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Optimizing product assortment, joint replenishments, and storage capacity allocation in a deteriorating inventory system

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Abstract: In this paper, we consider a single retailer who sells multiple products subject to decay, and that implements coordinated inventory replenishments among them. The overall available surface at the retailer is limited and is partitioned into two areas: the backroom facility and the display area. The demand rate of each product is a function of displayed quantity and location, and it also depends on the cross-elasticity among items. The objective is to find the product assortment, the replenishment policy of each product, the quantity to be displayed, and the surfaces assigned to the backroom and display areas that maximize the total profit per time unit. We develop the mathematical model and formulate the optimization problem. Finally, we investigate the model response by means of numerical experiments considering several problem instances.

Keywords: Inventory; Decay; Optimization; Assortment; Capacity allocation; Joint replenishment.

1. INTRODUCTION

Retail industry since many years has been struggling in an increasingly competitive market. This competition is further exacerbated by the external influence of online retail shopping which involves, by now, almost all product categories. In this scenario, greater and greater attention must be paid to how retail operations are carried out.

Several are the factors that influence retail business profitability and most of these factors depends upon each other (Eroglu et al., 2010). Hence, the corresponding management decisions turn out to be interdependent and, worst still, insisting on different planning horizons (Hübner & Kuhn, 2012). For instance, typical decisions span from assortment planning, which is traditionally in the domain of strategic planning, to shelf space planning that can be classified both as short-term and mid-term planning as it may combine operational decisions with tactical inventory management decisions. Moreover, all these decisions are to be made with an attentive scrutiny of spatial, logistical, budgetary and other operational constraints (Lotfi et al., 2011).

A recent literature review provides a very effective and all-encompassing classification framework of retail shelf space planning problems (Bianchi-Aguiar et al., 2021). Despite many of the existing researches address the shelf space planning problems in a siloed manner, Bianchi-Aguiar et al. (2021) call for more comprehensive and integrated decision models in planning retail activities.

Shelf space planning decisions involve space assignment to each product, its allocation to a vertical shelf and its horizontal location on the shelf. However, in order to maximize its profit, retailer must also determine the set of products that it intends to display on the shelves deployed in the store. Also, it has to decide for each product the timing and amount of replenishment orders to minimize the inventory related costs. Although in practice these decisions are taken in different moments and in a not coordinated fashion, in order to avoid predictable sub-optimal results various researches have proposed integrated decision models which coordinate at least two of the above decisions while considering demand effects, such as space elasticity or cross-space elasticity, out of stock substitution, and different kinds of constraints (Bianchi-Aguiar et al., 2021). For example, Hwang et al. (2005) proposed an integrated model that coordinates replenishment decisions with space assignment and vertical location decisions under space and cross-space elasticity effects. Hariga et al. (2007) extended the previous model by including the product assortment problem and considering showroom and backroom (storage) inventories separately.

Because of the objective of this paper, we focus here on the papers proposing integrated decision models which coordinate space assignment and/or allocation decisions with product selection and replenishment decisions.

Ramaseshan et al. (2009) proposed a model to maximize the total net profit while having product assortment, shelf space allocation review period, and order quantity as decision variables. The model considers several constraints such as available shelf and backroom space, retailer's financial resources. The rate of demand for products is based on shelf space allocation and competing products. Ramaseshan et al. (2008), while considering an independent franchise retailer, developed a Category Management Decision Support Tool (CMDST). This tool integrates the concept of category management and supports optimal product assortment decisions, space allocation and inventory quantities.

Lotfi & Torabi (2011) propose a fuzzy mixed integer non-linear goal programming model for the mid-term assortment planning of supermarkets while considering profitability,

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customer service, and space utilization as conflicting objectives. The model applies joint replenishment policy and accounts for the holding time limitation of perishable items. As a consequence of problem-specific rules, a heuristic method is developed to solve the problem.

Kim & Moon (2021) developed a mixed-integer non-linear programming (MINLP) model for shelf-space allocation with product selection and replenishment decisions to maximize the retailer's profit. It considers a two-dimensional display, and demand for each product is sensitive to space and cross-space elasticities and positioning effects. Two heuristic algorithms (tabu search and genetic) are proposed to solve the MINLP problem.

Even if it does not consider a coordinated space assignment, product selection and replenishment problem, we cite here the paper by Lofti et al. (2011) as it proposes a mixed integer non-linear goal programming model for space allocation and replenishment planning while having the order quantity and the cycle time of joint replenishments along with the amount of allocated showroom and backroom spaces, as key decision variables. The model considers budget, space, holding times of perishable items as constraints and has the objective of minimizing weighted deviations from the conflicting objectives of profitability and customer service level. In addition to inventory investment costs, replenishment costs, and inventory holding costs the model also includes costs related to non-productive use of space.

Literature on integrated decision models is still sparse but nevertheless worthwhile of being investigated because of the relevant benefits it can bring to retail industry and also to the whole supply chain (see, for instance, Sajadieh et al., 2010; Hariga & Al-Ahmari, 2013).

This paper contributes to the existing research on integrated decision models in retail operations management by proposing a mathematical model which jointly considers surface assignment, product assortment and coordinated replenishment decisions. Specifically, it considers shelf and backroom surfaces as variables of the problem along with variety of products to be displayed. Moreover, it specifies the replenishment problem as a joint replenishment with inventory cycle times and displayed quantities as variables. In addition, the model explicitly accounts for decaying items which is a condition valid for most all products of large-scaled distribution. The problem is to maximize the retailer's profit per time unit by determining optimal shelf and backroom surface, variety of displayed products, replenishment policy, and displayed quantity.

2. PRELIMINARIES

We consider a single retailer who sells multiple products subject to decay, which are replenished in a coordinated way. In particular, a periodic review policy is adopted for each item, in which the review period is an integer multiple of a basic cycle time. Every time an item is ordered, a minor ordering cost is paid that is independent of the other products. A major ordering cost is paid with frequency determined by the basic cycle time and is independent of the number of items ordered. The storage surface available at the retailer is limited and is partitioned into two areas: the backroom storage facility and the display area. The objective is to find the product assortment, the replenishment policy of each product, the quantity to be displayed, and the extension of the backroom and display areas that maximize the total profit per time unit.

2.1 Notation

The mathematical model uses the following notation:

Decision variables:

T	Basic cycle time [time unit]
k_i	Integer multiplier of the basic cycle time relevant to product *i*
q_i	Order quantity of product *i* [quantity unit]
s_i	Quantity of product *i* displayed on the shelf [quantity unit]
S_B	Surface extension of the backroom facility [unit surface]
S_D	Surface extension of the display area [unit surface]
$\mathcal{P}$	Set of products in the assortment

Parameters:

A	Major ordering cost [$/order]
a_i	Minor ordering cost of product *i*, which includes the cost of replenishing the shelf [$/order]
c_i	Purchase cost of product *i* [$/unit]
h_i	Stockholding cost rate of product *i* in the backroom facility [$/unit/unit time]
v_i	Stockholding cost rate of product *i* in the display area [$/unit/unit time]
p_i	Unit selling price of product *i* [$/unit]
θ_i	Decay rate of product *i*
ϕ_i	Quantity of product *i* that can be stored in the backroom facility per unit surface [quantity unit/unit surface]
ψ_i	Quantity of product *i* that can be stored in the display area per unit surface [quantity unit/unit surface]
S	Total surface available at the retailer [surface unit]

Sets:

$\mathbb{R}$	Real numbers
$\mathbb{N}$	Positive, integer numbers

Further notation will be introduced as needed.

2.2 Assumptions

The mathematical model is developed according to the following assumptions (some of them can also be found in, e.g., Hariga et al., 2007):

1. We consider *N* products.
2. Replenishments to the system are instantaneous.
3. For each *i*, the ordered quantity q_i arrives directly at the backroom facility, and s_i units are immediately transferred to the display area.

4. Each product occupies a single location in the display area, and the shelf is continuously replenished from the backroom facility to keep the maximum inventory level until the backroom becomes empty. Thereafter, the displayed inventory starts diminishing until the shelf is completely depleted.

5. For each $i \in \mathcal{P}$, the demand rate of product i is known and deterministic, and it is a function of both the displayed quantity of item i and the displayed inventory level of other products to take into account substitutions or complementarities. In particular, similarly to Hariga et al. (2007), the demand rate of product i is

$$D_i(t) = \alpha_i I_{Di}^{\beta_i}(t) \prod_{j \in \mathcal{P}, j \neq i} s_j^{\delta_{ij}}, \qquad (1)$$

where $I_{Di}(t)$ is the inventory level of the shelf at time t for product i, α_i is the space scale parameter of product i, β_i is the main space elasticity of product i, and δ_{ij} is the cross-elasticity between products i and j. Note that, at time $t = 0$, i.e., at the beginning of the inventory cycle, the demand rate is

$$D_{0i} = D_i(0) = \alpha_i s_i^{\beta_i} \prod_{j \in \mathcal{P}, j \neq i} s_j^{\delta_{ij}}. \qquad (2)$$

If we use (2) in (1), we get

$$D_i(t) = D_{0i}(I_{Di}(t)/s_i)^{\beta_i}. \qquad (3)$$

6. The inventory level of item i is reviewed every $T_i = k_i T$ time units, for each $i = 1,2,\dots,N$.

7. The minor ordering cost a_i is paid every T_i time units, for each $i = 1,2,\dots,N$, while the major ordering cost A is paid every T time units.

8. The decay rate is known and constant, and deteriorated items are not repaired or replaced.

3. MATHEMATICAL MODEL

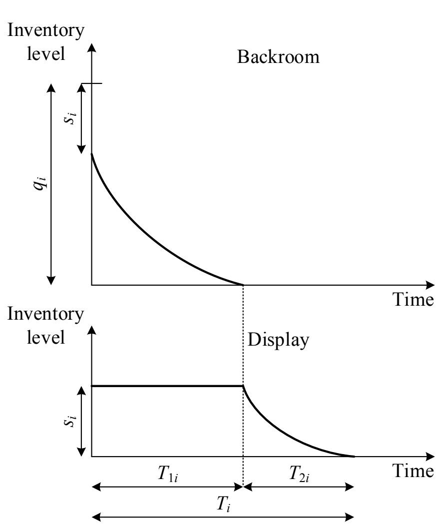

Figure 1. Inventory-time plot for a generic item i, with $i \in \mathcal{P}$.

Assume that $\mathcal{P} \neq \emptyset$ and consider a generic product i in the assortment, i.e., $i \in \mathcal{P}$. Figure 1 shows the inventory level dynamics in the backroom facility and in the display area for product i. The cycle time T_i is given by the sum of T_{1i} and T_{2i}, where the first one is the time needed to deplete the inventory in the backroom facility, and the second one is the time necessary to consume the stock in the display area, once the backroom becomes empty.

Assume that the inventory cycle begins at time $t = 0$. We let $I_{Bi}(t)$ be the inventory level in the backroom facility of item i. The time derivatives of $I_{Bi}(t)$ and $I_{Di}(t)$ are denoted by $\dot{I}_{Bi}(t)$ and $\dot{I}_{Di}(t)$, respectively. The ordinary differential equations (ODEs) capturing the inventory level dynamics in the backroom facility and in the display area are, respectively,

$$\dot{I}_{Bi}(t) = -\left(D_{0i} + \theta_i I_{Bi}(t)\right), \quad \text{with } t \in [0, T_{1i}], \qquad (4)$$

where $D_i(0)$ is given by (2), and

$$\dot{I}_{Di}(t) = -\left(D_i(t) + \theta_i I_{Di}(t)\right), \quad \text{with } t \in [T_{1i}, T_{1i} + T_{2i}], \qquad (5)$$

where $D_i(t)$ is given by (3). Note that, according to (3), $D_i(T_{1i} + T_{2i}) = 0$, as $I_{Di}(T_{1i} + T_{2i}) = 0$. Hence, the inventory system cannot observe backorders and/or lost sales.

Solving (4) with initial condition $I_{1i}(0) = q_i - s_i$ (the solution to this ODE can be found in, e.g., Luo, 1998), we get

$$I_{Bi}(t) = (q_i - s_i)\exp\{-\theta_i t\} - D_{0i}(1 - \exp\{-\theta_i t\})/\theta_i, \qquad (6)$$

with $t \in [0, T_{1i}]$. Since $I_{Bi}(T_{1i}) = 0$, from (6) we obtain

$$T_{1i} = \ln[1 + \theta_i(q_i - s_i)/D_{0i}]/\theta_i. \qquad (7)$$

Solving (5) with initial condition $I_{Di}(T_{1i}) = s_i$ (see, e.g., Polyanin and Zaitsev, 1995), we obtain

$$I_{Di}(t) = \left\{s_i^{1-\beta_i}\exp\{-\theta_i(t - T_{1i})(1 - \beta_i)\} + -D_{0i}(1 - \exp\{-\theta_i(t - T_{1i})(1 - \beta_i)\})/\left(\theta_i s_i^{\beta_i}\right)\right\}^{1/(1-\beta_i)}, \qquad (8)$$

with $t \in [T_{1i}, T_{1i} + T_{2i}]$. Using the condition $I_{Di}(T_{1i} + T_{2i}) = 0$, from (8) we obtain

$$T_{2i} = \ln(1 + \theta_i s_i/D_{0i})/[\theta_i(1 - \beta_i)]. \qquad (9)$$

Since $T_i = T_{1i} + T_{2i}$, then we have

$$T_i = \ln[1 + \theta_i(q_i - s_i)/D_{0i}]/\theta_i + \ln(1 + \theta_i s_i/D_{0i})/[\theta_i(1 - \beta_i)], \qquad (10)$$

which gives us

$$q_i = \left[\exp\{\theta_i T_i\} D_{0i}^{\frac{2-\beta_i}{1-\beta_i}} (D_{0i} + \theta_i s_i)^{-\frac{1}{1-\beta_i}} + -(D_{0i} - \theta_i s_i)\right]/\theta_i, \qquad (11)$$

where $T_i = k_i T$. Hence, we expressed q_i as a function of s_i, k_i, and T.

The average inventory levels in the backroom facility and in the display area are, respectively,

$$\bar{I}_{Bi} = \frac{1}{k_i T}\int_0^{T_{1i}} I_{Bi}(t)dt \quad (12)$$

and

$$\bar{I}_{Di} = \frac{1}{k_i T}\left(s_i T_{1i} + \int_{T_{1i}}^{T_{1i}+T_{2i}} I_{Di}(t)dt\right). \quad (13)$$

The average number of units that deteriorated in $[0, T_{1i}]$ is

$$\lambda_{1i} = q_i - s_i - D_{0i}T_{1i}, \quad (14)$$

being $D_{0i}T_{1i}$ the units demanded in $[0, T_{1i}]$. The average number of decayed units in $[T_{1i}, T_{1i} + T_{2i}]$ is

$$\lambda_{2i} = s_i - \int_0^{T_{2i}} D_i(t)dt\,, \quad (15)$$

as $\int_0^{T_{2i}} D_i(t)dt$ is the demand cumulated in $[T_{1i}, T_{1i} + T_{2i}]$. Hence, the average number of units lost because of deterioration in the inventory cycle is

$$\lambda_i = \lambda_{1i} + \lambda_{2i} = q_i - D_{0i}T_{1i} - \int_0^{T_{2i}} D_i(t)dt\,. \quad (16)$$

The ordering cost per cycle and the purchase cost per cycle are a_i and $c_i q_i$, respectively, while $p_i q_i$ is the revenue per cycle. Hence, the profit per time unit relevant to product $i \in \mathcal{P}$ is

$$\mathfrak{P}_i = \mathfrak{P}_i(T, s_i, k_i) = [(p_i - c_i)q_i - a_i - c_i\lambda_i]/(k_i T) + \\ -h_i\bar{I}_{Bi} - v_i\bar{I}_{Di}, \quad (17)$$

where q_i is a function of s_i, k_i, and T according to (11). Finally, considering the major ordering cost A, the total profit per time unit is

$$\mathfrak{P} = \mathfrak{P}(T, \mathbf{s}, \mathbf{k}) = \sum_{i\in\mathcal{P}} \mathfrak{P}_i - A/T, \quad (18)$$

where $\mathbf{s} = (s_i)_{i\in\mathcal{P}}$ and $\mathbf{k} = (k_i)_{i\in\mathcal{P}}$.

4. PROBLEM FORMULATION AND ANALYSIS

We first let $S_B = \rho S$, where ρ is a real number such that $\rho \in]0,1[$. Consequently, being $S = S_B + S_D$, we then have $S_D = (1-\rho)S$.

The optimization problem we tackle is the following:

$$\max\{\widehat{\mathfrak{P}}: \mathcal{P} \in \mathbb{P}, \rho \in]0,1[\}, \quad (19)$$

where $\mathbb{P}$ is the set of all possible product assortments, and $\widehat{\mathfrak{P}}$ is the optimal profit given a product assortment $\mathcal{P} \in \mathbb{P}$ and a fixed ρ. Given $\mathcal{P}$ and ρ, the optimal profit $\widehat{\mathfrak{P}}$ is obtained as follows:

$$\begin{aligned} \widehat{\mathfrak{P}} = \max_{(T,\mathbf{s},\mathbf{k})} \quad & \mathfrak{P} \\ \text{s.t.} \quad & q_i \geq s_i \ \forall\, i \in \mathcal{P} \\ & s_i > 0 \ \forall\, i \in \mathcal{P} \\ & T > 0 \\ & \sum_{i\in\mathcal{P}} s_i\psi_i^{-1} \leq S_D = (1-\rho)S \\ & \sum_{i\in\mathcal{P}} q_i\phi_i^{-1} \leq S_B = \rho S \\ & k_i \in \mathbb{N} \ \forall\, i \in \mathcal{P} \\ & q_i, s_i \in \mathbb{R} \ \forall\, i \in \mathcal{P} \\ & T \in \mathbb{R} \end{aligned}$$

We now make the following observations that are useful in the choice of a possible optimization method, in order to determine a heuristic solution. The optimization over $\mathbb{P}$ is a clear combinatorial optimization problem. For a given $\mathcal{P} \in \mathbb{P}$, the optimal $\mathbf{s}$ can be searched identifying lower and upper bounds for each s_i, with $i \in \mathcal{P}$: the upper bound is $\psi_i S_D$, while the lower bound is a sufficiently small, positive number, that is also smaller than $\psi_i S_D$. The optimization in ρ can be tackled through a line search over $]0,1[$.

From the first constraint, it is possible to get a lower bound for T. Using (11) (or (10)), the relation $q_i \geq s_i$ becomes

$$k_i T \geq \ln(1 + \theta_i s_i/D_{0i})\,/[\theta_i(1-\beta_i)] = T_{2i}, \quad (20)$$

which gives us

$$T \geq T_{2i}/k_i, \quad (21)$$

for each $i \in \mathcal{P}$. Then, the lower bound, $T_{\min}$, is thus given by

$$T_{\min} = \max\{T_{2i}/k_i : i \in \mathcal{P}\}. \quad (22)$$

We can then obtain an upper bound for T, denoted by $T_{\max}$, from the constraint on the available surface in the backroom facility. In particular, $T_{\max}$ is determined solving in T the equation

$$\sum_{i\in\mathcal{P}} q_i\phi_i^{-1} - S_B = 0\,, \quad (23)$$

where q_i is given by (11) with $T_i = k_i T$. Equation (23) can be solved only numerically.

Note that, for a given vector $(\mathbf{s}, \mathbf{k})$, it may happen that $T_{\max} < T_{\min}$. Evidently, this is a not admissible solution (independently of the capacity constraint in the display area).

5. NUMERICAL EXPERIMENTS

We carried out numerical experiments to investigate the response of the model in several problem instances. Experiments were carried out on a platform developed in MATLAB® R2020b on a machine with an Intel® Core™ i7-8750H CPU at 2.20GHz and 16GB RAM memory.

For each instance, we achieved a heuristic solution according to the following procedure. The optimization over ρ was carried out with a line search over $]0,1[$ with a 0.1 step size. For fixed ρ, a genetic algorithm (GA) was used to optimize both the product assortment and the vector $(\mathbf{s}, \mathbf{k})$, assuming that s_i was integer, for each $i \in \mathcal{P}$. GA was programmed in MATLAB® and operates similarly to the native GA in the Global Optimization Toolbox (e.g., a similar set of parameters was used). GA parameters were optimized by means of preliminary experiments (a detailed discussion is not possible because of the requirement on the maximum number of pages). A neighborhood search is also performed in each iteration of GA. For fixed $\mathcal{P} \in \mathbb{P}$, $(s_i)_{i\in\mathcal{P}}$, and $(k_i)_{i\in\mathcal{P}}$, the optimal basic cycle time was searched between the bounds $T_{\min}$ and $T_{\max}$ by means of the MATLAB® solver called with the function "fminbnd", which is included into Optimization Toolbox™, provided that $(s_i)_{i\in\mathcal{P}}$ and $(k_i)_{i\in\mathcal{P}}$ lead to a feasible solution, i.e., a solution such that $T_{\min} \leq T_{\max}$ and $\sum_{i\in\mathcal{P}} s_i\psi_i^{-1} \leq S_D$.

We consider a set of 7 products, i.e., $N = 7$. Reference parameter values are given in Tables 1 and 2. These values were randomly drawn within the intervals shown in Table 3. The presented problem instances are not representative of reality, but they only serve to analyze the model response. Costs/profits per time unit are expressed as yearly rates. Results of experiments are shown in Table 4 for two values of both the available surface, *S*, and the major ordering cost, *A*.

Table 1. Reference parameter values used in the experiments

	Product (*i*)						
	1	2	3	4	5	6	7
a_i	84	31	54	94	84	97	73
c_i	24	45	18	7	36	17	12
h_i	8	19	5	10	5	7	22
v_i	11	22	7	11	6	8	30
p_i	44	47	19	47	37	18	25
θ_i	0.1	0.4	0.5	0.4	0.4	0.4	0.3
ϕ_i	7	4	5	5	6	6	7
ψ_i	1	3	2	1	1	2	3
α_i	28	30	21	12	13	15	27
β_i	0.34	0.59	0.22	0.75	0.26	0.51	0.70

Table 2. Cross-elasticity matrix

	1	2	3	4	5	6	7
1	-	0.244	-0.350	0.251	-0.473	0.831	0.550
2	0.286	-	-0.568	0.054	0.779	0.130	-0.470
3	-0.337	-0.794	-	-0.602	0.654	-0.748	-0.083
4	-0.913	0.826	-0.996	-	-0.962	0.775	-0.869
5	-0.400	-0.800	-0.911	-0.264	-	0.580	0.145
6	-0.622	-0.513	-0.076	-0.123	-0.240	-	-0.945
7	-0.489	-0.900	0.111	-0.390	-0.404	0.132	-

Table 3. Intervals where parameter values were sampled

Parameter	Interval	Parameter	Interval
a_i	[20, 100]	ψ_i	[1, 3]
v_i/h_i	]1, 1.5[	ϕ_i	[4, 7]
p_i	[15, 50]	c_i/p_i	]0, 1[
θ_i	[0.1, 0.5]	h_i	[5, 25]
α_i	[10, 30]	δ_{ij}	]-1, 1[
β_i	]0, 1[		

We first observe that the solution always includes products 2, 4, and 6 into the assortment, except in the instance with *S* = 15 and *A* = 300, where only items 2 and 4 are part of the assortment. We then note that, for *A* = 50, the displayed quantity increases for almost all items as *S* becomes larger (s_4 is unchanged). Note that the order quantity, the basic cycle time, and the ratio ρ increase as well, while the integer multipliers reduce. For *A* = 300, a larger *S* makes the product assortment increased by one item, and the displayed quantity of products present in both cases, i.e., in cases *S* = 15 and *S* = 25, reduces (this may be related to the fact that a new item is included into the assortment). A clear trend in the order quantity is not evident, while the basic cycle time reduces, and the ratio ρ becomes larger.

We now discuss what happens for fixed *S*. For *S* = 15, an increment in *A* makes the product assortment reduced by one item (only products 2 and 4 remains in the assortment). Moreover, s_2 reduces, while s_4 increases. At the same time, q_2 increases, and q_4 becomes smaller (i.e., the changes in the displayed quantity and in the order quantity have different signs). When *A* increases, the basic cycle time becomes higher, and the integer multipliers reduce, as expected, while the ratio ρ does not modify. For *S* = 25, we do not observe any change in the product assortment. Moreover, s_2 reduces, s_4 increases, while s_6 is unchanged. The change in q_2 and q_4 is opposite to that observed for s_2 and s_4, while q_6, contrarily to the behavior of s_6, becomes larger. The behavior of the basic cycle, the integer multipliers, and the ratio ρ is similar to what observed for *S* = 15.

We finally comment on the change in the profit. For fixed *S*, the profit reduces as *A* increases, as expected. For fixed *A*, the profit is higher when *S* is greater, as larger product quantities are kept in inventory.

Table 4. Experimental results

			Solution					
S	*A*	Item	q_i	s_i	k_i	*T*	ρ	$\mathfrak{P}$
15		1	-	-	-			
		2	8.0	7	3			
		3	-	-	-			
	50	4	31.8	2	3	0.052	0.6	5950
		5	-	-	-			
		6	3.8	3	14			
		7	-	-	-			
		1	-	-	-			
		2	15.1	6	1			
		3	-	-	-			
	300	4	26.1	4	1	0.251	0.6	2271
		5	-	-	-			
		6	-	-	-			
		7	-	-	-			
25		1	-	-	-			
		2	11.2	9	2			
		3	-	-	-			
	50	4	69.1	2	2	0.085	0.7	14279
		5	-	-	-			
		6	5.2	5	11			
		7	-	-	-			
		1	-	-	-			
		2	11.3	5	1			
		3	-	-	-			
	300	4	57.5	3	1	0.181	0.7	9812
		5	-	-	-			
		6	19.0	5	9			
		7	-	-	-			

Now, assume S = 15 and A = 50, and consider the same parameter values as before (see Tables 1 and 2), except for θ_i, which is now increased by 50%, for each i. We aim at investigating the effect on the solution of increasing the decay rate. Results are given in Table 5.

Comparing results in Tables 4 and 5, we first note that product 6 was removed from the assortment. Moreover, both s_2 and s_4 increased, as well as the basic cycle time. For what concerns the order quantities, q_2 increased, while q_4 reduced. The integer multipliers and the ratio ρ decreased, as well as the profit.

Table 5. Results obtained increasing θ_i by 50%, for each i, with S = 15 and A = 50

Item	Solution					
	q_i	s_i	k_i	T	ρ	$\mathfrak{P}$
1	-	-	-			
2	12.2	11	2			
3	-	-	-			
4	14.7	5	1	0.102	0.4	4051
5	-	-	-			
6	-	-	-			
7	-	-	-			

6. CONCLUSIONS

This paper studied an inventory system in which a single retailer sells multiple products subject to decay and implements coordinated replenishments among them. The overall available surface is limited and is partitioned into two areas: the backroom facility and the display area. The problem was to find the product assortment, the replenishment policy of each product, the quantity to be displayed, and the surfaces assigned to the backroom and display areas that maximize the total profit per time unit.

Future research can be directed to investigate many questions related to the present study. For example, the properties of the profit function can be studied in order to develop an effective optimization algorithm. Moreover, further numerical experiments can be performed to get additional insights and to better evaluate the performance of the optimization algorithm. Finally, the model can be extended to include one or more actors in the upstream supply chain, such as multiple vendors.

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