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of the Informed Side of the Market

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Dynamic Adverse Selection and the Size of the Informed Side of the Market

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Abstract

In this paper we examine the problem of dynamic adverse selection in a stylized market where the quality of goods is a seller's private information. We show that in equilibrium all goods can be traded if a simple piece of information is made publicly available: the size of the informed side of the market. Moreover, we show that if exchanges can take place frequently enough, then agents roughly enjoy the entire potential surplus from exchanges. We illustrate these findings with a dynamic model of trade where buyers and sellers repeatedly interact over time. More precisely we prove that, if the size of the informed side of the market is a public information at each trading stage, then there exists a weak perfect Bayesian equilibrium where all goods are sold in finite time and where the price and quality of traded goods are increasing over time. Moreover, we show that as the time between exchanges becomes arbitrarily small, full trade still obtains in finite time – i.e., all goods are actually traded in equilibrium – while total surplus from exchanges converges to the entire potential. These results suggest two policy interventions in markets suffering from dynamic adverse selection: first, the public disclosure of the size of the informed side of the market in each trading stage and, second, the increase of the frequency of trading stages.

Key words: dynamic adverse selection; full trade; size of the informed side; frequency of exchanges; asymmetric information
JEL: D82, L15

1. Introduction

Since the publication of the seminal work by Akerlof (1970), the problem of adverse selection has been widely investigated by economic theorists. One quite recent development in this regard is the exploration of the dynamics of exchanges under asymmetric information, and in particular of the phenomenon of dynamic adverse selection (see e.g., Hendel and Lizzeri, 1999; Janssen and Roy, 2002; Hendel et al., 2005; Moreno and Wooders, 2010).[1] Although several important aspects of dynamic adverse selection have been investigated, so far no attention has been given to the consequences of the public access to information about the size of the informed side of the market, e.g., the number of goods or services on the market when their

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[1]A clear and formal account of dynamic adverse selection can be found in Bolton and Dewatripont (2005, Ch.9) where the consequences of multi-stage contracting are analyzed.

qualities are private information. The present paper explores this issue, identifying the potential benefits accruing from the public availability of such a piece of information.

In a market where trade can take place sequentially, the public access to information on the size of the informed side of the market can solve dynamic adverse selection problems. The mechanism behind this result is actually very simple and can be explained with a short example. Suppose that the informed side is constituted by two sellers, one who wants to sell one unit of a high quality good and the other who wants to sell one unit of a low quality good. Qualities are private information of the sellers. What does it mean to know the size of the informed side? Intuitively, this information tells agents how many goods – but not which qualities – are still circulating in the market. Upon arrival at the market, buyers are told that two goods are being supplied: one of high quality and the other of low quality. On the first day of the market, some buyers start offering a low price, certain that for a low price they can only buy the low quality good. This choice is reasonable since offering a high price would lead to expected losses because of information asymmetries. Given the low price, the seller of the high quality good will not accept as for her accepting would mean a certain loss. What about the seller of the low quality good? She could opt to wait in the hope of higher prices in the following days. However, if she also rejects the offer, then the size of the informed side does not change – two goods still circulate – and, hence, the following days buyers would face the same situation so that they would have no reasons to change their offer. Therefore, the seller of the low quality good finds it convenient to accept the low price offer, as the alternative would be indefinite waiting. As a result, the low quality good is sold and the size of the informed side shrinks by one unit. The second day of the market, upon disclosure of the fact that the size of the informed side has diminished by one unit, buyers still on the market find it reasonable to offer a high price as they know that the day before there were one high quality good and one low quality good, and that the previous low price offer could only be reasonably accepted by the low quality seller. Finally, since there is no good reason to expect a higher offer in the future, the high quality seller accepts the high price offer and the market clears.

The fact that the public knowledge of the size of the informed side of the market is a potential solution to dynamic adverse selection problems is an interesting finding from a theoretical perspective, but it might be of little relevance in practice if the size of the informed size can never be observed. Therefore, a natural question to ask is how likely it is that market participants have access to such information. At least in some circumstances, the size of the informed side can be quite a small piece of information to retrieve, e.g., when the market is small and all participants can directly observe goods. In the light of our analysis, in such circumstances we may expect that market forces lead to full trade, i.e., to the exchange of all goods brought to the market. However, not many real markets match this case, not even roughly. Large markets are costly to monitor. Moreover, the anonymity of market participants makes it hard to keep track of the number of goods circulating at any given time. In fact, the size of the informed side of the market is not an individual information, but an intrinsically global one. This last consideration suggests an interesting possibility. Even if the size of the informed side is not naturally available to market participants, it can be available to market authorities or external observers. Hence, such kind of information is a natural candidate for public disclosure by a dedicated authority, suggesting that our findings have an important normative insight. Indeed, in economic situations where adverse selection arises, a benevolent authority could induce additional desirable exchanges by providing a multi-stage environment for trade where at each stage information about the size of the informed side of the market is publicly disclosed.

Another important issue is whether the public knowledge of the number of goods circulating in the market is sufficient to let agents enjoy the whole potential surplus from exchanges, as would happen in the absence of asymmetric information on the quality of goods. Unfortunately, this desirable outcome in general is not guaranteed because sequential trade may take a substantial amount of time and, hence,

agents might be forced to wait long periods before enjoying their payoff which, reasonably, would be discounted accordingly. However, since the amount of time waited before exchanges take place has no special role in ensuring full trade, there is one straightforward way to increase total surplus: shortening the time between exchange opportunities. This implies that, if exchange opportunities are frequent enough, then agents roughly enjoy the whole potential surplus from exchanges.

Although these points are all conceptually simple, to prove them in a sufficiently general setup turns out to be quite a complicated task. In order to give readers the possibility to get the substance of our findings without forcing them to go through all technicalities and proofs, we begin the analysis (in section 4) by studying a simplified version of the model (whose general version is described in section 3). To keep things as simple as possible, we consider only two qualities (high and low) and assume that agents do not discount future payoffs. The structure of the game is as follows. The market exists for an infinite number of trading stages or until all goods are sold. Before the market begins the distribution of qualities brought to the market is announced. In each stage, firstly the number of goods still unsold is observed and, secondly, buyers make price offers at which they are willing to buy one good randomly chosen from any seller willing to sell; thirdly, sellers consider the list of price offers and decide whether to sell or not. We demonstrate that, in this setup, there exists a weak perfect Bayesian equilibrium leading to full trade in finite time.

The simplified model of section 4 shows with sufficient clarity why adverse selection can be solved by the possibility to make price offers conditional on the number of goods still on the market. However, the simple model is not rich enough to tackle other issues of interest which can arise when the number of qualities brought to the market is greater than two and future payoffs are discounted. In section 5 we study the general model presented in section 3 which can accommodate any number of qualities and the discounting of future payoffs. We show that also in this more general setup there exists a weak perfect Bayesian equilibrium leading to full trade. The proof of this result turns out to be substantially more complicated because, depending on qualities and discount factor, it can happen that goods of different qualities must be sold in equilibrium at the same price and in the same trading stage. Applying the model studied in section 5 we are also able (in section 6) to discuss issues related to welfare and, more specifically, to the losses due to the length of trading stages. In this regard we show that as the spell between exchanges tends to zero – or, equivalently, the discount factor tends to unity – full trade can still be obtained in finite time while total surplus tends to the entire potential surplus gainable from exchanges.

In section 7 we discuss a variety of issues related to the nature and robustness of our results. We begin by assessing some important features of full trade equilibria. Then, we indicate a few reasonable generalizations of the model which would not substantially modify our results. Moreover, we identify which assumptions – and, therefore, what market characteristics – are crucial for the emergence of full trade and the realization of the entire potential surplus from trade. Lastly, we discuss the role played by the possibility to condition current price offers to past price offers. We argue that this possibility brings in intertemporal competition among buyers as well as intertemporal competition between one buyer and herself in the past, potentially preventing full trade for reasons other than information asymmetries. We discuss in detail which are the cases where the different kinds of intertemporal competition lead to the existence of equilibria where not all goods are sold. We also argue that such equilibria are not intuitively very robust.

Finally, we refer the reader to the next section for a detailed discussion of the relation between the present paper and the existing literature on adverse selection, with special attention to dynamic and informational aspects.

2. Related literature

Dynamic adverse selection has recently been considered with an emphasis on the efficiency-enhancing role of multiple-stage contracting. In particular, it has been shown that if traders discount future payoffs then the delay of exchanges can fruitfully work as a screening device, potentially allowing market transactions that would never be made in the traditional static framework of Akerlof (1970). In this regard, an important contribution is provided by Janssen and Roy (2002) who show that delaying exchanges of high quality goods can effectively screen qualities and lead to full trade in a Walrasian dynamic setup (see also Janssen and Karamychev, 2002; Janssen and Roy, 2004, for an analysis of exchange cycles). More precisely, Janssen and Roy show that, under reasonable restrictions on agents' expectations about the minimum quality still on the market, every competitive equilibrium leads to full trade in finite time, with lower qualities being sold sooner and exchange prices increasing over time. The screening mechanism is based on the fact that sellers with higher-quality goods have greater incentive to wait for higher prices.

Our model and the one studied by Janssen and Roy (2002) are similar under many respects: there is a fixed set of potential sellers (each endowed with a unit of the good), the quality of goods varies across sellers, all buyers have the same valuations of goods, and trade always generate surplus. One difference is that Janssen and Roy (2002) focus mainly on the Walrasian approach, while we allow for strategic price setting.[2] The fundamental difference however lies in the mechanism which makes full trade possible: in our model the discounting of future payoffs is not crucial as the sorting of qualities over time is attained thanks to the possibility of conditioning price offers on the size of the informed side of the market.[3] The relevance of such a difference can be appreciated by considering the welfare loss due to the delay of exchanges as the discounting becomes arbitrarily small. In Janssen and Roy (2002) the welfare loss does not vanish since, as the discounting diminishes, the delay required for efficient sorting becomes consequently larger. Instead, as we show in section 6, in our model the welfare loss tends to disappear when the discount factor tends to unity.

A related body of literature has focused on the role of contract types (e.g., leasing rather than selling) and the interaction between new and used good markets. Hendel and Lizzeri (1999) investigate both the role and the existence of markets for used durables under dynamic adverse selection. They find that the used goods market never shuts down and, more importantly, that distortions due to information asymmetries are smaller than when re-sale is not allowed. Hendel and Lizzeri (2002) study the role of leasing contracts in durable goods markets. They show that, under dynamic adverse selection, leasing contracts for durables can improve welfare, although they remain imperfect tools. Johnson and Waldman (2003) find similar results, also showing that buybacks can further reduce inefficiencies due to asymmetric information.[4] Of particular interest here is the contribution by Hendel et al. (2005) who show that, if consumers observe the number of times a durable good has changed hand, then the combination of multiple secondary markets and endogenous assignment of new goods can completely eliminate the inefficiencies caused by asymmetric information. More precisely, they show that there exists a competitive equilibrium in which a menu of rental contracts induces precisely the same allocation that would prevail if quality were observable. Thus,

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[2]In the last part of the paper Janssen and Roy (2002) provide a game theoretical foundation of competitive equilibria by showing that the outcome of any such equilibrium can be sustained as weak perfect Bayesian equilibrium of a game where sellers make take-or-leave-it offers as well as of a game where buyers compete for the seller's good as in an auction.

[3]The fact that the costs of delay due to the discounting of future payoff are not responsible for the sorting of qualities over time can be neatly recognized by looking at the model presented in section 4, where there is no discounting of future payoffs.

[4]See also Johnson and Waldman (2010) for an extension of the model including moral hazard problems concerning the mainte-nance choices of owners of non-used goods.

if consumers can observe the times a good has been rented in the past, then asymmetric information can be fully overcome.

An important common element between our paper and Hendel et al. (2005) is that both provide a solution to dynamic adverse selection problems that does not exploit the delay in exchanges as a screening device (as, e.g., in Janssen and Roy, 2002). This is a relevant point because, as mentioned above, it affects welfare through the timing of exchanges. One difference with our approach is that Hendel et al. (2005) focus on competitive equilibria, disregarding strategic price setting (as done in Janssen and Roy, 2002). The most important difference, however, is that the mechanism underlying the provided solutions to dynamic adverse selection is based on different institutional and informational specifications. In Hendel et al. (2005) full trade arises only if consumers can publicly observe the vintage of used goods – i.e., the number of times the good has been rented in the past – that in equilibrium is correctly interpreted as a signal of goods' deterioration, which in turn allows the efficient sorting of consumers and used goods in secondary markets. In particular, the use of rental contracts in place of re-sale contracts is crucial because with rental contracts there is no incentive to keep used goods when they become of a lesser quality, so that vintages become an effective signal of current quality. In contrast, in our setup goods can change hand just once and only by standard sale contracts. Moreover, public information is not about vintages but about the number of goods still on the market, with straightforward consequences on incentives. In particular, the possibility to condition price offers on the number of goods still on the market gives to buyers an instrument to wipe out the benefits that a seller may expect to gain by refusing to sell at current prices in the hope of selling in the future at a higher price.

In recent years there has been an upsurge of interest in the relation between the overall informational structure and the likelihood of market failures due to adverse selection. Indeed, there are various aspects of the informational structure that can turn out to be relevant – e.g., the number and the quality of information, the distribution of information among agents – and it is not always straightforward to understand whether they mitigate or exacerbate adverse selection problems. Some basic facts, however, are now established. Kessler (2001) considers a lemons market where the seller can be uninformed with some probability and shows that welfare is non-monotonic in the amount of information on qualities. Levin (2001) shows that greater information asymmetries can reduce the gains from trade, although better information on the uninformed side unambiguously improves trade when demand is downward sloping. Creane (2008) proves that, in a pooling equilibrium where a monopolist sells a product of unknown quality to a group of consumers, welfare can be locally decreasing in the fraction of informed consumers. Sarath (1996) considers the issue of information disclosure to market participants and shows that entrusting the choice of (unverifiable) public information quality to traders who benefit from such information leads to inefficiencies, while delegating the choice of information quality to an independent agent who cannot share trade profits results in efficient implementation.[5]

Each of these contribution sheds some light on the role of informational structures in adverse selection problems, but none of them considers the case of dynamic adverse selection. As shown by Hörner and Vieille (2009) a dynamic environment can rise new and specific informational issues. More precisely, Hörner and Vieille (2009) compare the effects of public versus private price offers in a dynamic adverse selection model with information and payoff structures as in Akerlof (1970). Differently from our model, they consider a situation where there is a unique seller who bargains sequentially with potential buyers until agreement is reached, if ever. As in our (general) model, Hörner and Vieille (2009) allow for strategic price

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[5]Along this line of research there are also contributions which consider screening strategies. For instance, Lewis and Sappington (1994) show that, if the seller can make discriminating offers to the buyer, then the effects of a more informed buyer are ambiguous on the seller's welfare (see also Ottaviani and Prat, 2001).

setting (as well as discounting of future payoffs) and assume that buyers make a take-it-or-leave-it offer to the seller. Interestingly, Hörner and Vieille (2009) find that trade always eventually occurs when offers are private – i.e., buyers cannot observe past offers made by other buyers – while bargaining often ends at an impasse when offers are public. This happens because, with public offers, buyers can compete intertemporally deterring each other from making offers that lead to trade. This is a specific feature of dynamic settings with strategic price setting. Indeed, also in our model intertemporal competition among buyers can sustain weak perfect Beyesian equilibria where not all goods are sold in finite time. In addition, since in our model buyers stay on the market until they buy a good, there is also the possibility of intertemporal competition between a buyer now and herself in the future. We discuss in detail this issue in section 7 where we argue that equilibria sustained by intertemporal competition among buyers are intuitively not robust.

Further evidence of the importance of the observability of price offers is indirectly provided by Moreno and Wooders (2010) who show that in a dynamic model of decentralized trade all goods entering the market are sold at some stage, notwithstanding asymmetric information on goods quality. Decentralization of exchanges is modeled with a random matching mechanism where uninformed buyers make a take-it-or-leave-it offer to the informed seller they are matched with. If the seller accepts, then exchange takes place and both agents exit the market. If the seller rejects the offer, then both agents remain in the market and are randomly matched again. Note that such a matching mechanism entails private price offers, with the result that the problem of intertemporal competition mentioned above does not arise. Moreover, since agents are matched with different partners in each round they remain on the market, intertemporal competition of a single buyer with herself in the past does not arise (see on this our discussion in section 7). As in the competitive equilibria of Janssen and Roy (2002), full trade emerges thanks to the cost associated with the delay of exchanges that works as a screening device.[6] A relevant difference between our model and the one studied by Moreno and Wooders (2010) is that in the latter new cohorts of buyers and sellers can enter the market in each trading stage. This is similarly to what done in Janssen and Karamychev (2002) and Janssen and Roy (2004), although differently from them Moreno and Wooders only consider stationary competitive equilibria. Thus, since stationarity in our framework is meaningless, the comparison between our full trade equilibria and those of Moreno and Wooders is not of great interest.[7] Still, from a broader perspective, our model has something to say: thanks to the public knowledge of the informed side of the market, decentralization of trade – and in particular the privateness of price offers – is not required to approximate efficiency, not even under strategic price setting.

Decentralized trade in the form of random matching is also considered by Kultti et al. (2007) who study a dynamic adverse selection model where matching between sellers and buyers randomly generates a competitive situation that varies across different matchings. Interestingly, Kultti et al. (2007) find that a stationary equilibrium with equally frequent sales of all qualities is feasible for a narrower range of quality distributions than in Akerlof (1970). This happens when trading frictions – in the sense of Moreno and Wooders (2010) – are large enough because frictions increase buyers' ability to extract positive surplus from low quality goods when they are matched in a favorable competitive situation. In our model this effect is absent since trade is not decentralized and buyers have no chance to find themselves in a favorable competitive situation.

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[6]Moreno and Wooders (2010) refer to "frictions" as both the discounting of future gains and the possible delay in matching with a trading partner. In line with Janssen and Roy (2002), in Moreno and Wooders (2010) as frictions go to zero payoffs tend to the ones obtained in the single-period competitive equilibrium because, although traders become more patient, delay increases even more.

[7]We will come back to the issue of new buyers and sellers entering the market at later stages in section 7, where we discuss possible extensions of our model.

Another paper that deals with the role of public information in markets plagued by dynamic adverse selection is Daley and Green (2009). More precisely, Daley and Green (2009) investigate the market for financial assets under asymmetric information with the public disclosure, at each trading stage, of an information that affects the future value of the traded asset – as it typically happens in financial markets. Daley and Green (2009) show that, depending on beliefs, in equilibrium there can be immediate trade, no trade at all, or partial trade. There are various important differences between our model and this one – e.g., arrival of new buyers each at each stage, private price offers – but the most important one regards the nature of the information that is publicly disclosed at each trading stage. In Daley and Green (2009) what is disclosed is a piece of information that has actually to do with the value of the asset in the future – e.g., the current cash flow of a firm that has to be sold – and that is typically thought as a private information of the seller. In our model, instead, the piece of information that is disclosed to market participants has nothing to do with the actual value of the goods traded and is not a private information of the sellers but a characteristic of the market itself. In particular, the size of the informed side of the market is just one piece information even when sellers bring to the market a large numbers of qualities.

Finally, our paper is also partly related to the literature on sequential bargaining with one-sided incomplete information (see, among others, Fudenberg et al., 1985; Evans, 1989; Vincent, 1989; Deneckere and Liang, 2006). This literature studies a buyer and a seller who bargain over a unit of an indivisible good, with agents' valuations being a private information. The basic findings of this stream of literature are that the existence of a dynamic dimension together with some form of discounting of future payoffs can lead to intertemporal price discrimination and to the use of waiting times as screening devices (Gul and Sonnenschein, 1988). Thus, efficiency is in general not attained in these models. In particular, as discussed for Janssen and Roy (2002) and others, the welfare loss due to the discounting of future payoffs can persist even if the frequency of bargaining stages is increased since more stages are then needed to effectively screen qualities. In our model too there is price discrimination in full trade equilibria with prices (and qualities sold) increasing over time – a result often referred to as the skimming property. However, since delays are not used as screening devices, by making bargaining stages more frequent the efficient outcome can be reached in the limit.

3. A model of dynamic adverse selection with public information about the number of unsold goods

We model a situation of dynamic adverse selection where, in each trading stage, the size of the informed side of the market is observed by both buyers and sellers. We consider a market with n sellers and m buyers. We refer to the generic seller as seller i, with $i \in \{1, \ldots, n\}$, and to the generic buyer as buyer j, with $j \in \{1, \ldots, m\}$. We assume $m > n$ so that there is enough demand for all goods (for which $m \geq n$ is sufficient) and there is competition among buyers at every possible stage (which requires $m > n$). We will discuss this assumption in section 7. Each seller comes to the market with one good, and goods can differ by quality across sellers. The quality of a good is a private information of its seller, while the overall distribution of qualities is a public information. The market stays open from the first stage onwards, i.e., for periods $0, 1, 2, \ldots$, and buyers and sellers stay on the market until they complete a transaction.

In each time period the following things happen in order. Information about the current size of the informed side of the market – i.e., the number of goods still on the market – is disclosed to all market participants. Buyers simultaneously make price offers that are valid for the current period only and at which they are willing to buy one good. Price offers are public, i.e., observable by both buyers and sellers. After observing price offers, all sellers simultaneously decide whether to sell or not their goods (at the highest price offer). The allocation of goods is resolved as follows. The sellers who have accepted to sell are randomly paired with the buyers who have made the highest price offer: a transaction occurs for each

random pair of one seller and one buyer. Clearly, some sellers (buyers) will not be able to sell (buy) if the number of sellers wishing to sell is larger (smaller) than the number of buyers making the highest offer. When a transaction occurs between a seller and a buyer, both the buyer and the seller exit the market and the good that has been exchanged is no longer traded. We will discuss this important assumption in section 7. Figure 1 illustrates the timing of the game.

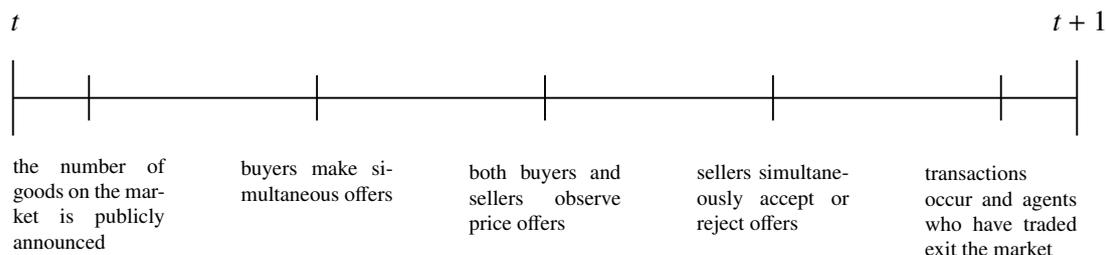|				
the number of goods on the market is publicly announced	buyers make simultaneous offers	both buyers and sellers observe price offers	sellers simultaneously accept or reject offers	transactions occur and agents who have traded exit the market

Figure 1: Timing of the stage game.

Payoffs are assigned as follows. Let $0 < \delta \leq 1$ be a discount factor common to all players, which can be interpreted as a measure of either progressive good deterioration or agents' impatience or a combination of both.[8] All buyers are identical so they have the same payoff structure. We denote with b_i the reservation price of good i for buyers and with s_i the reservation price of good i for seller i. A possible interpretation of b_i and s_i is that they are the present values of the stream of services granted to the owner of good i.[9] Alternatively, one can think that agents have appropriate options. We note that the payoff of buyers if they buy nothing is 0. To simplify notation, we also normalize at 0 the payoff of sellers in the case they do not sell the good – i.e., we systematically subtract s_i from the payoff of seller i. Therefore, considering present values at time t', a buyer who obtains good i for price p at time $t > t'$ evaluates the transaction $\delta^{t-t'}(b_i - p)$, while seller i evaluates the same transaction $\delta^{t-t'}(p - s_i)$. Without loss of generality, we assume that $i' > i$ implies $s_{i'} \geq s_i$, meaning that goods are ordered by increasing quality. Moreover, we impose that all goods brought to the market can potentially generate a surplus for both trading parties, i.e., $b_i > s_i$ for all i.

We introduce some further notation to simplify the exposition. We denote with G the set of all goods, with $G(t) \subseteq G$ the set of goods still unsold at time t, with $g^t = \|G(t)\|$ its cardinality, and with $S(t) \subseteq G$ the set of goods sold at time t. We refer to g^t equivalently as the size of the informed side of the market and as the number of goods still on the market at time t (we simply use g when we do not refer to a specific time period). A generic price offer observed in the market is denoted with $p \in \mathbf{R}_+$, a generic vector of price offers at time t (one for each buyer on the market) is denoted with $\boldsymbol{p} \in \mathbf{R}_+^{m-n+g(t)}$, and the maximum price offer in $\boldsymbol{p}$ is indicated with $p_{\max}(\boldsymbol{p})$.

The informational structure is as follows. At each stage buyers and sellers are informed about the number of goods still on the market. Moreover, they remember all previous offers. In addition, sellers also observe current offers before taking a decision about selling. Therefore, at each stage a buyer j knows $(\mathbf{g}^t, \mathbf{P}^{t-1})$: the sequence $\mathbf{g}^t = (g^0, g^1, \ldots, g^t)$ of observed numbers of unsold goods from time 0 to time t, and

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[8]Allowing heterogeneity of discount factors across agents is not conceptually demanding, but it makes the notation heavier. Since we do not find it particularly interesting to consider this heterogeneity, we stick to the case of a common discount factor and refer to section 7 for a brief discussion on the issue.

[9]We note that this interpretation does not work for $\delta = 1$ if goods provide an infinite stream of services.

the sequence $\mathbf{P}^{t-1} = (\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^{t-1})$ of buyers' price offers from time 0 to time $t-1$. At each stage a seller i knows $(\mathbf{g}^t, \mathbf{P}^t)$: the sequence $\mathbf{g}^t = (g^0, g^1, \ldots, g^t)$ of observed numbers of unsold goods from time 0 to time t, and the sequence $\mathbf{P}^t = (\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^t)$ of all price offers including the list of offers at time t.

A strategy for buyer j is a function π_j that assigns a price offer $p \in \mathbf{R}_+$ to each of her information sets. A strategy for seller i is a function ρ_i that assigns a response of either 1 or 0 to each of her information sets, where 1 means willingness to sell and 0 means refusal to sell. A strategy profile is $(\boldsymbol{\pi}, \boldsymbol{\rho})$, where $\boldsymbol{\pi}$ is the m-dimensional vector of buyers' offer functions and $\boldsymbol{\rho}$ is the n-dimensional vector of sellers's response functions. We say that a strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ leads to *full trade* when sooner or later every good is sold, that is, when there exists some $\hat{t}$ such that $\bigcup_{t \le \hat{t}} S(t) = G$.

Finally, throughout the paper we focus on the weak perfect Bayesian equilibrium (wPBE hereafter) as a solution concept. In brief, a strategy profile is wPBE if for every player, for every information she may obtain in the game, there exists a belief on the previous play of the game such that, in the presumption that subsequent play by other players unfolds as prescribed by the strategy profile, there does not exist a strictly convenient deviation from the behavior prescribed for the remaining part of the game. Beliefs are unrestricted everywhere but along the equilibrium path, where instead they are derived by means of Bayes rule (see e.g., Mas-Colell et al., 1995).

4. Existence of a weak perfect Bayesian equilibrium that leads to full trade for $n = 2$ and $\delta = 1$

In this section we illustrate how the information about the size of the informed side of the market can be exploited to obtain an equilibrium strategy profile that leads to full trade. The basic idea is that price offers are conditioned to the current number of unsold goods in such a way that, first, buyers can buy goods of the lowest possible quality without incurring in expected losses, second, only goods of lowest quality can be sold without incurring in losses, and, third, sellers of lowest quality cannot do better by refusing to sell. As a consequence of these price offers, sellers of lowest quality finds it optimal to sell so that the goods of lowest quality exit the market and the size of the informed side of the market shrinks accordingly. Iterating such price offers in the following stages we obtain that the quality of traded goods increases over time as well as the price at which transactions occur. We emphasize that, in the current setup with $\delta = 1$, observing the current number of unsold goods at the beginning of each trading stage is crucial for the obtainment of full trade.[10] The reason is that conditional price offers can dissuade sellers from refusing to sell in the hope of higher future offers since such higher offers will never arrive: if current offers are rejected then the number of goods still on the market stays put which implies that price offers will be the same in the next period.

The proof that this idea can actually work requires to tackle an issue raised by the presence of a time discount. As mentioned above, superior qualities are exchanged at higher prices in later periods. However, with the discounting of future payoffs, a price offer in the future – which is in any case bounded by buyers' valuations – may well not be enough to compensate the cost of waiting. This can result, depending on parameter values, in the impossibility to have an equilibrium where different qualities are sold at different times. In Section 5 we will cope with this problem by means of an algorithm dividing sellers in subsets, such that each subset comprises sellers that find it convenient to sell at the same time. In this section, however, we prefer to set the issue aside by assuming $\delta = 1$, so that there is no cost of waiting. This allows us to present the gist of our findings in a simpler and more intelligible way.

Furthermore, given the illustrative aim of the current section, we make an additional simplifying assumption: we consider only two sellers, i.e., $n = 2$, so that we may simply speak of a low quality good L

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[10]As already discussed, when $\delta < 1$ the waiting time can be used as a screening device to obtain full trade (see Janssen and Roy, 2002).

and a high quality good H (we also use L and H as identifying subscripts), with $s_L < s_H$. We will refer to the model with $\delta = 1$ and $n = 2$ as simplified model.

We are now ready to state our result for the simplified model.

Proposition 1

When $n = 2$ and $\delta = 1$, if the size of the informed side of the market is a public information then there exists a strategy profile that is wPBE and leads to full trade in 2 periods.

Proof. We now show that the following profile of strategies is a wPBE of the above game:

buyers: at every time period t, offer b_L if $g = 2$, and b_H if $g = 1$;

seller L: at every time period t, accept any offer larger or equal than b_L if $g = 2$, and accept any offer larger or equal than b_H if $g = 1$;

seller H: accept any offer larger or equal than b_H.

We consider decisions at a generic period and we call p_{max} the maximum price offer in that period. We begin the check from seller L. Consider first the case $g = 2$. If $p_{max} < b_L$, then seller L finds it strictly convenient to wait the next period, when she will receive an offer equal to b_L. If $b_L \leq p_{max} < b_H$, then seller L finds it convenient to accept (strictly convenient if $p_{max} > b_L$), since in case of refusal she will receive a price offer equal to b_L in the next period and at every future period if she goes on rejecting offers (because seller H always rejects offers strictly lower than b_H). If $p_{max} \geq b_H$, then seller L finds it convenient to accept (strictly convenient if $p_{max} > b_H$), since in case of refusal she will receive a price offer equal to b_H in the next period (because seller H accepts the current maximum price offer) and in every future period if she goes on rejecting offers. Now consider the case $g = 1$. The price offers in the next period – and in every future period if seller L goes on rejecting offers – will be equal to b_H. Therefore, if $p_{max} < b_H$ then seller L finds it strictly convenient to wait, while if $p_{max} > b_H$ then seller L finds it strictly convenient to accept, and she is indifferent between accepting and waiting in case $p_{max} = b_H$.

We now consider seller H. If $p_{max} < b_L$, then seller L finds it strictly convenient to wait for one period (if $g = 1$) or two periods (if $g = 2$): if good L is still on the market then this time no sale takes place, the next period good L will be sold at price b_L, and in the following period seller H will receive an offer equal to b_H; if instead good L has already been sold then this time no sale takes place, and the next period seller H will receive an offer equal to b_H. If $b_L \leq p_{max} < b_H$, then seller H finds it strictly convenient to delay selling at least until the next period: if good L is still on the market then it is sold at this time, otherwise no sale takes place; in any case, the next period seller H will receive an offer equal to b_H. If $p_{max} \geq b_H$, then seller H finds it convenient to accept (strictly convenient if $p_{max} > b_H$), since in case of refusal she will receive a price offer equal to b_H in the next period (because the number of goods still on the market will be 1 since seller L, if still on the market, accepts the current maximum price offer) and in every future period if she goes on rejecting offers.

Finally we consider a generic buyer at a generic information set in period t. If $g = 2$, then following the equilibrium strategy profile grants a payoff equal to 0, and no positive payoff can ever be obtained since seller L and seller H do not accept offers lower than b_L and b_H, respectively. If $g = 1$, then, if the information set is on the equilibrium path, then the only admissible belief requires that the good still on the market is H while, if the information set is not on the equilibrium path, then we choose the belief that the good still on the market is H. In any case, there is no strictly convenient deviation, since following the equilibrium strategy profile yields a null payoff, and no positive payoff can ever be obtained since seller H does not accept offers lower than b_H. $\square$

We end this section with two remarks. The first remark is on efficiency. In the simplified model every wPBE that leads to full trade is, obviously, efficient. However, this result crucially hinges on $\delta = 1$, i.e., on the absence of discounting of future payoffs. Indeed, when exchanges at later stages are worth less than exchanges in the current stage, equilibria where transactions occur sequentially do not allow to extract the whole surplus. In Section 6 we will focus on this issue showing that, if the time between any two subsequent rounds of market exchanges can be made arbitrarily small, then efficiency is recovered in the limit (Proposition 3).

The second remark is on the impossibility to obtain full trade when $\delta = 1$ and the size of the informed side of the market is not a public information. Consider the above setup with the only difference that the number of goods still on the market is not publicly observable. Intuitively, the exchange of the good of quality H encounters an unsurmountable obstacle which is represented by the impossibility for buyers to make offers that are conditional on the number of goods still on the market. In fact, a low current price offer combined with a high *unconditional* price offer in a later period – which is necessary to convince seller H to sell – will convince seller L to reject the current low price offer, since she knows that her current rejection will not prevent buyers from making the high offer in the future. This confirms that, when $\delta = 1$, in order to obtain full trade additional public information are required, such as the size of the informed side of the market or goods' vintage (Hendel et al., 2005) – in contrast, when $\delta < 1$ full trade can be obtained by using delays as screening (Janssen and Roy, 2002).

5. Existence of a weak perfect Bayesian equilibrium that leads to full trade for $n \geq 2$ and $\delta \leq 1$

In this section we remove the simplifying assumptions applied in section 4 and prove the same existence result for a more general model. Our objective is to assign the selling of each good in G to some time period – possibly having more goods sold at the same time – in such a way that the specified order is sustainable as wPBE, for a given value of the discount factor. As already anticipated in the previous section, in order to do this we rely on an algorithm which can be applied to any $G' \subseteq G$ yielding a partition of G' and a function $H(k, G')$ that assigns each element of the partition of G' to a distinct time period k, so that $H(0, G')$ is the subset of goods assigned to period 0 (current period), $H(1, G')$ is the subset of goods assigned to period 1 (next period), and so on and so forth. Intuitively, k indicates how many periods in the future goods in $H(k, G')$ will be sold. In this reagard it is also useful to define the function $\tau(i, G')$ indicating how many periods in the future good i will be sold, i.e. $\tau(i, G') = k$ such that $i \in H(k, G')$. The algorithm used for the construction of the partition of G' and function $H(k, G')$ is relegated to the Appendix. What matters here is that the algorithm allows us to establish some properties of function $H(k, G')$ (which are stated in Lemma 1 below) that turn out to be crucial for proving the existence of a wPBE that leads to full trade in this more general setup (see Proposition 2 below).

We also introduce some new notation. For any $G' \subseteq G$ subset of goods, we denote with $\beta(G')$ the average value for buyers of goods in G', i.e. $\beta(G') = \frac{\sum_{\text{good } i \in G'} b_i}{\|G'\|}$. Furthermore, we call $G^+(g)$ the set of g goods with highest index in G.

Lemma 1. *There exists a function $H(k, G')$ satisfying the following properties:*

> **Property 1.** *If good i belongs to $H(0, G')$, then $\beta(H(0, G)) - s_i \geq \delta^k(\beta(H(0, G''(k))) - s_i)$, with $G''(1) = (G' \setminus H(0, G')) \cup \{good\ i\}$ and $G''(k) = (G''(k-1) \setminus H(0, G''(k-1))) \cup \{good\ i\}$, for all k;*

> **Property 2.** *For every $k < \tau(i, G')$, $\beta(H(k, G')) - s_i < \delta^{\tau(i,G')-k}[\beta(H(\tau(i, G'), G')) - s_i]$.*

Proof. See the Appendix. $\square$

Property 1 concerns the non-optimality of deferring the time of selling, while Property 2 concerns the non-optimality of anticipating the time of selling. However, the meaning of these properties is better seen by considering the case where buyers, in a given time period, offer a price equal to the discounted average valuation of the goods associated with that time period, i.e., buyers offer $\delta^k \beta(H(k, G'))$ in k periods from now. In such a situation, Property 1 says that if seller i is among those who are selected to sell in the current time period, then she does not find convenient to reject current offers and sell in any future period. Property 2, instead, says that if seller i is selected to sell in k periods from now, then she does not find convenient to accept an offer made in previous time periods.

The following proposition is the counterpart of Proposition 1 but obtained in a more general setup with many sellers and discounting of future payoffs. In the proof we propose a strategy profile which leads to full trade and check that it is indeed a wPBE. Figure 2 illustrates what happens over time when buyers and sellers follow such equilibrium strategy profile.

Proposition 2
When $n \geq 2$ and $0 < \delta \leq 1$, if the size of the informed side of the market is a public information then there exists a strategy profile that is wPBE and leads to full trade in at most n periods.

Proof. Preliminarily, we let i^* be equal to i if $i \in G^+(g)$ and i^* be equal to the minimum index in $G^+(g)$ if $i \notin G^+(g)$ We consider $(\boldsymbol{\pi}, \boldsymbol{\rho})$ as the strategy profile proposed for equilibrium, where:

buyers: for every buyer j, for every information $(\mathbf{g}^t, \mathbf{P}^{t-1})$, buyer j makes the price offer $\pi_j(\mathbf{g}^t, \mathbf{P}^{t-1}) = \beta(H(0, G^+(g^t)))$;

sellers: for every seller i, for every information $(\mathbf{g}^t, \mathbf{P}^t)$, seller i accepts to sell, i.e., $\rho_i(\mathbf{g}^t, \mathbf{P}^t) = 1$, if and only if $p_{\max}(\boldsymbol{p}^t) \geq \delta[\beta(H(\tau(i^*, G^+(g^t)), G^+(g^t)))]$.

The strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ leads to sell some goods at every period until all goods are sold, which happens in at most n periods.

Check for buyers. Consider a generic buyer j and a generic information $(\mathbf{g}^t, \mathbf{P}^{t-1})$. If such information is along the equilibrium path, then Bayes rule forces buyer j to believe that the goods still on the market are $G^+(g)$. If instead the information is off-equilibrium, then we choose a belief for buyer j such that she thinks that the goods still on the market are $G^+(g)$. By the definition of the proposed strategy profile and by construction of function β, if buyer j follows the equilibrium strategy from now on, then she gains a null expected payoff. We have now to check that no deviation consisting in choosing a different action at this information set and/or at following information sets allows buyer j to gain a positive expected payoff.

We first consider deviations at $(\mathbf{g}^t, \mathbf{P}^{t-1})$. The strategy profile $(\boldsymbol{\pi}, \boldsymbol{\rho})$ requires other buyers (that exist since $m - n + g^t \geq 2$ for every t due to $m > n$) to offer $\beta(H(0, G^+(g)))$. Buyer j will buy nothing if she offers less, and she will realize a negative expected payoff if she offers more, because of sellers' proposed strategies and the construction of functions β and H.

We now turn our attention to future deviations. Consider a future information set $(\mathbf{g}^{t'}, \mathbf{P}^{t'-1})$ that can be reached from buyer j's belief at $(\mathbf{g}^t, \mathbf{P}^{t-1})$ through some sequence of actions by buyer j when other players follow their proposed strategies. Bayes rule implies – because a seller of lower quality following the proposed strategies surely accepts when a seller of higher quality accepts – that buyer j believes that the goods still on the market at $(\mathbf{g}^{t'}, \mathbf{P}^{t'-1})$ are $G^+(g^{t'})$. This allows us to conclude – similarly to what done when considering deviations at $(\mathbf{g}^t, \mathbf{P}^{t-1})$ – that deviations lead either to null or to negative expected payoffs.

Check for sellers. Consider a generic seller i and a generic information $(\mathbf{g}^t, \mathbf{P}^t)$. We treat beliefs similarly to what done for buyers. If information $(\mathbf{g}^t, \mathbf{P}^t)$ is along the equilibrium path, then Bayes rule forces seller i to believe that the goods still on the market are $G^+(g^t)$. If instead the information is off-equilibrium, then

we choose a belief for seller i such that she thinks that the goods still on the market are $G^+(g^t - 1) \cup \{\text{good } i\}$ (the difference with respect to the beliefs of buyers is due to the fact that seller i knows that her own good is still on the market).

We first consider deviations that anticipate the time of selling with respect to that in the proposed strategy profile. Suppose that $\delta\beta(H(h-1, G^+(g^t))) \le p_{\max}(\boldsymbol{p}^t) < \delta\beta(H(h, G^+(g^t)))$, with $h \le \tau(i, G^+(g^t))$ and where we set $\beta(H(-1, G)) = 0$ for notational consistency. If other sellers follow the proposed strategies, then sellers of goods in $H(0, G^+(g^t)) \cup \ldots \cup H(h-1, G^+(g^t))$ accept the current offer. If seller i rejects, then at next time buyers will offer $\beta(H(0, G^+(g^t) \setminus \bigcup_{r=0}^h H(r, G^+(g^t)))$, that is equal to $\beta(H(h, G^+(g^t)))$. Applying an analogous reasoning, we may extend this result to future time periods, obtaining that if seller i rejects for $\tau(i, G^+(g^t)) - h + 1$ time periods, then she will receive the following list of price offers in present value: $p_{\max}(\boldsymbol{p}^t), \delta\beta(H(h, G^+(g^t))), \delta^2\beta(H(h+1, G^+(g^t))), \ldots, \delta^{\tau(i,G^+(g^t))-h+1}\beta(H(k, G^+(g^t)))$. From $p_{\max}(\boldsymbol{p}^t) < \delta\beta(H(h, G^+(g^t)))$ and property 2 we get that any decision to accept when the proposed strategy prescribes to reject is not strictly convenient.

We now consider deviations that posticipate the acceptance time with respect to that in the proposed strategy profile. We first assume $i^* = i$. We consider any future $(\boldsymbol{g}^{t'}, \boldsymbol{P}^{t'})$ that can be reached from seller i's belief at $(\boldsymbol{g}^t, \boldsymbol{P}^t)$ through some sequence of actions by seller i (more precisely, rejections) when other players follow their proposed strategies. Bayes rule implies that seller i believes that the goods still on the market at $(\boldsymbol{g}^{t'}, \boldsymbol{P}^{t'})$ are $G^+(g^{t'} - 1) \cup \{\text{good } i\} = G^+(g^{t'})$ (the last inequality because $i^* = i$). Property 1 allows us to establish that rejecting a maximal price offer at the current or at any future information set when the proposed profile prescribes to accept cannot be strictly convenient for seller i. Finally, we note that when seller i is different from seller i^*, and in particular when $i < i^*$, to postpone selling is more costly so that the same conclusion holds a fortiori. $\qquad\square$

time period	0	1	...	k	...	$\tau(n, G)$
sold goods	$H(0, G)$	$H(1, G)$	...	$H(k, G)$	...	$H(\tau(n, G), G)$
price	$\beta(H(0, G))$	$\beta(H(1, G))$	...	$\beta(H(k, G))$	...	$H(\beta(\tau(n, G), G))$
buyers' payoff	0	0	...	0	...	0
sellers' payoff	$\beta(H(0, G)) - s_i$, for good $i \in H(0, G)$	$\delta(\beta(H(1, G)) - s_i)$, for good $i \in H(1, G)$	...	$\delta^k(\beta(H(k, G)) - s_i)$, for good $i \in H(k, G)$	...	$\delta^{\tau(n,G)}(\beta(H(\tau(n, G), G)) - s_i)$, for good $i \in H(\tau(n, G), G)$

Figure 2: Timing of the wPBE in Proposition 2.

6. Welfare analysis and the frequency of trading stages

In the previous two sections we have proven that, thanks to the public knowledge of the size of the informed side of the market, all goods brought to the market can be traded in finite time even in the presence of asymmetric information. Therefore, it seems natural to conclude that, at least in markets where such information is not naturally available, a benevolent public authority could contrast inefficiencies due to adverse selection by disclosing information about the size of the informed side of the market.

However, as we have already remarked in Sections 1 and 2, when $\delta < 1$ full trade can be obtained even if agents ignore the size of the informed side of the market and do not have any additional public information (such as goods' vintages, as in Hendel et al., 2005). Indeed, when agents discount future payoffs the waiting

time before exchanges take place can be used as a screening device, inducing sellers with lower qualities to sell first. In the light of this one might say that there is no need to disclose information about the size of the informed side of the market. Instead, the public knowledge of the informed size of the market is important because it often allows a superior outcome, because the use of the discounting of future payoffs to screen qualities imposes an intrinsically non-negligible and non-reducible cost.

The comparison between our result and what obtained in Janssen and Roy (2002) allows to appreciate the nature of the issue. In our model the discount factor is only an element of realism and does not play any essential role in allowing a full trade outcome. Proposition 2, where $\delta = 1$, substantiates this claim. Instead, in Janssen and Roy (2002) the discount factor plays a crucial role: lower quality sellers face a larger cost of waiting than high quality sellers, so that there always exists a delay of exchanges long enough to convince low quality sellers not to wait. This, however, imposes a non-negligible cost since part of the surplus must be burned to sort qualities over time. This cost is also non-reducible in a sensible way: reducing impatience or increasing the frequency of exchange opportunities – i.e., increasing the discount factor – implies that high quality sellers have to sell in more distant periods in order to convince low quality sellers not to wait. This is confirmed by Janssen and Roy (2002) who finds that, when the discount factor tends to 1, the length of time required for all exchanges to occur tends to infinity. This fact implies that, despite full trade, efficiency is lost and cannot be recovered even in the limiting case.

On the contrary, when the size of the informed side of the market is observable, full trade can lead to efficiency. This is evident for the case where $\delta = 1$, as summarized by the following straightforward result, that we give without proof.

Remark 1. *When $\delta = 1$, any full trade outcome is efficient.*

What is most important, however, is that we are able to get arbitrarily close to efficiency also for the case in which agents effectively discount future payoffs. In particular, in the following we argue that if the spell between any two subsequent stages of trading is made smaller and smaller, then efficiency is recovered in the limit.

If δ is the discount factor for one period of time and trading stages are Δt time distant from each other, then the discount factor between the current stage and the next stage is $\delta^{\Delta t}$. We define the *frequency of trading stages* as $1/\Delta t$. We measure the welfare of an outcome with the sum, over all agents (buyers and sellers), of individual payoffs. We refer to the maximum welfare that is attainable with an outcome supported by a wPBE as the *maximum attainable welfare*.[11] The entire potential surplus from exchange is equal to $\sum_{i=1}^{n} (b_i - s_i)$.

Proposition 3
As the frequency of trading stages increases, the maximum attainable welfare tends to the entire potential surplus from exchanges.

Proof. We note that in the wPBE that we have used in the proof of Proposition 2 all transactions occur within n periods. Since the surplus extracted from the exchange of good i is at least $\delta^n (b_i - s_i)$, it clearly converges to $b_i - s_i$ as the frequency of trading stages increases. □

Proposition 3 suggests an important line of intervention by a public authority. The disclosure of information about the size of the informed side of the market may be usefully combined with an increase in

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[11]Such maximum attainable welfare is well defined, since there is a finite number of outcomes with a welfare larger or equal than the welfare attained by the wPBE shown to exist in Proposition 2.

the frequency of trading stages. The former kind of intervention allows full trade to be sustained as wPBE without the use of the cost of waiting as in trade as screening device, while the latter kind of intervention allows total surplus to approach the efficient level, even when agents discount future payoffs.

7. Discussion

In this section we discuss several aspects and implications of our model. Since the arguments are both long and articulated, we find it convenient to organize the discussion in topic-specific paragraphs.

Features of full trade equilibria. In the following we comment on a couple of features that are evidently possessed by the wPBE in Proposition 2 as well as by any other wPBE leading to full trade, as it is intuitively understood.

A first important feature that characterizes wPBE profiles that lead to full trade is the so-called *skimming property*: lower quality goods are sold earlier and at a lower price than higher quality goods. Putting it differently, in each time period the lowest quality goods are sold and, as a consequence, the minimum quality of goods still on the market increases over time. This also implies that the actual selling price increases over time. The skimming property is a quite common feature in dynamic models of trade under asymmetric information.[12] The skimming property is inherently related to the type of strategic situation that we are modeling here, and is robust with respect to the details of modelization. Indeed, for buyers not to incur in losses it must be that lower qualities are sold separately from higher qualities and the only way to achieve this is to sell lower qualities first. In fact, if higher qualities were to be sold first then buyers would have to make high price offers when low quality goods are still on the market, and such high offers would induce the low quality sellers to accept, implying a negative expected payoff for buyers.

A second feature which characterizes full trade equilibria is that all surplus goes to sellers. This feature depends crucially on the fact that buyers always outnumber sellers so that there is competition among buyers whatever the number of goods still on market. We stress that different assumptions about the relative numerosity of buyers and sellers can lead to equilibria where buyers obtain part of the surplus. In the following paragraph about extensions, we briefly sketch the case of a monopsonistic buyer where her surplus is not zero.

Assumptions and extensions. A crucial assumption for the working of our model is that when a good is sold to a buyer, such good is no longer traded in that market. The reason why this assumption is crucial lies in the fact that the size of the informed side of the market only allows to separate lowest qualities from the rest of the goods, implying that if the lowest quality goods do not leave the market once sold then higher quality goods will never be sold. Of course, this does not mean that re-sales cannot take place but requires, as for instance in Hendel et al. (2005), that buyers can at least distinguish goods already sold once from goods never sold before in that market.

Another important assumption concerns the stationarity of the set of buyers and sellers. In our model sellers and buyers exit the market after completing a transaction, but no new seller or buyer enters the market. In particular, the arrival of new goods might represent a threat to full trade since, intuitively, it makes the size of the informed side of the market less informative about existing qualities. Indeed, changes in the number of goods still on the market would be a joint result of sales and arrival of new sellers – and, hence, of new goods. The issue here is how to let agents correctly infer the distribution of qualities induced

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[12]The property is usually stated in models of durable goods monopoly, see for instance Fudenberg et al. (1985) and Gul et al. (1986), but it can also be found in a literature closer to ours, see for instance Janssen and Roy (2002).

by the arrival of new goods. One possibility is to let agents know from the beginning the number of goods and the distribution of qualities entering in each period. This can be modeled with a grand set of sellers and a grand set of buyers with a subset of agents that are present from the initial period while additional sellers and buyers enter at known times with known distribution of qualities. Alternatively, we could have different markets for goods arrived at different dates. This can be easily obtained in the case agents are not anonymous with respect to the date of arrival.

There are many other details of our model which have a non-negligible role, but they can be considered as less important for the gist of our results. For instance, a different bargaining structure might be assumed (we might consider sellers making price offers or sellers and buyers alternating in making offers), price offers might not be a public information (we might use a matching model and only consider bilateral interactions), the relative numerosity of buyers and sellers might be different or buyers might be willing to buy more than one good (for instance, we might consider a monopsonistic buyer, as discussed below). We might also opt for a walrasian framework instead of considering strategic price setting. Each of these variants should be considered with care, and some technical issues certainly arise. However, the mechanism that exploits information on the size of the informed size of the market to enforce full trade is still applicable to such variants, although the policy implications might be different. For instance, consider the variant where there is a monopsonistic buyer who never exits the market, that makes a take-it-or-leave-it offer to sellers in every time period, and who is willing to buy whatever number of goods, provided the price is low enough. In this variant of our model we can find a set of strategy profiles that are wPBE and lead to full trade, with the only important difference with respect to what shown in this paper that the entire surplus would be earnt by the monopsonistic buyer. The different distribution of surplus is not surprising as it clearly depends on the absence of competition among buyers. The policy implications may however be different. Indeed, in the case of a monopsonistic buyer the usefulness of disclosing information on the number of goods still on the market is limited since the buyer can easily keep track of them. Our main result is better thought of as a normative suggestion to the monopsonistic buyer in favor of using a strategy that conditions price offers on the size of the informed side of the market.

Another assumption that might be easily dispensed with is the homogeneity of buyers' in their valuations of goods. The fact that different buyers have different valuations might reduce the competitive pressure, possibly allowing some buyers to obtain a positive surplus. Similarly, we may also let the discount factor vary across agents. This would have the intuitive consequence of creating differences in buyers' valuation that evolve over time. We note that the procedure associated with Lemma 1 would work in a similar way and with similar outcomes for what concerns the grouping of sellers over time.

Finally, we want to stress that the goods brought to the market in our model are best interpreted as durable goods, at least when $\delta < 1$. This implicitly points to a potentially interesting issue: how to extend our results to a model where agents discount future payoffs and goods have the standard features of non-durable goods – such as exhaustion upon consumption. Indeed, if the goods brought to the market are non-durable, then some of the arrangements which can reduce the loss due to adverse selection, such as leasing and secondary markets, become ineffective (see Waldman, 2003, for a comprensive discussion the functioning of real markets for durable goods). Moreover, delays in exchanges are intrinsically useless as screening devices since the seller's cost of delaying a sale is the same for high quality and low quality goods, because goods do not provide a stream of services over time. On the contrary, our mechanism based on the public knowledge of the size of the informed side of the market does not seem to suffer from this problem, and it may prove to be effective in obtaining both full trade and efficiency, provided that trading stages are made frequent enough. We are aware, however, that some futher reasons for the emergence of non-full trade equilibria could arise because of the features of non-durable goods, such as the possibility that a seller

consumes her own good at an initial stage of the game in the belief that no convenient offer will me made in the future.

Intertemporal competition and existence of non-full trade equilibria. In Propositions 1 and 2 we have shown that a wPBE exists where every good is traded. However, we have been silent on whether other wPBEs exist that do not lead to full trade. In the following we argue that such non-full trade equilibria might actually exist, and that their existence depends on intertemporal competition among buyers. Finally, we suggest that non-full trade wPBEs are not particularly robust.

There are two types of intertemporal competition that may arise in our framework. The first type is effective in preventing full trade only when the dicount factor is equal to 1. The following sketched example illustrates. Suppose that there exists a wPBE such that some goods remain unsold. We call i the minimum quality of unsold goods. Consider a time period where, according to the equilibrium strategies, no good is sold and will ever be sold. Consider then a buyer who is still on the market in such time period and who may offer a price p strictly comprised between s_i and b_i, in the hope to induce seller i to sell her good. Note that seller i would indeed profit from selling. However, such unexpectedly high offer is observable by other buyers, who may adopt a strategy that reacts to high offers by retaliating in the next period with a price offer equal to b_i. Such a retaliation induces seller i to reject the initial unexpectedly high offer. Note that this retaliation strategy does not lead to any loss for the buyers adopting it, and effectively makes the deviating buyer indifferent between deviating and not deviating (since in any case she makes a payoff equal to zero). This type of intertemporal competition sustains non-full trade wPBEs. However, it can be avoided if the time discount is strictly lower than 1. In such a case, in fact, a buyer could offer a price p strictly comprised between b_i and δb_i, in the attempt to gain from the purchase of unsold goods. This price offer grants to seller i a positive payoff in case of exchange, and it cannot be outperformed by any future retaliating price offer by other buyers, unless they offer more than b_i which would imply incurring in a loss.

The second type of intertemporal competition is more specific to our setup since it exploits the information about the size of the informed side of the market. As such, it can be considered a byproduct of the public knowledge of this piece of information and, unfortunately, it is also more robust to different degrees of discounting. The following sketched example illustrates. Suppose that in a wPBE there are two goods on the market that are never sold, call them L and H. Consider a buyer who deviates from her equilibrium strategy in the hope to gain some surplus from the purchase of unsold goods, and suppose that she makes a price offer strictly comprised between b_L and δb_L. Other buyers may react by offering $(b_L + b_H)/2$ in the next period, which in the case $(b_L + b_H)/2 > s_H$ may convince both seller L and seller H to reject the current offer and accept theirs. This kind of intertemporal competition could be contrasted by the initial buyer by considering a different behavior: making a price offer strictly comprised between $(b_L + b_H)/2$ and $\delta(b_L + b_H)/2$, which could result in the purchase of both goods. Indeed, other buyers cannot make more appealing offers in the next period without incurring in a loss, if they want to buy both goods. However, they may take advantage of the situation and try to buy only good H, by means of a reaction strategy that exploits the information about the number of goods still on the market. More precisely, they may offer b_H in the next period conditional on the fact that the number of goods still on the market decreases by one. Given these strategies, seller L obtains a positive payoff from accepting the initial deviation and cannot hope to earn more by rejecting it, since in such a case other buyers would not make a higher offer in the following period; instead, seller H finds it convenient to wait until such a higher offer, provided that the discount factor is close enough to 1. Note that the described reaction strategy is not costly for the buyers adopting it. The only one who loses is the buyer who attempts the initial deviation, that would be profitable (in expectation) in the case both seller L and seller H accept it, but it is not in the case only seller L accepts to sell. In conclusion, this type of intertemporal competition turns out to be effective in preventing the emergence of

full trade even in the presence of a time discount, although not for any value of the discount factor.

Note that when $\delta < 1$ the second type of intertemporal competition is the only cause of trade failure. More precisely, in any wPBE there can be periods of no trade – considering wPBEs that lead to full trade too – only if the second type of intertemporal competition is at work. When future payoffs are discounted, the minimum quality on the market is known in equilibrium and, hence, it is costly for both trading parties to renounce trading since there always exist mutually advantageous agreements. In order not to have one of such mutually advantageous agreements being exploited it must be that the minimum quality seller expects higher offers in the future. Therefore, with reference to the sellers of minimum quality goods that are not sold in a given period, buyers' equilibrium strategies must prescribe to make more appealing price offers in following periods whenever they observe a buyer deviating from her equilibrium strategy and offering a mutually advantageous proposal. At the same time, buyers' equilibrium strategies must prescribe price offers that are not above sellers' reservation price along the equilibrium path. In other words, in order to hinder trading of the lowest quality goods in any given period, buyers' strategy must prescribe price offers which are conditional on a buyer's deviation, that is, intertemporal competition must be at work.

Since intertemporal competition works through reaction strategies conditioned on the observation of deviations by some buyer, one might think that if price offers were observable only by sellers, and not by other buyers, then this type of intertemporal competition would be avoided, ensuring that all wPBEs lead to full trade. Unfortunately, that is not the case because a similar kind of intertemporal competition can exist between a buyer in the current period and the same buyer in later periods. Under perfect recall, every buyer remembers at least her own offers at previous times so that she can make offers that are conditional on that information. Therefore, the kind of reaction outlined above may come from the same buyer making the initial deviation. Indeed, such a behavior is not costly for her, since sellers will never accept lower offers in the conviction that future offers by the same buyer will be in accordance with her equilibrium strategy, and hence more convenient for them. Therefore, even if price offers cannot be observed by other buyers there might exist wPBEs that do not lead to full trade. It cannot be over-emphasized, however, that this possibility appears as crucially depending on the fact that buyers always earn a null payoff in equilibrium.

We also observe that there are important real situations where the type of intertemporal competition which can hinder full trade is not likely to emerge. A first case is what we may refer to as *extreme adverse selection*, i.e., a situation in which goods of different qualities cannot be sold in the same period because buyers' offers should be so high to convince sellers to accept that they would be unprofitable. In the above sketched example, extreme adverse selection would arise whenever $(b_L + b_H)/2 < s_H$. Under extreme adverse selection, only the first type of intertemporal competition is at work, so that the presence of a non-negligible discount factor is enough to ensure that every wPBE leads to full trade. We are aware that many real situations do not entail extreme adverse selection, but we observe that it is more likely to hold when market forces push buyers' valuations to be very close to sellers' valuations.

A second case is what we may call *extreme impatience*, i.e., a situation in which the time discount is so low that rejecting in order to accept higher offers in the next period turns out not to be convenient. More precisely, and referring to the usual example above, when buyers react to a deviation by means of an offer conditional on the number of goods still on the market, such a conditional offer is convenient for seller H only if the discount factor is close enough to 1. Therefore, when agents are sufficiently impatient – or equivalently when goods deteriorate quickly enough – seller H always prefers to accept the initial deviation making intertemporal competition ineffective.

Finally, let us stress that, even when they exist, non-full trade wPBEs are not particularly robust. In the first place, each non-full trade wPBE is strictly Pareto dominated by a full trade wPBE. In the light of this, one might argue that coordination is more likely to arise on the Pareto superior equilibria, thus suggesting

that the likelihood of non-full trade is low. A second argument, which is probably stronger, concerns the fact that in any non-full trade wPBE, deviating and making an offer slightly higher than s_i – where i denotes the lowest quality among unsold goods – is a weakly dominant choice for the buyers still on the market. Indeed, if sellers reject her offer she will still get a payoff equal to zero, while in the case sellers accept her offer (e.g., by mistake) the deviating buyer earns a strictly positive payoff. Note that if sellers actually reject the deviation, in the next period buyers actively competing across time will make a price offer that seller i will accept, deviating from the current equilibrium towards full trade. Otherwise, if buyers deviate and keep on making low offers after seller i has rejected profitable price offers in the first place, then sooner or later seller i's beliefs about future offers are likely to be revised, letting her accept current offers that are larger than s_i but lower than b_i, thus making current deviations a more attractive choice. This kind of reasoning provides a strong argument, we think, which goes against the robustness of non-full trade wPBE.

8. Conclusions

In this paper we have studied a dynamic model of trade under adverse selection, which emerges because the quality of goods is a seller's private information. We have shown that, if the size of the informed side of the market (that in our model corresponds to the number of goods still on the market) is public information, then conditional price offers can sustain a weak perfect Bayesian equilibrium where all goods brought to the market are exchanged in a bounded number of trading stages. Moreover, we have shown that this equilibrium tends to efficiency when the frequency of trading stages increases, even in the presence of a discount factor. This confirms the finding of Hendel et al. (2005) that additional information is needed to properly solve dynamic adverse selection problems – indeed, solutions obtained by exploiting the discounting of future payoffs as a screening device have been shown to be incapable of restoring efficiency (Janssen and Roy, 2002).

Our findings suggest a combination of two policy interventions whenever proper solutions to dynamic adverse selection do not arise endogenously. On the one hand, information about the size of the informed side of the market should be systematically disclosed to all market participants and, on the other hand, the frequency of trading stages should be rised as much as possible. By so doing, we expect to get close enough to efficiency, provided that intertemporal competition among buyers does not arise. In this regard, we have argued that intertemporal competition does not seem to be particularly effective in keeping agents from exploiting all trading opportunities. In particular, we have stressed that there are no strong reasons preventing buyers from making mutually advantageous offers to sellers of yet unsold goods. At any rate, a benevolent public authority could help agents to coordinate on Pareto superior equilibria where all goods are exchanged.

Appendix

Proof of Lemma 1. We use the following algorithm to build up a function $H(k, G')$ that satisfies the properties in Lemma 1.

Algorithm. The algorithm applies to any $G' \subseteq G$.

Step 0. Define a function τ which assigns to every good i in G' a natural number – interpreted as time – as follows: $\tau(i, G') = \|\{\text{good } \ell \in G' : \ell \leq i\}\|$. Initially this function simply assigns each good to a time period according to its index, but then, as the algorithm unfolds, some goods will be moved to earlier periods and grouped together with other goods. Let $H(k, G') = \{\text{good } i \in G' : \tau(i, G') = k\}$ be the function assigning to each time period a set of goods to be sold, as given by function τ (of whom $H(k, G')$ might

thought of as a sort of inverse). Note that function $\beta(H)$, as defined in Section 5, can be intepreted as the price offer whose expected value for buyers is zero if all and only the sellers in H accept the offer.

Step 1. Set k equal to the maximum value of function τ over G', and indicate with i the good with the minimum index in $H(k, G')$.

Step 2. Consider the following inequality, where $i \in H(k, G')$:

$$\beta(H(k-1, G') \cup \{\text{good } i\}) - s_i \geq \delta \left[\beta(H(k, G')) - s_i\right] .$$

Seller i is for the moment assigned to period k, i.e., $\tau(i, G') = k$. The left-hand side represents the value for seller i of accepting the price offer made in $k-1$ periods from now, while the right-hand side represents the value for seller i of accepting the price offer made in k periods from now.

Update and go back to step 1. If the inequality is satisfied, change the image of i according to τ from k to $k-1$, while leaving the image of other elements of G' unchanged, and then go back to step 1.

Update and go back to step 2. If the inequality is not satisfied and $k \geq 2$, then set k equal to $k-1$ and, subsequently, indicate with i the good with the minimum index in $H(k, G')$, and then go back to step 2.

Exit algorithm. If the inequality is not satisfied and $k = 1$, then exit the algorithm.

It is easy to check that a function τ is univocally selected when exiting the algorithm. As a consequence, a function H is selected as well.

We are now ready to check the properties stated in Lemma 1.

Check of property 1. We show that the inequality holds for $k = 1$, then the result can be extended to any k by iteration. The check trivially holds with equality if $\|H(0, G')\| = 1$, so we limit our attention to the case $\|H(0, G')\| \geq 2$. We first consider good i having the maximum index in $H(0, G')$, and we apply the algorithm to $G''(1)$. Clearly, good i belongs to $H(0, G''(1))$. We consider the following assignment of goods to time periods: the goods in $G' \setminus G''(1)$ are associated to the first $\|G' \setminus G''(1)\|$ periods, one per time period, so that good 1 is associated to time period 0, good 2 is associated to time period 1, ..., good $\|G' \setminus G''(1)\|$ is associated to time period $\|G' \setminus G''(1)\| - 1$, and then the subsets of goods found by means of the algorithm applied to G' are associated to the following time periods, so that $H(0, G''(1))$ is associated to time period $\|G' \setminus G''(1)\|$, $H(1, G''(1))$ is associated to time period $\|G' \setminus G''(1)\| + 1$, ... We note that this particular assignment of goods to time periods is an intermediate state in the algorithm applied to G' (for this it is crucial that good i has the maximum index in $H(0, G')$). Therefore, we know that the algorithm moves good i to a previous time period, and $\beta(\{\text{good } i - 1\} - s_i \cup \{\text{good } i\}) \geq \delta(\beta(H(0, G''(1))) - s_i)$. The algorithm will in general entail iterations of steps 1 and 2. Note that at every iteration of step 2, the discounted values of β computed on the subsets of goods associated with k and $k-1$ do not decrease (while the discounted values of β computed on all other subsets of goods are unaffected). So, focussing our attention on good i, we have a chain of inequalities of the form: $H(0, G') - s_i \geq \ldots \geq \delta(\beta(H(0, G''(1))) - s_i)$. Finally, when good i does not have minimum index in $H(0, G')$, the result holds a fortiori because the reservation value s_i is lower or equal (and hence waiting is more costly), and the average value $\beta(H(0, G''(1)))$ is easily seen to be lower or equal as well.

Check of property 2. When the algorithm terminates we have that $\beta(H(\tau(i, G') - 1) \cup \{\text{good } i\}) - s_i < \delta\left[\beta(H(\tau(i, G'))) - s_i\right]$ holds. Hence, $\beta(H(\tau(i, G') - 1)) - s_i < \delta\left[\beta(H(\tau(i, G'))) - s_i\right]$. Note that the latter inequality holds for any seller other than seller i too, and in particular it holds for any seller with an index lower than i that is associated to earlier time periods. So, if we consider good i' belonging to $H(\tau(i, G') - 1)$, it must be that $\beta(H(\tau(i, G') - 2)) - s_{i'} < \delta\left[\beta(H(\tau(i, G') - 1)) - s_{i'}\right]$. Note also that this same inequality holds a fortiori if we replace s_i with $s_{i'}$, implying that $\beta(H(\tau(i, G') - 2)) - s_i < \delta\left[\beta(H(\tau(i, G') - 1)) - s_i\right] < \delta^2\left[\beta(H(\tau(i, G'))) - s_i\right]$. Proceeding along this line of argument, the property is established. $\square$

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