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Community enforcement and the cost of cooperation

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Abstract

This paper studies how the cost of cooperation affects the effectiveness of community enforcement in a society of strangers, as in Kandori (Rev Econ Stud 59:63–80, 1992) and Ellison (Rev Econ Stud 61:567–588, 1994). We identify a critical threshold for the cost of cooperation, beyond which community punishment becomes a credible sanction to defectors, regardless of the population size and discount factor. Furthermore, we demonstrate that for any population size and discount factor, there exists a target cost of cooperation that makes community enforcement effective in supporting cooperation. This target cost can be used as a coordination device to sustain cooperation as an equilibrium outcome when the cost of cooperation is endogenous.

Keywords Community enforcement · Cost of cooperation · Random matching

JEL Classification C73 · C78 · D70

1 Introduction

In societies where access to information about individuals' past behavior is limited, community-sanctioning norms can help deter uncooperative actions. One way to establish this deterrence is by threatening to withdraw future cooperation in response to non-cooperative conduct. This mechanism has been extensively studied in the social sciences and applied in practical contexts.[1] In small communities with readily available stable partnerships, cooperative behavior can be enforced through direct sanctioning

[1] See for example Ostrom (1990), North (1990), and Greif (1994).

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(Rubinstein 1979; Axelrod 1981; Fudenberg and Maskin 1986). In large communities where individuals are strangers and stable partnerships are less likely to form, community-wide sanctioning schemes may become necessary to deter misbehavior (Kandori 1992; Ellison 1994).

The infinitely repeated Prisoner's Dilemma with anonymous random matching provides a natural framework for investigating cooperation, spurred by enforcement mechanisms, within groups of homogeneous strangers. In such an environment, cooperation can be supported by norms of community enforcement where a permanent punishment spreads within a group of anonymous individuals as a process of contagion ("contagious equilibrium"). The threat of a never-ending community punishment can sustain cooperative behavior if defections result in significant losses to cooperators and individuals are sufficiently patient (Kandori 1992). A more general result, which only depends on the value of discounting and the population size, emerges when single punishment periods are sporadically distributed among sufficiently long intervals of cooperation (Ellison 1994). The seminal papers of Kandori (1992) and Ellison (1994) have made it clear that contagious equilibrium hinges upon a delicate balance between two opposing forces, the magnitude of which depends on the discounting rate. If the discount factor is relatively low, individuals may give in to the temptation to behave opportunistically, a dilemma referred to as the "first-order social dilemma" (FOSD). On the other hand, if the discount factor is high, individuals may be reluctant to adhere to punishment schemes that ultimately undermine cooperative efforts, a dilemma referred to as the "second-order social dilemma" (SOSD).

In this paper, we address these two social dilemmas, but we take a different approach. We study how the cost of cooperation affects the effectiveness of community enforcement and, consequently, the existence of cooperative equilibrium, independent of the value of the discount factor and the size of the population. This aim leads us to shift the focus from the discount factor, which is a subjective preference parameter and, thus, an external obstacle to cooperation, to the cost of cooperation, which typically embodies environmental constraints that might be adjustable or even deliberately chosen.[2] This approach allows us to establish general results on the FOSD and the SOSD, and to extend the analysis of contagious equilibria to an economic environment where the cost and the benefit of cooperation are endogenous (Sect. 6).

Our model employs a canonical matching framework where individuals randomly and anonymously interact in pairs through a Prisoner's Dilemma and can rely on a social norm based on "grim" punishment, as in Kandori (1992) and Ellison (1994). In such an environment, we investigate how the relative cost of cooperation—i.e., the cost-to-benefit ratio—affects individuals' willingness to cooperate.[3] In doing so, we build upon the technique established by Camera and Gioffré (2014) and identify a class of functions capable of establishing conditions for cooperative equilibrium as the cost of cooperation varies. When the cost of cooperation is endogenous, one specific function within this class can be used as a private signal to select a common strategy for supporting cooperation.

[2] For example, this occurs when the cost of cooperation is associated with productivity or effort costs. In Sect. 6, we present an extended version of our model, where the cost of cooperation is a choice variable.

[3] We refer to the relative cost of cooperation as the ratio between the cost incurred in providing a benefit to another individual and the benefit received from this individual.

Our analysis yields two new results. The first result regards the SOSD. We offer a clear-cut solution for the SOSD independent of the discount factor and, more surprisingly, the population size. We establish that if the cost of cooperation exceeds a particular threshold value, engaging in grim punishment schemes becomes the best response to non-cooperative behavior. Importantly, we demonstrate that the SOSD manifests only within a narrow range of interactions, i.e., those where the relative cost of cooperation falls below a threshold that never exceeds one half. The second result regards both the SOSD and the FOSD. We demonstrate that for any population size and discount factor, a target value of the cost of cooperation exists that resolves both the first- and the second-order social dilemma. In interactions where the cost of cooperation approaches this target value, community enforcement becomes effective in fostering cooperation. This finding proves particularly valuable in scenarios where cooperation costs are endogenously determined. In such instances, the target cost of cooperation can act as a tool for coordination, effectively converting the infinitely repeated Prisoner's Dilemma from a social dilemma into a coordination problem. We present an extension of the baseline model to illustrate how to achieve coordination on cooperation when the cost of cooperation is a choice variable that is privately observed.

Overall, our results provide new insights into the behavior of anonymous individuals in cooperative settings, particularly regarding the inverse relationship between the propensity and the cost of cooperation. Our perspective complements the conventional view in the indirect reciprocity theory, which posits that high costs of cooperation undermine cooperative behavior (Nowak 2006). We find that a proper increase in cooperation costs can bolster cooperation, while lenient behaviors are more likely to prevail when cooperation costs are low. From a methodological standpoint, our analysis offers a simple framework for establishing cooperation costs that effectively support cooperative equilibrium.[4]

The literature on repeated games with random matching is voluminous and spans several categories based on the extent of information transmission, an extensive review of which is beyond the scope of this paper.[5] Our paper lies in the category of private monitoring. Within this strand of literature, our work is also close to Harrington (1995), who studies cooperation in a population where agents hold private information on the counterparts' identity met in the past, and to Okuno-Fujiwara and Postlewaite (1995), who introduce the concept of norm equilibrium in random matching games that are characterized by incomplete information. Our model is also related to Ghosh and Ray (1996), who study cooperation in a model of heterogeneous agents without information flows. More recent contributions include Takahashi (2010), who applies belief-free equilibria to repeated games with random matching where players can observe their opponent's past play, Annen (2011), who investigates sequential equilibrium supported by community enforcement norms when players can share past experience, Deb and Gonzalez-Diaz (2019), who show that cooperation can be generally sustained in random matching games under community enforcement and trust-building mechanisms, and Sugaya and Wolitzky (2021), who show that communication is essential

4 Our methodology holds potential applicability in designing laboratory experiments to induct cooperation among anonymous individuals, through community enforcement norms.

5 For a more general survey of the literature on repeated games with random matching we refer the reader to Wolitzky (2022) for theoretical predictions, and to Dal Bó and Fréchette (2018) for experimental results.

for community enforcement when players are types with a certain probability and can remember their partners identities. Unlike their settings, we remain confined to a model where agents can neither observe their opponents' past plays nor identify their identities.

To some degree, our study is also related to the literature on compensation mechanisms (Cho and Kreps 1987; Varian 1994; Charness et al. 2007). In Sect. 6, we consider an extended version of the model where the cost of cooperation is endogenous. We show that choosing an inefficient cooperation cost can help individuals coordinate on cooperation. In particular, players can use payoffs to signal their willingness to cooperate.[6]

Finally, there exists recent literature on indirect reciprocity based on an evolutionary approach See, for example, Nowak and Sigmund (1998), Nowak and Sigmund (2005), Ohtsuki and Iwasa (2004), and Sommerfeld et al. (2007). Ye et al. (2016) show that the SOSD may be resolved if the benefit of cooperation exhibits increasing returns to scale. Yet, the absence of reputation is a critical assumption in our model (large economy). It is worth noting that some studies have cast doubts on the role of reputation in the endurance of cooperation schemes. In particular, Suzuki and Kimura (2013) shows that introducing arbitrarily low costs for building a reputation makes cooperation impossible. Along this line, recent studies on indirect reciprocity have examined the impact of noisy reputation systems on communities (Hilbe et al. 2018).

The rest of the paper is organized as follows. Section 2 illustrates the baseline model. Section 3 provides an equilibrium analysis, which expands in Section 4 and 5 to establish our main results. Section 6 presents an application of the main results when the cost of cooperation is endogenous and information is incomplete. Concluding remarks are in Section 7.

2 The model

The economy is populated by $N = 2n \geq 4$ infinitely-lived players. Time is discrete and lasts forever. In each period $t = 0, 1, 2, \ldots$, players randomly interact in pairs, formed through a uniformly random matching process. In each pair, interactions take the form of the Prisoner's Dilemma in Table 1a. Players can cooperate, action C, or defect, action D. As standard, we assume that cooperating generates an individual cost, c, and provides a benefit, b, to the counterpart, with $b > c > 0$. In contrast, defecting results in no individual cost or benefit to the counterpart. To simplify the analysis, we normalize the payoff matrix presented in Table 1a by dividing each payoff by b, as shown in Table 1b. Here, the cost-to-benefit ratio $\gamma := \frac{c}{b} \in (0, 1)$ corresponds to the cost of cooperation.[7]

[6] A more general setting in which communication is carried out through actions that impact payoffs is described in Deb et al. (2020).

[7] The general results that follow in this paper apply to any payoff matrix of the Prisoner's Dilemma. In particular, a payoff matrix where the sucker's payoff does not mirror the cost of cooperation would not qualitatively alter our main findings, as explained in the remark at the end of Sect. 5. Therefore, working with the payoff matrix in Table 1b allows for simplifying the analysis by focusing on just one parameter (γ).

Table 1 Interaction between player i and j

(a) Payoff matrix

		Player j	
		C	D
Player i	C	$b-c,\ b-c$	$-c,\ b$
	D	$b, -c$	$0, 0$

(b) Normalized payoff matrix

		Player j	
		C	D
Player i	C	$1-\gamma,\ 1-\gamma$	$-\gamma,\ 1$
	D	$1,\ -\gamma$	$0,\ 0$

Players are anonymous and can only monitor their counterparts' actions privately. Anonymity prevents direct sanctioning, as players cannot track their counterparts' past actions. Private monitoring prevents the possibility of simultaneous, collective responses, as actions and payoffs are observable inside the meetings but not outside. A common discount factor $\delta \in (0, 1)$ to all players completes our economic environment.

We assume that players follow the grim strategy defined next. Since this strategy is adopted by all players we refer to it as a *social norm*.

Definition 1 (*Grim strategy*) In $t = 0$, the player cooperates. In $t > 0$, the player cooperates unless a defection is observed, in which case the player defects in all periods following the observed defection.

The grim strategy in Definition 1 identifies two states of behavior: *cooperation* and *punishment*. In the cooperation state, the player chooses action C in each period unless a defection is observed, in which case, the player switches to the punishment state. In the punishment state, the player chooses D in all subsequent periods. We refer to a player in the cooperation state as a "cooperator" and in the punishment state as a "defector."

It is well-established that the threat of a permanent punishment state ("always defect") allows sustaining cooperation in equilibrium if players are sufficiently patient (Kandori 1992). This classic result holds for general payoffs if players adopt a strategy where single punishment periods are scattered among sufficiently long intervals of cooperation (Ellison 1994). However, contagious equilibrium may collapse if the discount factor is not sufficiently high.The discount factor is a primitive preference parameter that may impede the standard grim strategy from sustaining cooperative equilibrium. If the discount factor is too low, players might find it optimal to defect and avoid the cost of cooperation while in the cooperation state. Conversely, if the discount factor is too high, players may see an advantage in slowing down the contagion process by choosing to cooperate instead of defecting during the punishment state. Incentives to punish are restored when the defections result in significant losses to cooperators (Kandori 1992) or when punishment is properly modulated over time (Ellison 1994). In those cases, community enforcement effectively sustains cooperation so long as players are sufficiently patient.

However, what happens if players are not sufficiently patient? We turn the problem around by focusing on the cost of cooperation, which, instead, is often determined by environmental factors that may be adequately addressed or even chosen. Shifting our focus allows us to understand better the frictions that generally hinder cooperation for any degree of patience.

3 Equilibrium analysis

This section outlines the methodology we use to derive our main results. We begin by establishing a closed-form solution of the relevant payoff functions required for our analysis: (*i*) the expected payoffs in the punishment state and (*ii*) the expected payoffs resulting from a one-shot deviation from the punishment state. Then, we present sufficient conditions for the cost of cooperation that ensure the existence of cooperative equilibrium.

Assume that players adopt the grim strategy in Definition 1. The expected payoff to a defector depends on the current and future number of defectors in the economy. Since monitoring is private and meetings are formed under a uniform probability distribution, cooperators become defectors via a contagious process of defection that is described by the following $N \times N$ Markov matrix:

$$\mathcal{Q}_N := \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & Q_{22} & 0 & Q_{24} & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{34} & 0 & Q_{36} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & Q_{N-2,N-2} & 0 & Q_{N-2,N} \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}.$$

The generic element $Q_{kk'} := Q_{kk'}(N)$ of matrix $\mathcal{Q}_N$ is the probability of transitioning from k to k' defectors across two periods. In other words, it represents the probability that, given k defectors today, the economy will end up with $k' \geq k$ defectors tomorrow due to the grim punishment scheme adopted by the players (Definition 1).[8]

3.1 Expected payoff in the punishment state

Let v_k denote the expected payoff to a defector when there are k defectors in the current period. We have

$$v_k = \sigma_k + \delta \sum_{k'=k}^{N} Q_{kk'}(N) v_{k'}, \quad \text{with } k = 1, \dots, N. \tag{1}$$

[8] Formally, $Q_{kk'}(N) := \frac{(k'-k)!\binom{k}{k'-k}\binom{N-k}{k'-k}(2k-k'-1)!!(N-k'-1)!!}{(N-1)!!}$, where the number $k' - k$ of additional defectors created in a period takes values in $\{0, 2, 4, \dots, \min(k, N-k)\}$ if k is even, and in $\{1, 3, 5, \dots, \min(k, N-k)\}$ if k is odd. See Camera and Gioffré (2014) for further details.

To explain the expression above, start by noticing that, in the current period, the defector meets a cooperator with probability $\sigma_k := \frac{N-k}{N-1}$, in which case she earns payoff 1, while she meets another defector with probability $1-\sigma_k$, in which case she earns payoff 0. The expected continuation payoff $\sum_{k'=k}^{N} Q_{kk'}(N)v_{k'}$ is then discounted by δ. This expected payoff depends on the probabilities $Q_{kk'}(N)$ of having k' defectors in the next period, with $k' = k, \ldots, N$.

Letting $\boldsymbol{v} := (v_1, \ldots, v_N)^\top$ and $\boldsymbol{\sigma} := (\sigma_1, \ldots, \sigma_N)^\top$ be the two column vectors containing all possible values of v_k and σ_k, respectively, as k varies from 1 to N, we can rewrite (1) in vector form as

$$\boldsymbol{v} = \boldsymbol{\sigma} + \delta \mathcal{Q}_N \boldsymbol{v},$$

which yields

$$\boldsymbol{v} = (\mathcal{I}_N - \delta \mathcal{Q}_N)^{-1}\boldsymbol{\sigma}.$$

Denoting by $\boldsymbol{e}_k^\top = (0, \ldots, 1, \ldots, 0)$ the row vector with 1 in the k^{th} entry and 0 everywhere else, each element v_k of $\boldsymbol{v}$ is

$$v_k = \frac{\phi_k(\delta)}{1-\delta}, \tag{2}$$

where $\phi_k(\delta) := \boldsymbol{e}_k^\top(1-\delta)(\mathcal{I}_N - \delta\mathcal{Q}_N)^{-1}\boldsymbol{\sigma} \in (0,1)$ can be interpreted as the expected rate of encounters between the defector and cooperators in the continuation game, given that there are k defectors in the current period. In other words, $\frac{\phi_k(\delta)}{1-\delta}$ is the number of cooperators that the defector expects to meet in the future. Importantly, $\phi_k(\delta)$ is a continuous, decreasing function of $\delta \in (0,1)$, and a decreasing function of k, with $\phi_k(\delta) \le \sigma_k$ (Camera and Gioffré 2014).

3.2 Expected payoffs from a deviation from the punishment state

Suppose that there are k defectors in the current period, and let $\tilde{v}_k$ denote the expected payoff to a defector, say player i, who takes a one-shot deviation from the punishing state, by cooperating in the current period and returning to defecting from the next period. The payoff $\tilde{v}_k$ is given by

$$\begin{aligned}\tilde{v}_k = \sigma_k &\left[1-\gamma+\delta\sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\, v_{k'+1}\right] \\ &+ (1-\sigma_k)\left[-\gamma+\delta\sum_{k'=k-2}^{N-2} Q_{k-2k'}(N-2) v_{k'+2}\right].\end{aligned} \tag{3}$$

To clarify expression (3), note that with probability σ_k, player i meets a cooperator. In this case, her current payoff is $1-\gamma$, while her continuation payoff $v_{k'+1}$ depends on the number $k'+1$ of defectors in the next period. Since we have fixed the meeting between player i and her counterpart, the number of defectors next period is given by the k' defectors exiting the meetings between the remaining $N-2$ players ($N-k-1$ cooperators and $k-1$ defectors)—which explains the argument $N-2$ (instead of N) in the transition probabilities—plus one additional defector, player i, who returns to the punishment state from next period—which explains the number 1 in the subscript of $v_{k'+1}$.[9] With probability $1-\sigma_k$, player i meets a defector. In this case, her current payoff is $-\gamma$, while next period there will be k' defectors exiting the meetings between the remaining $N-2$ players ($N-k$ cooperators and $k-2$ defectors), plus two additional defectors, player i and her counterpart, who is already in the punishment state—this explains the number 2 in the subscript of $v_{k'+2}$.

3.3 Cooperative equilibrium

Here we show that the cooperative equilibrium supported by the grim strategy in Definition 1 is confined to a narrow path of the space (δ, γ), which expands as γ and δ becomes large.

First, we need an auxiliary result. Letting

$$\Phi_k(\delta) := \frac{\delta}{1-\delta}[\phi_k(\delta) - \phi_{k+1}(\delta)], \qquad \text{with } k = 1, \ldots, N-1, \tag{4}$$

we have the following:

Lemma 1 *For $k = 1, \ldots, N-1$, $\Phi_k(\delta)$ takes values in $(0, 1)$, it is a continuous, increasing function of $\delta \in (0, 1)$, and a decreasing function of k.*

Proof We begin by characterizing $\Phi_k(\delta)$ to make the difference between $\phi_k(\delta)$ and $\phi_{k+1}(\delta)$ explicit. Suppose that in the current period, say period 0, there are $k = 1, \ldots, N-1$ defectors. Let $\mathcal{C}_k(t)$ denote the set of cooperators at the beginning of period $t \geq 0$ when there are k initial defectors in period 0. The complementary set $\overline{\mathcal{C}}_k(t)$ accounts for all defectors at the beginning of period t. Since defection is an absorbing state (Definition 1), $\mathcal{C}_k(t+1) \subseteq \mathcal{C}_k(t)$. Consider now a new scenario where there is an additional initial defector, player h. In this case, the initial defectors in period 0 are $k+1$. We want to track the set $\mathcal{D}^h(t)$ of defectors entering period t as a direct or indirect consequence of player h being an initial defector instead of an initial cooperator. In this new scenario, player $h \in \overline{\mathcal{C}}_{k+1}(0)$. In period 0, we have $\mathcal{D}^h(0) = \{h\}$, while in all other periods $t > 0$, we have $\mathcal{D}^h(t) = \mathcal{D}^h(t-1) \cup (\overline{\mathcal{C}}_{k+1}(t) \cap \mathcal{C}_k(t))$. The set $\mathcal{D}^h(t)$ contains all players that were in $\mathcal{D}^h(t-1)$ plus the new defectors generated at the end of period $t-1$ as a consequence of making h an initial defector. This set of *additional* defectors in period t is the intersection between the set of defectors at the beginning of period t when h is an initial defector, set $\overline{\mathcal{C}}_{k+1}(t)$, and the set of cooperators at the beginning of period t when h is an initial cooperator, set $\mathcal{C}_k(t)$. In other words, if player h were not a defector in period 0, then the players in $\mathcal{D}^h(t)$ would be cooperators in t. Since being a defector is an absorbing state, we have $\mathcal{D}^h(t) \subseteq \mathcal{D}^h(t+1)$. Consider then player $i \in \overline{\mathcal{C}}_k(0)$. For this initial defector, $\frac{1}{1-\delta}[\phi_k(\delta) - \phi_{k+1}(\delta)]$ is the expected

[9] If player i cooperates in the current period, her counterpart does not switch to the punishment state next period.

number of additional cooperators that player i would encounter (over her lifetime), if agent h were a cooperator instead of being a defector in period 0 (i.e., if we started with k instead of $k+1$ defectors). Therefore

$$\phi_k(\delta) - \phi_{k+1}(\delta) = (1-\delta)\sum_{t=0}^{\infty}\delta^t \sum_{k'=k}^{N}(\mathcal{Q}^t)_{kk'}\Pr[o_i(t)\in\mathcal{D}^h(t)|k'], \tag{5}$$

where $\Pr[o_i(t)\in\mathcal{D}^h(t)|k']$ is the probability that, t periods forward, player i meets someone in $\mathcal{D}^h(t)$, denoted by $o_i(t)$, conditioning on having k' defectors in that period.

Consider now the case in which, other than h, there is another additional defector in period 0, say player j. In this scenario, the set of defectors generated at the beginning of each period t, as a direct or indirect consequence of players h and j being initial defectors instead of being initial cooperators, is

$$\mathcal{D}^{h,j}(t) = \mathcal{D}^h(t)\cup\mathcal{D}^j(t),$$

where $\mathcal{D}^j(t)$ is equal to $\mathcal{D}^h(t)$ as we replace h with j. We have

$$\phi_k(\delta) - \phi_{k+2}(\delta) = (1-\delta)\sum_{t=0}^{\infty}\delta^t \sum_{k'=k}^{N}(\mathcal{Q}^t)_{kk'}\Pr[o_i(t)\in\mathcal{D}^{h,j}(t)|k']$$

where

$$\begin{aligned}\Pr[o_i(t)\in\mathcal{D}^{h,j}(t)|k'] &= \Pr[o_i(t)\in\mathcal{D}^h(t)|k'] + \Pr[o_i(t)\in\mathcal{D}^j(t)|k'] \\ &\quad - \Pr[o_i(t)\in\mathcal{D}^h(t)\cap\mathcal{D}^j(t)|k'].\end{aligned}$$

Notice that $\Pr[o_i(t)\in\mathcal{D}^h(t)|k'] = \Pr[o_i(t)\in\mathcal{D}^j(t)|k']$ since the random matching process is uniform. Therefore

$$\begin{aligned}\phi_{k+1}(\delta) - \phi_{k+2}(\delta) &= [\phi_k(\delta) - \phi_{k+2}(\delta)] - [\phi_k(\delta) - \phi_{k+1}(\delta)] \\ &= (1-\delta)\sum_{t=0}^{\infty}\delta^t \sum_{k'=k}^{N} Q^t_{kk'}\Pr[o_i(t)\in\mathcal{D}^h(t)|k'] \\ &\quad - (1-\delta)\sum_{t=0}^{\infty}\delta^t \sum_{k'=k}^{N} Q^t_{kk'}\Pr[o_i(t)\in\mathcal{D}^h(t)\cap\mathcal{D}^j(t)|k'] \\ &\leq \phi_k(\delta) - \phi_{k+1}(\delta).\end{aligned}$$

Hence, $\Phi_k(\delta)$ is decreasing in k.

Using the definition of $\Phi_k(\delta)$ and (5), we also have

$$\Phi_k(\delta) = \sum_{t=0}^{\infty}\delta^{t+1}\sum_{k'=k}^{N}(\mathcal{Q}^t)_{kk'}\Pr[o_i(t)\in\mathcal{D}^h(t)|k'] > 0.$$

From the expression above, it immediately follows that $\Phi_k(\delta)$ is a continuous function of δ with $\Phi_k'(\delta) > 0$. Moreover, expression (1) for $k = 1$ becomes $v_1 = 1 + \delta v_2$ since $\sigma_1 = 1$, $Q_{11} = 0$, and $Q_{12} = 1$. Using (2) and (4), $v_1 = 1 + \delta v_2$ can be rewritten as

$$\Phi_1(\delta) = 1 - \phi_1(\delta).$$

Therefore, since $\phi_1(\delta) \in (0, 1)$, it follows that $\Phi_1(\delta) \in (0, 1)$ as well. Noting that $\Phi_k(\delta) > 0$ for all $k = 1, \ldots, N - 1$ and $\Phi_k(\delta)$ is decreasing in k, it follows that $\Phi_k(\delta) \in (0, 1)$ for all $k = 1, \ldots, N - 1$. □

Using Lemma 1, we can establish sufficient conditions for the cost of cooperation γ that ensure the existence of cooperative equilibrium when players adopt the grim strategy in Definition 1.

Proposition 1 *If*

$$\sigma_2\Phi_3(\delta) \leq \gamma \leq \Phi_1(\delta),$$

then the social norm in Definition 1 supports cooperation as a subgame perfect equilibrium.

Proof We need to demonstrate that neither deviations from the equilibrium path (where everyone is in the cooperation state) nor deviations off the equilibrium path (where someone is in the punishment state) are profitable.

Deviations from the equilibrium path. Suppose that every player is in the cooperation state. The expected payoff to a player in this state is

$$v_0 := \frac{1 - \gamma}{1 - \delta}. \tag{6}$$

Deviating from the cooperation state is never optimal if $v_0 \geq v_k$ for all $k = 1, \ldots, N$. Recall that the expected payoff v_k decreases with k. Therefore, it is sufficient to ensure

$$v_0 \geq v_1. \tag{7}$$

Using (2) and the definition of v_0, expression (7) becomes

$$1 - \phi_1(\delta) \geq \gamma. \tag{8}$$

As we also showed in the proof of Lemma 1, expression (1) for $k = 1$, which is $v_1 = 1 + \delta v_2$, can be rewritten, using (2) and (4), as

$$1 - \phi_1(\delta) = \Phi_1(\delta), \tag{9}$$

where, from (4), $\Phi_1(\delta) = \frac{\delta}{1-\delta}[\phi_1(\delta) - \phi_2(\delta)]$. Therefore, using (8) and (9), condition (7) is equivalent to

$$\Phi_1(\delta) \geq \gamma. \tag{10}$$

Therefore, since $\Phi_1(\delta)$ is a continuous, increasing function of $\delta \in (0, 1)$, which takes values in $(0, 1)$ (Lemma 1), given $\delta \in (0, 1)$, it follows that if γ is sufficiently small, then (10) holds, and a deviation from the equilibrium path is suboptimal. Alternatively, given $\gamma \in (0, 1)$, there exists a value $\underline{\delta} := \Phi_1^{-1}(\gamma)$ such that, for $\delta \geq \underline{\delta}$, condition (10) holds and a deviation from the cooperation state is suboptimal.[10]

Deviations off the equilibrium path. Consider now a player in the punishment state, say player i. This player might find it profitable to deviate by cooperating in the current period and returning to punish from the next period on. One-shot deviations from the punishment state are costly but reduce contagion and increase the expected value of future gains. The trade-off between the current cost of cooperating today and the future gain from returning to punish tomorrow is what we must assess to determine whether or not deviating from the punishment state is profitable.

If the defector does not deviate from the punishment state, her expected payoff is v_k in (2), if she does, then her expected payoff is $\tilde{v}_k$ in (3). To facilitate the direct comparison between v_k and $\tilde{v}_k$, it is convenient to rewrite the payoff v_k by fixing the current meeting with a counterpart. We have

$$v_k = \sigma_k \left[1 + \delta \sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\, v_{k'+2} \right] + (1-\sigma_k)\, \delta \sum_{k'=k-2}^{N-2} Q_{k-2k'}(N-2) v_{k'+2}. \tag{11}$$

To explain the expression above, fix the meeting between player i and her opponent, as in expression (3). With probability σ_k, player i meets a cooperator, and her current payoff is $1-\gamma$, while her continuation payoff $v_{k'+2}$ depends on the number k' of defectors exiting the meetings between the remaining $N-2$ players ($N-k-1$ cooperators and $k-1$ defectors) plus two additional defectors, player i, who remains in the punishment state, and her opponent, who becomes a defector next period. With probability $1-\sigma_k$, instead, player i meets a defector. In this case, her current payoff is 0, while in the next period, there will be k' defectors exiting the meetings between the remaining $N-2$ players ($N-k$ cooperators and $k-2$ defectors) plus two additional defectors, player i and her counterpart, who both are in the punishment state.

Deviating from the punishment state is never optimal if

$$v_k \geq \tilde{v}_k, \qquad \text{for } k = 2, \ldots, N. \tag{12}$$

Replacing (2) into (3) and (11), $v_k \geq \tilde{v}_k$ becomes

$$\sigma_k \sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\Phi_{k'+1}(\delta) \leq \gamma, \qquad \text{for } k = 2, \ldots, N-1, \tag{13}$$

where $\Phi_{k'+1}(\delta) := \frac{\delta}{1-\delta}[\phi_{k'+1}(\delta) - \phi_{k'+2}(\delta)]$ is the expected gain in the number of encounters with cooperators if the defector implements a one-shot deviation from the punishment state. Such a gain materializes only if the deviation from the punishment state takes place in a meeting with a cooperator—which occurs with probability σ_k.

[10] The value $\underline{\delta}$ corresponds to the lower-bound of δ in Kandori (1992).

From Lemma 1, it is sufficient to restrict condition (13) to the most stringent case. Since σ_k and $\Phi_k(\delta)$ are decreasing in k, the most stringent case for (13) is $k = 2$, which provides a sufficient condition for all $k \geq 2$. Letting $k = 2$, (13) becomes

$$\sigma_2 \Phi_3(\delta) \leq \gamma, \tag{14}$$

where $\Phi_3(\delta) := \frac{\delta}{1-\delta}[\phi_3(\delta) - \phi_4(\delta)]$ represents the increase in the number of encounters with cooperators that a defector expects to experience from slowing-down the contagion, when $k = 2$. Since this increase occurs with probability σ_2, the expression $\sigma_2 \Phi_3(\delta)$ is then the expected gain of a one-shot deviation from the punishment state when initially there are only $k = 2$ defectors. Therefore, condition (13) has a straightforward interpretation. Such a deviation is never optimal if the cost of deviating from the punishment state (right-hand side) exceeds the expected gain that this deviation generates (left-hand side).

Hence, noting that $\Phi_1(\delta) > \Phi_3(\delta)$ (Lemma 1) and $\sigma_2 < 1$ concludes the proof.[11] □

The intuition of Proposition 1 is straightforward. For any given $\delta \in (0, 1)$, there always exists a corresponding $\hat{\gamma} \in (0, 1)$ such that, for $\gamma = \hat{\gamma}$, (7) holds with equality, thus making a player indifferent between cooperating or defecting on the equilibrium path.[12] This is because sustaining the cost $\hat{\gamma}$ is completely outweighed by the expected gain from cooperating instead of defecting—from (10) we get $\hat{\gamma} = \Phi_1(\delta)$. Off-equilibrium, since at least one player is defecting, the continuation payoff from cooperating instead of defecting is lower than that in equilibrium. Therefore, if on the equilibrium path, cooperating and defecting are equally attractive when $\gamma = \hat{\gamma}$, on the off-equilibrium path, they are not, as cooperating becomes less attractive than defecting. Hence, when (7) holds with equality then (12) holds with strict inequality ($\sigma_2 \Phi_3(\delta) < \hat{\gamma} = \Phi_1(\delta)$). This intuition is also discussed in Wolitzky (2013).[13]

Proposition 1 shows that the range of cooperation costs for which the grim strategy in Definition 1 is effective in supporting the cooperative equilibrium depends only on the discount factor and the population size. Figure 1 provides an illustration for a given population size, $N = 500$. The area above the bottom curve, $\sigma_2 \Phi_3(\delta)$, contains the points (δ, γ) that solve the SOSD. The area below the top curve, $\Phi_1(\delta)$, contains the points that solve the FOSD. The gray-colored area represents the equilibrium set, that is, the locus of points (δ, γ) that sustain cooperation as an equilibrium outcome. As Fig. 1 displayed, the existence of the cooperative equilibrium is confined to a narrow path in the space (δ, γ) which expands as the values of δ and γ increase.[14] A balanced relationship between these two quantities is essential for supporting cooperative equilibrium. However, since the cost of cooperation and the discount factor are two independent, exogenous variables, achieving such balance is generally incidental.

[11] We emphasize that the result in Proposition 1 was already recognized in Ellison (1994, Lemma 2 and Proposition 4), although with a different approach.

[12] This $\hat{\gamma}$ always exists because for $\delta \in (0, 1)$, $\lim_{\gamma \to 1} v_0 < v_1$ and $\lim_{\gamma \to 0} v_0 > v_1$. By continuity, there exists $\gamma = \hat{\gamma}$ such that $v_0(\hat{\gamma}) = v_1(\hat{\gamma})$.

[13] We thank an anonymous Referee for raising this point.

[14] It can be also shown that the entire equilibrium set squeezes as the size of the population N increases.

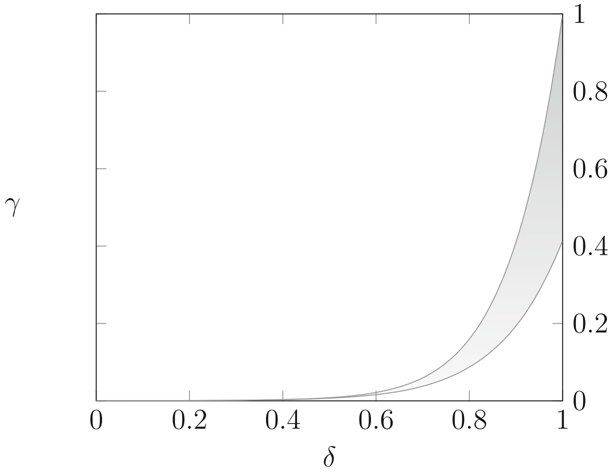

Fig. 1 The cooperative equilibrium set. Population size $N = 500$. The gray-colored area contains the points (δ, γ) that sustain cooperative equilibrium. Formally, it corresponds to the set $\{(\delta, \gamma) \in (0, 1) \times (0, 1) | \sigma_2 \Phi_3(\delta) \leq \gamma \leq \Phi_1(\delta)\}$. The bottom curve is the function $\sigma_2 \Phi_3(\delta)$, the top curve is the function $\Phi_1(\delta)$

Things are different if the cost of cooperation is a choice variable. In the next sections, we exploit the properties of functions $\Phi_k(\delta)$ to demonstrate that if the cost of cooperation is equal or greater than a threshold whose value is below one half, then community punishing is always individually optimal, for any value of the discount factor and any size of the population (Sect. 4). Moreover, for any discount factor and population size we identify a target cost of cooperation that enables the social norm in Definition 1 to support cooperation as an equilibrium outcome (Sect. 5). Finally, we use the target cost of cooperation to show how players can achieve the cooperative equilibrium when the cost of cooperation is endogenous (Sect. 6).

4 Solving the SOSD

The condition stated in Eq. (14) is not generally satisfied for high discount factor values. If γ is sufficiently small, moving away from the punishment state can become profitable when the discount factor is high, thus undermining the credibility of the grim punishment. We have referred to this issue as the second-order social dilemma (SOSD).

In this section, we show that the SOSD arises only in a small sample of interactions, that is, interactions where the cost of cooperation is below a threshold that never exceeds one half. If the cost of cooperation is above such a threshold value, then community enforcement is always incentive-compatible. Importantly, this result holds regardless of the discount factor and the population size.

Theorem 1 *If players adopt the social norm in Definition 1, then there exists* $\overline{\gamma} \in (0, \frac{1}{2})$*, such that if* $\gamma \geq \overline{\gamma}$*, punishing is optimal.*

Proof To prove the statement of Theorem 1, we need to demonstrate that there exists $\overline{\gamma} \in (0, \frac{1}{2})$ such that if $\gamma \geq \overline{\gamma}$, then (14) holds for any value of the discount factor δ and any population size N. To proceed, we start by demonstrating that

$$\frac{\delta}{1-\delta}\left[\phi_2(\delta) - \phi_4(\delta)\right] = 1 - \frac{\phi_2(\delta)}{\sigma_2}, \qquad \forall \delta \in (0, 1). \tag{15}$$

Consider (1) for $k = 2$. We have

$$v_2 = \sigma_2 + \delta\left(Q_{22}v_2 + Q_{24}v_4\right).$$

Using (2), the expression above becomes

$$\frac{\phi_2(\delta)}{1-\delta} = \sigma_2 + \delta\left[Q_{22}\frac{\phi_2(\delta)}{1-\delta} + Q_{24}\frac{\phi_4(\delta)}{1-\delta}\right]. \tag{16}$$

Notice that $Q_{24} = \sigma_2$ and $Q_{22} = 1 - \sigma_2$. Therefore, Eq. (16) can be rewritten as

$$\frac{\delta}{1-\delta}\phi_2(\delta) + \phi_2(\delta) = \sigma_2 + \delta(1-\sigma_2)\frac{\phi_2}{1-\delta} + \delta\sigma_2\frac{\phi_4(\delta)}{1-\delta}.$$

Simplifying and rearranging it, we get (15).

Now consider the following identity:

$$\frac{\delta}{1-\delta}\left[\phi_2(\delta) - \phi_4(\delta)\right] = \Phi_2(\delta) + \Phi_3(\delta).$$

Since $\Phi_2(\delta) > \Phi_3(\delta)$ (Lemma 1), it follows that $\frac{\delta}{1-\delta}\left[\phi_2(\delta) - \phi_4(\delta)\right] > 2\Phi_3(\delta)$, which using (15) becomes

$$\Phi_3(\delta) < \frac{1}{2}\left[1 - \frac{\phi_2(\delta)}{\sigma_2}\right] < \frac{1}{2}, \tag{17}$$

where the last inequality holds since $\phi_2 < \sigma_2$.

Recalling that $\Phi_k(\delta)$ increases in δ and decreases in k (Lemma 1), from (17) we have

$$\tilde{\gamma}(N) := \sup_{\delta\in(0,1)} \sigma_2(N)\Phi_3(\delta, N) < \frac{1}{2},$$

where in the expression $\sigma_2(N)\Phi_3(\delta, N)$ we have made the population size explicit. Given N, if $\gamma \geq \tilde{\gamma}(N)$, then condition (14) holds for any $\delta \in (0, 1)$. From (17), it follows that $\lim_{N\to\infty}\tilde{\gamma}(N) = \overline{\gamma} \in (0, \frac{1}{2})$. Hence, if $\gamma \geq \overline{\gamma}$ then (14) holds for any size $N = 2n \geq 4$ of the population, which concludes the proof of Theorem 1. □

Theorem 1 states that if the cost-to-benefit ratio reaches a threshold value $\overline{\gamma}$, which never exceeds one half, the incentives to punish are fully restored, independent of the value of the discount factor and the size of the population. Therefore, if $\gamma \geq \overline{\gamma}$ community punishment is always credible. Figure 2 provides a numerical illustration.

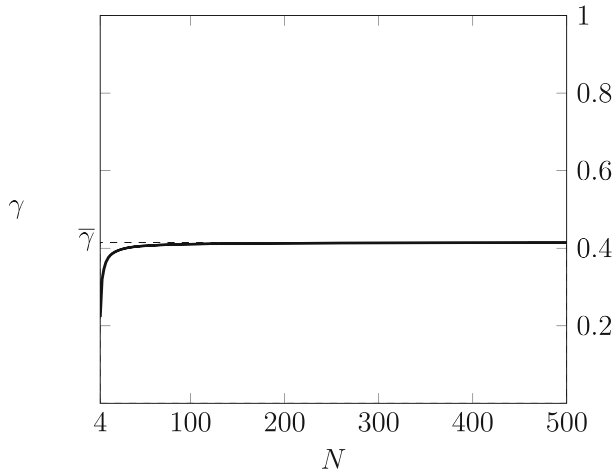

Fig. 2 The cost function $\tilde{\gamma}(N)$ and the threshold $\overline{\gamma}$. The solid line represents $\tilde{\gamma}(N)$ as a function of N. As the population size increases from 4 to 500 players $\tilde{\gamma}(N)$ converges to $\overline{\gamma} < \frac{1}{2}$ (dashed line). All values $\gamma \geq \overline{\gamma}$ solve the SOSD for any $\delta \in (0, 1)$ and for any $N \geq 2n$

The solid line is the function $\tilde{\gamma}(N)$, which rapidly converges to $\overline{\gamma} < \frac{1}{2}$ as the population grows.

Since $\Phi_3(\delta)$ is a continuous, increasing function of $\delta \in (0, 1)$, we can also derive the upper-bound of the discount factor below which punishing defectors is always optimal. Formally, we have

$$\bar{\delta} := \begin{cases} \Phi_3^{-1}\left(\dfrac{\gamma}{\sigma_2}\right), & \text{if } \gamma < \bar{\gamma} \\ 1 & \text{if } \gamma \geq \bar{\gamma}. \end{cases} \tag{18}$$

and for $\delta \leq \bar{\delta}$ no deviations from the punishment state are profitable. If $\gamma \geq \bar{\gamma}$, then $\bar{\delta} = 1$ and the grim punishment is credible for any value of the discount factor and any size of the population.

Here, two remarks are noteworthy. First, Theorem 1 reveals that the SOSD arises in a small segment of all possible interactions, i.e., interactions where the cost of cooperation is less than $\overline{\gamma}$, which is well below one half. Second, Theorem 1 has also a straightforward implication. In interactions where $\gamma < \bar{\gamma}$, increasing the cost of cooperation is beneficial to the credibility of community enforcement, and, therefore, to the cooperative equilibrium, as long as such additional costs do not destroy the incentives to cooperate (FOSD).

Theorem 1 holds potential applicability in economic environments where players have different discount factors, where cost-to-benefit ratios vary, or where population size is non-stationary. In all those scenarios, evaluating the effectiveness of community punishment responses can be a cumbersome task. Theorem 1 suggests that players can solve the SOSD if they are enabled to increase the cost of cooperation. It is advantageous because it simplifies the decision-making process. Once the SOSD is removed, players only need to evaluate their incentives to cooperate (FOSD).[15]

Corollary 1 *If $\gamma \geq \bar{\gamma}$, then the social norm in Definition 1 supports cooperation as a subgame perfect equilibrium if $\delta \geq \Phi_1^{-1}(\gamma)$.*

5 The target cost of cooperation

In economic environments where cooperation incurs a selectable cost, individuals can use the magnitude of that cost as a signal to disclose both their willingness to cooperate and their commitment to punish those who deviate. To achieve this, they need a recognizable value of the cost of cooperation to effectively coordinate their actions. One way to proceed is to link the cost of cooperation to the environmental constraints that individuals face, such as the population size and the discount factor, other than the random matching process. This is reasonable because, on the one hand, the larger the population size, the harder cooperating becomes, and on the other hand, the higher the discount factor, the greater the tendency to delay gratification from the present to the future.

In this section, we demonstrate that for any population size and discount factor, there exists a specific value of the cost of cooperation—referred to as the target cost of cooperation—that enables the social norm defined in Definition 1 to sustain cooperation in equilibrium. Specifically, we identify the target cost of cooperation as the cost associated with missing out on the expected gains that result from deviating from the punishment state. As the value of discounting rises, this target cost of cooperation also increases because deviating from the punishment state becomes more profitable. Formally:

Definition 2 Given $\delta \in (0, 1)$, we define $\gamma = \Phi_2(\delta)$ as the target cost of cooperation.

The target cost of cooperation described above reflects the idea that as a player becomes more patient and is more willing to forgo immediate rewards in favor of future benefits, the cost of cooperating increases. This is because the benefits that the player must give up to cooperate are greater when they have a stronger tendency to delay gratification.[16]

We then have the following:

[15] The result in Theorem 1 expands to any value of $\gamma \in (0, 1)$ if players can observe their opponents' past actions, as in Takahashi (2010). To understand the reasoning behind this, notice that in a setting where players can observe their opponents' past actions, the grim strategy in Definition 1 incorporates that information to potentially trigger the punishment phase. Specifically, assume, that after matches are formed and right before actions are taken, each player receives a report about her current opponent's history of plays (up to the previous period). In this situation, if a player in the punishment state, say player i, deviates off equilibrium, that deviation generates a current loss (-γ) without slowing down the contagion process. This occurs because, as a defector, player i's opponent defects independent of player i's history, while, as a cooperator, she observes a defection in player i's history and responds by defecting as the strategy in Definition 1 prescribes. As a result, deviating from the punishment state is never optimal for any value of $\gamma \in (0, 1)$. Notice also that the same result holds if information about opponents' past plays is limited to the very last period.

[16] This positive relationship between the target cost of cooperation and the value of discounting is consistent with recent empirical evidence indicating that sacrificing immediate gains for future rewards, which is what

Table 2 Payoff matrix with the target cooperation cost

		Player j	
		C	D
Player i	C	$1-\Phi_2(\delta),\ 1-\Phi_2(\delta)$	$-\Phi_2(\delta),\ 1$
	D	$1,\ -\Phi_2(\delta)$	$0,\ 0$

Theorem 2 *For any $\delta \in (0,1)$, if γ is the target cost of cooperation, then the strategy in Definition 1 sustains cooperation as a subgame perfect equilibrium.*

Proof From Lemma 1 immediately follows that $\Phi_2(\delta) := \frac{\delta}{1-\delta}[\phi_2(\delta) - \phi_3(\delta)]$ is such that

$$\Phi_3(\delta) < \Phi_2(\delta) < \Phi_1(\delta).$$

Therefore, if $\gamma = \Phi_2(\delta)$, then the condition in Proposition 1 is fulfilled, since $\sigma_2 < 1$. □

To explain the result above, letting $\gamma = \Phi_2(\delta)$, the payoff matrix in Table 1b becomes that in Table 2. For any value of the discount factor in the unit interval, no deviations from the strategy in Definition 1 are profitable. Formally, this means that (14) and (8) must hold once $\gamma = \Phi_2(\delta)$. Start by considering (14), which now becomes $\Phi_2(\delta) \geq \sigma_2\Phi_3(\delta)$. This inequality holds since $\Phi_2(\delta) > \Phi_3(\delta)$ and $\sigma_2 < 1$. Now consider (8), which becomes $\Phi_2(\delta) \leq 1 - \phi_1(\delta)$. This inequality holds because $1 - \phi_1(\delta) = \Phi_1(\delta) > \Phi_2(\delta)$. Hence, for any $\delta \in (0,1)$, a cost of cooperation set to $\Phi_2(\delta)$ solves both the FOSD and the SOSD, and the social norm in Definition 1 supports cooperation as an equilibrium outcome.

In Fig. 3 we run a numerical simulation to evaluate how the target cost of cooperation $\Phi_2(\delta)$ depends on the players' discount factor, for a given size of the population ($N = 500$). The figure reports the existence domain of cooperative equilibrium (gray-colored area) and the target cost of cooperation $\Phi_2(\delta)$ (solid line), which is an increasing and convex function of δ, mapping $(0, 1)$ into $(0, \bar{\Phi}_2)$, with $\bar{\Phi}_2 < 0.6$. The analysis shows that if $\gamma = \Phi_2(\delta)$, then patient players have a higher target cost of cooperation than impatient players. To understand why this happens, notice that in environments where cooperation relies on decentralized sanctioning schemes, individuals pay a reliability cost as they are called upon to punish. This cost arises because punishment requires the sacrifice of present gains for future benefits. Therefore, complying with the designed sanctioning scheme represents an opportunity cost, which increases with the value of discounting as the potential loss becomes more valuable.

Notice that any point within the grey-colored area of Fig. 3 corresponds to a suitable cost that satisfies the condition outlined in Corollary 1. This is especially true for the function $\Phi_3(\delta)$, which is lower than $\Phi_2(\delta)$. However, it is important to highlight that setting a target cost for cooperation at $\Phi_3(\delta)$ would destabilize the punishment scheme in larger societies. As the population size N increases, $\Phi_3(\delta)$ approaches $\sigma_2\Phi_3(\delta)$. Consequently, having $\gamma = \Phi_3(\delta)$ would make defectors indifferent between punishing

cooperation entails, imposes an immediate psychological cost on individuals, thus making cooperation more costly for patient players (Giuliano and Sapienza 2020).

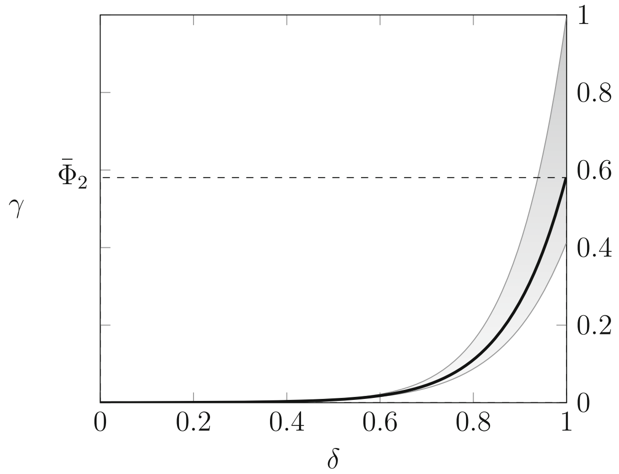

Fig. 3 The target cost of cooperation. Population size $N = 500$. The gray-colored area contains the points (δ, γ) that sustain cooperative equilibrium. The area above the bottom line (function $\sigma_2\Phi_3(\delta)$) contains the points that solve the SOSD. The area below the upper line (function $\Phi_1(\delta)$) contains the points that solve the FOSD. The solid line is the target cost of cooperation $\Phi_2(\delta)$

and cooperating when the population grows, thereby putting their compliance with the social norm at risk.[17]

The cost of cooperation γ lies within the unit interval, which can be partitioned into three subsets:

$$(0, 1) = \mathcal{A} \sqcup \mathcal{B} \sqcup \mathcal{C}, \tag{19}$$

where $\mathcal{A} := (0, \sigma_2\Phi_3(\delta))$, $\mathcal{B} := [\sigma_2\Phi_3(\delta), \Phi_1(\delta)]$, and $\mathcal{C} := (\Phi_1(\delta), 1)$. If $\gamma \in \mathcal{B}$, cooperative equilibrium exists (Proposition 1). If $\gamma \in \mathcal{A}$, the cost of cooperation is too low and cooperating in the cooperative state is incentive-compatible, but defecting in the punishment state is not. If $\gamma \in \mathcal{C}$, the cost of cooperation is too high and cooperating in the cooperation state is not incentive-compatible, while defecting in the punishment state is. Therefore, if $\gamma \in \mathcal{A} \cup \mathcal{C}$, the social norm in Definition 1 does not support cooperation in equilibrium.[18] In this case, a compensation mechanism capable of lowering or increasing the cost of cooperation in order to reach a value that lies in the set $\mathcal{B}$, would ensure the existence of cooperative equilibrium.[19] The key message of Proposition 1 is, in fact, that cooperation is quite rare in the repeated Prisoner's Dilemma among strangers, also under the threat of a grim punishment scheme. Generally, players have an incentive to defect, even though defecting generates

[17] See Levine et al. (2022) for a more general discussion on the limitations of collective punishment in a large society.

[18] It is worth noting that adopting a strategy where punishment is less severe than "grim" would not qualitatively change the partition in (19). A more lenient punishment scheme would only make the size of set $\mathcal{B}$ smaller and shift its values toward the upper-bound 1.

[19] Compensation mechanisms that promote cooperation are discussed, for example, in Cho and Kreps (1987), Varian (1994), Van Damme (1989), Ben-Porath and Dekel (1992), Charness et al. (2007), Dickinson et al. (2015).

Table 3 Normalized Prisoner's Dilemma

		Player j	
		C	D
Player i	C	$1-\gamma,\ 1-\gamma$	$1-\eta,\ 1$
	D	$1,\ 1-\eta$	$0,\ 0$

less gains than mutual cooperation. It is then desirable to design mechanisms that can help players to attain the efficient outcome.

In the following section, we build upon the original model introduced in Sect. 2 by considering an environment where the benefit to the counterpart depends on the magnitude of the cost of cooperation, which, in turn, becomes a choice variable. In this revised context, the target cost of cooperation can act as a signal to encourage cooperative efforts.

Remark We emphasize that the results in Proposition 1, Theorems 1 and 2 do not depend on our working assumption about the payoff matrix in Table 1. They can also be obtained by considering a canonical Prisoner's Dilemma with a reward payoff R, a temptation payoff T, a sucker's payoff S, and a punishment payoff P, where $T > R > P > S$. If we use the normalization adopted in Table 1b, we obtain the payoff matrix in Table 3, where we have redefined $\gamma := \frac{T-R}{T-P}$ and $\eta := \frac{T-S}{T-P}$, with $\gamma < 1 < \eta$. Repeating the same arguments of Sect. 3.3, it follows that condition (8) does not depend on η, while condition (13) becomes

$$\sigma_k \sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\Phi_{k'+1}(\delta) \leq \sigma_k \gamma + (\eta-1)(1-\sigma_k), \qquad \text{for } k = 2, \ldots, N-1.$$

Since $\eta - 1 > 0$, it is sufficient to show that $\sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\Phi_{k'+1}(\delta) \leq \gamma$ holds. For $k = 2$, this condition becomes $\Phi_3(\delta) \leq \gamma$, which holds for all $\gamma \geq \overline{\gamma}$.

Finally, we also emphasize that the result in Proposition 2 can be extended to a scenario where the payoff matrices take the form of that in Table 3. However, in that case we have one more degree of freedom, given by the parameter η.

6 Endogenous cost of cooperation

Consider a game similar to that presented in Sect. 2, where now there is an initial stage in which each player i chooses the level of the cost $c_i > 0$ she intends to sustain in each interaction. We assume that the benefit b_j that player i generates to her counterpart j is an increasing function of the cost c_i chosen by i in the initial stage, i.e., $b_j = b(c_i)$.[20] We assume that all players have the same production function $b(c)$, with $b(c)$ being

[20] The cost c_i can be interpreted as the effort that player i must exert to generate the benefit b_j to the counterpart j.

Table 4 Payoff matrix with endogenous costs

		Player j	
		C	D
Player i	C	$b_j - c_i,\ b_i - c_j$	$-c_i,\ b_i$
	D	$b_j,\ -c_j$	$0,\ 0$

a continuous function satisfying the following standard conditions: (i) $b(0) = 0$, (ii) $b'(c) > 0$, (iii) $\lim_{c\to 0} b'(c) = +\infty$ and $\lim_{c\to +\infty} b'(c) = 0$ and (iv) $b''(c) < 0$. Assumptions (i)-(iv) describe a standard production process with decreasing returns to scale. These assumptions ensure that there exists a value $\bar{c} > 0$ such that, for $c \in (0, \bar{c})$, $b(c) > c$, with $\frac{c}{b(c)}$ mapping the interval $(0, \bar{c})$ into $(0, 1)$. As a result, if all players choose the same cost $c \in (0, \bar{c})$, the payoff matrix in Table 4 represents a symmetric Prisoner's Dilemma as that described in Table 1a. However, if players choose asymmetric costs, then the payoff matrix in Table 4 is match-dependent and may vary over time.

In the initial stage—just before the realization of the first matching in period 0—players simultaneously and non-cooperatively choose the cost c they are willing to sustain to generate the benefit $b(c)$ to their counterparts in all future meetings. We refer to this cost determination stage as Γ_0. Once the costs are chosen in Γ_0, players randomly meet in each period and face the payoff matrices that those costs determine. We assume that players cannot withdraw their choices. To explain, if players i and j choose c_i and c_j in Γ_0, they establish the benefits $b(c_i)$ and $b(c_j)$ that their counterparts will receive. If these two players meet at some point, their interaction is described by the payoff matrix in Table 4, with $b_j = b(c_i)$ and $b_i = b(c_j)$. Adding the initial stage Γ_0 to the original game in Sect. 2 expands the action set, which now consists of the cost determination c in Γ_0, and the actions C and D in Γ^∞. Moreover, this new stage transforms the infinitely repeated Prisoner's Dilemma in an infinite sequence of potentially different and asymmetric stage games, denoted by Γ^∞.[21] We let Γ be the game that comprises Γ_0 and Γ^∞. The time of events is illustrated in Fig. 4. The initial period $t = 0$ is divided into two subperiods: in the first subperiod, players simultaneously and non-cooperatively choose the cost $c \geq 0$ (stage Γ_0); in the second subperiod, interactions begin and players play an infinite sequence of games that are now match-dependent (Γ^∞). As in the original model described in Sect. 2, monitoring remains private. However, unlike that model players are aware that they will face a game structured as in Table 4 in each period, but they do not know the specific payoff matrix they encounter until meetings are formed.

Given the above considerations, what cost of cooperation should the players select in Γ_0? What strategy should they adopt in Γ^∞ to support cooperation? Choosing a sufficiently high cost of cooperation may encourage opponents to cooperate, but it can also discourage those bearing that cost from doing so. Furthermore, the cost should remain reasonable to maintain incentives for punishment if needed.

[21] We emphasize that Γ^∞ is an infinite repeated Prisoner's Dilemma only if all players choose the same cost $c \in (0, \bar{c})$ in Γ_0. If choices in Γ_0 are asymmetric, then Γ^∞ is an infinite sequence of games, whose payoff matrix generally differs over time and across players.

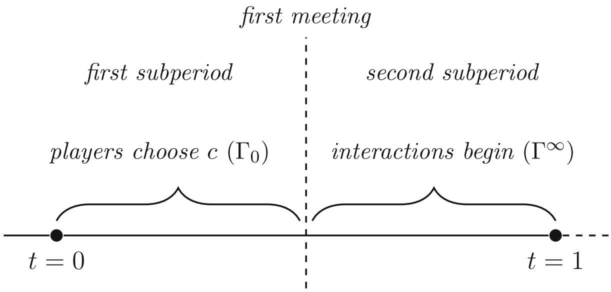

Fig. 4 Timing of events

Below, we show that players can use the initial stage Γ_0 as a means of coordination. More precisely, players can choose the target cost of cooperation to signal their willingness to cooperate. Then, they can combine this signal with the grim trigger strategy to support cooperation as an equilibrium outcome. What is needed is an extended norm of community enforcement that accounts for possible deviations from the target cost of cooperation in Γ_0. This is done next.

Let c^* be such that $\frac{c^*}{b(c^*)} = \Phi_2(\delta)$.[22] We assume that the players adopt the following social norm:

Definition 3 (*"Extended" grim strategy*) In Γ_0, the player chooses $c = c^*$. In Γ^∞, if the player observes a symmetric cost c^* and no defection in past periods, then she cooperates; otherwise, she defects.

As for the standard grim strategy in Definition 1, the strategy in Definition 3 identifies two states of behavior: *cooperation* and *punishment*. In the cooperation state, the player chooses c^* in Γ_0 and plays C in each period of Γ^∞, unless a deviation from one of these two actions is observed, in which case, the player switches permanently to the punishment state. In the punishment state, the player, who has experienced a deviation from the cooperation state (either $c \neq c^*$ or D), chooses D in the current and all future periods. Again, we refer to a player in the cooperation state as a cooperator and in the punishment state as a defector.

Unlike the standard grim strategy in in Definition 1, the strategy in Definition 3 encapsulates a coordination mechanism on the cost of cooperation chosen by the players. If a player selects a cost of cooperation $c = c^*$ in Γ_0, then this choice signals the counterpart the intention to cooperate in Γ^∞—as long as a defection is not observed, in which case the player starts to punish in all future periods of Γ^∞. Conversely, if a player chooses $c \neq c^*$ in Γ_0, then this choice is interpreted by the counterpart as a signal that the player does not intend to cooperate in Γ^∞, and such a deviation from the target cost of cooperation is directly and indirectly punished in Γ^∞.[23] In other words, the trigger is now either observing $c \neq c^*$ or a defection happening.

[22] With a slight abuse of notation, we also refer to c^* as the target cost of cooperation when the payoff matrix is not normalized and takes the form shown in Table 4.

[23] Directly punished because, in that case, the counterpart will play D in the current period, and indirectly punished because she will also play D also in all future periods.

The following result shows that the "extended" grim strategy defined above ensures cooperation for any discount factor.

Proposition 2 *For any* $\delta \in (0, 1)$*, the social norm in Definition 3 supports cooperation in* Γ *as a subgame perfect equilibrium.*

Proof It is sufficient to demonstrate that no deviations from the cooperation state in the equilibrium path (i.e., everyone is in the cooperation state) and no deviations from the punishment state off the equilibrium path (i.e., someone is in the punishment state) are profitable.

Deviations from the equilibrium path. Suppose that all players are in the cooperation state and that player i deviates from that state. Here, there are two possible deviations to consider: (i) player i chooses $c_i \neq c^*$ in Γ_0 and D in Γ^∞; and (ii) player i chooses $c_i = c^*$ in Γ_0 and D in Γ^∞.[24] Consider (i). Since choosing $c_i \neq c^*$ generates in every match between i and her counterparts an asymmetric payoff matrix, player i and her opponents respond to this deviation by playing D in the current and future periods of Γ^∞, as the strategy prescribes. These responses generate zero payoffs to player i in all meetings. Conversely, if player i does not deviate from the cooperation state, her expected (equilibrium) payoff is

$$\frac{b(c^*) - c^*}{1 - \delta} > 0. \tag{20}$$

Therefore, deviating from the equilibrium path by choosing $c_i \neq c^*$ is clearly suboptimal. Now consider (ii). Deviating by playing D after choosing $c_i = c^*$ in Γ_0 generates a current gain of $b(c^*)$ to player i but triggers the contagion process of defections described by matrix $\mathcal{Q}_N$ in Sect. 3. Therefore, the expected payoff to player i that this deviation generates is

$$\frac{\phi_1(\delta)}{1 - \delta} b(c^*). \tag{21}$$

To see whether such a deviation is suboptimal, we need to ensure that (21) is not larger than (20). Formally, we must show that

$$\frac{\phi_1(\delta)}{1 - \delta} b(c^*) \leq \frac{b(c^*) - c^*}{1 - \delta}. \tag{22}$$

Using (9), expression (22) can be rewritten as

$$\Phi_2(\delta) \leq 1 - \phi_1(\delta) = \Phi_1(\delta),$$

which holds for all $\delta \in (0, 1)$ (Lemma 1). Hence, no deviations are profitable on the equilibrium path.

[24] Observing a deviation means that the counterpart or the player herself has chosen an action that differs from what the strategy prescribes.

Deviations off the equilibrium path. Suppose that player i and other $k-1$ players, with $k \geq 2$, are in the punishment state. We now consider a one-shot deviation from the punishment state for player i, consisting on playing C in one period and reverting to punish in all subsequent periods. We want to make sure that such deviation is never optimal. We distinguish three cases: (i) everyone chose c^* in Γ_0; (ii) some player $j \neq i$ chose $c_j \neq c^*$ in Γ_0; (iii) player i chose $c_i \neq c^*$ in Γ_0.

Consider case (i). Here, everyone chose c^* in Γ_0 but someone (possibly player i) defected in Γ^∞. The expected payoff in the punishment state to player i is

$$\begin{aligned} v_k(c^*) = \sigma_k \left[b(c^*) + \delta \sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\, v_{k'+2}(c^*) \right] \\ + (1-\sigma_k)\,\delta \sum_{k'=k-2}^{N-2} Q_{k-2k'}(N-2) v_{k'+2}(c^*). \end{aligned} \tag{23}$$

Similarly, the expected payoff to player i, when she takes a one-shot deviation from the punishing state, is

$$\begin{aligned} \tilde{v}_k(c^*) = \sigma_k \left[b(c^*) - c^* + \delta \sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\, v_{k'+1}(c^*) \right] \\ + (1-\sigma_k) \left[-c^* + \delta \sum_{k'=k-2}^{N-2} Q_{k-2k'}(N-2) v_{k'+2}(c^*) \right]. \end{aligned} \tag{24}$$

Therefore, deviating from the punishment state is never optimal if

$$v_k(c^*) \geq \tilde{v}_k(c^*), \qquad \text{for } k = 2, \ldots, N,$$

which can be rewritten as

$$\sigma_k \sum_{k'=k-1}^{N-2} Q_{k-1k'}(N-2)\Phi_{k'+1}(\delta) \leq \frac{c^*}{b(c^*)}, \qquad \text{for } k = 2, \ldots, N-1.$$

The most stringent case is $k = 2$, which returns

$$\sigma_2 \Phi_3(\delta) \leq \frac{c^*}{b(c^*)} = \Phi_2(\delta).$$

From Lemma 1, the inequality above holds for all $\delta \in (0, 1)$, and therefore, in this case, a deviation from the punishment state is suboptimal.

Consider case (ii). Suppose that among the k defectors there are $\ell < k$ players who have not chosen c^*. When a cooperator meets one of these ℓ players, the cooperator switches to the punishment state at the beginning of the period—i.e., before taking action. When a cooperator meets one of the $k-\ell$ defectors who have chosen c^*, the cooperator becomes a defector at the end of the period—i.e., after taking action. Although these two types of meetings generate different payoffs in the current period, the contagion process of defections progresses as described by matrix $\mathcal{Q}_N$ in Sect. 3. The reason for this is that cooperators would also switch state if the defectors they meet had chosen c^* (defectors would play D anyway). As a result, the expected payoff to player i in the punishment state is still given by (23), while deviating to C in symmetric matches—i.e., player i and her opponent chose c^* in Γ_0—generates an expected payoff to player i that is still given by (24). Therefore, in symmetric matches deviating from the punishment state is not profitable. Now suppose that player i deviates in an asymmetric match, where her opponent did not choose c^* in Γ_0. In this case, such a deviation generates a loss of $-c^*$ and has no impact on the contagion process. Such a deviation is clearly suboptimal because

$$\overbrace{0+\delta\sum_{k'=k}^{N}\mathcal{Q}_{kk'}(N)v_{k'}(c^*)}^{i\text{ is in the punishment state}} \geq \overbrace{-c^*+\delta\sum_{k'=k}^{N}\mathcal{Q}_{kk'}(N)v_{k'}(c^*)}^{i\text{ deviates from the punishment state}}.$$

Consider case (iii). Here, a one-shot deviation from the punishment state generates a current loss of $-c_i \neq -c^*$, independent of meeting a defector or a cooperator (because c^* was not symmetrically observed), and a payoff equal to zero in all future meetings. Such a deviation is suboptimal as player i could avoid the loss $-c_i$ by playing D in the current period, as the strategy prescribes, and earning a zero payoff. □

Proposition 2 shows that the players can use the target cost of cooperation c^* to signal their counterparts the willingness to cooperate. Once the players choose c^* in Γ_0, the community enforcement encapsulated in the "extended" grim strategy (Definition 3) is effective to support cooperation in Γ^∞. Conversely, if c^* is not chosen by all players, a player who observes a cooperation cost that differs from c^* responds by punishing her opponent (direct punishment) and by defecting in all future meetings to signal the observed deviation from c^* to the entire community (indirect punishment). As a result, the coordination mechanism comprised in Definition 3 induces players to sustain the target cost of cooperation in Γ_0 and enforces cooperation in Γ^∞. Proposition 2 reveals that, despite players holding incomplete information about future interactions (costs are privately observed), a social norm that incorporates direct and indirect punishment (Definition 3) can support cooperation as an equilibrium outcome.

We emphasize that there are alternative "extended" grim strategies that can support a cooperative equilibrium. As we discussed in Sect. 3, given $\delta \in (0, 1)$, any value of $\gamma \in [\sigma_2\Phi_3(\delta), \Phi_1(\delta)]$ enables the "standard" grim strategy to sustain a cooperative equilibrium in the original game (Table 1b). In the extended game (Table 4), to any cost $c' \neq c^*$, such that $\frac{c'}{b(c')} \in [\sigma_2\Phi_3(\delta), \Phi_1(\delta)]$, is associated an extended grim strategy that can sustain a cooperative equilibrium. In that case, the equilibrium strategy is that described in Definition 3, where $c = c'$ is chosen in Γ_0.

We also stress that c^* is not the efficient cost—not necessarily. The equilibrium payoff is maximized at $\hat{c} := b'^{-1}(1) \neq c^*$, which generally does not coincide with c^*.[25] So why in the extended grim strategy don't we consider a cost that is more efficient than c^*? To answer this question, we first need to recognize that $\hat{c}$ is a feasible cost to the extended grim strategy only if $\frac{\hat{c}}{b(\hat{c})} \in [\sigma_2\Phi_3(\delta), \Phi_1(\delta)]$. But, the latter condition does not generally hold, since it depends on the production function $b(c)$. Therefore, players cannot generally coordinate on $\hat{c}$. As for other costs c' that are feasible and more efficient than c^*, a coordination problem may arise. To see why, suppose that a cost $c' \neq c^*$ is chosen, such that $\frac{c'}{b(c')} \in [\sigma_2\Phi_3(\delta), \Phi_1(\delta)]$. Now it turns out that other equilibrium strategies exist based on punishment schemes less severe than "grim," for which c' may also be feasible.[26] In that case, although players face a feasible, symmetric cost c', they might not coordinate on the same strategy. Conversely, identifying c^* as the *target cost of cooperation*—although this cost is generally inefficient—solves both the issues above. Specifically, for any given production function $b(c)$, with the standard properties (i)-(iv), the cost c^* is fully determined by the discount factor and the size of the population. Moreover, c^* signals a unambiguous choice on the strategy adopted by the players, as its value is uniquely determined by the contagious process generated by the grim strategy. The net loss $b(c^*) - c^* - [b(\hat{c}) - \hat{c}]$ represents the coordination cost to make community punishment effective when monitoring is limited and individuals are unlikely to encounter the same counterparts in the future. It can be also interpreted as the cost that individuals are willing to bear to ensure that defectors are punished, even when punishment is not observed.[27]

7 Conclusions

The theory of cooperation in the repeated Prisoner's Dilemma with random matching has made it clear that community enforcement can support cooperative equilibrium among strangers if individuals are sufficiently patient (Kandori 1992; Ellison 1994). What happens if individuals are impatient? In such case, the size of the cooperation cost generates some restrictions on the effectiveness of community enforcement. If cooperation costs are high, individuals may be tempted to free ride, while if they are limited, individuals may hesitate to punish free riders. The strength of these two social dilemmas moves in opposite directions as the cost of cooperation varies.

[25] We let $\hat{c} := \arg\max_{[0,\bar{c}]}[b(c) - c]$. Given properties (i)-(iv) of the production function $b(c)$, the problem $\max_{c\in[0,\bar{c}]}[b(c) - c]$ is well-defined and the FOC reads $b'(c) - 1 = 0$, which returns $\hat{c} = b'^{-1}(1)$. It is worth noting that properties (i)-(iv) exclude the case of $b(c)$ being linear. If we consider a linear production function, say $b(c) = \alpha c$, with $\alpha > 1$ (to ensure positive payoffs in equilibrium), any value of $c > 0$ would generate the same cost-to-benefit ratio $\frac{c}{b(c)} = \frac{1}{\alpha}$. Therefore, the result in Proposition 2 holds if $\frac{1}{\alpha} \in [\sigma_2\Phi_3(\delta), \Phi_1(\delta)]$. However, since, in this case, the production function exhibits constant returns to scale and the cost function is linear, the players are incentivized to choose the highest possible cost, which in our model is unbounded. This is why we have excluded the case of a linear production function.

[26] For example, a punishment scheme implemented every two periods would generate an equilibrium path that partially overlaps that in Fig. 1. The possibility of multiple strategies is also discussed in Camera and Gioffré (2018).

[27] Experimental studies suggest that individuals tend to opt for costly punishment mechanisms when they are available (Fudenberg and Pathak 2010).

In our analysis we have provided two new findings to address these social dilemmas. First, we have demonstrated that if the relative cost of cooperation (namely, the cost-to-benefit ratio) is greater or equal to a threshold that never exceeds one half, the "grim" punishment is always individually optimal and community punishment becomes a credible threat to defectors. The result shows that not only is there a subset of all possible interactions where the credibility of engaging in indiscriminate punishment schemes is never affected by the value of the discount factor or the population size, but it also reveals that such a subset comprises the majority of the interactions. This finding suggests that increasing the cost of cooperation may benefit cooperation, since it reinforces the credibility of the sanctioning scheme to punish defectors. However, these extra costs must be mild enough to contain the temptation to behave opportunistically when cooperating.

Second, we have demonstrated that for any value of discounting and for any size of the population, there exists a target cost of cooperation capable of making community enforcement effective, thus solving both the first- and second-order social dilemma. We have then extended the original game by adding an initial stage in which players choose the size of the cost of cooperation they intend to sustain. We have demonstrated that in such a scenario the target cost of cooperation can be used as a coordination device to enforce individual contributions and support cooperation. Our analysis shows that cooperation among strangers is more likely to occur when the cost of cooperation is endogenous. This is because players can hedge against any value of discounting. In that case, the equilibrium cost is an increasing and convex function of the value of discounting and mirrors the insight, outlined in recent empirical studies, according to which a psychological cost may emerge as individuals' patience increases (Giuliano and Sapienza 2020).

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