

This is a pre print version of the following article:

A survey of attended home delivery and service problems with a focus on applications / Cordeau, Jean-François; Iori, Manuel; Vezzali, Dario. - In: 4OR. - ISSN 1619-4500. - 21:4(2023), pp. 547-583.
[10.1007/s10288-023-00556-2]

Terms of use:

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

28/02/2025 02:34

(Article begins on next page)

A Survey of Attended Home Delivery and Service Problems with a Focus on Applications

Jean-François Cordeau¹, Manuel Iori², Dario Vezzali^{2,3*}

¹Department of Logistics and Operations Management, HEC Montréal, 3000 Chemin de la Côte-Sainte-Catherine, Montréal, H3T 2A7, Canada.

²Department of Sciences and Methods for Engineering, University of Modena and Reggio Emilia, Via Amendola 2, Reggio Emilia, 42122, Italy.

³“Marco Biagi” Foundation, University of Modena and Reggio Emilia, Largo Marco Biagi 10, Modena, 41121, Italy.

*Corresponding author(s). E-mail(s): dario.vezzali@unimore.it;

Contributing authors: jean-francois.cordeau@hec.ca;

manuel.iori@unimore.it;

Abstract

The research field of Attended Home Delivery (AHD) and Attended Home Service (AHS) problems has experienced fast growing interest in the last two decades, with the rapid growth of online platforms and e-commerce transactions. The COVID-19 pandemic has reinforced that interest, raising further challenges and opportunities that have to be tackled by innovative methodologies and policies. The aim of this work is to provide an extensive literature review on the state of the art for AHD and AHS problems, with a particular focus on real-world applications. A discussion of promising future research directions is also provided.

Keywords: Attended Home Delivery, Attended Home Service, Demand Management, Routing, Integrated Demand Management and Routing

MSC Classification: 90-02 , 90B06

1 Introduction

Attended Home Delivery (AHD) and Attended Home Service (AHS) are last-mile operations where the customer must be present at home for the delivery of goods, the

1 execution of a service or, in some cases, both the delivery of goods and the execu-
2 tion of an additional service (Agatz et al., 2008a; Ehmke, 2012). Examples of AHD
3 and AHS are, among others, the delivery of groceries directly at home, the delivery
4 and installation of large furniture and appliances, or the provision of home health-
5 care therapies. By definition, they differ from Unattended Home Delivery (UHD) and
6 Unattended Home Service (UHS) operations, which can be fulfilled without the cus-
7 tomer being present at home. Examples of UHD and UHS are the delivery of parcels
8 right in front of the door or inside a nearby parcel locker, or the reading of a meter in-
9 stalled outside a house. To limit the research area, in this work we focus only on those
10 operations that are *attended* by the customers. For a detailed review on last-mile de-
11 livery concepts we refer the interested reader to Boysen et al. (2021). We are neither
12 interested in surveying the class of Same-Day Delivery (SDD) problems, for which we
13 refer to Voccia et al. (2019), nor in recent trends in last-mile delivery, such as the use
14 of drones and autonomous delivery robots or crowdshipping, which are also reviewed
15 by Boysen et al. (2021).

16 AHD problems originated in the context of e-grocery (see, e.g., Punakivi and
17 Saranen 2001 and Lin and Mahmassani 2002 for seminal ideas) and, more generally,
18 e-fulfillment (see, e.g., Agatz et al. 2008b for an in-depth introductory review). Since
19 the first definition found in the work by Campbell and Savelsbergh (2006), they have
20 seen a continuous increase not only in terms of interest in the research community,
21 but also in terms of importance in many business sectors. The COVID-19 pandemic
22 has just fostered the demand for AHD services, as confirmed by the Organisation for
23 Economic Co-operation and Development (2020). In particular, during the first and
24 second quarters of 2020 online retail sales have registered a worldwide increase of
25 14.8% to 16% in the United States and 30% in the 27 member countries of the Euro-
26 pean Union, with a similar trend in the Asia-Pacific countries. How long this growth
27 will last and whether we will ever return to the pre-pandemic levels is still matter for
28 debate (Wang et al., 2021). In the meantime AHD has already triggered irreversible
29 changes in the logistics of our cities (The Guardian, 2019), and new trends emerging
30 in large metropolitan areas are posing further challenges (Bloomberg, 2021). Among
31 these trends, we mention the delivery of building materials to contractors directly on
32 site and the recent phenomenon of ultra-fast delivery of groceries in as little as 15
33 minutes. A further indication that AHD and AHS problems are drawing increasing at-
34 tention is represented by an analysis that we performed on Scopus and whose results
35 are reported in Figure 1. We looked at the number of documents per year where the
36 entries “attended home delivery”, “attended home service”, “attended home deliver-
37 ies”, or “attended home services” appeared between 2006 and 2022. The results show
38 a slightly yet constantly growing trend between 2006 and 2017, followed by a notable
39 increase between 2017 and 2022.

40 As mentioned before, AHD problems are directly linked to the growth of the *e-*
41 *grocery* business model, where a fierce competition has arisen around the logistical
42 challenges offered by this particular sector, like the perishability of goods, the unpre-
43 dictability of demand, the narrow time windows made available to customers for the
44 delivery, and the low profit margins. Even more challenging is the practice of *meal de-*
45 *livery*, which has become increasingly popular in the last years. Another sector that
46
47
48

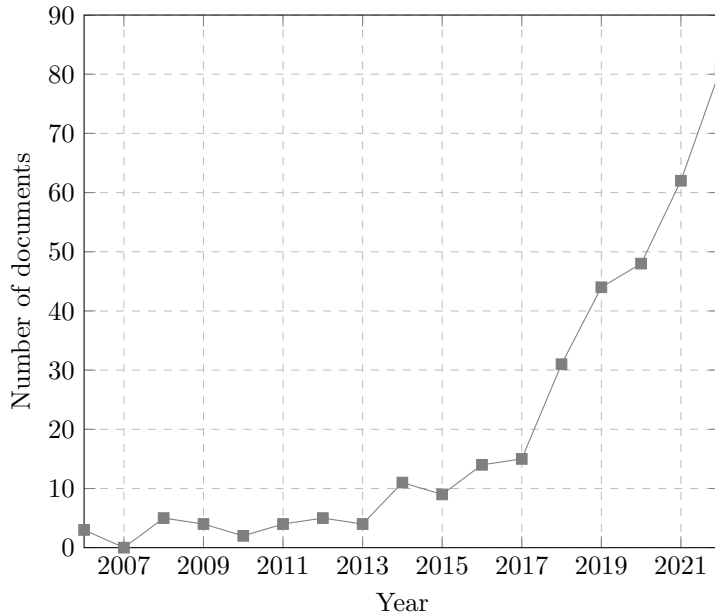


Figure 1: Documents per year on AHD and AHS published between 2006 and 2022

is commonly associated with AHD is the *online retail* of so-called “dry” goods, where the perishability is not an issue, but the parcels may be fragile and require a careful handling, the demand volume can be very high and unpredictable, the goods need to be moved rapidly along the supply chain, and, lastly, the customer might not be at home during the delivery, thus causing additional routing costs and further congestion in city road networks. More traditional sectors are those of *large appliances and furniture*, which usually combine the delivery of goods with the additional installation service. In this sense, we can situate them at the intersection of AHD and AHS problems. Typically, these operations might require a careful handling due to the fragility of some appliances and furniture, but they usually benefit from a larger planning horizon.

The field of AHS itself has received less attention from the research community compared with AHD, but still includes some essential activities like *home healthcare services*, that are important not only to efficiently manage the capability of hospitals but especially to guarantee high-quality therapies to patients who cannot move from home. In this context, we should distinguish between *ordinary* and *extraordinary* care services. The first can be planned over a larger planning horizon, while the latter deal with emergencies and must provide an immediate response. This leads to different problems from an operational research perspective. AHS problems typically arise also in the context of *utilities* (e.g., electricity, gas and water distribution companies, internet and telecommunications service providers, and so forth), where companies might be required by local authorities (see, e.g., [Bruck et al. 2018, 2020](#)) to give customers the opportunity to book their installation or maintenance services within publicly

1 available time slots. As for home care services, we should distinguish these ordinary
2 booking activities from extraordinary ones (e.g., a gas leakage) that require an immediate
3 response. So far, we have mentioned only business-to-consumer sectors, but many
4 observations also hold in a business-to-business environment. Indeed, *on-site main-*
5 *tenance and repair services* present similar characteristics to many AHS operations,
6 including the distinction between ordinary and extraordinary services.

7 Addressing real-world AHD and AHS problems is challenging, as it typically implies
8 solving a multi-stage problem: firstly, a *demand management problem*, and conse-
9 quently, a *routing problem*, where the decisions taken in the previous stage can greatly
10 affect the feasibility as well as the economic profitability of the following decisions.

11 As described in the recent surveys by [Nguyen et al. \(2018\)](#) and [Waßmuth et al. \(2023\)](#),
12 on the *demand side* companies must be able to find effective ways to efficiently
13 leverage the demand of customers by putting into action Revenue Management (RM)
14 principles. Initially borrowed from the airline industry, the practice of RM has become
15 increasingly popular for AHD and AHS problems. Examples of RM decisions in the
16 context of AHD and AHS problems might regard the basic offering and pricing of time
17 slots, their length, the choice of overlapping versus non-overlapping time slots, or the
18 capacity allocated to each of them. These are typically static decisions. More complex
19 decisions are required in a dynamic environment, where a company might be willing
20 to frequently adjust the offering and pricing of time slots, or increase/decrease the
21 capacity allocated based on the actual demand of customers. The complexity of these
22 decisions is also affected by the immediate responsiveness they typically require.

23 On the *supply side*, companies seek to limit the operational costs by applying tra-
24 ditional routing techniques, which have been widely studied in the Vehicle Routing
25 Problem (VRP) literature. The degree of complexity of these techniques is affected by
26 the decisions taken at the demand management stage, and by the possible inclusion
27 of stochastic and dynamic routing aspects. In addition, AHD and AHS problems re-
28 quire considerable “back-end” activities in terms of inventory management and order
29 assembly, which are out of scope of this work.

30 Finally, a meet-in-the-middle approach that is worth considering is to integrate
31 demand management and vehicle routing, as discussed in the recent survey by [Fleck-](#)
32 [enstein et al. \(2023\)](#). Such an integration requires the anticipation of some routing
33 aspects at the demand management stage, which is complex since the VRP is NP-hard.

34 AHD and AHS problems can also be classified according to the planning horizon
35 of the decisions that must be taken. Long-term decisions typically dealing with the
36 setup of business (i.e., with lasting effects from months to years), like the opening of
37 new facilities or the creation of demand clusters given an extended geographical area,
38 are taken at a *strategic* level. Medium-term decisions typically dealing with the sizing
39 of business (i.e., with lasting effects from weeks to months), like the design of basic
40 model-weeks for each demand cluster or the allocation of capacity to each single time
41 slot, are taken at a *tactical* level. Finally, short-term decisions typically dealing with
42 the management of business (i.e., with lasting effects of a few hours to a few days), like
43 the dynamic adjustment of the basic time slot offering and pricing or the definition of
44 detailed routing plans for the delivery of goods or the execution of services, are taken
45 at an *operational* level.
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

Our work makes a number of valuable contributions, namely:

- it extensively reviews the academic literature by distinguishing for the first time between AHD and AHS problems;
- it identifies three classes of problems depending on the extent of the integration between the demand management and the routing stages;
- it looks at these relevant classes of problems through the lens of real-world applications, with the aim of highlighting the main managerial leverages to set up and maintain a profitable business;
- it underlines the most significant future research directions in the AHD and AHS research field.

The remainder of the paper is organized as follows. Mathematical models and solution methods for demand management, routing, and integrated demand management and routing problems in AHD and AHS are reviewed in Sections 2, 3 and 4, respectively. Then, in Section 5 we draw some conclusions on the state of the art of AHD and AHS problems and discuss possible future research directions.

2 Demand Management Problems in AHD and AHS

The practice of Demand Management (DM) refers to those structural, price and quantity decisions that need to be taken in a business context. Similarly with the previously mentioned RM, DM has its origin in the early 1980s, when Robert Crandall, then American Airline's vice president of marketing, introduced the first principles of DM in the airline industry (Talluri and Van Ryzin, 2004). Since then, other industries adopted (and adapted) DM techniques, in sectors such as hospitality, transportation, and energy. As explained by Talluri and Van Ryzin (2004), all of these industries share similar conditions that motivate the adoption of DM: *customer heterogeneity, demand variability and uncertainty, production inflexibility, data and information system infrastructure*, and *management culture*. Many of these conditions may well be found in AHD and AHS systems, which probably explains why in recent years the practice of DM has become common in this industry.

A widely accepted classification of demand management decisions in AHD and AHS is the one proposed by Agatz et al. (2013). On one dimension, the authors distinguish between *slotting* and *pricing* decisions, that deal with the proposal of time slots to customers and the definition of prices for each time slot, respectively. On the other dimension, they distinguish between *differentiated* (or *static*) and *dynamic* decisions, where the former are taken off-line and are usually based on forecasts, while the latter are taken in real time.

The main difference between DM in traditional industries, where costs are generally supposed to be fixed, and DM in AHD and AHS, is that decisions taken at this level greatly affect the resulting routing costs. Therefore, even at an early stage, it is necessary to seek a trade-off between revenue maximization and cost balance, which is not trivial.

In this section, we review several demand management models proposed in the literature on AHD and AHS problems, where a routing part may be considered but is not the core of the research. An overview of the main characteristics of the reviewed

1 articles is provided in Table 1. A particular emphasis is put on real-world applications.
2 In addition, we highlight that column “Cost Estimation” includes both rather simple
3 methods, used to compute the additional routing cost while accepting an incoming
4 request, and more sophisticated methods, used to estimate the opportunity cost of ac-
5 cepting an incoming request and forgoing a potentially more profitable future request.

6
7 For a more detailed study on DM/RM, we refer the interested reader to the reviews
8 by [Strauss et al. \(2018\)](#) and [Klein et al. \(2020\)](#), where in the latter a specific section
9 is dedicated to innovative applications of RM in AHD.

10 2.1 Slotting Problems

11 Following the research avenue opened by [Asdemir et al. \(2009\)](#), [Yang et al.](#)
12 [\(2016\)](#), [Yang and Strauss \(2017\)](#), and [Klein et al. \(2018\)](#), which is discussed in
13 the subsequent sections on pricing and integrated demand management and rout-
14 ing problems, [Mackert \(2019\)](#) proposed a new approach for the dynamic Time Slot
15 Management Problem (TMSM), a tactical problem in AHD aimed at determining an
16 efficient set of time slots for each region within a delivery area with the objective of
17 minimizing the delivery costs while satisfying given service requirements. In particu-
18 lar, the author was the first to introduce a customer-choice model in the context of
19 slotting problems; namely, he used a General Attraction Model (GAM) (see, e.g., [Gal-
20 lego et al. 2015](#)), of which the Multinomial Logit (MNL), largely found in the stream
21 of literature on pricing problems, is a special case. The advantage of using the GAM,
22 instead of the MNL, is to avoid a potential overestimation of the choice probabilities
23 in particular settings. Another noteworthy contribution of this work is the definition
24 of a novel Mixed Integer Linear Programming (MILP) model to approximate the value
25 function, hence the opportunity costs, of the Dynamic Programming (DP) framework
26 underlying the slotting problem. In doing so, the author built upon the work of [Klein
27 et al. \(2018\)](#), combining insertion heuristics, for the computation of the routing costs
28 associated to already accepted orders, and a dynamic seed-based scheme, to estimate
29 the delivery costs of expected future orders. The resulting online slotting problem is
30 solved through a Linear Programming (LP) formulation derived from a Non-Linear
31 Binary Program. In the computational experiments performed using relaxed versions
32 of the proposed MILP model to favor real-time decisions, the results show a potential
33 increase of 4 to 7% in terms of average profit compared to benchmark policies.

34
35 The idea of adding flexibility to the slotting problem was introduced in the work
36 of [Köhler et al. \(2020\)](#), where the authors presented four alternative algorithmic ap-
37 proaches to derive the time slot offering for each incoming customer request. Their
38 main contribution was to investigate the effect of proposing both long time windows
39 (i.e., of 4 hours), to preserve a certain flexibility in building the tentative routing plan
40 during the booking horizon (especially in the early phases), and short time windows
41 (i.e., of 30 minutes), which are commonly used in the e-grocery business sector. The
42 results obtained on different demand scenarios (one derived from a German e-grocer)
43 were greatly affected by the customers’ willingness to accept long time windows,
44 but they showed a clear potential in terms of increased number of accepted orders
45 compared to the benchmark approach in which only short time windows are offered.
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

Table 1: Overview of the main characteristics of demand management problems in AHD and AHS

Sector	Real-World Application	Degree of Dynamism	Problem	Planning Horizon	Objective	Main Framework	Choice Model	Cost Estimation	Reference
E-grocery	No	Dynamic	Slotting	Operational	Max PR	LP	GAM	SB, MILP	Mackert (2019)
E-grocery	Yes	Dynamic	Slotting	Operational	Max AR	SIM	SP	IH	Köhler et al. (2020)
E-grocery	Yes	Dynamic	Slotting	Operational	Max RV	SIM	MNL	ADP	Lang et al. (2021a)
E-grocery	No	Dynamic	Slotting	Tact./Oper.	Multiple	SIM	MNL	IH	Lang et al. (2021b)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	DP	MNL	–	Asdemir et al. (2009)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	ADP	MNL	IH, SB	Klein et al. (2018)
E-grocery	Yes	Static	Pricing	Tactical	Max PR	MILP	GNR	SB	Klein et al. (2019)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	QP	SP	ADP	Vinsensius et al. (2020)
Large appl.	No	Static	Pricing	Operational	Min TC	DP	AP	–	Yilduz and Savelsbergh (2020)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	LP	MNL	CR	Strauss et al. (2021)

List of abbreviations Approximate Dynamic Programming (ADP), Acceptance Probabilities (AP), Number of Accepted Requests (AR), Cluster-first, Route-second (CR), Dynamic Programming (DP), General Attraction Model (GAM), General Nonparametric Rank-Based (GNR), Insertion Heuristics (IH), Linear Programming (LP), Mixed Integer Linear Programming (MILP), Multinomial Logit (MNL), Profit (PR), Quadratic Program (QP), Revenue (RV), Seed-Based (SB), Simulation (SIM), Selection Probabilities (SP), Total Cost (TC).

1 In the first of a series of papers on dynamic slotting, [Lang et al. \(2021a\)](#) studied
2 incremental modular approaches that rely on the idea of anticipating, through simula-
3 tion during an offline phase preceding the booking horizon, the information on delivery
4 schedules and opportunity cost. In particular, the authors solve a Team Orienteering
5 Problem with Multiple Time Windows to build anticipatory schedule patterns, while
6 they apply an Approximate Dynamic Programming (ADP) to estimate the oppor-
7 tunity cost (taking inspiration from the work of [Yang and Strauss 2017](#) on dynamic
8 pricing that is reviewed in the following section). During the online booking phase, an
9 Assortment Optimization Problem is solved to derive the set of time slots proposed
10 for each incoming request, adding a Theft-based mechanism to dynamically adjust de-
11 livery capacity by “stealing” extra capacity from neighboring areas of the previously
12 determined schedule patterns.

13 In their following work, [Lang et al. \(2021b\)](#) were the first to introduce the Multi-
14 Criteria Dynamic Slotting Problem, where they seek to (i) maximize revenue, (ii)
15 maximize the visibility of branded trucks, and (iii) maximize the social influence
16 produced by the most influencing groups of customers, using a scalarized objec-
17 tive function. The last two objectives are in line with marketing principles, but the
18 proposed approach is flexible and adaptable to other sets of criteria.

20 2.2 Pricing Problems

21 [Asdemir et al. \(2009\)](#) developed a dynamic pricing model that dynamically adjusts the
22 delivery prices of multiple delivery options over a discrete booking horizon according
23 to the remaining time, the residual capacity, and the affinity of customers with a par-
24 ticular class (which characterizes their arrival probability, expected profit, predictable
25 utility for each delivery option and price sensitivity). The authors adopt a Logit-based
26 model to reproduce the customer-choice behavior and a discrete-time, discrete-state
27 Markov Decision Process (MDP) to set the pricing decisions of the e-grocer. Using
28 simple examples, they demonstrate how optimal prices may change over time and how
29 an increase or decrease in terms of capacity can influence them, even in the case when
30 more than one class of customers is considered.

31 [Klein et al. \(2018\)](#) presented a novel MILP formulation to approximate the oppor-
32 tunity costs in dynamic pricing problems. In the proposed approach, which is repeated
33 in an iterative way for each customer request received within a discrete booking hori-
34 zon, the authors combine insertion heuristics (to compute the delivery cost for already
35 accepted orders), an MNL model (to anticipate expected customers’ reactions to future
36 pricing decisions and, consequently, estimate future revenues), a dynamic seed-based
37 approximation (to estimate the delivery costs of expected future orders), and the
38 MILP formulation (to approximate the value function of a customer request in a DP
39 framework). The results show an average increase in terms of total profits compared
40 to common policies (e.g., fixed price and order value-based), as well as the “Foresight
41 Policy” by [Yang et al. \(2016\)](#), which is considered as a benchmark policy. The so-
42 obtained total profit is on average 5.5% higher in the first case, and 2.3% higher in
43 the latter case. In addition, they find that a regular recalculation of the opportunity
44 costs is preferable rather than a periodic, less frequent recalculation.

1 Klein et al. (2019) were the first to address the problem of pricing from a tactical
2 perspective, proposing different variants of an exact MILP formulation for the Differentiated Time Slot Pricing Problem (DTSPP). In their work, motivated by an industrial
3 partnership with a German e-grocer, the customer-choice behavior is modeled using a
4 general nonparametric rank-based approach where the preferences of customers (as-
5 suming that all customers in a particular segment share the same preferences) are
6 expressed through simple preference lists of slot-price tuples. The restrictions imposed
7 by the DM problem are embedded into the MILP formulation in a first group of
8 constraints, while the restrictions imposed by the routing problem (namely, route con-
9 struction, demand and capacity, and time windows) are embedded into a second group
10 of constraints. Given the NP-hardness of the DTSPP, the authors proposed two alter-
11 native model-based approximations for the routing constraints, one seed-based (Fisher
12 and Jaikumar, 1981) while the other adapting and extending the approach found
13 in Agatz et al. (2011). After an extensive computational study, the authors show that
14 at a tactical level it is preferable to adopt model-based approaches that embed rout-
15 ing constraints. In fact, an early approximation of the delivery costs results in higher
16 profits compared to diffused practical pricing approaches. In this sense, a trade-off
17 between more accurate formulations, where the delivery cost approximation is more
18 elaborate at the expense of an increase in the integrality gap, and less accurate for-
19 mulations, where the delivery cost approximation is particularly rough but optimality
20 can be reached, needs to be found.

21
22 Vinsensius et al. (2020) developed an Incentive-Routing Optimization framework
23 for solving the dynamic pricing problem in AHD, where the pricing problem itself is
24 formulated as a Quadratic Programming (QP) model with the objective of maximiz-
25 ing the total expected profits. As in Campbell and Savelsbergh (2006), the authors
26 adopt a simple model to shape the customer-choice behavior, based on selection prob-
27 abilities and a linear response to incentives. The QP formulation receives as an input
28 the marginal fulfillment cost of each incoming order, which is computed through an
29 ADP mechanism. The boundary condition for the ADP is obtained by solving an
30 independent VRP with Service Choice for each time slot; to reduce the computa-
31 tional time, this particular sub-problem is solved using a minimum-regret construction
32 heuristic (Pisinger and Ropke, 2007). Compared to a “Free Choice” policy, where the
33 customers are free to choose their preferred time slot, and a “Myopic Incentive” pol-
34 icy, where the incentives are set based only on the QP model (with a myopic marginal
35 cost anticipation), the “ADP Incentive” approach proposed by the authors shows bet-
36 ter results in terms of total costs and fulfilled orders. The results are confirmed by a
37 sensitivity analysis on some parameters (e.g., order density, arrival probability, and
38 number of vehicles).

39
40 Yıldız and Savelsbergh (2020) studied the Pricing for Delivery Flexibility Problem
41 where, unlike in other reviewed articles, they seek to minimize the total expected cost
42 (which comprises both the delivery costs and the discounts offered to customers for
43 changing the delivery day). The idea is to increase the delivery flexibility by proposing
44 a discount to those customers that accept a different delivery day than the preferred
45 one, with the objective to reduce the delivery costs. To solve the problem, the authors
46 implemented an exact DP algorithm where the customer-choice behavior is modeled
47

1 through acceptance probabilities. Several computational experiments were performed
2 to evaluate the potential of cost reduction in the presence of different properties. The
3 results show an expected cost reduction of more than 30% in the best cases, albeit a
4 similar approach may be applicable only to those cases where the level of detail is the
5 delivery day and the demand volume is not too high (e.g., large appliances).

6 The opportunity of proposing flexible time slots (either adjacent or non-adjacent)
7 compared to single standard time slots is investigated in the work by [Strauss et al.](#)
8 [\(2021\)](#), where a dynamic pricing approach based on an LP formulation is developed.
9 The authors show how the offering of flexible time slots to customers may be beneficial
10 for companies in reducing delivery costs, as it gives them more flexibility to build
11 their routes. An additional and interesting insight regards the composition of the
12 proposed flexible time slots. Indeed, a combination of more popular and less popular
13 non-adjacent time slots is able to generate higher total profits compared to adjacent
14 time slots, especially when the capacity is tight relative to the demand.

15 A promising work that is worth mentioning and might open new directions for dy-
16 namic pricing implementations is the one by [Lebedev et al. \(2021\)](#), where the authors
17 studied several mathematical properties of the pricing problem, in the context of AHD,
18 that can be used to find closer approximations of the value function in DP algorithms.

20 **3 Routing Problems in AHD and AHS**

21 In the broad sense, the VRP consists in determining a set of minimum-cost routes
22 to serve a set of customer requests, given a starting depot, a fleet of vehicles, and
23 specific constraints depending on the application at hand. A rich body of literature
24 on the family of VRPs is available, as these problems have been widely studied for
25 more than 60 years and represent one of the main application areas in combinatorial
26 optimization. We refer to [Toth and Vigo \(2014\)](#) for an extensive review on the VRP
27 and its main variants, and to [Wang and Wasil \(2021\)](#) for a recent survey.

28 Given that they are associated with last-mile delivery operations, AHD and AHS
29 problems are strongly related to city logistics, as the majority of deliveries is nat-
30 urally condensed in populated urban areas. A detailed overview of VRPs arising in
31 city logistics is provided by [Cattaruzza et al. \(2017\)](#). In recent years, we have also
32 seen the emergence of new VRP variants in line with the increasing complexity and
33 variety of real-world applications; a brief overview of this topic can be found in the
34 survey of [Vidal et al. \(2020\)](#), where the authors focus on emerging metrics to evaluate
35 VRP solutions (which may give several hints for novel multi-criteria formulations),
36 integrated approaches where the VRP is linked to upstream decisions and sometimes
37 conceived as an evaluation tool for these decisions (which, to some extent, can be the
38 case of AHD and AHS applications), and refinements of existing models.

39 When we consider the routing stage of AHD and AHS problems, we are interested
40 in solving a Vehicle Routing Problem with Time Windows (VRPTW), in which ca-
41 pacity constraints are typically not binding if compared to time window constraints.
42 For state-of-the-art works on the VRPTW we refer to [Bräysy and Gendreau \(2005a\)](#)
43 for route construction methods and local search algorithmic techniques, [Bräysy and](#)
44 [Gendreau \(2005b\)](#) for metaheuristic algorithms, [Kallehauge \(2008\)](#) and [Baldacci et al.](#)
45

(2012) for exact solution approaches, Vidal et al. (2013) for an efficient hybrid genetic algorithm, and Desaulniers et al. (2014) for mathematical formulations, as well as exact and heuristic methods. Recently, new VRPTW extensions have emerged, by considering stochastic service times (Errico et al., 2018), multiple trips per vehicle and time-dependent travel times (Pan et al., 2021), as well as synchronized visits (see, e.g., Liu et al. 2019 and Polnik et al. 2021). In addition, the Electric VRPTW has received much attention for its practical implications (see, e.g., Schneider et al. 2014, Desaulniers et al. 2016, Hiermann et al. 2016, Keskin and Çatay 2016, 2018, Keskin et al. 2019, 2021, Duman et al. 2022, and Lam et al. 2022).

In multi-stage AHD and AHS problems, the VRPTW may be used as a boundary condition in a DP framework, where the selected customer-choice model most of the times is an MNL model and a VRPTW must be solved for each state to update such boundary condition. However, this makes the AHD/AHS problem intractable due to the NP-hardness of the VRPTW (see, e.g., Savelsbergh 1985). This drawback can be partially overcome, at the expense of optimality, by applying approximate techniques.

The anticipation of the routing costs during the demand management stage is another critical aspect in AHD and AHS problems. As described in more detail in Section 4.1, an early approximation of the routing cost leads to higher profits compared to pure revenue management approaches that are still diffused in practice. This idea was also investigated by Bühler et al. (2016), who proposed four MILP models, all based on the Set Covering formulation for the VRP. The four models are conceived to be integrated into more developed DM models as “plug-in” modules to anticipate the estimation of the routing costs. The results show that the proposed models, decremental in terms of decision variables and constraints, approximate well the routing costs (i.e., the overestimation is no more than 10% compared to benchmark exact models, and slightly less than 3% compared to benchmark heuristics) in an acceptable computational time, thus being promising for real-world applications and suitable for decision support at a tactical level. In the aforementioned work by Klein et al. (2019), the authors built on these preparatory findings by introducing a routing module into their MILP formulation for the DTSP.

Since a detailed review of routing problems would be too ambitious, we limit the scope of this section to the main routing models developed to solve specific AHD and AHS problems. An overview of the main characteristics of the reviewed articles is provided in Table 2. We remark that a particular emphasis is put on real-world applications.

3.1 Routing Problems in AHD

In the first work of a series of articles on VRPTW variants for AHD problems, Azi et al. (2007) defined the Single-Vehicle Routing Problem with Time Windows and Multiple Routes (S-VRPMTW), where during a typical workday a single vehicle performs multiple routes of short duration for the delivery of perishable goods. Given the impossibility of serving all customers within the required time window, the multiple objectives are to maximize the number of customers served and to minimize the

Table 2: Overview of the main characteristics of routing problems in AHD and AHS

Sector	Real-World Application	Planning Horizon	Objective	Model Structure	Constraints	Solution Method	Reference
E-grocery	No	Operational	Multiple	MILP	CP, TW	2-SA	Azi et al. (2007)
E-grocery	No	Operational	Multiple	MILP	CP, TW	BP	Azi et al. (2010)
E-grocery	No	Operational	Multiple	MILP	CP, TW	ALNS	Azi et al. (2014)
E-grocery	No	Operational	Max PR	MILP	CP, TW	ALNS	Azi et al. (2012)
Multiple	No	Operational	Multiple	-	CP, TW	TS, LP	Jabali et al. (2015)
Online retail	No	Operational	Multiple	MILP	TW	ALNS	Ozariq et al. (2021)
Meal delivery	Yes	Operational	Min CCo	MILP	S-CS, T-CS, Ctd	CRG	Yildiz and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min Ctd	MILP	S-CS, T-CS	CRG	Yildiz and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min Rtd	MILP	S-CS, T-CS, Ctd	CRG	Yildiz and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min C+DO	MILP	S-CS, T-CS	CRG	Yildiz and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min Rtp	MILP	S-CS, T-CS, Ctd	CRG	Yildiz and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min ESD	MDP	DD	ACA	Ulmer et al. (2021)
Large appl.	Yes	Operational	Min TC	MILP	CP, TW, PC, SYN	ALNS	Ali et al. (2021)
Home health.	No	Operational	Multiple	MILP	TW, PC, SYN	LBH	Bredström and Rönnqvist (2008)
Home health.	Yes	Tact./Oper.	Maxmin	MILP	SK, CCa, WL	PGP	Cappanera and Scutellà (2015)
Home health.	Yes	Tact./Oper.	Minmax	MILP	SK, CCa, WL	PGP	Cappanera and Scutellà (2015)
Home health.	Yes	Tact./Oper.	Minmax	MILP	SK, CCa, WL	PGP, RO	Cappanera et al. (2018)
Home health.	No	Operational	Min TC	MILP	TW, WT, OT	TS	Zhan and Wan (2018)
Home health.	Yes	Operational	Min TC	MILP	SK, TW, OT, IT	LNS, SPP	Grenouilleau et al. (2019)
Home health.	No	Operational	Min TC	MILP	TW, WT, OT	LM	Zhan et al. (2021)
Maintenance	Yes	Operational	Min TC	MILP	TW, TB, SK	ALNS	Kovacs et al. (2012)
Maintenance	No	Operational	Min Makespan	MDP	EL, PC	RTR	Chen et al. (2016)
Maintenance	Yes	Operational	Min TC	MILP	TW, TB, SK, WT, OT	BP	Zamorano and Stolletz (2017)
Maintenance	Yes	Operational	Multiple	MILP	SK, PC, IN, DT, TW, BR	BB	Mathlouthi et al. (2018)
Maintenance	Yes	Operational	Multiple	MILP	SK, PC, IN, DT, TW, BR	BP	Mathlouthi et al. (2021a)
Maintenance	Yes	Operational	Multiple	MILP	SK, PC, IN, DT, TW, BR	TS	Mathlouthi et al. (2021b)

List of abbreviations Two-phase Solution Approach (2-SA), Anticipatory Customer Assignment (ACA), Adaptive Large Neighborhood Search (ALNS), Branch-and-Bound (BB), Branch-and-Price (BP), Breaks (BR), Continuity of Care (CCa), Courier Compensation (CCo), Capacity (CP), Column- and Row-Generation (CRG), Click-to-Door Time (Ctd), Click-to-Door Coverage (CtDO), Delivery Deadline (DD), Distance Traveled (DT), Experience Level Constraints (EL), Expected Sum of the Delay (ESD), Inventory (IN), Idle Time (IT), Local Branching Heuristic (LBH), L-shaped Method (LM), Large Neighborhood Search (LNS), Linear Programming (LP), Markov Decision Process (MDP), Mixed Integer Linear Programming (MILP), Overtime (OT), Precedence (PC), Pattern Generation Policy (PGP), Profit (PR), Robust Optimization (RO), Ready-to-Door Time (Rtd), Ready-to-Pickup Time (RtP), Record-To-Record Travel Algorithm (RTR), Spatial Consistency (S-CS), Skill (SK), Set Packing/Partitioning Problem (SPP), Synchronization (SYN), Time Consistency (T-CS), Team Building (TB), Total Cost (TC), Tabu Search (TS), Time Windows (TW), Workday Length (WL), Waiting Time (WT).

1 total distance (for the same number of customers served). The problem is solved using
2 a two-phase solution approach based on the exact algorithm for the Elementary
3 Shortest Path Problem (ESPP) proposed by [Feillet et al. \(2004\)](#).

4 In their second paper, [Azi et al. \(2010\)](#) defined a multiple-vehicle generalization
5 of the S-VRPMTW, named the Vehicle Routing Problem with Time Windows and
6 Multiple Routes (VRPMTW). Here, the multiple objectives are to maximize the total
7 revenue and to minimize the total distance, and the problem is solved via Branch-and-
8 Price (BP). In particular, the primary problem is a Set Partitioning Problem (SPP)
9 formulation solved through column generation, while the pricing subproblem is an
10 ESPP solved using the aforementioned algorithm by [Feillet et al. \(2004\)](#).

11 A few years later, [Azi et al. \(2014\)](#) solved the VRPMTW by means of an Adaptive
12 Large Neighborhood Search (ALNS) algorithm. Interestingly, the authors demon-
13 strate the advantage of applying destruction and insertion operators at different levels
14 (customer, route, and workday) instead of using only customer-based operators.

15 Building upon their previous results, [Azi et al. \(2012\)](#) solved the dynamic
16 VRPMTW, where the source of dynamicity is given by the arrival of new customer
17 requests during the operational horizon. Note that such requests are inserted in future
18 routes, as the current ones are fixed. Compared to the previously mentioned ALNS, a
19 dynamic environment (in which the acceptance rule is slightly modified to take care
20 of dynamicity) and an event management mechanism (to handle different types of
21 events) were added. The results show that the proposed non-myopic approach (where
22 future requests are considered) outperforms the myopic approach (where future re-
23 quests are not considered) in terms of profit (computed as the total revenue associated
24 with the served customers minus the total distance), percentage of served customers,
25 number of routes per day, and number of customers per route, at the expense of con-
26 siderably higher computational times (however acceptable and compatible with the
27 response time required by an offline real-world application).

28 An interesting characteristic introduced by [Jabali et al. \(2015\)](#) is the use of self-
29 imposed endogenous time windows rather than the exogenous ones typically considered
30 in the VRPTW literature. Those self-imposed time windows are assigned to the cus-
31 tomers by the company which, in turn, is committed to respecting them. A similar
32 approach may be applicable to sectors like online retail, large appliances and furniture,
33 as well as utilities. Another important feature included in this work is the presence of
34 stochastic travel times that are dependent on a random variable representing a non-
35 negative delay. Such delay is added to the base travel time. To solve the problem, the
36 authors proposed a collaborative two-stage hybrid algorithm. First, the routing part
37 is solved via Tabu Search (TS) using three alternative criteria for choosing a move.
38 Second, the scheduling part, which takes as an input the solution found at the previ-
39 ous stage, is solved through an LP formulation that includes buffer times to handle
40 the uncertainty given by the adoption of stochastic travel times. From a practical per-
41 spective, the use of self-imposed time windows may represent an unconventional policy
42 (compared to the common practice of letting customers select their favorite time win-
43 dows) to lighten the time window constraints, thus reducing the operating costs while
44 keeping a certain service level.
45
46
47
48
49

1 Resuming the idea originally proposed by [Pan et al. \(2017\)](#) of using customer-
2 related data to improve the effectiveness of AHD systems, [Ózarık et al. \(2021\)](#)
3 defined the Vehicle Routing and Scheduling Problem with Time-Dependent Costs
4 (VRSPDTC). The problem is a variant of the VRPTW, as it adds a time-dependent
5 penalty cost to the objective function. Such penalty cost is directly linked to the so-
6 called “customer availability profiles” (introduced for the first time by [Florio et al.](#)
7 [2018](#)) that identify, for each customer, the probability of being present at home when
8 the delivery is performed. In case the customer is absent during the first attempt of de-
9 livery, the authors assume that the next attempt is outsourced to an external courier,
10 thus causing additional costs. From a practical perspective, the issue of low hit rates
11 (i.e., frequent unsuccessful deliveries due to the absence of customers) is still one of
12 the most significant problems in last-mile delivery. The VRSPDTC is solved using an
13 ALNS-based metaheuristic algorithm with several removal and insertion operators.
14 The results indicate the existence of a trade-off between the minimization of travel
15 costs and the increase of hit rates. However, by taking advantage of customer-related
16 data, it is possible to reach relevant cost savings. In particular, introducing the infor-
17 mation on customer availability, in combination with the practice of waiting before
18 serving a customer, may generate up to 40% in cost savings. Last but not least, the
19 ALNS-based algorithm produced good results in comparison with a state-of-the-art
20 MILP solver, and showed short computational times, which is desirable for a potential
21 real-world application.
22

23 **3.1.1 A Focus on the Meal Delivery Routing Problem**

24
25 Given the outstanding expansion of the food delivery sector in the last few years, a
26 necessary exception from the main scope of our work is required for the Meal Delivery
27 Routing Problem (MDRP). This problem is part of AHD (in the sense that the cus-
28 tomer must be present at home for the delivery of food), but it also comprises typical
29 elements of SDD (with new requests coming during the operational horizon) as well
30 as the use of innovative practices arising in last-mile logistics, like crowdshipping and
31 bundle generation. For an overview on last-mile delivery challenges and, in particular,
32 routing problems with crowdshipping we refer to [Archetti and Bertazzi \(2021\)](#), while
33 for a recent work on routing with bundle generation and occasional drivers we refer
34 to [Mancini and Gansterer \(2022\)](#).
35

36 Among the first to study the MDRP, [Yıldız and Savelsbergh \(2019\)](#) introduced a
37 mathematical formulation which is adaptable to different objectives that may be worth
38 considering for an online food ordering and delivery platform (e.g., courier compensa-
39 tion, click-to-door time, ready-to-door time, click-to-door overage, and ready-to-pickup
40 time). Interestingly, their work is based on the concept of work package, which is
41 a possible way to serve a bundle of orders. To solve the problem, the authors im-
42 plemented a column- and row-generation algorithm, enhanced by a selective column
43 inclusion scheme, that proved to be effective on the MDRPLIB instance set publicly
44 made available by Grubhub (an American online ordering and delivery platform and a
45 subsidiary of Just Eat Takeaway). In addition, a noteworthy analysis reported by the
46 authors demonstrates that guaranteeing a minimum-pay to couriers does not cause a
47 dramatic increase in terms of total cost (i.e., 9% in the worst case); to the contrary,
48
49

1 it ensures a large availability of couriers. In our opinion, such an analysis may well
2 contribute to the wide debate on policies for platform workers.

3 The Restaurant Meal Delivery Problem (RMDP) was addressed by [Ulmer et al.](#)
4 [\(2021\)](#). Inspired by the previous work of [Ulmer et al. \(2020\)](#), the authors defined the
5 RMDP as a route-based MDP, solving it by means of an Anticipatory Customer As-
6 signment (ACA) heuristic algorithm. Such an approach was strengthened by the use
7 of time buffering and postponement to soften the effects of stochasticity and dynamic-
8 ity. The proposed policy was tested in an extensive computational study on real-world
9 data from Iowa City. In comparison with the common-sense benchmark policy of as-
10 signing an incoming order to the driver that is able to deliver it as fast as possible,
11 which is typically used in current practice, the results show that the ACA, relying on
12 both time buffering and postponement, achieves strong improvements in terms of total
13 delay. In particular, the use of time buffering itself produces significant improvements,
14 as it decreases the effects of uncertain events. With the addition of postponement,
15 it is also possible to take advantage of newly collected information which favor the
16 assignment, as well as the bundling, of orders. From a practical perspective, the pro-
17 posed algorithm proved to be robust in the presence of variability and suitable to solve
18 real-world problems.

21 **3.2 Routing Problems in AHS**

22 In this section, we are interested in reviewing some recent articles on routing problems
23 for AHS.

24 A particularly interesting problem at the intersection between AHD and AHS is the
25 Delivery Installation and Routing Problem (DIRP) investigated by [Ali et al. \(2021\)](#).
26 The DIRP is inspired by a real-world application encountered in the sector of large ap-
27 pliances and furniture, where the deliveries and the installations are performed by two
28 heterogeneous fleet of deliverymen and installers, respectively. This particular applica-
29 tion requires the synchronization of worker skills and is characterized by the presence
30 of temporal precedence constraints (i.e., an installer must wait for a deliveryman to
31 complete the delivery service before reaching the location of a customer and starting
32 the installation service). In some cases, the installation may be directly performed by
33 the deliveryman (with a lower efficiency as such figure is less specialized than an in-
34 staller). The authors defined the DIRP using a flexible MILP formulation, from which
35 specific variants of the VRP can be easily derived (i.e., in case all the installations are
36 performed only by deliverymen we refer to the VRP with time windows and driver-
37 specific times, while in case all the installations are performed only by installers we
38 refer to the VRP with multiple synchronization constraints). In addition, a variant of
39 the DIRP was discussed in which the deliveryman and the installer can perform an
40 installation together (instead of assuming that only one worker can perform the in-
41 stallation, as in the previous case). To solve the problem, the authors implemented an
42 ALNS algorithm and compared its performance with that of the MILP formulation
43 solved by a commercial solver. The results show that the ALNS algorithm is able to
44 find good-quality solutions in short computing times both for test instances, as well as
45 for real-world instances obtained from an industrial partner. Two noticeable insights
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

1 emerged from the sensitivity analysis performed by the authors. The first is that using
2 two heterogeneous fleets of deliverymen and installers has a positive impact in
3 terms of total routing cost reduction. The second demonstrates the existence of a cor-
4 relation between the efficiency of the deliverymen and the percentage of installations
5 performed by the installers.

6 **3.2.1 A Focus on the Home Healthcare Routing and Scheduling** 7 **Problem**

8
9 Given their practical implications, we cannot forget to mention relevant works, in the
10 context of home care services, on service planning and patient-to-nurse assignment.
11 Among these, we refer to [Eveborn et al. \(2006, 2009\)](#), where the authors described a
12 DSS developed for the Swedish healthcare system, which is based on an SPP formu-
13 lation and a repeated matching algorithm for optimizing the generation of attended
14 home visiting schedules. Another noticeable work is that of [Duque et al. \(2015\)](#), where
15 the case of *Landelijke Thuiszorg*, a Belgian home care service provider, is described.
16 For what concerns the assignment of patients to traveling nurses, [Hertz and Lahrichi](#)
17 [\(2009\)](#) developed an assignment algorithm to solve a real-world problem arising in a
18 small area of Montréal (Québec), while [Carello and Lanzarone \(2014\)](#) and [Lanzarone](#)
19 [and Matta \(2014\)](#) addressed the robust nurse-to-patient assignment problem by fo-
20 cusing on structural policies to guarantee the continuity of care (which means that a
21 patient must be visited by a restricted group of caregivers). For more references on
22 routing and scheduling problems in home healthcare we refer the interested reader to
23 the surveys by [Fikar and Hirsch \(2017\)](#) and by [Euchi et al. \(2022\)](#).

24
25 Starting from the real-world application described by [Eveborn et al. \(2006,](#)
26 [2009\)](#), [Bredström and Rönnqvist \(2008\)](#) defined a novel MILP formulation for the
27 Vehicle Routing and Scheduling Problem with Time Windows (VRSPWTW). The pe-
28 culiarity of the VRSPWTW is given by the presence of pairwise temporal precedence
29 constraints and pairwise synchronization constraints. As discussed by the authors,
30 similar constraints may be found in homecare staffing and scheduling problems, where
31 different staff members are required to visit a patient one after the other or simultane-
32 ously. The problem was solved using a local branching heuristic inspired by [Fischetti](#)
33 [et al. \(2004\)](#). This solution method was tested by considering alternative objective
34 functions (i.e., minimization of preferences, minimization of traveling time, minimiza-
35 tion of maximal difference in workload among staff members, or minimization of a
36 weighted sum of multiple objectives).

37
38 [Cappanera and Scutellà \(2015\)](#) addressed the Palliative Home Care Problem
39 (PHCP), an important problem arising in home healthcare that refers to the provision
40 of palliative therapies to terminal patients. The authors modeled the PHCP through
41 an MILP formulation where assignment, scheduling and routing decisions are taken in
42 an integrated fashion. Two alternative objective functions, *maxmin* (which balances
43 the operator workload by maximizing the minimum utilization factor) and *minmax*
44 (which balances the operator workload by minimizing the maximum utilization fac-
45 tor), were defined and used to guide the solution process. The MILP formulation was
46 strengthened with the addition of symmetry breaking constraints and valid inequali-
47 ties. To solve the PHCP, the authors implemented three alternative pattern generation
48

1 policies (a greedy heuristic procedure, a realistic procedure based on current practice,
2 and a flow-based model), where patterns are alternative schedules of visits that are
3 generated a priori for each patient. The generated patterns are given as input to the
4 MILP formulation that solves the original PHCP. This approach proved to be effective
5 on different sets of realistic instances. From a practical perspective, it is worth
6 highlighting that the selection of *maxmin* as the objective function of the MILP formulation
7 produces more balanced solutions in terms of workload among operators. On the contrary,
8 the selection of *minmax* produces less costly solutions, as the total travel time for the operators is minimized.
9

10 Extending the previous work by [Cappanera and Scutellà \(2015\)](#), [Cappanera et al. \(2018\)](#)
11 generalized the Home Care Problem (HCP) by taking into account demand uncertainty. In particular,
12 the authors adopted the cardinality-constrained framework proposed by [Bertsimas and Sim \(2004\)](#)
13 to define the sequence-preserving Γ -Robust Home Care Problem (sRHC $_{\Gamma}$). In this robust version of the HCP,
14 uncertainty is handled by considering additional uncertain requests; among these, at most Γ requests
15 must be inserted into each solution tour (where Γ is a given parameter). The decisions
16 of the sRHC $_{\Gamma}$ are guided by the aforementioned *minmax* objective function. The proposed
17 approach turned out to produce more robust solutions compared to the nominal formulation,
18 showing a high degree of fairness in terms of caregiver utilization factor and a low approximation error.
19 The authors also experimented with a decomposition approach by fixing the scheduling decisions. This
20 approach proved to be suitable for solving larger instances.
21

22 [Zhan and Wan \(2018\)](#) defined the Routing and Appointment Scheduling with Team
23 Assignment (RASTA) problem, which arises in the context of home healthcare and
24 integrates decisions on team assignment, routing and scheduling. The authors formulated
25 the RASTA as an MILP model and solved it by implementing a TS algorithm, where the initial
26 feasible routing schedule is built using a modified parallel savings algorithm. This initial
27 solution is then improved by invoking classical local search operators (e.g., 2-opt, relocate and Or-opt)
28 until a termination criterion is reached, while the customers' appointment times are determined
29 by solving a scenario-based LP formulation (which considers the routing schedule as an input).
30 The stochastic information on service times was estimated based on the results found by
31 [Lanzarone et al. \(2010\)](#). The proposed methodology proved to be effective on small sets of
32 randomly generated instances, leaving room for potential extensions. In their following
33 work, [Zhan et al. \(2021\)](#) focused on the Routing and Appointment Scheduling problem
34 by defining a novel MILP formulation and solving it via the L-shaped method. Additionally,
35 a heuristic algorithm to handle large-size instances was also developed.
36

37 Motivated by a collaboration with Alayacare, a Canadian start-up based in
38 Montréal (Québec), [Grenouilleau et al. \(2019\)](#) studied the Home Health Care Routing
39 and Scheduling Problem (HHCRSP). In particular, the authors defined the problem
40 as an SPP with the objective of selecting the best daily routes for each caregiver. Such
41 routes are built by taking into account the patients' mandatory requirements, the caregivers'
42 skills, and the required time windows. Several objectives, such as the number of missing
43 optional requirements, the travel time, the continuity of care, and a penalty for non-compliance
44 with minimum and maximum working hours, are inserted
45
46
47
48

1 into the weighted sum cost function that is computed for each route. The weekly over-
2 time and idle time for each caregiver, and the number of unscheduled visits are then
3 added in the overall objective function of the SPP formulation as additional objec-
4 tives. A Large Neighborhood Search (LNS) algorithm is used to find the set of feasible
5 routes that are given as input to a relaxed version of the SPP, after which a construc-
6 tive heuristic algorithm is called to rebuild the integrality of solutions. Interestingly,
7 the proposed approach outperformed Alayacare’s current solution by 37% in terms of
8 total travel time and 16% in terms of continuity of care, thus proving to be effective
9 in solving real-world instances. The HHCRSP with temporal dependencies under uncer-
10 tainty was later addressed in the work of [Shahnejat-Bushehri et al. \(2021\)](#), where
11 the authors defined the problem using a robust optimization model and solved it by
12 implementing three alternative metaheuristic algorithms.

13 **3.2.2 A Focus on the Technician Routing and Scheduling Problem**

14 Starting from the problem formulation given by [Cordeau et al. \(2010\)](#) and motivated
15 by a collaboration with an infrastructure service provider, [Kovacs et al. \(2012\)](#) were
16 among the first to address the Service Technician Routing and Scheduling Problem
17 (STRSP). In particular, the authors presented an MILP formulation for the STRSP
18 and implemented two alternative versions of an ALNS algorithm, one without team
19 building and the other with team building. Both ALNS algorithms rely on several de-
20 stroy and repair operators from the literature. The proposed algorithms were tested on
21 benchmark instances as well as on real-world instances, showing a significant average
22 cost reduction of almost 11% compared to the manual plans adopted by the company.
23 Other pioneering works on the Technician Routing and Scheduling Problem (TRSP)
24 and the Technician Dispatching Problem (TDP) that are worth mentioning are those
25 by [Pillac et al. \(2013\)](#) and [Cortés et al. \(2014\)](#), respectively.

26 Later, [Chen et al. \(2016\)](#) studied a novel problem variant, named Technician Rout-
27 ing and Scheduling Problem with Experienced-based Service Times. Here, the authors
28 formally described the problem as an MDP, and developed a myopic solution approach
29 based on a daily routing problem solved with a metaheuristic algorithm. The notewor-
30 thy contribution of this work is to consider, for the first time in the routing literature,
31 different learning curves and heterogeneity of technicians and to derive some “rules
32 of thumb” that can be used from a managerial perspective. In particular, the results
33 demonstrate the advantage of considering learning curves and heterogeneity of techni-
34 cians instead of static productivity. In addition, the authors emphasize the idea that
35 the routing aspect should be favored in the presence of fast-learning and experienced
36 technicians, while the scheduling aspect should be favored in the presence of slow-
37 learning and inexperienced technicians. In their following works, [Chen et al. \(2017,](#)
38 [2018\)](#) addressed the Multi-period Technician Scheduling Problem with Experienced-
39 based Service Times and Stochastic Customers by focusing on the problem of assigning
40 tasks to technicians and omitting the routing component. In particular, the authors
41 proposed an ADP-based solution approach, in which the so-called “cost-to-go” is
42 computed by looking ahead both one period and over the entire planning horizon.

43 Motivated by the real-world case of an external maintenance provider specialized
44 in electric forklifts, [Zamorano and Stolletz \(2017\)](#) defined the Multiperiod Technician
45
46
47
48

1 Routing and Scheduling Problem (MPTRSP) and solved it using two alternative BP
2 algorithms based on different decomposition schemes (i.e., a day decomposition and
3 a team-day decomposition). Compared to the literature on Workforce Scheduling and
4 Routing, of which the MPTRSP is a generalization, the novel contribution of this
5 work is to consider multiple periods and team building simultaneously. The numerical
6 experiments conducted on test instances show that the BP algorithm based on the
7 team-day decomposition scheme, which results in more but easier-to-solve subprob-
8 lems, performs better in terms of computing times and gap to optimality. The same
9 experiments are repeated on real-world as well as larger instances, confirming the ef-
10 fectiveness of the proposed solution approach. Additional experiments conducted on
11 other test instances indicate a negative correlation between time window length and
12 overall costs, which is noticeable from a practical perspective, and a positive correlation
13 between time window length and computing times.

14 In the first of a series of papers on technician routing and scheduling, [Mathlouthi
15 et al. \(2018\)](#) presented a novel MILP formulation for a Multi-attribute Technician
16 Routing and Scheduling Problem (MATRSP) solving it using a commercial solver.
17 This work is motivated by a real-world application arising at a company providing
18 maintenance and repair services for electronic transaction equipment. The noteworthy
19 contribution of this work is to accurately define a complex problem by combining a
20 number of heterogeneous characteristics (required skills, precedence constraints for
21 special parts, inventory levels for spare parts, maximum traveled distance, breaks, and
22 time windows). Several computational experiments are performed to assess the effect
23 of certain parameter variations, such as the percentage of special parts, technician
24 skills, the impact of service times, and the number of technicians.

25 In their following work, [Mathlouthi et al. \(2021a\)](#) implemented a BP algorithm
26 to solve the MATRSP. As in [Azi et al. \(2010\)](#), the primary problem is formulated as
27 an SPP, while the pricing subproblem is an Elementary Shortest Path Problem with
28 Resource Constraints (ESPPRC) which is solved using both the algorithm by [Feillet
29 et al. \(2004\)](#) and the Decremental State-Space Relaxation (DSSR) algorithm by [Righ-
30 ini and Salani \(2008\)](#). Also, two alternative branching strategies are presented here.
31 The results demonstrate that the DSSR implementation with the ternary branching
32 strategy obtains the best results. In addition, the BP algorithm proved to solve to op-
33 timality larger instances (with up to 45 tasks) as compared to the MILP formulation
34 presented in [Mathlouthi et al. \(2018\)](#) and solved with a commercial solver.

35 [Mathlouthi et al. \(2021b\)](#) developed a TS metaheuristic algorithm with adaptive
36 memory for the MATRSP. Interestingly, the algorithm found the same optimal values
37 as the exact method by [Mathlouthi et al. \(2021a\)](#) for instances with up to 45 tasks
38 and solved instances with up to 200 tasks within 2 hours, which is compatible with
39 practical implementations.
40
41

42 **4 Integrated Demand Management and Routing** 43 **Problems in AHD and AHS**

44 Many authors have been approaching the field of integrated demand management and
45 routing from their methodological backgrounds since the mid-2000s. In Section 4.1,
46
47
48

1 we review the most relevant articles in the literature on AHD and AHS and give an
2 overview of their main characteristics in Table 3. In section 4.2, we then focus on the
3 Time Window Assignment Vehicle Routing Problem (TWAVRP).

4 4.1 Integrated Problems 5

6 Although the authors do not refer directly to the problem of integrating demand
7 management and routing, the paper of [Bent and Van Hentenryck \(2004\)](#) may be
8 considered a pioneering work in this area, as it anticipates the idea of using stochastic
9 information in the decision to accept or reject a request. Indeed, the Scenario Based
10 Planning Approach (SBPA) to dynamic stochastic VRPTW they proposed fits well
11 with the ordering phase of AHD problems that precedes the cutoff time, when the order
12 requests arrive and must be accepted or rejected. Also, the SBPA may be applied for
13 practical implementations of maintenance and repair services, where it is not known
14 a priori when the next call will arrive. The basic principle of SBPA is to keep in
15 memory a set of routing plans that are updated at each execution step. These routing
16 plans are generated by considering information on already known requests as well as
17 possible future requests. The plan to be implemented is then selected by means of a
18 so-called consensus function. The experimental results show that the SBPA performs
19 well compared to less sophisticated methodologies in terms of number of customers
20 served and number of vehicles used.
21

22 Among the first to see a potential in the integration between order promise and
23 order delivery phases, [Campbell and Savelsbergh \(2005\)](#) proposed several insertion-
24 based heuristics for AHD problems. In particular, the authors developed a number
25 of probability-based heuristics where the information on potential future orders is
26 considered in the decision to either accept or reject an order. Compared to the common
27 practice of accepting a fixed number of orders per time slot and using simple dynamic
28 insertion heuristics, the proposed probability-based heuristics are constantly more
29 efficient in capturing the economic profitability of incoming requests. The authors
30 extensively tested such heuristics by varying some experimental characteristics. In
31 many cases, the probability-based heuristics were able to come very close to the results
32 obtained in the presence of perfect information and, except in one case, they showed
33 computational times that are compatible with practical implementations.
34

35 Building upon their previous work (i.e., [Campbell and Savelsbergh 2005](#)), [Camp-](#)
36 [bell and Savelsbergh \(2006\)](#) addressed the use of incentive schemes to steer customer
37 behavior in AHD services. In particular, the authors propose two alternative LP for-
38 mulations to solve the Home Delivery Problem with Time Slot Incentives and the
39 Home Delivery Problem with Wider Slot Incentives, respectively, that do not incorpo-
40 rate a proper customer-choice model but use, instead, simple selection probabilities.
41 In both formulations, an estimation of the delivery costs of accepted orders, performed
42 using a combination of insertion heuristics and randomization, is inserted in the objec-
43 tive function. In addition, the feasibility of the routes under construction is checked.
44 Interestingly, the results show that companies could take advantage from the use of
45 incentive schemes to reduce delivery costs and, consequently, increase profits even in
46 the early stages of the decision process. The authors also demonstrate that developing
47 incentives schemes for wider time slots is easier and has the potential to produce an
48

Table 3: Overview of the main characteristics of integrated demand management and routing problems in AHD and AHS

Sector	Real-World Application	Planning Horizon	Objective	Modeling Approach	Constraints	Choice Model	Solution Method	Reference
Maintenance	No	Operational	Max AR	-	TW	-	SBPA	Bent and Van Hentenryck (2004)
E-grocery	No	Operational	Max PR	-	TW, IL	RP	IH	Campbell and Savelsbergh (2005)
E-grocery	No	Operational	Max PR	LP	CP, TW	SP	LP, IH	Campbell and Savelsbergh (2006)
E-grocery	Yes	Tactical	Min RC	CA	CP, TW	-	GH, CR	Agatz et al. (2011)
E-grocery	Yes	Tactical	Min RC	ILP	CP, TW	-	ILP, SB	Agatz et al. (2011)
Online retail	Yes	Operational	Max AR	-	TW	-	SIM, IH	Ehmke and Campbell (2014)
E-grocery	Yes	Operational	Max PR	DP	-	MNL	GH, IH	Yang et al. (2016)
Large appl.	No	Tactical	Min RC	MILP	CP, TW	-	TS	Hernandez et al. (2017)
Online retail	No	Operational	Multiple	MILP, SDP	CP, TW	-	TS, ADP	Han et al. (2017)
E-grocery	Yes	Operational	Max PR	DP	-	MNL	ADP, CR	Yang and Strauss (2017)
Utilities	Yes	Tactical	Min RC	ILP	CP, TW	SS	LNS, ILP	Bruck et al. (2018)
Online retail	Yes	Tact./Oper.	Min TC	2-SP	CP, TW, DT, NB	-	MLM	Restrepo et al. (2019)
Utilities	Yes	Strat./Tact.	Min RC	MILP	CP, TW	-	LNS, ILP	Bruck et al. (2020)
E-grocery	No	Operational	Max PR	DP	-	MNL	ADP, IH, GH	Koch and Klein (2020)
E-grocery	Yes	Operational	Max PR	DP	-	MNL	ADP, IH, SA	Abdollahi et al. (2023)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	-	BFC	Spliet and Gabor (2015)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	-	BFC	Spliet and Desaulniers (2015)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	-	BFC	Spliet et al. (2018)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	-	BC	Dalmeijer and Spliet (2018)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW, MP, SD	-	3-SA	Neves-Moreira et al. (2018)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	-	SDA	Subramanyam et al. (2018)
Maintenance	No	Tact./Oper.	Multiple	2-SP	CP, TW	-	ALNS	Vareias et al. (2019)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	-	BFC	Dalmeijer and Desaulniers (2021)
Multiple	No	Tact./Oper.	Min TWVI	RO	CP, TW	-	BC	Hoogeboom et al. (2021)

List of abbreviations Two-stage Stochastic Programming (2-SP), Three-phase Solution Approach (3-SA), Approximate Dynamic Programming (ADP), Adaptive Large Neighborhood Search (ALNS), Number of Accepted Requests (AR), Branch-and-Cut (BC), Branch-Price-and-Cut (BPC), Continuous Approximation (CA), Capacity (CP), Cluster-first, Route-second (CR), Dynamic Programming (DP), Distance Traveled (DT), Expected Total Cost (ETC), Greedy Heuristic (GH), Insertion Heuristics (IH), Incentive Limit (IL), Integer Linear Programming (ILP), Large Neighborhood Search (LNS), Linear Programming (LP), Mixed Integer Linear Programming (MILP), Multicut L-shaped Method (MLM), Multinomial Logit (MNL), Multiple Product (MP), Neighboring (NB), Profit (PR), Routing Costs (RC), Robust Optimization (RO), Realization Probabilities (RP), Simulated Annealing (SA), Seed-Based (SB), Scenario Based Planning Approach (SBPA), Split Deliveries (SD), Scenario Decomposition Algorithm (SDA), Stochastic Dynamic Programming (SDP), Simulation (SIM), Selection Probabilities (SP), Simulation Strategies (SS), Total Cost (TC), Tabu Search (TS), Time Windows (TW), Time Window Violation Index (TWVI).

1 increase in profits as well (additionally determining a benefit in terms of flexibility in
2 building efficient routes).

3 A milestone in the field of AHD is the work of [Agatz et al. \(2011\)](#), where the TMSP
4 in AHD was defined for the first time. The authors studied the particular TMSP arising
5 at Albert.nl, the leading Dutch e-grocer at the time, and proposed two alternative
6 formulations for the problem, in which the expected delivery costs are minimized.
7 The first extends the Continuous Approximation (CA) approach found in [Daganzo
8 \(1987\)](#); in particular, the authors start from a base schedule (e.g., the one adopted
9 by the company) and iteratively improve it until the expected routing costs do not
10 decrease anymore or a maximum number of iterations is reached. In this formulation,
11 a “cluster-first, route-second” strategy is used to approximate the delivery costs. The
12 second formulation is an Integer Linear Programming (ILP) model that relies on the
13 seed-based scheme originally proposed by [Fisher and Jaikumar \(1981\)](#) to approximate
14 the routing costs. As shown by the computational experiments both formulations pro-
15 duce high-quality schedules, resulting in a slight reduction of delivery costs compared
16 to the schedule used by the company. But the greatest potential generated by the two
17 formulations is that of automating the schedule design process; in this sense, the CA
18 approach is better than the ILP model as it requires shorter computational times.
19 Further remarkable findings are presented in the what-if analyses conducted by the au-
20 thors, where the effects of potential changes (increase of demand, increase or decrease
21 of vehicle capacity, increase or reduction of service level, and use of alternative time
22 slot templates) are investigated. Among them, they remark the existence of a trade-
23 off between the time slot length and the routing efficiency (with an increase of up to
24 25% in delivery costs going from an entire shift length to a two-hour length). Also,
25 they highlight the idea that introducing a demand clustering may have a beneficial
26 effect of approximately 10% reduction in terms of delivery costs.

27 Building upon the work of [Campbell and Savelsbergh \(2005\)](#) as well as the re-
28 sults previously found by [Ehmke et al. \(2012a,b\)](#), [Ehmke and Campbell \(2014\)](#)
29 developed and compared novel customer acceptance mechanisms for AHD applica-
30 tions in metropolitan areas. The innovative idea behind their work is represented
31 by the introduction of time-dependent and stochastic travel time information in the
32 decision-making process of accepting or rejecting an incoming order request. In par-
33 ticular, to take care of possible lateness, due to variable travel times in rush hours,
34 and the so-called “lateness propagation” effect, which depends on accumulated travel
35 time variations during the execution of delivery routes, the authors included a thor-
36 ough computation of individual buffer times. Such computation was integrated in a
37 time-dependent variant of the I1 insertion heuristic algorithm originally developed
38 by [Solomon \(1987\)](#). The results obtained from several rounds of simulation show that
39 the proposed acceptance mechanism generally outperforms alternative approaches,
40 both static and dynamic, in terms of the number of accepted requests and potential
41 to avoid lateness. The authors also investigated the effect of changes in some input
42 parameters (e.g., distribution of customer locations between downtown and subur-
43 ban areas, service times, time window length, lateness avoidance, and confluence of
44 requests in popular time slots) and provided meaningful practical insights.

1 Yang et al. (2016) defined a DP framework for the dynamic pricing of delivery
2 time slots based on a thorough demand model, where the arrival of customers for a
3 single delivery day is estimated using a time-dependent Poisson process, while the
4 selection of time slots within a given delivery day is modeled through an MNL model.
5 The dynamic program is defined to gain insights for the development of good pricing
6 policies, as it is not solvable in short computing times due to the curse of dimensionality
7 and the VRPTW that must be solved at each stage. To overcome this problem, during
8 the online booking phase an approximation of the routing costs is computed based
9 on the insertion heuristics by Campbell and Savelsbergh (2006) and an online pricing
10 problem is solved. As a valuable result, the authors show that a dynamic pricing
11 policy that includes an estimation of the delivery costs for expected future orders,
12 instead of focusing only on already accepted orders, is preferable. Moreover, they
13 show how a similar policy produces a remarkable increase in terms of total profits
14 (i.e., 3.8% on average) compared to the common industrial practices of using static
15 prices or order-based prices for time slots. This effect is even more evident when
16 capacity is scarce. The work was motivated by an industrial partnership with a major
17 e-grocer in the United Kingdom that provided anonymized booking data that were
18 used to train the models and perform different runs of simulation. Building upon
19 their previous work and using the same sample data provided by a major e-grocer
20 operating in the Greater London area, Yang and Strauss (2017) developed an APD
21 procedure. In particular, the proposed approach adopts a dynamic pricing policy that
22 incorporates both approximated delivery costs (obtained by applying the “cluster-
23 first, route-second” approach originally proposed by Daganzo 1987) and estimated
24 revenues to compute the opportunity costs from expected future orders. Remarkably,
25 the results show an average total profit increase of more than 2% compared to base
26 policies where no opportunity cost is considered, and a computational time compatible
27 with real-world applications.

28 A different interpretation of the Tactical Time Slot Management Problem
29 (TTSM) was given in the work of Hernandez et al. (2017), where the authors defined
30 the TTSM through a MILP formulation and solved it heuristically. In particular, two
31 alternative heuristics were proposed. The first heuristic relies on a three-phase decom-
32 position, that initially solves a Periodic Vehicle Routing Problem (PVRP), in which
33 the time slots in the TTSM correspond to the periods in the PVRP, subsequently
34 merges the routes obtained from Phase 1 over each day, and, finally, solves a VRPTW
35 for each day in the planning horizon (i.e., optimizes the routes merged during Phase
36 2). The second heuristic interprets the TTSM as a Periodic Vehicle Routing Prob-
37 lem with Time Windows (PVRPTW), in which the days in the TTSM correspond
38 to the periods in the PVRPTW while the time slots correspond to the time windows.
39 Both problems were solved using a TS algorithm that has proven to be efficient for
40 these problems (see, e.g., Cordeau et al. 1997, 2001). Although the first heuristic is
41 competitive for being more generic and tractable with state-of-the-art techniques and
42 available software, it is generally outperformed by the second heuristic both in terms
43 of computational times and solution quality.

44 Inspired by the work of Schmid and Doerner (2014), Han et al. (2017) developed an
45 integrative approach for solving the appointment scheduling and routing problem in
46 the context of AHD. What characterizes this work is the inclusion of random customer
47
48

1 behavior in the proposed model by considering no-show probabilities and random
2 response times during the delivery phase. Such randomness typically represents a
3 remarkable issue in real-world applications, frequently causing inefficient re-routing,
4 potential disruptions, and extra costs. To solve the problem, the authors implemented
5 a hybrid heuristic algorithm, which iteratively combines a TS metaheuristic, for solving
6 the routing part, and an approximate DP algorithm, for solving the scheduling
7 part. The results show how the proposed integrative approach outperforms a tradi-
8 tional hierarchical approach. However, the computational times obtained on large
9 instances warn against a potentially low compatibility with real-world cases, as the
10 developed algorithm took almost 20 hours to solve instances with up to 5 vehicle and
11 50 customers.

12 In their work at the border between AHD and SDD, [Restrepo et al. \(2019\)](#) in-
13 troduced for the first time the Integrated Shift Scheduling and Load Assignment
14 Problem. The problem, originating from a real-world start-up company offering last-
15 mile delivery services in many cities of France, is formulated as a two-stage Stochastic
16 Programming model. In particular, the first stage aims at designing tactical schedules
17 for couriers, which are allocated to a restricted number of geographic areas, while the
18 second stage defines the assignment of customer orders to couriers. In this work, we
19 have a co-presence of stochasticity (given a portion of stochastic orders generated using
20 a Poisson distribution) and dynamicity (given a portion of orders that must be
21 fulfilled according to a same-day delivery policy). To solve the problem, the authors
22 implemented a multicut L-shaped method with some additional algorithmic refine-
23 ments to generate initial cuts and derive valid inequalities. The main idea underlying
24 this work is represented by the opportunity of using the tactical model to compare
25 alternative policy offerings and to evaluate their impact on total cost and solution
26 quality. In addition, the results show the advantage of including uncertainty when
27 generating tactical solutions.

28 A very interesting real-world application of differentiated slotting in the context of
29 utilities was studied in the work of [Bruck et al. \(2018\)](#). Here, the authors addressed a
30 particular problem arising from an Italian gas distribution company, named *IRETI*,
31 in which the required Quality of Service (QoS) level is exogenously fixed by the public
32 authority that regulates the market, so there is no opportunity to influence the demand
33 of customers using RM principles. As a consequence, the design of good quality time
34 slot tables is fundamental to limit the routing costs generated after the actual demand
35 is revealed. For doing so, the authors developed a three-step approach having at its core
36 an LNS algorithm that iteratively improves an initial set of time slot tables by means of
37 destroy and repair methods. Interestingly, the customer-choice behavior in the process
38 of booking the preferred time slot for the execution of a service was reproduced using
39 four alternative simulation strategies. The cost of the solutions computed by the LNS
40 algorithm is evaluated through a Multidepot multiple Traveling Salesman Problem
41 (MmTSP), which relies on a time-extended network. Note that a different MmTSP is
42 solved for each day in the booking horizon. The results obtained on real-case instances
43 showed an expected reduction of routing costs in the order of 5% to 15% compared to
44 the company's solution.
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

1 Addressing the same real-world application described by [Bruck et al. \(2018\)](#), [Bruck](#)
2 [et al. \(2020\)](#) developed a Decision Support System (DSS) to solve the practical prob-
3 lem of defining the organizational model for a so-called “minimum territorial area”
4 (ATEM), given the QoS levels imposed by the public authority regulating the gas
5 distribution market. The DSS is intended to support *IRETI* in solving a three-stage
6 problem, in which the decisions are sequential. In the first stage, a number of municipi-
7 talities are clustered by solving a p-Median Facility Location Problem; in the second
8 stage, an initial model-week is generated for each cluster by using an improved ILP
9 formulation compared to the one in [Bruck et al. \(2018\)](#) and an LNS algorithm; in the
10 third stage detailed technician routing plans are created by solving an MmTSP for
11 each day in the simulating horizon and several key performance indicators are pro-
12 vided in output to the decision makers. Interestingly, dynamic changes are made to the
13 model-weeks during the simulation, thus reproducing a common practice to address
14 demand fluctuations. Also, it is worth noting that the DSS has integrated a machine
15 learning submodule that gives the opportunity to design solutions in the presence
16 of missing information (i.e., by predicting the demand of partially known or totally
17 unknown ATEMs).

18 Extending previous works and combining them with ideas from recent streams of
19 literature on the VRP, [Koch and Klein \(2020\)](#) proposed a route-based ADP approach
20 for dynamic pricing, where the opportunity cost due to the displacement of poten-
21 tial future orders is carefully estimated through a route-based formulation borrowed
22 from the Stochastic Dynamic VRP literature (see, e.g., [Ulmer et al. 2020](#)). In par-
23 ticular, the authors used artificial routes to improve the estimation of future routing
24 costs and introduced a time window budget approach to better evaluate the idle time
25 of vehicles within the time windows. These features serve as an input for the online
26 pricing problem, which is solved using an efficient heuristic algorithm. Computational
27 experiments show that the performance of the route-based ADP approach with time
28 window budget is superior compared not only to another ADP approach with waiting
29 time (proposed by the same authors), but also to other policies adapted from the liter-
30 ature (among which the one by [Yang and Strauss 2017](#)). Such superiority is expressed
31 both in terms of average profit and number of served customers. Another valuable
32 change that the authors introduced in this work, compared to the previous literature,
33 is represented by the use of a finite-mixture MNL model as the customer-choice model.

34 Following up on the works by [Yang et al. \(2016\)](#) and [Yang and Strauss \(2017\)](#), [Ab-](#)
35 [dollahi et al. \(2023\)](#) presented a new dynamic pricing approach in which the
36 opportunity cost estimation is based on a combination of actual orders with time win-
37 dows and forecast orders without time windows. Interestingly, each time an incoming
38 requests is accepted and inserted in a route, a forecast order is removed from that
39 route and the underlying dynamic VRPTW is re-optimized to adjust the pricing offer
40 for future requests. Compared to commonly used static pricing policies, the proposed
41 approach performed better in terms of total profits, with an increase between 13.57%
42 and 21.43%.
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

4.2 The Time Window Assignment Vehicle Routing Problem

In this section, we survey a particular class of integrated demand management and routing problems, the TWAVRP, in which time windows must be assigned to customers before demand is known, followed by the creation of routing schedules that minimize the expected routing costs.

The TWAVRP was introduced for the first time in the paper of [Spliet and Gabor \(2015\)](#), where the authors presented a compact MILP formulation which considers multiple scenarios corresponding to different realizations of demand. In particular, they distinguished between exogenous and endogenous time windows to identify, respectively, time windows imposed by an external stakeholder and time windows agreed upon by the customer and supplier. To solve the problem, the authors proposed a Branch-Price-and-Cut (BPC) algorithm, in which the restricted primary problem is solved via column generation while the secondary pricing problem, an ESPPRC in which vehicle capacity and time windows are the resource constraints, is decomposed by scenario and solved using basic route relaxation techniques (i.e., allowing all cyclic routes but eliminating 2-cycle routes). An acceleration strategy and some valid inequalities were also proposed. The computational experiments proved that the proposed BPC algorithm can solve to optimality instances with up to 25 customers and 3 demand scenarios. Interestingly, the authors compared the results found by the BPC algorithm for the TWAVRP with those obtained by a heuristic procedure to solve the VRPTW with average demand (i.e., which corresponds to a one-scenario TWAVRP), showing that the routing costs of VRPTW solutions with average demand are on average 1.85% higher.

In a follow-up work, [Spliet and Desaulniers \(2015\)](#) defined the Discrete Time Window Assignment Vehicle Routing Problem (DTWAVRP), which differs from the TWAVRP in that a finite set of candidate time windows is given for each customer. Building upon their previous approach, the authors proposed an exact BPC algorithm, in which the secondary pricing problem is solved using the ng -route relaxation technique by [Baldacci et al. \(2011\)](#), with $\Delta_{ng} \in \{1, 5, n\}$. Also, five column generation heuristics (i.e., one restricted master heuristic, two diving heuristics, and two rounding heuristics) were developed. When solving the DTWAVRP using the exact BPC algorithm, the authors demonstrate how the configuration with $\Delta_{ng} = 5$ represents a good compromise between short computing times and solution quality if compared to the configurations allowing all cyclic routes (i.e., when $\Delta_{ng} = 1$) and elementary paths only (i.e., when $\Delta_{ng} = n$). The five column generation heuristics, in turn, proved to find solutions with relatively small gap to optimality (i.e., between 0.29% and 4.30%) for instances with up to 25 customers and 5 demand scenarios, while they were able to solve instances with up to 60 customers, although without proving optimality. Among them, the so-called TWDiving-Tabu heuristic produced the best results. Additional experiments were performed to compare a multiple-scenario TWDiving-Tabu heuristic with a single-scenario average demand based TWAVRP. These experiments confirmed the potential of the TWDiving-Tabu heuristic in generating solutions with lower expected routing costs as well as the advantage of considering multiple scenarios.

A novel formulation of the TWAVRP with time-dependent travel times was presented in the work of [Spliet et al. \(2018\)](#), where the authors developed an innovative

1 labeling algorithm to solve the secondary pricing problem based on the contributions
2 of [Ioachim et al. \(1998\)](#) and [Feillet et al. \(2004\)](#), and built upon the TS column gener-
3 ator originally proposed by [Spliet and Desaulniers \(2015\)](#). Also, new arc-synchronization
4 inequalities were formulated to strengthen the BPC algorithm used to solve the
5 problem.

6 In their paper, [Dalmeijer and Spliet \(2018\)](#) defined an alternative MILP formu-
7 lation for the TWAVRP based on the two-commodity network flow approach for the
8 Capacitated VRP by [Baldacci et al. \(2004\)](#) and the well-known MTZ-inequalities. The
9 authors solved the problem via Branch-and-Cut (BC) with the addition of a tailored
10 class of valid inequalities for the TWAVRP (i.e., the precedence inequalities) and the
11 introduction of a new branching rule. The results show that the proposed BC algo-
12 rithm clearly outperforms the BPC algorithm of [Spliet and Gabor \(2015\)](#) in terms of
13 computing times and gap to optimality. More interestingly, the BC algorithm is able
14 to solve to optimality larger instances with up to 35 customers and 3 scenarios, while
15 showing small optimality gap for instances with up to 40 customers.

16 Starting from a real-world application and data provided by a large European food
17 retailer, an extended version of the TWAVRP with product dependent time windows
18 was studied by [Neves-Moreira et al. \(2018\)](#). The impact of realistic features like multi-
19 product deliveries and fleet requirements (e.g., temperature at which products are kept
20 during transportation and compatibility between vehicle and retail site capacities)
21 were also investigated by the authors. To solve the problem, a three-phase approach
22 consisting of (i) route generation, (ii) initial solution construction, and (iii) improve-
23 ment matheuristic (see, e.g., [Boschetti and Maniezzo 2022](#) for an overview of this
24 topic) was developed. The benefit from considering multi-product deliveries, instead
25 of single-product deliveries only, was confirmed by the computational experiments in
26 which an average saving of 6.44% in terms of total routing costs was achieved thanks
27 to the additional flexibility of multi-product deliveries. Furthermore, in line with the
28 results obtained by [Spliet and Gabor \(2015\)](#), the authors demonstrate that a stochas-
29 tic multiple-scenario approach is preferable to a deterministic single-scenario approach
30 with average demand (with the former that outperforms the latter by 5.3% on aver-
31 age). Some useful managerial insights were also derived from a sensitivity analysis. In
32 particular, the authors proved that further savings can be achieved by increasing the
33 time window length and product flexibility (in terms of the minimum quantity of the
34 main product that must be delivered in multi-product deliveries).
35

36 In their work, [Subramanyam et al. \(2018\)](#) took advantage from the similarities
37 between the TWAVRP and the Consistent VRP (see, e.g., [Kovacs et al. 2014](#) for an
38 overview of this problem) to adapt the decomposition algorithm previously proposed
39 by [Subramanyam and Gounaris \(2018\)](#) for the Consistent TSP. Such an algorithm
40 turned out to outperform state-of-the-art solution methods both for the TWAVRP
41 and the DTWAVRP, thus demonstrating a good efficiency and versatility in solving
42 problem of this class.

43 In [Vareias et al. \(2019\)](#), a TWAVRP with stochastic travel times is solved with the
44 goal of designing routes having minimum traveling distance and minimum earliness
45 and lateness penalty costs due to time windows violation. The problem is solved by
46 means of two mathematical models and an ALNS.
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

1 Building upon the work of Dalmeijer and Spliet (2018), Dalmeijer and Desaulniers
2 (2021) introduced an edge-based branching method to eliminate orientation symmetry
3 from the search tree of a BPC, and they presented enhancements to make this method
4 efficient in practice. They consistently reduced the number of explored nodes and
5 solved 25 TWAVRP benchmark instances to proven optimality for the first time.

6 A robust formulation of the TWAVRP for solving problems in which the probability
7 distribution of travel and service times is partially unknown was presented in Hooge-
8 boom et al. (2021). Their formulation is based on a time window violation index that
9 measures the risk associated with the violation of the time windows assigned to des-
10 tination nodes. This index is inspired by the Requirements Violation Index originally
11 proposed by Jaillet et al. (2016). The problem was solved via BC and the results were
12 compared with those obtained by a stochastic variant of the TWAVRP in which the
13 probability distribution of travel times is known.

14 5 Conclusions and Future Research Directions

15 This work has provided a detailed literature review on the state of the art for Attended
16 Home Delivery (AHD) and Attended Home Service (AHS) problems, a research field
17 that is experiencing increasing attention, as confirmed by the fast-growing number of
18 documents published each year on this class of problems. Given its strong practical
19 relevance, a particular focus has been put on real-world applications with the purpose
20 of gaining useful managerial insights. Indeed, AHD and AHS problems owe their pop-
21 ularity to the rapid diffusion of online platforms, where a particularly high demand is
22 registered for e-grocery and online retail transactions.

23 Since the seminal works in this topic, an increased awareness of the multi-stage
24 nature of AHD and AHS problems, where the decisions taken at the first level greatly
25 affect the feasibility as well as the economic profitability of the decisions taken at
26 the second level, has emerged. Demand management and routing are well-established
27 research fields per se, but the integration of demand management and routing decisions
28 represents the complex part of solving real-world AHD and AHS problems, as these
29 decisions are affected by uncertainty.

30 Many authors have proposed several sophisticated methods to solve alternately
31 demand management problems (where the information related to the routing subprob-
32 lem is estimated or forecast) or routing problems (where the information related to
33 the demand management subproblem is oversimplified and used as an input or, once
34 again, forecast), but the search for a more effective integration of these two stages may
35 represent one of the most significant future research directions in AHD and AHS.

36 In this sense, a promising approach may be that of using Dynamic Programming
37 as the main framework, but great efforts are needed to overcome the issues of dimen-
38 sionality and complexity of solving a Vehicle Routing Problem with Time Windows as
39 the boundary condition for each state. An alternative approach may be that of bor-
40 rowing some ideas from the Stochastic Dynamic Vehicle Routing Problem literature to
41 roughly solve the online demand management problem by anticipating some routing
42 aspects that must be fine-tuned offline.

1 The sustainability of AHD and AHS systems is another relevant topic having
2 received little attention as compared to the wide literature on AHD and AHS prob-
3 lems. The recent work of [Agatz et al. \(2021\)](#) presents an interesting discussion on the
4 effectiveness of using “green” incentives to steer customer choices, along with tradi-
5 tional price incentives. As sustainability may represent for AHD and AHS problems an
6 additional objective, which may be conflicting with profit maximization or cost min-
7 imization, the benefit from introducing multi-criteria problem formulations is worth
8 exploring. Also, further objectives may emerge and be considered in the future. For
9 this reason, the introduction of Multi-Criteria Decision Analysis for solving AHD and
10 AHS problems may represent another future research directions in this field.

11 Other interesting research directions may include the use of machine learning tech-
12 niques to support online time slot decisions (see, e.g., [van der Hagen et al. 2022](#)),
13 the extensive adoption of data science approaches to analyze large amounts of his-
14 torical order data and, consequently, better understand the preferences of customers
15 (see, e.g., [Köhler et al. 2023](#)), and the exploitation of opportunity sales to generate
16 additional profits (see, e.g., [Ötken et al. 2023](#)).

17 Finally, we have seen that real-world AHD and AHS applications may be encoun-
18 tered in heterogeneous business sectors, although the problem at its core maintains
19 a similar structure (with some exceptions). In upcoming years, we expect a denser
20 transfer of ideas and technologies among different sectors as well as the emergence of
21 innovative areas of application.

22 **Statements and Declarations**

23 **Authors’ Contribution**

24 All authors contributed to the conception and design of the survey. The literature
25 search was performed by Dario Vezzali. The first draft of the manuscript was written
26 by Dario Vezzali and revised by Jean-François Cordeau and Manuel Iori. All authors
27 read and approved the final manuscript.

28 **Conflict of Interest**

29 The authors declare that they have no conflict of interest.

30 **Data Availability**

31 The authors did not analyze or generate any data sets, because the work proceeds
32 within a theoretical approach.

33 **Funding**

34 The authors gratefully acknowledge financial support under the National Recovery and
35 Resilience Plan (NRRP), Mission 04 Component 2 Investment 1.5–NextGenerationEU,
36 Call for tender n. 3277 dated 30/12/2021, Award Number: 0001052 dated 23/06/2022.

References

- 1
2 Abdollahi M, Yang X, Nasri MI, Fairbank M (2023) Demand management in time-
3 slotted last-mile delivery via dynamic routing with forecast orders. *European*
4 *Journal of Operational Research* 309(2):704–718
5
- 6 Agatz N, Campbell AM, Fleischmann M, Savelsbergh M (2008a) Challenges and op-
7 portunities in attended home delivery. In: Golden BL, Raghavan S, Wasil E (eds)
8 *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer,
9 Boston, p 379–396
10
- 11 Agatz N, Fleischmann M, van Nunen J (2008b) E-fulfillment and multi-channel
12 distribution – A review. *European Journal of Operational Research* 187(2):339–356
13
- 14 Agatz N, Campbell AM, Fleischmann M, Savelsbergh M (2011) Time slot management
15 in attended home delivery. *Transportation Science* 45(3):435–449
16
- 17 Agatz N, Campbell AM, Fleischmann M, van Nunen J, Savelsbergh M (2013) Revenue
18 management opportunities for internet retailers. *Journal of Revenue and Pricing*
19 *Management* 12(2):128–138
20
- 21 Agatz N, Fan Y, Stam D (2021) The impact of green labels on time slot choice and op-
22 erational sustainability. *Production and Operations Management* 30(7):2285–2303
23
- 24 Ali O, Côté JF, Coelho LC (2021) Models and algorithms for the delivery and instal-
25 lation routing problem. *European Journal of Operational Research* 291(1):162–177
26
- 27 Archetti C, Bertazzi L (2021) Recent challenges in routing and inventory routing: E-
28 commerce and last-mile delivery. *Networks: An International Journal* 77(2):255–268
29
- 30 Asdemir K, Jacob VS, Krishnan R (2009) Dynamic pricing of multiple home delivery
31 options. *European Journal of Operational Research* 196(1):246–257
32
- 33 Azi N, Gendreau M, Potvin JY (2007) An exact algorithm for a single-vehicle routing
34 problem with time windows and multiple routes. *European Journal of Operational*
35 *Research* 178(3):755–766
36
- 37 Azi N, Gendreau M, Potvin JY (2010) An exact algorithm for a vehicle routing problem
38 with time windows and multiple use of vehicles. *European Journal of Operational*
39 *Research* 202(3):756–763
40
- 41 Azi N, Gendreau M, Potvin JY (2012) A dynamic vehicle routing problem with
42 multiple delivery routes. *Annals of Operations Research* 199(1):103–112
43
- 44 Azi N, Gendreau M, Potvin JY (2014) An adaptive large neighborhood search for a
45 vehicle routing problem with multiple routes. *Computers & Operations Research*
46 41:167–173
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1 Baldacci R, Hadjiconstantinou E, Mingozzi A (2004) An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation. *Operations Research* 52(5):723–738
- 2
- 3
- 4 Baldacci R, Mingozzi A, Roberti R (2011) New route relaxation and pricing strategies for the vehicle routing problem. *Operations Research* 59(5):1269–1283
- 5
- 6
- 7 Baldacci R, Mingozzi A, Roberti R (2012) Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints. *European Journal of Operational Research* 218(1):1–6
- 8
- 9
- 10
- 11 Bent RW, Van Hentenryck P (2004) Scenario-based planning for partially dynamic vehicle routing with stochastic customers. *Operations Research* 52(6):977–987
- 12
- 13
- 14 Bertsimas D, Sim M (2004) The price of robustness. *Operations Research* 52(1):35–53
- 15
- 16 Bloomberg (2021) The dark side of 15-minute grocery delivery. <https://www.bloomberg.com/news/articles/2021-12-07/what-instant-delivery-services-could-do-to-cities>, [Online; Accessed 22 September 2023]
- 17
- 18
- 19
- 20
- 21 Boschetti MA, Maniezzo V (2022) Matheuristics: using mathematics for heuristic design. *4OR* 20(2):173–208
- 22
- 23
- 24 Boysen N, Fedtke S, Schwerdfeger S (2021) Last-mile delivery concepts: a survey from an operational research perspective. *OR Spectrum* 43(1):1–58
- 25
- 26
- 27 Bräysy O, Gendreau M (2005a) Vehicle routing problem with time windows, part I: Route construction and local search algorithms. *Transportation Science* 39(1):104–118
- 28
- 29
- 30
- 31 Bräysy O, Gendreau M (2005b) Vehicle routing problem with time windows, part II: Metaheuristics. *Transportation Science* 39(1):119–139
- 32
- 33
- 34 Bredström D, Rönnqvist M (2008) Combined vehicle routing and scheduling with temporal precedence and synchronization constraints. *European Journal of Operational Research* 191(1):19–31
- 35
- 36
- 37
- 38 Bruck BP, Cordeau JF, Iori M (2018) A practical time slot management and routing problem for attended home services. *Omega* 81:208–219
- 39
- 40
- 41 Bruck BP, Castegini F, Cordeau JF, Iori M, Poncemi T, Vezzali D (2020) A decision support system for attended home services. *INFORMS Journal on Applied Analytics* 50(2):137–152
- 42
- 43
- 44
- 45 Bühler D, Klein R, Neugebauer M (2016) Model-based delivery cost approximation in attended home services. *Computers & Industrial Engineering* 98:78–90
- 46
- 47
- 48
- 49
- 50
- 51
- 52
- 53
- 54
- 55
- 56
- 57
- 58
- 59
- 60
- 61
- 62
- 63
- 64
- 65

- 1 Campbell AM, Savelsbergh M (2005) Decision support for consumer direct grocery
2 initiatives. *Transportation Science* 39(3):313–327
- 3 Campbell AM, Savelsbergh M (2006) Incentive schemes for attended home delivery
4 services. *Transportation Science* 40(3):327–341
- 5
6 Cappanera P, Scutellà MG (2015) Joint assignment, scheduling, and routing mod-
7 els to home care optimization: A pattern-based approach. *Transportation Science*
8 49(4):830–852
- 9
10 Cappanera P, Scutellà MG, Nervi F, Galli L (2018) Demand uncertainty in robust
11 home care optimization. *Omega* 80:95–110
- 12
13 Carello G, Lanzarone E (2014) A cardinality-constrained robust model for the assign-
14 ment problem in home care services. *European Journal of Operational Research*
15 236(2):748–762
- 16
17 Cattaruzza D, Absi N, Feillet D, González-Feliu J (2017) Vehicle routing problems for
18 city logistics. *EURO Journal on Transportation and Logistics* 6(1):51–79
- 19
20 Chen X, Thomas BW, Hewitt M (2016) The technician routing problem with
21 experience-based service times. *Omega* 61:49–61
- 22
23 Chen X, Thomas BW, Hewitt M (2017) Multi-period technician scheduling with
24 experience-based service times and stochastic customers. *Computers & Operations*
25 *Research* 82:1–14
- 26
27 Chen X, Hewitt M, Thomas BW (2018) An approximate dynamic programming
28 method for the multi-period technician scheduling problem with experience-based
29 service times and stochastic customers. *International Journal of Production Eco-*
30 *nomics* 196:122–134
- 31
32 Cordeau JF, Gendreau M, Laporte G (1997) A tabu search heuristic for periodic
33 and multi-depot vehicle routing problems. *Networks: An International Journal*
34 30(2):105–119
- 35
36 Cordeau JF, Laporte G, Mercier A (2001) A unified tabu search heuristic for vehicle
37 routing problems with time windows. *Journal of the Operational Research Society*
38 52(8):928–936
- 39
40 Cordeau JF, Laporte G, Pasin F, Ropke S (2010) Scheduling technicians and tasks in
41 a telecommunications company. *Journal of Scheduling* 13:393–409
- 42
43 Cortés CE, Gendreau M, Rousseau LM, Souyris S, Weintraub A (2014) Branch-
44 and-price and constraint programming for solving a real-life technician dispatching
45 problem. *European Journal of Operational Research* 238(1):300–312
- 46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1 Daganzo CF (1987) Modeling distribution problems with time windows: Part I.
2 Transportation Science 21(3):171–179
- 3 Dalmeijer K, Desaulniers G (2021) Addressing orientation symmetry in the time
4 window assignment vehicle routing problem. INFORMS Journal on Computing
5 33(2):495–510
- 6
7 Dalmeijer K, Spliet R (2018) A branch-and-cut algorithm for the time window
8 assignment vehicle routing problem. Computers & Operations Research 89:140–152
9
- 10 Desaulniers G, Madsen OB, Ropke S (2014) Chapter 5: The vehicle routing problem
11 with time windows. In: Toth P, Vigo D (eds) Vehicle Routing: Problems, Methods,
12 and Applications. SIAM, Philadelphia, p 119–159
- 13
14 Desaulniers G, Errico F, Irnich S, Schneider M (2016) Exact algorithms for electric
15 vehicle-routing problems with time windows. Operations Research 64(6):1388–1405
16
- 17 Duman EN, Taş D, Çatay B (2022) Branch-and-price-and-cut methods for the electric
18 vehicle routing problem with time windows. International Journal of Production
19 Research 60(17):5332–5353
20
- 21 Duque PM, Castro M, Sørensen K, Goos P (2015) Home care service planning. The
22 case of Landelijke Thuiszorg. European Journal of Operational Research 243(1):292–
23 301
24
- 25 Ehmke JF (2012) Integration of information and optimization models for routing in
26 city logistics. Springer, New York
27
- 28 Ehmke JF, Campbell AM (2014) Customer acceptance mechanisms for home deliveries
29 in metropolitan areas. European Journal of Operational Research 233(1):193–207
30
- 31 Ehmke JF, Meisel S, Mattfeld DC (2012a) Floating car based travel times for city
32 logistics. Transportation Research Part C: Emerging Technologies 21(1):338–352
33
- 34 Ehmke JF, Steinert A, Mattfeld DC (2012b) Advanced routing for city logistics service
35 providers based on time-dependent travel times. Journal of Computational Science
36 3(4):193–205
37
- 38 Errico F, Desaulniers G, Gendreau M, Rei W, Rousseau LM (2018) The vehicle routing
39 problem with hard time windows and stochastic service times. EURO Journal on
40 Transportation and Logistics 7(3):223–251
41
- 42 Euchi J, Masmoudi M, Siarry P (2022) Home health care routing and scheduling
43 problems: a literature review. 4OR 20(3):351–389
44
- 45 Eveborn P, Flisberg P, Rönnqvist M (2006) LAPS CARE—an operational system for
46 staff planning of home care. European Journal of Operational Research 171(3):962–
47 976
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
- Eveborn P, Rönnqvist M, Einarsdóttir H, Eklund M, Lidén K, Almroth M (2009) Operations research improves quality and efficiency in home care. *Interfaces* 39(1):18–34
- Feillet D, Dejax P, Gendreau M, Gueguen C (2004) An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems. *Networks: An International Journal* 44(3):216–229
- Fikar C, Hirsch P (2017) Home health care routing and scheduling: A review. *Computers & Operations Research* 77:86–95
- Fischetti M, Polo C, Scantamburlo M (2004) A local branching heuristic for mixed-integer programs with 2-level variables, with an application to a telecommunication network design problem. *Networks: An International Journal* 44(2):61–72
- Fisher ML, Jaikumar R (1981) A generalized assignment heuristic for vehicle routing. *Networks: An International Journal* 11(2):109–124
- Fleckenstein D, Klein R, Steinhardt C (2023) Recent advances in integrating demand management and vehicle routing: A methodological review. *European Journal of Operational Research* 306(2):499–518
- Florio AM, Feillet D, Hartl RF (2018) The delivery problem: Optimizing hit rates in e-commerce deliveries. *Transportation Research Part B: Methodological* 117:455–472
- Gallego G, Ratliff R, Shebalov S (2015) A general attraction model and sales-based linear program for network revenue management under customer choice. *Operations Research* 63(1):212–232
- Grenouilleau F, Legrain A, Lahrichi N, Rousseau LM (2019) A set partitioning heuristic for the home health care routing and scheduling problem. *European Journal of Operational Research* 275(1):295–303
- van der Hagen L, Agatz N, Spliet R, Visser TR, Kok L (2022) Machine learning-based feasibility checks for dynamic time slot management. *Transportation Science*
- Han S, Zhao L, Chen K, Luo Zw, Mishra D (2017) Appointment scheduling and routing optimization of attended home delivery system with random customer behavior. *European Journal of Operational Research* 262(3):966–980
- Hernandez F, Gendreau M, Potvin JY (2017) Heuristics for tactical time slot management: a periodic vehicle routing problem view. *International Transactions in Operational Research* 24(6):1233–1252
- Hertz A, Lahrichi N (2009) A patient assignment algorithm for home care services. *Journal of the Operational Research Society* 60(4):481–495

- 1 Hiermann G, Puchinger J, Ropke S, Hartl RF (2016) The electric fleet size and mix ve-
2 hicle routing problem with time windows and recharging stations. *European Journal*
3 *of Operational Research* 252(3):995–1018
- 4 Hoogeboom M, Adulyasak Y, Dullaert W, Jaillet P (2021) The robust vehicle routing
5 problem with time window assignments. *Transportation Science* 55(2):395–413
6
- 7 Ioachim I, Gelinass S, Soumis F, Desrosiers J (1998) A dynamic programming algorithm
8 for the shortest path problem with time windows and linear node costs. *Networks:*
9 *An International Journal* 31(3):193–204
10
- 11 Jabali O, Leus R, Van Woensel T, de Kok T (2015) Self-imposed time windows in
12 vehicle routing problems. *OR Spectrum* 37(2):331–352
13
- 14 Jaillet P, Qi J, Sim M (2016) Routing optimization under uncertainty. *Operations*
15 *Research* 64(1):186–200
16
- 17 Kallehauge B (2008) Formulations and exact algorithms for the vehicle routing
18 problem with time windows. *Computers & Operations Research* 35(7):2307–2330
19
- 20 Keskin M, Çatay B (2016) Partial recharge strategies for the electric vehicle rout-
21 ing problem with time windows. *Transportation Research Part C: Emerging*
22 *Technologies* 65:111–127
23
- 24 Keskin M, Çatay B (2018) A matheuristic method for the electric vehicle routing
25 problem with time windows and fast chargers. *Computers & Operations Research*
26 100:172–188
27
- 28 Keskin M, Laporte G, Çatay B (2019) Electric vehicle routing problem with time-
29 dependent waiting times at recharging stations. *Computers & Operations Research*
30 107:77–94
31
- 32 Keskin M, Çatay B, Laporte G (2021) A simulation-based heuristic for the electric ve-
33 hicle routing problem with time windows and stochastic waiting times at recharging
34 stations. *Computers & Operations Research* 125:105060
35
- 36 Klein R, Mackert J, Neugebauer M, Steinhardt C (2018) A model-based approximation
37 of opportunity cost for dynamic pricing in attended home delivery. *OR Spectrum*
38 40(4):969–996
39
- 40 Klein R, Neugebauer M, Ratkovitch D, Steinhardt C (2019) Differentiated time slot
41 pricing under routing considerations in attended home delivery. *Transportation*
42 *Science* 53(1):236–255
43
- 44 Klein R, Koch S, Steinhardt C, Strauss AK (2020) A review of revenue management:
45 Recent generalizations and advances in industry applications. *European Journal of*
46 *Operational Research* 284(2):397–412
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1 Koch S, Klein R (2020) Route-based approximate dynamic programming for dy-
2 namic pricing in attended home delivery. *European Journal of Operational Research*
3 287(2):633–652
- 4 Köhler C, Ehmke JF, Campbell AM (2020) Flexible time window management for
5 attended home deliveries. *Omega* 91:102023
- 6
7 Köhler C, Campbell AM, Ehmke JF (2023) Data-driven customer acceptance for
8 attended home delivery. *OR Spectrum*
- 9
10 Kovacs AA, Parragh SN, Doerner KF, Hartl RF (2012) Adaptive large neighborhood
11 search for service technician routing and scheduling problems. *Journal of Scheduling*
12 15:579–600
- 13
14 Kovacs AA, Golden BL, Hartl RF, Hartl RF (2014) Vehicle routing problems in which
15 consistency considerations are important: A survey. *Networks: An International*
16 *Journal* 64(3):192–213
- 17
18 Lam E, Desaulniers G, Stuckey PJ (2022) Branch-and-cut-and-price for the elec-
19 tric vehicle routing problem with time windows, piecewise-linear recharging and
20 capacitated recharging stations. *Computers & Operations Research* 145:105870
- 21
22 Lang MA, Cleophas C, Ehmke JF (2021a) Anticipative dynamic slotting for attended
23 home deliveries. *Operations Research Forum* 2(4):70
- 24
25 Lang MA, Cleophas C, Ehmke JF (2021b) Multi-criteria decision making in dynamic
26 slotting for attended home deliveries. *Omega* 102:102305
- 27
28 Lanzarone E, Matta A (2014) Robust nurse-to-patient assignment in home care ser-
29 vices to minimize overtimes under continuity of care. *Operations Research for Health*
30 *Care* 3(2):48–58
- 31
32 Lanzarone E, Matta A, Scaccabarozzi G (2010) A patient stochastic model to support
33 human resource planning in home care. *Production Planning & Control* 21(1):3–25
- 34
35 Lebedev D, Goulart P, Margellos K (2021) A dynamic programming framework
36 for optimal delivery time slot pricing. *European Journal of Operational Research*
37 292(2):456–468
- 38
39 Lin II, Mahmassani HS (2002) Can online grocers deliver?: Some logistics considera-
40 tions. *Transportation Research Record* 1817(1):17–24
- 41
42 Liu R, Tao Y, Xie X (2019) An adaptive large neighborhood search heuristic for the
43 vehicle routing problem with time windows and synchronized visits. *Computers &*
44 *Operations Research* 101:250–262
- 45
46 Mackert J (2019) Choice-based dynamic time slot management in attended home
47 delivery. *Computers & Industrial Engineering* 129:333–345
- 48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1 Mancini S, Gansterer M (2022) Bundle generation for last-mile delivery with occasional
2 drivers. *Omega* 108:102582
- 3 Mathlouthi I, Gendreau M, Potvin JY (2018) Mixed integer linear programming for
4 a multi-attribute technician routing and scheduling problem. *INFOR: Information*
5 *Systems and Operational Research* 56(1):33–49
- 6
7 Mathlouthi I, Gendreau M, Potvin JY (2021a) Branch-and-price for a multi-attribute
8 technician routing and scheduling problem. *Operations Research Forum* 2(1):1
9
- 10 Mathlouthi I, Gendreau M, Potvin JY (2021b) A metaheuristic based on tabu search
11 for solving a technician routing and scheduling problem. *Computers & Operations*
12 *Research* 125:105079
- 13
14 Neves-Moreira F, Da Silva DP, Guimarães L, Amorim P, Almada-Lobo B (2018) The
15 time window assignment vehicle routing problem with product dependent deliveries.
16 *Transportation Research Part E: Logistics and Transportation Review* 116:163–183
17
- 18 Nguyen DH, de Leeuw S, Dullaert WE (2018) Consumer behaviour and order fulfil-
19 ment in online retailing: A systematic review. *International Journal of Management*
20 *Reviews* 20(2):255–276
21
- 22 Organisation for Economic Co-operation and Development (2020) E-commerce
23 in the times of COVID-19. Available at [https://www.oecd.org/coronavirus/
24 policy-responses/e-commerce-in-the-time-of-covid-19-3a2b78e8/](https://www.oecd.org/coronavirus/policy-responses/e-commerce-in-the-time-of-covid-19-3a2b78e8/), [Online;
25 Accessed 22 September 2023]
26
- 27 Ötken ÇN, Yıldız B, Arslan O, Laporte G (2023) Making opportunity sales in attended
28 home delivery. *Computers & Operations Research* 160:106362
29
- 30 Özarkıç SS, Veelenturf LP, Van Woensel T, Laporte G (2021) Optimizing e-commerce
31 last-mile vehicle routing and scheduling under uncertain customer presence. *Trans-
32 portation Research Part E: Logistics and Transportation Review* 148:102263
33
- 34 Pan B, Zhang Z, Lim A (2021) Multi-trip time-dependent vehicle routing problem
35 with time windows. *European Journal of Operational Research* 291(1):218–231
36
- 37 Pan S, Giannikas V, Han Y, Grover-Silva E, Qiao B (2017) Using customer-related
38 data to enhance e-grocery home delivery. *Industrial Management & Data Systems*
39 *117(9):1917–1933*
40
- 41 Pillac V, Gueret C, Medaglia AL (2013) A parallel matheuristic for the technician
42 routing and scheduling problem. *Optimization Letters* 7:1525–1535
43
- 44 Pisinger D, Ropke S (2007) A general heuristic for vehicle routing problems.
45 *Computers & Operations Research* 34(8):2403–2435
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1 Polnik M, Riccardi A, Akartunah K (2021) A multistage optimisation algorithm for the
2 large vehicle routing problem with time windows and synchronised visits. *Journal*
3 *of the Operational Research Society* 72(11):2396–2411
- 4 Punakivi M, Saranen J (2001) Identifying the success factors in e-grocery home
5 delivery. *International Journal of Retail & Distribution Management* 29(4):156–163
6
- 7 Restrepo MI, Semet F, Pocreau T (2019) Integrated shift scheduling and load assign-
8 ment optimization for attended home delivery. *Transportation Science* 53(4):1150–
9 1174
- 10
- 11 Righini G, Salani M (2008) New dynamic programming algorithms for the resource
12 constrained elementary shortest path problem. *Networks: An International Journal*
13 51(3):155–170
14
- 15 Savelsbergh MW (1985) Local search in routing problems with time windows. *Annals*
16 *of Operations Research* 4(1):285–305
17
- 18 Schmid V, Doerner KF (2014) Examination and operating room scheduling including
19 optimization of intrahospital routing. *Transportation Science* 48(1):59–77
20
- 21 Schneider M, Stenger A, Goeke D (2014) The electric vehicle-routing problem with
22 time windows and recharging stations. *Transportation Science* 48(4):500–520
23
- 24 Shahnejat-Bushehri S, Tavakkoli-Moghaddam R, Boronoos M, Ghasemkhani A (2021)
25 A robust home health care routing-scheduling problem with temporal dependencies
26 under uncertainty. *Expert Systems with Applications* 182:115209
27
- 28 Solomon MM (1987) Algorithms for the vehicle routing and scheduling problems with
29 time window constraints. *Operations Research* 35(2):254–265
30
- 31 Spliet R, Desaulniers G (2015) The discrete time window assignment vehicle routing
32 problem. *European Journal of Operational Research* 244(2):379–391
33
- 34 Spliet R, Gabor AF (2015) The time window assignment vehicle routing problem.
35 *Transportation Science* 49(4):721–731
36
- 37 Spliet R, Dabia S, Van Woensel T (2018) The time window assignment vehicle routing
38 problem with time-dependent travel times. *Transportation Science* 52(2):261–276
39
- 40 Strauss A, Gülpınar N, Zheng Y (2021) Dynamic pricing of flexible time slots for
41 attended home delivery. *European Journal of Operational Research* 294(3):1022–
42 1041
43
- 44 Strauss AK, Klein R, Steinhardt C (2018) A review of choice-based revenue
45 management: Theory and methods. *European Journal of Operational Research*
46 271(2):375–387
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

- 1 Subramanyam A, Gounaris CE (2018) A decomposition algorithm for the consistent
2 traveling salesman problem with vehicle idling. *Transportation Science* 52(2):386–
3 401
- 4 Subramanyam A, Wang A, Gounaris CE (2018) A scenario decomposition algorithm
5 for strategic time window assignment vehicle routing problems. *Transportation*
6 *Research Part B: Methodological* 117:296–317
- 7
8 Talluri KT, Van Ryzin GJ (2004) *The theory and practice of revenue management*.
9 Springer, New York
- 10
11 The Guardian (2019) How our home delivery habit reshaped
12 the world. [https://www.theguardian.com/technology/2019/nov/21/
13 how-our-home-delivery-habit-reshaped-the-world](https://www.theguardian.com/technology/2019/nov/21/how-our-home-delivery-habit-reshaped-the-world), [Online; Accessed 22 September
14 2023]
- 15
16 Toth P, Vigo D (2014) *Vehicle routing: Problems, methods, and applications*. SIAM,
17 Philadelphia
- 18
19 Ulmer MW, Goodson JC, Mattfeld DC, Thomas BW (2020) On modeling stochastic
20 dynamic vehicle routing problems. *EURO Journal on Transportation and Logistics*
21 9(2):100008
- 22
23 Ulmer MW, Thomas BW, Campbell AM, Woyak N (2021) The restaurant meal deliv-
24 ery problem: Dynamic pickup and delivery with deadlines and random ready times.
25 *Transportation Science* 55(1):75–100
- 26
27 Vareias AD, Repoussis PP, Tarantilis CD (2019) Assessing customer service reliabil-
28 ity in route planning with self-imposed time windows and stochastic travel times.
29 *Transportation Science* 53(1):256–281
- 30
31 Vidal T, Crainic TG, Gendreau M, Prins C (2013) A hybrid genetic algorithm with
32 adaptive diversity management for a large class of vehicle routing problems with
33 time-windows. *Computers & Operations Research* 40(1):475–489
- 34
35 Vidal T, Laporte G, Matl P (2020) A concise guide to existing and emerging vehicle
36 routing problem variants. *European Journal of Operational Research* 286(2):401–416
- 37
38 Vinsensius A, Wang Y, Chew EP, Lee LH (2020) Dynamic incentive mechanism for
39 delivery slot management in e-commerce attended home delivery. *Transportation*
40 *Science* 54(3):567–587
- 41
42 Voccia SA, Campbell AM, Thomas BW (2019) The same-day delivery problem for
43 online purchases. *Transportation Science* 53(1):167–184
- 44
45 Wang X, Wasil E (2021) On the road to better routes: Five decades of pub-
46 lished research on the vehicle routing problem. *Networks: An International Journal*
47

- 1
2 Wang XC, Kim W, Holguín-Veras J, Schmid J (2021) Adoption of delivery services in
3 light of the COVID pandemic: Who and how long? *Transportation Research Part*
4 *A: Policy and Practice* 154:270–286
5
- 6 Waßmuth K, Köhler C, Agatz N, Fleischmann M (2023) Demand management for
7 attended home delivery – A literature review. *European Journal of Operational*
8 *Research* 311(3):801–815
9
- 10 Yang X, Strauss AK (2017) An approximate dynamic programming approach to
11 attended home delivery management. *European Journal of Operational Research*
12 263(3):935–945
13
- 14 Yang X, Strauss AK, Currie CS, Eglese R (2016) Choice-based demand management
15 and vehicle routing in e-fulfillment. *Transportation Science* 50(2):473–488
16
- 17 Yıldız B, Savelsbergh M (2019) Provably high-quality solutions for the meal delivery
18 routing problem. *Transportation Science* 53(5):1372–1388
19
- 20 Yıldız B, Savelsbergh M (2020) Pricing for delivery time flexibility. *Transportation*
21 *Research Part B: Methodological* 133:230–256
22
- 23 Zamorano E, Stolletz R (2017) Branch-and-price approaches for the multiperiod tech-
24 nician routing and scheduling problem. *European Journal of Operational Research*
25 257(1):55–68
26
- 27 Zhan Y, Wan G (2018) Vehicle routing and appointment scheduling with team
28 assignment for home services. *Computers & Operations Research* 100:1–11
29
- 30 Zhan Y, Wang Z, Wan G (2021) Home service routing and appointment scheduling
31 with stochastic service times. *European Journal of Operational Research* 288(1):98–
32 110
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65