjugate. Similarly, the cross-PSD of two signals is given by:

$$S_{oi}(i\omega) = \overline{Y}_o(i\omega) X_i(i\omega) \quad (3.2)$$

The PSD matrix is obtained assembling the PSD functions (on the diagonal elements) and the cross-PSD functions (off-diagonal elements) as follows:

$$\mathbf{S}_{\mathbf{xx}}(i\omega) = \begin{bmatrix} \overline{X}_1(i\omega)X_1(i\omega) & \overline{X}_1(i\omega)X_2(i\omega) & \dots & \overline{X}_1(i\omega)X_{N_i}(i\omega) \\ \overline{X}_2(i\omega)X_1(i\omega) & \overline{X}_2(i\omega)X_2(i\omega) & \dots & \overline{X}_2(i\omega)X_{N_i}(i\omega) \\ \dots & \dots & \dots & \dots \\ \overline{X}_{N_i}(i\omega)X_1(i\omega) & \overline{X}_{N_i}(i\omega)X_2(i\omega) & \dots & \overline{X}_{N_i}(i\omega)X_{N_i}(i\omega) \end{bmatrix} = \overline{\mathbf{X}}(i\omega)\mathbf{X}(i\omega)^T \quad (3.3)$$

$$\mathbf{S}_{\mathbf{yy}}(i\omega) = \begin{bmatrix} \overline{Y}_1(i\omega)Y_1(i\omega) & \overline{Y}_1(i\omega)Y_2(i\omega) & \dots & \overline{Y}_1(i\omega)Y_{N_o}(i\omega) \\ \overline{Y}_2(i\omega)Y_1(i\omega) & \overline{Y}_2(i\omega)Y_2(i\omega) & \dots & \overline{Y}_2(i\omega)Y_{N_o}(i\omega) \\ \dots & \dots & \dots & \dots \\ \overline{Y}_{N_o}(i\omega)Y_1(i\omega) & \overline{Y}_{N_o}(i\omega)Y_2(i\omega) & \dots & \overline{Y}_{N_o}(i\omega)Y_{N_o}(i\omega) \end{bmatrix} = \overline{\mathbf{Y}}(i\omega)\mathbf{Y}(i\omega)^T \quad (3.4)$$

where the superscript T denotes the transpose. $\mathbf{S}_{\mathbf{xx}}(i\omega)$ is the $N_i \times N_i$ PSD matrix of the inputs and $\mathbf{S}_{\mathbf{yy}}(i\omega)$ is the $N_o \times N_o$ PSD matrix of the responses. These matrices are both Hermitian, i.e. matrices with complex entries equal to their own conjugate transpose.

The PSD matrices of the unknown inputs and the measured responses can be related through the $N_o \times N_i$ FRF matrix [15]:

$$\mathbf{S}_{\mathbf{yy}}(i\omega) = \overline{\mathbf{H}}(i\omega)\mathbf{S}_{\mathbf{xx}}(i\omega)\mathbf{H}(i\omega)^T \quad (3.5)$$

The FRF can be written in the pole/residue form as [19]:

$$\mathbf{H}(i\omega) = \sum_{j=1}^N \frac{R_j}{i\omega - \lambda_j} + \frac{\overline{R}_j}{i\omega - \overline{\lambda}_j} \quad (3.6)$$

where N is the number of modes, λ_j is the pole and R_j is the residue:

$$R_j = \varphi_j \gamma_j^T \quad (3.7)$$

with φ_j and γ_j the mode shape vector and the modal participation vector, respectively. When the input is a white noise, its PSD is a constant matrix, i.e. $\mathbf{S}_{xx}(i\omega) = C$. Hence, Eq.(3.5) becomes:

$$\mathbf{S}_{yy}(i\omega) = \sum_{j=1}^N \sum_{s=1}^N \left[\frac{R_j}{i\omega - \lambda_j} + \frac{\bar{R}_j}{i\omega - \bar{\lambda}_j} \right] C \left[\frac{R_s}{i\omega - \lambda_s} + \frac{\bar{R}_s}{i\omega - \bar{\lambda}_s} \right]^H \quad (3.8)$$

where superscript H denotes complex conjugate and transpose. Multiplying the two partial fraction factors and adopting the Heaviside partial fraction theorem, the PSD of the responses can be reduced to the pole/residue form [19]:

$$\mathbf{S}_{yy}(i\omega) = \sum_{j=1}^N \frac{\mathbf{A}_j}{i\omega - \lambda_j} + \frac{\bar{\mathbf{A}}_j}{i\omega - \bar{\lambda}_j} + \frac{\mathbf{B}_j}{-i\omega - \lambda_j} + \frac{\bar{\mathbf{B}}_j}{-i\omega - \bar{\lambda}_j} \quad (3.9)$$

being $\mathbf{A}_j$ the j th residual matrix of the output PSD. Just like the output PSD itself, the residue matrix is a $N_o \times N_o$ Hermitian matrix given by:

$$\mathbf{A}_j = R_j C \left[\sum_{s=1}^N \frac{\bar{R}_s^T}{-\lambda_k - \bar{\lambda}_s} + \frac{R_s^T}{-\lambda_k - \lambda_s} \right] \quad (3.10)$$

The contribution to the residue form from the j th mode is given by:

$$\mathbf{A}_j = \frac{R_j C \bar{R}_j}{2\alpha_j} \quad (3.11)$$

with α_j being minus the real part of the pole $\lambda_j = -\alpha_j + i\omega$. When the system is lightly-damped, the term of Eq.(3.11) becomes the dominant one, and thus the residue results proportional to the mode shape vector:

$$A_j \propto R_j C \bar{R}_j = \varphi_j \gamma_j^T C \varphi_j \gamma_j^T = d_j \varphi_j \varphi_j^T \quad (3.12)$$

where d_j is a scalar factor. At a certain frequency ω , only a few modes (typically one or two) contribute significantly, denoted by $Sub(\omega)$. Hence, when the system is lightly-damped, the response PSD matrix can be written:

$$\mathbf{S}_{yy}(i\omega) = \sum_{j \in Sub(\omega)} \frac{d_j \varphi_j \varphi_j^T}{i\omega - \lambda_j} + \frac{\bar{d}_j \bar{\varphi}_j \bar{\varphi}_j^T}{i\omega - \bar{\lambda}_j} \quad (3.13)$$

Starting from the recorded signals, the output PSD matrix is defined at discrete frequencies $\omega = \omega_z$ and then decomposed taking the SVD of the matrix:

$$\mathbf{S}_{yy}(i\omega_z) = \begin{bmatrix} \mathbf{u}_{z1} & \mathbf{u}_{z2} & \dots & \mathbf{u}_{zN_o} \end{bmatrix} \begin{bmatrix} s_{z1} & & & \\ & s_{z2} & & \\ & & \dots & \\ & & & s_{zN_o} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{z1}^H \\ \mathbf{u}_{z2}^H \\ \dots \\ \mathbf{u}_{zN_o}^H \end{bmatrix} = \mathbf{U}_z \mathbf{\Sigma}_z \mathbf{U}_z^H \quad (3.14)$$

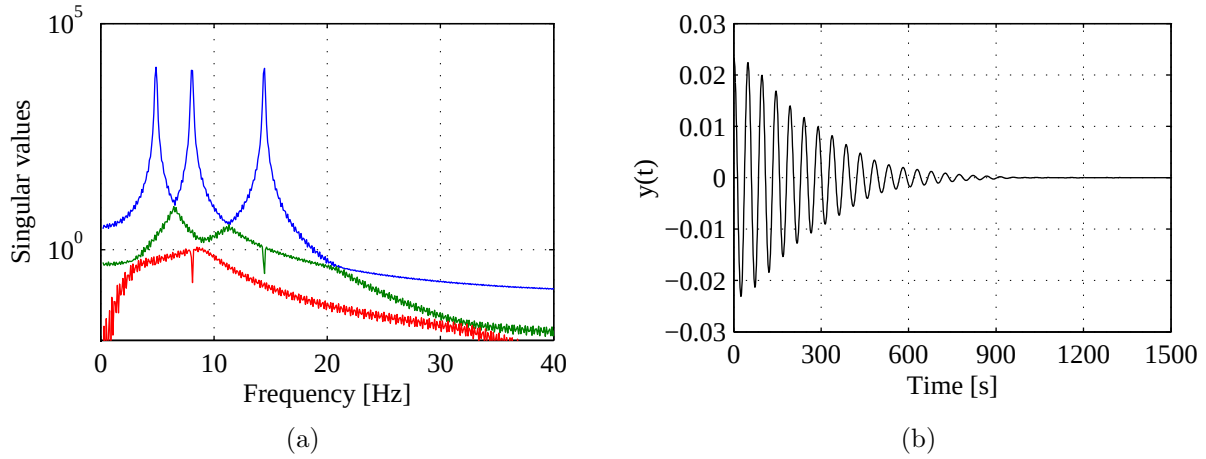


Figure 3.2: (a) Singular value functions and (b) SDOF auto correlation function

where Σ_z is a $N_o \times N_o$ diagonal matrix whose elements are the singular values s_{zi} and $\mathbf{U}_z$ is a $N_o \times N_o$ matrix containing the corresponding singular vectors $\mathbf{u}_z$. The matrix $\mathbf{U}_z$ is an unitary matrix, namely $\overline{\mathbf{U}}_z \mathbf{U}_z = \mathbf{U}_z \overline{\mathbf{U}}_z = \mathbf{I}$.

Thanks to the singular value decomposition of the PSD matrix $\mathbf{S}_{yy}$, the information contained in every output signal $y_o(t)$ can be grouped together in the plot of the singular values, as the one shown in Figure 3.2(a). As the singular values are sorted in descending order, i.e. $s_{z1} > s_{z2} > \dots > s_{zN_o}$, the blue line corresponds to the first singular value, the green line to second one and the red line to the third one.

Near a peak corresponding to the j th mode in the spectrum, this mode (or a possible close mode) is dominating, and thus, only one term remains in Eq.(3.13):

$$\mathbf{S}_{yy}(i\omega)_{\omega \rightarrow \omega_j} = d_j \varphi_j \overline{\varphi_j}^T \quad (3.15)$$

and Eq.(3.14) becomes:

$$\mathbf{S}_{yy}(i\omega_z)_{\omega_z \rightarrow \omega_j} = s_{z,1} \mathbf{u}_{z,1} \overline{\mathbf{u}}_{z,1}^T \quad (3.16)$$

where the first singular value is the highest one. Comparing Eqs.(3.15) and (3.16), it can be deduced that the singular vector is an estimate of the j th mode shape:

$$\hat{\varphi}_j = \mathbf{u}_{z,1} \quad (3.17)$$

and the corresponding singular value $s_{z,1}$ is the PSD function of the corresponding SDOF system, which represents the structure response amplification under dynamic loads (resonance in correspondence of a natural mode). The PSD function of the corresponding SDOF system is identified around the peak by comparing the estimated mode shape $\hat{\varphi}_j$ with the singular vectors for the frequency lines around the peak. As long as a singular vector is found that has high MAC [46] value with $\hat{\varphi}_j$, the corresponding singular values belongs to the SDOF density function [19].

To have a more stable and reliable estimation of the mode shape, all the singular vectors corresponding to the singular values of the SDOF density function are considered and the mode shape is obtained as the average of them. The average is weighted considering each singular values as a weighting factor for the corresponding singular vector such that the

singular vectors close to the peak have greater importance.

The natural frequency of the j th natural mode is obtained from the peak value of the SDOF density function (peak picking).

Since $s_{z1} > s_{z2} > \dots > s_{zN_o}$ for all frequency lines, the natural modes can be estimated from the plot of the first singular values. Normally, only the first singular value can be considered, although, when two natural modes are characterized by the same frequency, the first two singular values and corresponding singular vectors have to be accounted for. Finally, the SDOF density function around the peak is taken back to the time domain by Inverse Fast Fourier Transform (IFFT), obtaining the corresponding SDOF auto correlation function. An example of the SDOF auto correlation function is shown in Figure 3.2(b). From the time domain function, the damping is estimated via the logarithmic decrement [18, 19], measuring the amplitudes of two following maxima ($y_{max}(a)$ and $y_{max}(a + 1)$):

$$\xi_j = \frac{1}{2\pi} \ln \frac{y_{max}(a)}{y_{max}(a + 1)} \quad (3.18)$$

Finally, from the *crossing times* of the autocorrelation function, the natural frequency of the j th mode can be evaluated and compared with the one obtained from the peak of the SDOF PSD function.

3.3 Finite element model updating

Model updating procedures allow to obtain numerical models that can accurately simulate the dynamic behaviour of real structures. A FE model of the structure is built and then adjusted by means of a calibration procedure so that a better correlation between numerical and experimental frequencies and mode shapes can be achieved. In the present study, the optimization process is carried out through an improved surrogate-assisted evolutionary algorithm. Surrogate-assisted evolutionary strategies use efficient computational models, as response surface or high polynomial functions, to approximate the objective function. Recently, they received considerably increasing interest in reducing the computational effort in optimization problems, mainly when the evaluation of the objective function is highly time consuming [69]. Here, the so-called DE-Q algorithm is adopted [52, 111], which combines the robustness of the DE algorithm [96], thanks to the simultaneous use of multiple search points, with the computational efficiency due to a second-order surrogate approximation of the objective function. The optimal parameters are obtained through the minimization of an objective function defined as the relative error between modal frequencies and mode shapes obtained experimentally ($\bar{\omega}_j, \bar{\varphi}_j$) and those obtained adopting a given set of identification parameters (ω_j, φ_j) in the numerical model, i.e.:

$$H(\mathbf{x}) = \sum_{j=1}^N \left[w_1 \left(\frac{\omega_j - \bar{\omega}_j}{\bar{\omega}_j} \right)^2 + w_2 \text{NMD}_j^2 \right] \quad (3.19)$$

where $\mathbf{x}$ denotes the D -dimensional vector of unknown mechanical parameters and NMD is the so called *Normalized Modal Difference* [53], defined as:

$$\text{NMD}_j = \sqrt{\frac{1 - \text{MAC}(\varphi_j, \bar{\varphi}_j)}{\text{MAC}(\varphi_j, \bar{\varphi}_j)}} \quad (3.20)$$

In Eq.(3.19), N is the number of mode shapes considered and w_1 and w_2 are weight constants. The optimization procedure is performed adopting as unknown parameters those that mainly influence the structural behavior of the footbridge, such as the mass of the deck or the stiffness of some elements.

3.4 The Pasternak footbridge

In this section, the Pasternak footbridge, a deck curved cable-stayed footbridge, is described and analysed. Over the last few decades, the advantages of cable-stayed footbridges have become known worldwide and thus they have gained much popularity [97, 98]. Some advantages of these kinds of structures are in the reduction of the deck bending moments thanks to the load transfer to the cables and the ease of construction due to the same flow of forces present during free-cantilever construction stages as well as after completion. Moreover, compared to suspension bridges, cable-stayed bridges are characterized by greater inherent stiffness and higher eigenfrequencies [26]. However, cable-stayed footbridges are generally very slender and thus very sensitive to dynamic actions induced by wind and pedestrians. This feature is also increased by the inherent scarce damping properties of these kinds of structures [35, 95].

Hence, due to its inherent features, the Pasternak footbridge is very sensitive to human-induced loadings. Furthermore, its curved shape makes it significantly more sensitive to vertical actions rather than horizontal ones [11, 12].

3.4.1 Description of the footbridge

The Pasternak footbridge (Figures 3.3) was built in 2008 and overpasses an important free-way crossroad in Modena (Italy). The footbridge is about 270 meters long and composed of a curved central span 60 meters long and two access ramps with bent layout. The main span is sustained by cables linked with two V-shaped steel towers, each of them composed of inclined masts connected with each other by X-bracing (Figures 3.3(c)). Each mast is approximately 18 meters high with an inclination of about 15 degrees with respect to the vertical. Lateral access ramps are composed of several spans supported by V-shaped precast concrete piers. The structure is about 3 meters wide and has a constant radius of approximately 32 meters. The deck is composed of steel trusses that support a concrete slab. The main span stiffening spatial truss girder (Figure 3.3(d)) is composed of circular hollow profiles, connected with the steel-concrete deck. At the deck level, trusses are made up of two open-shaped concrete-filled chords connected in plane with H- and L-shaped profiles. The masts are constituted by prismatic hollow profiles with variable sections linked with each other through H-shaped bracing by means of bolted connections. Each cable is composed of two or three elements connected with each other and anchored to the towers with plates and bolts. Finally, a steel parapet is present along the entire length of the footbridge.

The following investigations and analyses are focused on the behaviour of the central span of the footbridge.



(a)



(b)



(c)



(d)

Figure 3.3: The Pasternak footbridge: (a,b) general views and details of the (c) V-shaped steel tower and (d) spatial truss girder

3.4.2 The dynamic tests

3.4.2.1 Experimental setup

The dynamic tests were performed by means of the acquisition system described in section 3.1, composed of the main unit and 12 sensing units. To estimate as many natural modes as possible, the sensors were arranged into several setups, shown in Figure 3.4. They were installed on the vertical elements of the steel parapets at the deck level by means of magnets. Sensors were mainly placed in the curved central span to acquire accelerations in the vertical and transverse horizontal direction. One sensor was located in correspondence of a tower, to measure accelerations in the radial (transverse horizontal) and longitudinal directions. In addition, a few sensors in the main span were installed to record vertical and longitudinal accelerations. However, longitudinal accelerations are found to be significantly lower than the others, with amplitudes almost comparable to the instrument noise. In the

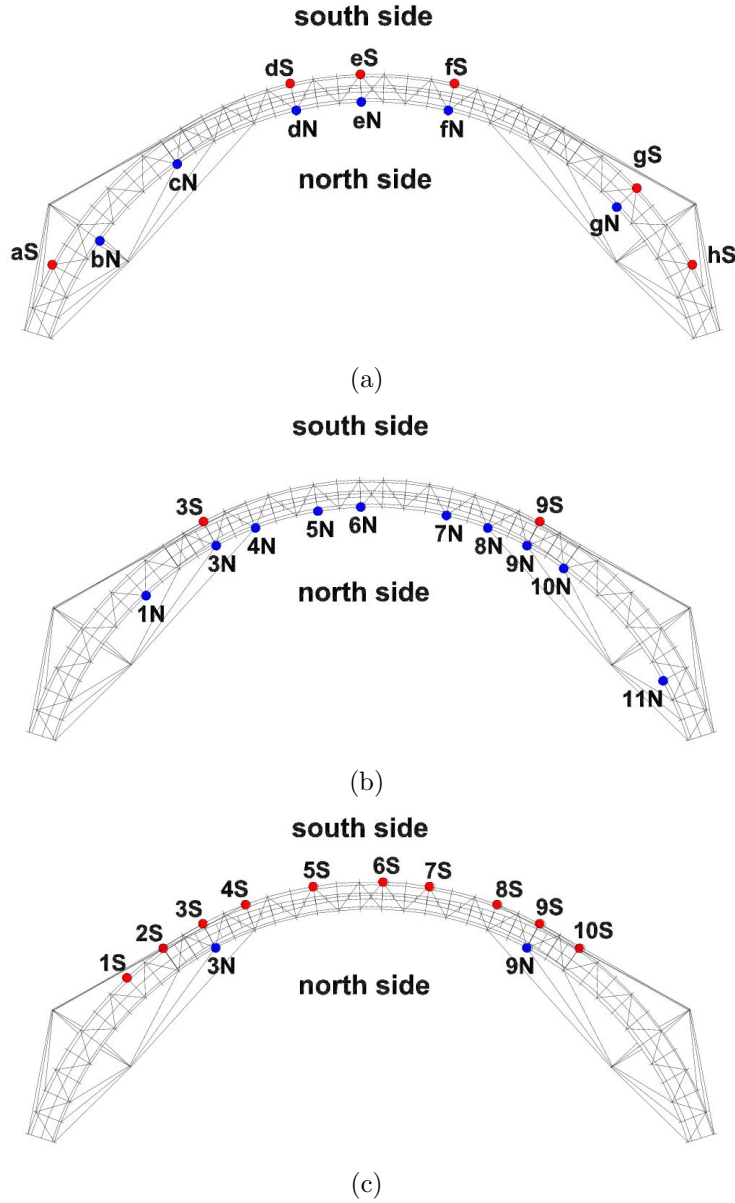


Figure 3.4: Experimental setup no. (a) 1, (b) 2 and (c) 3

first setup, sensors were arranged quite equally spread on both deck sides along the whole central span. In the second and third setup, sensors were mainly located on the north or the south side of the central span to better identify the greater number of modes. For all test arrangements, some sensors were placed at the same locations to combine data obtained from different setups. Mode shapes are defined by combining mode shape components obtained from the second and third setup, for a total amount of 20 positions. For this reason, the reference numeration is related to these two setups.

During the tests, both ambient and human-induced accelerations have been acquired. The former are adopted to identify the modal properties of the footbridge while the latter are the basis for comparison with the numerically predicted accelerations.

3.4.2.2 Accelerations recorded in operational conditions

An example of accelerations recorded in operational conditions is reported in Figure 3.5. The vertical acceleration caused by wind and traffic reaches values of about 5 mg while the transverse horizontal acceleration is about 1.5 mg. In fact, due to the curved shape of the footbridge, vibrations in transverse direction are lower than those in the vertical one. Longitudinal horizontal accelerations are not reported as they have negligible amplitude compared to the vertical and transverse ones.

3.4.2.3 Pedestrian-induced accelerations

To study the dynamic behaviour of the footbridge subjected to pedestrian loads, several experimental tests were performed with pedestrians crossing the footbridge running and free or synchronized walking with different pacing frequencies (Figure 3.6). To achieve a better synchronization among pedestrians, a megaphone and metronome were always used to control the step frequency, which generally ranged between 1.2 Hz and 2.2 Hz. A number of crowd tests were carried out with different-sized groups of pedestrians walking on the footbridge, some of which are presented in the following.

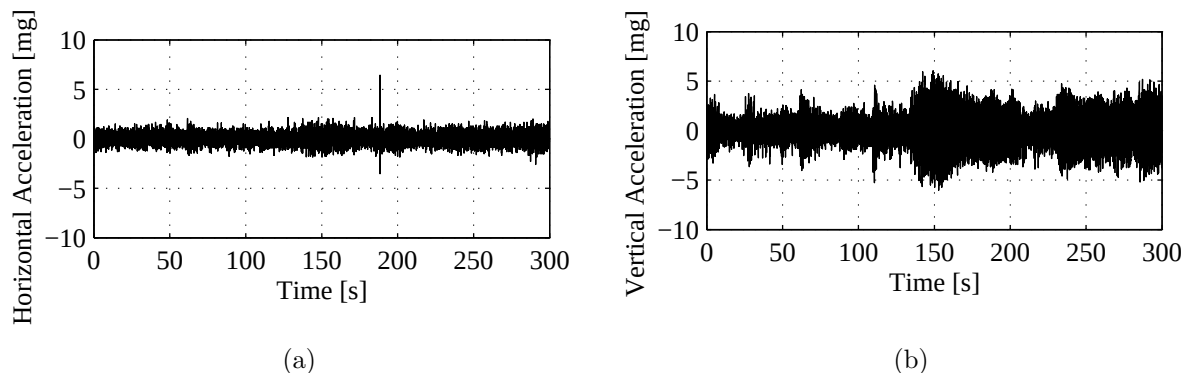


Figure 3.5: (a) Horizontal and (b) vertical midspan acceleration recorded in operational conditions



Figure 3.6: Examples of dynamic tests: (a) 12 pedestrians walking in single file on the north side of the deck and (b) 44 pedestrians walking in a closed loop on the main span

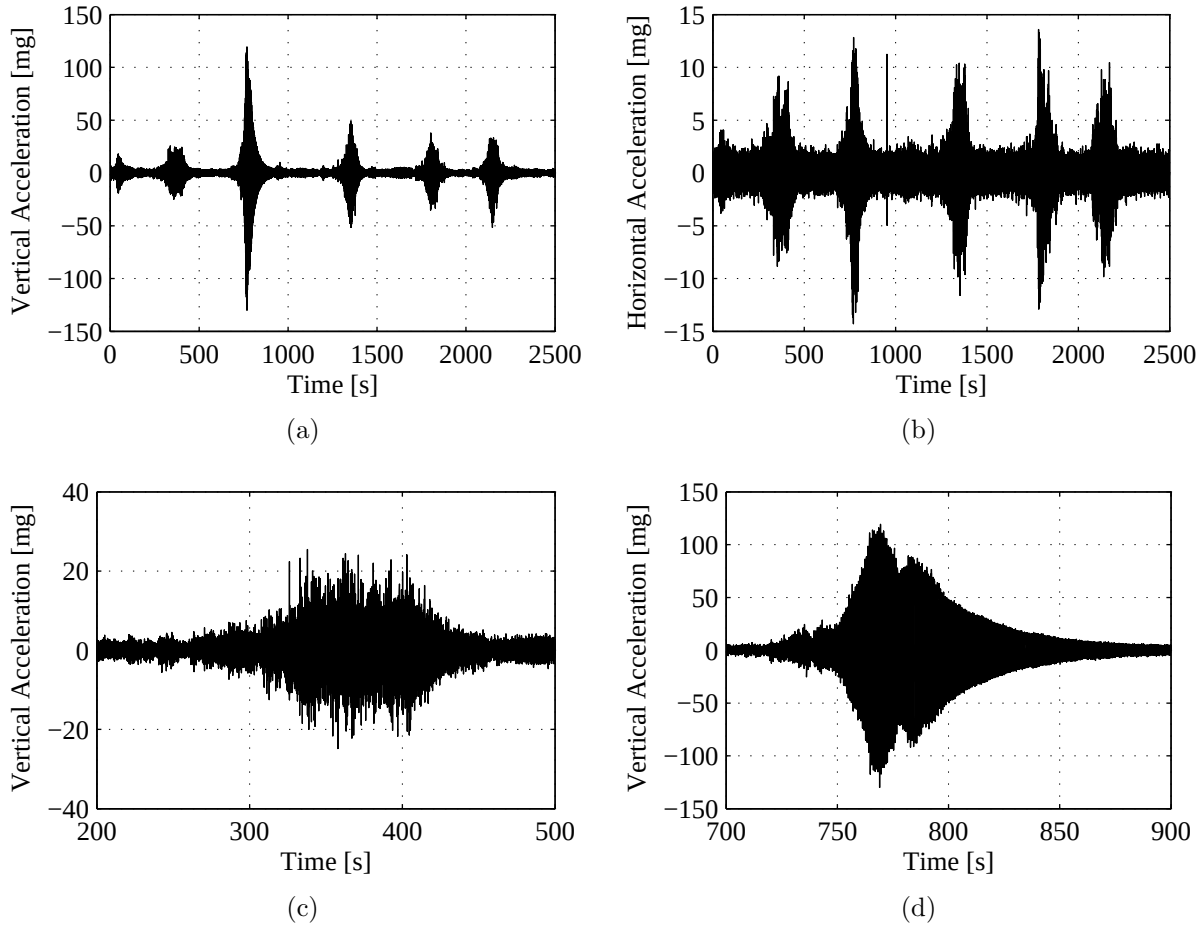


Figure 3.7: (a) Vertical and (b) transverse horizontal accelerations measured by the sensor 6N caused by 12 pedestrian crossing the footbridge at 1.2, 1.45, 1.6, 1.6 and 1.8 Hz and details of the footbridge crossing at (c) 1.2 Hz and (d) 1.45 Hz

Figures 3.7(a) and 3.7(b) present the vertical and transverse horizontal acceleration caused by successive footbridge crossings of 12 pedestrians walking in single file, on the north or south side. In these tests pedestrians walked with a step frequency of 1.2, 1.45, 1.6, 1.6 and 1.8 Hz, respectively. The step frequency of 1.45 Hz was chosen as near-resonant with the first vertical mode (see section 3.4.2.4). The vertical acceleration caused by pedestrians walking at 1.45 Hz reaches amplitudes higher than 100 mg (where g indicates the gravity acceleration) corresponding to 1 m/s^2 . On the contrary, it is significantly lower in the other cases ($\approx [30, 50] \text{ mg}$), without any evidence of excessive vibration amplitude. Moreover, the transverse horizontal acceleration shows values of one order of magnitude lower than the vertical one, evidence that the footbridge is more sensitive in the vertical direction than in the lateral one. Details of the tests with pedestrians walking at 1.2 and 1.45 Hz are reported in Figures 3.7(c) and 3.7(d). Maximum accelerations of $[20, 25] \text{ mg}$ are reached when walking at 1.2 Hz.

In addition, several tests were carried out arranging a stream of 44 walking pedestrians that completely filled the deck. For instance, Figures 3.8(a) and 3.8(b) present the vertical and transverse acceleration caused by the pedestrian group walking at 1.2 and 1.45 Hz, respectively. Also in this case the highest acceleration is obtained when the step frequency

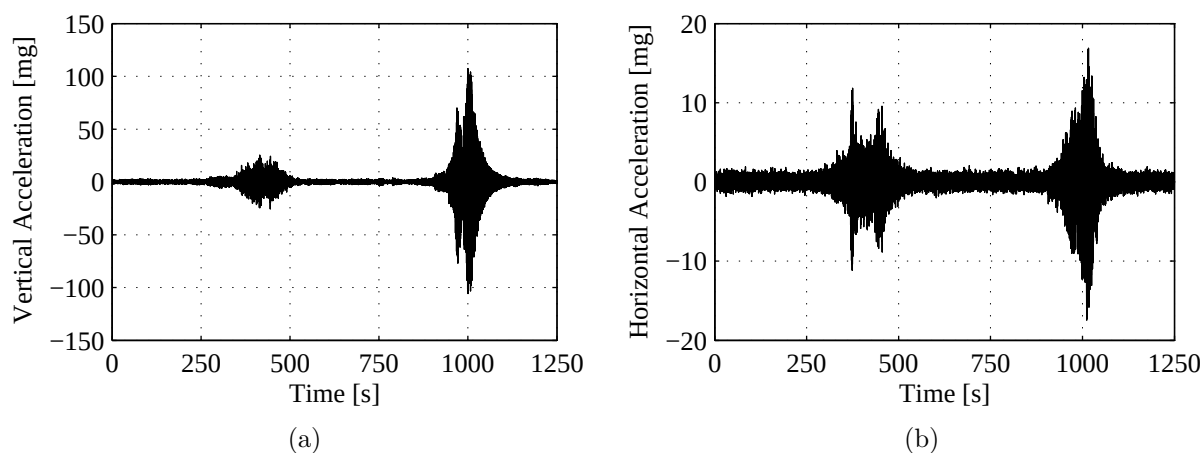


Figure 3.8: (a) Vertical and (b) transverse horizontal accelerations measured by the sensor 6N caused by 44 pedestrians walking at a step frequency of 1.2 and 1.45 Hz

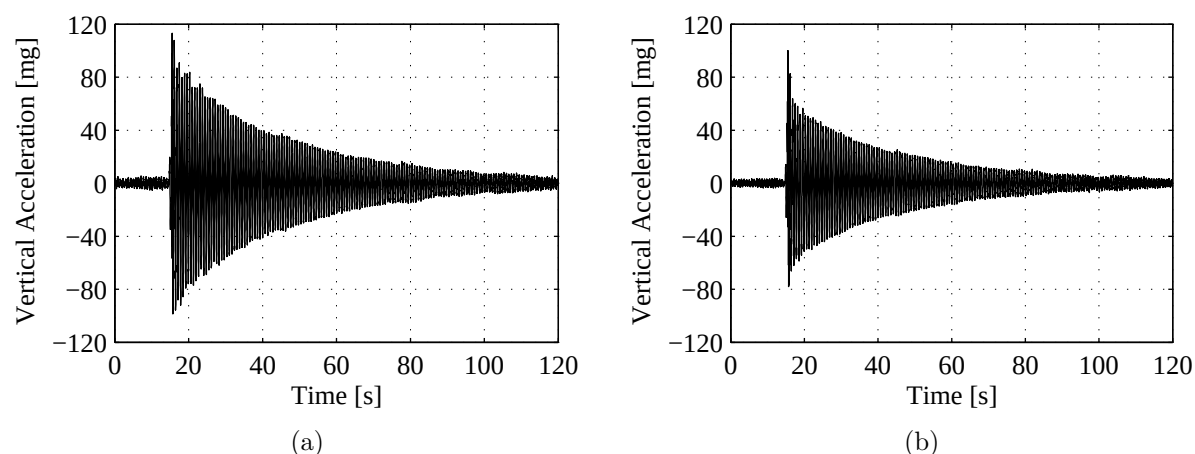


Figure 3.9: Vertical midspan acceleration due to a single jump measured on the (a) south and (b) north side

is near-resonant with the first vertical mode.

Figure 3.9 shows the vertical midspan acceleration caused by a single jump and measured on the opposite deck sides. It can be observed that the acceleration of the external (south) side is higher than that of the inner (north) side.

Some tests were carried out considering few people (from 1 up to 4) running at about 2.2 Hz, with the aim of exciting the second vertical mode. An example of these tests is reported in Figure 3.10(a), where accelerations of about 5 mg (comparable to the ambient accelerations) are reached.

Then, midspan acceleration induced by 44 pedestrians walking in a closed loop on the main span with a step frequency of 1.45 Hz is evaluated in Figure 3.10(b). Midspan accelerations greater than 120 mg are reached only about 25 seconds after the test start. These high accelerations produced feelings of discomfort for pedestrians and thus, the test was immediately suspended. After that, accelerations gradually decreased up to 20 mg

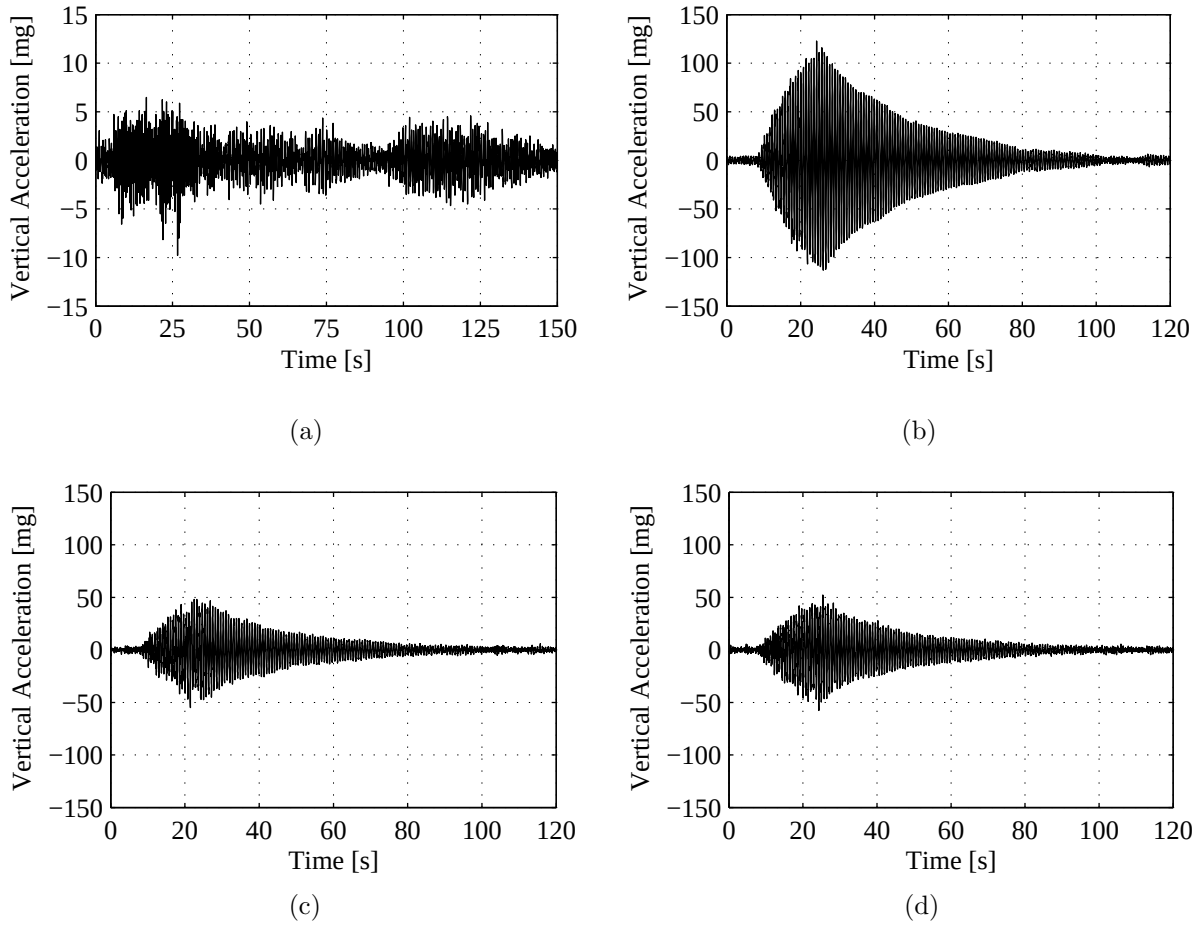


Figure 3.10: (a) Vertical midspan acceleration due to a single pedestrian running at 2.2 Hz and vertical accelerations caused by 44 pedestrians walking at 1.45 Hz in a closed loop on the main span measured at positions (b) 6N, (c) 3N and (d) 9N

in about 30 seconds. The long period spent to reach low accelerations confirms the small values of damping ratio obtained from the identification procedure (see section 3.4.2.4). Even larger vibration amplitudes may have been measured if people had been allowed to keep walking.

Accelerations due to the same loading condition but measured at the two quarters of the central span are shown in Figures 3.10(c) and 3.10(d). These accelerations are basically the same as they are recorded in symmetric positions about the midspan. Moreover, being the first mode mainly excited, accelerations at the two quarter spans are less than half of those at midspan.

To make a qualitative assessment of the vibration level, peak values of the vertical acceleration recorded by the monitoring system during the dynamic tests are compared with the vibration comfort criteria provided by design codes. In particular, comfort criteria given by Eurocode EN 1990/A1 [44], international standard for buildings and walkways ISO 10137:2007 [67], Sétra [99] and HiVoSS [89] guidelines are evaluated.

According to EN 1990/A2, verification of vibration comfort criteria should be performed, at the design stage, when the fundamental frequency of the footbridge is less than 5 Hz for vertical vibrations and 2.5 Hz for lateral and torsional vibrations.

Step Frequency	[Hz]	1.2	1.45	1.6	1.8
Peak acceleration	[mg]	25	130	52	53

Table 3.1: Peak acceleration in the vertical direction measured at midspan caused by 12 pedestrians walking at different step frequencies

Human sensitivity to vibrations depends mainly on the vibration amplitude, frequency characteristics of the vibrations, vibration direction, exposure time and human activity [82]. Despite this, Eurocode EN 1990/A1 recommendations neglect the influence of the vibration frequencies and the kind and duration of the human activity. It defines absolute values of maximum acceptable acceleration for any part of the deck as: 70 mg (0.70 m/s^2) for vertical vibrations, 20 mg for horizontal vibrations in normal use and 40 mg for exceptional crowded conditions.

Sétra and HiVoSS guidelines present four ranges of acceleration, independent on the human activity and the frequency characteristics of vibrations, corresponding to different comfort levels, namely maximum, mean, minimum and unacceptable comfort. Each comfort level is allowed in specific traffic conditions; for instance, acceleration corresponding to a minimum comfort level are allowed in very dense traffic conditions.

Finally, the international standard ISO 10137:2007 provides vertical acceleration limits which depend on the vibration frequency and the human activity (walking or standing still person). Hence, three comfort levels are identified: maximum comfort level, representing acceptable acceleration for a standing still person, mean comfort level, i.e. unacceptable acceleration for a still person but acceptable for a walking pedestrian, and the unacceptable level. Threshold levels are defined depending on the frequency characteristics of vibrations. ISO 10137:2007 provides limit values of vibrations in terms of RMS accelerations. However, requirements of ISO 10137:2007 can be converted to peak accelerations through the relation $a_{rms} = 0.707a_{peak}$, which is valid in case of harmonic vibrations, i.e. vibrations caused by people [82].

Peak acceleration values caused by 12 pedestrians walking at step frequencies of 1.2, 1.45, 1.6 and 1.8 Hz are reported in Table 3.1. These values are compared with the threshold levels in Figure 3.11.

EN 1990/A1 considers acceptable all induced vertical accelerations except for the peak acceleration of 130 mg caused by 12 pedestrians crossing the footbridge at a pacing frequency of 1.45 Hz. This peak exceeds also comfort standards prescribed in ISO 10137:2007. ISO 10137:2007 defines threshold levels depending on the frequency of vibration, which is,

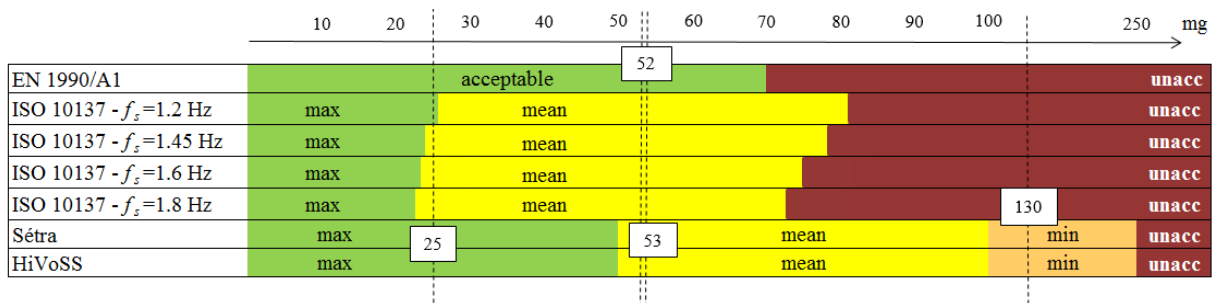


Figure 3.11: Comparison between comfort criteria and measured peak accelerations

at the design stage, usually assumed equal to the pacing frequency and the fundamental frequency of the footbridge (hypothesis of resonance). In this case, tests are performed considering different pacing frequencies, not always resonant with the fundamental frequency, and consequently different threshold levels are obtained. For Sétra and HiVoSS guidelines, the peak acceleration of 130 mg satisfies a minimum comfort level. However, such a high acceleration is allowed in very dense traffic situation, which is not the case presented here.

3.4.2.4 Results of the identification process

Starting from accelerations acquired in operational conditions, dynamic properties are identified through the algorithm described in section 3.2. Examples of PSD functions obtained from several sensor in the vertical and radial direction are reported in Figure 3.12. The relative amplitude of the PSDs in the vertical direction is significantly higher than that in the radial direction, as the footbridge is mainly flexible in the vertical direction. In fact, due to the curved shape, lateral modes are stiffer and characterized by higher eigenvalues. Ten natural modes are clearly identified, characterized by the natural frequencies and damping ratios summarized in Table 3.2. The corresponding mode shapes are presented in Figures 3.13 and 3.14. Six of the identified modes are verticals and four torsionals, with frequencies ranging between 1.462 Hz and 12.061 Hz, while damping ratios are in the range $[0.12, 0.67]$ %. To better read the results, experimental components of the mode shapes are presented together with the mode shapes obtained from the FE model A3 (see section 3.4.3). The lateral components, shown for the first two natural modes, have lower amplitude than the vertical ones, amplitude which is not well represented by the FE model. Because of the curved shape, displacements of the two deck sides are not symmetrical and the external side (south side) is characterized by higher amplitude vibrations. Moreover, Figure 3.12 shows that the PSDs of the transverse horizontal accelerations have peaks of amplitude at frequencies in the range $[3, 4]$ Hz. Mode shape components, obtained from the sensors arranged according to the second setup, associated to two of those peaks are shown in Figure 3.15. These modes are characterized by significant lateral component amplitude even if the main contribution comes from the vertical direction. These peaks cannot be associated specific modes, neither vertical or lateral.

3.4.3 Finite element model and model updating

A preliminary FE model of the footbridge is developed representing the curved main span and portions of the lateral spans (Figure 3.16). To accurately model the steel-concrete deck and the towers, beam and shell elements are used. In particular, steel members and cables are modelled adopting 1D elements while the concrete slab and the towers are modelled with 2D elements. Constraints are applied at the base of towers and piers.

The modal analysis provides frequencies and mode shapes. The first four mode shapes are shown in Figure 3.17. Numerical frequencies of the preliminary FE model (called FE Model A1 in the following) are compared to the experimental ones in Table 3.3. Numerical results match quite well the experimental modes with reference to the vertical modes, with frequency errors in the range $[2, 4]$ %. Torsional modes show errors of about 10% while MAC values exhibit values greater than 80%.

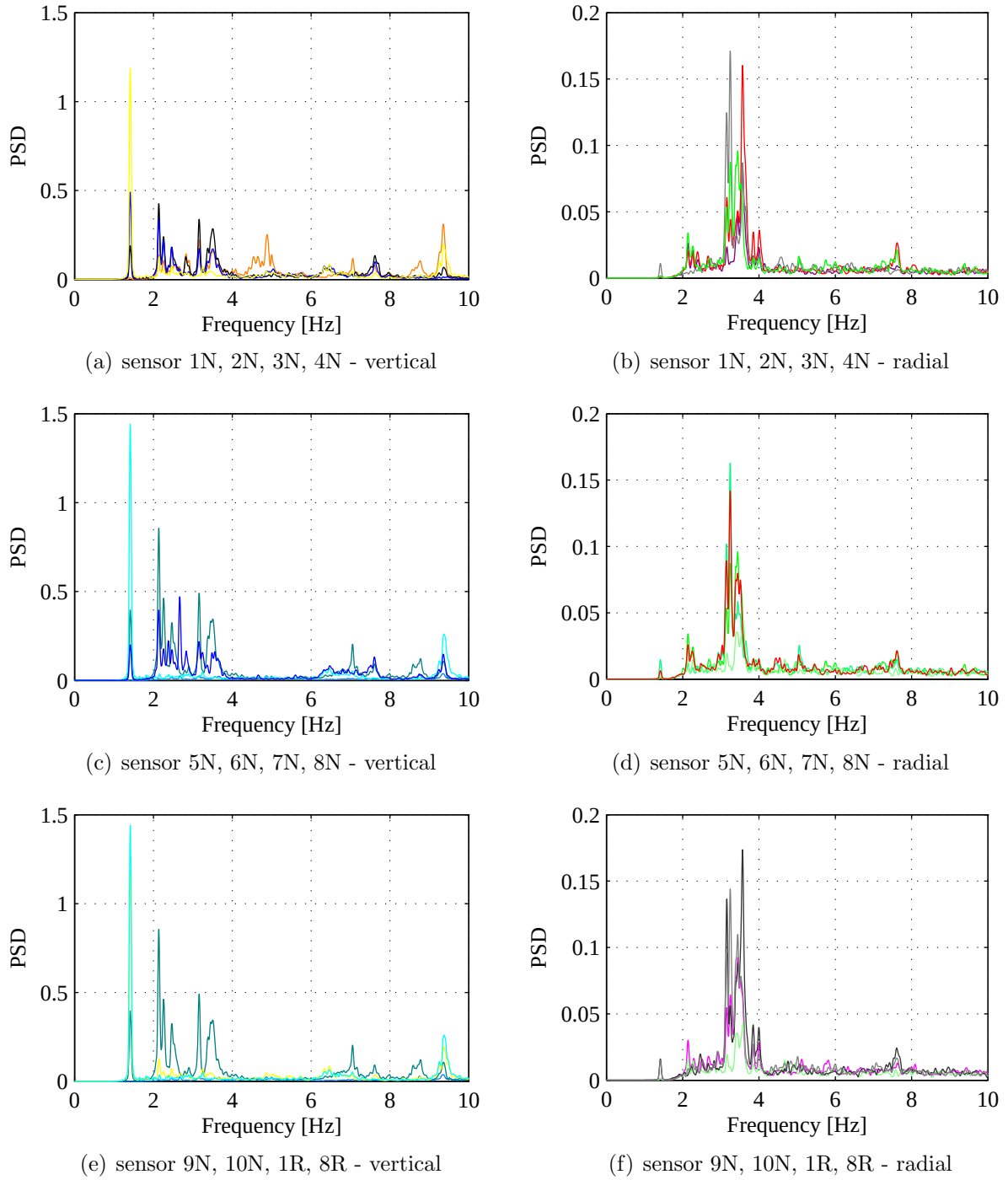


Figure 3.12: PSD functions obtained in the vertical and radial direction

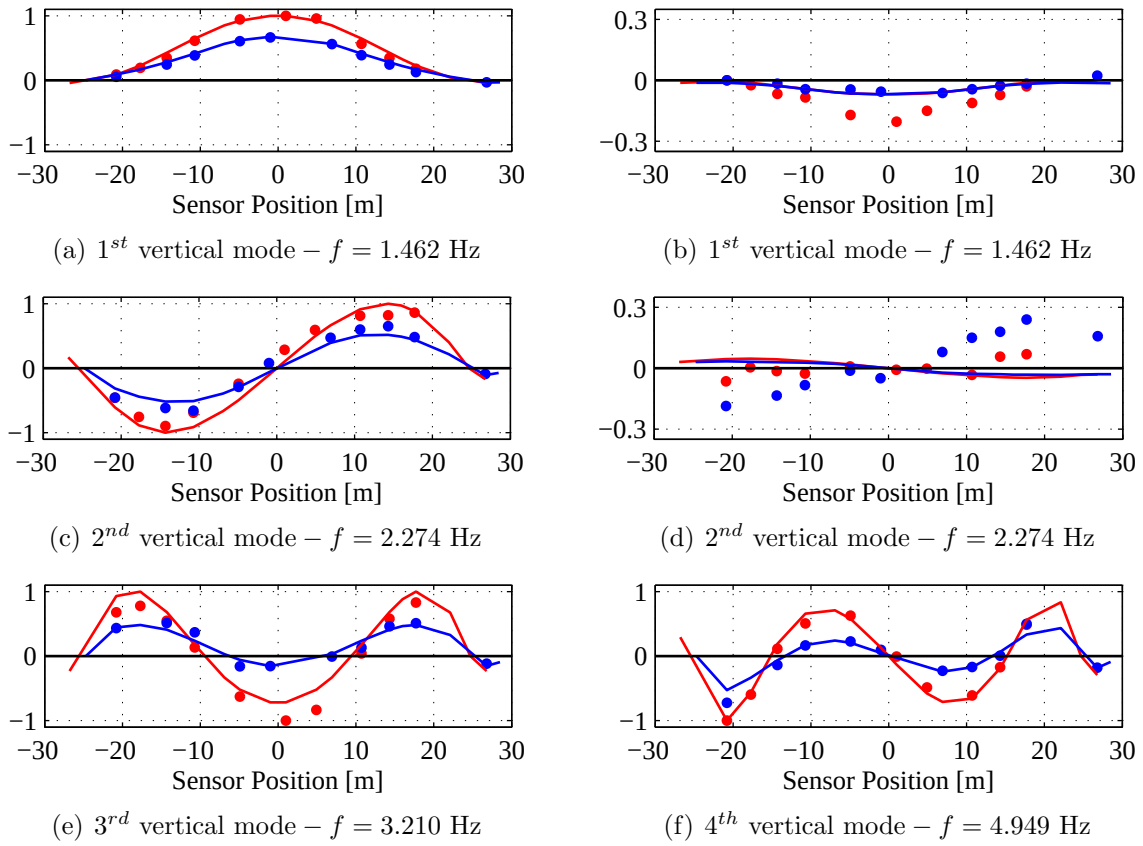


Figure 3.13: (a,c,e,f) Vertical and (b,d) radial components of the experimental (dots) and numerical (lines) mode shapes [1, 4]. Blue line: north side; red line: south side

Then, the FE model is adjusted by means of the model updating procedure presented in section 3.3, so that a better correlation between numerical and experimental frequencies and mode shapes can be achieved. The optimization procedure is performed adopting as unknown parameters those that mainly influence the structural behaviour of the footbridge.

Mode type	Mode type	Experimental Frequency [Hz]	Damping ratio [%]
1	1 st Vertical	1.462	0.67
2	2 nd Vertical	2.274	0.56
3	3 rd Vertical	3.210	0.39
4	4 th Vertical	4.949	0.40
5	1 st Torsional	6.551	0.30
6	2 nd Torsional	7.605	0.15
7	6 th Vertical	8.772	0.22
8	3 rd Torsional	9.409	0.24
9	7 th Vertical	11.453	0.18
10	4 th Torsional	12.061	0.12

Table 3.2: Experimentally identified natural modes

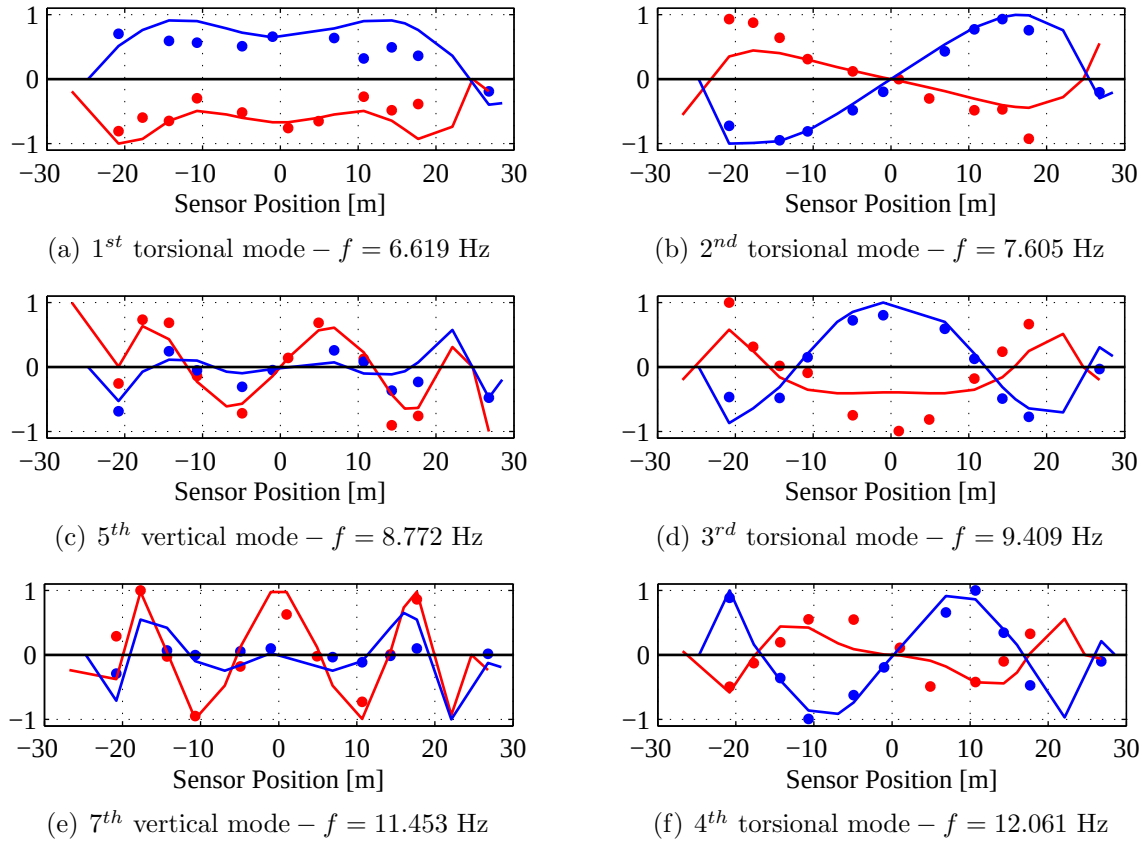


Figure 3.14: Vertical components of the experimental (dots) and numerical (lines) mode shapes [5,10]. Blue line: north side; red line: south side

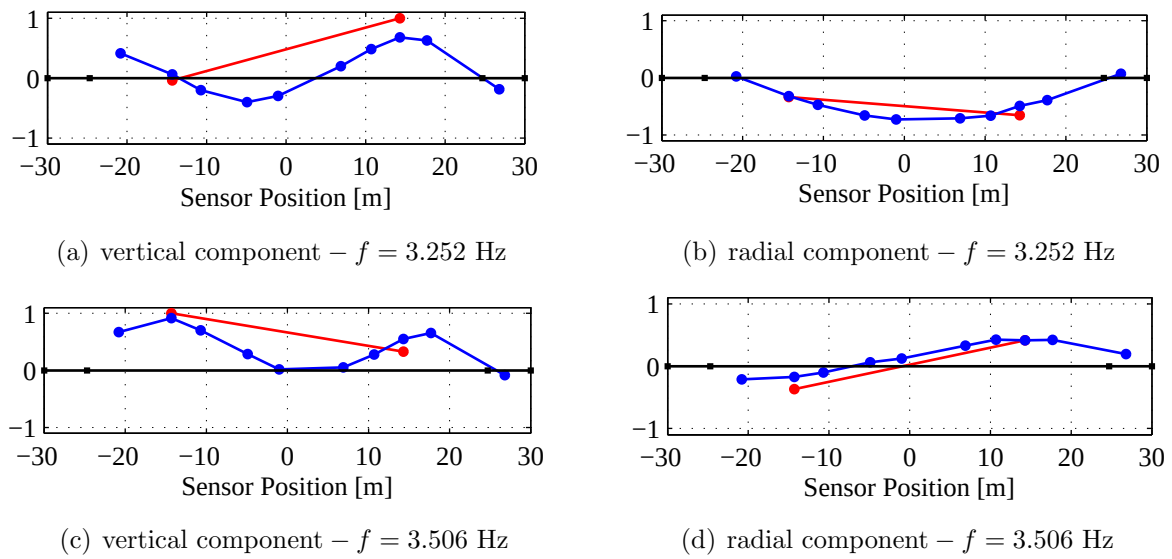


Figure 3.15: (a,c) Vertical and (b,d) radial components of two experimental modes

The selected identification parameters are: (i) the stiffness of cables (introduced in the model by means of the Young's modulus, due to the unknown actual tensile stress), (ii) the equivalent density of longitudinal chords defined considering the weight increment due to concrete fillings and (iii) the stiffness of the parapets.

The first four vertical modes, with frequencies in the range $[1.462, 4.949]$ Hz, are selected for the preliminary optimization process. The same weight constant is set for each mode. To obtain a reliable comparison, numerical and experimental mode shapes are coupled by using the MAC. In Table 3.3, frequencies of the preliminary updated model (FE Model A2) are reported and compared with the experimental results. Regarding the first vertical modes, the numerical frequencies match very well the experimental ones, with error values smaller than 1% for the first 2 modes and about 1% for the third and fourth mode. On the contrary, torsional modes exhibit error values greater than 9%. MAC reaches values greater than 90% considering only the vertical components of the mode shapes. Instead,

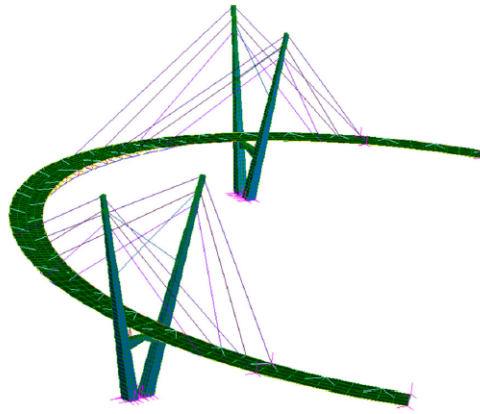


Figure 3.16: FE model of the footbridge

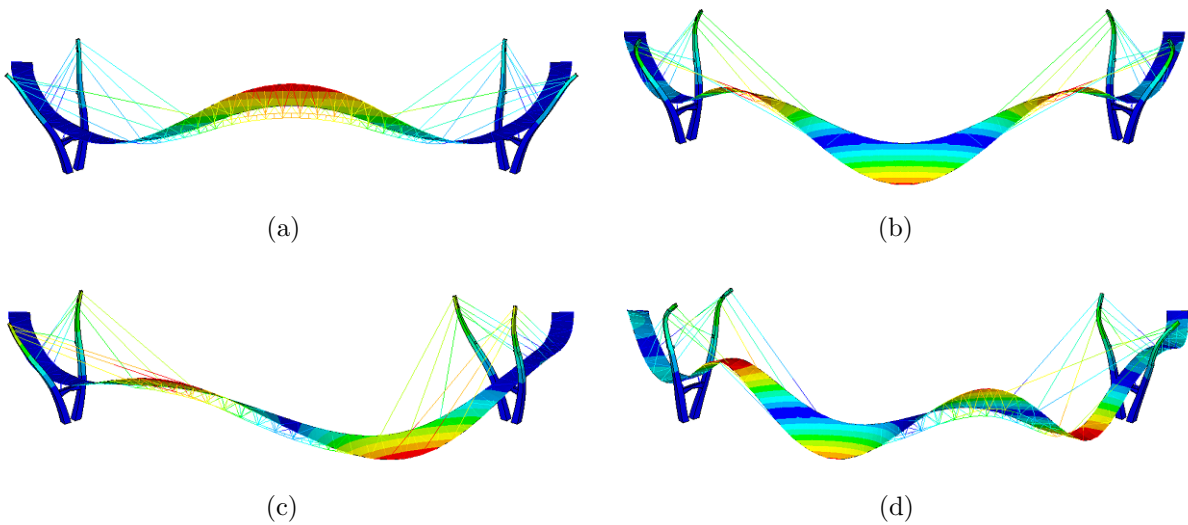


Figure 3.17: Numerical mode shapes: (a) first, (b) second, (c) third and (d) fourth vertical mode

Mode type	Experimental	FE Model A1		FE Model A2		FE Model A3		
	Freq. [Hz]	Freq. [Hz]	Error [%]	Freq. [Hz]	Error [%]	Freq. [Hz]	Error [%]	MAC [%]
1 st Vertical	1.462	1.419	-2.94	1.455	-0.51	1.465	+0.16	98.5
2 nd Vertical	2.274	2.365	+4.00	2.269	-0.23	2.257	-0.75	90.4
3 rd Vertical	3.210	3.249	+1.21	3.256	+1.42	3.303	+2.88	79.9
4 th Vertical	4.949	4.837	-2.25	4.904	-0.91	4.960	+0.22	93.8
1 st Torsional	6.551	5.977	-8.76	5.916	-9.69	6.475	-1.15	79.9
2 nd Torsional	7.605	6.479	-14.81	6.484	-14.74	7.297	-4.05	81.2

Table 3.3: Experimental and numerical modes

the transverse mode shape components are not accurately appreciated by the model. As a matter of fact, the model exhibits transverse deformations considerably higher than those obtained experimentally, particularly referring to the first mode.

Afterwards, a further optimization procedure is performed to assure a near-perfect correlation with reference to the first vertical mode as well as to improve the numerical estimation of the torsional and transverse behaviour of the footbridge. The latter is mainly influenced by the effect of cable prestress and parapet stiffness. For this reason, the stiffness of internal cables is kept distinct from the stiffness of external cables in the definition of parameters. Moreover, in the optimization process, the first torsional mode is considered in addition to the first four vertical modes. Higher weight constants are applied to the first vertical mode and the first torsional one. The updated model is referred to as FE Model A3 in the following.

After the updating procedure, errors in term of frequencies exhibit values of about 0.16% for the first vertical mode and lower than 1.2% for other modes save for the third vertical one (about 3%) and the second torsional one (about 4%). A great correlation between numerical and experimental mode shapes is achieved, with MAC values greater than 90% save for the third vertical mode and the torsional modes with MAC values of about 80%. Figures 3.13 and 3.14 show the transverse and vertical components of the identified mode shapes compared with the corresponding numerical modes. Generally, the correlation between numerical and experimental mode shapes is almost perfect as regards the vertical components. However, horizontal mode shape components are not so accurately appreciated by the numerical model. For instance, the first mode exhibits experimental transverse deformations of the south side higher than those measured on the north side whereas the numerical model shows comparable transverse components on both sides. With reference to the second mode, numerical transverse deformations reach values about one order lower than those identified experimentally. These disagreements are probably related to the complex curved shape of the structure that may increase difficulties in structural modelling and simulation, especially concerning the transverse deformations of the north and south side.

3.4.4 Simulation of the footbridge response

The dynamic response of the Pasternak footbridge to vertical human-induced loads is predicted adopting both the single step force model proposed by Li et al. [73] (see section 2.3) and the multi-harmonic force model proposed by Živanović et al. [115] (see section 2.4). Because of their intrinsic qualities, the two models are suited to different simulation procedures, defined as follows:

- **Simulation Model I (SM-I)** Numerical dynamic analyses are performed with the footbridge FE model and the pedestrian action is simulated through the single step force model proposed by Li et al. [73]. Depending on the assigned step frequency, the single step force is calculated and consequently the walking force induced by a single pedestrian is represented as a series of moving single foot forces. By doing so, the walking force turns out to be a deterministic periodic force. Analysis are carried out considering the FE models described in section 3.4.3, namely:
 - **FE model A1** Uncalibrated;
 - **FE model A2** Calibrated with reference to the first four vertical modes;
 - **FE model A3** Calibrated with reference to the first four vertical modes and the first torsional one;

According to [73], the first five harmonics of the walking force are accounted for in the definition of the single step force. The five harmonics are characterized by fundamental frequencies $0.5f_s$, f_s , $1.5f_s$, $2.0f_s$ and $2.5f_s$, being f_s the assigned step frequency. As regards the dynamic analysis, all the relevant modes are assumed to contribute to the structural response.

- **Simulation Model II (SM-II)** The footbridge dynamic response to pedestrian loads is investigated solving the equation of motion via modal decomposition, as described in section 2.4.2. The modal force acting on each SDOF (i.e. each structural mode) is defined through the multi-harmonic force model proposed by Živanović et al. [115]. In particular, the contribution of the first four harmonics (i.e. the first two harmonics and subharmonics according to the classification adopted in [115]) of the walking force is considered. These are characterized by fundamental frequencies of $0.5f_s$, f_s , $1.5f_s$ and $2.0f_s$. In the dynamic analysis, the first three structural modes are assumed to contribute to the structural response.

As the pedestrian load model relies on a uniformly distributed random phase, the structural response is to be evaluated through a Monte Carlo analysis, performing several simulations with different random phase vectors. Then, the structural response is evaluated as the average of all the responses obtained from the different simulations.

Simulations are performed considering groups of pedestrians walking at defined pacing frequencies to reproduce the in-situ test conditions. The step length l_s is deterministically imposed proportional to the step frequency and the pedestrian height (Eq.(2.2)). In the case of synchronized pedestrians, the inter-subject variability is neglected and all pedestrians are given the same step frequency, weight, arrival time, DLFs and phase angle. Thus,

the acceleration caused by n pedestrians is obtained as n times the acceleration of a single pedestrian. In this case, only the intra-subject variability is taken into account with the Simulation model II. On the contrary, with the Simulation model I, the intra-subject variability is neglected because of the deterministic nature of the model. To simulate the imperfect synchronization among pedestrians, the inter-subject variability of the walking force is taken into account with the Simulation model II via probability-based modelling of the walking force parameters.

Numerical results are compared with those recorded experimentally based on the root-mean-square (RMS) value of the vertical acceleration, which is a commonly used method to quote vibration magnitude. The RMS acceleration is the square root of the mean value of the square acceleration during time record [54], given by the expression:

$$\text{RMS} = \sqrt{\frac{\int_{t_1}^{t_2} \ddot{y}(t)^2 dt}{t_2 - t_1}} \quad (3.21)$$

with $\ddot{y}(t)$ the acceleration time-history and $t_2 - t_1$ the time interval considered. In the following, an averaging period of 1 s is adopted.

Two types of results are presented: (i) the maximum acceleration obtained at a specific location on the footbridge caused by a single pedestrian crossing the main span with different step frequencies and (ii) the maximum acceleration obtained at the different footbridge locations caused by a specific loading condition (defined in terms of step frequency, pedestrian number and weight). For the latter case, specific loading conditions are defined corresponding to some in-situ test conditions assumed as the basis for comparison, which are:

- **Case 1** a stream of 44 pedestrians walking at a step frequency of 1.2 Hz, with a mean weight equal to 750 N;
- **Case 2** 12 synchronized pedestrians walking in single file at a step frequency of 1.45 Hz, with a mean weight equal to 750 N;
- **Case 3** a single pedestrian that weighs 650 N running at a step frequency of 2.2 Hz.

Experimental accelerations caused by these loading conditions are presented in Figure 3.18, along with the RMS acceleration traces calculated with an averaging period of 1 s.

Vertical accelerations are predicted in locations corresponding to the sensor positions (Figure 3.4) as well as in correspondence of the towers, where the vertical mode shape components are assumed to be null. Accelerations of the Case 1 and 2 have been recorded with sensors arranged in configuration 2, while for the Case 3 they were arranged in configuration 3. This is why numerical results related to different cases are given for slightly different footbridge positions.

In the following, the footbridge dynamic response in the vertical direction is predicted according to the Simulation model I and II in section 3.4.4.1 and then numerical accelerations are compared with the experimental ones in section 3.4.4.2.

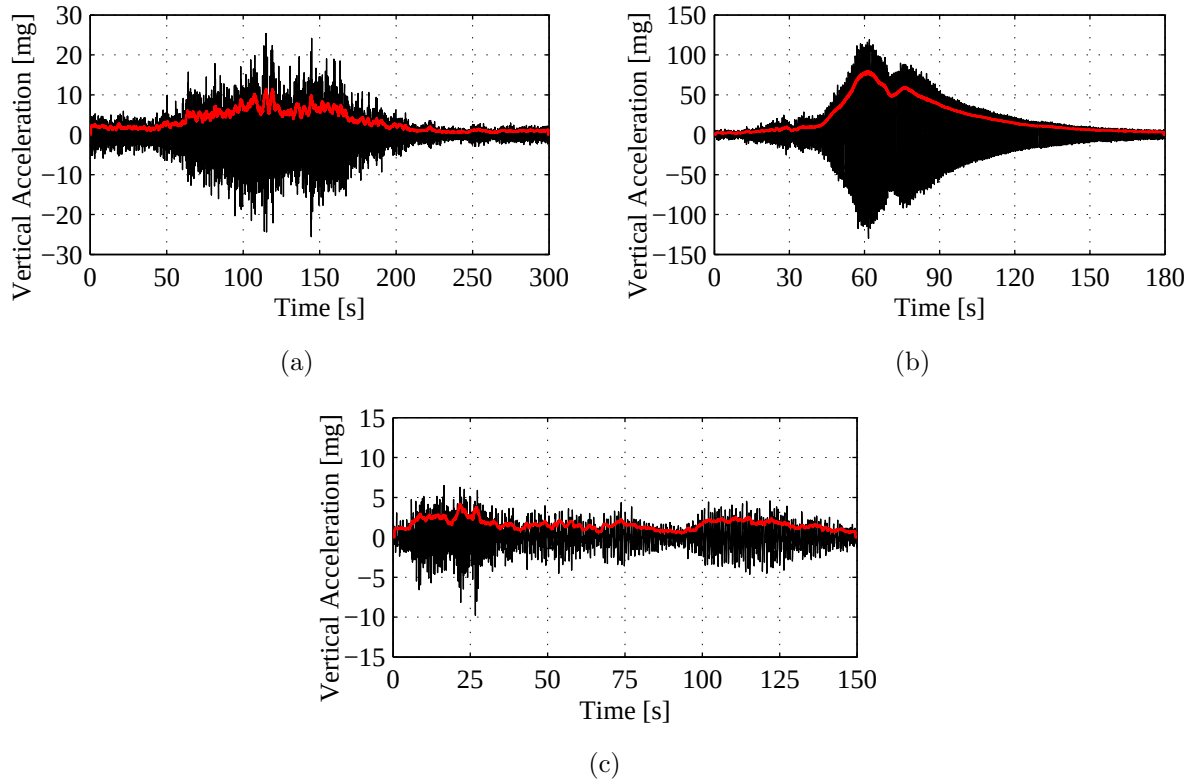


Figure 3.18: Midspan accelerations and RMS acceleration traces due to different loading conditions: (a) a stream of 44 pedestrians walking at a pacing frequency of 1.2 Hz, (b) 12 pedestrians walking at 1.45 Hz in single file and (c) a single individual running at 2.2 Hz. Accelerations (a,b) were measured from the sensor 6N while acceleration (c) from the sensor 6S

3.4.4.1 Numerical accelerations induced by pedestrians

Simulation Model I

As a starting point, structural responses obtained from the Simulation model I with the three FE models (A1, A2, A3) are compared with each others.

Figure 3.19(a) presents a comparison among the amplitude of the maximum accelerations obtained on the north side at a quarter of the central span (position 3N) and caused by a single pedestrian walking with a step frequency ranging between 1 and 3.5 Hz.

It has to be stressed that in the Simulation model I, based on the pedestrian model proposed by Li et al. [73], the DLFs are defined for step frequencies in the range [1.6, 2.4] Hz. Nevertheless, numerical simulations are performed considering wider step frequency ranges, in order to investigate the footbridge dynamic behaviour close to the first natural frequency, simulate in-situ test conditions and evaluate the effect of several mode shapes. However, this could lead to inaccuracies in the response simulated for step frequencies outside the range [1.6, 2.4] Hz.

A good correlation between results obtained performing dynamic analyses with the updated FE models A2 and A3 can be noted. In fact, even if the FE model A2 does not allow an accurate representation of the torsional behaviour of the structure, it can appreciate

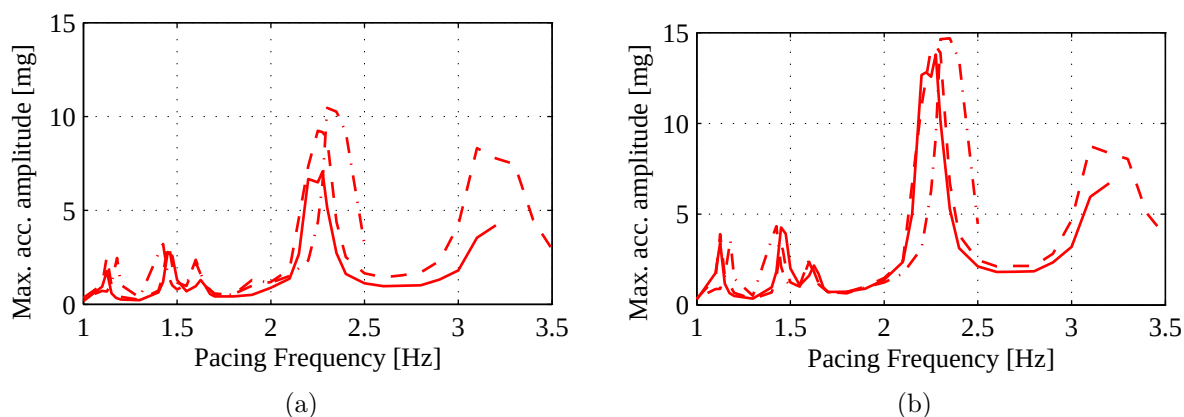


Figure 3.19: Amplitude of the maximum acceleration vs. pacing frequency obtained at position (a) 3N and (b) 3S considering the FE Model A1 (dash-dotted line), A2 (dashed line) and A3 (solid line) - Simulation model I

very well the first vertical modes, which provide the greatest contribution to the dynamic response. On the contrary, more significant differences are obtained considering the uncalibrated FE model A1.

Results show the great dependence of the structural response on the pacing frequency. Significant response amplifications are obtained for the five values of step frequencies reported in Table 3.4. Particularly, for each peak are reported: (i) the step frequency range in which the peak is obtained, (ii) the frequency range of the five harmonics of the walking force, (iii) the harmonic causing resonance and (iv) the resonant mode. Note that peaks are identified in ranges of step frequencies such as to include resonant frequencies from all three FE models. In fact, the three FE models, as a result of different calibrations, are characterized by natural modes with slightly different frequencies, leading to response amplifications obtained for different step frequencies.

Two peaks are identified for step frequencies corresponding to the first two natural modes, while other two are obtained for step frequencies which are half the frequencies of two natural modes, and thus resonant with the fourth harmonic. Finally, for the fifth peak, the second and the fourth harmonics are characterized by frequencies close to the third vertical mode and the first torsional one, both contributing to the structural response.

Similar results are reported in Figure 3.19(b), where the amplifications factors obtained on the south side at a quarter of the central span (position 3S) are shown. Dynamic amplifications are achieved for the same step frequencies of the previous case, but in this case higher response amplitudes are obtained. In fact, due to the curved shape of the footbridge, the south side is characterized by higher displacements and accelerations than the north one.

Figure 3.19 is referred to positions at quarter span, where a substantial number of natural modes are clearly recognized. On the contrary, Figure 3.20 shows that considering a position at midspan, the main dynamic amplification is due to the first vertical mode. Also in this case, dynamic amplifications obtained on the south side are higher than those on the north side. In addition, considering the south side, response amplifications are obtained for pacing frequencies higher than 3 Hz, due to the contribution of the third vertical mode

	# Harmonic	1	2	3	4	5		
# Peak	f_s range [Hz]	$0.5f_s$ [Hz]	$1.0f_s$ [Hz]	$1.5f_s$ [Hz]	$2.0f_s$ [Hz]	$2.5f_s$ [Hz]	Res. har.	Res. mode
1	1.125-1.180	0.563-0.590	1.125-1.180	1.688-1.770	2.250-2.360	2.813-2.950	#4	2 nd Vert.
2	1.425-1.475	0.713-0.738	1.425-1.475	2.138-2.213	2.850-2.950	3.563-3.688	#2	1 st Vert.
3	1.600-1.625	0.800-0.813	1.600-1.625	2.400-2.438	3.200-3.250	4.000-4.063	#4	3 rd Vert.
4	2.200-2.350	1.100-1.175	2.200-2.350	3.300-3.525	4.400-4.700	5.500-5.875	#2	2 nd Vert.
5	3.100-3.300	1.550-1.650	3.100-3.300	4.650-4.950	6.200-6.600	7.750-8.250	#2-4	3 rd Vert.-1 st Tors.

Table 3.4: Response amplification: pacing frequency range, frequency ranges of the five harmonics, resonant harmonic and structural mode

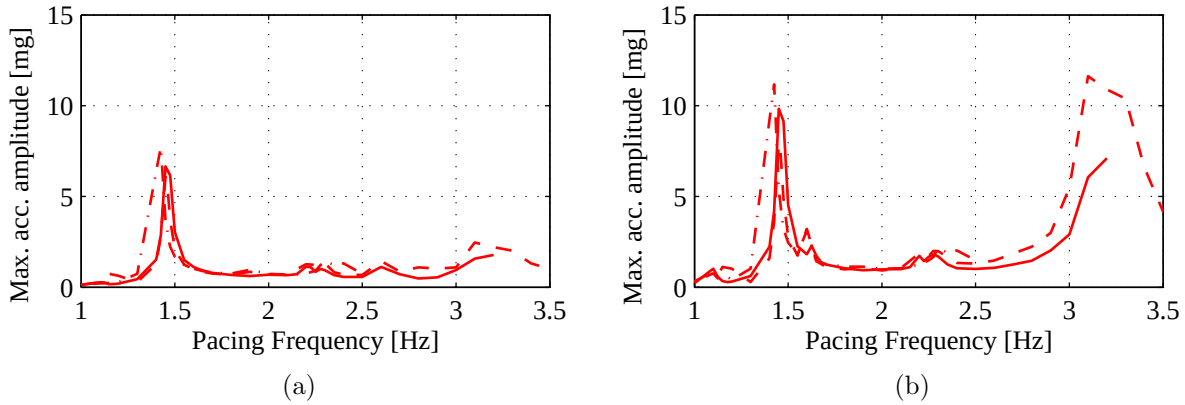


Figure 3.20: Amplitude of the maximum acceleration vs. pacing frequency obtained at position (a) 6N and (b) 6S considering the FE Model A1 (dash-dotted line), A2 (dashed line) and A3 (solid line) - Simulation model I

and the first torsional one.

Thereafter, the maximum RMS acceleration caused by the loading conditions referred to as Case 2 and Case 3 is estimated for each position of the midspan and results obtained from the three FE models are compared.

Figure 3.21(a) presents the maximum RMS acceleration obtained on the north side caused by a single pedestrians walking at a pacing frequency of 1.45 Hz. In this case, as the step frequency is basically resonant with the first vertical mode (1.462 Hz), the highest RMS acceleration is reached at midspan and then it decreases up to zero at positions 0 m and 60 m, corresponding to the towers. Accelerations obtained with the FE models A2 and A3 are highly comparable, as they are both well calibrated with respect to the first vertical mode (see Table 3.3), which provides the largest contribution. On the contrary, the FE

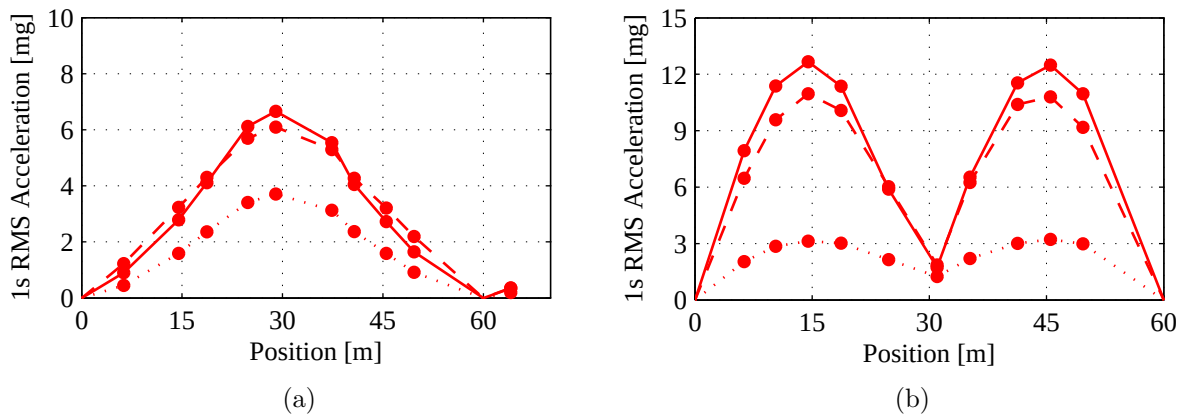


Figure 3.21: RMS acceleration levels at the different locations obtained (a) on the north side due to a single pedestrian walking at a step frequency of 1.45 Hz and (b) on the south side caused by a single individual running at a step frequency of 2.2 Hz. Accelerations are simulated with the FE model A1 (dotted line), A2 (dashed line) and A3 (solid line) - Simulation model I

model A1 is characterized by a first vertical mode with a natural frequency of 1.419 Hz and thus dynamic amplifications are lower.

As the 12 pedestrians are assumed to walk synchronized, the RMS acceleration caused by the pedestrian group is 12 times the acceleration of a single pedestrian.

Maximum RMS acceleration obtained on the south side caused by one pedestrian running at a step frequency of 2.2 Hz is presented in Figure 3.21(b). In this case, the step frequency is close to the frequency of the second vertical mode (2.274 Hz) and thus the RMS acceleration presents maximum values at the two quarter spans and lowest values at midspan. Even in this case, accelerations obtained with the FE models A2 and A3 are highly comparable, being the two models characterized by low errors in terms of frequency with respect to the experimentally identified second vertical mode. On the contrary, the FE model A1 is characterized by a second vertical mode with a natural frequency 4% higher than the experimental one, resulting in lower dynamic amplifications.

In the previous cases, results obtained with the FE models A2 and A3 are basically compa-

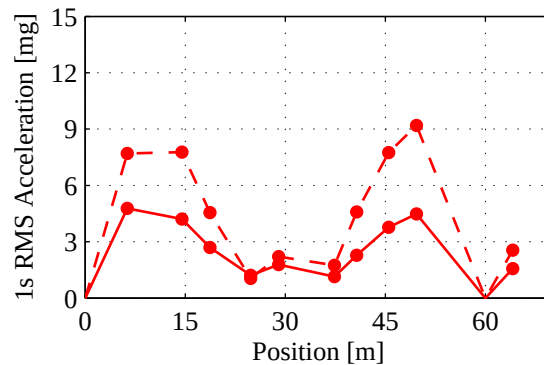


Figure 3.22: RMS acceleration levels at the different locations obtained on the north side due to a single pedestrian running at a step frequency of 3.2 Hz. Accelerations are simulated with the FE model A2 (dashed line) and A3 (solid line) - Simulation model I

able as maximum accelerations are due to the contribution of the first two natural modes, against which both models are well calibrated. The main difference between the FE model A2 and A3 lies in the fact that, besides the first four vertical modes, the FE model A3 is calibrated also with reference to the first torsional mode. Hence, to evaluate the effect of the first torsional mode, a pedestrian running at a pacing rate of 3.2 Hz is considered (Figure 3.22). Thus, the second harmonic of the walking force is characterized by a frequency of 3.2 Hz, near-resonant with the third vertical mode. In addition, the fourth harmonic shows a frequency of 6.4 Hz, close to the identified first torsional mode. Concerning these two natural modes, FE models A2 and A3 are characterized by different frequency errors with respect to the identified parameters. Consequently, responses obtained from the two FE models show more significant differences than those achieved in the previous cases.

Simulation Model II

Structural accelerations caused by the loading conditions referred to as Case 2 and Case 3 are estimated through the Simulation Model II. In addition, quantities playing a key role in the response evaluation are thoroughly analysed with reference to a single response simulation, i.e. generating a single random phase vector. Then, to obtain the mean response, a Monte Carlo analysis is to be performed.

In the Simulation model II, according to [70], the DLFs are defined for step frequencies in the range $[1, 2.65]$ Hz. Nevertheless, also in this case numerical simulations are performed considering wider step frequency ranges, which could lead to inaccuracies in the simulated response.

Starting from an assigned step frequency and the pedestrian's weight, the pedestrian induced force is calculated from Eq.(2.19). As regards the Case 2, the force induced by a single pedestrian walking at a step frequency of 1.45 Hz is shown in Figure 3.23(a). The dynamic effect makes the pedestrian force fluctuates around its static counterpart, i.e. the pedestrian's weight of 750 N. The force is defined for a duration equal to the crossing time T_{cr} (Eq.(2.13)), which is in turn calculated from the step length l_s (Eq.(2.5)):

$$l_s = 0.24 h f_s = 0.24 \cdot 1.70 \cdot 1.45 = 0.59 \text{ m} \quad (3.22)$$

$$T_{cr} = \frac{L}{l_s f_s} = \frac{64}{0.59 \cdot 1.45} = 74.90 \text{ s} \quad (3.23)$$

Note that the footbridge length equal to 64 m corresponds to the distance between the first and the last sensor in configuration 2 (Figure 3.4(b)).

Then, the modal force corresponding to each contributing natural mode is obtained as (Eq.(2.26)):

$$F_j(t) = \frac{f(t)}{m_j} \varphi_j(t) \quad j = 1, 2, 3 \quad (3.24)$$

Since the modal masses are not available, experimental mode shapes are normalized to the masses, in such a way that $m_j = 1$. In order to normalize mode shapes to the masses, they are scaled over the amplitude of the numerical mode shapes, which are mass-normalized. Modal forces obtained for the first three identified modes are reported in Figures 3.23(b), 3.23(c) and 3.23(d).

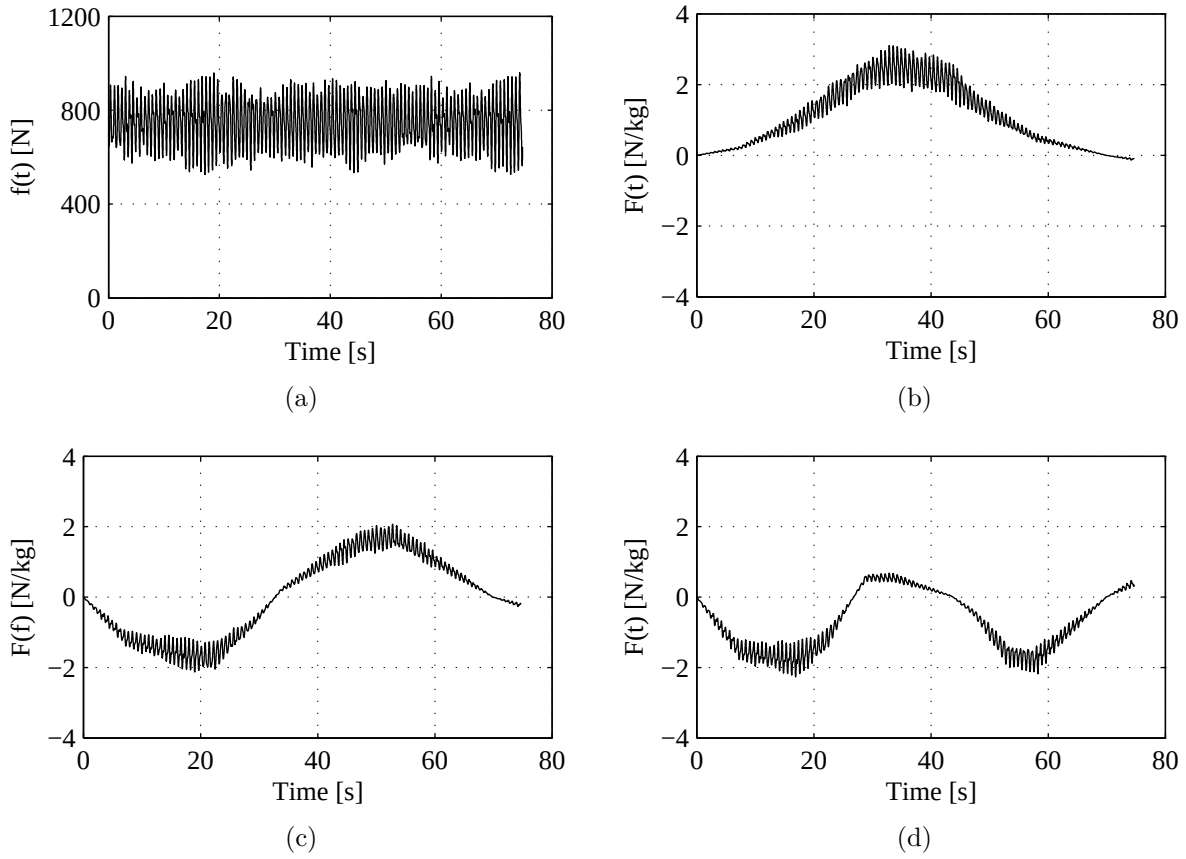


Figure 3.23: (a) Pedestrian-induced force and modal force corresponding to the (b) first, (c) second and (d) third structural mode - Simulation model II

Afterwards, the modal acceleration response corresponding to each natural mode is achieved by solving the equation of motion for the j th decoupled system (Eq.(2.28)). The acceleration response in physical coordinates at a specific footbridge location is obtained multiplying the modal acceleration by the mode shape amplitude in that location (Eq.(2.22)). Acceleration responses obtained close to the midspan on the north side (position 6N) are shown in Figures 3.24(a), 3.24(b) and 3.24(c). Maximum accelerations are obtained from the contribution of the first natural mode, due to both the position (close to the midspan the contribution of the first mode is the principal one) and the near-resonance with the step frequency (1.45 Hz). The acceleration due to second natural mode is almost null as the amplitude of the mode shape close to the midspan is almost zero.

The RMS values of the acceleration responses at position 6N are calculated with an averaging period of 1 s and represented by red lines in Figure 3.24. The maximum RMS acceleration corresponding to each natural mode is then evaluated:

$$\text{RMS}_{\max,1} = 5.164 \text{ mg}$$

$$\text{RMS}_{\max,2} = 0.122 \text{ mg}$$

$$\text{RMS}_{\max,3} = 0.023 \text{ mg}$$

Finally, the total response (in terms of maximum RMS acceleration) is obtained by adding the responses for individual vibration modes according to the mode superposition principle.

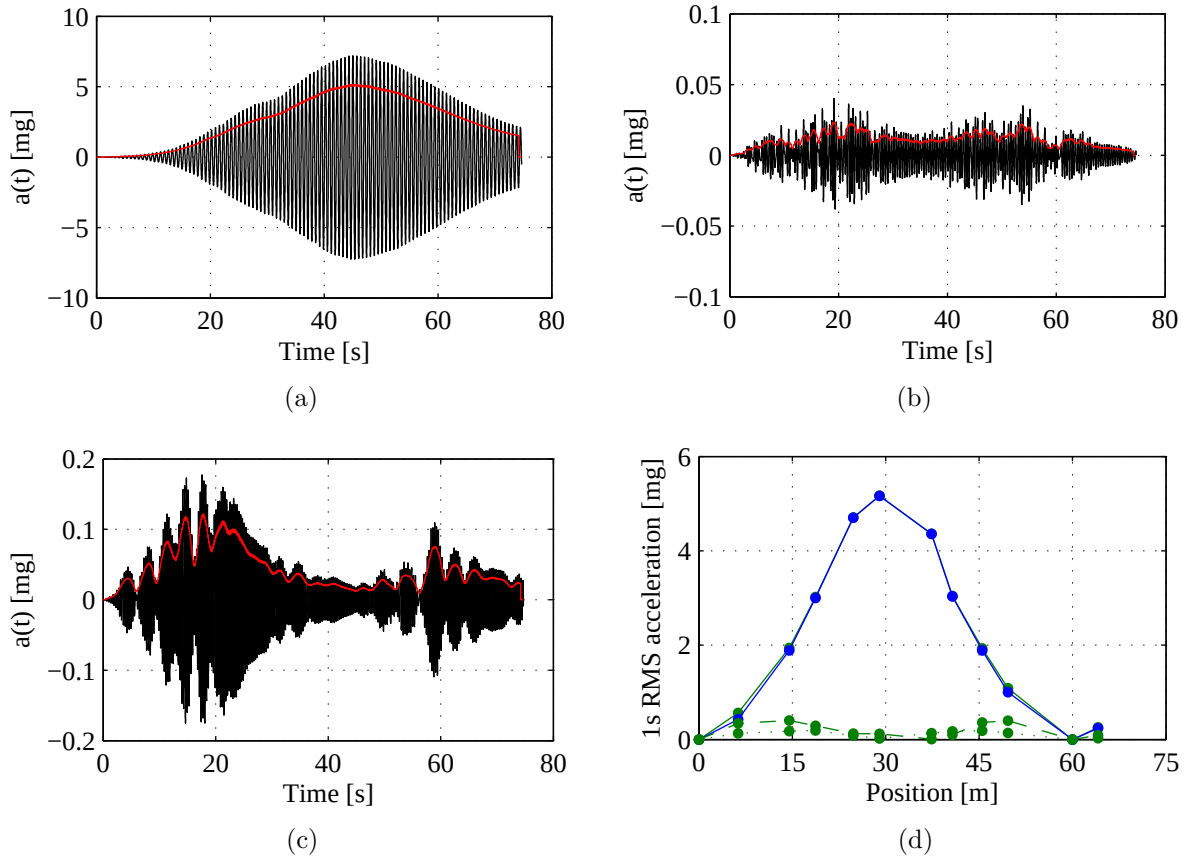


Figure 3.24: Vertical accelerations and RMS accelerations predicted at position 6N due to the (a) first, (b) second and (c) third structural mode. (d) Maximum RMS accelerations predicted at different footbridge locations: total (blue solid line) and due to the contribution of the first (green solid line), second (green dotted line) and third (green dashed line) structural mode - Simulation model II

To account for a more realistic response evaluation, maximum RMS accelerations are added together through the SRSS (Square Root Sum of Square) combination:

$$\text{RMS}_{\text{tot}} = \sqrt{\text{RMS}_{\text{max},1}^2 + \text{RMS}_{\text{max},2}^2 + \text{RMS}_{\text{max},3}^2} = 5.164 \text{ mg} \quad (3.25)$$

With reference to the position 6N, the contribution of the first mode to the total maximum RMS acceleration is basically the only relevant one.

Figure 3.24(d) presents the maximum RMS acceleration at the different footbridge locations. Position 0 m and 60 m represents the towers, where the maximum vertical RMS is null, as it is assumed that in those positions mode shape components are null as well. The maximum RMS acceleration increases moving towards the central positions until it reaches its highest value at midspan. Although due to the near-resonance with the step frequency the contribution of the first natural mode is always the principal one, the contribution of the second and third mode assumes a slight importance close to the two quarter spans. Due to the assumption of synchronized pedestrians, the maximum RMS acceleration caused by 12 pedestrians is 12 times the acceleration of the single pedestrian.

The same results are reported considering the loading condition referred to as Case 3, i.e.

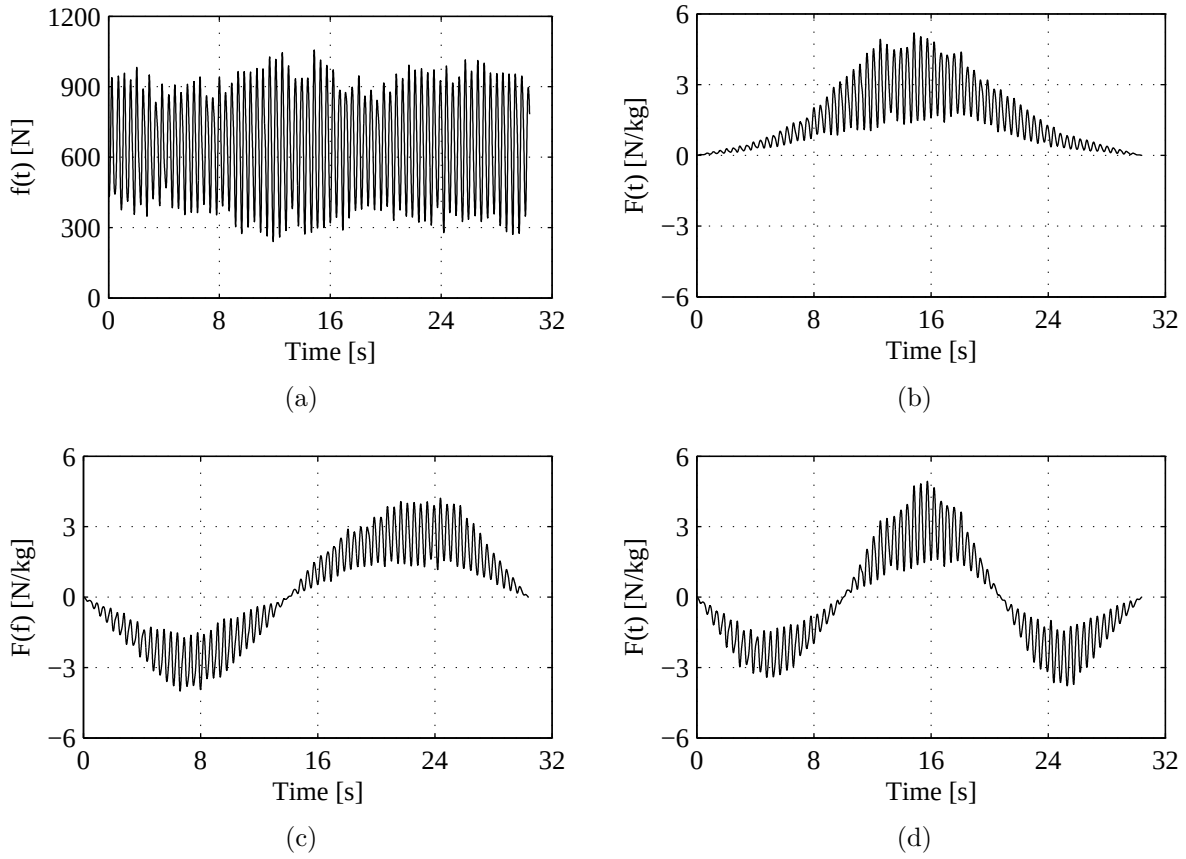


Figure 3.25: (a) Pedestrian-induced force and modal force corresponding to the (b) first, (c) second and (d) third structural mode - Simulation model II

a single pedestrian running a pacing frequency equal to 2.2 Hz. The step length l_s and the crossing time T_{cr} are evaluated as:

$$l_s = 0.24 h f_s = 0.24 \cdot 1.70 \cdot 2.2 = 0.90 \text{ m} \quad (3.26)$$

$$T_{cr} = \frac{L}{l_s f_s} = \frac{60}{0.90 \cdot 2.2} = 30.40 \text{ s} \quad (3.27)$$

The footbridge length equal to 60 m corresponds to the distance between the towers, as in in configuration 3 there are no sensors in the side spans (Figure 3.4(c)).

The pedestrian-induced force is reported in Figure 3.25(a). In this case the dynamic amplification with respect to the static weight is higher than the one obtained in the previous case. In fact, with increasing step frequency, the dynamic load factor of the first harmonic increases (according to the DLF proposed by Kerr [70] and reported in Table 2.2), causing higher dynamic amplifications. Figures 3.25(b), 3.25(c) and 3.25(d) present the modal force corresponding to each contributing natural mode.

Afterwards, the modal acceleration response is calculated. From that, the acceleration response in physical coordinates in a specific footbridge location is obtained considering the mode shape amplitude in that location. Acceleration responses obtained close to the midspan on the south side (position 6S) are shown in Figures 3.26(a), 3.26(b) and 3.26(c). In this case, the higher contribution comes from the second structural mode, due to the

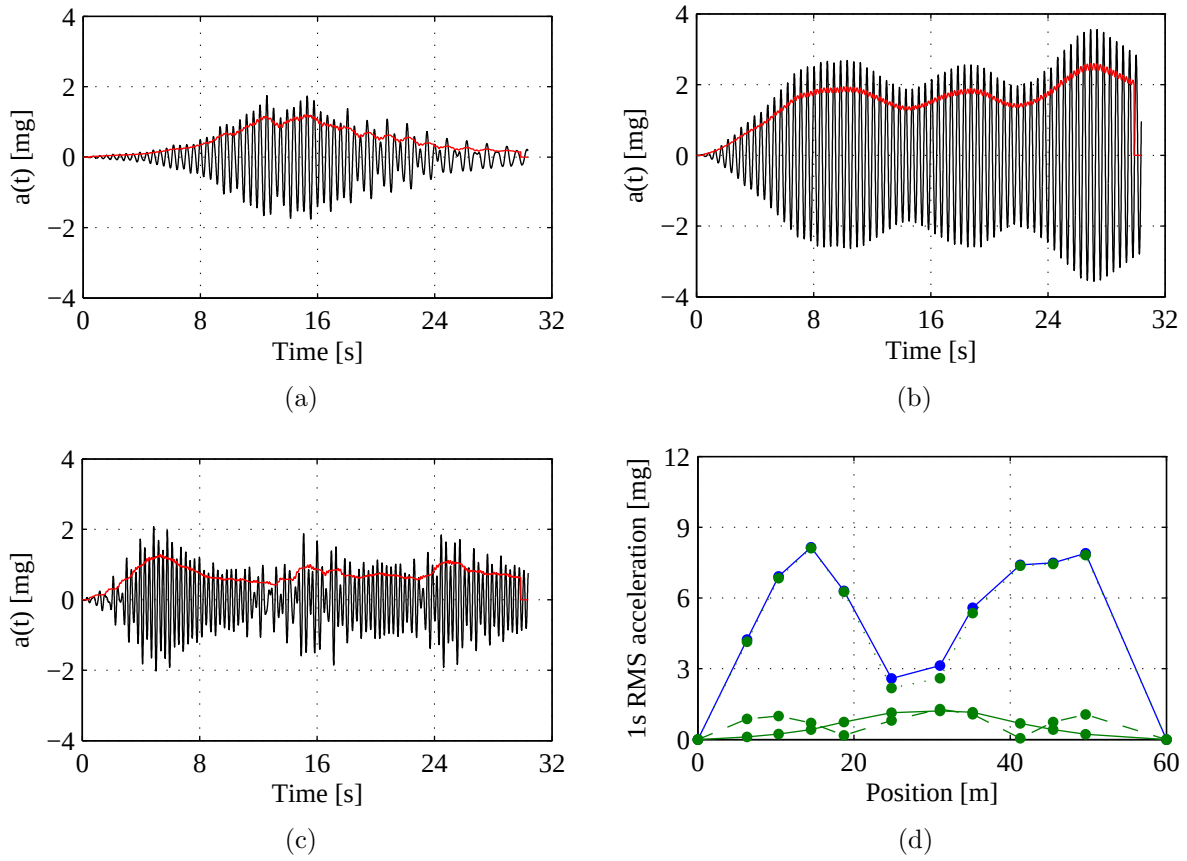


Figure 3.26: Vertical accelerations and RMS accelerations predicted at position 6N due to the (a) first, (b) second and (c) third structural mode. (d) Maximum RMS accelerations predicted at different footbridge locations: total (blue solid line) and due to the contribution of the first (green solid line), second (green dotted line) and third (green dashed line) structural mode - Simulation model II

resonance with the step frequency. However, accelerations given by the first three mode shapes are of the same order of magnitude. In fact, the mode shape component of the second vertical mode close to the midspan is the lowest one. If a position at quarter span was considered, acceleration due to the second mode would be significantly higher than the other ones.

RMS accelerations, represented by red lines in Figure 3.26, are calculated and then, the maximum RMS acceleration for the position close to the midspan and corresponding to each natural mode is evaluated:

$$\text{RMS}_{\max,1} = 1.207 \text{ mg}$$

$$\text{RMS}_{\max,2} = 2.596 \text{ mg}$$

$$\text{RMS}_{\max,3} = 1.284 \text{ mg}$$

Finally, the total response (in terms of maximum RMS acceleration) is obtained by adding the responses for individual vibration modes through the SRSS (Square Root Sum of Square) combination:

$$\text{RMS}_{\text{tot}} = \sqrt{\text{RMS}_{\max,1}^2 + \text{RMS}_{\max,2}^2 + \text{RMS}_{\max,3}^2} = 3.137 \text{ mg} \quad (3.28)$$

With reference to the position close to the midspan, all three modes contribute to the structural response, with the second mode being the principal one due to the near resonance with the step frequency. On the contrary, close to the quarter span, the second mode provides the main contribution. Maximum RMS acceleration obtained at position 3S corresponding to each natural mode are the following:

$$\text{RMS}_{\max,1} = 0.420 \text{ mg}$$

$$\text{RMS}_{\max,2} = 8.109 \text{ mg}$$

$$\text{RMS}_{\max,3} = 0.703 \text{ mg}$$

while the total response is:

$$\text{RMS}_{\text{tot}} = \sqrt{\text{RMS}_{\max,1}^2 + \text{RMS}_{\max,2}^2 + \text{RMS}_{\max,3}^2} = 8.150 \text{ mg} \quad (3.29)$$

Maximum RMS acceleration at the different footbridge locations is presented in Figure 3.26(d). Maximum RMS acceleration shows a trend in line with the shape of the second natural mode, with highest values at the two quarter spans and lowest one at midspan.

Furthermore, the amplitude of the maximum acceleration caused by a single pedestrian crossing the footbridge with step frequencies in the range $[1, 3.5]$ Hz is evaluated. Response amplifications obtained on the north and south side are compared with each other in Figure 3.27. Response amplitude on the north side is higher than that on the north side, especially regarding the second vertical mode.

The structural response is evaluated through a Monte Carlo analysis, performing 100 response simulations. The mean response with increasing number of simulations obtained close to the midspan and caused by a pedestrian walking at 2 Hz is shown in Figure 3.28. The response at the m th simulation is evaluated as the average of the responses obtained from the previous $m - 1$ simulations.

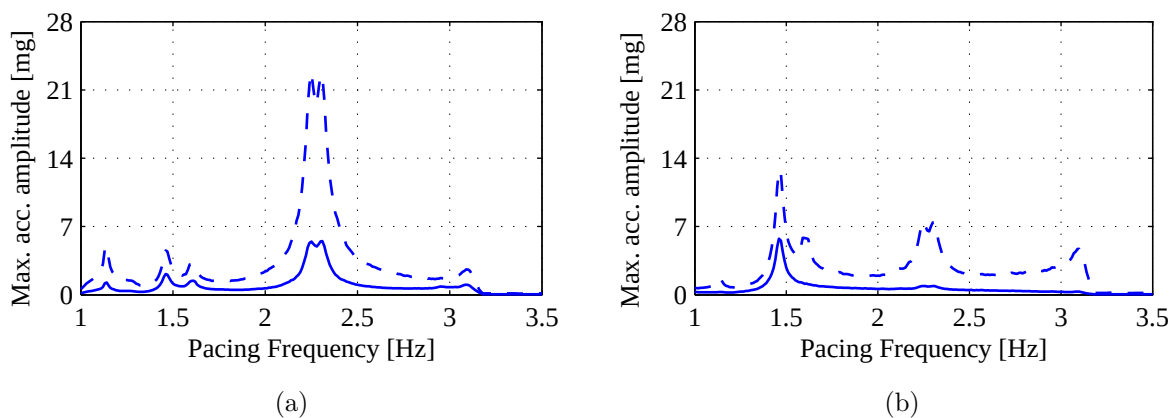


Figure 3.27: Amplitude of the maximum acceleration vs. pacing frequency obtained on the north (solid lines) and the south (dashed lines) side with reference to a position (a) at quarter span and (b) close to the midspan - Simulation model II

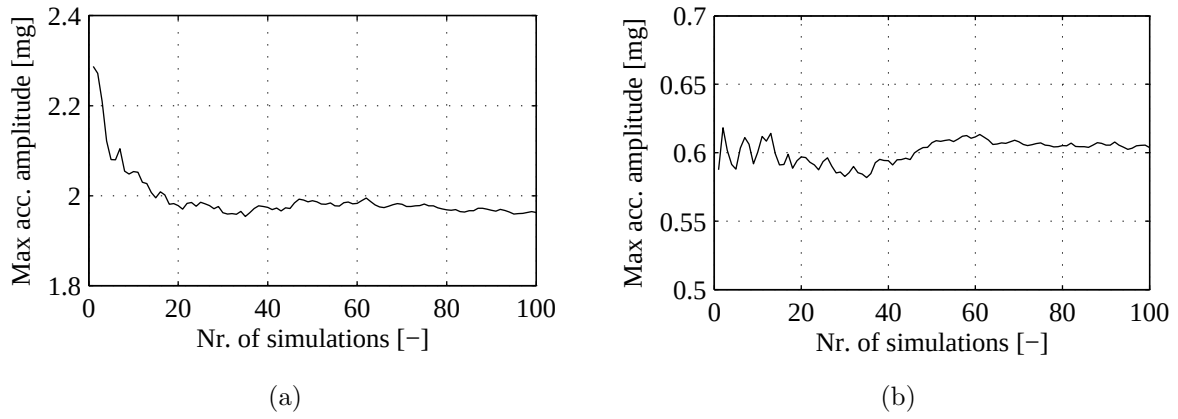


Figure 3.28: Amplitude of the maximum acceleration close to the midspan on the (a) north and (b) south side for increasing number of simulations caused by a single pedestrian walking at a step frequency of 2 Hz - Simulation model II

Comparison between Simulation Model I and II

In this section, results obtained from the Simulation Model I and II are compared with each other. Analyses with the Simulation Model I are performed employing the FE model A3, considered the one that can represent more accurately the footbridge behaviour, including in terms of the torsional behaviour.

Amplitude of the maximum acceleration obtained from the Simulation model I and II at positions close to the midspan and quarter span on the north and south side is shown in Figure 3.29. As regards the Simulation model II, dynamic analyses are performed by solving the equation of motion in modal coordinates, adopting the experimentally identified modal properties. For that reason, dynamic amplifications are obtained for pacing frequencies corresponding to the experimentally identified natural frequencies and their multiples. However, with the Simulation model I, response amplifications are obtained for almost the same frequencies, as the FE model A3 is characterized by low frequency

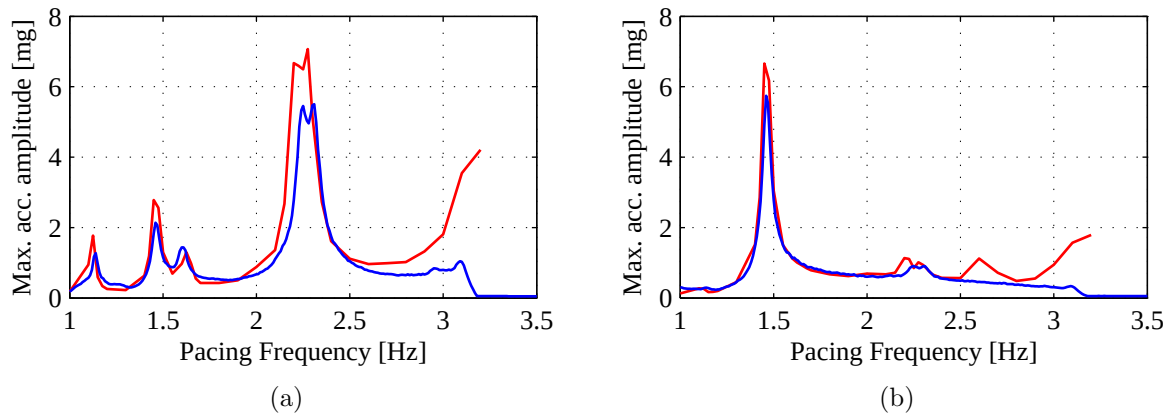


Figure 3.29: Amplitude of the maximum acceleration vs. pacing frequency obtained from SM-I (red line) and SM-II (blue line) at position (a) 3N and (b) 6N

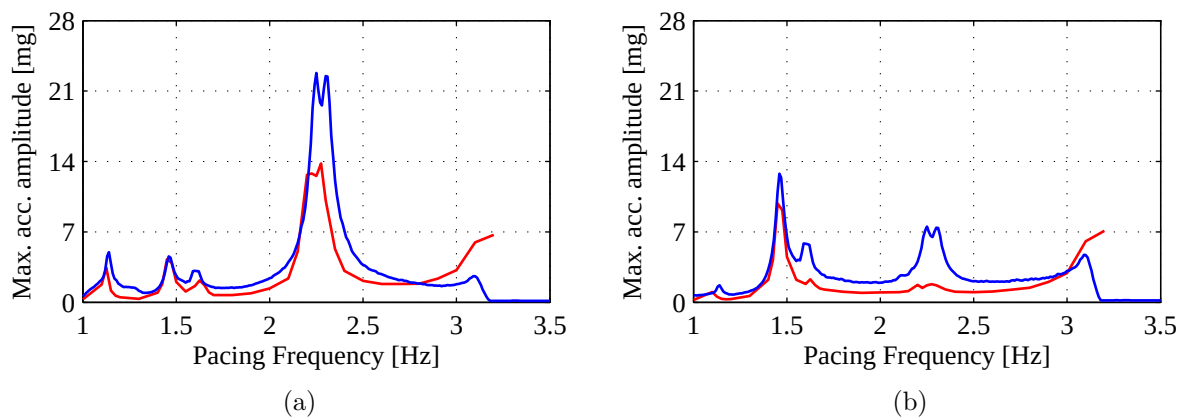


Figure 3.30: Amplitude of the maximum acceleration vs. pacing frequency obtained from SM-I (red line) and SM-II (blue line) at position (a) 3S and (b) 6S

errors with respect to the experimental results. When it comes to the north side, response amplifications obtained from the Simulation model I and II are basically the same until the step frequency is lower than 3 Hz, while more significant differences are obtained for higher step frequencies. It is worthy to highlight again that responses obtained when the step frequency is higher than about 2.5 Hz may not be accurate as the DLFs are defined for lower values of step frequencies.

Slightly higher differences among the results from the Simulation model I and II are obtained when the south side of the deck is considered (Figure 3.30), especially concerning the second vertical mode ($f = 2.274$ Hz).

3.4.4.2 Comparison between numerical and experimental accelerations induced by pedestrians

Vertical accelerations recorded at the different measuring positions and caused by the loading conditions referred to as Case 1, Case 2 and Case 3 are compared with those obtained from Simulation model I and II.

Maximum RMS values of the vertical accelerations caused by 12 pedestrians walking at a step frequency of 1.45 Hz are presented in Figure 3.31(a). Results obtained from the SM-I match almost perfectly the experimental ones, while they are slightly underestimated by the SM-II.

In the same way, the maximum RMS accelerations caused by one pedestrian walking at a step frequency of 2.2 Hz, close to the frequency of the second natural mode of the structure, is shown in Figure 3.31(a). In this case, numerical results from the Simulation model I and II are quite comparable and slightly higher than the experimental ones.

Finally, experimental accelerations caused by 44 pedestrians walking at a pacing frequency of 1.2 Hz are compared with the numerical ones (Figure 3.32). In this case, the maximum response is evaluated considering different values of synchronization factor: n (pedestrian number), representative of a perfect synchronization, $\sqrt{n}$, which represent absence of synchronization and $n/2$, for a rough estimation of an intermediate synchronization level. In fact, the frequency of 1.2 Hz indicates a very slow pedestrian velocity, typical of a crowded

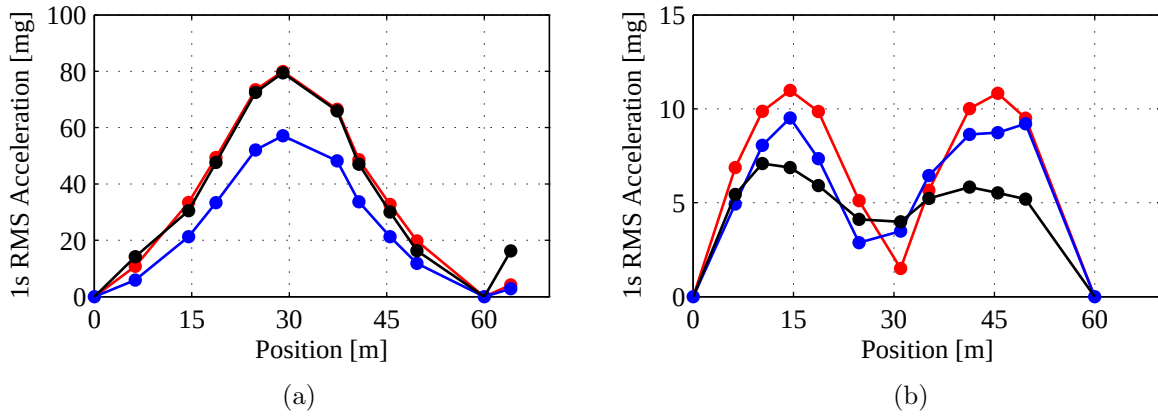


Figure 3.31: RMS acceleration levels at different measuring positions and for different test conditions: (a) 12 pedestrians walking at a step frequency of 1.45 Hz and (b) a single pedestrian running at a pacing frequency of 2.2 Hz. Black line: experimental RMS acceleration; red line: RMS acceleration from SM-I adopting the FE Model A3; blue line: RMS acceleration from SM-II

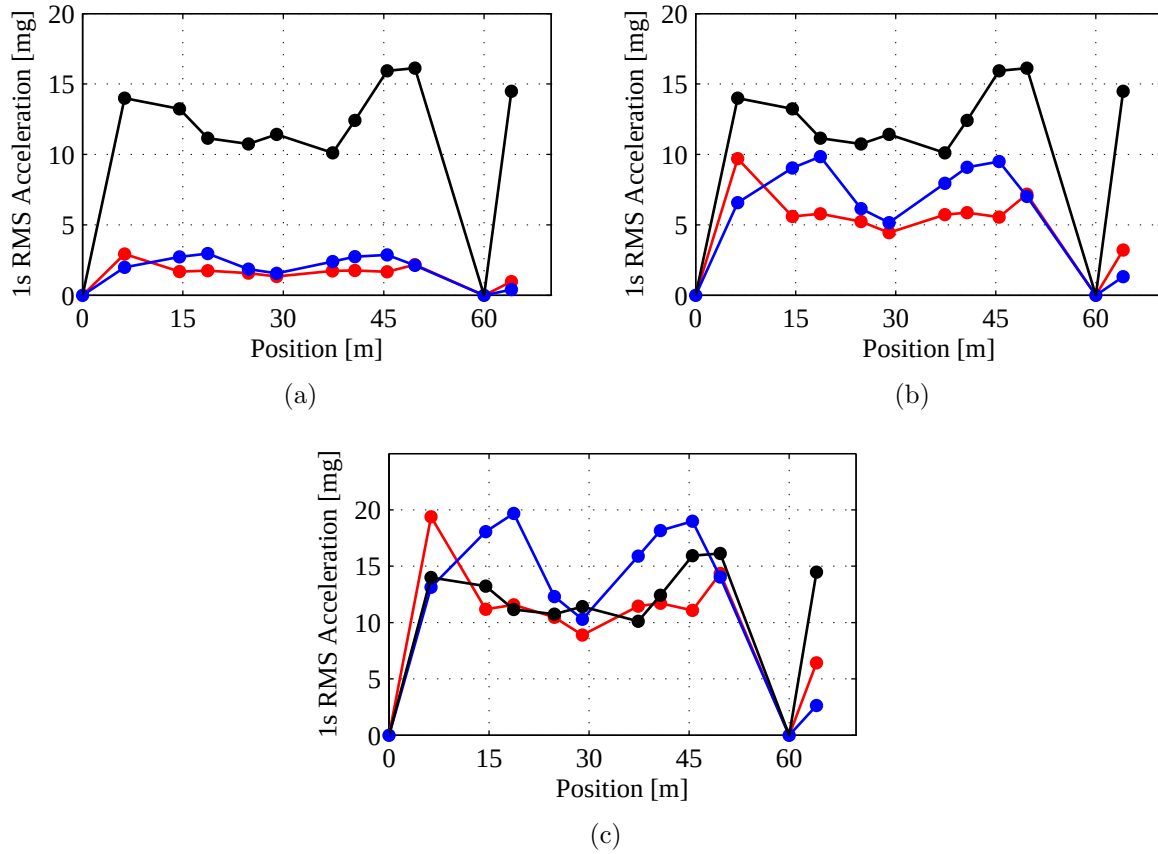


Figure 3.32: RMS acceleration levels at different measuring positions caused by 44 pedestrians walking at a step frequency of 1.2 Hz obtained considering a synchronization factor equal to (a) $\sqrt{n}$, (b) $n/2$ and (c) n . Black line: experimental RMS acceleration; red line: RMS acceleration from SM-I adopting the FE Model A3; blue line: RMS acceleration from SM-II

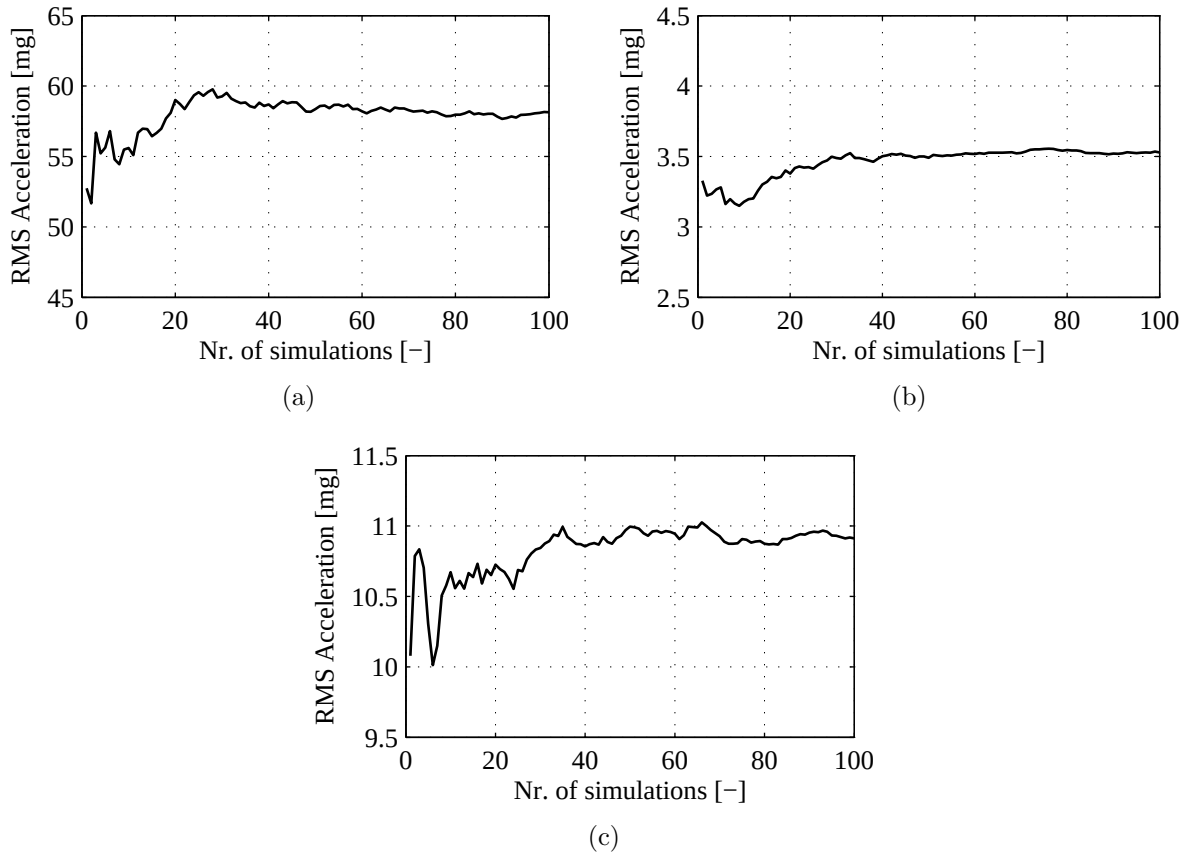


Figure 3.33: Maximum RMS acceleration close to the midspan for increasing number of simulations caused by (a) 12 pedestrians walking at a step frequency of 1.45 Hz, (b) a single pedestrian running at a pacing frequency of 2.2 Hz and (c) 44 pedestrians walking at a step frequency of 1.2 Hz - Simulation model II

situation. Hence, it is reasonable to assume that, even if pedestrians were guided by the metronome signal, the synchronization was not perfectly obtained because of the slow velocity imposed. However, the assumption of total absence of synchronization leads to a significant underestimation of the induced accelerations. Even the synchronization factor equal to $n/2$ leads to an underestimation of the response, evidence that pedestrians were almost synchronized. Considering a perfect synchronization, maximum accelerations estimated through the Simulation model I are quite comparable with the experimental ones, while accelerations obtained from the Simulation model II are slightly higher.

The mean value of the RMS acceleration estimated close to the midspan through the SM-II with increasing number of simulations is shown in Figure 3.33. The response at the m th simulation is evaluated as the average of the responses obtained from the previous $m - 1$ simulations.

To account for the fact that a perfect synchronization is difficult to achieve for a large group of pedestrians walking at a low velocity, instead of evaluating the synchronization factor a slight variation in the step frequency or in the phase of each pedestrian can be introduced.

Accelerations obtained considering a standard deviation of the step frequency of 0.002 Hz

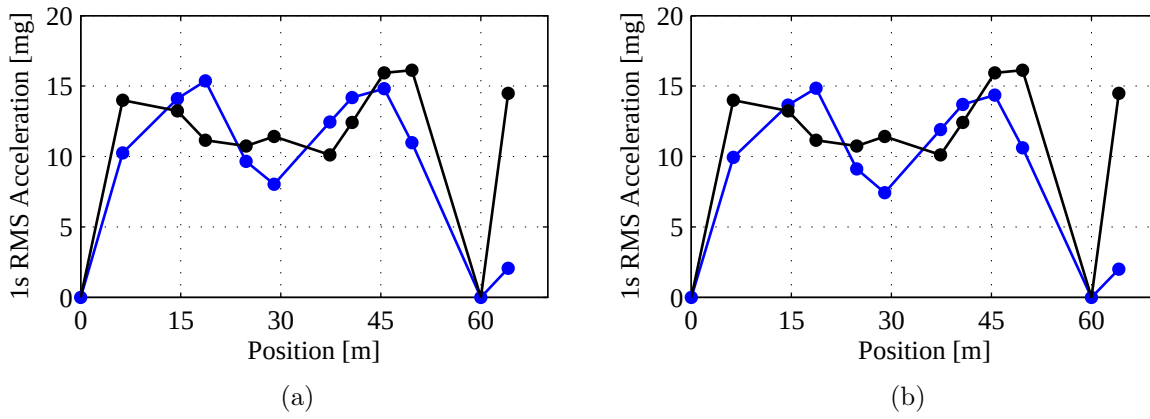


Figure 3.34: RMS acceleration levels at different measuring positions caused by 44 pedestrians walking at a step frequency of 1.2 Hz obtained considering a standard deviation (a) of the step frequency equal to 0.002 Hz and (b) of the phase equal to 0.85 rad. Black line: experimental RMS acceleration; blue line: RMS acceleration from SM-II

with respect to its mean value (1.2 Hz) is shown in Figure 3.34(a).

To consider the variation in the phase angle, in each simulation a “mean” phase vector is generated (representing the phase for each frequency line in the force spectrum) and a standard deviation with respect to the mean value is assigned to each pedestrian. Thus, each pedestrian is characterized by a slightly different phase angle, to account for the not perfect synchronization with the others. Accelerations obtained considering a standard deviation of the phase angle of 0.85 rad with respect to its mean value are shown in Figure 3.34(b).

It can be observed that the introduction of a small variability of the walking parameters leads to a response closer to the experimental one, evidence that pedestrians were almost synchronized during the experimental test.

3.5 The Correggio footbridge

The present section describes and analyses the dynamic behaviour of the Correggio footbridge, a steel footbridge composed of five simple-supported spans. The footbridge is sensitive to vibrations in both the vertical and lateral direction, as its natural frequencies fall within the typical frequency range of pedestrian-induced dynamic actions [110].

3.5.1 Description of the footbridge

The Correggio footbridge (Figure 3.35) was built in 2011 in Reggio Emilia (Italy). It is 168 m long and composed of five simple-supported spans linked at lower-floor level. The central span is 40 m long while the lateral ones are 32 m long each one. The footbridge is supported by 4 piles; the central and lateral piles are about 5.20 m and 2.80 m high, respectively. The structure has a box cross-section 3.00 m x 2.85 m composed of truss girders (Figure 3.35(c)). Piles and trusses are made up by rectangular hollow profiles and



(a)



(b)



(c)

Figure 3.35: The Correggio footbridge: (a,b) general views and (c) detail of the spatial truss girder

L-shaped profiles are used for the lower lateral bracing system. The deck is 3 m wide and consists of a wooden plank resting on a corrugated sheet. The whole structure is slender, lightweight and characterized by sensitive vibrations due to pedestrian, cyclist and wind effects.

3.5.2 The dynamic tests

3.5.2.1 Experimental setup

The aim of dynamic tests is to evaluate both the modal properties of the footbridge and the effect of pedestrian dynamic actions, recording accelerations caused by ambient vibrations and by people walking, running and jumping. The footbridge dynamic behaviour was monitored by means of the monitoring system described in section 3.1, composed of the main unit and 10 sensors. To estimate as many natural modes as possible, sensors were arranged into 3 different setups. Since each sensor can record accelerations along two orthogonal axes, 20 measurement channels for each setup (10 in the vertical and 10 in the transverse direction) were available. Positions of sensors in the different setups are shown in Figure 3.36, where the letter denotes the sensor and the number represents the global position. In the first two setups, sensors were mainly located in the central span, placed on both the upper and the lower chord of the main truss girders and on both deck sides,

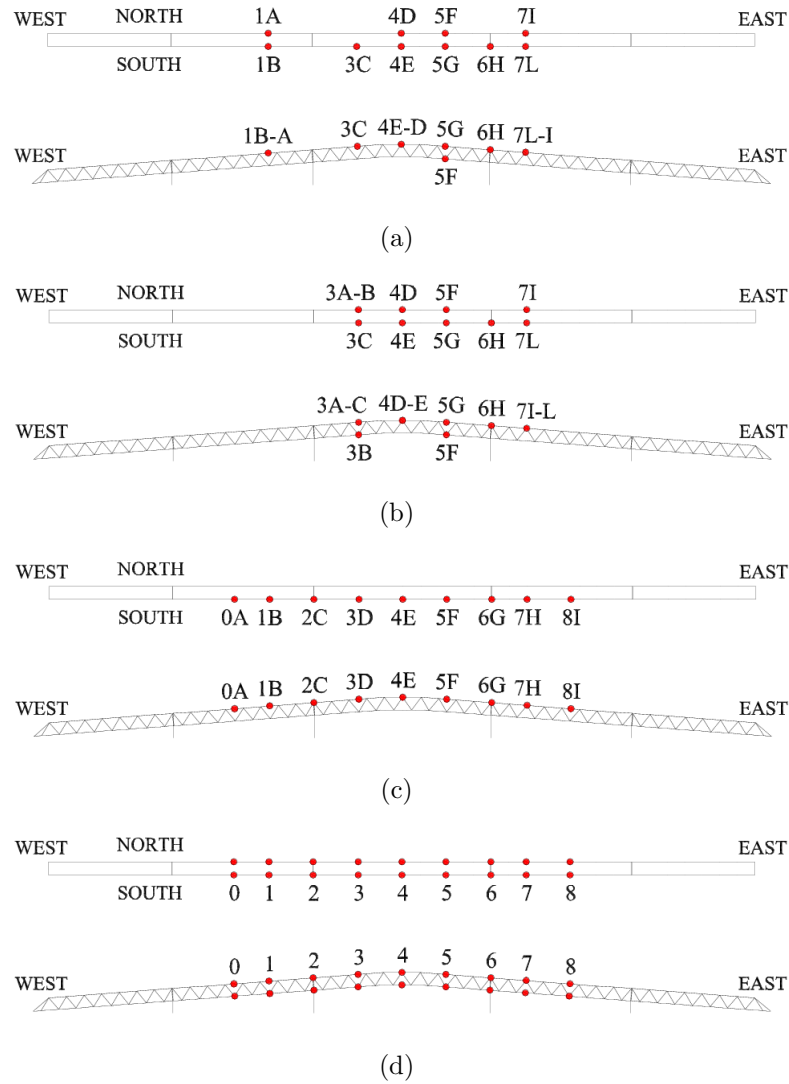


Figure 3.36: Experimental setup no. (a) 1, (b) 2, (c) 3 and (d) position numbering

to evaluate the relative lateral displacement of the cross-section and identify also torsional modes. In the third configuration, sensors were all placed at the upper chord level of the south side, to cover the maximum possible length of the footbridge. In the latter setup, sensors were located at midspan, at quarter spans and close to the piles of the three central spans.

3.5.2.2 Pedestrian-induced accelerations

Experimental measurements show that vertical accelerations caused by few people walking are in the range $[5, 6]$ mg ($[0.05, 0.06]$ m/s²). In Figure 3.37, vertical and lateral accelerations induced by people running and jumping are reported. Vertical accelerations caused by two people running are mainly in the range $[40, 60]$ mg, while lateral accelerations show values of about 30 mg. Effects of 20 seconds of jumping and a single jump are shown in Figures 3.37(c) and 3.37(d). Vertical and horizontal accelerations reach values of about

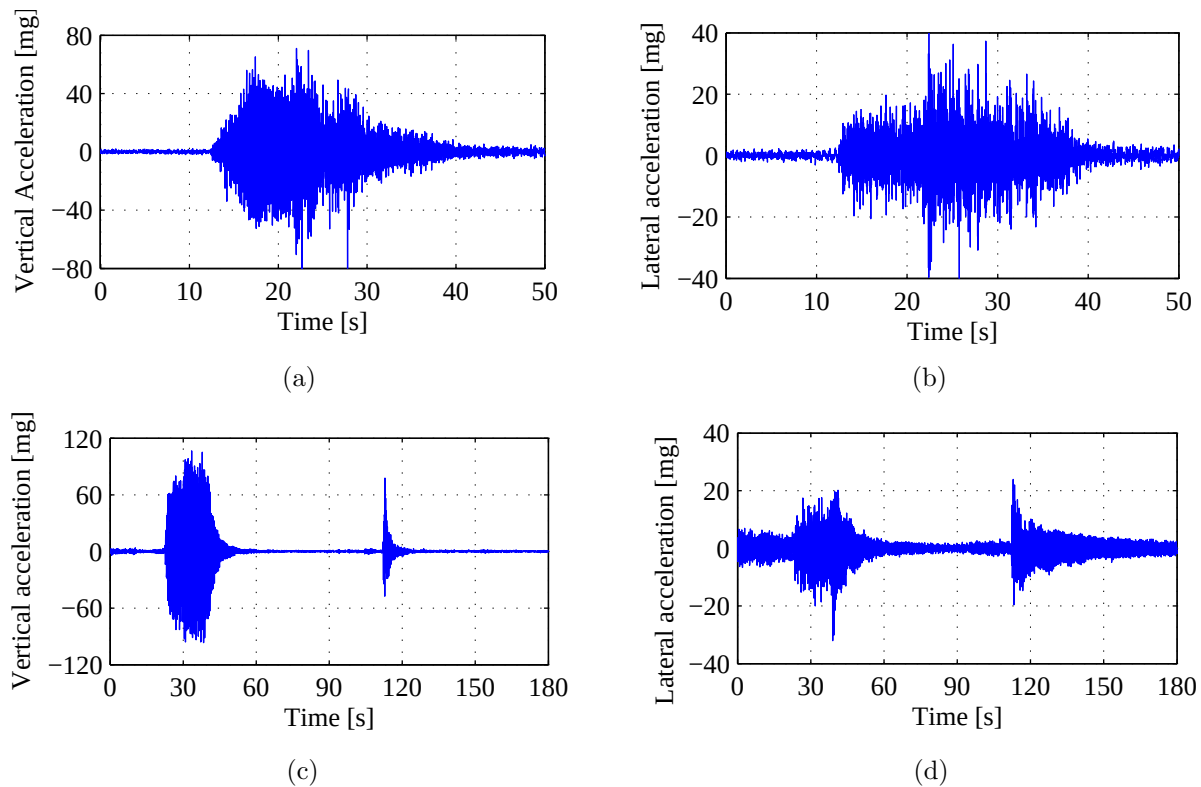


Figure 3.37: Vertical and lateral midspan acceleration due to (a,b) running and (c,d) 20 seconds of jumping and a single jump

100 mg and 20 mg, respectively. Note that after each jump, vertical accelerations damp in few seconds while not negligible horizontal accelerations are measured even 50 seconds after the jump, denoting very low damping ratios.

3.5.2.3 Results of the identification process

Starting from the accelerations recorded in operational conditions, the footbridge modal properties are identified through the algorithm described in section 3.2. Examples of PSD functions obtained from vertical and transverse horizontal accelerations are reported in Figure 3.38. A total of 10 modes (6 lateral modes, 2 verticals and 2 torsionals) with frequencies in the range $[1.67, 12.08]$ Hz are clearly identified. Frequencies of identified modes are reported in Table 3.5, alongside a comparison with those obtained from the finite element model of the footbridge (see section 3.5.3). Mode shape components of the identified modes are shown in Figures 3.39, 3.40 and 3.41. The first four modes show deformations mainly in the horizontal direction and the first vertical mode has a frequency almost 2.5 times higher than the first lateral one. This displays the high flexibility of the footbridge in the horizontal direction.

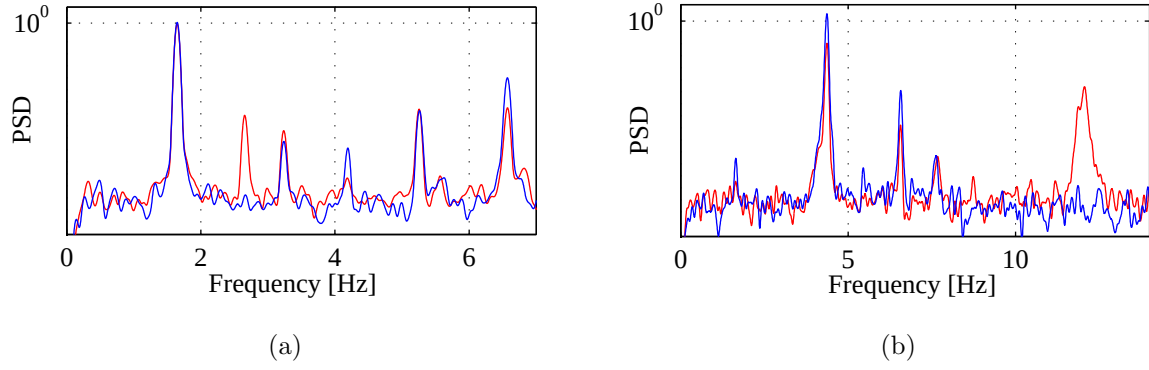


Figure 3.38: PSDs obtained from the (a) lateral and (b) vertical accelerations recorded by the sensor 5G (red line) and 4E (blue line)

Mode no.	Mode type	Experimental Frequency [Hz]	Numerical Frequency [Hz]	Error [%]	MAC [%]
1	1 st Lateral	1.674	1.676	+0.12	99.04
2	2 nd Lateral	2.663	2.702	+1.46	98.43
3	3 rd Lateral	3.082	3.103	+0.68	83.72
4	4 th Lateral	3.567	3.489	-2.19	93.24
5	1 st Vertical	4.392	4.293	-2.25	96.62
6	Lateral - central span	5.276	5.236	-0.76	86.39
7	Lateral - side spans	5.970	6.096	+2.11	65.78
8	1 st Torsional	6.596	6.521	-1.14	85.87
9	Torsional - central span	7.690	7.918	+2.96	85.61
10	2 nd Vertical	12.085	11.063	-8.46	96.68

Table 3.5: Experimental and numerical modes

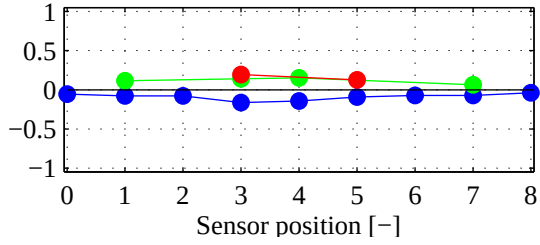
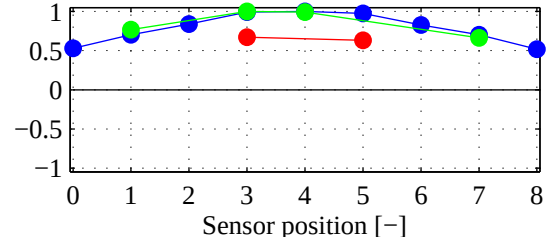
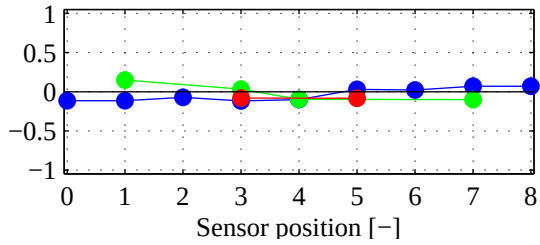
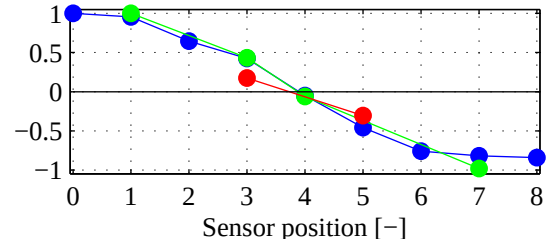
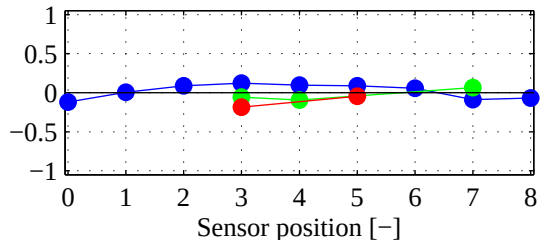
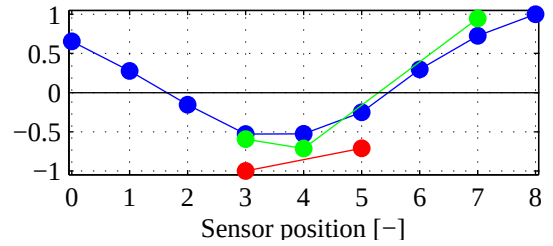
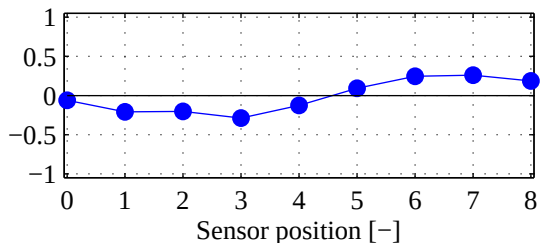
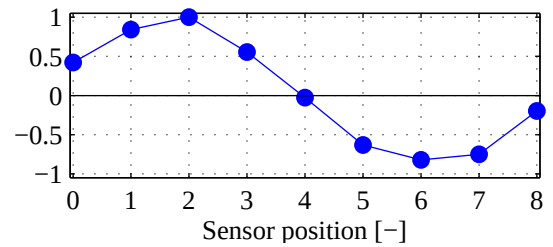
(a) 1st mode - vertical comp. - $f = 1.647$ Hz(b) 1st mode - lateral comp. - $f = 1.647$ Hz(c) 2nd mode - vertical comp. - $f = 2.663$ Hz(d) 2nd mode - lateral comp. - $f = 2.663$ Hz(e) 3rd mode - vertical comp. - $f = 3.082$ Hz(f) 3rd mode - lateral comp. - $f = 3.082$ Hz(g) 4th mode - vertical comp. - $f = 3.567$ Hz(h) 4th mode - lateral comp. - $f = 3.567$ Hz

Figure 3.39: Vertical and lateral components of the experimental modes [1,4]. Blue line: upper chord level of the south side, green line: upper chord level of the north side, red line: lower chord level of the north side

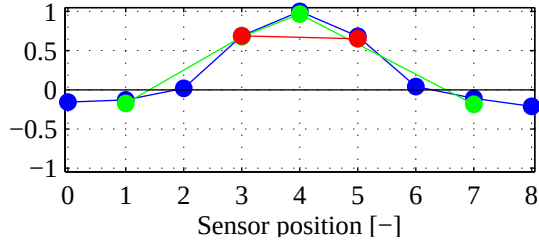
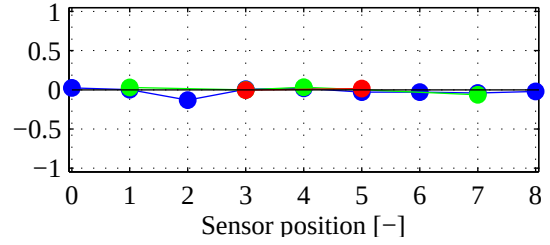
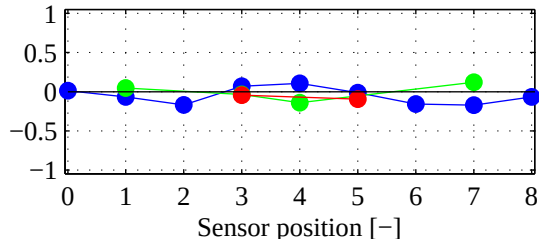
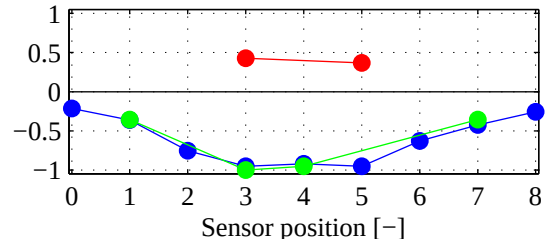
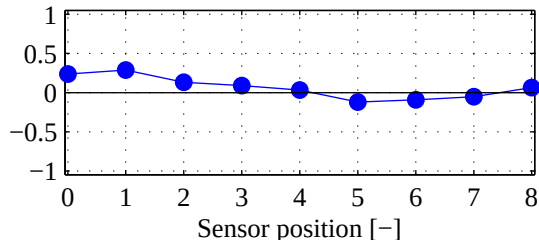
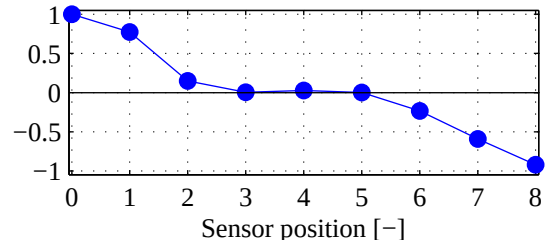
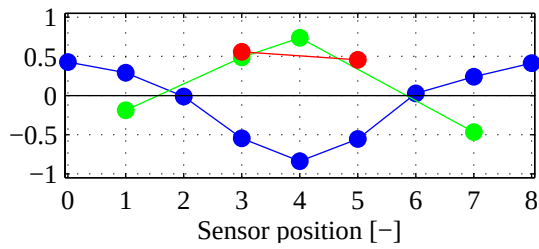
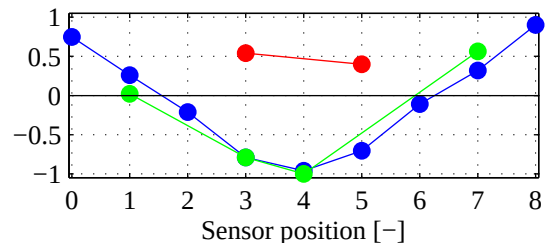
(a) 5th mode - vertical comp. - $f = 4.293$ Hz(b) 5th mode - lateral comp. - $f = 4.293$ Hz(c) 6th mode - vertical comp. - $f = 5.236$ Hz(d) 6th mode - lateral comp. - $f = 5.236$ Hz(e) 7th mode - vertical comp. - $f = 6.096$ Hz(f) 7th mode - lateral comp. - $f = 6.096$ Hz(g) 8th mode - vertical comp. - $f = 6.521$ Hz(h) 8th mode - lateral comp. - $f = 6.521$ Hz

Figure 3.40: Vertical and lateral components of the experimental modes [5,8]. Blue line: upper chord level of the south side, green line: upper chord level of the north side, red line: lower chord level of the north side

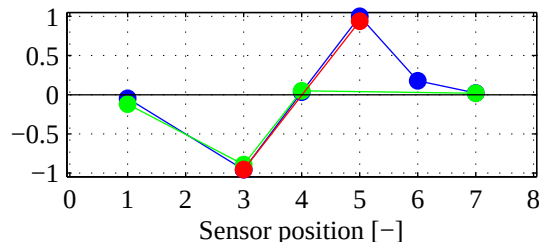
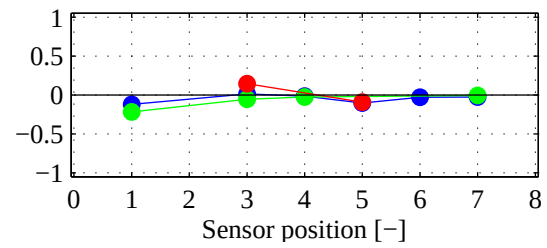
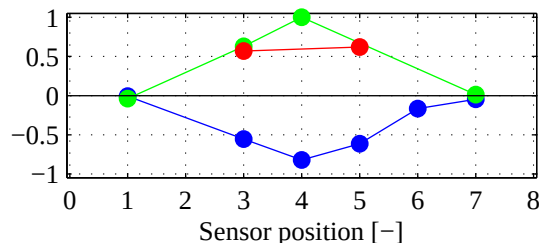
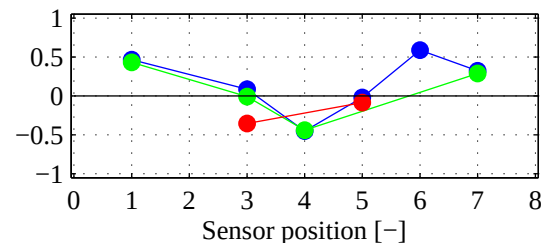
(a) 9th mode - vertical comp. - $f = 7.918$ Hz(b) 9th mode - lateral comp. - $f = 7.918$ Hz(c) 10th mode - vertical comp. - $f = 11.063$ Hz(d) 10th mode - lateral comp. - $f = 11.063$ Hz

Figure 3.41: Vertical and lateral components of the experimental modes [9,10]. Blue line: upper chord level of the south side, green line: upper chord level of the north side, red line: lower chord level of the north side

3.5.3 Finite element model and model updating

The finite element model of the footbridge, shown in Figure 3.42, is built and calibrated. Truss and beam elements are adopted for girders and piles, while constraints are applied at the base of piles and at the lateral span ends. Then, the FE model is calibrated according to the procedure summarized in section 3.3, so that the numerical results correspond as close as possible to the experimental ones. For this purpose, a set of unknown model parameters are adjusted such that frequencies of modes no. 1, 2, 5 and 8 match the experimental ones. The selected model parameters are the mass of the members (to take into account additional masses of plates and bolts) and the mass of the deck. The first four numerical mode shapes are shown in Figure 3.43. In Table 3.5, the numerical frequencies obtained after the model calibration are compared with those identified experimentally while the mode shapes are compared using the MAC. Numerical results match accurately the experimental ones with error never greater than 3% as for the first 9 frequencies and about 8% for the tenth. MAC values are also good, except for the seventh mode, where the MAC value is only the 65%. This poor correlation is due to some disagreements in the horizontal components of the central span: experimental values are close to zero while not negligible values are obtained in the numerical model.

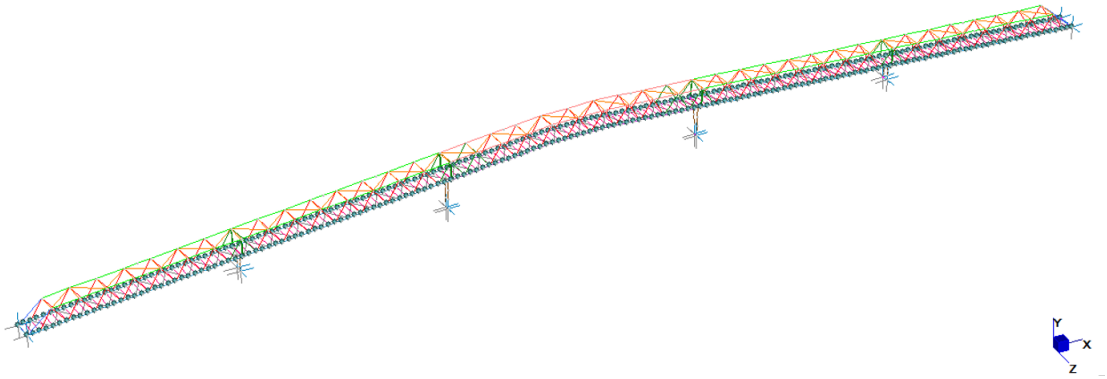
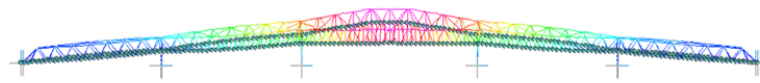
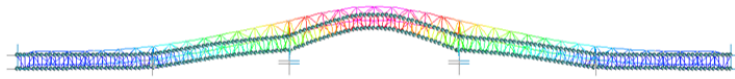


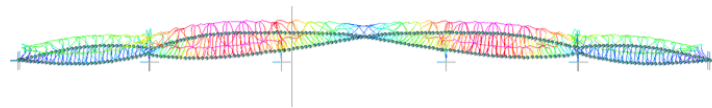
Figure 3.42: FE model of the footbridge



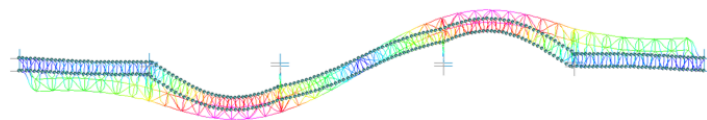
(a) First mode - view from the side



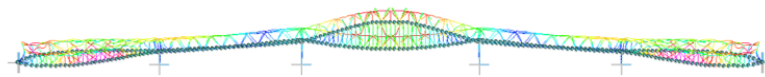
(b) First mode - view from above



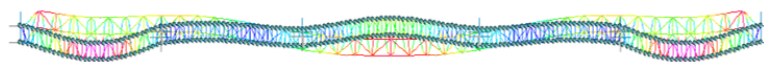
(c) Second mode - view from the side



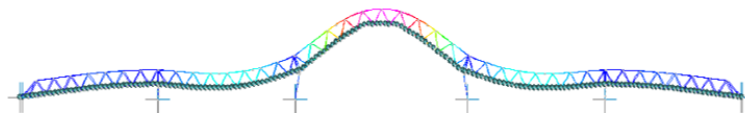
(d) Second mode - view from above



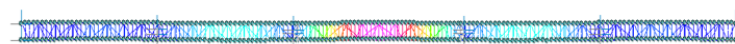
(e) Third mode - view from the side



(f) Third mode - view from above



(g) Fourth mode - view from the side



(h) Fourth mode - view from above

Figure 3.43: View form the side and from above of the first four numerical mode shapes

3.5.4 Simulation of the footbridge response

The Correggio footbridge is sensitive to human-induced vibrations in both vertical and lateral directions, but especially in the lateral one. For this reason, even though the simulation of pedestrian action in the lateral direction is not addressed in the present study, simple simulations of the structural response in the lateral direction are also performed. The pedestrian-induced lateral force is represented, in a simplified way, as an harmonic force with fundamental frequency half the step frequency and a DLF equal to 0.039. The latter is in line with the proposal of Bachmann and Ammann [6]. The human-induced vertical force is represented according to the model proposed by Young [120] and dynamic analyses are performed with the FE model described in section 3.5.3.

Numerical analyses are performed considering one or more pedestrians crossing the footbridge with different velocities and some results are described in the following. Figure 3.44 presents accelerations and displacements predicted at midspan and in a side span caused by two people running at a pacing frequency of 3 Hz. Small values of displacement (less than 1 mm) are obtained in both vertical and lateral directions. Numerical accelerations are compared with those recorded experimentally caused by two people running but without monitoring the effective pacing frequency (Figure 3.45). Maximum predicted vertical acceleration is comparable with the experimental ones, with values of about 40 mg (0.4 m/s^2). On the contrary, more significant differences are obtained for lateral accel-

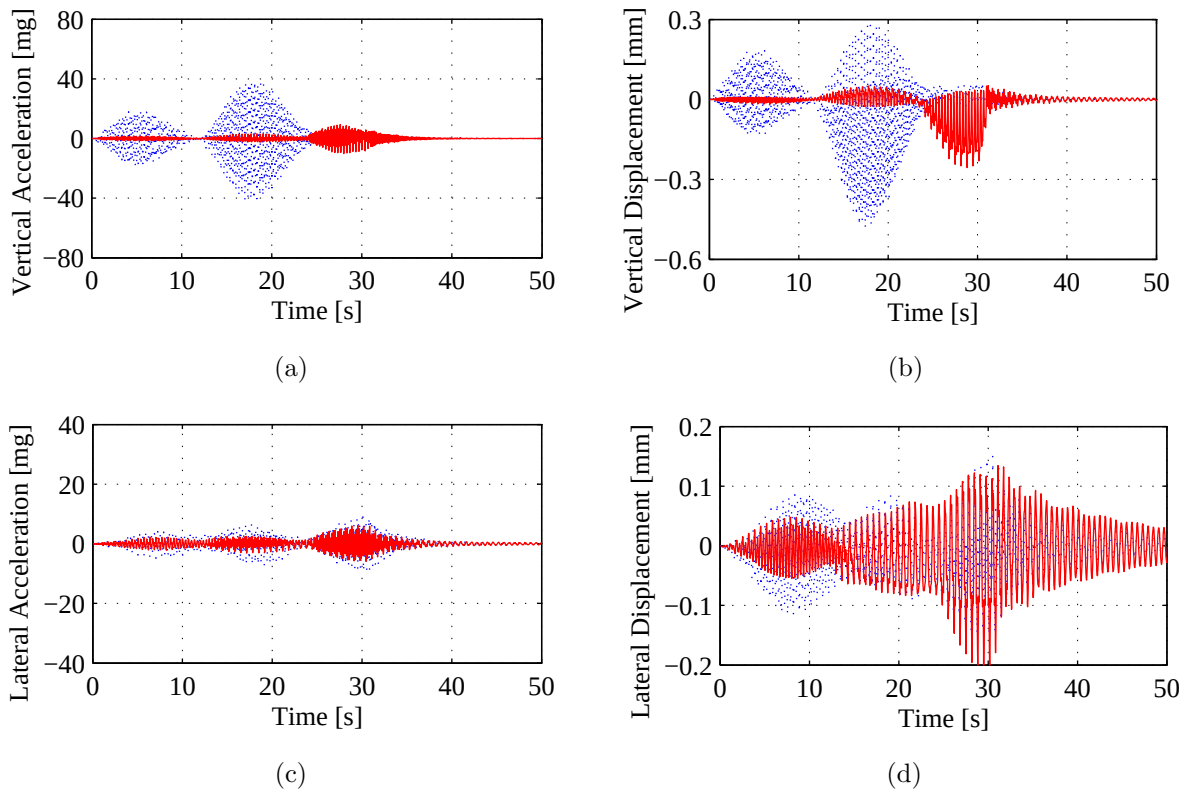


Figure 3.44: Numerical (a,b) vertical and (c,d) lateral accelerations and displacements caused by 2 people running at a pacing frequency of 3 Hz obtained at midspan (red line) and in a side span (blue line)

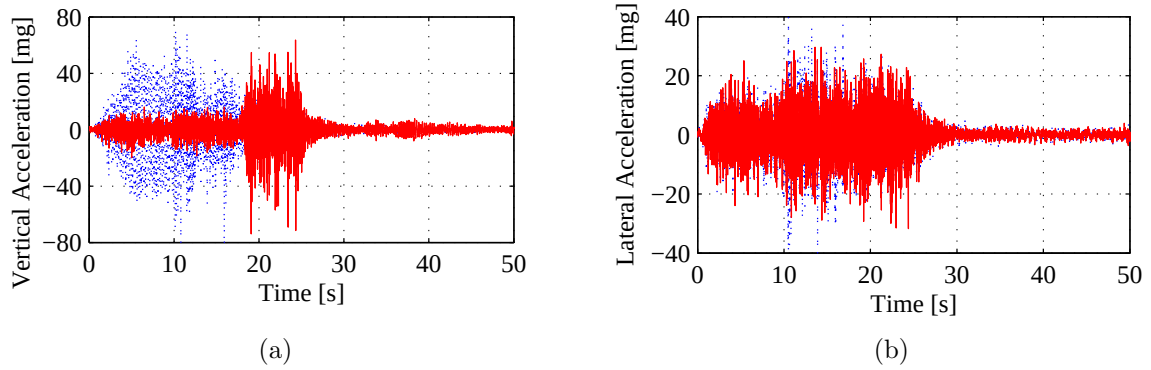
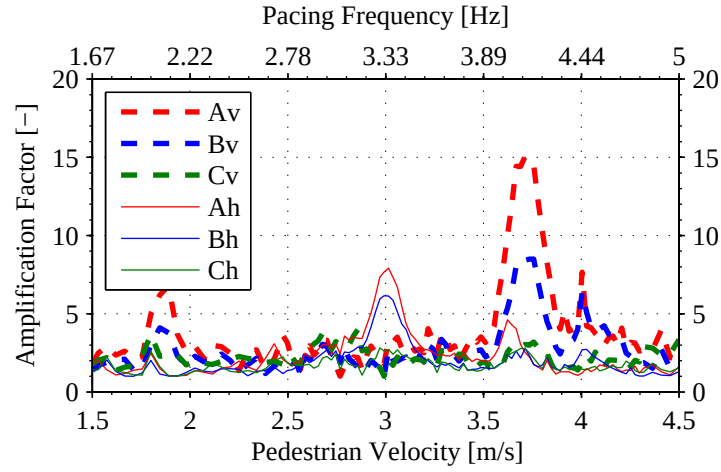


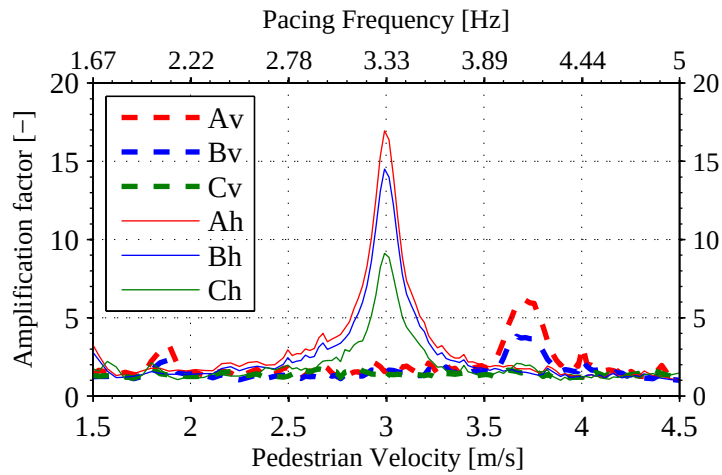
Figure 3.45: Experimental (a) vertical and (b) lateral accelerations caused by 2 people running measured at midspan (red line) and in a side span (blue line)

erations: maximum values in the range $[8, 10]$ mg are predicted while greater values are measured experimentally (about 30 mg). It is worth noting that these values strongly depend on the imposed numerical damping ratio; thus, the experimental identification of damping ratios should be as much accurate as possible to obtain a reliable comparison between experimental and numerical results.

Finally, in Figures 3.46(a) and 3.46(b) the dynamic amplification factors in terms of acceleration and displacement are presented. Amplification factors are obtained considering a pedestrian crossing the footbridge with velocity in the range $[1.5, 4.5]$ m/s. Adopting the relation $c_s = 0.9f_s$ proposed in [20], the velocity range corresponds to the step frequency range $[1.67, 5]$ Hz. Significant lateral amplifications are observed for a pacing frequency of about 3.33 Hz, corresponding to a horizontal forcing frequency of 1.665 Hz, i.e. close to the frequency of the first lateral mode. Vertical amplifications are obtained for a pacing frequency of about 4.15 Hz, quite close to the first vertical mode of the structure.



(a)



(b)

Figure 3.46: Amplification factor in terms of (a) acceleration and (b) displacement in the vertical (dashed lines) and horizontal (solid lines) direction. A: node at midspan of the central span; B: node at a quarter of the central span; C: node at midspan of a side span

Chapter 4

Spectral model for pedestrian-induced loading

4.1 Frequency domain models

Whereas the first studies on the human-induced action were mainly focused on the analysis and modelling in the time domain, in the early 1980s Ohlsson [81] took an alternative approach to the problem. He was interested more in the energy and frequency content than in the exact time history of the vertical force, introducing a rather new concept at the time for mathematical modelling of the human-induced force. Namely, he produced continuous walking force based on a measured single step force and the assumption of perfect periodicity. Then, he studied that force by determining its power-spectral density (PSD). He observed that the single step force had most of its energy in the frequency range from 0 to 6 Hz, even if he focused on the high-frequency content of the PSD because he was investigating the behaviour of high-frequency building floors made of timber [114]. Over the next 10-15 years, this approach was further developed and extended to low-frequency floors. In this context, Eriksson [45], who was Ohlsson's student, focused on low-frequency floors making use of the PSD. He measured a continuous walking force indirectly and presented it in terms of its PSD, observing that it was a narrow band random process. Eriksson explained that the fact that each peak of the spectrum has some width means that human walking cannot be perfectly periodic and thus cannot be accurately described by a time domain model. The fact that the walking force is a narrow band process has been widely confirmed by studies conducted over the years. For instance, Brownjohn et al. [23] and Van Nimmen et al. [103] showed that in contrast to perfectly periodic forces, imperfect real walking results in a distribution of forces around the dominant harmonics [65, 85]. This leads to significant differences between responses due to the imperfect and the equivalent periodic walking force. Therefore, the random nature of human walking is more suited to stochastic analysis, which can be performed by representing the walking forces as PSDs [23]. This procedure is based on the assumption that human-induced force can be treated as a stationary random process [114].

Ohlsson and Eriksson paved the way to the vibration serviceability assessment based on the theory of stationary random process. It is known that for a stationary random process

the PSD of the response $S_y(\omega)$ can be calculated as [80]:

$$S_y(\omega) = |H(\omega)|^2 S_x(\omega) \quad (4.1)$$

where $H(\omega)$ is the frequency response function (FRF) of the system and $S_x(\omega)$ is the single-sided PSD of the input force. Then, the mean square value of the response $E[y^2]$ can be calculated as the area under the spectral density curve of the response (see [80] and appendix A):

$$E[y^2] = \int_0^{+\infty} S_y(\omega) d\omega \quad (4.2)$$

From the mean square value of the response, the root-mean square (RMS) or peak acceleration can be obtained and compared to the design values, usually based on comfort criteria [114]. The aim of spectral design methods has become to find a simple way to describe the stochastic pedestrian loading and system response. Eriksson [45] proposed a force mean square design model with a constant mean square in the range of the first harmonic and frequency dependent for higher harmonics. He also proposed a multiplication factor for a single person RMS force dependent on the number of people in the group. Mouring [78] and Mouring and Ellingwood [79] modelled the PSD of the modal force due to a pedestrian crowd as the product of the number of people and the PSD of a single pedestrian. This is equivalent to assume that the acceleration response due to a group of n uncorrelated people is $\sqrt{n}$ times higher than the acceleration due to a single person, as obtained by Matsumoto et al. [76] in the time domain. Hansen and Sørensen [57] defined jumping crowd load in terms of its harmonics, accounting for crowd effect and lack of synchronization among people by a crowd reduction factor.

Brownjohn et al. [23] focused on the spread of energy around the harmonics and showed that this effect is more emphasized for higher harmonics. Based on direct measurements of the vertical force time histories for three test subjects walking on a treadmill, they proposed a model which describes a forcing function for the first six harmonics. Furthermore, the evaluation of the degree of correlation among pedestrians in a crowd is an important task for the calculation of the bridge response. For this purpose, the approach proposed by Brownjohn et al. [23] considers the correlation among pedestrians at different locations on a structure in a similar way the dynamic response of structures to turbulent buffeting by wind is estimated. That kind of approach can be used for single pedestrians as well as crowd loading, based on the definition of correlation among pedestrians and statistics of their pacing rates [47]. The frequency domain model proposed by Brownjohn et al. [23] for a realistic treatment of vertical forces induced by groups of normally walking pedestrians accounts for the imperfection of individual walking excitation as well as the statistical distribution of pacing rates in the walking crowd. However, the model can only be adopted to assess the effect of low level pedestrian traffic, as the correlation among pedestrians is not accounted for.

In the present study, the spectral model proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] for the modal force induced by pedestrian groups modelled as a stationary random process is analysed. The model relies on an analytical definition of the power spectral density (PSD) of the modal force that can be adopted in unrestricted traffic conditions, as formulated in [83]. In addition, it can also be adopted in crowded

conditions, but in absence of human-structure interaction, thanks to the generalized formulation proposed in [47]. The latter is based on a physically-based expression of the coherence function, which accounts for the increased synchronization among pedestrians with increasing pedestrian density.

In the present chapter, the procedure proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] for the evaluation of the maximum dynamic response is described. Pedestrian traffic conditions are modelled probabilistically and based on such a probabilistic model and the analysis of peculiar non-dimensional parameters, the spectral model for the modal force induced by pedestrian groups modelled as a stationary random process is analytically deduced. Starting from the analytical spectral model of the load, the maximum dynamic response is evaluated through simple closed-form expressions with the methods of linear random dynamics.

4.2 Evaluation of the maximum dynamic response based on the spectral model for pedestrian induced loading

The procedure proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] for the serviceability analysis of footbridges, is based on the estimation of the maximum dynamic response through the methods of linear random dynamics (e.g., [41]). In particular, the acceleration response of a system subjected to a random excitation, such as the pedestrian groups modelled as stationary random process, is statistically characterized. When a lightly-damped linear system is subjected to random excitation, the characteristics of the excitation are modified by the system, which behaves as a filter, and the resulting response is a narrow band random process [80, 119]. As described in appendix B, the mean value of the distribution of maxima can be obtained from the standard deviation of the output acceleration $\sigma_{\ddot{p}_j}$ and a peak coefficient $g_{\ddot{p}_j}$, Eq.(B.32):

$$\ddot{p}_{j_{max}} = g_{\ddot{p}_j} \sigma_{\ddot{p}_j} \quad (4.3)$$

where $\ddot{p}_{j_{max}}$ is the maximum accelerations of the principal coordinates [83]. Then, considering that the structural response is dominated by the j th natural mode shape, the maximum acceleration in physical coordinates can be estimated as:

$$\ddot{q}_{max}(x) = \varphi_j(x) \ddot{p}_{j_{max}} \quad (4.4)$$

where $\varphi_j(x)$ is the structural mode shape and x represents the bridge location. The standard deviation of the output acceleration is estimated from the acceleration variance, which in turn is calculated as (see appendix A, Eq.(A.2)):

$$\sigma_{\ddot{p}_j}^2 = \int_0^\infty \omega^4 |H_B(\omega)|^2 S_{F_j}(\omega) d\omega \quad (4.5)$$

where $S_{F_j}(\omega)$ is the single-sided PSD function of the modal force induced by pedestrians and $H_B(\omega)$ is the frequency response function of the footbridge relating the harmonic input excitation to the acceleration response. For lightly-damped systems and when the

response may be assumed mainly resonant, the acceleration variance can be evaluated as (see appendix A, Eq.(A.9)):

$$\sigma_{\ddot{p}_j}^2 = \frac{\pi\omega_j}{4\xi_j m_j} S_{F_j}(\omega_j) \quad (4.6)$$

where ω_j , ξ_j and m_j are the structural circular frequency in rad/s, the damping ratio [%] and the modal mass [kg], respectively.

The spectral model proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] relies on an analytical definition of the PSD of the modal force. A summary of the procedure for the derivation of the analytical expression of the spectral load is given here. First, the pedestrian-induced force is probabilistically modelled and the modal force is calculated. Second, the analytical expression of the PSD of the modal force is obtained for both the unrestricted and crowded traffic conditions in section 4.2.3. The peak coefficient is described in section 4.2.4. Finally, applications of the spectral load model proposed in [83] and [47] are presented and discussed.

4.2.1 Probabilistic modelling of pedestrian-induced forces

A pedestrian walking on a structure produces a dynamic force that has components in the vertical, lateral and longitudinal direction. The present study is focused on the simulation of the human-induced forces in the vertical direction. As reported in chapter 2, the vertical force caused by a single pedestrian can be expressed in the classic form of a periodic force described by a Fourier series: [1, 28, 49, 99]:

$$f(t) = G + \sum_{k=1}^n G\alpha_k \sin(2\pi k f_s t + \Psi_k) \quad (4.7)$$

where G [N] is the pedestrian's weight, k is the number of the harmonic, α_k [-] and Ψ_k are the Fourier's coefficient (i.e. the DLF) and the phase angle of the k th harmonic, n is the total number of contributing harmonics and f_s is the step frequency. Considering a loading scenario characterized by pedestrians arriving randomly, each of them with his own characteristic in terms of loading amplitude, frequency, velocity and phase, the force induced by N_P pedestrians can be expressed, accounting only for the first harmonic of the walking force, as:

$$f(x, t) = \sum_{z=1}^{N_P} \alpha_z G_z \sin(\Omega_z(t - \tau_z) + \Psi_z) \delta[x - c_z(t - \tau_z)] \left[H(t - \tau_z) - H\left(t - \tau_z - \frac{L}{c_z}\right) \right] \quad (4.8)$$

where, $\alpha_z G_z$ [N], Ω_z [rad/s], Ψ_z [rad], c_z [m/s] and τ_z [s] are the force amplitude, the step circular frequency, the phase angle, the velocity and the arrival time of the z th pedestrian, while δ and H represent the Dirac and Heaviside functions, respectively. All these quantities have to be probabilistically modelled to account for the variability in walking forces induced by different pedestrians. According to experimental measurements and different models available in literature (see chapter 2 for a brief summary and [87, 114] for comprehensive literature reviews), the step circular frequency, the pedestrian weight, the DLF and the walking velocity are considered as Gaussian random variables. The phase angle among different pedestrians is usually assumed as a random variable uniformly distributed in the interval $[0, 2\pi]$. Finally, pedestrian arrivals can be considered as Poisson events [76].

4.2.2 Definition of the modal force

The spectral model of the force induced by pedestrian flows is introduced from the analysis of the modal force, which is derived from the pedestrian force modelled as a series of impulsive functions (Eq.(4.8)).

Considering the structure a linear system with classic damping, the equation of motion of the j th principal coordinate p_j is written as:

$$m_j \ddot{p}_j(t) + 2\xi_j m_j \omega_j \dot{p}_j(t) + m_j \omega_j^2 p_j(t) = \int_0^L f(x, t) \varphi_j(x) dx \quad (4.9)$$

Then, the following non-dimensional quantities are introduced:

$$\tilde{t} = \omega_j t, \quad \tilde{p}_j(t) = \frac{p_j(t) m_j \omega_j^2}{\alpha_m G_m}, \quad \tilde{f}(\tilde{x}, \tilde{t}) = \frac{f(x, t) L}{\alpha_m G_m}, \quad \tilde{x} = \frac{x}{L} \quad (4.10)$$

where the superscript $\sim$ denotes the dimensionless parameters. The variables of Eq.(4.9) can be expressed depending on the non-dimensional quantities as follows:

$$p_j(t) = \tilde{p}_j(t) \frac{\alpha_m G_m}{m_j \omega_j^2}, \quad f(x, t) = \tilde{f}(\tilde{x}, \tilde{t}) \frac{\alpha_m G_m}{L}, \quad dx = d\tilde{x} L \quad (4.11)$$

with $\tilde{f}(\tilde{x}, \tilde{t})$ the force per unit-length exerted by pedestrians and $\alpha_m G_m$ the mean amplitude of the vertical force from a single pedestrian. Substituting the expressions of Eq.(4.11) into Eq.(4.9), the latter becomes:

$$m_j \ddot{\tilde{p}}_j(t) \frac{\alpha_m G_m}{m_j \omega_j^2} + 2\xi_j m_j \omega_j \dot{\tilde{p}}_j(t) \frac{\alpha_m G_m}{m_j \omega_j^2} + m_j \omega_j^2 \tilde{p}_j(t) \frac{\alpha_m G_m}{m_j \omega_j^2} = \int_0^1 \tilde{f}(\tilde{x}, \tilde{t}) \frac{\alpha_m G_m}{L} \varphi_j(\tilde{x}) d\tilde{x} L \quad (4.12)$$

$$\ddot{\tilde{p}}_j(t) \frac{1}{\omega_j^2} + 2\xi_j \dot{\tilde{p}}_j(t) \frac{1}{\omega_j} + \tilde{p}_j(t) = \int_0^1 \tilde{f}(\tilde{x}, \tilde{t}) \varphi_j(\tilde{x}) d\tilde{x} \quad (4.13)$$

Moreover, considering the relation $\tilde{t} = \omega_j t$:

$$\dot{\tilde{p}}_j(\tilde{t}) = \frac{d\tilde{p}_j}{d\tilde{t}} = \frac{d\tilde{p}_j}{dt \omega_j} = \frac{\dot{\tilde{p}}_j(t)}{\omega_j} \rightarrow \dot{\tilde{p}}_j(t) = \dot{\tilde{p}}_j(\tilde{t}) \omega_j \quad (4.14)$$

$$\ddot{\tilde{p}}_j(\tilde{t}) = \frac{d^2 \tilde{p}_j}{d\tilde{t}^2} = \frac{d^2 \tilde{p}_j}{dt^2 \omega_j^2} = \frac{\ddot{\tilde{p}}_j(t)}{\omega_j^2} \rightarrow \ddot{\tilde{p}}_j(t) = \ddot{\tilde{p}}_j(\tilde{t}) \omega_j^2 \quad (4.15)$$

Finally, introducing Eqs.(4.14) and (4.15) into Eq.(4.13):

$$\ddot{\tilde{p}}_j(\tilde{t}) \omega_j^2 \frac{1}{\omega_j^2} + 2\xi_j \dot{\tilde{p}}_j(\tilde{t}) \omega_j \frac{1}{\omega_j} + \tilde{p}_j(\tilde{t}) = \int_0^1 \tilde{f}(\tilde{x}, \tilde{t}) \varphi_j(\tilde{x}) d\tilde{x} \quad (4.16)$$

$$\ddot{\tilde{p}}_j(\tilde{t}) + 2\xi_j \dot{\tilde{p}}_j(\tilde{t}) + \tilde{p}_j(\tilde{t}) = \tilde{F}_j(\tilde{t}) \quad (4.17)$$

where the non-dimensional modal force has been introduced depending on the force exerted by pedestrians and the mode shape:

$$\tilde{F}_j(\tilde{t}) = \int_0^1 \tilde{f}(\tilde{x}, \tilde{t}) \varphi_j(\tilde{x}) d\tilde{x} \quad (4.18)$$

Then, by substituting Eqs.(4.8) and (4.10) into Eq.(4.18), the non-dimensional modal force due to a probabilistically-modelled group of pedestrians is obtained:

$$\tilde{F}_j(\tilde{t}) = \sum_{z=1}^{N_P} \tilde{\alpha}_z \tilde{G}_z \sin(\tilde{\Omega}_z(\tilde{t} - \tilde{\tau}_z) + \Psi_z) \varphi_j(\tilde{\Omega}_{cz}(\tilde{t} - \tilde{\tau}_z)) \left[H(\tilde{t} - \tilde{\tau}_z) - H\left(\tilde{t} - \tilde{\tau}_z - \frac{1}{\tilde{\Omega}_{cz}}\right) \right] \quad (4.19)$$

where the non-dimensional parameters are defined as:

$$\tilde{\alpha}_z = \frac{\alpha_z}{\alpha_m}, \quad \tilde{G}_z = \frac{G_z}{G_m}, \quad \tilde{\Omega}_z = \frac{\Omega_z}{\omega_j}, \quad \tilde{\Omega}_{cz} = \frac{c_z}{\omega_j L}, \quad \tilde{\tau}_z = \omega_j \tau_z \quad (4.20)$$

The non-dimensional random variables $\tilde{\alpha}_z$, $\tilde{G}_z$, $\tilde{\Omega}_z$, $\tilde{\Omega}_{cz}$, and $\tilde{\tau}_z$ should be probabilistically characterized; however, Piccardo and Tubino [83] assume that the randomness of $\tilde{\alpha}_z$, $\tilde{G}_z$ and $\tilde{\Omega}_{cz}$ is neglected and they are modelled as deterministic variables coincident with their mean value. Also the randomness in pedestrian arrivals is neglected and they are assumed to arrive equally spaced in time. Under these hypothesis, the non-dimensional parameters become:

$$\tilde{\alpha}_z = 1, \quad \tilde{G}_z = 1, \quad \tilde{\Omega}_{cz} = \tilde{\Omega}_{cm} = \frac{c_m}{\omega_j L}, \quad \tilde{\tau}_z = \frac{z}{\tilde{\Omega}_{cm} N_P} \quad (4.21)$$

This corresponds to focus on the randomness of the step frequency only, which is represented as a normal distributed random variable:

$$\tilde{\Omega}_z = \tilde{\Omega}_m(1 + V_\Omega N(0, 1)) = \frac{\Omega_m}{\omega_j}(1 + V_\Omega N(0, 1)) \quad (4.22)$$

where $N(0, 1)$ is an independent random variable with zero mean and unitary variance and V_Ω represents the coefficient of variation of the step frequency. Another simplifying assumption adopted in [83] regards the hypothesis of a sinusoidal structural mode shape:

$$\varphi_j(\tilde{x}) = \sin(j\pi\tilde{x}) = \sin(j\pi\tilde{\Omega}_{cm}\tilde{t}) \quad (4.23)$$

Under the above mentioned assumptions, Eq.(4.19) becomes:

$$\tilde{F}_j(\tilde{t}) = \sum_{z=1}^{N_P} \sin(\tilde{\Omega}_z\tilde{t} + \Psi_z) \sin(j\pi\tilde{\Omega}_{cm}\tilde{t}) \left[H(\tilde{t} - \tilde{\tau}_z) - H\left(\tilde{t} - \tilde{\tau}_z - \frac{1}{\tilde{\Omega}_{cm}}\right) \right] \quad (4.24)$$

Adopting the trigonometric identities, Eq.(4.24) can be rewritten as [83]:

$$\tilde{F}_j(\tilde{t}) = \sum_{z=1}^{N_P} \frac{1}{2} \left\{ \cos[(\tilde{\Omega}_z + j\pi\tilde{\Omega}_{cm})\tilde{t} + \Psi_z] + \cos[(\tilde{\Omega}_z - j\pi\tilde{\Omega}_{cm})\tilde{t} + \Psi_z] \right\} \left[H(\tilde{t} - \tilde{\tau}_z) - H\left(\tilde{t} - \tilde{\tau}_z - \frac{1}{\tilde{\Omega}_{cm}}\right) \right] \quad (4.25)$$

Piccardo and Tubino [83] observe that $\tilde{\Omega}_{cm}$ is very small compared to $\tilde{\Omega}_z$ for realistic values of the involved parameters, so that it can be assumed:

$$\tilde{\Omega}_{cm} = \varepsilon \tilde{\Omega}_z \quad (4.26)$$

being ε a small parameter. The assumption corresponds to the hypothesis that the time required for a pedestrian to cross the footbridge is long compared to the time required to take a step. The latter is equal to the natural period of vibration of the system when the response can be considered mainly resonant. Hence, Eq.(4.25) becomes:

$$\tilde{F}_j(\tilde{t}) = \sum_{z=1}^{N_P} \frac{1}{2} \left\{ \cos[\tilde{\Omega}_z(1+\varepsilon j\pi)\tilde{t} + \Psi_z] + \cos[\tilde{\Omega}_z(1-\varepsilon j\pi)\tilde{t} + \Psi_z] \right\} \left[H(\tilde{t} - \tilde{\tau}_z) - H\left(\tilde{t} - \tilde{\tau}_z - \frac{1}{\varepsilon \tilde{\Omega}_z}\right) \right] \quad (4.27)$$

Taking into account that ε is a small parameter, it is stated that $\varepsilon \rightarrow 0$, which implies that the finite duration of the footbridge crossing becomes nonessential if the crossing time is much longer than the natural period of vibration. Hence, Eq.(4.27) can be rewritten as follows:

$$\tilde{F}_j(\tilde{t}) \simeq \sum_{z=1}^{N_P} \cos[\tilde{\Omega}_z \tilde{t} + \Psi_z] \quad (4.28)$$

The non-dimensional force of Eq.(4.28) is the sum of the forces exerted by each single z th pedestrian. Defining $p_{\tilde{\Omega}}(\tilde{\Omega}_z)$ the probability density function of the non-dimensional pedestrian step frequency, the circular frequency range can be divided into $N_{\tilde{\Omega}}$ uniformly-spaced intervals of amplitude $\Delta\tilde{\Omega}$, with:

$$\tilde{\Omega}_k = k\Delta\tilde{\Omega}, \quad k = 1, \dots, N_{\tilde{\Omega}}$$

The probability P that a pedestrian is walking with a step frequency $\tilde{\Omega}_z \in [\tilde{\Omega}_k - \Delta\tilde{\Omega}/2, \tilde{\Omega}_k + \Delta\tilde{\Omega}/2]$ can be expressed by the product $p_{\tilde{\Omega}}(\tilde{\Omega}_k)\Delta\tilde{\Omega}$. Hence, the number of pedestrian N_k walking with circular frequency $\tilde{\Omega}_z \in [\tilde{\Omega}_k - \Delta\tilde{\Omega}/2, \tilde{\Omega}_k + \Delta\tilde{\Omega}/2]$ is given by:

$$N_k = N_P P[\tilde{\Omega}_z \in [\tilde{\Omega}_k - \Delta\tilde{\Omega}/2, \tilde{\Omega}_k + \Delta\tilde{\Omega}/2]] = N_P p_{\tilde{\Omega}}(\tilde{\Omega}_k) \Delta\tilde{\Omega} \quad (4.29)$$

Therefore, the total number of pedestrians can be divided into $N_{\tilde{\Omega}}$ subsets of N_k pedestrians walking with circular frequency $\tilde{\Omega}_i \in [\tilde{\Omega}_k - \Delta\tilde{\Omega}/2, \tilde{\Omega}_k + \Delta\tilde{\Omega}/2]$, and the non-dimensional modal force can be rewritten as:

$$\tilde{F}_j(\tilde{t}) \simeq \sum_{k=1}^{N_{\tilde{\Omega}}} \sum_{z=1}^{N_k} \cos[\tilde{\Omega}_k \tilde{t} + \Psi_z] \quad (4.30)$$

The force exerted by N_k pedestrians walking with frequency $\tilde{\Omega}_k$ and uniformly distributed random phases Ψ_z is given by:

$$\sum_{z=1}^{N_k} \cos[\tilde{\Omega}_k \tilde{t} + \Psi_z] = \sqrt{N_k} \cos[\tilde{\Omega}_k \tilde{t} + \tilde{\Psi}_k] = \sqrt{N_P p_{\tilde{\Omega}}(\tilde{\Omega}_k) \Delta\tilde{\Omega}} \cos[\tilde{\Omega}_k \tilde{t} + \tilde{\Psi}_k] \quad (4.31)$$

Therefore, the non-dimensional modal force assumes the final expression:

$$\tilde{F}_j(\tilde{t}) \simeq \sum_{k=1}^{N_{\tilde{\Omega}}} \sqrt{N_P p_{\tilde{\Omega}}(\tilde{\Omega}_k) \Delta\tilde{\Omega}} \cos[\tilde{\Omega}_k \tilde{t} + \tilde{\Psi}_k] \quad (4.32)$$

In summary, to obtain the expression of the modal force reported in Eq.(4.32), the following assumptions are made:

- Only the resonant structural mode shape is considered;
- The mode shape is sinusoidal;
- Only the randomness of the step frequency is considered:
 - α and G are assumed as deterministic variables coincident with their mean value;
 - pedestrians are assumed to arrive equally spaced in times;
- Only the first harmonic of the walking force is considered;
- The time required to a pedestrian to cross the footbridge is long compared to the period required to take one step (i.e. the natural period of vibration of the system in the resonance case), $\tilde{\Omega}_{cm} = \varepsilon \tilde{\Omega}_z$.

4.2.3 Spectral model of the modal force

According to Piccardo and Tubino [83], the PSD of the modal force of Eq.(4.32) can be expressed depending on the pedestrian number N_P and the probability distribution of the non-dimensional step frequency $p_{\tilde{\Omega}}(\tilde{\Omega})$ as:

$$S_{\tilde{F}_j}(\tilde{\Omega}) \simeq \frac{N_P}{4} p_{\tilde{\Omega}}(\tilde{\Omega}) \quad (4.33)$$

Starting from the definition of the PSD function of the modal force, in the following sections the spectral model of the pedestrian loading is defined for both unrestricted [83] and crowded pedestrian traffic conditions [47].

4.2.3.1 Spectral model for unrestricted pedestrian traffic conditions

In the present section, the spectral model of the force induced by flows of pedestrians is introduced for the case of low pedestrian density, assuming perfectly uncorrelated pedestrians. According to Eq.(4.33), the PSD of the modal force can be expressed depending only on the pedestrian number and the probability distribution of the step frequency. Alternatively, with the aim of considering also mode shapes which are not sinusoidal, the power spectral density function of $\tilde{F}_j(\tilde{t})$ can be obtained from its definition:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = \int_0^1 \int_0^1 S_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' \quad (4.34)$$

where $S_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}')$ is the cross-PSD of the non-dimensional force $\tilde{f}(\tilde{x}, \tilde{t})$:

$$S_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') = \sqrt{S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}) S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}')} \text{Coh}_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') \quad (4.35)$$

In Eq.(4.35), $S_{\tilde{f}}(\tilde{\Omega}, \tilde{x})$ is the PSD of the non-dimensional force while $\text{Coh}_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}')$ is the coherence function. Assuming that the non-dimensional force has a PSD which is independent on the position:

$$S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}) = S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}') = S_{\tilde{f}}(\tilde{\Omega}) \quad (4.36)$$

and that pedestrians are perfectly uncorrelated (free walking hypothesis):

$$\text{Coh}_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') = \delta(\tilde{x} - \tilde{x}') \quad (4.37)$$

Eq.(4.34) becomes:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = S_{\tilde{f}}(\tilde{\Omega}) \int_0^1 \int_0^1 \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' \quad (4.38)$$

Moreover, in [83] it is stated that:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = S_{\tilde{f}}(\tilde{\Omega}) \int_0^1 \int_0^1 \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' \simeq S_{\tilde{f}}(\tilde{\Omega}) \int_0^1 \varphi_j^2(\tilde{x}) d\tilde{x} \quad (4.39)$$

which becomes, when the mode shape is sinusoidal:

$$S_{\tilde{F}_j}(\tilde{\Omega}) \simeq \frac{S_{\tilde{f}}(\tilde{\Omega})}{2} \quad (4.40)$$

Note that the relation:

$$\int_0^1 \int_0^1 \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' = \int_0^1 \varphi_j^2(\tilde{x}) d\tilde{x}$$

can be considered valid when dealing with the first sinusoidal mode shape, although also in this case the error is about 20%, as:

$$\begin{aligned} \int_0^1 \sin(\pi \tilde{x})^2 d\tilde{x} &= 0.5 \\ \int_0^1 \int_0^1 \sin(\pi \tilde{x}) \sin(\pi \tilde{x}') d\tilde{x} d\tilde{x}' &= 0.405 \end{aligned}$$

By comparing Eqs.(4.33) and (4.40), the PSD of the force induced by a group of uncorrelated pedestrians can be obtained as:

$$S_{\tilde{F}}(\tilde{\Omega}) = \frac{N_P}{2} p_{\tilde{\Omega}}(\tilde{\Omega}) \quad (4.41)$$

The spectral model of Eq.(4.41) can be adopted in case of low pedestrian density and when the non-dimensional walking velocity $\tilde{\Omega}_{cm}$ is sufficiently small compared to the non-dimensional step frequency, $\tilde{\Omega}_{cm} = \varepsilon \tilde{\Omega}_i$. The model of Eq.(4.41) is derived assuming a sinusoidal mode shape. However, Piccardo and Tubino [83] performed numerical simulations considering the mode shape of a double-clamped beam and showed that, in such case, Eq.(4.41) is valid independently on the structural mode shape.

Finally, the PSD of the modal force is obtained from Eqs.(4.39) and (4.41) as a function of the structural mode shape:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = \frac{N_P}{2} p_{\tilde{\Omega}}(\tilde{\Omega}) \int_0^1 \varphi_j^2(\tilde{x}) d\tilde{x} \quad (4.42)$$

The analytical formulation of the spectral load model in Eq.(4.42), has been obtained considering the simplifying assumptions of Eq.(4.21). Namely the variability of the DLF, pedestrian's weight, velocity and arrival times is neglected, and only the randomness of the step frequency and the phase is considered. However, in order to evaluate the applicability of Eq.(4.42) even when the simplifying assumptions do not hold, Piccardo and Tubino [83] performed probabilistic numerical simulations of the modal force considering as random all the involved parameters. Simulations have been carried out assuming two different structural schemes, a simple-supported beam and a double-clamped beam. Then, the results of the numerical simulations are compared with the analytical expression of the PSD of the modal force of Eq.(4.42). Results show that, for those two cases, the analytical expression of the PSD can be adopted with a good accuracy also when a probabilistic modelling of the walking parameters is assumed [83].

4.2.3.2 Generalized spectral model for the human-induced loading

In this section, the generalized spectral model for pedestrian-induced forces proposed by Ferrarotti and Tubino [47], which can be applied in case of both low pedestrian density and crowded conditions, is presented. In section 4.2.3.1, the perfect uncorrelation among pedestrians is obtained by modelling the coherence function as the Dirac function (Eq.(4.37)). This corresponds to assume that the pedestrian is perfectly correlated with himself ($\delta(\tilde{x} - x) = 1$ if $\tilde{x} = x$) while he is totally uncorrelated with other pedestrians ($\delta(\tilde{x} - x) = 0$ if $\tilde{x} \neq x$). Actually, a pedestrian moving in a crowd is influenced and constrained by the presence of the others. This results in the human-human interaction effect, i.e a decrease in the mean velocity when the pedestrian density grows up. To model the increasing correlation among pedestrians with increasing pedestrian density, Ferrarotti and Tubino [47] proposed a physically-based expression for the coherence function that depends on both the pedestrian density and the distance among pedestrians. The step frequency, pedestrian velocity and pedestrian density are strictly correlated and thus the harmonic content of pedestrian induced forces is also influenced by the pedestrian density. The pedestrian density-speed relations proposed in literature show a monotonic decreasing trend of the mean velocity c_m with increasing pedestrian density ρ . Here, the density-speed relation presented by Venuti and Bruno [108] is adopted:

$$c_m(\rho) = c_u \left\{ 1 - \exp \left[- \gamma \rho_{jam} \left(\frac{1}{\rho} - \frac{1}{\rho_{jam}} \right) \right] \right\} \quad (4.43)$$

where c_u is the free-flow speed, γ is a parameter depending on the travel purpose and ρ_{jam} is the jam density, i.e the pedestrian density associated with completely stopped traffic flow. The free-flow speed c_u , i.e. the mean pedestrian velocity in unrestricted traffic conditions, is a random variable that follows a Gaussian distribution characterized by the mean value $c_{um} = 1.34$ m/s and the standard deviation $\sigma_{c_u} = 0.26$ m/s [25]. Eq.(4.43) is a generic form of the velocity-density relation originally proposed by Weidmann, the so called *Kladdek formula* [116]. By assuming $\gamma = 0.354$, $\rho_{jam} = 5.4$ ped/m² and $c_u = 1.34$ m/s, Eq.(4.43) matches the original one [47]. Figure 4.1(a) shows the trend of the mean velocity with increasing pedestrian density obtained considering the three values of free-flow speed: $c_{um} = 1.34$ m/s, $c_{um} + \sigma_{c_{um}} = 1.6$ m/s and $c_{um} - \sigma_{c_{um}} = 1.08$ m/s. For low pedestrian densities, i.e. $\rho \leq \rho_u$ where ρ_u is the upper bound of the pedestrian density

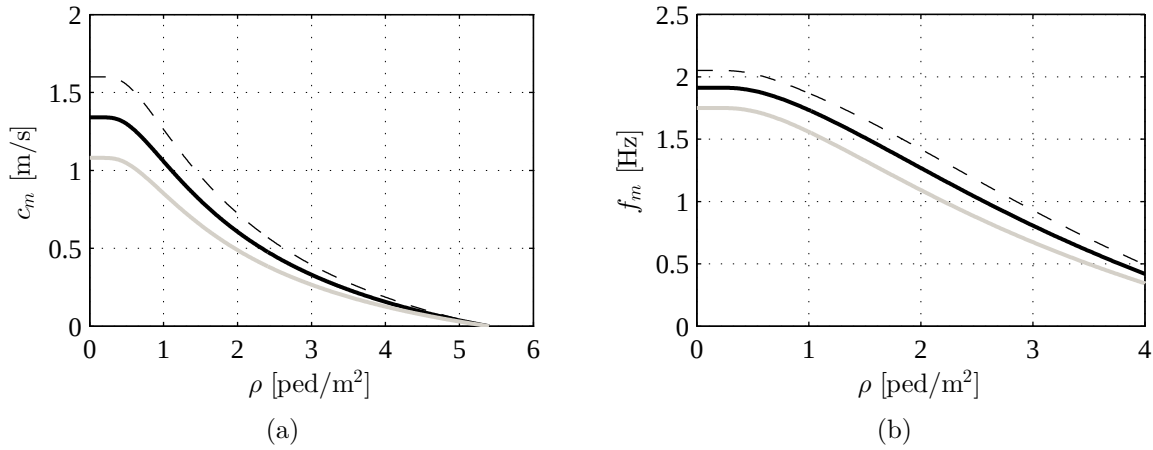


Figure 4.1: Trend of (a) mean velocity and (b) mean step frequency with increasing pedestrian density for different values of free-flow speed: 1.08 m/s (bold grey line), 1.34 m/s (bold black line) and 1.6 m/s (dashed black line)

range corresponding to free and unrestricted walking behaviour, the mean velocity is equal to the mean free-flow speed. Then, for higher pedestrian densities ($\rho \geq \rho_u$) the walking speed gradually decreases until the jam density ρ_{jam} is reached, for which people cannot keep walking.

Starting from the mean pedestrian velocity, Venuti and Bruno [24] propose a relation, based on a cubic curve fitting of the experimental data presented in [16], to evaluate the mean step frequency:

$$f_m = 0.35c_m^3 - 1.59c_m^2 + 2.93c_m \quad (4.44)$$

The applicability limits for this empirical relation can be assumed either as [0.2; 2.5] m/s or [0.23; 2.2] m/s, in accordance with [24] or [16], respectively. Through Eqs.(4.43) and (4.44), the mean step frequency can be related to the pedestrian density, obtaining the trend shown in Figure 4.1(b) for the three different values of free-flow speed. The lower limit of the pedestrian velocity 0.2 m/s corresponds to a pedestrian density of about 4 ped/m² (slightly differences depend on the free-flow speed value) and thus the mean step frequency is defined depending on pedestrian densities lower than 4 ped/m². Once the frequency content of the walking force is characterized depending on the pedestrian density, Ferrarotti and Tubino [47] propose a physically-based expression for the coherence among pedestrians. The latter depends on being able to see and keep up with the others and it is likely to drop to zero beyond a short separation distance. The proposed expression is composed of an unitary step function for small distance and an exponentially decreasing function for greater distance:

$$\text{Coh}_{ff}(x, x', \varepsilon, C) = \begin{cases} H\left(\frac{|x - x'|}{\varepsilon}\right) - H\left(\frac{|x - x'|}{\varepsilon} - 1\right), & \text{if } |x - x'| \leq \varepsilon \\ \exp\left[-C\left(\frac{|x - x'|}{\varepsilon} - 1\right)\right], & \text{otherwise} \end{cases} \quad (4.45)$$

where x and x' are the current abscissa, ε is the width of the initial unitary step function and C is a correlation coefficient. The width of the initial unitary step function ε is the

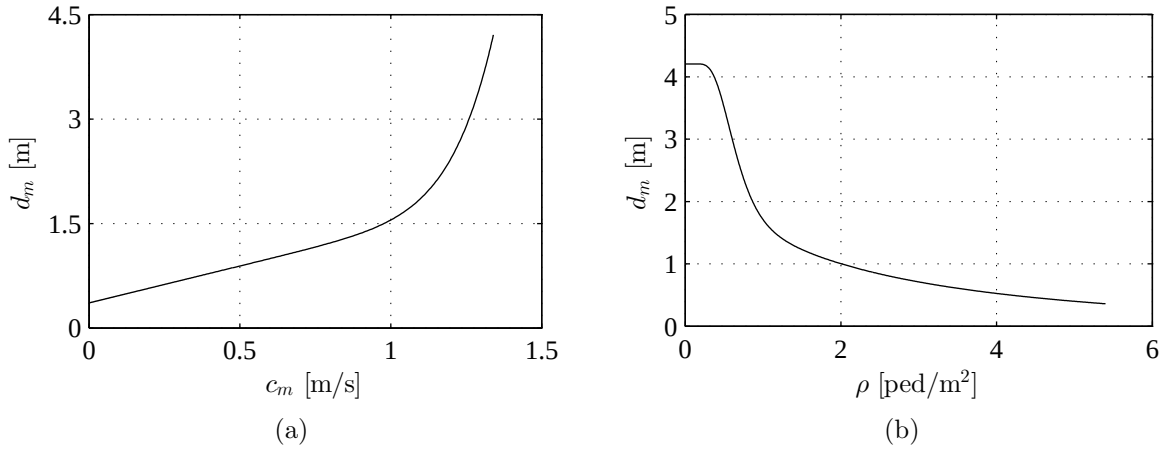


Figure 4.2: Trend of the mean forward distance with (a) mean pedestrian velocity and (b) pedestrian density, obtained considering $c_u = c_{um} = 1.34$ m/s

distance that each pedestrian interposes with others in order to avoid contact. It can be evaluated starting from the definition of the space occupied by a pedestrian during his motion, namely the Pedestrian Area Module (PAM). The PAM is defined as $S = wd$, being d the forward distance and w the lateral space [108]. According to Venuti and Bruno [108], both these quantities, in terms of mean values, can be considered as the sum of the dimension of a motionless pedestrian and a dimension defined as a function of the mean walking velocity thanks to experimental results:

$$w_m(c_m) = w_0 \left(1 + 0.62 \frac{c_m}{c_u} \right) \quad (4.46)$$

$$d_m(c_m) = d_0 + 1.06c_m + bc_m^{10} \quad (4.47)$$

where $b = (2.08c_u - d_0)/c_u^{10}$ and $w_0 = 0.45$ m and $d_0 = 0.36$ m are the average width and depth of the human body. The trend of the mean forward distance with pedestrian velocity and density is shown in Figure 4.2. By looking at the trend of the forward distance with the pedestrian density (Figure 4.2(b)), it is possible to identify three domains in which the law $d_m - \rho$ is almost linear, according to the same criteria adopted by Seyfried and Steffen [94]:

Unconstrained traffic $\rho \leq \rho \leq \rho_u$. Pedestrians' movements are completely free and the forward distance among pedestrians is constant and equal to its maximum value.

Correlated traffic $\rho_u < \rho \leq \rho_{LS}$. The forward distance d_m has a linear trend with pedestrian velocity for speed values smaller than about 0.9 m/s, assumed as lock-step speed c_{LS} . Corresponding value in terms of pedestrian density is referred as ρ_{LS} . In this domain, pedestrians' movements are not completely free and the forward distance among pedestrians quickly decreases with increasing pedestrian density.

Constrained traffic $\rho > \rho_{LS}$. Pedestrians' movements are completely constrained by the presence of other pedestrians and the forward distance is small and almost linear with pedestrian density, slowly decreasing with increasing pedestrian density.

The separation distance between two pedestrians moving on a linear domain is strictly connected to the forward distance d_m . Taking into account that each pedestrian is modelled as a moving harmonic punctual load (Eq.(4.8)), the separation distance between two

pedestrians becomes $d_m - d_0/2$. The width of the initial unitary step function ε is identified as the forward distance corresponding to the mean free-flow speed $c_u = c_{um} = 1.34$ m/s [47]:

$$\varepsilon = d_m(c_{um}) - \frac{d_0}{2} \simeq 4 \text{ m} \quad (4.48)$$

The trend of the coherence function obtained for different pedestrian densities is shown in Figure 4.4. The coherence reaches unitary values for distances less than the width of the unitary step function ε and then it decreases on increasing pedestrian distance. How fast the coherence function drops to zero depends on the pedestrian density through the correlation coefficient C , which models the increase in synchronization that occurs when the pedestrian density grows up. Ferrarotti and Tubino [47] propose an expression of the correlation coefficient as a function of the mean step frequency (which itself is related to the pedestrian density via the pedestrian velocity):

$$C(f_m) = \exp(C_1 f_m + C_2) \quad (4.49)$$

The two coefficients C_1 and C_2 are obtained by enforcing the conditions $C(f_u) = C_{max} = 10$ and $C(f_{LS}) = C_{min} = 0.1$, where f_u is the pacing rate of unconstrained pedestrians and f_{LS} is the lock-step frequency. The limit values for the coefficients are assumed based on the results of numerical simulations performed by Ferrarotti and Tubino [47]. The latter show how the exponential portion of the coherence function becomes negligible for values of the exponent greater than 10 and becomes almost unitary for exponents less than 0.1. The unconstrained step frequency f_u and the lock-step frequency f_{LS} are obtained as follows:

- The mean value of the unconstrained step frequency f_u is obtained from the mean step frequency corresponding to the mean free-flow speed $c_{um} = 1.34$ m/s:

$$f_u = 0.35c_{um}^3 - 1.59c_{um}^2 + 2.93c_{um} = 1.9133 \text{ Hz} \quad (4.50)$$

The corresponding value of the critical density ρ_u , below which pedestrian traffic is unrestricted, is obtained as the inverse of the area occupied by a pedestrian when his velocity is equal to the mean free-flow speed $c_{um} = 1.34$ m/s:

$$\rho_u = \frac{1}{w_m(c_{um}) d_m(c_{um})} = 0.326 \text{ ped/m}^2 \quad (4.51)$$

- By looking at the trend of the forward distance d_m with the pedestrian mean speed c_m (Figure 4.2), the lock-step speed c_{LS} is assumed equal to 0.9 m/s. Corresponding values in terms of step frequency and pedestrian density are the following:

$$f_{LS} = 0.35c_{LS}^3 - 1.59c_{LS}^2 + 2.93c_{LS} = 1.6043 \text{ Hz} \quad (4.52)$$

$$\rho_{LS} = \rho(c_{LS}) = 1.3025 \text{ ped/m}^2 \quad (4.53)$$

From the lock-step frequency and the unconstrained step frequency, the coefficients C_1 and C_2 are calculated as follows:

$$\begin{cases} C(f_u) = \exp(C_1 f_u + C_2) = 10 \\ C(f_{LS}) = \exp(C_1 f_{LS} + C_2) = 0.1 \end{cases} \quad (4.54)$$

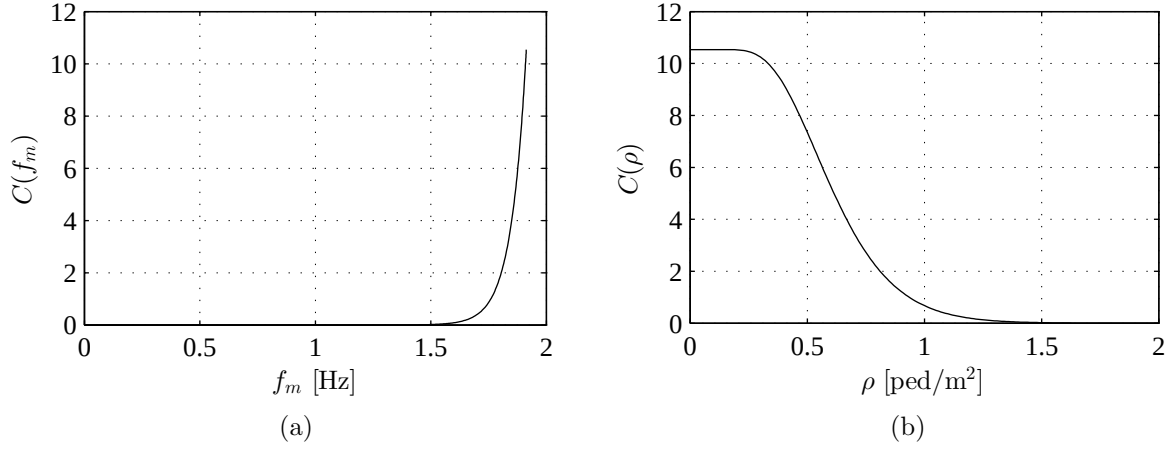


Figure 4.3: Correlation coefficient of the coherence function with (a) mean step frequency and (b) pedestrian density, obtained considering $c_u = c_{um} = 1.34$ m/s

$$\begin{cases} C_1 = 15.0641 \\ C_2 = -26.4403 \end{cases} \quad (4.55)$$

The trend of the exponential decay coefficient with the mean step frequency and pedestrian density obtained with $C_1 = 15.0461$ s⁻¹ and $C_2 = -26.4403$ is reported in Figures 4.3(a) and 4.3(b), respectively.

In summary, in the case of uncorrelated pedestrian load, the pedestrian density is low, the pedestrian velocity and the step frequency increase and the coherence function drops to zero very fast, hence the exponential coefficient reaches high values. On the contrary, when the pedestrian density is high, corresponding pedestrian density and velocity are low, pedestrians' movements are completely constrained and the coherence function reaches unitary values, hence the exponential coefficient drops to zero.

The coherence function may be rewritten in the following non-dimensional form:

$$\text{Coh}_{ff}(\tilde{x}, \tilde{x}', \tilde{\varepsilon}, C) = \begin{cases} H\left(\frac{|\tilde{x} - \tilde{x}'|}{\tilde{\varepsilon}}\right) - H\left(\frac{|\tilde{x} - \tilde{x}'|}{\tilde{\varepsilon}} - 1\right), & \text{if } |\tilde{x} - \tilde{x}'| \leq \tilde{\varepsilon} \\ \exp\left[-C\left(\frac{|\tilde{x} - \tilde{x}'|}{\tilde{\varepsilon}} - 1\right)\right], & \text{otherwise} \end{cases} \quad (4.56)$$

where the non-dimensional width of the initial unitary step function $\tilde{\varepsilon}$ is obtained as ε/L . The trend of the coherence function in the non-dimensional form for different values of pedestrian density is shown in Figure 4.4, where $\tilde{\varepsilon} = 0.1$ is obtained considering a foot-bridge length L equal to 40 m.

Once the coherence function is characterized, the PSD of the modal force, which accounts for the effective synchronization among pedestrians, can be obtained. According to Eq.(4.34), the PSD of the modal force is defined as:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = \int_0^1 \int_0^1 S_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' \quad (4.57)$$

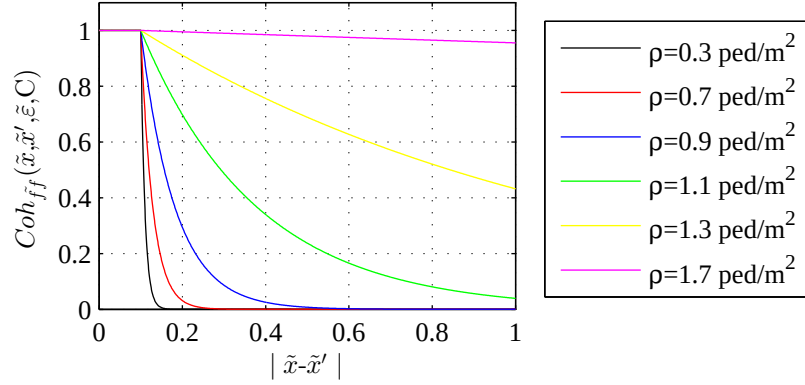


Figure 4.4: Coherence function in the non-dimensional form for different values of pedestrian density

where $S_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}')$ is the cross-PSD of the non-dimensional force:

$$S_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') = \sqrt{S_{\tilde{f}}(\tilde{\Omega}, \tilde{x})S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}')} \text{Coh}_{\tilde{f}\tilde{f}}(\tilde{\Omega}, \tilde{x}, \tilde{x}') \quad (4.58)$$

Assuming that the non-dimensional force has a PSD which is independent on the position:

$$S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}) = S_{\tilde{f}}(\tilde{\Omega}, \tilde{x}') = S_{\tilde{f}}(\tilde{\Omega}) \quad (4.59)$$

the PSD of the non-dimensional modal force can be rewritten as follows:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = S_{\tilde{f}}(\tilde{\Omega})\chi_j(\tilde{\epsilon}, C) \quad (4.60)$$

where:

$$\chi_j(\tilde{\epsilon}, C) = \int_0^1 \int_0^1 \text{Coh}_{\tilde{f}\tilde{f}}(\tilde{x}, \tilde{x}', \tilde{\epsilon}, C) \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' \quad (4.61)$$

is the admittance function. The integrand of Eq.(4.61) is shown in Figure 4.5 for a sinusoidal mode shape and pedestrian densities of 0.1 ped/m² and 1 ped/m², respectively. When the pedestrian density is low, the coherence function drops to zero very quickly and the non-null contribution to the admittance function comes from pedestrian distances lower than $\tilde{\epsilon}$, where the coherence function reaches unitary values. On the contrary, for high pedestrian density, the function is non-null even for higher distance, due to the mutual influence among pedestrians.

Ferrarotti and Tubino [47] state that for unrestricted pedestrian traffic condition (Figure 4.5(a)), the admittance function assumes the following simplified expression:

$$\chi_j(\tilde{\epsilon}, C) \simeq \chi_{uj}(\tilde{\epsilon}) \simeq 2\tilde{\epsilon} \int_0^1 \varphi_j^2(\tilde{x}) d\tilde{x} \quad (4.62)$$

with a generic mode shape, and:

$$\chi_j(\tilde{\epsilon}, C) \simeq \chi_{uj}(\tilde{\epsilon}) \simeq \tilde{\epsilon} \quad (4.63)$$

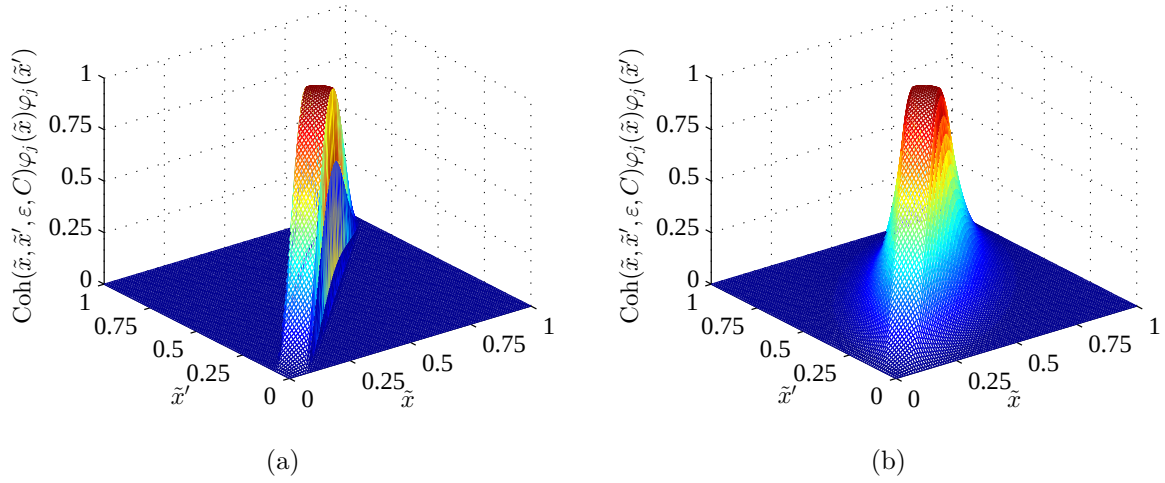


Figure 4.5: Integrand of $\chi(\tilde{\varepsilon}, C)$ for pedestrian density of (a) 0.1 ped/m² and (b) 1 ped/m² and sinusoidal mode shape

when the mode shape is sinusoidal. Hence, Eq.(4.60) becomes:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = S_{\tilde{f}}(\tilde{\Omega})\tilde{\varepsilon} \quad (4.64)$$

Considering Eqs.(4.60) and (4.33), the PSD of the pedestrian induced force for unrestricted pedestrian traffic conditions is given by:

$$S_{\tilde{f}}(\tilde{\Omega}) = \frac{N_P}{4\tilde{\varepsilon}} p_{\tilde{\Omega}}(\tilde{\Omega}) \quad (4.65)$$

Finally, the psdf of the modal force for the different pedestrian traffic conditions is obtained as:

$$S_{F_j}(\tilde{\Omega}) = \frac{N_P}{4\tilde{\varepsilon}} p_{\tilde{\Omega}}(\tilde{\Omega}) \chi_j(\tilde{\varepsilon}, C) \quad (4.66)$$

4.2.4 Peak coefficient

The peak coefficient, adopted in Eq.(4.3) to calculate the maximum dynamic response, is given by (see Eq.(B.31) and [36, 37]):

$$g_{\tilde{p}_j} = \left(\sqrt{2 \ln(2\tilde{v}_e \tilde{T})} + \frac{0.5772}{\sqrt{2 \ln(2\tilde{v}_e \tilde{T})}} \right) \quad (4.67)$$

In Eq.(4.67), $\tilde{v}_e$ is the modified non-dimensional expected frequency of the response and $\tilde{T}$ is the conventional non-dimensional time interval in which the maximum response is evaluated:

$$\tilde{T} = \omega_j N \frac{L}{c_m} = \frac{N}{\tilde{\Omega}_{cm}} \quad (4.68)$$

where L/c_m is the mean duration of the footbridge crossing and N represents the number of groups of N_P pedestrians that subsequently cross the footbridge in order to make the process stationary. As reported in Eq.(B.29), the modified expected frequency $\tilde{\nu}_e$ is expressed as [37]:

$$\tilde{\nu}_e = (1.63q^{0.45} - 0.38)\tilde{\nu}_0 \quad (4.69)$$

where $\tilde{\nu}_0$ is the non-dimensional average frequency of the system $\tilde{\nu}_0 = \omega_j/(2\pi\omega_j) = 1/(2\pi)$, and q is the spectral bandwidth parameter, given by [106]:

$$q = \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_1}\right)^{1/2} \quad (4.70)$$

with $\lambda_n = \int_0^\infty \tilde{\Omega}^n S(\tilde{\Omega}) d\tilde{\Omega}$. For a lightly-damped system, the spectral bandwidth parameter q can be approximated by Eq.(B.26), here reported:

$$q \approx 2\left(\frac{\xi}{\pi}\right)^{1/2} \quad (4.71)$$

According to [37], eqs. (4.67) and (4.69) are applicable in the range $0.1 \leq q \leq 1$ and $5 \leq \nu_0 T \leq 1000$.

Piccardo and Tubino [83] highlight that a delicate point of the procedure regards the choice of N , the number of pedestrian groups that cross the footbridge, in order to make the process stationary. In [83] it is stated that a value of N of about 10 leads to good results regarding the applications they proposed and for small values of non-dimensional step frequency. Nevertheless, they highlight that concerning this parameter further validations are required. In the present work, the number of pedestrian groups that cross the footbridge is assumed equal to 10.

4.2.5 Outline of the procedure

In the present section, the main stages of the procedure for the estimation of the maximum dynamic response are summarized through the flow chart presented in Figure 4.6. First, the pedestrian density ρ is assigned and from that, for a defined value of free-flow speed c_u , the mean velocity c_m and mean step frequency f_m are achieved. Then, from the mean step frequency and its standard deviation σ_{f_m} , the PDF related to the first harmonic of the walking force is obtained $p_{\tilde{\Omega}}(\tilde{\Omega})$. The PSD of the modal force $S_{\tilde{F}_j}(\tilde{\Omega})$ is estimated from the generalized model of Eq.(4.66), which can be adopted for different loading conditions, from unrestricted to crowded. Different loading conditions are modelled by the admittance function $\chi_j(\tilde{\varepsilon}, C)$, which also depends on the pedestrian density. The acceleration variance $\sigma_{\tilde{p}_j^2}$ is then calculated through the FRF of the system. The non-dimensional variance is obtained from the dimensional one divided by m_j^2 . From that, along with the peak coefficient, the maximum non-dimensional response of the principal coordinates $\tilde{\tilde{p}}_j$ is achieved from Eq.(4.3). Note that Eq.(4.3) is defined for the dimensional parameters, but it can be equally applied considering the non-dimensional acceleration $\tilde{\tilde{p}}_j$ instead of $\ddot{p}_j$ and the non-dimensional peak coefficient of Eq.(4.67). By considering the force amplitude $\alpha_m G_m$,

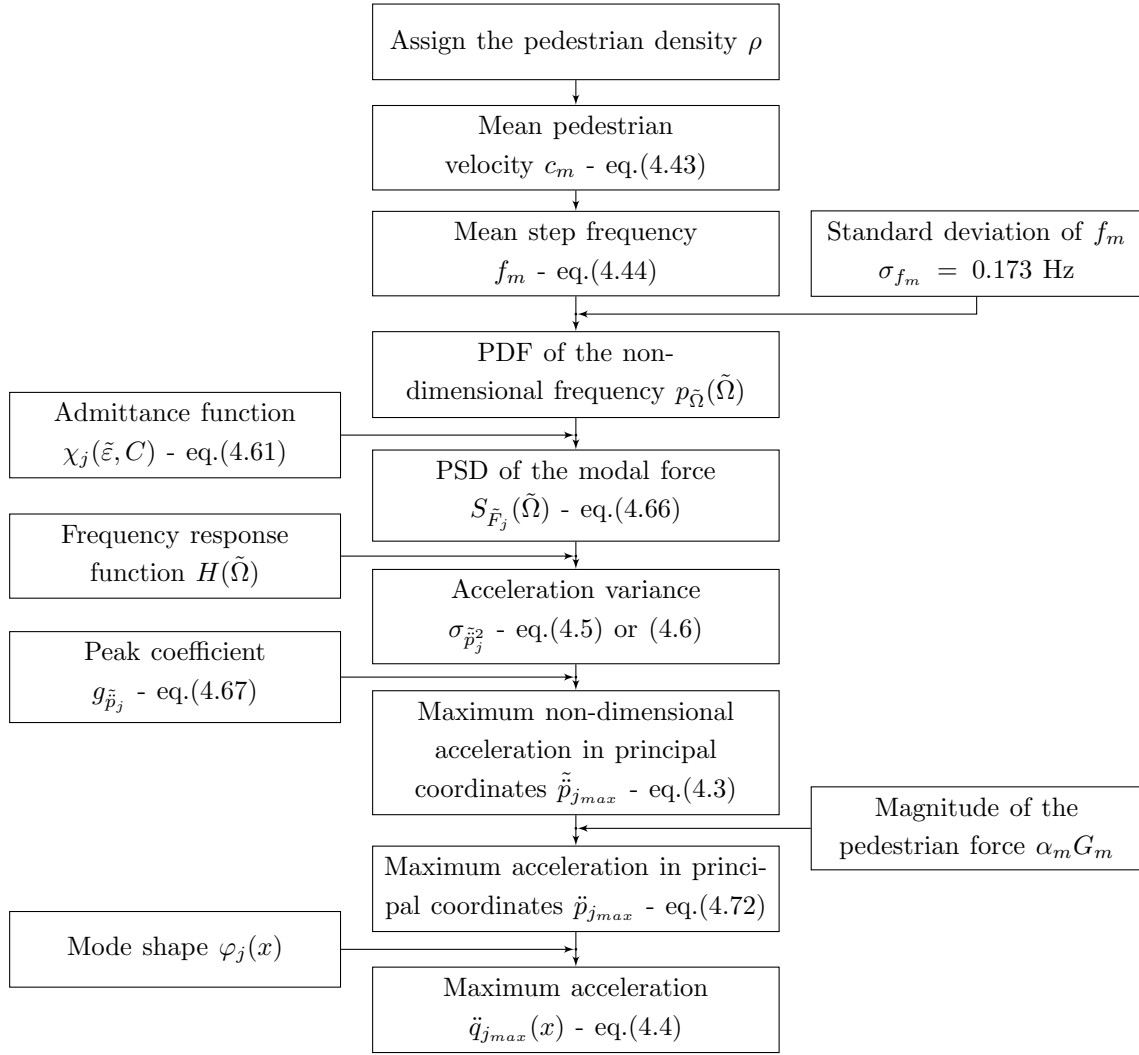


Figure 4.6: Flow chart of the procedure for estimating the maximum dynamic acceleration

being α_m the mean DLF of the first harmonic, the maximum acceleration of the principal coordinate $\ddot{p}_{j_{max}}$ can be rewritten in dimensional form [83]:

$$\ddot{p}_{j_{max}} = \frac{\alpha_m G_m}{m_j} \ddot{p}_{j_{max}} \quad (4.72)$$

Then, considering that the structural response is dominated by the j th natural mode, the maximum acceleration given by the first harmonic, in physical coordinates $\ddot{q}_{max}(x)$ is estimated from Eq.(4.4).

The relations for the evaluation of the peak coefficient presented in section 4.2.4, are defined for narrow band processes, as the output response of a lightly-damped system subjected to random excitation. However, in the present work, these relations are adopted even in the presence of human-structure interaction (see section 5.2), which can cause a significant increase in the damping of the system. As a consequence, the estimation of the peak coefficient can be less accurate. On the contrary, the analytical definition of the PSD of the modal force is valid with and without the human-structure interaction effects.

4.2.6 Applications of the spectral load model

In the present section, the applications of the spectral load model proposed in [83] and [47] are presented and discussed. Applications concern the evaluation of the maximum structural response based on the equivalent spectral load model for unrestricted traffic conditions, presented in section 4.2.3.1, and the generalized spectral model for both low-density and crowded conditions, described in section 4.2.3.2.

4.2.6.1 Maximum dynamic response for unrestricted pedestrian traffic conditions

In the following examples, the maximum response is evaluated considering the spectral model of the modal force presented in [83] and described section 4.2.3.1, which can only be adopted in unrestricted pedestrian traffic conditions.

Ideal footbridge

In the first application, the procedure proposed in [83] is applied to estimate the maximum dynamic response of an ideal footbridge to unrestricted pedestrian traffic conditions. The footbridge is a simply supported beam for which only the contribution of the fundamental mode is considered:

- $f_j = 1.8$ Hz, natural frequency;
- $m_j = 40 \times 10^3$ kg, modal mass;
- $\xi_j = 0.005$, modal damping;
- $\varphi_j(x) = \sin(\pi/L x)$, sinusoidal mode shape;
- $L = 80$ m, footbridge length;

while the parameters of the walking force are defined as:

- $\alpha_m = 0.4$, mean dynamic load factor;
- $G_m = 700$ N, mean pedestrian's weight;
- $f_m = 1.8$ Hz, mean step frequency, assumed resonant with the footbridge natural frequency;
- $\tilde{\Omega}_m = 1$, non-dimensional mean step frequency;
- $\sigma_{f_m} = 0.173$ Hz, standard deviation of the step frequency;
- $V_{\Omega} = 0.09$, coefficient of variation of the step frequency;
- $c_m = 0.9 f_m = 1.62$ m/s, mean pedestrian speed;
- $N_P = 10$, pedestrian number;

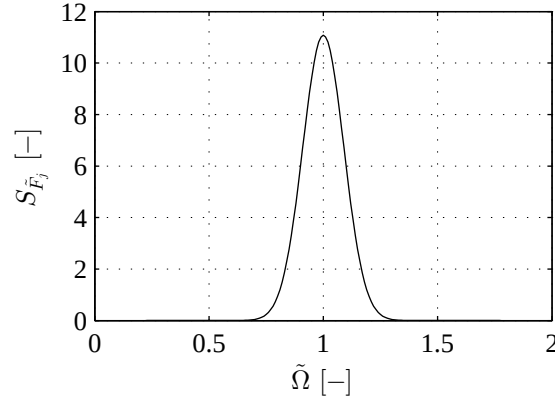


Figure 4.7: PSD function of the modal force

- $N = 10$, number of pedestrian groups.

Starting from the probability distribution function of the non-dimensional step frequency, the average number of pedestrians at the same time on the footbridge and the structural mode shape, the non-dimensional PSD function of the modal force is obtained through Eq.(4.42). The non-dimensional PSD is reported in Figure 4.7, where the non-dimensional frequency domain is normalized with respect to the footbridge natural frequency ($\tilde{\Omega} = \Omega/f_1$). As in this case the pedestrian action is assumed resonant with the footbridge natural frequency, the PSD of the modal force has its maximum value for $\tilde{\Omega} = 1$. Moreover, for the same reason, the variance of the acceleration can be calculated considering only the contribution of S_F in correspondence of the footbridge natural frequency (Eq.(4.6)):

$$\sigma_{\ddot{p}_j}^2 = \frac{\pi}{4\xi} S_{\tilde{F}_j}(1) = 1740.8$$

$$\sigma_{\ddot{p}_j} = 41.7$$

From the expected frequency, the spectral bandwidth parameter and the non-dimensional duration of the process, the peak factor is $g_{\ddot{p}_j} = 3.5$ (Eq.(4.67)). Thus, the maximum non-dimensional acceleration of the principal coordinate is calculated from Eq.(4.3):

$$\ddot{p}_{jmax} = g_{\ddot{p}_j} \sigma_{\ddot{p}_j} = 3.5 \cdot 41.7 = 146$$

and, finally, through Eqs.(4.72) and (4.4), the mean value of the maximum midspan acceleration is estimated:

$$\ddot{q}_{max} = \varphi_j(L/2) \ddot{p}_{jmax} = \varphi_j(L/2) \frac{\alpha_m G_m}{m_j} \ddot{p}_{jmax} = 1 \frac{0.4 \cdot 700}{40000} 146 = 1.022 \text{ m/s}^2$$

Real footbridge

The second application of the spectral load model proposed in [83], regards the estimation of the maximum acceleration of a real footbridge in Milan, considering different values of mean step frequency. Characteristics of the fundamental mode and dimensions of the footbridge are the following:

- $f_j = 2.17$ Hz, natural frequency;
- $m_j = 51 \times 10^3$ kg, modal mass;
- $\xi_1 = 0.005$, modal damping;
- $\varphi_j(x) = \sin(\pi/L x)$, sinusoidal mode shape;
- $L = 60$ m, footbridge length;
- $B = 2$ m, footbridge width.

while the parameters of the walking force are defined as:

- $\alpha_m = 0.4$, dynamic load factor;
- $G_m = 600$ N, pedestrian weight;
- $f_m = [1.8, 2]$ Hz, mean step frequency;
- $\Omega_m = [0.83, 0.92]$ Hz, non-dimensional mean step frequency;
- $\sigma_{f_m} = 0.173$ Hz, standard deviation of the step frequency;
- $V_\Omega = 0.09$, coefficient of variation of the step frequency;
- $c_m = 0.9 f_m$ m/s, mean pedestrian speed;
- $N_P = 15$, pedestrian number;
- $N = 10$, number of pedestrian groups.

The procedure is repeated for different values of the mean step frequency ranging between 1.8 and 2 Hz. Figure 4.8(a) shows the PSD for mean step frequencies of 1.8 Hz, 1.9 Hz and 2.0 Hz. As the PSD obtained from Eq.(4.42) depends only on the pedestrian number and the PDF of the step frequency, PSD functions have the same shape and the maximum in correspondence of the corresponding non-dimensional mean step frequency.

In these cases the pedestrian action is not perfectly resonant with the footbridge natural frequency but still near resonant. Thus, the variance of the acceleration can be estimated considering the PSD evaluated in $\Omega = 1$ (Eq.(4.6)), i.e. considering only the resonant part of the response, as $S_{\tilde{F}}(1)$ does not correspond to the maximum value of the PSD.

The maximum midspan acceleration obtained for the different mean step frequencies is shown in Figure 4.8(b). It can be observed that the maximum acceleration increases with increasing mean step frequency. In fact, when the mean step frequency increases, it approaches the footbridge natural frequency and the maximum response increases due to the near resonance. In other words, $S_{\tilde{F}}(1)$ is higher for higher values of mean step frequency.

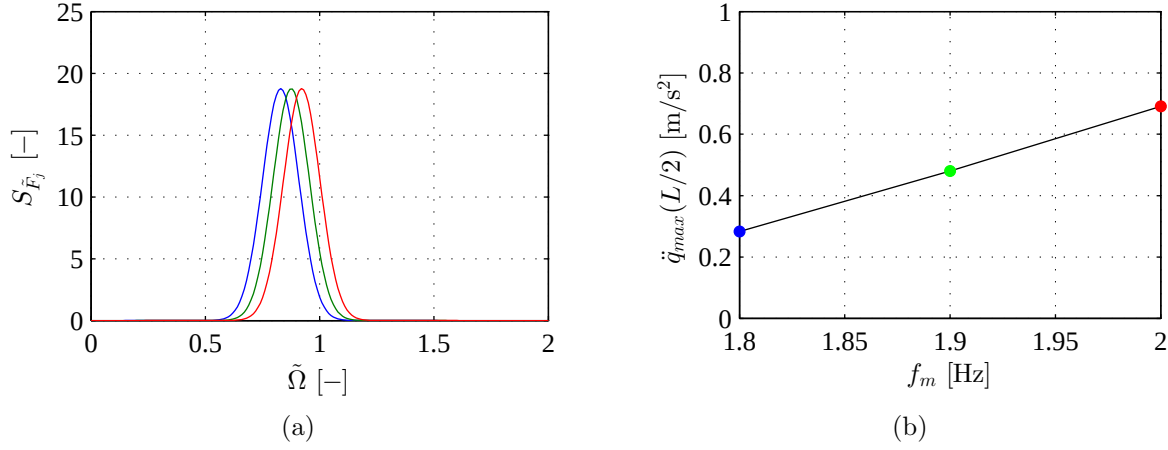


Figure 4.8: (a) PSD function of the modal force for different mean step frequencies: 1.8 Hz (blue line), 1.9 Hz (green line) and 2.0 Hz (red line). (b) Maximum midspan acceleration for different mean step frequencies

4.2.6.2 Maximum dynamic response for crowded conditions

The generalized spectral load model (proposed in [47] and described in section 4.2.3.2) is applied to estimate the maximum structural dynamic response to various pedestrian traffic conditions, from completely unrestricted to fully-restricted. The footbridge is the same analysed in section 4.2.6.1, referred to as ‘Real footbridge’. The parameters of the walking force are the following:

- $\alpha_m = 0.4$, dynamic load factor;
- $G_m = 700$ N, pedestrian weight;
- $c_m(\rho) = c_u \left\{ 1 - \exp \left[-\gamma \rho_{max} \left(\frac{1}{\rho} - \frac{1}{\rho_{max}} \right) \right] \right\}$ m/s, mean velocity depending on the pedestrian density;
- $c_u = 1.34$ m/s, mean free-flow speed;
- $\sigma_{c_u} = 0.26$ m/s, standard deviation of the free-flow speed;
- $f_m(c_m) = 0.35c_m^3 - 1.59c_m^2 + 2.93c_m$ Hz, mean step frequency depending on the mean velocity;
- $\sigma_{f_m} = 0.173$ Hz, standard deviation of the step frequency;
- $N_P(\rho) = \rho BL$, pedestrian number;
- $N = 10$, number of pedestrian groups.

The dynamic response of the footbridge is evaluated for different values of free-flow speed in the range $[c_{um} \pm \sigma_{c_u}]$ and pedestrian densities ranging from 0 to 2.5 ped/m², through the procedure summarized in section 4.2.5.

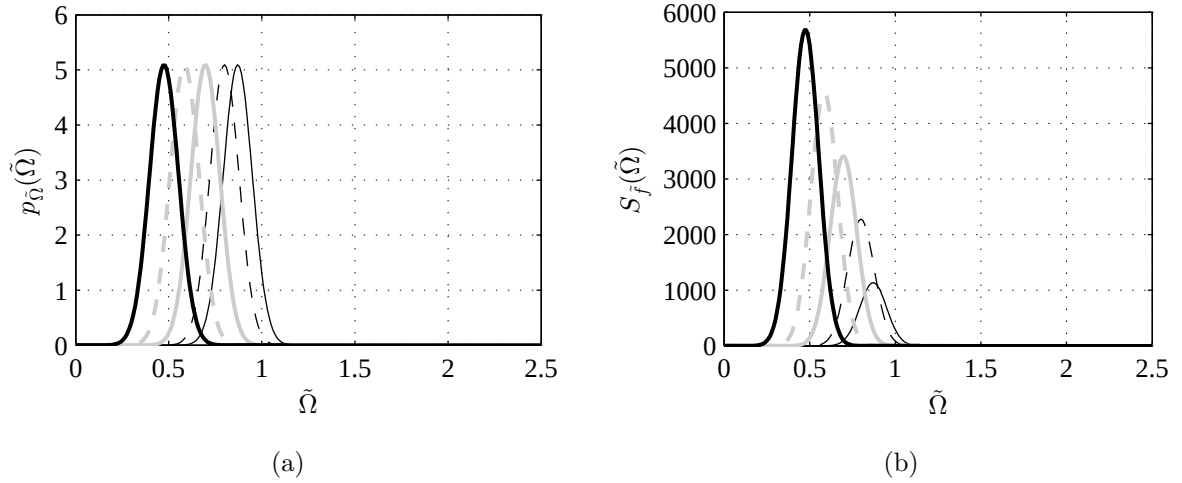


Figure 4.9: (a) PDF of the step frequency and (b) PSD of the pedestrian force for different values of pedestrian density: 0.5 ped/m² (solid black line), 1.0 ped/m² (dashed black line), 1.5 ped/m² (bold grey line), 2.0 ped/m² (dashed bold grey line) and 2.5 ped/m² (bold black line) - $c_u = c_{um} = 1.34$ m/s

Figure 4.9(a) shows the PDF of the step frequency $p_{\tilde{\Omega}}(\tilde{\Omega})$ for different pedestrian densities. When the pedestrian density increases, the mean step frequency decreases and thus the center value of the distribution, while the shape of the PDF remains the same due to the assumption of constant standard deviation, $\sigma_{f_m} = 0.173$ Hz.

Then, the PSD of the pedestrian force $S_{\tilde{f}}(\tilde{\Omega})$ is estimated from Eq.(4.65) and shown in Figure 4.9(b). As the PSD of the pedestrian force is proportional to the pedestrian number, its amplitude increases on increasing pedestrian density. The admittance function, obtained from Eq.(4.61) for different values of pedestrian density, is reported in Figure 4.10. It increases on increasing pedestrian density and, in particular, it reaches the constant values 0.405 for pedestrian density higher than about 1.5 ped/m². This is due to the fact that for high pedestrian densities the coherence function tends to one and thus the admittance function is given by the double integral of the sine function:

$$\chi_j(\tilde{\varepsilon}, C) = \int_0^1 \int_0^1 \text{Coh}_{\tilde{f}\tilde{f}}(\tilde{x}, \tilde{x}', \tilde{\varepsilon}, C) \sin(\pi\tilde{x}) \sin(\pi\tilde{x}') d\tilde{x} d\tilde{x}' = 0.405$$

when $\text{Coh}_{\tilde{f}\tilde{f}}(\tilde{x}, \tilde{x}', \tilde{\varepsilon}, C) \rightarrow 1$

Figure 4.11 shows the PSD of the modal force, obtained from the admittance function and the PSD of the pedestrian force. Finally, the variance of the acceleration is calculated and from that, together with the peak coefficient, the maximum midspan acceleration (Figure 4.12). Ferrarotti and Tubino [47] propose values of the constants C_1 and C_2 (adopted to calculate the correlation coefficient C) equal to $C_1 = 22$ s⁻¹ and $C_2 = -38$. In section 4.2.3.2, they are evaluated as $C_1 = 15.0461$ s⁻¹ and $C_2 = -26.4403$. The maximum acceleration is estimated considering both values of the constants C_1 and C_2 . It can be observed that the maximum accelerations reached in the two cases are basically comparable. Hence, the following analyses (chapter 5) are performed considering the values of C_1 and C_2 estimated in the present study ($C_1 = 15.0461$ s⁻¹ and $C_2 = -26.4403$).

To better understand the trend of the maximum acceleration with increasing pedestrian

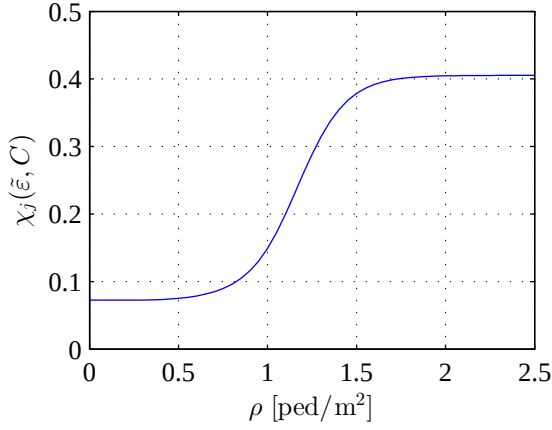


Figure 4.10: Trend of the admittance function with increasing pedestrian density - $c_u = c_{um} = 1.34$ m/s

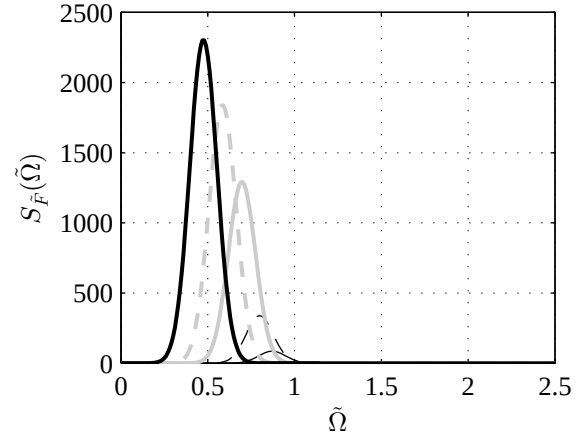


Figure 4.11: PSD of the modal force for different values of pedestrian density: 0.5 ped/m² (solid black line), 1.0 ped/m² (dashed black line), 1.5 ped/m² (bold grey line), 2.0 ped/m² (dashed bold grey line) and 2.5 ped/m² (bold black line) - $c_u = c_{um} = 1.34$ m/s

density, the trend of the mean step frequency compared with the footbridge natural frequency is to be analysed. Figure 4.13 shows that the mean step frequency decreases with increasing pedestrian density and it ranges between highest values for highest free-flow speed (namely it ranges in $[0.87, 1.75]$ Hz when $c_u = 1.08$ m/s and in $[1.17, 2.05]$ Hz when $c_u = 1.60$ m/s). However, the mean step frequency is always lower than the footbridge natural frequency, thus, the maximum resonance is obtained for low pedestrian densities. Then, when the pedestrian density increases, the mean step frequency moves far from the footbridge natural frequency.

This behaviour is reflected in the trend of the maximum acceleration. For low pedestrian densities, the dynamic response of the footbridge increases on increasing pedestrian density due to the growth of the number of pedestrians exciting the footbridge. On the contrary, for high values of pedestrian density, the structural response decreases, despite the number of pedestrians is still growing. Moreover, highest accelerations are obtained corresponding to the highest free-flow speed value, as in that case the mean step frequency is closer to the footbridge natural frequency.

Note that depending on the footbridge natural frequency the maximum acceleration can be obtained for different values of free-flow speed. Hence, the variability of the free-flow speed has to be accounted for, to consider all the possible mean step frequency ranges.

Finally, the relevant parameters for the evaluation of the peak coefficient are analysed. The spectral bandwidth parameter q is obtained from Eq.(4.71):

$$q \approx 2 \left(\frac{\xi}{\pi} \right)^{1/2} = 2 \left(\frac{0.005}{\pi} \right)^{1/2} = 0.080$$

while the product $\tilde{v}_0 \tilde{T}$ assumes the values presented in Figure 4.14. When the pedestrian density increases, the mean velocity decreases and thus the time required to cross the

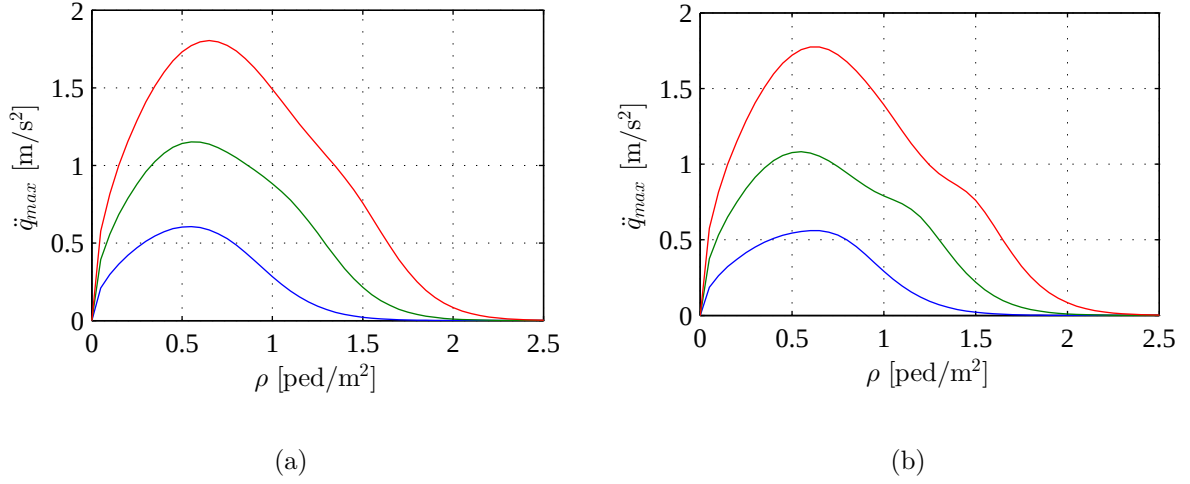


Figure 4.12: Maximum midspan acceleration for different values of free-flow speed: $c_u = 1.08$ m/s (blue line), $c_u = 1.34$ m/s (green line) and $c_u = 1.60$ m/s (red line). The acceleration is estimated considering the constants C_1 and C_2 (a) calculated in the present study and (b) proposed by Ferrarotti and Tubino

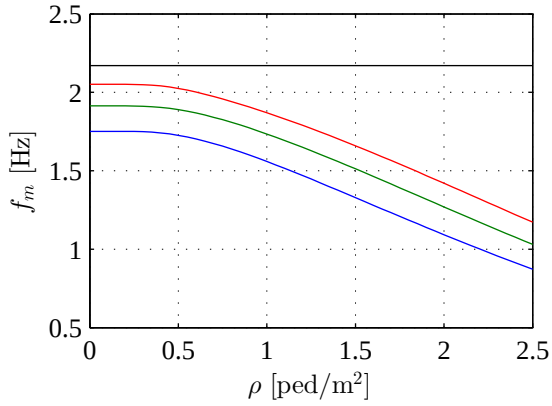


Figure 4.13: Mean step frequency for different values of free-flow speed: $c_u = 1.08$ m/s (blue line), $c_u = 1.34$ m/s (green line) and $c_u = 1.60$ m/s (red line). Black line: foot-bridge natural frequency

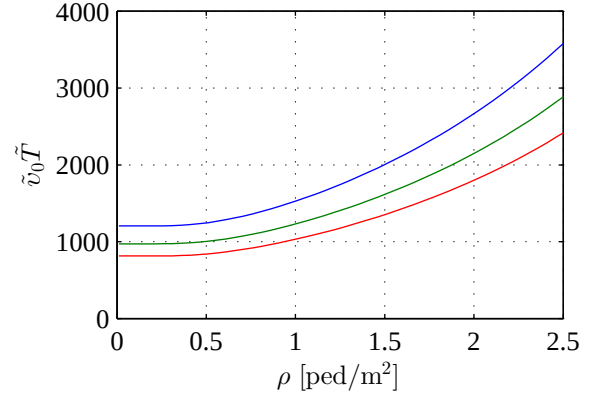


Figure 4.14: $\tilde{v}_0 \tilde{T}$ for different values of pedestrian density and free-flow speed

footbridge increases. Consequently, the time $\tilde{T}$, obtained from the time required to cross the footbridge and the number of subsequent crossings N , increases. These values are outside the applicability ranges defined in [37], i.e. $0.1 \leq q \leq 1$ and $5 \leq v_0 T \leq 1000$. These parameters are evaluated in more detail in section 5.2.2.3.

Chapter 5

Developments of the spectral load model

In the present chapter, some considerations and developments of the spectral model for the pedestrian-induced loading are presented and discussed. First, in section 5.1, the maximum structural response is predicted considering different relevant parameters for the footbridge and the pedestrian action. Moreover, the presence of a crowd of pedestrians can cause significant changes in the modal characteristics of the coupled crowd-structure system with respect to those of the empty footbridge. Therefore, to allow for a more accurate prediction of the maximum structural response, the present study in addition accounts for the vertical mechanical interaction between pedestrians, represented by simple lumped parameter models, and the supporting structure. The maximum structural response in presence of human-structure interaction (HSI) is investigated in section 5.2.

Under the assumption that the structural response is dominated by the j th natural mode, in the following analyses the maximum structural acceleration is calculated for a reference footbridge presenting the following characteristics:

- $f_j=2$ Hz, natural frequency;
- $\omega_j=12.57$ rad/s, circular natural frequency;
- $m_j = 25 \times 10^3$ kg, modal mass;
- $\xi_j=0.004$, structural damping;
- $\varphi_j(x)$, sinusoidal mode shape;
- $L = 50$ m, footbridge length;
- $B = 3$ m, footbridge width.

Pedestrian densities ranging between 0.1 ped/m^2 and 2.5 ped/m^2 are considered, corresponding to a total number of pedestrians from 15 to 375. Analyses are performed considering the mean value of the free-flow speed, $c_u = c_{um} = 1.34 \text{ m/s}$ that leads to a mean step frequency f_m , evaluated from Eq.(4.44), decreasing from 1.91 Hz to 1 Hz with increasing pedestrian density. However, to have a complete overview of the maximum response, the

variability of the free-flow speed c_u has to be considered, in order to account for all the possible ranges of mean step frequency. The standard deviation of the step frequency is assumed equal to 0.173 Hz, as suggested by Matsumoto et al. [76]. Moreover, a mean pedestrian's weight of 700 N is considered, as well as a number N of pedestrian groups the subsequently cross the footbridge equal to 10.

5.1 Parametric study

First, the influence of the natural frequency of the footbridge on the predicted structural response is analysed. The application presented in section 4.2.6.2 is referred to a footbridge with natural frequency of 2.17 Hz. As earlier reported, considering the mean free-flow speed equal to $c_{um} = 1.34$ m/s and a pedestrian density ranging between 0.1 ped/m² and 2.5 ped/m², the mean step frequency decreases from 1.91 Hz to 1 Hz. This implies that the maximum resonance between the step frequency and the natural frequency is obtained for small values of pedestrian densities. Hence, the structural response is evaluated considering different natural frequencies, resonant with mean step frequencies corresponding to higher pedestrian density and in the typical range of the pedestrian dynamic action (section 5.1.1). Moreover, the model proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] and described in section 4, accounts only for the contribution of the first harmonic of the walking force, with a dynamic load factor assumed constant. Here, the contribution of the resonant harmonic on the maximum response is evaluated (section 5.1.3) as well as a more realistic definition of the dynamic load factors (section 5.1.2).

5.1.1 Influence of the natural frequency of the footbridge on the maximum dynamic response

5.1.1.1 Worst-case scenario for different pedestrian densities

In the present section, the maximum structural response is predicted for footbridges with different natural frequencies. The natural frequencies are chosen as resonant with the mean step frequency corresponding to fixed pedestrian densities, representing the worst-case scenario for different values of pedestrian density. In particular, five reference values of pedestrian density are adopted, namely 0.5, 1.0, 1.5, 2.0 and 2.5 ped/m². For each density ρ , the corresponding mean step frequency is calculated through Eqs.(4.43) and (4.44). Then, the maximum dynamic response is estimated considering the footbridge natural frequency equal to the mean step frequency. The damping is assumed constant and equal to 0.004. Table 5.1 presents the mean step frequencies corresponding to the reference values of pedestrian density.

The structural response is predicted considering the natural frequencies from Table 5.1 as well as the reference natural frequency of 2 Hz. Figure 5.1(a) shows the accelerations predicted considering the contribution of the first harmonic of the walking force and the corresponding DLF equal to 0.4 (according to [47, 83]). Accelerations are calculated considering the response as mainly resonant, i.e. calculating the variance of the acceleration from Eq.(4.6). In Figure 5.1(b), the natural frequencies for the different cases are compared

ρ [ped/m ²]	c_m [m/s]	f_m [Hz]
0.5	1.30	1.89
1.0	1.06	1.74
1.5	0.81	1.51
2.0	0.61	1.27
2.5	0.45	1.03

Table 5.1: Mean velocity and mean step frequency for different pedestrian densities

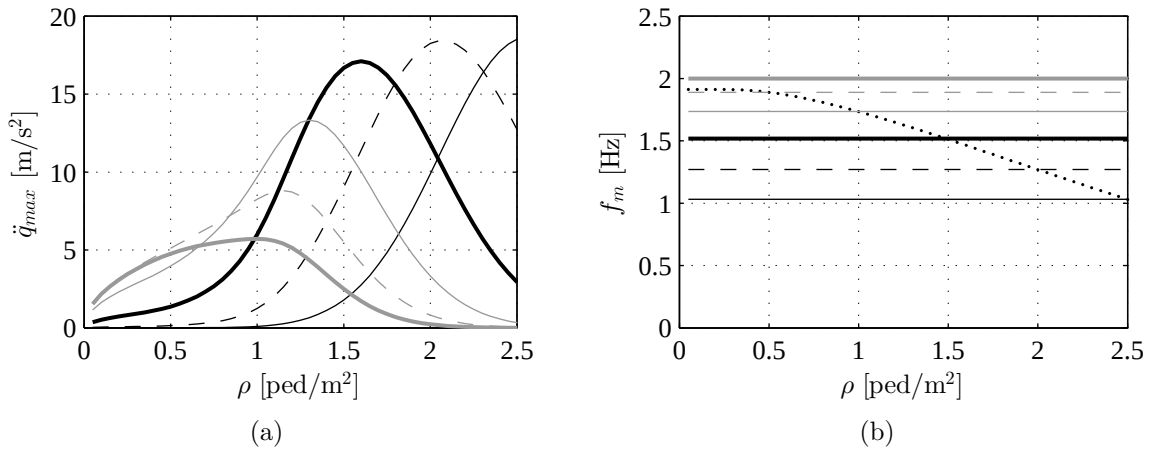


Figure 5.1: (a) Maximum midspan acceleration of footbridges with different natural frequency: 1.03 Hz (black line), 1.27 Hz (dashed black line), 1.52 Hz (bold black line), 1.74 Hz (grey line), 1.89 Hz (dashed grey line) and 2 Hz (bold grey line). The acceleration is obtained considering the response as mainly resonant and the contribution of the first harmonic, with a DLF $\alpha_1 = 0.4$. (b) Mean step frequency for the different pedestrian densities (dotted line) and different footbridge natural frequencies (horizontal lines)

with the mean step frequency. When the natural frequency decreases, the resonance with the pedestrian action is caused by increasing pedestrian density. Moreover, for decreasing structural frequencies, the maximum acceleration amplitude increases because the resonance is caused by an higher number of pedestrian. It can also be observed that for lower natural frequencies (1.03, 1.27 and 1.52 Hz), the maximum acceleration is obtained for the pedestrian density causing the resonance with the mean step frequency. On the contrary, for higher natural frequencies (1.74, 1.89 and 2 Hz) the maximum acceleration is given by a pedestrian density higher than the one causing the resonance, due to the effect of the increasing pedestrian number.

5.1.1.2 Typical range of human-induced dynamic action

In section 5.1.1.1, the maximum structural response is evaluated for natural frequencies in the range between 1 and 2 Hz, chosen as representative of the worst-case scenario for different pedestrian densities. Furthermore, in the present section, the maximum response is evaluated considering also natural frequencies within the range of the typical pedestrian

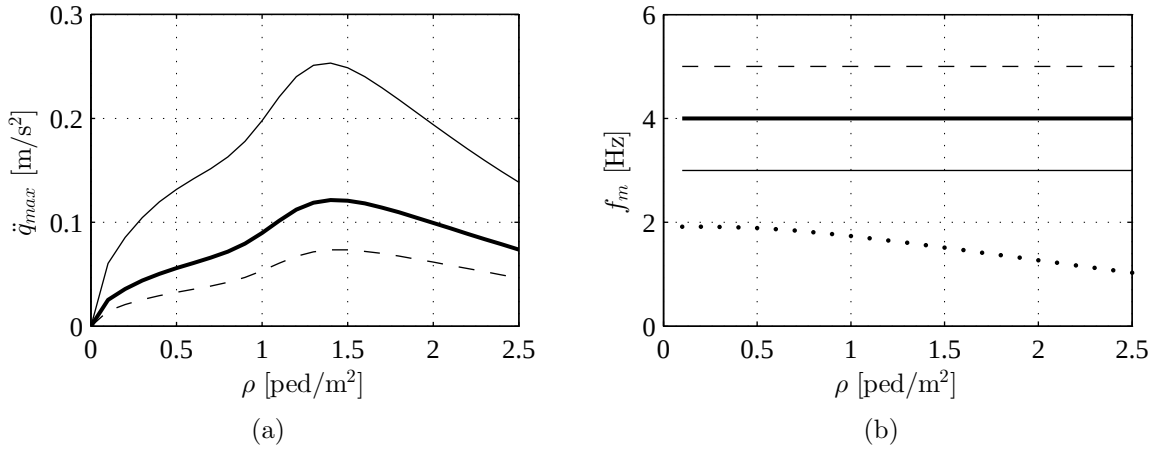


Figure 5.2: (a) Maximum midspan acceleration of the footbridge with different natural frequencies: 3 Hz (solid black line), 4 Hz (bold black line), 5 Hz (dashed black line). The acceleration is obtained considering the contribution of S_F from all the frequency domain and the contribution of the first harmonic, with a DLF $\alpha_1 = 0.4$. (b) Mean step frequency for the different pedestrian densities (dotted line) and different footbridge natural frequencies (horizontal lines)

action, between 1 and 5 Hz. For this purpose, the maximum structural response is evaluated considering footbridges natural frequencies equal to 3, 4 and 5 Hz (as the natural frequencies of 1 and 2 Hz are investigated in section 5.1.1.1). In these cases, the footbridge natural frequencies and step frequency are no longer near-resonant. Hence, the variance of the acceleration has to be evaluated considering the contribution of $S_F(\omega)$ from all the frequency domain, as reported in Eq.(4.5), rather than from Eq.(4.6). The maximum structural response obtained when the contribution of $S_F(\omega)$ from all the frequency domain is accounted for is presented in Figure 5.2(a). It has to be pointed out that accelerations are so small due to the fact that only the contribution of the first harmonic of the walking force is accounted for, while higher harmonics would give a more significant contribution. The effect of higher harmonics of the walking force is analysed in section 5.1.3.

As explained in appendix A.2.3 and section 4.2, to consider the response as mainly resonant means to consider the contribution of $S_F(\omega)$ from only the frequencies close to the natural frequency:

$$\sigma_{\ddot{p},res}^2 = S_F(\omega_j) \int_0^{+\infty} \omega^4 |H(\omega)|^2 d\omega = S_F(\omega_j) \int_0^{+\infty} |H_B(\omega)|^2 d\omega \quad (5.1)$$

with $H(\omega)$ and $H_B(\omega)$ the FRFs of the system relating the harmonic input excitation to the displacement and acceleration response, respectively. On the contrary, when the assumption of near-resonance between the pedestrian excitation and the structural response is no longer valid, the variance of the acceleration of the j th principal coordinate $\sigma_{\ddot{p}_j}$ is to be evaluated as:

$$\sigma_{\ddot{p},int}^2 = \int_0^{+\infty} \omega^4 |H(\omega)|^2 S_F(\omega) d\omega = \int_0^{+\infty} |H_B(\omega)|^2 S_F(\omega) d\omega \quad (5.2)$$

To better understand what considering $\sigma_{\ddot{p},res}^2$ or $\sigma_{\ddot{p},int}^2$ implies, quantities playing a key role in the definition of the variance of the acceleration are examined with reference to the

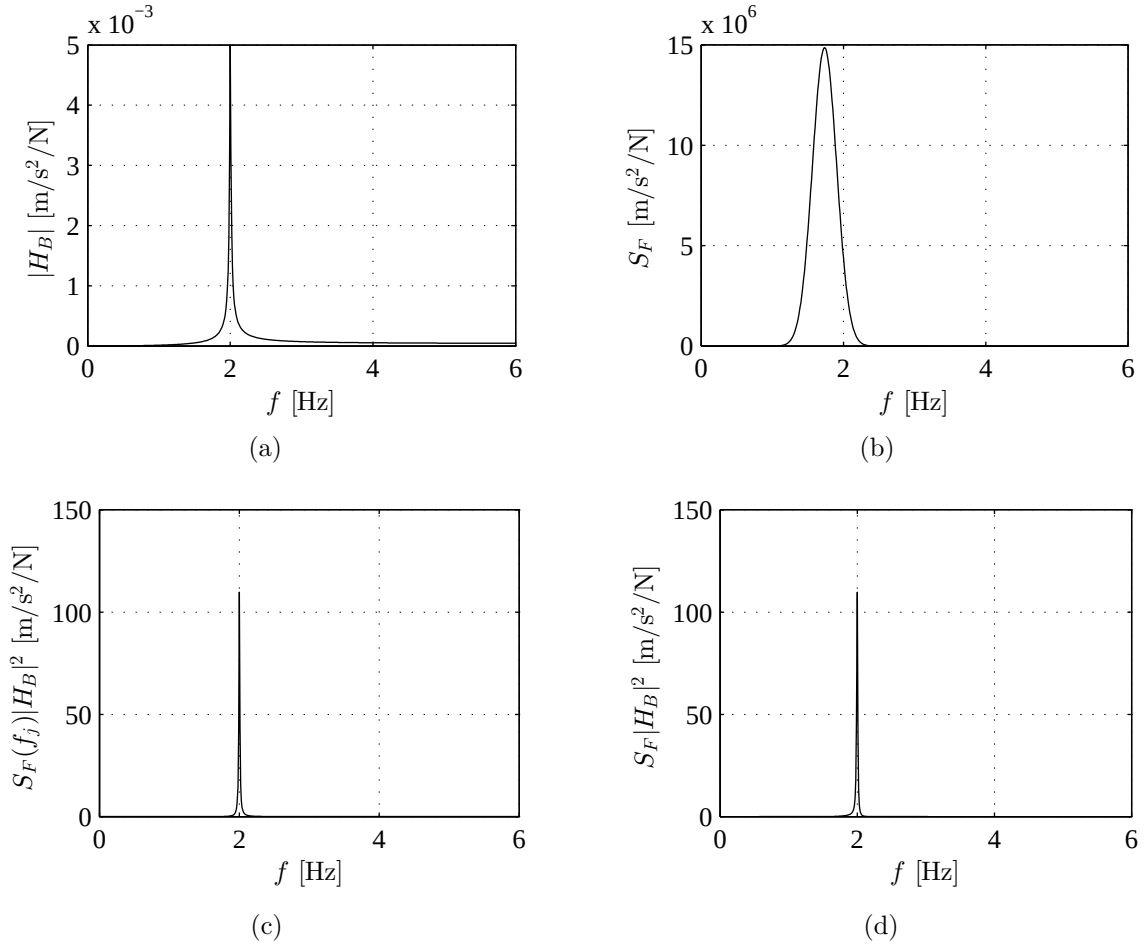


Figure 5.3: Results obtained for $\rho = 1 \text{ ped/m}^2$ and $f_j = 2 \text{ Hz}$: (a) acceleration FRF, (b) PSD of the modal force and product between $|H|^2$ and S_F (c) in the resonant case and (d) when the contribution of S_F from all the frequency domain is considered

natural frequencies of 2 and 3 Hz. Relevant quantities for the definition of the acceleration variance are: the acceleration FRF, the PSD of the modal force and the product between $|H|^2$ and S_F in the resonant case and when the contribution of S_F from all the frequency domain is considered. These quantities are reported, with reference to a pedestrian density of 1 ped/m^2 , in Figures 5.3 and 5.4, for the natural frequency of 2 Hz and 3 Hz, respectively. The values of the variance of the acceleration and the maximum acceleration, obtained from the former via the peak coefficient, are reported in Table 5.2. As the pedestrian excitation is near-resonant with the footbridge natural frequency of 2 Hz, $\sigma_{p,int}^2$ and $\sigma_{p,res}^2$ are almost the same in that case. On the contrary, the natural frequency of 3 Hz is significantly far from the mean step frequency, resulting in an important difference between $\sigma_{p,int}^2$ and $\sigma_{p,res}^2$.

As regards the natural frequencies considered in section 5.1.1.1, the difference between $\sigma_{p,int}^2$ and $\sigma_{p,res}^2$ is negligible, since all of those frequencies are near-resonant with the mean step frequencies.

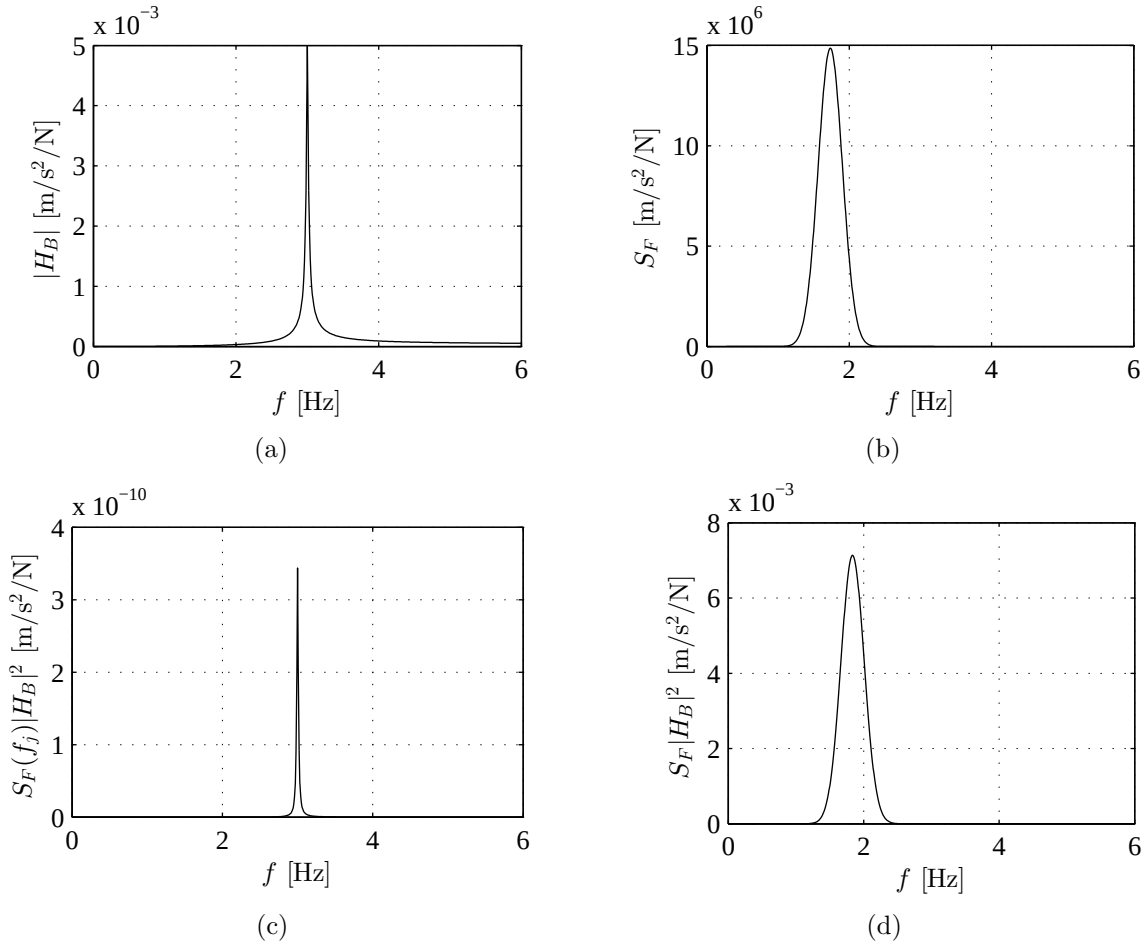


Figure 5.4: Results obtained for $\rho = 1 \text{ ped/m}^2$ and $f_j = 3 \text{ Hz}$: (a) acceleration FRF, (b) PSD of the modal force and product between $|H|^2$ and S_F (c) in the resonant case and (d) when the contribution of S_F from all the frequency domain is considered

f_j	[Hz]	2	3
ρ	[ped/m ²]	1	1
$\sigma_{\ddot{p},res}^2$	[(m/s ²) ²]	2.76	1.3e-11
$\sigma_{\ddot{p},int}^2$	[(m/s ²) ²]	2.64	0.003
$\sigma_{\dot{p},res}$	[(m/s ²)]	1.66	3.6e-6
$\sigma_{\dot{p},int}$	[(m/s ²)]	1.62	0.055
g_{peak}	[-]	3.46	3.58
$\ddot{q}_{max,res}$	[m/s ²]	5.75	1.29e-05
$\ddot{q}_{max,int}$	[m/s ²]	5.62	0.20

Table 5.2: Variance of the acceleration for different natural frequencies

5.1.2 Influence of the dynamic load factor on the maximum dynamic response

In the examples proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] and presented in section 4.2.6, the dynamic load factor of the first harmonic of the walking force α_1 is assumed constant and equal to 0.4. To adopt the same dynamic load factor $\alpha_1 = 0.4$ for different natural frequencies in the range $[1, 2]$ Hz, implies to obtain accelerations higher than 15 m/s^2 , as shown in Figure 5.1. In fact, considering for instance $f_j = 1.03 \text{ Hz}$ and $\rho = 2.5 \text{ ped/m}^2$, it is assumed that pedestrians walking at a mean step frequency of 1 Hz cause a dynamic force proportional to the 40% of their own weight. This is unrealistic as the force induced at such a low velocity is almost static. Hence, to take into account that the amplitude of the dynamic component of the walking force depends on the walking speed, the DLF of the first harmonic is calculated as function of the mean step frequency, as proposed by Kerr in [70]:

$$\alpha_1 = -0.2649f_m^3 + 1.3206f_m^2 - 1.7597f_m + 0.7613, \quad 1 \leq f_m \leq 2.65 \text{ Hz} \quad (5.3)$$

The trend of α_1 with increasing step frequency is shown in Figure 5.5. Note that the value $\alpha_1 = 0.4$ proposed in [47, 83] corresponds to α_1 estimated from Eq.(5.3) for a mean step frequency of about 2 Hz. As in the presented analyses the mean step frequency ranges between 1.9 and 1 Hz, Eq.(5.3) can be adopted for all pedestrian density values.

With reference to the same cases presented in section 5.1.1.1, the maximum acceleration obtained considering the mean DLF depending on the step frequency, is shown in Figure 5.6(a). It can be observed how the maximum acceleration significantly decreases due to the reduction of the DLF when the step frequency is low.

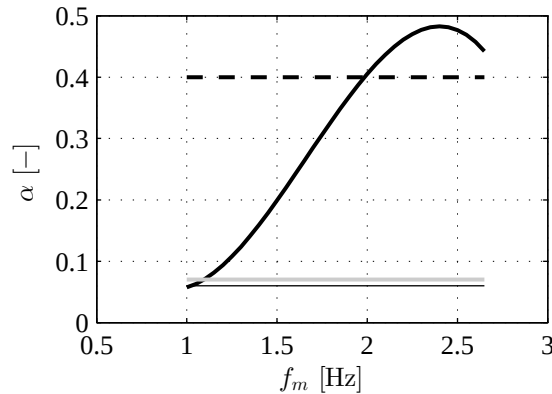


Figure 5.5: Dynamic load factors of the first (bold black line), second (bold grey line) and third (solid black line) harmonic proposed by Kerr and constant value (dashed bold black line) adopted by Piccardo and Tubino

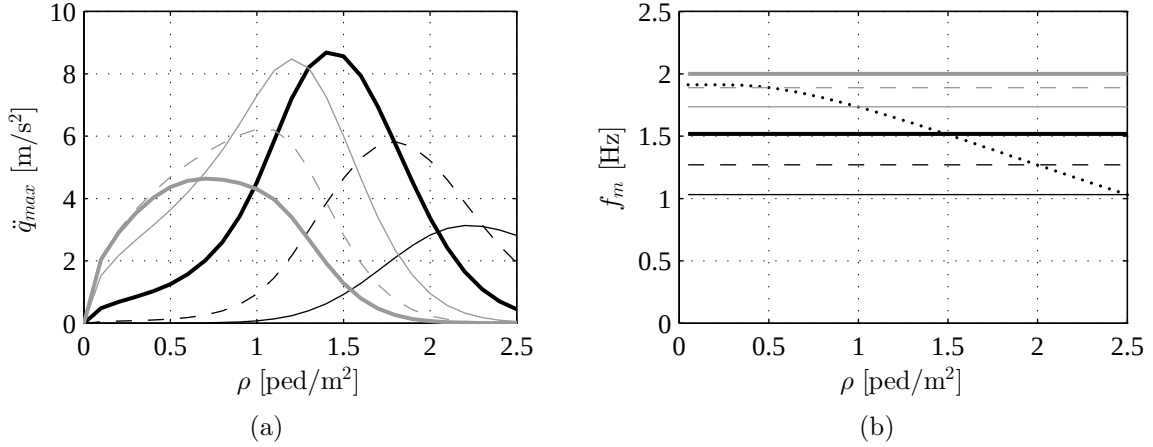


Figure 5.6: (a) Maximum midspan acceleration of the footbridge with different natural frequencies: 1.03 Hz (black line), 1.27 Hz (dashed black line), 1.52 Hz (bold black line), 1.74 Hz (grey line), 1.89 Hz (dashed grey line) and 2 Hz (bold grey line). The acceleration is obtained considering the response as mainly resonant and the contribution of the first harmonic, with α_1 depending on the mean step frequency. (b) Mean step frequency for the different pedestrian densities (dotted line) and different natural frequencies (horizontal lines)

5.1.3 Contribution of the resonant harmonic to the maximum dynamic response

The spectral load model proposed by Piccardo and Tubino [83] and Ferrarotti and Tubino [47] focuses only on the first harmonic of the walking force exerted by pedestrians, while the contribution of higher harmonics is neglected. This hypothesis is reliable when the natural frequency of the footbridge is near resonant with the mean step frequency. On the contrary, it implies a significant underestimation of the maximum response when the natural frequency is far from the mean step frequency, as for the cases reported in section 5.1.1.2. In the present study, the contribution of the resonant harmonic of the walking force is accounted for by defining an adequate probability density function (PDF) of the frequency related to each harmonic. The frequency PDF $p_f(f)$ for the k th harmonic is given by:

$$p_f(f) = \mathcal{N}(k f_m(\rho), k 0.173) [\text{Hz}] \quad (5.4)$$

$$p_{\tilde{\Omega}}(\tilde{\Omega}) = \mathcal{N}(k f_m(\rho)/f_j, k 0.173/f_j) [-] \quad (5.5)$$

in the dimensional and non-dimensional form, respectively. In the following analyses, the contribution of the first three harmonics of the walking force is account for. Figure 5.7(a) presents the mean frequency of the first three harmonics compared with the natural frequencies of 2 Hz, 3 Hz, 4 Hz and 5 Hz. Of course, the second and the third harmonics are resonant with higher natural frequencies of the footbridge. The DLFs of the second and third harmonic (Figure 5.5) are assumed, as proposed by Kerr [70], equal to:

$$\alpha_2 = 0.07, \alpha_3 = 0.06$$

The spectral load model allows to take into account one harmonic of the walking force at time. Hence, the procedure for the evaluation of the maximum midspan acceleration is

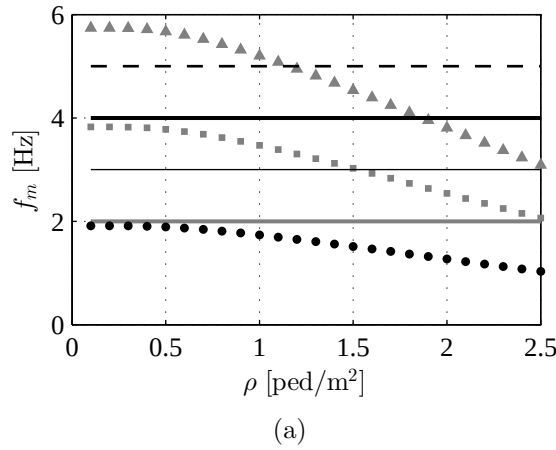


Figure 5.7: Mean frequency of the first (black dots), second (grey squares) and third (grey triangles) harmonic compared to the natural frequencies of 2 Hz (bold grey line), 3 Hz (solid black line), 4 Hz (bold black line) and 5 Hz (dashed black line).

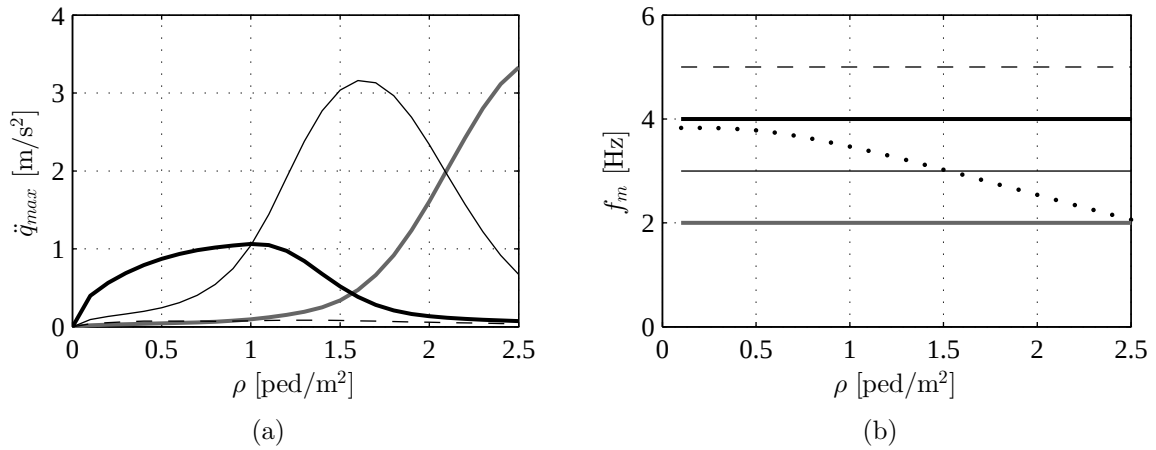


Figure 5.8: (a) Maximum midspan acceleration of the footbridge with different natural frequencies: 2 Hz (bold grey line), 3 Hz (solid black line), 4 Hz (bold black line) and 5 Hz (dashed black line), obtained considering the contribution of the second harmonic of the walking force. (b) Mean step frequency for the different pedestrian densities (dotted line) and different footbridge natural frequencies (horizontal lines) - $\alpha_2 = 0.07$

repeated considering the contribution of the first, second (Figure 5.8) and third (Figure 5.9) harmonic for natural frequencies in the range [2, 5] Hz. Moreover, in Figure 5.10 the contributions of the first, second and third harmonic of the walking force are compared for each natural frequency of the footbridge.

Finally, the maximum response is obtain as the envelope of the responses given by each harmonic. This amounts to consider, for each pedestrian density, the contribution of the resonant harmonic of the walking force. As regards the natural frequency of 2 Hz, the contribution of the first harmonic is the most significant for pedestrian densities lower than 1.5 ped/m² and then the second harmonic becomes the resonant one. For natural frequencies of 3 and 4 Hz the main contribution comes from both the second and third harmonic, while the natural frequency of 5 Hz is always resonant with the third one.

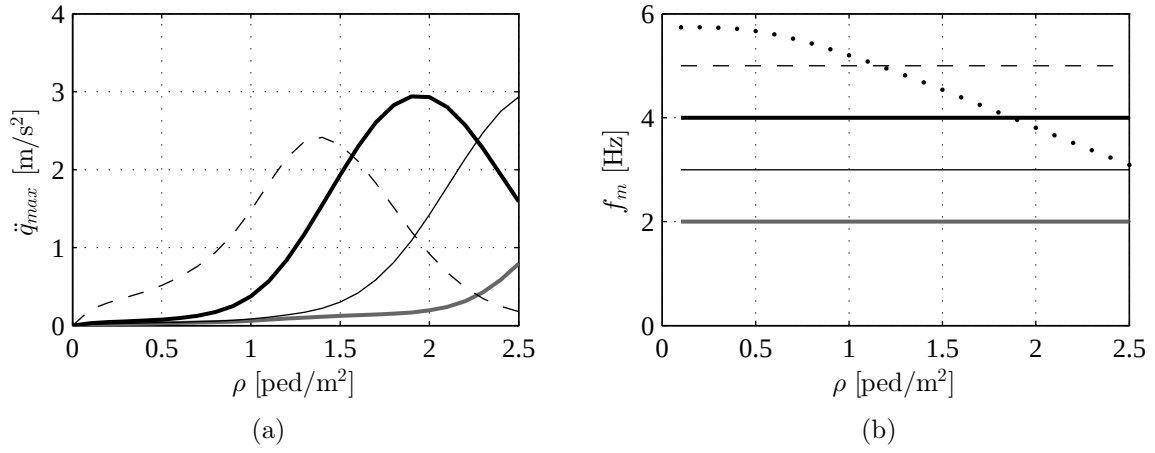


Figure 5.9: (a) Maximum midspan acceleration of the footbridge with different natural frequencies: 2 Hz (bold grey line), 3 Hz (solid black line), 4 Hz (bold black line) and 5 Hz (dashed black line), obtained considering the contribution of the third harmonic of the walking force. (b) Mean step frequency for the different pedestrian densities (dotted line) and different footbridge natural frequencies (horizontal lines) - $\alpha_3 = 0.06$

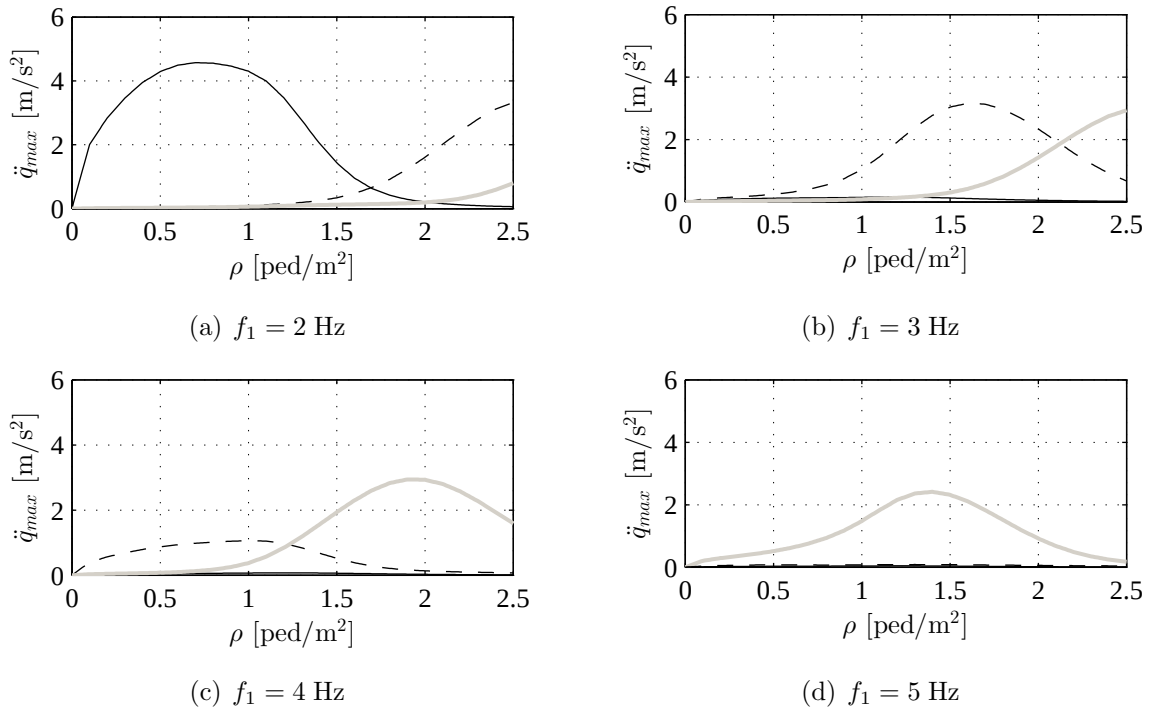


Figure 5.10: Maximum midspan acceleration due to the contribution of the first (solid black line), second (dashed black line) and third (solid grey line) harmonic of the walking force for different natural frequencies

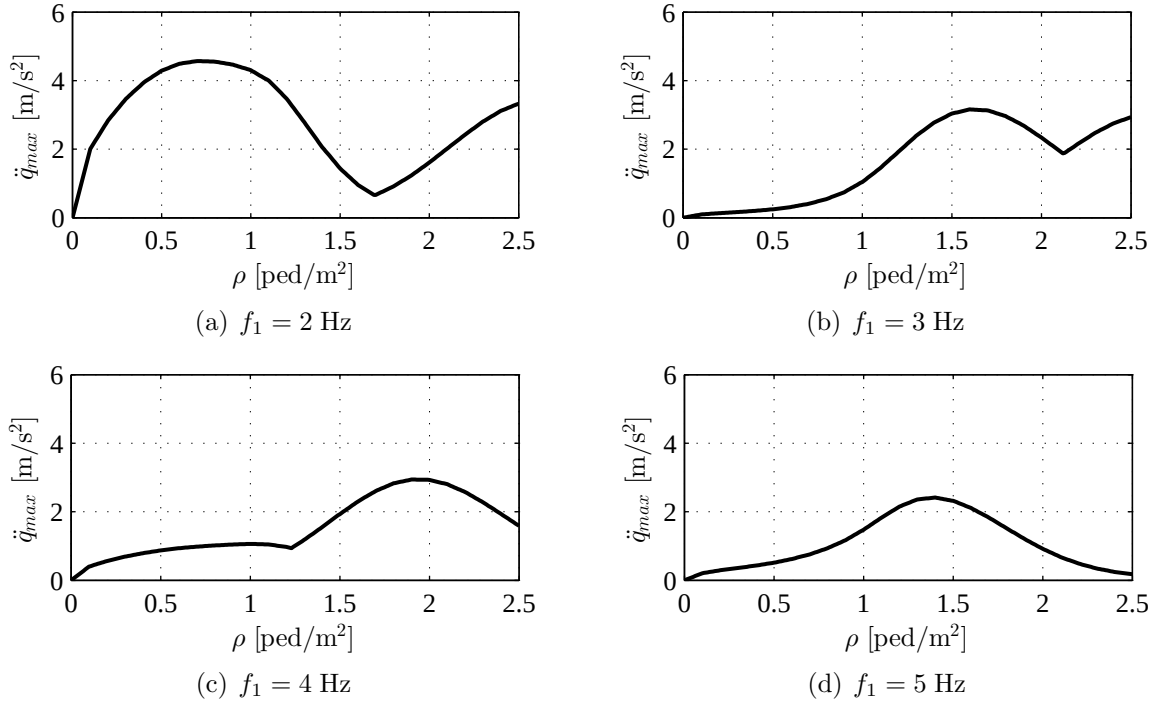


Figure 5.11: Maximum midspan acceleration due to the resonant harmonic of the walking force for different natural frequencies

5.1.4 Updated outline of the procedure

The present section sums up the main stages of the improved procedure for the estimation of the maximum dynamic response considering the contribution of the resonant harmonic. The updated procedure is presented in the flow chart of figure 5.12. The difference with the original procedure summarized in section 4.2.5 is in the evaluation of the contribution of the resonant harmonic of the walking force and the definition of the DLF of the first harmonic depending on the mean step frequency. Considering the first three harmonics of the walking force, the procedure has to be repeated three times to evaluate the maximum response due to each harmonic. For each analysis, the PDF of the frequency and the DLF related to the considered harmonic are calculated. Once the maximum acceleration in physical coordinates given by the k th harmonic $\ddot{q}_{jk_{max}}(x)$ is estimated, the maximum acceleration $\ddot{q}_{max}(x)$ is obtained as the maximum among the responses due to the three harmonics:

$$\ddot{q}_{j_{max}}(x) = \max \ddot{q}_{jk_{max}}(x) \quad (5.6)$$

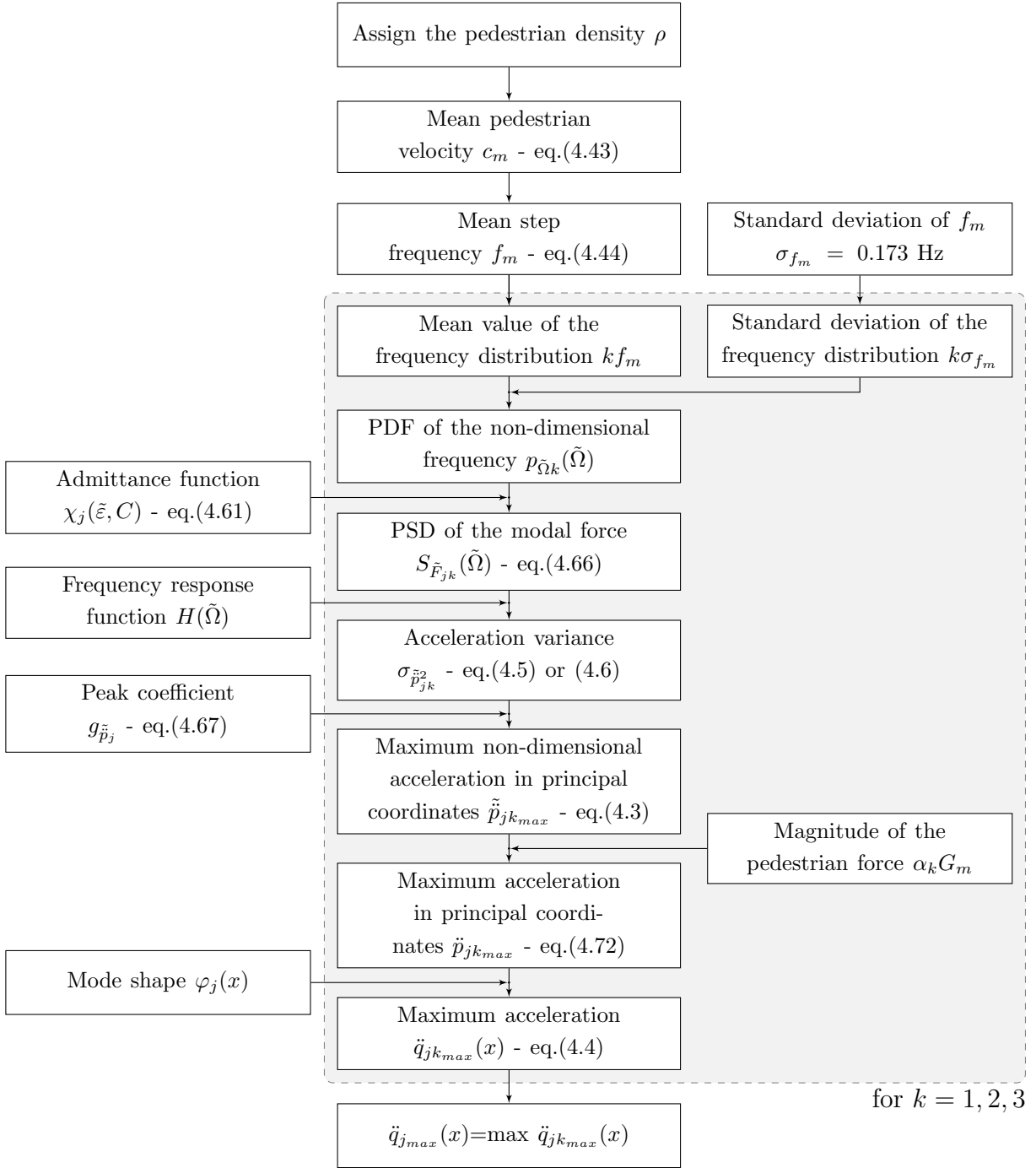


Figure 5.12: Flow chart of the updated procedure for estimating the maximum expected acceleration

5.2 Human-structure interaction

Pedestrians are mechanical systems which interact with the structure that is supporting them. Hence, during footbridge vibrations some kind of human-structure interaction (HSI) is almost unavoidable. Often, this interaction is neglected, but it is becoming common that, in some cases, its effects can significantly influence the structural response and cannot be neglected [114]. Due to the presence of stationary pedestrians, the human-structure interaction leads to changes in the modal properties of the dynamic system composed of the footbridge and the pedestrians. These changes are mainly in the damping and natural frequency that can significantly differ from those of the empty footbridge. The degree to which the dynamic behaviour is modified is expected to increase with an increasing crowd to structural mass ratio, making these effects non-negligible for lightweight footbridges [105]. In particular, the damping of the coupled human-structure system increases compared with that of the empty footbridge [42, 43, 68, 90]. Hence, the human body behaves like a damped dynamic system interacting with the main structural system. Although the human body is actually a complex MDOF system, the low-frequency ($[0, 10]$ Hz) behaviour of the human body in the vertical direction can be represented by a highly-damped single degree of freedom (SDOF) system [75, 105].

In the present section, the influence of the HSI on the maximum dynamic response based on the spectral load model is evaluated. The HSI-effects are accounted for by characterizing the dynamic behaviour of the coupled system composed of the footbridge and the pedestrians modelled as an ensemble of SDOF systems, each characterized by its own dynamic properties (mass, stiffness, damping). The resulting coupled system composed of the crowd and the footbridge is a MDOF system, where the number of DOFs (n_m) is given by the sum of the pedestrian number and the natural modes considered. In section 5.2.1, the dynamic behaviour of the coupled system is characterized, based on the study performed by Van Nimmen et al. [101, 104, 105]. Finally, the maximum response in presence of HSI-effects is evaluated and discussed in section 5.2.2.

5.2.1 Dynamic characteristics of the coupled crowd-structure system

For the empty footbridge, i.e. when the HSI-effects are not taken into account, the maximum steady-state amplitude $\ddot{u}_{B,max}$ to harmonic excitation is given by:

$$\ddot{u}_{B,max} \sim \frac{1}{2\xi_j m_j} = |H_B(\omega_j)| \quad (5.7)$$

with $H_B(\omega)$ the frequency response function (FRF) of the footbridge relating the harmonic input excitation to the acceleration response and ω_j the natural circular frequency of the footbridge in rad/s. The key parameters determining the structural dynamic response are the natural frequency f_j , the modal damping ratio ξ_j and the modal mass m_j . Similarly, the FRF of the coupled system relating the harmonic input excitation to the acceleration response is referred to as $H_{HB}(\omega)$. With reference to the footbridge described at the beginning of the chapter, Figure 5.13(a) presents a comparison between the FRF of the empty footbridge and that of the coupled system with 150 people. In this case, interaction effects

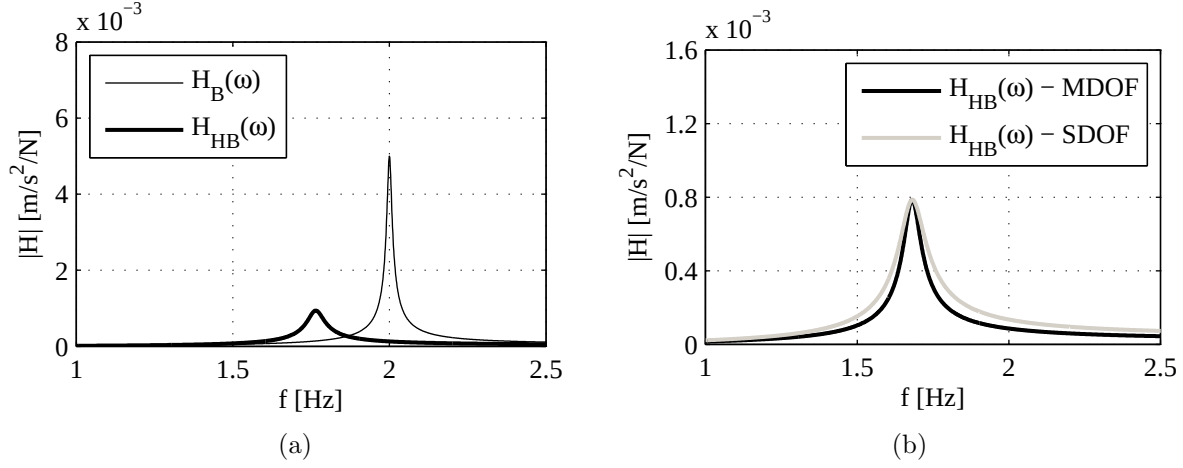


Figure 5.13: (a) Amplitude of the FRF of the empty footbridge (solid line) and the footbridge occupied by 150 people (bold line). (b) Amplitude of the FRF of the MDOF coupled system (black line) and the equivalent SDOF system (grey line)

are in the reduction of the peak value and corresponding frequency (abscissa) of $H_{HB}(\omega)$. The peak value of $H_{HB}(\omega)$ determines the amplitude of the steady-state acceleration response of the coupled system $\ddot{u}_{HB,max}$ when subjected to harmonic excitation:

$$\ddot{u}_{HB,max} \sim |H_{HB}(\omega_{HB})| \quad \text{with} \quad \omega_{HB} = \arg_{\omega} \max |H_{HB}(\omega)| \quad (5.8)$$

From the analysis of the FRF $H_{HB}(\omega)$, the HSI-effects can be quantified by introducing the effective frequency and damping ratio of the coupled system. The effective natural frequency of the coupled system is evaluated as the frequency corresponding to the peak of $H_{HB}(\omega)$:

$$f_{\text{eff}} = \frac{\omega_{HB}}{2\pi} \quad (5.9)$$

while the effective damping ratio ξ_{eff} is defined as a measure for the change in maximum steady state acceleration response in relation to that of the empty footbridge:

$$\ddot{u}_{HB,max} \sim \frac{1}{2\xi_{\text{eff}}m_j} \rightarrow \xi_{\text{eff}} = \frac{\ddot{u}_{B,max}}{\ddot{u}_{HB,max}} \xi_j = \frac{|H_B(\omega_B)|}{|H_{HB}(\omega_{HB})|} \xi_j \quad (5.10)$$

Furthermore, the coupled crowd-footbridge system can be reduced to an equivalent SDOF system characterized by the effective frequency and damping ratio defined in Eqs.(5.9) and (5.10), while the modal mass is assumed equal to that of the empty footbridge. The FRFs of the MDOF coupled system and that of the equivalent SDOF system are characterized by the same peak amplitude and position, but they can have different shape, as shown in Figure 5.13(b).

As regards the parameters of the SDOF system representing the human body, extensive experimental investigations are performed by Van Nimmen et al. [101, 104] to derive a reasonable range of human body model parameter values. The mechanical properties of the SDOF system largely depend on the posture. In [101, 104], stationary human body models representative of a body posture with one or two legs slightly bent are considered as an approximation of the postures assumed during the walking cycle. In addition, the

intra-subject variability is accounted for by assuming a Gaussian distribution of the natural frequency and modal damping ratio of the human body model. Moreover, all SDOF system are given the same mass, equal to the mean value of 70 kg. The mean values and coefficient of variations are based on the results reported in [75] and the results of the experimental studies performed in [101]:

$$f_{H1,k} \sim \mathcal{N}(3.25, 0.32) \text{ [Hz]} \quad (5.11)$$

$$\xi_{H1,k} \sim \mathcal{N}(0.30, 0.05) \text{ [-]} \quad (5.12)$$

According to the parametric study developed in [101], the level of interaction between the two subsystems, i.e. the crowd and the footbridge, depends on the ratio of the natural frequency of the human body f_{H1} to that of one of the relevant mode of vibration of the empty footbridge f_j :

- $f_{H1}/f_j < 0.5$ For low frequency ratios the crowd is a much more flexible system than the supporting structure and the movements of the two subsystems remain strongly uncoupled;
- $0.5 < f_{H1}/f_j < 1.5$ For intermediate frequency ratios the coupled system is characterized by a high level of interaction between the two subsystems, which is reflected in a significant increase in the damping ratio and the change in the peak position of the FRF of the coupled system;
- $f_{H1}/f_j > 1.5$ For high frequency ratios the crowd is much stiffer than the supporting structure and, for the first mode of vibration of the coupled system, the crowd moves in phase and along with the footbridge.

5.2.2 HSI-effects on the maximum dynamic response

The present section presents the influence of the HSI on the maximum midspan acceleration estimated based on the spectral model for the modal force. The structural response is evaluated through the updated procedure summarized in section 5.1.4, i.e. considering the contribution of the resonant harmonic among the first three. Moreover, the analyses are performed considering the mean free-flow speed $c_u = c_{um} = 1.34$ m/s and the spectral bandwidth parameter is calculated from Eq.(4.71).

The footbridge considered here is the one described at the beginning of the chapter, i.e. a simply supported beam with a span of 50 m and a bridge deck width of 3 m. Only the contribution of the fundamental mode is considered, characterized by sinusoidal mode shape, a modal damping ratio ξ_B of 0.4% and a modal mass m_B of 25×10^3 kg. Simulations are performed considering the natural frequency in the typical range of the pedestrian action, namely f_B of 2, 3, 4 and 5 Hz. The footbridge natural frequency of 1 Hz is not analysed here, due to its frequency ratio of 3.25, significantly outside the range of maximum interaction.

HSI-effects are accounted for considering both the MDOF coupled system and the equivalent SDOF characterized by the effective parameters. Then, accelerations obtained in presence of HSI-effects are compared with those of the empty footbridge.

The variance of the acceleration, adopted to calculate the maximum response through

Eq.(4.3), can generally be evaluated from Eq.(4.5), considering the contribution of $S_F(\omega)$ from all the frequency domain. Otherwise, for lightly-damped system and when the step frequency is close to the footbridge natural frequency, the response may be considered as mainly resonant and the variance of the acceleration calculated considering only the contribution of $S_F(\omega)$ in correspondence of the footbridge natural frequency (Eq.(4.6)).

In the following analyses the variance of the acceleration is calculated considering both the response as mainly resonant, $\sigma_{\ddot{p},res}^2$, and the contribution of $S_F(\omega)$ from all the frequency domain, $\sigma_{\ddot{p},int}^2$. Note that when the HSI-effects cause a significant increase in the effective damping of the coupled system, the hypothesis of lightly-damped system is no longer valid and the variance of the acceleration has to be calculated considering the contribution of $S_F(\omega)$ from all the frequency domain.

In summary, considering (i) the footbridge without HSI-effects, (ii) the MDOF coupled system and (iii) the equivalent SDOF system, the FRF and the variance of the acceleration assume the following expressions:

- **Footbridge without HSI-effects** Assuming that the structural response is dominated by the j th natural mode, the acceleration FRF of the empty footbridge is defined as:

$$H_B(\omega) = \frac{1}{m_j} \frac{\omega^2}{\omega_j^2 - \omega^2 + 2i\xi_j\omega\omega_j} \quad (5.13)$$

where ω_j , m_j and ξ_j are the modal parameters of the empty footbridge. According to appendix A.2, the variance of the acceleration is estimated as:

$$\sigma_{\ddot{p},int}^2 = \int_0^{+\infty} S_F(\omega) |H_B(\omega)|^2 d\omega = \frac{1}{m_j^2} \int_0^{+\infty} \frac{S_F(\omega) \omega^4}{(\omega_j^2 - \omega^2)^2 + 4\xi_j^2\omega^2\omega_j^2} d\omega \quad (5.14)$$

when the contribution of $S_F(\omega)$ from all the frequency domain is considered, and:

$$\sigma_{\ddot{p},res}^2 = \frac{S_F(\omega_j)}{m_j^2} \int_0^{+\infty} \frac{\omega^4}{(\omega_j^2 - \omega^2)^2 + 4\xi_j^2\omega^2\omega_j^2} d\omega = S_F(\omega_j) \frac{\pi\omega_j}{4\xi_j m_j^2} \quad (5.15)$$

when the response is assumed as mainly resonant.

- **Coupled system - Equivalent SDOF** When the HSI-effects are considered by defining the effective frequency $\omega_{j,eff}$ and damping $\xi_{j,eff}$ of the equivalent SDOF system, the acceleration FRF of the coupled system is given by:

$$H_{HB,eq}(\omega) = \frac{1}{m_j} \frac{\omega^2}{\omega_{j,eff}^2 - \omega^2 + 2i\xi_{j,eff}\omega\omega_{j,eff}} \quad (5.16)$$

The variance of the acceleration is obtained as:

$$\sigma_{\ddot{p},int}^2 = \int_0^{+\infty} S_F(\omega) |H_{HB,eq}(\omega)|^2 d\omega = \frac{1}{m_j^2} \int_0^{+\infty} \frac{S_F(\omega) \omega^4}{(\omega_{j,eff}^2 - \omega^2)^2 + 4\xi_{j,eff}^2\omega^2\omega_{j,eff}^2} d\omega \quad (5.17)$$

when the contribution of $S_F(\omega)$ from all the frequency domain is considered, and:

$$\sigma_{\ddot{p},res}^2 = \frac{S_F(\omega_{j,eff})}{m_j^2} \int_0^{+\infty} \frac{\omega^4}{(\omega_{j,eff}^2 - \omega^2)^2 + 4\xi_{j,eff}^2 \omega^2 \omega_{j,eff}^2} d\omega = S_F(\omega_{j,eff}) \frac{\pi \omega_{j,eff}}{4\xi_{j,eff} m_j^2} \quad (5.18)$$

when the response is assumed as mainly resonant.

- **Coupled system - MDOF** The FRF of the coupled crowd-footbridge system is obtained considering the contribution of all the DOFs (n_m =number of pedestrians+number of modes) as:

$$H_{HB,kl}(\omega) = \sum_{j=1}^{n_m} \frac{\varphi_{jk} \varphi_{jl} \omega^2}{\omega_j^2 - \omega^2 + 2i\xi_j \omega \omega_j} \quad (5.19)$$

with $\varphi_j \mathbf{M} \varphi_j = 1$, being $\mathbf{M}$ the mass matrix of the coupled system. The variance of the acceleration is obtained as:

$$\sigma_{\ddot{y},int}^2 = \int_0^{+\infty} S_F(\omega) |H_{HB,kl}(\omega)|^2 d\omega = \int_0^{+\infty} \sum_{j=1}^{n_m} \frac{\varphi_{jk} \varphi_{jl} S_F(\omega) \omega^4}{(\omega_j^2 - \omega^2)^2 + 4\xi_j^2 \omega^2 \omega_j^2} d\omega \quad (5.20)$$

when the contribution of $S_x(\omega)$ from all the frequency domain is considered, and:

$$\sigma_{\ddot{y},res}^2 = S_F(\omega_{j,eff}) \int_0^{+\infty} \sum_{j=1}^{n_m} \frac{\varphi_{jk} \varphi_{jl} \omega^4}{(\omega_j^2 - \omega^2)^2 + 4\xi_j^2 \omega^2 \omega_j^2} d\omega \quad (5.21)$$

when the response is assumed as mainly resonant. Note that the variance of the acceleration for the resonant case is obtained considering the amplitude of the PSD of the modal force in correspondence of the effective natural frequency.

First, the effective parameters of the coupled crowd-footbridge system are analysed with reference to the different natural frequencies. The effective damping ratios and frequencies for the different cases are shown in Figures 5.14(a) and 5.14(b), respectively.

The effective damping ratio increases monotonically with the pedestrian density but is highly dependent on the natural frequency of the footbridge. Increases of effective damping ratio up to 12%, 25% and 17% are obtained for natural frequencies of 3, 4 and 5 Hz, respectively, while it reaches maximum value of 3% for the natural frequency of 2 Hz.

Furthermore, due to increasing number of pedestrians, the effective frequency of the coupled system decreases for the structures with natural frequency of 2 and 3 Hz, lower than the mean natural frequency of the human body ($f_{H1} = 3.25$ Hz). On the contrary, the effective frequency of the coupled system increases on increasing pedestrian density for the natural frequencies of 4 Hz and 5 Hz, higher than f_{H1} .

The level of interaction can be approximately estimated through the analysis of the frequency ratio, i.e. the ratio of the human body model natural frequency f_{H1} to the footbridge natural frequency f_j . Frequency ratios for the different natural frequencies are summarized in Table 5.3. Maximum levels of interaction, in terms of increasing effective damping ratio, are reached when the frequency ratio f_{H1}/f_B is in the range $[0.5; 1.5]$, as happens for the natural frequencies of 3, 4 and 5 Hz. On the contrary, for the footbridge with natural

f_j [Hz]	2	3	4	5
f_{H1} [Hz]	3.25	3.25	3.25	3.25
f_{H1}/f_B [-]	1.625	1.08	0.812	0.65

Table 5.3: Frequency ratios for different natural frequencies

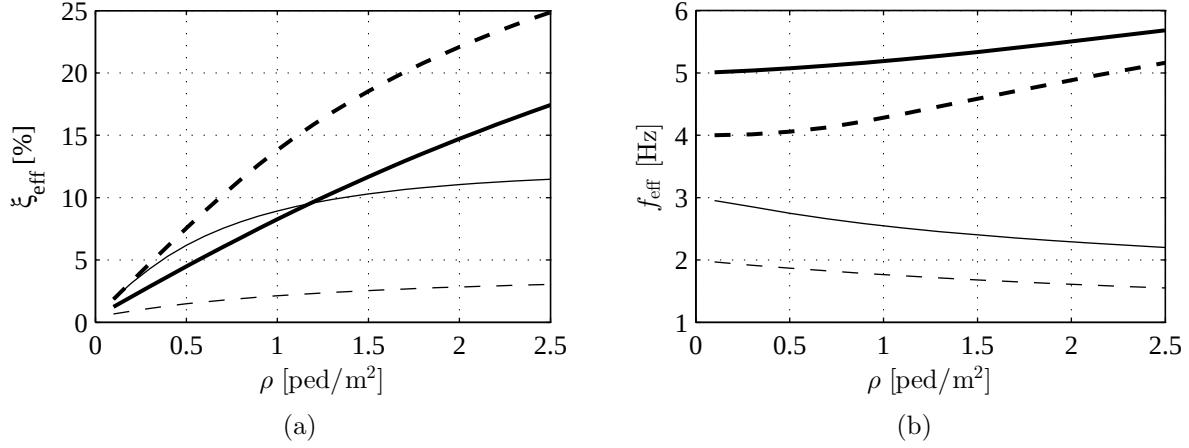


Figure 5.14: Trend of the (a) effective damping ratio and (b) effective frequency for different natural frequencies: 2 Hz (dashed line), 3 Hz (solid line), 4 Hz (dashed bold line) and 5 Hz (solid bold line)

frequency of 2 Hz, the frequency ratio is higher than 1.5, implying a lower increase in the effective damping ratio.

In the following, the maximum structural response of the coupled crowd-footbridge system is evaluated considering (i) each pedestrians having the same dynamic characteristics, i.e. neglecting the inter-subject variability (section 5.2.2.1) and (ii) considering the variability of the human body model parameters (section 5.2.2.2).

5.2.2.1 No intra-subject variability of the human body model parameters

The present section shows the effects of the HSI when pedestrians are SDOF systems characterized by the same dynamic characteristics (mass, stiffness and damping), i.e. in absence of intra-subject variability.

First, the behaviour of the footbridge with natural frequency of 2 Hz is analysed. Figure 5.15(a) shows the maximum acceleration of the empty footbridge, while in Figure 5.15(b) the footbridge natural frequency and damping ratio are reported, together with the mean step frequency. The mean step frequency represents also the frequency of the first harmonic of the walking force, whose contribution to the maximum acceleration is the resonant one for pedestrian densities lower than 1.5 ped/m². Then, the acceleration grows up again due to the contribution of the second harmonic. The maximum acceleration obtained in presence of HSI-effects is presented in Figure 5.15(c), while Figure 5.15(d) shows the effective frequency and damping ratio of the coupled system. Due to the HSI-effects, the effective frequency of the coupled system decreases on increasing pedestrian density, similarly to

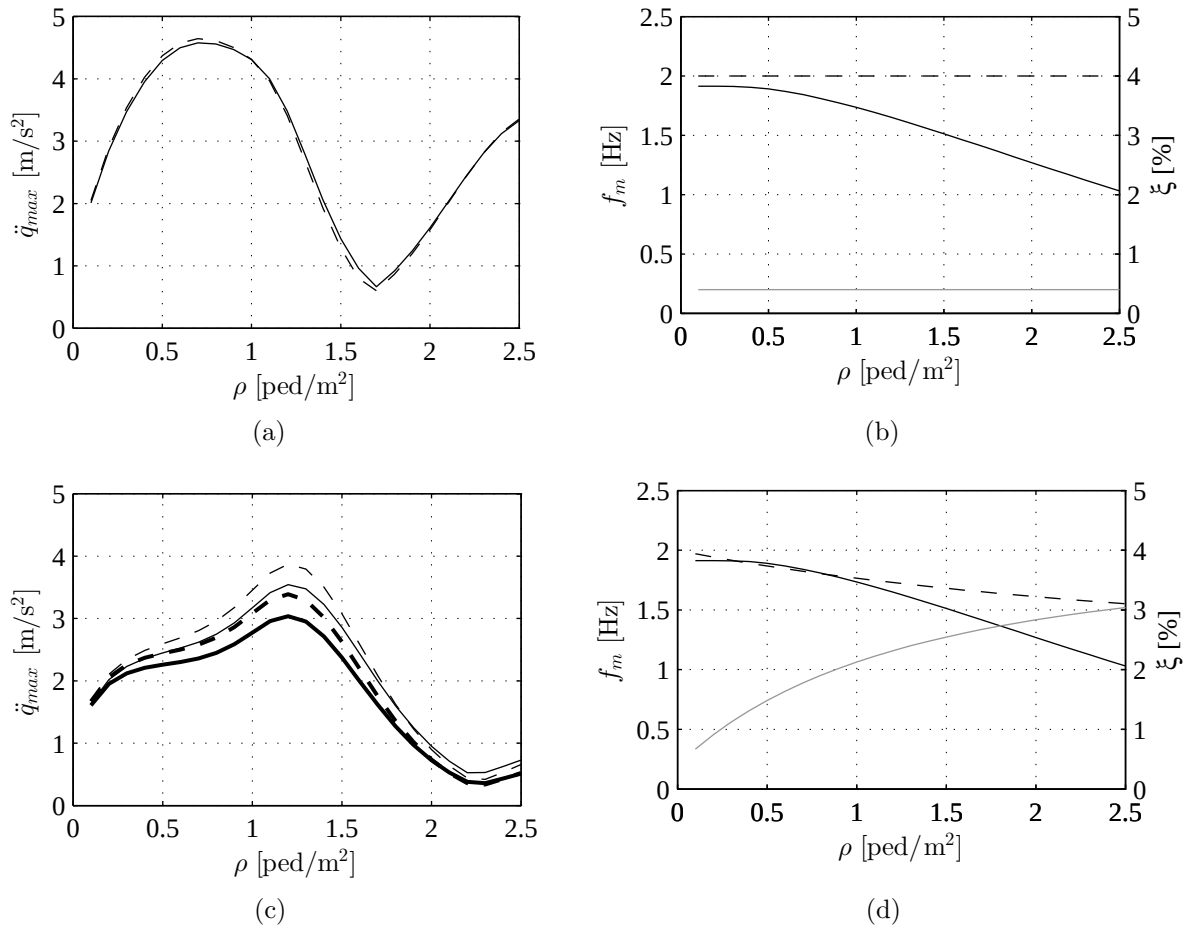


Figure 5.15: Maximum midspan acceleration of the footbridge with natural frequency $f_j = 2$ Hz, obtained (a) without the HSI-effects and (c) with the HSI-effects, accounted for through the MDOF coupled system (bold lines) and the equivalent SDOF system (solid lines). Accelerations are obtained considering both the contribution of $S_F(\omega)$ from the whole frequency domain (solid lines) and the response as mainly resonant (dashed lines). (b,d) Mean step frequency (black line) and natural/effective frequency (dashed line) and damping ratio (grey line) of the empty footbridge/coupled system

the mean step frequency. Hence, the excitation is near-resonant with the coupled system for higher pedestrian densities. As a consequence, the maximum acceleration is caused by a pedestrian density of about 1.2 ped/m^2 , differently from the empty footbridge, for which the maximum acceleration is reached for a pedestrian density of about 0.75 ped/m^2 .

To distinguish between the influence of the effective frequency and damping ratio on the structural response, the maximum acceleration of the equivalent SDOF system is calculated considering separately the contribution of the effective damping ratio (Figure 5.16(a)) and the effective frequency (Figure 5.16(c)). When only the effective damping ratio of the equivalent SDOF system is considered, while the frequency and modal mass are assumed equal to those of the empty footbridge, the increase in the effective damping ratio causes a significant decrease of the maximum acceleration. On the contrary, when only the effective frequency of the equivalent SDOF system is considered, while damping ratio and modal mass are those of the empty footbridge, acceleration significantly higher than that of the

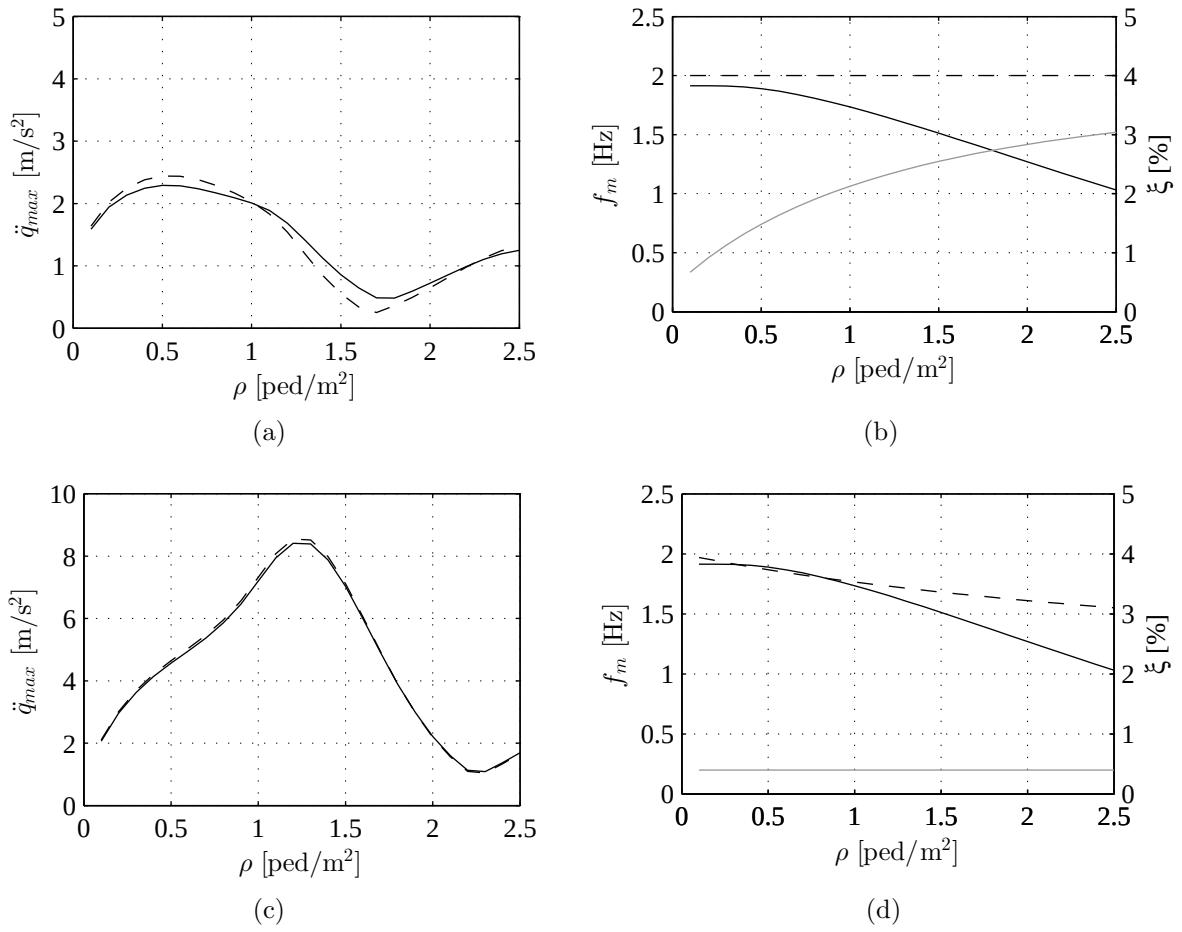


Figure 5.16: Maximum midspan acceleration of the footbridge with natural frequency $f_j = 2$ Hz, obtained with HSI-effects accounting only for (a) the effective damping ratio effect and (c) the effective frequency effect. Acceleration of the equivalent SDOF system is obtained considering the response as mainly resonant (dashed line) and the contribution of S_F from the whole frequency domain (solid lines). (b,d) Mean step frequency (black line) and natural/effective frequency (dashed line) and damping ratio (grey line) of the coupled system

empty footbridge is obtained. The decrease in the effective frequency causes near-resonance between the excitation and the structure for higher pedestrian densities than with the empty footbridge. Therefore, the maximum acceleration increases as it caused by an higher number of pedestrians. Considering only the effect of the effective frequency, the maximum acceleration is obtained for a pedestrian density of about 1.2 ped/m^2 and reaches value of almost 9 m/s^2 . When also the effect of the effective damping is taken into account (Figure 5.15(c)), the maximum response is still obtained for the pedestrian density of about 1.2 ped/m^2 , but the maximum amplitude is between 4 and 5 m/s^2 . This is due to the increasing damping from 0.4% of the empty footbridge to 1.2% for the coupled system. Moreover, it is observed that in the pedestrian density range $[1.2, 1.8] \text{ ped/m}^2$, the maximum acceleration of the coupled system is higher than the one obtained without the HSI-effects, as in that range the effect of the effective frequency prevails on the effect of the effective damping ratio.

The effect of obtaining, for certain values of pedestrian density, higher accelerations when

the HSI is accounted for rather than when it is not, can be observed because the analyses are performed considering a fixed footbridge natural frequency and a mean step frequency varying depending on the pedestrian density. Based on a conservative approach, simplified vibration serviceability checks proposed by the codes are generally carried out considering resonance between the step frequency and the footbridge. In the latter case, only a general reduction of the structural response due the increasing effective damping ratio would be observed. In fact, the latter approach does not account for the mutual relationship between the frequency of the footbridge (natural or effective) and the step frequency, which can lead to this non-negligible effect.

Afterwards, accelerations obtained considering the response as mainly resonant ($\ddot{q}_{max,res}$) or the contribution of $S_F(\omega)$ from the whole frequency domain ($\ddot{q}_{max,int}$) are compared. First, it can be qualitatively observed that differences between $\ddot{q}_{max,res}$ and $\ddot{q}_{max,int}$ are lower when the damping ratio of the empty footbridge is considered (Figures 5.15(a) and 5.16(c)) rather than when dealing with the effective damping ratio (Figures 5.15(c) and 5.16(a)). In fact, $\ddot{q}_{max,res}$ provides an accurate estimation of $\ddot{q}_{max,int}$ when the system is lightly-damped and the response resonant. Thus, the lower the structural damping, the more accurate is the response evaluated with the resonance assumption.

In addition, differences between $\ddot{q}_{max,res}$ and $\ddot{q}_{max,int}$ are quantitatively evaluated through the relative error $\varepsilon_{\ddot{q}_{max}}$:

$$\varepsilon_{\ddot{q}_{max}} = \frac{\ddot{q}_{max,int} - \ddot{q}_{max,res}}{\ddot{q}_{max,int}} 100 \quad (5.22)$$

The trend of $\varepsilon_{\ddot{q}_{max}}$ with the pedestrian density is shown in Figure 5.17(a). When it comes to the empty footbridge, the error is very low for pedestrian densities lower than 1 ped/m² and higher than 2 ped/m², where the contribution of the first and second harmonic, respectively, is mainly resonant. On the contrary, it reaches values of about 15% for intermediate pedestrian densities, farther from resonance. Regarding the coupled system (MDOF and equivalent SDOF) the error between $\ddot{q}_{max,int}$ and $\ddot{q}_{max,res}$ is more significant than that of the empty footbridge, due to the increasing effective damping ratio of the system, which

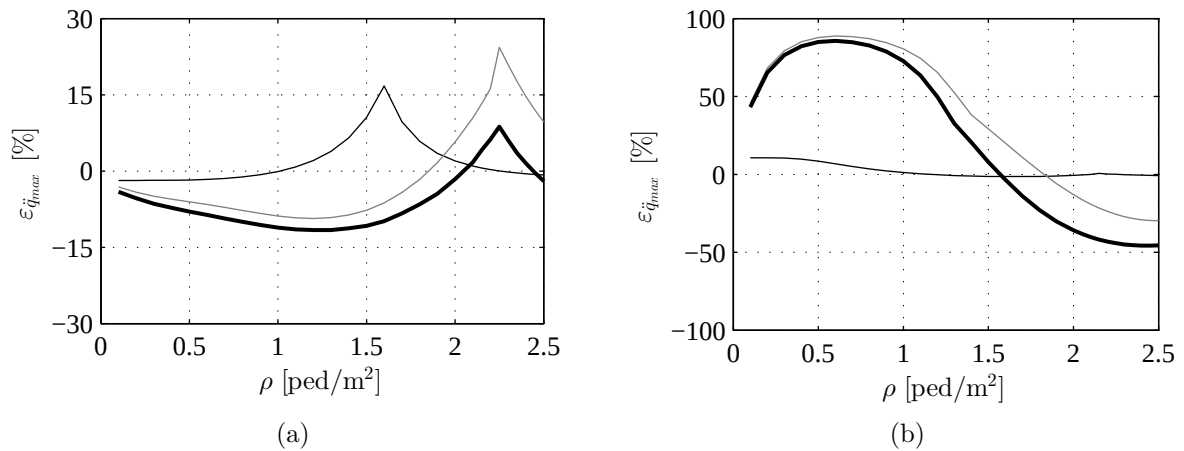


Figure 5.17: Trend of $\varepsilon_{\ddot{q}_{max}}$ with pedestrian density for (a) $f_j = 2$ Hz and (b) $f_j = 3$ Hz, without HSI-effects (solid black line) and with the HSI-effects accounted for through the MDOF coupled system (bold black line) and the equivalent SDOF system (solid grey line)

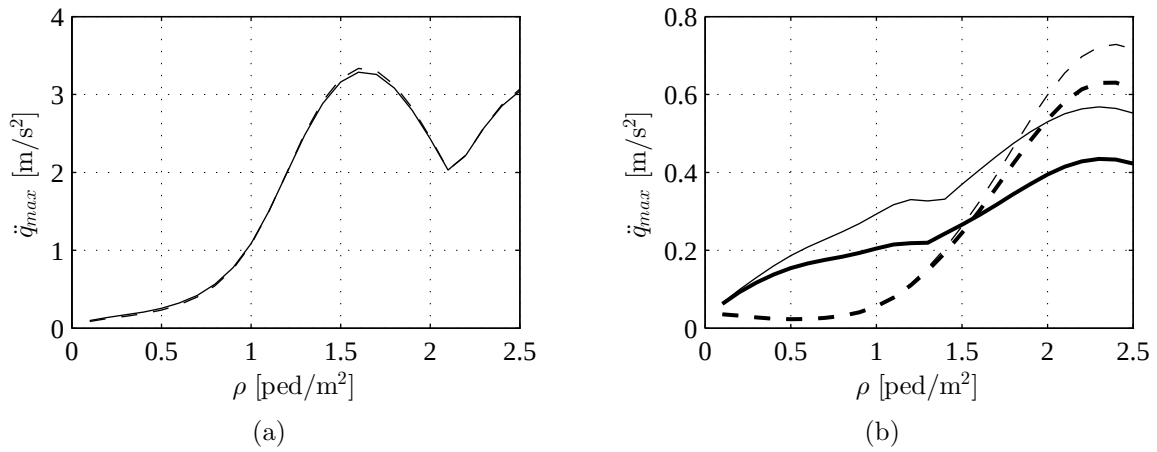


Figure 5.18: Maximum midspan acceleration of the footbridge with natural frequency $f_j = 3$ Hz, obtained (a) without the HSI-effects and (b) with the HSI-effects, accounted for through the MDOF coupled system (bold lines) and the equivalent SDOF system (solid lines). Accelerations are obtained considering both the contribution of $S_F(\omega)$ from the whole frequency domain (solid lines) and the response as mainly resonant (dashed lines)

makes the assumption of lightly-damped system less accurate. However, the error is still limited to 25% as the increase in the effective damping ratio is not that high. Moreover, due to the decrease in the effective frequency, the contribution of the first harmonic of the walking force is the most relevant one till pedestrian density of about 2 ped/m².

Then, the same comparison is made considering the natural frequency of 3 Hz, representative of an higher level of interaction in terms of effective damping ratio. In this case, the effective damping ratio increases significantly and thus the hypothesis of lightly-damped system is no longer valid. Figure 5.17(b) shows that $\varepsilon_{\ddot{q}_{max}}$ reaches values of almost 100% when the HSI-effects are taken into account. For this reason, when dealing with natural frequencies of 3, 4 and 5 Hz, characterized by significant effective damping ratio, the maximum response is estimated considering the contribution of $S_F(\omega)$ from the whole frequency domain. The accelerations $\ddot{q}_{max,res}$ and $\ddot{q}_{max,int}$ of the coupled system are reported in Figure 5.18(b). As regards the empty footbridge (Figure 5.18(a)), $\ddot{q}_{max,res}$ and $\ddot{q}_{max,int}$ are highly comparable and $\varepsilon_{\ddot{q}_{max}}$ is limited to 10%. This is due to the fact that the empty footbridge is a lightly-damped system and the response is resonant with the second and third harmonic of the walking force.

Figure 5.19 presents a comparison among the maximum accelerations obtained without and with the HSI-effects, accounted for through the equivalent SDOF system and the coupled MDOF system, for the different natural frequencies. For natural frequencies of 3, 4 and 5 Hz, the main effect comes from the significant increase in the effective damping ratio. It causes a decrease of the maximum response of the coupled system in comparison to that of the empty footbridge from values of the order of 3 m/s² up to about 0.3 m/s². On the contrary, for the case of 2 Hz, the increase in the damping ratio is not as high as in previous cases and the drop of the maximum acceleration is less sharp, decreasing from about 5 m/s² up to 3 m/s². As previously mentioned, in the pedestrian density range [1.2, 1.8] ped/m², the maximum acceleration of the coupled system is higher than the one obtained without the HSI-effects, due to the effect of the effective frequency which prevails

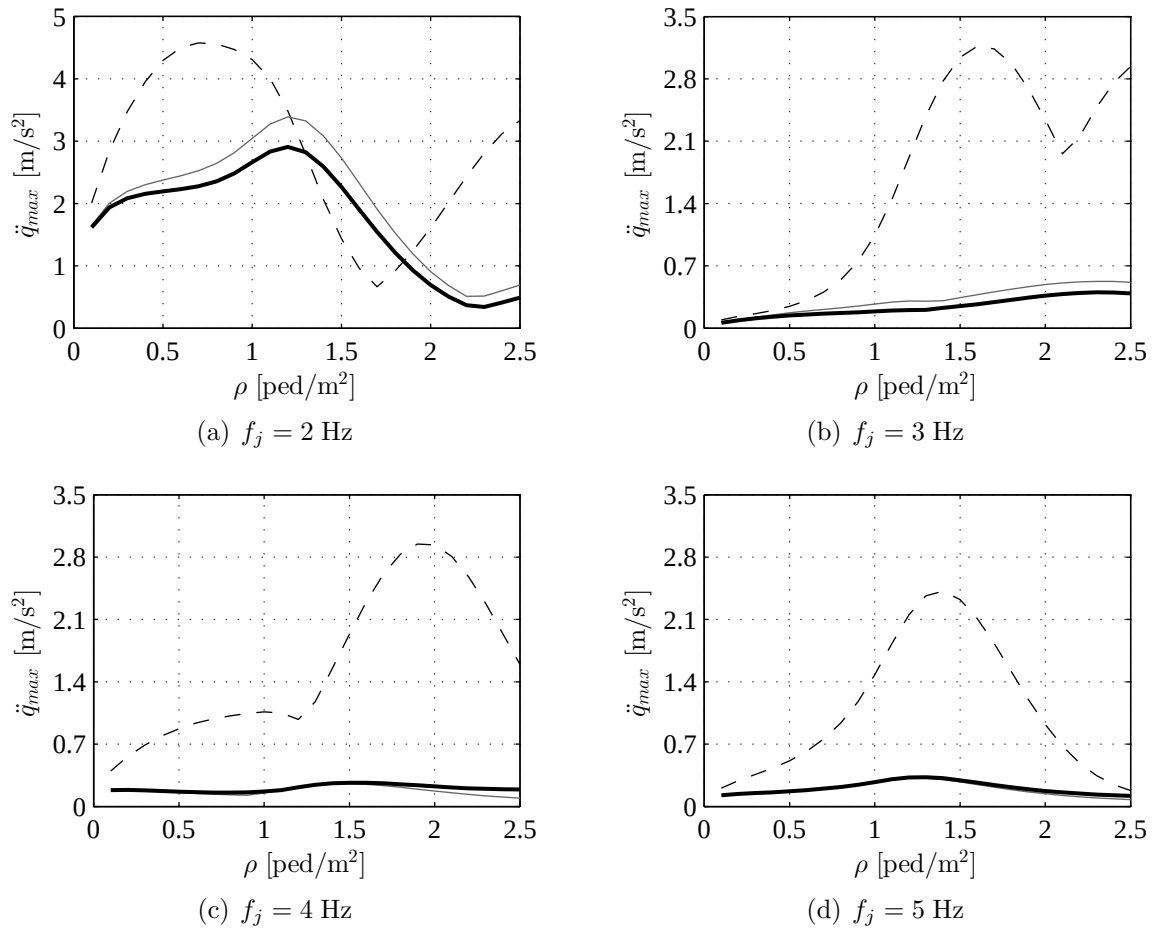


Figure 5.19: Maximum midspan acceleration of the empty footbridge (dashed lines), the MDOF coupled system (bold lines) and the equivalent SDOF system (solid lines), for different natural frequencies

on the effect of the effective damping ratio.

Accelerations obtained from the coupled MDOF system and the equivalent SDOF are characterized by similar trend and slightly different amplitudes, due to the differences in the corresponding FRFs (see section 5.2.1).

Effects of the effective frequency and damping are evaluated separately in Figure 5.20. The effective frequency causes a change in the peak position. In particular, for the natural frequencies of 2 and 3 Hz, the effective frequency decreases with increasing pedestrian density causing near-resonance and consequently higher accelerations for higher values of pedestrian density. On the contrary, for the natural frequencies equal to 4 and 5 Hz, the effective frequency increases with increasing pedestrian density causing near-resonance, and consequently higher accelerations, for lower values of pedestrian density. The effect of the effective damping is a general reduction of the maximum response, especially for natural frequencies of 3, 4 and 5 Hz, characterized by an high increase in the effective damping.

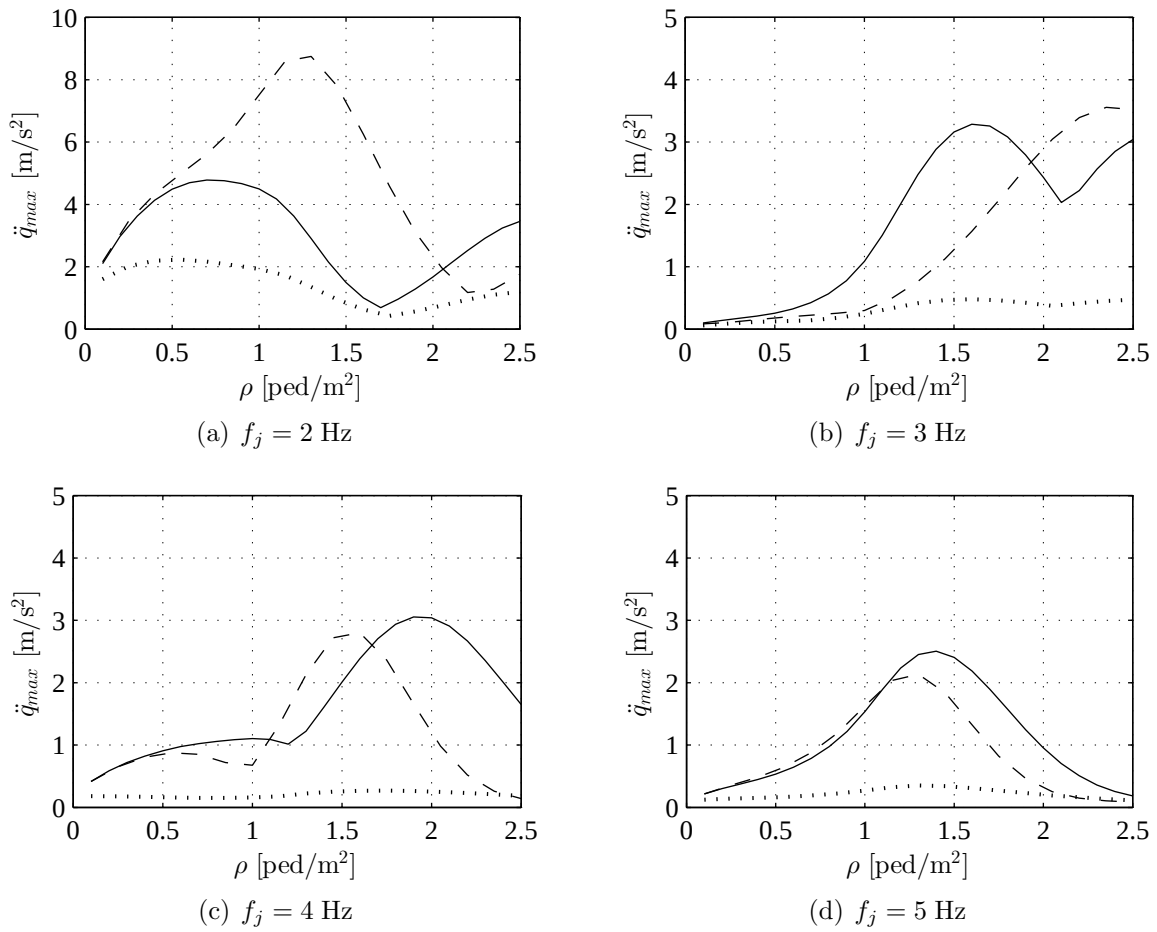


Figure 5.20: Maximum midspan acceleration of the empty footbridge (solid lines) and the equivalent SDOF when only the effective frequency is accounted for (dashed lines) or only the effective damping ratio (dotted lines), for different natural frequencies

5.2.2.2 Intra-subject variability of the human body model parameters

Finally, the maximum response is evaluated considering the intra-subject variability of the human body model parameters. First, for each pedestrian a natural frequency f_{H1} and a modal damping ξ_{H1} are randomly extracted from the PDFs of Eqs.(5.11) and (5.12). Then, the analyses are repeated considering many possible realizations and the mean response is evaluated. Accelerations obtained with and without the intra-subject variability are compared in Figure 5.21. The procedure for the estimation of the structural response based on the spectral load model leads to the mean value of the maximum response. Hence, the mean response obtained considering the intra-subject variability tends to the response obtained considering the mean values of the human body model parameters. Figure 5.22 shows two examples of the convergence of the response to its mean value. At the m th simulation, the response is evaluated as the average of the responses obtained from the previous $m - 1$ simulations.

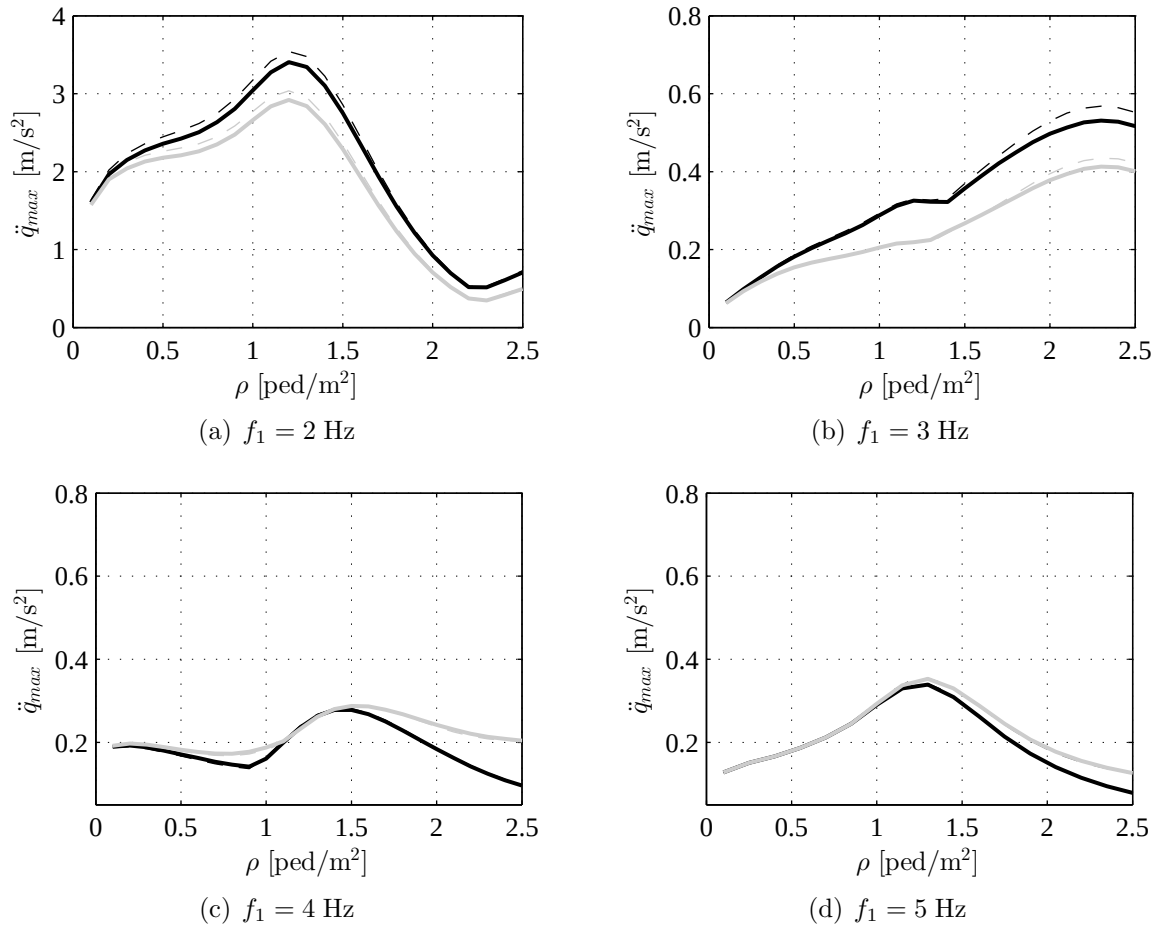


Figure 5.21: Maximum midspan acceleration of the footbridge with different natural frequencies obtained considering (bold lines) or not (dashed lines) the intra-subject variability of the human body model parameters. Grey lines represent the coupled MDOF system while black lines represent the equivalent SDOF system

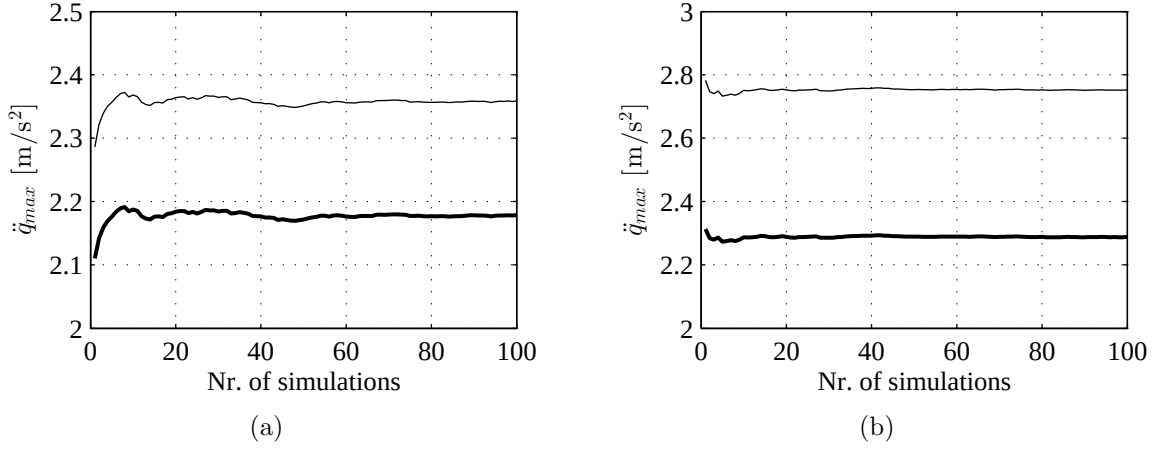


Figure 5.22: Maximum midspan acceleration for increasing number of simulations, considering the natural frequency $f_j = 2 \text{ Hz}$ and pedestrian density (a) $\rho = 0.5 \text{ ped/s}^2$ and (b) $\rho = 1.5 \text{ ped/s}^2$

5.2.2.3 Analysis of the parameters for the evaluation of the peak coefficient

In the present section, the influence of the key parameters for the evaluation of the peak coefficient (described in section 4.2.4) and the maximum response are analyzed. Parameters to be analyzed are the following:

- The spectral bandwidth parameter q . For lightly-damped system it can be approximated by Eq.(4.71):

$$q \approx 2 \left(\frac{\xi}{\pi} \right)^{1/2} \quad (5.23)$$

The hypothesis of lightly-damped system can be no longer valid when dealing with HSI-effects. In that case, the spectral bandwidth parameter can be calculated from its definition, Eq.(4.70):

$$q = \left(1 - \frac{\lambda_1^2}{\lambda_0 \lambda_1} \right)^{1/2} \quad (5.24)$$

where

$$\lambda_n = \int_0^\infty \omega^n S(\omega) d\omega \quad (5.25)$$

As the expressions for the calculation of the peak coefficient are valid in the range $0.1 \leq q \leq 1$ [37], this condition is also checked;

- The product $\tilde{v}_0 \tilde{T}$ and the condition $5 \leq \tilde{v}_0 \tilde{T} \leq 1000$ [37];
- The condition $\tilde{\Omega}_{cm} = \varepsilon \tilde{\Omega}_i$ (where ε is a parameter which has to be small enough), on which the definition of the modal force of Eq.(4.32) is based.

The trend of the above mentioned parameters is evaluated considering the reference cases of natural frequency of 2 and 4 Hz, chosen as representative of different levels of interaction between the crowd and the footbridge.

Natural frequency of 2 Hz

First, the trend of $\tilde{v}_0\tilde{T}$ with and without the HSI-effects is analysed in Figure 5.23(a). It can be observed that for pedestrian densities higher than about $[1, 1.5]$ ped/m² the product $\tilde{v}_0\tilde{T}$ reaches values higher than 1000, outside the applicability range. This is due to the decreasing mean velocity that causes an increase in the time for the footbridge crossing. Moreover, the product $\tilde{v}_0\tilde{T}$ is lower for the coupled system than for the empty footbridge. In fact, the effective frequency of the system decreases with respect to that of the empty footbridge as well as the term $\tilde{v}_0$.

The ratio $\tilde{\Omega}_{cm}/\tilde{\Omega}_i$, reported in Figure 5.23(b), ranges from 1.4% and 0.8%, confirming the hypothesis of ε being a small parameter.

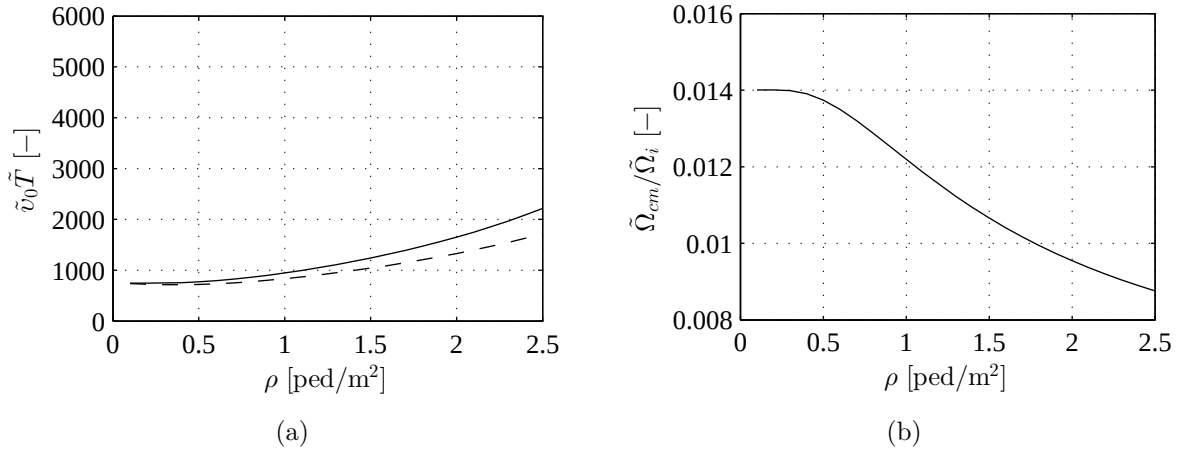


Figure 5.23: Trend of (a) the product $\tilde{v}_0\tilde{T}$ and (b) $\varepsilon = \tilde{\Omega}_{cm}/\tilde{\Omega}_i$ with increasing pedestrian density. In (a), dashed and solid lines represent the footbridge with and without the HSI-effects, respectively - $f_j = 2$ Hz

Figure 5.24(a) presents the trend of the spectral bandwidth parameter q with the pedestrian density for different cases:

- black lines: $q \approx 2\left(\frac{\xi}{\pi}\right)^{1/2}$, Eq.(5.23);
- red lines: $q = \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_1}\right)^{1/2}$, Eq.(5.24);
- blue lines: $q = \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_1}\right)^{1/2}$, Eq.(5.24), enforcing the condition $0.1 \leq q \leq 1$.

For the empty footbridge, the spectral bandwidth parameter q calculated with the approximation for the lightly-damped system (Eq.(5.23)) is constant, as it depends only on

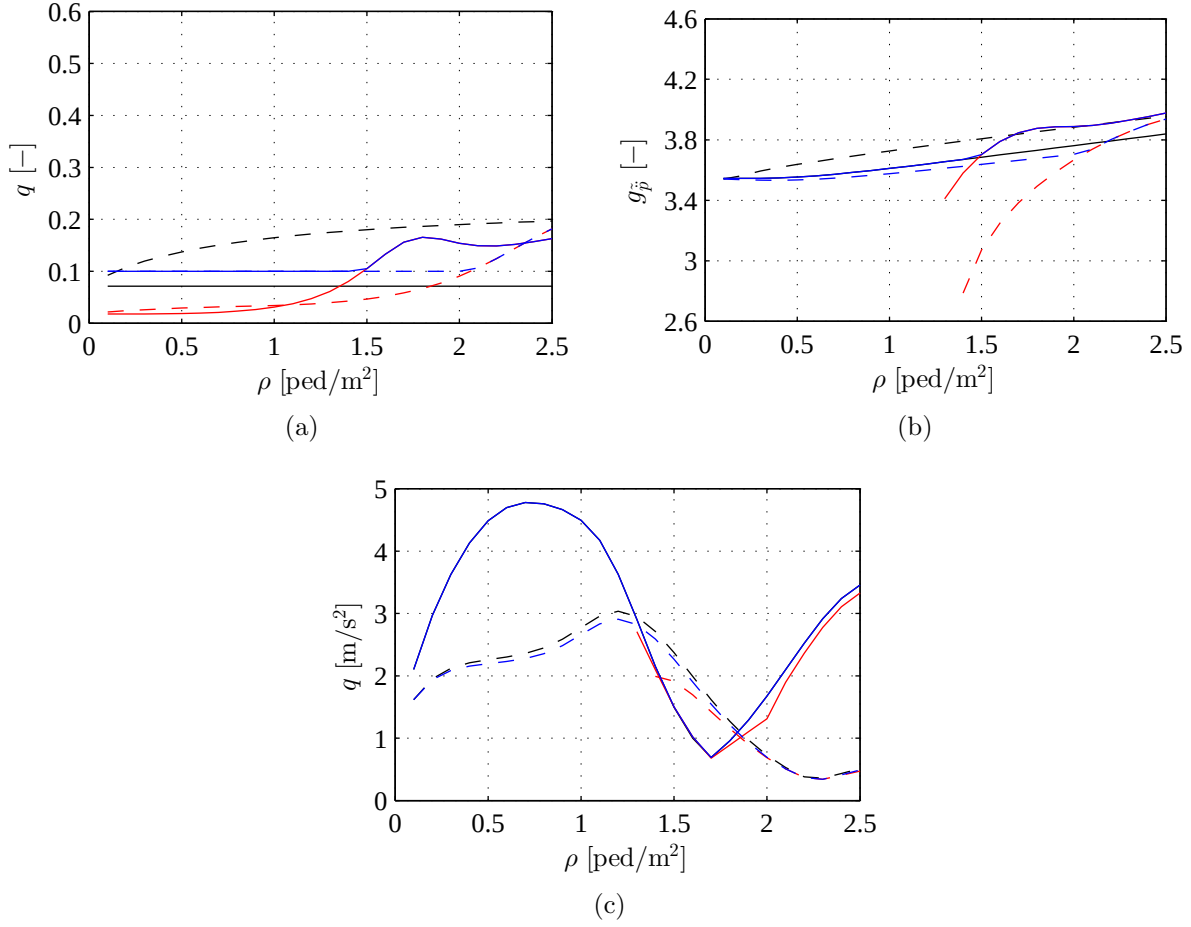


Figure 5.24: Trend of the (a) spectral bandwidth parameter, (b) peak coefficient and (c) maximum acceleration with increasing pedestrian density. Dashed and solid lines represent the system with and without the HSI-effects, respectively - $f_j = 2$ Hz

the modal damping ratio. It has constant value equal to $q = 0.08$, outside the validity range $[0.1; 1]$, but quite close to the lower bound. On the contrary, it is within the validity range when the HSI-effects are accounted for, due to the increasing effective damping ratio. When the spectral bandwidth parameter is calculated from Eq.(5.24) without enforcing the condition $0.1 \leq q \leq 1$, it exhibits values significantly lower than 0.1. Too low values of q imply that the modified expected frequency $\tilde{v}_e$ can become negative and thus the peak coefficient can become an imaginary number. In particular:

$$v_e = (1.63q^{0.45} - 0.38)v_0 < 0, \quad \text{when} \quad (1.63q^{0.45} - 0.38) < 0 \rightarrow q < \sqrt[0.45]{\frac{0.38}{1.63}} \rightarrow q < 0.039$$

Hence, when $q < 0.039$, the peak coefficient cannot be estimated as well as the maximum acceleration.

Figure 5.24(b) shows the peak coefficient estimated for the different cases. When q is calculated from its definition without enforcing the applicability condition, real values of the peak coefficient are reached for pedestrian densities higher than about 1.3 ped/m², i.e. when q reaches values higher than 0.039.

Finally, maximum accelerations obtained for the different cases are shown in Figure 5.24(c).

It can be observed that the responses obtained calculating q from Eq.(5.23) or Eq.(5.24) enforcing the condition $0.1 \leq q \leq 1$ are almost the same as q ranges between 0.1 and 0.2. On the contrary, to be significantly outside the validity range of q implies that in such case the maximum acceleration cannot be estimated.

Natural frequency of 4 Hz

Afterwards, parameters determining the peak coefficient are analysed with reference to the natural frequency of 4 Hz, characterized by a more significant level of interaction (in terms of effective damping ratio) between the crowd and the footbridge. In this case the product $\tilde{v}_0\tilde{T}$ (Figure 5.25) is twice the one obtained for a natural frequency of 2 Hz and is always outside the validity range [5,1000]. It could be reduced by reducing the number N of footbridge crossings in order to make the process stationary. Considering that the maximum value reached by the product $\tilde{v}_0\tilde{T}$ is about 6000, the number N of footbridge crossings should be reduced by 6 times to obtain $\tilde{v}_0\tilde{T} \sim 1000$. However, as the number N of footbridge crossings is assumed equal to 10, to reduce it by 6 times implies to consider 1.6 pedestrian groups crossing the footbridge, which does not ensure the stationariness of the process.

The term $\tilde{\Omega}_{cm}/\tilde{\Omega}_i$, not reported here, has the same trend of the previous case, as it depends only on the mean step frequency and the footbridge length, and not directly on the natural frequency.

The trend of the spectral bandwidth parameter q , peak coefficient and corresponding acceleration is shown in Figure 5.26. In this case, q is always higher than 0.039 and thus the response can be estimated for each pedestrian density. Figure 5.26(a) shows that the spectral bandwidth parameter calculated from its definition is characterized by the same trend whether the HSI-effects are accounted for or not. Figure 5.26(c) shows that the maximum responses obtained from the three cases are basically comparable.

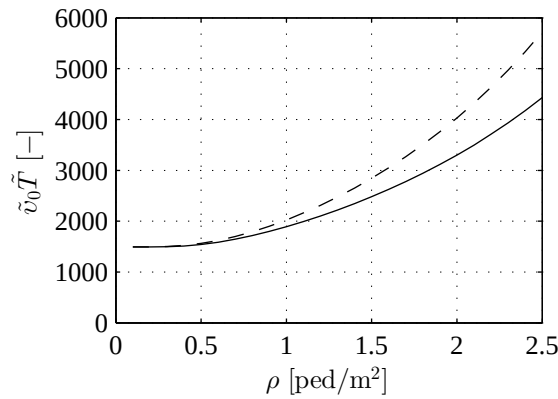


Figure 5.25: Trend of the product $\tilde{v}_0\tilde{T}$ with increasing pedestrian density. Dashed and solid lines represent the footbridge with and without the HSI-effects, respectively - $f_j = 4$ Hz

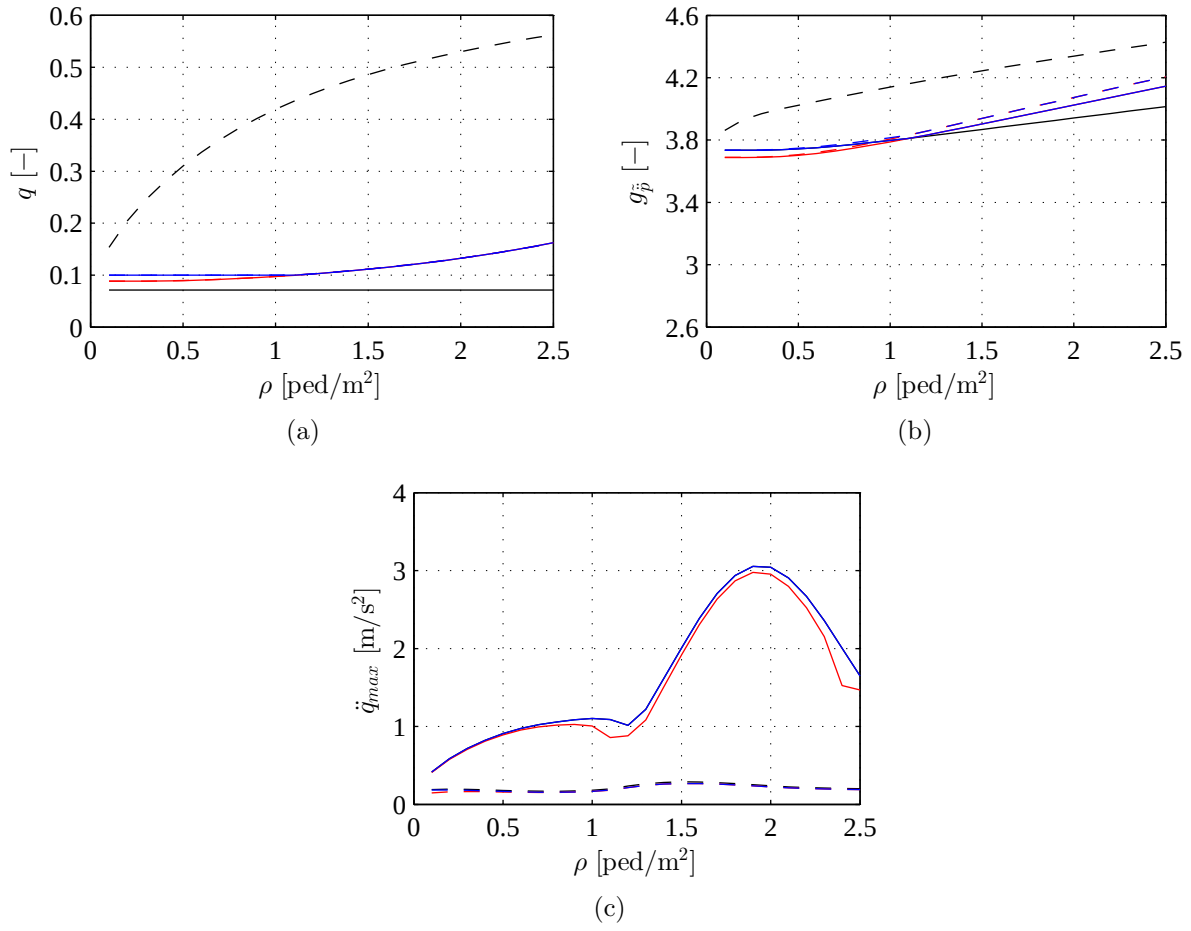


Figure 5.26: Trend of the (a) spectral bandwidth parameter, (b) peak coefficient and (c) maximum acceleration with increasing pedestrian density. Dashed and solid lines represent the system with and without the HSI-effects, respectively - $f_j = 4$ Hz

Chapter 6

Conclusions

The present chapter summarizes the main conclusion of this work. First, the main stages of the procedure for the evaluation of the structural response to human-induced loading are characterized. The procedure includes experimental investigations to identify the footbridge dynamic behaviour and characterize human-induced actions, calibration of structural numerical models and adoption of suitable mathematical models to simulate pedestrian loads. The performance of the different pedestrian load models are assessed comparing numerical and experimental accelerations. The accuracy of the predicted structural response depends on both the adoption of adequate pedestrian load models and the capability of representing the structural dynamic behaviour. Experimental campaigns have been performed on the Pasternak and Correggio footbridge, to characterize their dynamic behaviour. Thanks to the acquired accelerations, several natural modes of the two footbridges are clearly identified by means of the classic Enhanced Frequency Domain Decomposition method.

Among the various simulation methods for the evaluation of the structural response to pedestrian excitation, some of them require the adoption of FE models, while others rely only on the identified dynamic properties. In the first case, model updating procedures are to be employed in order to achieve a reliable description of the dynamic behaviour.

During the experimental campaigns, several dynamic tests with different-sized groups of pedestrians moving across the footbridge at different step frequencies were performed to investigate the dynamic response of the structure to pedestrian loads. Then, the footbridge response to vertical pedestrian load is simulated adopting two mathematical models in the time domain. The first simulation method, based on a periodic load model, is adopted to perform dynamic analyses with both the updated and non-calibrated FE models. In the second approach, the equation of motion is solved via modal decomposition, considering experimental modal parameters and the multi-harmonic force model. Simulation results display the strong dependency of the footbridge acceleration level on the step frequency of the pedestrian. Accelerations obtained through the mathematical approaches and the experimental tests are generally in very good agreements and both simulation models well represent the dynamic behaviour of the footbridge. As regards the Simulation model II, good agreement between the simulated and measured response can be noted, mainly due to the direct use of experimental natural frequencies and mode shapes. Moreover, the possibility to take into account the uncertainties in the step frequencies and the walking

force (i.e. the inter- and intra-subject variability during walking) is appreciable. As for the Simulation model I, the good agreement between the numerical and experimental accelerations is also related to the adoption of an accurately calibrated FE model.

Furthermore, a spectral model for vertical pedestrian-induced loading proposed in literature is thoroughly analysed and developed. Being a spectral model, it allows a simple description of the stochastic load and structural response with no need of time analysis and it accounts for the fact that the pedestrian force is a narrow band process. In addition, the model can represent the pedestrian-induced loading in both unrestricted and crowded pedestrian traffic conditions, but in absence of HSI, thanks to the definition of a physically-based function describing the correlation among pedestrians with increasing pedestrian density. However, the presence of a crowd of pedestrians can change the dynamic properties of the coupled system composed of the structure and the crowd. Hence, in the present study, HSI-effects on the structural response based on the spectral load model are evaluated. The HSI-effects are accounted for by characterizing the dynamic behaviour of the coupled crowd-structure system.

First, a parametric study is performed to evaluate the maximum structural response considering different pedestrian and footbridge parameters. In particular, footbridges with natural frequency in the typical range of pedestrian dynamic actions (from 1 to 5 Hz) are considered. Moreover, simulations in sparse ($\rho < 1$ ped/m²) and dense ($\rho > 1$ ped/m²) crowd conditions are performed. In the model proposed by Ferrarotti and Tubino [83], the contribution of the first harmonic of the walking force and a constant DLF are considered. On the contrary, the present study in addition accounts for the contribution of the resonant harmonic as well as a more realistic definition of the DLF depending on the step frequency. Especially for low natural frequencies, the near-resonance with the step frequency is caused by high pedestrian densities. When the pedestrian density is high, the mean velocity and step frequency are low. Thus, to consider that the DLF decreases for decreasing step frequency allows to avoid overestimation of the structural response. In addition, when the natural frequency of the footbridge is far from the resonance with the fundamental frequency of the pedestrian action, the contribution of the higher harmonics becomes the principal one.

Finally, the maximum structural response in the presence of HSI is evaluated. Results show that effects of the HSI are highly dependent on the natural frequency of the footbridge. In fact, for footbridges characterized by frequency ratios f_{H1}/f_B within the range of maximum interaction, the increase in the effective damping ratio of the coupled system is such that the maximum response is drastically reduced. On the contrary, for the footbridge with natural frequency of 2 Hz, there is a range of pedestrian density in which the maximum acceleration obtained with the HSI-effects is higher than that of the empty footbridge. This is due to the effect of the change in the frequency of the coupled system which is, in that range, more significant than the effect of the increasing damping ratio.

When compared to design checking methods (based on the resonance hypothesis), to account for HSI leads to a general reduction of the structural response. In the present study, a more realistic situation in which the pedestrian action may or may not be resonant with the structure depending on the pedestrian density is considered. In this case, conditions in which the HSI causes an increase in the structural response with respect to that of the empty footbridge can be observed.

Appendix A

Random vibration of structures

Structural engineers often deal with the analysis and design of structures for dynamic load, such as wind forces on tall buildings and bridges, strong earthquake motion effects on buildings [119], including human-induced loading on footbridges. In the dynamic analysis as well as in the design, the first step is to determine the dynamic load. The dynamic load can be represented as a definite function of time characterized by controlling parameters such as amplitudes, frequency, period and phase. Once the dynamic load or input excitation is defined, then the dynamic response of the structure can be determined. The complicated issue in the structural analysis and design for dynamic loads lies in the difficulty to determining the design dynamic loads accurately. It is indeed characterized by a large uncertainty in peak values, vibration characteristics (frequency contents and phase angles) and time duration. The uncertainties of the dynamic load must be built into a new class of indefinite or random time functions, known as random process. For a random process, the characteristics are no longer definite but are in statistical and probabilistic terms. Treating the input dynamic excitations as random time functions, leads to structural responses that are also random time functions [119]. The study of random vibrations of structures relies on the characterization of input random excitation and the determination of random structural response. The characteristic features of the random process are introduced in section A.1, while the response to random excitation is presented in section A.2.

A.1 Statistics of random process

For a quantitative description of random excitations, a large number of random time functions has to be defined. The ensemble of the sample functions is the so called random process and each sample function represents a possible realization of the process. A random variable X can be completely characterized through its probability density function $p(x)$ (see [119]). However, it is neither easy or necessary to have such a complete characterization of the random variable. Rather, a number of gross descriptors known as statistics are sufficient. For a random variable X , the important statistics are the mean, the mean square, the variance and the standard deviation, together with the autocorrelation and the power spectral density [119].

A.1.1 Stationary random processes

A random process is stationary when its statistics are stationary with time. The statistics of the stationary random process as well as the autocorrelation and the power spectral density are defined in the following.

Mean

$$E(X) = \int_{-\infty}^{+\infty} xp(x)dx$$

Mean square

$$E(X^2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^2 p(x) dx_1 dx_2$$

Variance Mean square minus the square of the mean

$$\sigma_x^2 = E[X^2] - [E(X)]^2$$

Thus, for a zero mean process

$$\sigma_x^2 = E[X^2]$$

Standard deviation Measure of the spread of the random variable about the mean

$$\sigma_x = \sqrt{\sigma_x^2}$$

Autocorrelation function

$$R_x(\tau) = E[X(t)X(t + \tau)]$$

For $\tau = 0$

$$R_x(0) = E[X(t)^2]$$

Power spectral density

$$G_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_x(\tau) e^{-i\omega\tau} d\tau$$

Then

$$R_x(\tau) = \int_{-\infty}^{+\infty} G_x(\omega) e^{i\omega\tau} d\omega$$

A.1.2 Ergodic processes

As it is not always possible to define an infinitely large number of sample functions, the probabilistic and statistical properties of a random process should be approximately estimated from the available single sample function. For certain stationary random process,

the statistics extracted from a single but long sample of time, called temporal statistics, approximate the ensemble statistics quite well. These kinds of stationary random processes are called ergodic processes. The temporal statistics are the following.

Temporal mean

$$\langle x(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt$$

Temporal mean square

$$\langle x(t)^2 \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)^2 dt$$

Temporal autocorrelation

$$\Phi_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau) dt$$

Temporal spectral density

$$G_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi_x(\tau) e^{-i\omega\tau} d\tau$$

Then

$$\Phi_x(\tau) = \int_{-\infty}^{+\infty} G_x(\omega) e^{i\omega\tau} d\omega$$

When $\tau = 0$

$$\Phi_x(0) = \langle x(t)^2 \rangle = \int_{-\infty}^{+\infty} G_x(\omega) d\omega \quad (\text{A.1})$$

A.2 Random excitation and response

Once the random excitation is characterized via the double-sided spectral density function $G_x(\omega)$, then the relation between the input excitation and the response of a linear vibrating system with SDOF is to be analysed. The output response of a linear system excited by a random excitation can be in general a non-stationary random process. In the following, the case of a stationary random response is considered. Dealing with zero mean processes, the variance of the stationary random response can be evaluated as [80, 41, 119]:

$$\sigma_y^2 = E[y(t)^2] = R_y(0) = \int_{-\infty}^{+\infty} G_y(\omega) d\omega$$

where the power spectral density function of the response $G_y(\omega)$ is obtained starting from the PSD of the excitation $G_x(\omega)$ and the displacement transfer function of the vibrating system $H(\omega)$:

$$G_y(\omega) = |H(\omega)|^2 G_x(\omega)$$

Furthermore, it can be demonstrated [80] that the spectral density of the derived process is ω^2 times the spectral density of the original process:

$$G_{\dot{y}}(\omega) = \omega^2 G_y(\omega)$$

$$G_{\ddot{y}}(\omega) = \omega^2 G_{\dot{y}}(\omega) = \omega^4 G_y(\omega)$$

Hence:

$$\sigma_y^2 = E[y(t)^2] = \int_{-\infty}^{+\infty} |H(\omega)|^2 G_x(\omega) d\omega$$

$$\sigma_{\dot{y}}^2 = E[\dot{y}(t)^2] = \int_{-\infty}^{+\infty} \omega^2 |H(\omega)|^2 G_x(\omega) d\omega$$

$$\sigma_{\ddot{y}}^2 = E[\ddot{y}(t)^2] = \int_{-\infty}^{+\infty} \omega^4 |H(\omega)|^2 G_x(\omega) d\omega$$

If the single-sided PSD $S_x(\omega)$ is considered instead of the double-sided $G_x(\omega)$, the previous relations can be rewritten as follows:

$$\begin{aligned} \sigma_y^2 &= E[y(t)^2] = \int_0^{+\infty} |H(\omega)|^2 S_x(\omega) d\omega \\ \sigma_{\dot{y}}^2 &= E[\dot{y}(t)^2] = \int_0^{+\infty} \omega^2 |H(\omega)|^2 S_x(\omega) d\omega \\ \sigma_{\ddot{y}}^2 &= E[\ddot{y}(t)^2] = \int_0^{+\infty} \omega^4 |H(\omega)|^2 S_x(\omega) d\omega \end{aligned} \quad (\text{A.2})$$

Starting from its definition, the variance of the response of a SDOF vibrating system is analysed considering different kinds of input $F(t)$: band limited white noise (Case 1), ideal white noise (Case 2) and non-white noise (Case 3) [41]. Considering the SDOF vibrating system with mass m , spring constant k , damping coefficient c and forcing function $F(t)$, the equation of motion in terms of the displacement Y , velocity $\dot{Y}$ and acceleration $\ddot{Y}$ may be given in the form:

$$\ddot{Y} + 2\xi\omega_0\dot{Y} + \omega_0^2 Y = X(t)$$

where the natural frequency ω_0 , damping ratio ξ , critical damping coefficient c_c and the excitation $X(t)$ are given by

$$\omega_0 = \frac{k}{m}, \quad \xi = \frac{c}{c_c}, \quad c_c = 2\omega_0 m, \quad X(t) = \frac{F(t)}{m}$$

A.2.1 Case 1: $F(t)$ band limited white noise

Assuming that $F(t)$ is a band-limited white noise, the spectral density of the input is given by

$$G_x(\omega) = \begin{cases} G_0, & |\omega| \leq \omega_c \\ 0, & \text{otherwise} \end{cases}$$

Hence, the spectral density of the displacement is found as

$$G_y(\omega) = \begin{cases} \frac{1}{m^2} \frac{G_0}{|\omega_0^2 - \omega^2 + 2\xi i \omega \omega_0|^2} = \frac{1}{m^2} \frac{G_0}{(\omega_0^2 - \omega^2)^2 + 4\xi^2 \omega^2 \omega_0^2}, & |\omega| \leq \omega_c \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.3})$$

Starting from Eq.(A.3), the mean square of displacement, velocity and acceleration can be obtained.

Mean square of the displacement

$$\sigma_y^2 = \frac{G_0}{m^2} \int_{-\omega_c}^{+\omega_c} \frac{d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2 \omega^2 \omega_0^2}$$

According to the solution proposed in [41]

$$\sigma_y^2 = \frac{G_0 \pi}{2\xi \omega_0^3 m^2} I_0\left(\frac{\omega_c}{\omega_0}, \xi\right)$$

where

$$I_0\left(\frac{\omega_c}{\omega_0}, \xi\right) = \frac{\xi}{2\pi\sqrt{1-\xi^2}} \ln \frac{(\omega_c/\omega_0 + \sqrt{1-\xi^2})^2 + \xi^2}{(\omega_c/\omega_0 - \sqrt{1-\xi^2})^2 + \xi^2} + \frac{1}{\pi} \left[\tan^{-1} \left(\frac{\omega_c/\omega_0 - \sqrt{1-\xi^2}}{\xi} \right) + \tan^{-1} \left(\frac{\omega_c/\omega_0 + \sqrt{1-\xi^2}}{\xi} \right) \right]$$

Mean square of the velocity

$$\sigma_{\dot{y}}^2 = \frac{G_0}{m^2} \int_{-\omega_c}^{+\omega_c} \frac{\omega^2 d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2 \omega^2 \omega_0^2}$$

According to the solution proposed in [41]

$$\sigma_{\dot{y}}^2 = \frac{G_0 \pi}{2\xi \omega_0^3 m^2} I_1\left(\frac{\omega_c}{\omega_0}, \xi\right)$$

where

$$I_1\left(\frac{\omega_c}{\omega_0}, \xi\right) = \frac{\xi}{2\pi\sqrt{1-\xi^2}} \ln \frac{(\omega_c/\omega_0 + \sqrt{1-\xi^2})^2 + \xi^2}{(\omega_c/\omega_0 - \sqrt{1-\xi^2})^2 + \xi^2} + \frac{1}{\pi} \left[\tan^{-1} \left(\frac{\omega_c/\omega_0 - \sqrt{1-\xi^2}}{\xi} \right) + \tan^{-1} \left(\frac{\omega_c/\omega_0 + \sqrt{1-\xi^2}}{\xi} \right) \right]$$

Mean square of the acceleration

$$\sigma_{\ddot{y}}^2 = \frac{G_0}{m^2} \int_{-\omega_c}^{+\omega_c} \frac{\omega^4 d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2 \omega^2 \omega_0^2}$$

According to the solution proposed in [41]

$$\sigma_{\ddot{y}}^2 = \frac{2G_0 \omega_c}{m^2} \left\{ 1 + \frac{1-4\xi^2}{8\sqrt{1-\xi^2}} \left(\frac{\omega_c}{\omega_0} \right) \ln \frac{(\omega_c/\omega_0 - \sqrt{1-\xi^2})^2 + \xi^2}{(\omega_c/\omega_0 + \sqrt{1-\xi^2})^2 + \xi^2} + \frac{1-4\xi^2}{4\xi} \left(\frac{\omega_c}{\omega_0} \right) \left[\tan^{-1} \left(\frac{\omega_c/\omega_0 - \sqrt{1-\xi^2}}{\xi} \right) + \tan^{-1} \left(\frac{\omega_c/\omega_0 + \sqrt{1-\xi^2}}{\xi} \right) \right] \right\}$$

A.2.2 Case 2: $F(t)$ ideal white noise

If the input is an ideal white noise, the mean squares of the random process can be obtained from the results of Case 1 when $\omega_c \rightarrow \infty$ [41]

Mean square of the displacement

$$\sigma_{y-case2}^2 = \lim_{\omega_c \rightarrow \infty} \sigma_{y-case1}^2 = \frac{G_0 \pi}{2\xi \omega_0^3 m^2} \quad (\text{A.4})$$

as

$$\lim_{A \rightarrow \infty} \ln(A) = 0$$

$$\lim_{A \rightarrow \infty} \tan^{-1}(A) = \frac{\pi}{2}$$

$$\lim_{\omega_c \rightarrow \infty} = I_0\left(\frac{\omega_c}{\omega_0}, \xi\right) = 1$$

Mean square of the velocity

$$\sigma_{\dot{y}-case2}^2 = \lim_{\omega_c \rightarrow \infty} \sigma_{\dot{y}-case1}^2 = \frac{G_0 \pi}{2\xi \omega_0 m^2} \quad (\text{A.5})$$

as

$$\lim_{A \rightarrow \infty} \ln(A) = 0$$

$$\lim_{A \rightarrow \infty} \tan^{-1}(A) = \frac{\pi}{2}$$

$$\lim_{\omega_c \rightarrow \infty} = I_1\left(\frac{\omega_c}{\omega_0}, \xi\right) = 1$$

Mean square of the acceleration

By comparing Eqs. (A.4) and (A.5), it is obtained that the mean square of the velocity is ω_0^2 times higher than the mean square of the displacement:

$$\sigma_{\dot{y}-case2}^2 = \omega_0^2 \sigma_{y-case2}^2$$

Extending this result to a general relation between the mean square of the derived process and the original ones, the mean square of the acceleration can be expressed as

$$\sigma_{\ddot{y}-case2}^2 = \omega_0^2 \sigma_{\dot{y}-case2}^2 = \frac{G_0 \pi \omega_0}{2\xi m^2}$$

A.2.3 Case 3: $F(t)$ coloured noise

For a lightly-damped system, the term

$$\frac{1}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2}$$

exhibits very sharp peaks in the vicinity of $\pm\omega_0$ so that the main contribution to the mean square derives from the frequencies close to $\pm\omega_0$ [41].

Mean square of the displacement

$$\sigma_y^2 = \frac{1}{m^2} \int_{-\infty}^{+\infty} \frac{G_F(\omega) d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2}$$

The dominant contribution to the integral comes from the values of the integrand function close to those of ω for which the integrand is maximized. These values can be the minimum points $\omega = \pm\omega_0$ of the frequency function in the denominator if $G_F(\omega)$ is smooth in the neighbourhood of $\pm\omega_0$ [41].

$$\sigma_y^2 \simeq \frac{1}{m^2} \int_{-\omega_0-\epsilon}^{-\omega_0+\epsilon} \frac{G_F(\omega) d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2} + \frac{1}{m^2} \int_{\omega_0-\epsilon}^{\omega_0+\epsilon} \frac{G_F(\omega) d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2}$$

then, since $G_F(\omega)$ is an even function (i.e. $G_F(\omega_0) = G_F(-\omega_0)$), it becomes

$$\begin{aligned} \sigma_y^2 &\simeq \frac{G_F(\omega_0)}{m^2} \left[\int_{-\omega_0-\epsilon}^{-\omega_0+\epsilon} \frac{d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2} + \int_{\omega_0-\epsilon}^{\omega_0+\epsilon} \frac{d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2} \right] = \\ &\quad \frac{G_F(\omega_0)}{m^2} \int_{-\infty}^{+\infty} \frac{d\omega}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega^2\omega_0^2} = \frac{G_F(\omega_0)\pi}{2\xi\omega_0^3 m^2} \quad (\text{A.6}) \end{aligned}$$

The mean square of the velocity and acceleration is then obtained considering the relation between the mean square of the derived process and the original ones shown in Case 2.

Mean square of the velocity

$$\sigma_{\dot{y}}^2 = \omega_0^2 \sigma_y^2 = \frac{G_F(\omega_0)\pi}{2\xi\omega_0 m^2} \quad (\text{A.7})$$

Mean square of the acceleration

$$\sigma_{\ddot{y}}^2 = \omega_0^2 \sigma_{\dot{y}}^2 = \frac{G_F(\omega_0)\pi\omega_0}{2\xi m^2} \quad (\text{A.8})$$

Finally, considering the single-sided PSD $S_F(\omega)$ instead of the double-sided $G_F(\omega)$, the mean square of the acceleration assumes the expression:

$$\sigma_{\ddot{y}}^2 = \frac{S_F(\omega_0)\pi\omega_0}{4\xi m^2} \quad (\text{A.9})$$

being $S_F(\omega) = 2G_F(\omega)$ the relation between the single- and double-sided PSD function.

Appendix B

Statistics of narrow band processes

Appendix A shows the general relations between the input and the output of a linear vibrating SDOF system subjected to stationary random excitation. The characteristics of the excitation are modified by the system which acts as a filter. In most vibration problems, a SDOF system has one resonant frequency at which large amplitude are generated by small input, while at the other frequencies the transmission is reduced. For this reason, the output spectrum is confined to a narrow band of frequencies close to the resonant frequency. Thus, the response $y(t)$ is a narrow band random process and its typical time history resembles a sine wave of varying amplitude and phase. If the excitation is a Gaussian process, the probability distribution for the response $y(t)$ can be obtained from its statistics (mean, mean square, variance..). Moreover, it is also possible to obtain important information about the probability distribution of peak values, that is to say information about the amplitude of the sine wave that makes up the process [80]. In the following sections, some formulations presented in literature for the distribution of maxima of a random function are summarized.

B.1 Distribution of the largest value of a random function

Davenport [36] proposes the following procedure for the characterization of the distribution of maxima of a random process. A stationary Gaussian random process $Y(t)$ having a zero mean value and single-sided spectral density function $S(\omega)$ is considered. Indicating with $\bar{y}$ and σ_y the mean and the standard deviation of the random function $y(t)$ and defining the reduced variate $\eta = (y - \bar{y})/\sigma_y$, the probability distribution of the function $y(t)$ is:

$$P(\eta) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\eta^2\right) \quad (\text{B.1})$$

The cumulative probability distribution $R(\eta)$ is defined as the probability of the function $y(t)$ exceeding the value η :

$$R(\eta) = \int_{\eta}^{\infty} P(\eta) d\eta \quad (\text{B.2})$$

Indicated with $p(\eta)$ the probability distribution of the maxima of the random function $y(t)$, the cumulative probability distribution $r(\eta)$ of the maxima of the function is expressed as [36]:

$$r(\eta) = \int_{\eta}^{\infty} p(\eta) d\eta \approx (1 - c^2)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\eta^2\right) \quad (\text{B.3})$$

where:

$$c = \sqrt{1 - \frac{\lambda_2^2}{\lambda_0\lambda_4}} \quad (\text{B.4})$$

and λ_i represents the moments of the process:

$$\lambda_n = \int_0^{\infty} \omega^n S(\omega) d\omega \quad (\text{B.5})$$

Concerning not with the distribution of all the maxima which occur within a certain period of time but only with the largest one, the probability density for the largest maximum can be defined. Considering a sample of N maxima, the probability $P_{max}(\eta)$ that the largest of them has the value η is the probability that one of the maxima has this value and the others are smaller:

$$P_{max} = N[1 - r(\eta)]^{N-1} p(\eta) \quad (\text{B.6})$$

Considering large values of N and introducing the term l , the cumulative probability distribution of the maxima $r(\eta)$ can be expressed:

$$r(\eta) = l/N, \quad 0 \leq l \leq N \quad (\text{B.7})$$

The number of maxima N during a period T , when N is large, is given by [36]:

$$N = \left(\frac{\lambda_4}{(2\pi)^2\lambda_2}\right)^{1/2} T = \frac{1}{2\pi} \left(\frac{\lambda_4}{\lambda_2}\right)^{1/2} T \quad (\text{B.8})$$

Hence:

$$\begin{aligned} l = Nr(\eta) &= \frac{1}{2\pi} \left(\frac{\lambda_4}{\lambda_2}\right)^{1/2} T (1 - c^2)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\eta^2\right) = \\ &= \frac{1}{2\pi} \left(\frac{\lambda_4}{\lambda_2}\right)^{1/2} T \left(1 - \left(1 - \frac{\lambda_2^2}{\lambda_0\lambda_4}\right)\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\eta^2\right) = \\ &= \frac{1}{2\pi} \left(\frac{\lambda_4}{\lambda_2}\right)^{1/2} T \left(\frac{\lambda_2^2}{\lambda_0\lambda_4}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\eta^2\right) = \\ &= \frac{1}{2\pi} T \left(\frac{\lambda_2}{\lambda_0}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\eta^2\right) = v_0 T \exp\left(-\frac{1}{2}\eta^2\right) \end{aligned} \quad (\text{B.9})$$

where the quantity:

$$v_0 = \frac{1}{2\pi} \left(\frac{\lambda_2}{\lambda_0}\right)^{1/2} \quad (\text{B.10})$$

has been introduced. Dealing with large values of N , the asymptotic form, in the limit, can be written:

$$P_{max}d\eta = d(1 - r(\eta))^{N-1} = d\left(1 - \frac{l}{N}\right)^{N-1} \quad (\text{B.11})$$

By definition of the Euler's number:

$$e = \lim_{N \rightarrow \infty} \left(1 + \frac{1}{N}\right)^N \quad (\text{B.12})$$

and:

$$e^x = \lim_{N \rightarrow \infty} \left(1 + \frac{x}{N}\right)^N \quad (\text{B.13})$$

Being N a large value:

$$\left(1 + \frac{(-l)}{N}\right)^{N-1} \simeq \left(1 + \frac{(-l)}{N}\right)^N \quad (\text{B.14})$$

Thus:

$$\lim_{N \rightarrow \infty} \left(1 + \frac{-l}{N}\right)^N = e^{(-l)} \quad (\text{B.15})$$

From Eq.(B.15), Eq.(B.11) becomes:

$$P_{max}d\eta = d\left(1 - \frac{l}{N}\right)^{N-1} = d \exp(-l) = d \exp(-Nr(\eta)) \quad (\text{B.16})$$

Substituting Eq.(B.9) into Eq.(B.16):

$$\begin{aligned} P_{max}(\eta)d\eta &= d \exp \left[-v_0 T \exp \left(-\frac{1}{2}\eta^2 \right) \right] = \\ &\left[-v_0 T \exp \left(-\frac{1}{2}\eta^2 \right) \right] \exp \left[-v_0 T \exp \left(-\frac{1}{2}\eta^2 \right) \right] (-\eta)d\eta = -l \exp(-l)(-\eta)d\eta = \\ &l\eta \exp(-l)d\eta \end{aligned} \quad (\text{B.17})$$

In addition:

$$P_{max}(\eta)d\eta = d[\exp(-l)] = -\exp(-l)dl \quad (\text{B.18})$$

Therefore, the mean of the distribution of the largest maximum can be obtained as:

$$\bar{\eta}_{max} = \int_{-\infty}^{\infty} \eta P_{max}(\eta)d\eta = \int_{-\infty}^{\infty} \eta \exp(-l)dl \quad (\text{B.19})$$

where the reduced variate η can be expressed, from Eq.(B.9), as:

$$\eta = \sqrt{2 \ln(v_0 T) - 2 \ln l} = \sqrt{2 \ln(v_0 T) \left(1 - \frac{\ln l}{\ln(v_0 T)}\right)} = \sqrt{2 \ln(v_0 T)} - \frac{\ln l}{\sqrt{2 \ln(v_0 T)}} - \frac{1}{2} \frac{\ln^2 l}{\sqrt{(2 \ln(v_0 T))^3}} + \dots \quad (\text{B.20})$$

Obtaining:

$$\bar{\eta}_{max} = \int_{-\infty}^{\infty} \left[\left(\sqrt{2 \ln(v_0 T)} - \frac{\ln l}{\sqrt{2 \ln(v_0 T)}} - \frac{1}{2} \frac{\ln^2 l}{\sqrt{(2 \ln(v_0 T))^3}} + \dots \right) \exp(-l) \right] dl \quad (\text{B.21})$$

Neglecting terms of order $(2 \ln(v_0 T))^{3/2}$, and using the standard integral:

$$\int_0^{\infty} \ln l \exp(-l) dl = -\gamma \quad (\text{B.22})$$

where $\gamma = -0.5772$ is the Euler's constant, the mean value becomes:

$$\begin{aligned} \bar{\eta}_{max} &= \int_{-\infty}^{\infty} \sqrt{2 \ln(v_0 T)} \exp(-l) dl - \int_{-\infty}^{\infty} \frac{\ln l \exp(-l) dl}{\sqrt{2 \ln(v_0 T)}} = \\ &= \sqrt{2 \ln(v_0 T)} \int_{-\infty}^{\infty} \exp(-l) dl + \frac{\gamma}{\sqrt{2 \ln(v_0 T)}} = \sqrt{2 \ln(v_0 T)} + \frac{\gamma}{\sqrt{2 \ln(v_0 T)}} \end{aligned} \quad (\text{B.23})$$

The quantity v_0 of Eq.(B.10) can be interpreted physically as the frequency at which most of the energy in the spectrum is concentrated that is, for lightly-damped systems, generally close to the natural frequency of the system.

Eq.(B.23) defines the mean value of the probability distribution of the largest maximum normalized with respect to the standard deviation of the process, hence, for a stationary random process with standard deviation σ_y the mean value of the largest maximum is obtained as:

$$\bar{y}_{max} = \bar{\eta}_{max} \sigma_y \quad (\text{B.24})$$

B.2 Properties of spectral moments with applications to random vibrations

In [106], Vanmarcke provides a comprehensive investigation on the distribution of maxima of a random function as well as an analytical formulation of that distribution. In the present section, some key parameters of the distribution are briefly introduced. Considering the stationary Gaussian random process $Y(t)$ having a zero mean value and single-sided spectral density function $S(\omega)$, the principal parameters of $S(\omega)$ can be introduced:

- $\lambda_0 = \int_0^{\infty} S(\omega) d\omega = \sigma_Y^2$, the area under $S(\omega)$;

- $\omega_n = \left(\frac{\lambda_n}{\lambda_0}\right)^{1/i}$;
- $q = \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}\right)^{1/2} = \frac{(\omega_2^2 - \omega_1^2)^{1/2}}{\omega_2}$, the spectral bandwidth parameter, which is a measure of the variability of the frequency content.

Spectral bandwidth parameter The spectral parameter is related to the definition and properties of the envelope $R(t)$ of the random process $Y(t)$. It can be demonstrated that [106]:

$$q = \frac{\sigma_{\dot{R}}}{\sigma_{\dot{Y}}} \quad (\text{B.25})$$

where $\sigma_{\dot{Y}}$ is the RMS of the slope of the process and $\sigma_{\dot{R}}$ is the RMS of the slope of its envelope. q is the unitless measure of the spectral bandwidth, that is the spread of frequencies about their central value. It is always between 0 and 1, being relatively small for narrow band processes and relatively large for wide band processes. When the damping $\xi \rightarrow 0$, q can be approximated as:

$$q \approx 2\left(\frac{\xi}{\pi}\right)^{1/2} \quad (\text{B.26})$$

Average threshold crossing rates for Gaussian processes

v_a = mean rate of up-crossings of a level a by the stationary random Gaussian process $Y(t)$.

$$v_a = \frac{1}{2\pi} \left(\frac{\lambda_2}{\lambda_0}\right)^{1/2} \exp\left(-\frac{a^2}{2\lambda_0}\right) \quad (\text{B.27})$$

When the threshold level is equal to 0, the mean rate of up-crossings represents the average frequency:

$$v_0 = \frac{1}{2\pi} \left(\frac{\lambda_2}{\lambda_0}\right)^{1/2} = \frac{\omega_2}{2\pi} \quad (\text{B.28})$$

B.3 Structural response to stationary excitation

Finally, Der Kiureghian [37] compared the proposals of Davenport [36] and Vanmarcke [106] for the distribution of maxima. In particular, the mean value of the cumulative distribution of Y_τ (maximum absolute value of the process over the duration τ) can be numerically evaluated by using the cumulative distribution proposed by Vanmarcke or with the asymptotic expression proposed by Davenport, Eq.(B.23), both normalized with respect to σ_Y . Davenport's expression is based on the Poisson model of crossings while in deriving his formulation Vanmarcke has included the dependence between barrier crossings through the spectral bandwidth parameter q . The bandwidth parameter is inversely proportional to

the number (clump size) of barrier crossings of the process which immediately follows each barrier crossing by its envelope. For narrow band processes, q is small, indicating a large clump size and therefore a significant dependence between crossings. Results of the comparison show that the distribution proposed by Davenport generally tends to overestimate the mean value of the distribution of maximum obtained from Vanmarcke. For this reason, if the Poisson model of crossings is to be used, a reduced crossing rate v_e , representing an equivalent rate of statistically independent crossings, would be appropriate. An empirical relation for v_e in terms of v_0 and q is proposed in [37]:

$$v_e = (1.63q^{0.45} - 0.38)v_0 \quad (\text{B.29})$$

In summary, from the expression proposed by Davenport (Eq.(B.23)), the reduced crossing rate (Eq.(B.29)) and considering the relation in Eq.(B.24), the mean value of the maximum of $y(t)$ is found as:

$$\bar{y}_{max} = \left(\sqrt{2 \ln(v_e T)} + \frac{0.5772}{\sqrt{2 \ln(v_e T)}} \right) \sigma_y \quad (\text{B.30})$$

Eq. (B.30) is applicable in the ranges $0.1 \leq q \leq 1$ and $5 \leq (v_0 T) \leq 1000$, as Eq.(B.29) are calibrated from simulations in those ranges [37].

Finally, considering bilateral thresholds, the mean rate of up-crossings is twice the value for single barrier (i.e. a single threshold up-crossing). Thus, the mean value of the maximum assumes the following expressions [83]:

$$\bar{y}_{max} = \left(\sqrt{2 \ln(2v_e T)} + \frac{0.5772}{\sqrt{2 \ln(2v_e T)}} \right) \sigma_y \quad (\text{B.31})$$

The term in brackets can be considered as a peak coefficient or a crest factor, representing how much the standard deviation of the response (representative of a mean response) must be incremented to obtain the peak value:

$$\bar{y}_{max} = g_{y_{max}} \sigma_y \quad (\text{B.32})$$

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