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Limit Pricing and Strategic Investment

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Abstract

We study an entry model where an incumbent privately informed about costs can make a cost-reducing investment choice, along with a pricing decision, in order to prevent a competing firm from entering the market. We show that if limit pricing per se can not deter profitable entry, the opportunity to undertake a strategic investment does not provide an additional instrument for the achievement of this goal to the incumbent.

JEL Classification Numbers: D42, D82, L12.
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1 Introduction

The analysis of signalling effects and strategic effects in two periods models have been and still are two important, although separated, pillars of research and teaching in the field of industrial organization. The analysis of their interaction in determining market outcomes is arguably very important. Here we provide a short contribution for the analysis of such interaction in the context of a workhorse model of entry deterrence. Consider, for simplicity, the classical model of Milgrom and Roberts (1982) entry model where an incumbent, who has private information about costs, may prevent a competing firm from entering the market even when entry would be profitable by setting a limit price which does not convey any new piece of information to the entrant. We ask whether and how market outcomes are modified if we entertain the possibility that a cost-reducing investment is available to the incumbent. We will study the specific case in which entrant's expected profits are assumed to be positive, rather than negative, if no signalling effect emerges in the entry stage. We will show that deterrence of profitable entry through strategic investment and limit pricing is a possible equilibrium outcome, but not the most plausible.

In the next section we provide a more complete description of the model. In section 3 the analysis of Perfect Bayesian Equilibria is provided and it is shown that pooling equilibria do not satisfy the Intuitive Criterion.

2 An entry problem with investment and private information

We consider a two periods entry model where an incumbent firm faces the potential entry of a competing firm in a market for a homogeneous good. In the first period the incumbent (he), who has private information about his costs of production, decides how much to produce and how much to invest in a cost-reducing technology. In the second period the entrant (she), after observing incumbent's choices, decides whether to enter the market. If entry occurs, the entrant pays an entry fee, learns incumbent's costs and firms compete in quantities. Otherwise, the incumbent remains a monopolist and the potential entrant stays out.

There are two types of incumbent, type L with low costs and type H with high costs. The prior probability that the incumbent has high costs is denoted by β. Fixed costs are set to zero for convenience, while marginal costs are constant and are affected by investment, e.[1] Investment

[1]Investment can also be interpreted as an expansion to a related market which allows the incumbent to benefit from economies of scope as in Pires and Catalão-Lopes (2020).

is undertaken by the incumbent in the first period and reduces marginal costs in both periods. The incumbent has access to the cost-reducing technology $\theta(e)$, a strictly decreasing and convex differentiable function with $\theta(0) = 0$. Marginal costs are $c_t + \theta(e) > 0$, with $t = L, H$ and $c_H > c_L$, and are incumbent's private information.

Inverse market demand in each period is $p(q) = 1 - q$, where q is the quantity sold on the market, and incumbent's first period profits (gross of investment) are $\pi_t(e, q) = [p(q) - c_t - \theta(e)]q$. For each level of e the monopoly quantity is denoted by $m_t(e)$ and the associated profit by $M_t(e) = \pi_t(e, m_t(e))$.

Incumbent's profits in the second period depend on the entrant decision denoted by $y \in \{0, 1\}$, with $y = 1$ if entry takes place and 0 otherwise. If $y = 0$ the incumbent remains a monopolist and earns $M_t(e)$, while if $y = 1$ the two firms compete in quantities. Incumbent's duopoly profits are $D_t(e)$ and depend on first period investment. Both $M_t(e)$ and $D_t(e)$ are increasing functions of e. The incumbent's decision about first period output q and investment e, is based on *total profits* over the two periods that, assuming no time discounting, are given by

$$\Pi_t(e, q; y) = \pi_t(e, q) - e + yD_t(e) + (1 - y)M_t(e). \tag{1}$$

The entrant earns zero profits if she stays out of the market. If entry takes place, entrant's profits $\Pi_E(e, t)$ depend on the type of incumbent she faces and on his investment. Clearly, $\Pi_E(e, t)$ is decreasing in e and $\Pi_E(e, H) > \Pi_E(e, L)$. After observing incumbent's choice, the entrant makes an inference about incumbent's costs and enters if her expected profits are strictly positive.

In this environment we introduce a few assumptions to provide the additional structure that make the analysis possible. As a benchmark, let us denote by e_L^* and m_L^* the monopoly choices by L in the absence of any entry threat, i.e. $(e_L^*, m_L^*) = \operatorname{argmax} \Pi_L(e, q; 0)$.

A.1. Entry is not profitable against L i.e. $\Pi_E(e_L^*, L) < 0$.

A.2. The entrant's expected profits are strictly positive if she has prior beliefs and she observes e_L^*, i.e. $\beta\, \Pi_E(e_L^*, H) + (1 - \beta)\, \Pi_E(e_L^*, L) > 0$.

A.3. Entry can be deterred by H if investment is high enough, i.e. there exists a level of investment e^D such that $\Pi_E(e^D, H) = 0$. The incumbent profits when e^D is chosen will be denoted by $\Pi_H^D = \Pi_H(e^D, m_H(e^D); 0)$.

A.4. Type H has an incentive to mimic L, i.e. $\Pi_H(e_L^*, m_L^*; 0) > \Pi_H^A$, where $\Pi_H^A = \Pi_H(e^A, m_H(e^A); 1)$ is the accommodation profit and e^A is the solution to $\max_e \Pi_H(e, m_H(e); 1)$.

The above assumptions describe a situation where the entrant does not enter if convinced that the incumbent is L, she enters if no additional information emerges at the entry stage. Moreover, H has an incentive to mimic L and the strategic effect provides H with a non empty threat to deter entry by investing.

3 Equilibria of the signalling game

Notice first that the entry problem as outlined above shares features with models with multiple signals as in Bagwell and Ramey (1988) and Bagwell (2007) where, however, the strategic effect is shut down. Some of the arguments provided in those analyses will be used to derive our results in a context where the strategic effect of observable investment is operative. Nature moves at time zero and chooses the type of incumbent. In the first period, after observing his type, the incumbent decides his output and investment, i.e. the two signals to send. In the second period the potential entrant, who does not know the type of incumbent, observes the signals and decides whether or not to enter the market. A pure strategy for the incumbent consists of two pairs, (e_H, q_H) and (e_L, q_L), which associate with each type a level of investment and a first period quantity. A pure strategy for the entrant is a function $y(e, q) \in \{0, 1\}$ which associates the entry decision to any incumbent's observable choice. The incumbent's payoffs are given by (1) and the entrant's payoffs are $\Pi_E(e, t)$ in the case of entry and zero otherwise.

The solution concept of *Perfect Bayesian Equilibrium* (PBE) is used and the Intuitive Criterion will be applied to refine equilibria.[2] The entrant's beliefs that associate the ex-post probability of H with any observed incumbent's choice is denoted by $\hat{\beta}(e, q)$.

Let us recall how the Intuitive Criterion is applied. Let Π_t^* denote the payoff to type t in a given equilibrium (e_t, q_t), then a deviation $(\tilde{e}, \tilde{q})$ from (e_t, q_t) is *equilibrium dominated for* t if $\Pi_t^* > \Pi_t(\tilde{e}, \tilde{q}; 0)$. A given equilibrium satisfies the Intuitive Criterion if, whenever a deviation is equilibrium dominated for t and strictly more profitable for t', the entrant's beliefs supporting the equilibrium do not assign the deviation to t.

Suppose that $(\tilde{e}, \tilde{q})$ is equilibrium dominated for H and strictly more profitable for L, i.e.

$$\Pi_H(e_H, q_H; y(e_H, q_H)) > \Pi_H(\tilde{e}, \tilde{q}; 0) \tag{2}$$

$$\Pi_L(\tilde{e}, \tilde{q}; 0) > \Pi_L(e_L, q_L; y(e_L, q_L)). \tag{3}$$

[2]See Fudenberg and Tirole (1991) and Cho and Kreps (1987).

The Intuitive Criterion requires the belief to be $\hat{\beta}(\tilde{e}, \tilde{q}) = 0$. If the entrant has this belief, however, the incumbent's equilibrium strategy is not optimal because L is better off by deviating. Hence, if (2) and (3) hold, the beliefs supporting the equilibrium strategy can not satisfy the Intuitive Criterion.

Non intuitive PBE. *A PBE, (e_t, q_t) and $y(e, q)$, with $t = H, L$, does not satisfy the Intuitive Criterion if there exists a deviation $(\tilde{e}, \tilde{q}) \neq (e_t, q_t)$ such that (2) and (3) hold.*

For convenience of exposition we will split the analysis of pure strategy equilibria into two parts and deal first with the case $\Pi_H^A \geq \Pi_H^D$.

3.1 The case $\Pi_H^A \geq \Pi_H^D$

Suppose that H prefers entry accommodation to deterrence because, for example, investment marginal benefits are small (as for a flat $\theta(e)$ function), i.e. $\Pi_H^A > \Pi_H^D$. Both separating and pooling are possible equilibrium outcomes. In a separating equilibrium the two types of incumbent make different choices and the entrant is fully informed. H will accommodate entry by choosing $e_H = e^A$ and $q_H = m_H(e^A)$, while L will signal his type by choosing e_L and q_L so as to satisfy the incentive compatibility constraint, $\Pi_H(e_L, q_L; 0) \leq \Pi_H^A$. Furthermore, as L profits at equilibrium must be greater than the profits he may earn by accommodating entry, denoted by Π_L^A, the choices (e_L, q_L) must also satisfy the 'participation constraint' $\Pi_L(e_L, q_L; 0) \geq \Pi_L^A$. The equilibrium choices will be distorted in excess with $e_L > e_L^*$ and $q_L > m_L^*$, i.e. L will over invest and set a limit price. The entrant will infer the type from the signal and entry will take place only when profitable, i.e. with probability β. The 'least cost' equilibrium, where the distortions of L choices are minimized, is the unique separating equilibrium satisfying the Intuitive Criterion and can be shown to exist under standard conditions by using similar arguments as those in Bagwell and Ramey (1988).

Let us turn to pooling equilibria, i.e. situations where all types select the same investment and first period output and the entrant, who does not receive any new piece of information, stays out. In a pooling equilibrium all types of incumbent are required to *over invest*, i.e. to invest in excess of e_L^*. To see this point notice that observing e_L^* would not dissuade the entrant from entering, because, by A.4, type H has an incentive to mimic L and expected profits from entry are positive due to A.2. As entrant's profits are negatively related to incumbent's costs, the incumbent is required to invest more to hold the entrant out. In fact, it is easily seen that there is a level of investment e_0 which drives entrant's *expected profits* to zero, i.e.

$$\beta\, \Pi_E(e_0, H) + (1 - \beta)\, \Pi_E(e_0, L) = 0 \tag{4}$$

and that $e_0 > e^*$.[3] As a result, any pooling equilibrium (e^P, q^P) is characterized by $e^P \geq e_0$. Furthermore, if the equilibrium is intuitive, $q^P \geq m_L(e_0)$, otherwise the deviation $\tilde{q} = m_L(e_0)$ satisfies (2) and (3).

Pooling equilibrium. *A pooling equilibrium (e^P, q^P) obeying the Intuitive Criterion must satisfy $e^P \geq e_0$, $q^P \geq m_L(e_0)$ and the participation constraints $\Pi_t(e^P, q^P; 0) \geq \Pi_t^A$.*

At a pooling equilibrium the incumbent over invests and sets a limit price, entry never takes place and profitable entry is deterred with probability β, the probability of H. The pooling equilibrium where both types invest e_0 and set the quantity $m_L(e_0)$ is the best candidate to be the solution to the entry problem, because it Pareto dominates any other equilibrium from the point of view of the incumbent. The question, however, is whether this equilibrium is also a *plausible* solution or, in other words, whether both types of incumbent will be willing to use investment strategically to hold the entrant out. Here, we show that this can not be the case because pooling equilibria do not satisfy the Intuitive Criterion. Indeed, as in a pooling equilibrium each type over invests, the marginal benefits of investment must be negative so that decreasing investment increases incumbent's profits. On the other hand, due to the difference in marginal costs, an increase of first period output cuts H profits more than L profits. This suggests that a deviation $\tilde{e} < e^P$ and $\tilde{q} > q^P$ from the pooling equilibrium can be found which violates the Intuitive Criterion.

Proposition. *No pooling equilibrium satisfies the Intuitive Criterion.*

Proof. Let us focus on the pooling equilibrium $(e^P, q^P) = (e_0, m_L(e_0))$ and find a deviation $(\tilde{e}, \tilde{q})$ which satisfies (2) and (3). The argument for other pooling equilibria is similar. For small changes of e and q around e_0 and $m_L(e_0)$ let us consider the changes in incumbent profits for types H and L and specifically

$$\begin{aligned}
&[\Pi_H(e, q; 0) - \Pi_H(e_0, m_L(e_0); 0)] - [\Pi_L(e, q; 0) - \Pi_L(e_0, m_L(e_0); 0)] \\
&= [\pi_H(e, q) - \pi_L(e, q)] - [\pi_H(e_0, m_L(e_0)) - \pi_L(e_0, m_L(e_0))] + [M_H(e) - M_L(e)] - [M_H(e) - M_L(e)] \\
&= (c_L - c_H)q - (c_L - c_H)m_L(e_0) + (m_H(e)^2 - m_L(e)^2) - (m_H(e_0)^2 - m_L(e_0)^2) \\
&= (c_L - c_H)[q - m_L(e_0)] + \frac{c_L - c_H}{2}[m_H(e) - m_H(e_0) + m_L(e) - m_L(e_0)] \\
&= (c_H - c_L)(m_L(e_0) - q) - (c_H - c_L)(m_L(e) - m_L(e_0)) \\
&= (c_H - c_L)(2m_L(e_0) - m_L(e) - q) \qquad (5)
\end{aligned}$$

where in the second and third equalities we have used linearity of demand and additivity of marginal

[3] This follows from A.1, A.2 and A.3.

costs.[4] Let us consider the pairs $(\bar{e}, \bar{q})$, with $\bar{e} < e_0$ and $\bar{q} > m_L(e_0)$, which leave L total profits unchanged, i.e. such that

$$\Pi_L(\bar{e}, \bar{q}; 0) - \Pi_L(e_0, m_L(e_0); 0) = 0. \tag{6}$$

After simple manipulations of (6) we obtain the equation

$$\bar{q}^2 - 2m_L(\bar{e})q - 2(m_L(\bar{e})^2 - m_L(e_0)^2) + m_L(\bar{e})^2 - (e_0 - \bar{e}) = 0.$$

Solving for $\bar{q}$ yields

$$\bar{q} = m_L(\bar{e}) + \sqrt{\Delta}, \tag{7}$$

where $\Delta = (e_0 - \bar{e}) + 2(m_L(\bar{e})^2 - m_L(e_0)^2)$. Finally, by (5), (6) and (7) we have

$$\Pi_H(\bar{e}, \bar{q}; 0) - \Pi_H(e_0, m_L(e_0); 0) = (c_H - c_L)\{2\,[m_L(e_0) - m_L(\bar{e})] - \sqrt{\Delta}\} \tag{8}$$

The sign of (8) is the same as the sign of the term in braces which, in turn, is the same as the sign of[5]

$$\begin{aligned} g(\bar{e}) &= [2(m_L(e_0) - m_L(\bar{e}))]^2 - \Delta \\ &= 4[m_L(e_0) - m_L(\bar{e})]^2 - (e_0 - \bar{e}) - 2(m_L(\bar{e})^2 - m_L(e_0)^2) \\ &= \bar{e} - e_0 + 2[m_L(e_0) - m_L(\bar{e})][3m_L(e_0) - m_L(\bar{e})] \\ &= \bar{e} - e_0 + [\theta(\bar{e}) - \theta(e_0)]2m_L(e_0) + \frac{[\theta(\bar{e}) - \theta(e_0)]^2}{2} \end{aligned}$$

As $g(e_0) = 0$, if we show that $g'(e_0) > 0$ then, by continuity, we can find $\bar{e}' < e_0$ such that $g(\bar{e}') < 0$, which means that also (8) is strictly negative. Taking the first derivative of $g(\bar{e})$ evaluated at e_0 yields

$$g'(e_0) = 1 + \theta'(e_0)2m_L(e_0) = -\lambda > 0$$

where $\lambda < 0$ is the Lagrange multiplier of the problem $\max_{e,q} \Pi_L(e, q; 0)$ subject to $e \geq e_0$. Therefore, for $\bar{e}'$ and $\bar{q}' = m_L(\bar{e}') + \sqrt{(e_0 - \bar{e}') + 2(m_L(\bar{e}')^2 - m_L(e_0)^2)}$ we have $\Pi_L(\bar{e}', \bar{q}'; 0) - \Pi_L(e_0, m_L(e_0); 0) = 0$ and $\Pi_H(\bar{e}', \bar{q}'; 0) - \Pi_H(e_0, m_L(e_0); 0) < 0$. By continuity of profit functions and by $\partial \pi_L(\bar{e}', \bar{q}')/\partial q < 0$, we can slightly cut the output $\bar{q}'$ so that the deviation $\tilde{e} = \bar{e}'$ and $\tilde{q} = \bar{q}' - \epsilon$ for $\epsilon > 0$ small enough will increase the profits of L while holding H profits below the equilibrium level. Hence, the deviation satisfies (2) and (3) and this completes the proof. $\square$

[4] Recall that $m_t(e) = (1 - c_t - \theta(e))/2$, $M_t(e) = [m_t(e)]^2$, $m_H(e) - m_L(e) = (c_L - c_H)/2$ and $m_t(e) - m_t(e_0) = [\theta(e_0) - \theta(e)]/2$.

[5] Notice that $2\,[m_L(e_0) - m_L(\bar{e})] + \sqrt{\Delta} > 0$.

3.2 The case $\Pi_H^D > \Pi_H^A$ and extensions

Let us turn briefly to the case where H prefers entry deterrence to accommodation, i.e. $\Pi_H^D > \Pi_H^A$. At a separating equilibrium H will invest to deter entry, i.e. $e_H = e^D$ and $q_H = m_H(e^D)$, while L will signal his type and choose $e_L \geq e_L^*$ and $q_L \geq m_L^*$ so as to satisfy incentive compatibility and participation constraints. Entry will never take place because it is never profitable to the entrant. Pooling equilibria do not exist if $\Pi_H^D > \Pi_H(e_0, m_L(e_0); 0)$, because H would rather invest to deter entry than pool with L at $(e_0, m_L(e_0))$. Conversely, if $\Pi_H(e_0, m_L(e_0); 0) \geq \Pi_H^D$ pooling equilibria may exist and have the same characteristics as those studied in Section 3.1. In particular, $e^P \geq e_0$, $q^P \geq m_L(e_0)$ and no pooling equilibrium satisfies the Intuitive Criterion. This result shows that strategic investment and limit pricing will be hardly used to deter profitable entry in the model described in section 2.

Finally, if some of the assumptions of section 2 are relaxed the analysis of equilibrium of the entry problem is quite standard. Indeed, if A.2 does not hold, so that entrant's expected profits are negative when the incumbent acts as an L type monopolist under no entry threat, the analysis of equilibrium is very much the same as that in Milgrom and Roberts (1982), i.e. the 'least cost' separating equilibrium is intuitive and also pooling equilibria satisfy the Intuitive Criterion. On the other hand, if A.4 does not hold, i.e. H has no incentive to mimic L, there are only trivial cases of separating equilibria. L will make his monopoly choices while H will either accommodate entry if $\Pi_H^A \geq \Pi_H^D$ or invest to deter entry otherwise.

4 Conclusions

We analysed an entry problem with private information about costs, in an environment where the incumbent has the opportunity to undertake a cost-reducing investment with a commitment value. We asked whether a strategic use of investment *cum* limit pricing could allow the incumbent to deter profitable entry even when entrant's expected profits are positive. We have shown that although this is a possible equilibrium outcome, nevertheless it is not the most plausible one, because pooling equilibria do not satisfy the Intuitive Criterion. An implication for business strategy analysis is that in market situations where an incumbent cannot deter profitable entry by using a limit pricing strategy and where opportunities of strategic investments are present, the incumbent will be able to deter only unprofitable entry, either by using aggressive investment strategy or by setting a low limit price. This result implies that deterrence of profitable entry only occurs under the same kind of conditions found in Milgrom and Roberts (1982), i.e. only when the entrant's holds pessimistic prior beliefs and expected profits are negative. The inclusion of strategic investment surprisingly does not modify that result in this case.

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