An Iterated Dual Substitution Approach for Binary Integer Programming Problems under the Min–Max Regret Criterion

A Preprint

Wei Wu
Shizuoka University
Hamamatsu, Japan
goi@shizuoka.ac.jp

Manuel Iori
University of Modena and Reggio Emilia
Reggio Emilia, Italy
manuel.iori@unimore.it

Silvano Martello
University of Bologna
Bologna, Italy
silvano.martello@unibo.it

Mutsunori Yagiura
Nagoya University
Nagoya, Japan
yagiura@nagoya-u.jp

December 15, 2020

Abstract

We consider binary integer programming problems with the min–max regret objective function under interval objective coefficients. We propose a new heuristic framework, which we call the iterated dual substitution (iDS) algorithm. The iDS algorithm iteratively invokes a dual substitution heuristic and excludes from the search space any solution already checked in previous iterations. In iDS, we use a best-scenario-based lemma to improve performance. We apply iDS to four typical combinatorial optimization problems: the knapsack problem, the multidimensional knapsack problem, the generalized assignment problem, and the set covering problem. For the multidimensional knapsack problem, we compare the iDS approach with two algorithms widely used for problems with the min–max regret criterion: a fixed-scenario approach, and a branch-and-cut approach. The results of computational experiments on a broad set of benchmark instances show that the proposed iDS approach performs best on most tested instances. For the knapsack problem, the generalized assignment problem, and the set covering problem, we compare iDS with state-of-the-art results. The iDS algorithm successfully updates best known records for a number of benchmark instances.

Keywords Combinatorial optimization · Min–max regret · Heuristics · Iterated dual substitution

1 Introduction

Incomplete information, poor predictions of future events, or mismeasurements can make it difficult to provide accurate input parameters for real-world optimization problems. Robust optimization is a common approach for solving optimization problems under uncertainty. The min–max regret criterion is a representative criterion for robust optimization in which value or cost parameters in the objective function are inaccurate in the optimization phase. When a set of value or cost parameters (called a *scenario*) is revealed, the *regret* of a solution is defined as the difference between the resulting objective value of the solution and the value of the optimal solution to the revealed scenario. In this paper, we focus on the minimization of the maximum regret (min–max regret) over all scenarios.

The study of min–max regret strategy is motivated by practical applications in which the decision maker has to make a decision and the regret for the decision in case of a wrong choice is crucial. Recently, models and solution methods to the min–max regret problems have been applied to a number of real-world cases in many fields, such as water resources management (see Poorsepahy-Samian et al. 2012), greenhouse gas abatement (see Loulou and Kanudia 1999), semiconductor manufacturing (see Chien and Zheng 2012), assembly line worker assignment and balancing (see Pereira 2018), power management (see Dong et al. 2011), revenue management (see Ayvaz-Cavdaroglu et al. 2016), and portfolio optimization (see Xidonas et al. 2017).

The min–max regret versions of important combinatorial optimization problems such as the assignment problem (Pereira and Averbakh 2011), the shortest path problem (Karaşan et al. 2001), the 0-1 knapsack problem (Furini et al. 2015), the generalized assignment problem (Wu et al. 2018a), the traveling salesman problem (Montemanni et al. 2007), and the set covering problem (Pereira and Averbakh 2013) have been recently studied, and both exact and heuristic algorithms have been proposed for this kind of problems. Several attempts at heuristic methods have been made, including constructive heuristics based on the solution for a fixed scenario (see Kasperski and Zieliński 2006) or on the inclusion of a dual relaxation component (see Furini et al. 2015), as well as more elaborated metaheuristics such as genetic algorithms or filter-and-fan methods (see Pereira and Averbakh 2013). Concerning exact algorithms, Benders-like decomposition methods iteratively solve a relaxed master problem in which only a subset of scenarios is considered for constraints. Scenarios corresponding to violated constraints are computed by solving a slave problem and possibly added to its master problem until a solution satisfying all constraints with respect to the scenario is found (see, e.g., Montemanni 2006). Branch-and-cut methods, which extend Benders-like decomposition by including the solution of the slave problem at all nodes of the enumeration tree, have been applied to several min–max regret problems (see Pereira and Averbakh 2013). Other authors have instead used the structure of a min–max regret problem to devise specifically tailored combinatorial branch-and-bound algorithms (see Montemanni and Gambardella 2005). For general introductions to this area, we refer to Aissi et al. (2009), Candia-Véjar et al. (2011), Kasperski (2008), and Kouvelis and Yu (2013).

In this paper, we consider binary integer programming problems with the min–max regret objective function under interval objective coefficients. In Section 2, we describe the problem and we introduce a mixed integer programming model for it. In Section 3, we provide a brief introduction to classical heuristics for this kind of problems. In Section 4, we propose a new heuristic framework, which we call the *iterated dual substitution* (iDS) algorithm. To exclude already checked solutions from the search space, the iDS approach considers two kinds of constraints: those for pruning checked solutions based on the Hamming distance and those using a lemma based on the best scenario. We apply the proposed approach to the min–max regret version of four representative combinatorial optimization problems: the knapsack problem, the multidimensional knapsack problem, the generalized assignment problem, and the set covering problem. In Section 5, we describe these four problems, and we present computational results for their min–max regret versions in Section 6. For the multidimensional knapsack problem, we compare the iDS approach with two algorithms widely applied to problems with the min–max regret criterion: a fixed-scenario approach and a branch-and-cut approach. We evaluate these algorithms through computational experiments on a broad set of benchmark instances (available online). The proposed iDS algorithm performs best for most tested instances. For the knapsack problem, the generalized assignment problem, and the set covering problem, we compare iDS with state-of-the-art results. The results show that the iDS algorithm is highly efficient as a heuristic, successfully updating the best known records for a number of benchmark instances.

2 Problem Description

Let $I = \{1, 2, \ldots, m\}$ and $J = \{1, 2, \ldots, n\}$. In this paper, we consider a *binary integer programming* problem (BIP) with a linear objective function and linear constraints defined as

$$\max \quad \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \forall i \in I, \tag{2}$$

$$x_j \in \{0, 1\} \qquad \forall j \in J, \tag{3}$$

where $\boldsymbol{x}$ represents the vector of variables, $\boldsymbol{c}$ and $\boldsymbol{b}$ are vectors of coefficients, and $\boldsymbol{A} = (a_{ij})$ is a matrix of coefficients. For convenience, we define the set of all feasible solutions as

$$X_0 = \{\boldsymbol{x} \mid \boldsymbol{x} \text{ satisfies constraints (2)–(3)}\}. \tag{4}$$

Note that minimization problems and problems with equality constraints can be rewritten as an equivalent problem in the form of Eqs. (1)–(3). This form includes several classical combinatorial optimization problems such as the knapsack problem, the assignment problem, the shortest path problem, and the set covering problem.

2.1 Min–Max Regret Criterion

In many real-world applications, the objective coefficients c_j are affected by various factors and therefore may be inaccurate. Here, we assume that every c_j can take any value within the range $[c_j^-, c_j^+]$. A scenario s is defined as a vector of costs c_j^s satisfying $c_j^s \in [c_j^-, c_j^+], \forall j \in J$. We denote the objective value of a solution $\boldsymbol{x}$ under scenario s as

$$z^s(\boldsymbol{x}) = \sum_{j=1}^{n} c_j^s x_j,$$

and the optimal value under scenario s as

$$z_*^s = \max_{\boldsymbol{y} \in X_0} z^s(\boldsymbol{y}).$$

The *regret* $r^s(\boldsymbol{x})$ corresponding to a solution $\boldsymbol{x}$ under scenario s is then the difference between these two values:

$$r^s(\boldsymbol{x}) = z_*^s - z^s(\boldsymbol{x}).$$

We define $S = \left\{ s \mid c_j^s \in [c_j^-, c_j^+], \forall j \in J \right\}$ to be the set of all possible scenarios. The *maximum regret* $r_{\max}(\boldsymbol{x})$ of a solution $\boldsymbol{x}$ is then the maximum $r^s(\boldsymbol{x})$ value over all scenarios:

$$r_{\max}(\boldsymbol{x}) = \max_{s \in S} r^s(\boldsymbol{x}). \tag{5}$$

The *binary integer programming problem under min–max regret* criterion (MMR-BIP) is the problem of finding a feasible solution $\boldsymbol{x}$ that minimizes the maximum regret:

$$\begin{aligned} \min_{\boldsymbol{x} \in X_0} r_{\max}(\boldsymbol{x}) &= \min_{\boldsymbol{x} \in X_0} \max_{s \in S} r^s(\boldsymbol{x}) \\ &= \min_{\boldsymbol{x} \in X_0} \max_{s \in S} \left\{ \max_{\boldsymbol{y} \in X_0} \sum_{j=1}^{n} c_j^s y_j - \sum_{j=1}^{n} c_j^s x_j \right\}. \end{aligned} \tag{6}$$

The following lemma is a classic general result proposed by Aissi et al. (2009) (based on Yaman et al. (2001)).

Lemma 2.1. *The regret of a solution $\boldsymbol{x} \in X_0$ is maximized under the scenario $\sigma(\boldsymbol{x})$ defined as*

$$c_j^{\sigma(\boldsymbol{x})} = \begin{cases} c_j^- & \text{if } x_j = 1 \\ c_j^+ & \text{otherwise} \end{cases} \qquad \forall j \in J. \tag{7}$$

From Lemma 2.1, the MMR-BIP can be rewritten as

$$\min_{\boldsymbol{x} \in X_0} r_{\max}(\boldsymbol{x}) = \min_{\boldsymbol{x} \in X_0} \left\{ \max_{\boldsymbol{y} \in X_0} \sum_{j=1}^{n} c_j^{\sigma(\boldsymbol{x})} y_j - \sum_{j=1}^{n} c_j^- x_j \right\}. \tag{8}$$

The MMR-BIP is Σ_2^p-hard, because it has as a special case the interval min–max regret knapsack problem (Furini et al. 2015), which has been proved by Deineko and Woeginger (2010) to be Σ_2^p-hard (see Garey and Johnson 1979, Chap. 7).

2.2 Mixed Integer Programming Model

This section presents a *mixed integer programming* (MIP) model for the MMR-BIP. By introducing a new continuous variable λ, along with constraints that force λ to satisfy $\lambda \geq z_*^s$, $\forall s \in S$, the MMR-BIP can be formulated as the following MIP model (MIP-MMR-BIP):

$$\min \quad \lambda - \sum_{j=1}^{n} c_j^- x_j \tag{9}$$

$$\text{s.t.} \quad \lambda \geq \sum_{j=1}^{n} c_j^{\sigma(\boldsymbol{x})} y_j = \sum_{j=1}^{n} \left(c_j^+ + \left(c_j^- - c_j^+ \right) x_j \right) y_j \qquad \forall \boldsymbol{y} \in X_0, \tag{10}$$

$$\boldsymbol{x} \in X_0. \tag{11}$$

Observe that this model has an exponential number of constraints (10). In the following, we call constraints (10) *Benders cuts*.

3 Classical Heuristics

In this section, we present classical algorithms for the MMR-BIP, including a fixed-scenario algorithm and a branch-and-cut framework.

3.1 Fixed-Scenario Algorithm

The fixed-scenario algorithm is designed based on the observation that the feasible region of the MMR-BIP is the same as that of the classical BIP. This implies that we can obtain a feasible solution to an MMR-BIP instance by fixing a scenario, solving the resulting BIP instance to optimality, and using (8) to evaluate the maximum regret of the obtained solution. This approach has been used for solving a number of interval min–max regret problems (see, e.g., Kasperski and Zieliński 2006, Pereira and Averbakh 2013, and Wu et al. 2018a).

Among the three most commonly used scenarios, namely, lowest value $\boldsymbol{c}^-$, highest value $\boldsymbol{c}^+$, and median value $(\boldsymbol{c}^- + \boldsymbol{c}^+)/2$, in this paper we consider the median-value scenario, for which the following general result, proved by Kasperski and Zieliński (2006), can be directly applied to the MMR-BIP.

Lemma 3.1. *Let $\widetilde{s}$ be the median-value scenario, i.e., $c_j^{\widetilde{s}} = (c_j^- + c_j^+)/2,\ \forall j \in J$, and let $\widetilde{\boldsymbol{x}}$ be an optimal solution to the BIP under $\widetilde{s}$. Then, $r_{\max}(\widetilde{\boldsymbol{x}}) \leq 2r_{\max}(\boldsymbol{x})$ holds for all $\boldsymbol{x} \in X_0$.*

Note that Lemma 3.1 also holds if the original BIP is a minimization problem, that is,

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} c_j x_j \\ \text{s.t.} \quad & \text{(2) and (3)}. \end{aligned} \tag{12}$$

3.2 Branch-and-Cut Algorithm

Branch-and-cut is a standard exact approach widely applied to interval min–max regret problems (see Furini et al. 2015, Montemanni et al. 2007, Pereira and Averbakh 2013, and Wu et al. 2018a). We introduce an implementation of this algorithm using Benders cuts (10) based on a basic branch-and-cut framework.

We define $P(X)$ as a relaxation of the MIP-MMR-BIP, in which set X_0 in constraints (10) is replaced by a subset $X \subseteq X_0$:

$$\lambda \geq \sum_{j=1}^{n} \left(c_j^+ + \left(c_j^- - c_j^+\right) x_j\right) y_j \qquad \forall y \in X. \tag{13}$$

For a feasible solution $(\boldsymbol{x}^*, \lambda^*)$ to problem $P(X)$, we define a *slave problem* $Q(\boldsymbol{x}^*)$ as

$$\max_{\boldsymbol{y} \in X_0} \sum_{j=1}^{n} \left(c_j^+ + \left(c_j^- - c_j^+\right) x_j^*\right) y_j. \tag{14}$$

Let $\boldsymbol{y}^*$ be an optimal solution to $Q(\boldsymbol{x}^*)$ and $q(\boldsymbol{y}^*)$ be the corresponding objective value. If $q(\boldsymbol{y}^*) > \lambda^*$ holds, then the constraint (10) induced by $\boldsymbol{y}^*$ is violated by the current optimal solution $(\boldsymbol{x}^*, \lambda^*)$ of $P(X)$.

We solve $P(X)$ by a branch-and-cut framework. When an integer-feasible solution candidate $\boldsymbol{x}^*$ has been identified, we check a violated constraint (10) by solving the slave problem $Q(\boldsymbol{x}^*)$. If such a violated constraint is found, we discard the candidate solution $\boldsymbol{x}^*$, adding the solution $\boldsymbol{y}^*$ to X and continuing the branch-and-cut with the new $P(X)$; otherwise, we update the incumbent solution and prune the node.

The general branch-and-cut framework can be implemented in many ways. In our implementation, we maintain the set X for constraints (13) as follows. To prevent the LP relaxation $\bar{P}(X)$ at the root node from being unbounded, we start with set $X = \{\widetilde{\boldsymbol{x}}\}$, where $\widetilde{\boldsymbol{x}}$ is the optimal solution obtained by the fixed-scenario heuristic under the median-value scenario. All cuts added to X at any active node are used throughout the remaining computations, and so contribute to all other active nodes.

The branch-and-cut algorithm usually obtains feasible solutions during the search process, and hence it can also be used as a heuristic by prematurely terminating its execution.

4 Iterated Dual Substitution Heuristic

In this section, we propose an approach obtained by iterating a dual substitution heuristic.

4.1 Dual Substitution Heuristic

The *dual substitution* (DS) heuristic is an algorithm based on a MIP model in which a subproblem is replaced by the dual counterpart of its linear relaxation. Kasperski (2008) provided a general technique, and similar ideas of using MIP models were used in other min–max regret problems (see Furini et al. 2015, Karaşan et al. 2001, Kasperski and Zieliński 2006, Wu et al. 2018a, and Yaman et al. 2001). In this paper, we use it as a heuristic for the MMR-BIP.

For every fixed $\boldsymbol{x}$, the maximization problem over $\boldsymbol{y}$ in (8) is a classical BIP that can be expressed as

$$\max \quad \sum_{j=1}^{n} c_j^{\sigma(\boldsymbol{x})} y_j \tag{15}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} y_j \le b_i \qquad \forall i \in I, \tag{16}$$

$$y_j \in \{0,1\} \qquad \forall j \in J. \tag{17}$$

We consider a corresponding linear program, obtained by replacing the integrality constraint (17) with

$$0 \le y_j \le 1 \qquad \forall j \in J. \tag{18}$$

We define two types of dual variables: $u_i \ (i \in I)$ for constraints (16) and $v_j \ (j \in J)$ for the ≤ 1 constraints in (18). The dual of problem (15), (16), (18) can be formulated as

$$\min \quad \sum_{i=1}^{m} b_i u_i + \sum_{j=1}^{n} v_j \tag{19}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} a_{ij} u_i + v_j \ge c_j^{\sigma(\boldsymbol{x})} = c_j^{+} + \left(c_j^{-} - c_j^{+}\right) x_j \qquad \forall j \in J, \tag{20}$$

$$u_i \ge 0 \qquad \forall i \in I, \tag{21}$$

$$v_j \ge 0 \qquad \forall j \in J. \tag{22}$$

We then embed the dual counterpart into (8). By replacing $\boldsymbol{y} \in X_0$ with constraints (20)–(22), and by replacing the inner maximization problem (15) with (19), we obtain the following *dual substitution model* (D-MMR-BIP):

$$\min \quad \sum_{i=1}^{m} b_i u_i + \sum_{j=1}^{n} v_j - \sum_{j=1}^{n} c_j^{-} x_j \tag{23}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} a_{ij} u_i + v_j \ge c_j^{+} + \left(c_j^{-} - c_j^{+}\right) x_j \qquad \forall j \in J, \tag{24}$$

$$u_i \ge 0 \qquad \forall i \in I, \tag{25}$$

$$v_j \ge 0 \qquad \forall j \in J, \tag{26}$$

$$\boldsymbol{x} \in X_0. \tag{27}$$

The dual substitution heuristic exactly solves the D-MMR-BIP and outputs the obtained solution. The D-MMR-BIP is not easier than the BIP, since it contains all the BIP constraints.

Remark 4.1. *The optimal value of the D-MMR-BIP is an upper bound on the optimal value for the MMR-BIP.*

In addition, for any instance, a tighter upper bound can be obtained as follows.

Remark 4.2. *The bound obtained by evaluating the maximum regret of any optimal solution to the D-MMR-BIP is at least as good as the optimal value to the D-MMR-BIP.*

Our computational experiments showed that the DS heuristic obtains better solutions than does the fixed-scenario heuristic for most instances, although it does not guarantee solution quality for some problems, including the generalized assignment problem (Wu et al. 2018a) and the multidimensional knapsack problem (Wu et al. 2016b).

4.2 Best Scenario

Before describing the iterated dual substitution heuristic, we introduce some relevant properties of the BIP. For a solution $\boldsymbol{x}$, let us consider the scenario

$$\phi(\boldsymbol{x}) = \begin{cases} c_j^+ & \text{if } x_j = 1 \\ c_j^- & \text{otherwise,} \end{cases}$$

opposite to the worst-case scenario (7), which we call the *best scenario* for solution $\boldsymbol{x}$.

Lemma 4.1. *If $\boldsymbol{x}^*$ is an optimal solution of the BIP* (1)–(3) *under a scenario $s \in S$, then $\boldsymbol{x}^*$ is also an optimal solution under its best scenario $\phi(\boldsymbol{x}^*)$.*

Proof. In $\phi(\boldsymbol{x}^*)$ the c_j value of the items selected (resp. not selected) in $\boldsymbol{x}^*$ can only increase (resp. decrease). □

Note that even if the original BIP is a minimization problem (12), (2), (3), Lemma 4.1 holds with a slight modification, that is, the best scenario of $\boldsymbol{x}^*$ becomes $\sigma(\boldsymbol{x}^*)$ in (7).

Lemma 4.2. *For any feasible pair of solutions $\bar{\boldsymbol{x}}, \hat{\boldsymbol{x}} \in X_0$ of the BIP* (1)–(3)*, if*

$$z^{\phi(\bar{\boldsymbol{x}})}(\hat{\boldsymbol{x}}) \geq z^{\phi(\bar{\boldsymbol{x}})}(\bar{\boldsymbol{x}}) \tag{28}$$

holds, then we have $r_{\max}(\hat{\boldsymbol{x}}) \leq r_{\max}(\bar{\boldsymbol{x}})$.

Proof. From (28) we obtain the following inequalities:

$$\begin{aligned} \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=1}} c_j^+ + \sum_{\substack{j:\,\bar{x}_j=0,\\ \hat{x}_j=1}} c_j^- &\geq \sum_{j:\,\bar{x}_j=1} c_j^+ \\ \sum_{\substack{j:\,\bar{x}_j=0,\\ \hat{x}_j=1}} c_j^- &\geq \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=0}} c_j^+. \end{aligned} \tag{29}$$

From (5) we get

$$r_{\max}(\hat{\boldsymbol{x}}) = z_*^{\sigma(\hat{\boldsymbol{x}})} - z^{\sigma(\hat{\boldsymbol{x}})}(\hat{\boldsymbol{x}}) = z_*^{\sigma(\hat{\boldsymbol{x}})} - \sum_{j:\,\hat{x}_j=1} c_j^- = z_*^{\sigma(\hat{\boldsymbol{x}})} - \sum_{\substack{j:\,\bar{x}_j=0,\\ \hat{x}_j=1}} c_j^- - \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=1}} c_j^-. \tag{30}$$

From (29) we obtain

$$r_{\max}(\hat{\boldsymbol{x}}) \leq z_*^{\sigma(\hat{\boldsymbol{x}})} - \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=0}} c_j^+ - \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=1}} c_j^- = z_*^{\sigma(\hat{\boldsymbol{x}})} - \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=0}} \left(c_j^+ - c_j^-\right) - \sum_{j:\,\bar{x}_j=1} c_j^-. \tag{31}$$

Letting x^* be an optimal solution to the BIP under $\sigma(\hat{\boldsymbol{x}})$, inequality (31) can be rewritten as

$$\begin{aligned} r_{\max}(\hat{\boldsymbol{x}}) &\leq \sum_{\substack{j:\,x_j^*=1,\\ \hat{x}_j=1}} c_j^- + \sum_{\substack{j:\,x_j^*=1,\\ \hat{x}_j=0}} c_j^+ - \sum_{\substack{j:\,\bar{x}_j=1,\\ \hat{x}_j=0}} \left(c_j^+ - c_j^-\right) - \sum_{j:\,\bar{x}_j=1} c_j^- \\ &= \sum_{\substack{j:\,x_j^*=1,\\ \bar{x}_j=1}} c_j^- + \sum_{\substack{j:\,x_j^*=1,\\ \hat{x}_j=0,\\ \bar{x}_j=0}} c_j^+ + \sum_{\substack{j:\,x_j^*=1,\\ \hat{x}_j=1,\\ \bar{x}_j=0}} c_j^- + \sum_{\substack{j:\,x_j^*=0,\\ \hat{x}_j=0,\\ \bar{x}_j=1}} \left(c_j^- - c_j^+\right) - \sum_{j:\,\bar{x}_j=1} c_j^- \\ &\leq \sum_{\substack{j:\,x_j^*=1,\\ \bar{x}_j=1}} c_j^- + \sum_{\substack{j:\,x_j^*=1,\\ \hat{x}_j=0,\\ \bar{x}_j=0}} c_j^+ + \sum_{\substack{j:\,x_j^*=1,\\ \hat{x}_j=1,\\ \bar{x}_j=0}} c_j^+ - \sum_{j:\,\bar{x}_j=1} c_j^- \\ &= z^{\sigma(\bar{\boldsymbol{x}})}(\boldsymbol{x}^*) - \sum_{j:\,\bar{x}_j=1} c_j^- \leq z_*^{\sigma(\bar{\boldsymbol{x}})} - \sum_{j:\,\bar{x}_j=1} c_j^- = r_{\max}(\bar{\boldsymbol{x}}). \end{aligned}$$

□

It is not difficult to adapt the proof of the previous lemma to show that the property also holds if the original BIP is a minimization problem i.e.,

Lemma 4.3. *For any feasible pair of solutions* $\bar{\boldsymbol{x}}, \hat{\boldsymbol{x}} \in X_0$ *of problem* (12), (2), (3), *if*

$$z^{\sigma(\bar{\boldsymbol{x}})}(\hat{\boldsymbol{x}}) \leq z^{\sigma(\bar{\boldsymbol{x}})}(\bar{\boldsymbol{x}}) \tag{32}$$

holds, then we have $r_{\max}(\hat{\boldsymbol{x}}) \leq r_{\max}(\bar{\boldsymbol{x}})$.

4.3 Iterated Dual Substitution

Computational experiments on specific min–max regret optimization problems (see, e.g., Furini et al. 2015, Wu et al. 2018a) have shown that the DS heuristic performs better than the fixed scenario algorithm in most cases, but it can, in some cases, produce unsatsfactory solutions. To overcome this, we propose an iterative method, which we call the *iterated dual substitution* (iDS), that improves the performance of the DS heuristic through iterative application, excluding from the search space the solutions that were already checked in previous iterations.

We define $\hat{X}$ ($\subseteq X_0$) as the set of already obtained feasible solutions, and denote by D-MMR-BIP($\hat{X}$) the problem obtained by adding constraints based on $\hat{X}$ to the D-MMR-BIP (23)–(27) in the way described below. The iDS algorithm starts with an empty set $\hat{X} = \emptyset$. It then solves the D-MMR-BIP($\hat{X}$), obtaining a solution $\hat{\boldsymbol{x}}$, and evaluates its maximum regret $r_{\max}(\hat{\boldsymbol{x}})$. Since the D-MMR-BIP($\hat{X}$) contains all constraints of the original BIP, any feasible solution $\hat{\boldsymbol{x}}$ to the D-MMR-BIP($\hat{X}$) is also feasible to the MMR-BIP. At each iteration, iDS adds $\hat{\boldsymbol{x}}$ to $\hat{X}$ and solves the updated D-MMR-BIP($\hat{X}$). This process is repeated until the search space of D-MMR-BIP($\hat{X}$) becomes empty, or a termination condition (here, a time limit) is satisfied.

To remove already checked feasible regions, we consider two kinds of constraints based on $\hat{X}$: a Hamming-distance constraint and a best-scenario constraint.

Hamming-Distance Constraint.

We consider the constraint

$$\sum_{j:\, \hat{x}_j=0} x_j + \sum_{j:\, \hat{x}_j=1} (1 - x_j) \geq d \qquad \forall \hat{\boldsymbol{x}} \in \hat{X}, \tag{33}$$

where d ($\geq$1) is an integer parameter. Any solution $x \in X_0$ that satisfies (33) for a solution $\hat{\boldsymbol{x}}$ has a Hamming distance larger than or equal to d from $\hat{\boldsymbol{x}}$. Our aim is to find a good solution to the D-MMR-BIP that has a Hamming distance greater than or equal to d for every solution in $\hat{X}$, and hence solutions within a distance less than d from a solution in $\hat{X}$ are removed from the feasible region.

Note that Hamming-distance constraints (33) are also valid when the original BIP is a minimization problem.

Best-Scenario Constraint.

Lemma 4.2 indicates that a feasible solution $\hat{\boldsymbol{x}}$ dominates all solutions in

$$\left\{ \boldsymbol{x} \in X_0 \;\middle|\; z^{\phi(\boldsymbol{x})}(\hat{\boldsymbol{x}}) \geq z^{\phi(\boldsymbol{x})}(\boldsymbol{x}) \right\}. \tag{34}$$

For every solution $\hat{\boldsymbol{x}} \in \hat{X}$, we consider a constraint for removing those solutions that are dominated by $\hat{\boldsymbol{x}}$, namely,

$$z^{\phi(\boldsymbol{x})}(\hat{\boldsymbol{x}}) < z^{\phi(\boldsymbol{x})}(\boldsymbol{x}) \qquad \forall \hat{\boldsymbol{x}} \in \hat{X}. \tag{35}$$

Note that the constraint (35) of each $\hat{\boldsymbol{x}}$ excludes $\hat{\boldsymbol{x}}$ itself from the feasible region. The linear expression of (35) is

$$\begin{aligned} &\sum_{j:\, \hat{x}_j=1} \left(c_j^+ x_j - c_j^- x_j + c_j^- \right) < \sum_{j=1}^{n} c_j^+ x_j && \forall \hat{\boldsymbol{x}} \in \hat{X} \\ &\sum_{j:\, \hat{x}_j=1} c_j^- x_j + \sum_{j:\, \hat{x}_j=0} c_j^+ x_j > \sum_{j:\, \hat{x}_j=1} c_j^- && \forall \hat{\boldsymbol{x}} \in \hat{X}. \end{aligned} \tag{36}$$

Given a feasible solution set $\hat{X}$, best-scenario constraints (36) dominate Hamming-distance constraints when $d = 1$, because Hamming-distance constraints with $d = 1$ can only exclude all solutions in $\hat{X}$ from the search space.

If the original BIP is a minimization problem, the constraints corresponding to (36) become

$$\sum_{j:\,\hat{x}_j=1} c_j^+ x_j + \sum_{j:\,\hat{x}_j=0} c_j^- x_j < \sum_{j:\,\hat{x}_j=1} c_j^+ \qquad \forall \hat{\boldsymbol{x}} \in \hat{X}. \tag{37}$$

If all the values of c_j^- and c_j^+ are integers, we can replace (36) and (37), respectively, with

$$\sum_{j:\,\hat{x}_j=1} c_j^- x_j + \sum_{j:\,\hat{x}_j=0} c_j^+ x_j \geq \sum_{j:\,\hat{x}_j=1} c_j^- + 1 \qquad \forall \hat{\boldsymbol{x}} \in \hat{X},$$

and

$$\sum_{j:\,\hat{x}_j=1} c_j^+ x_j + \sum_{j:\,\hat{x}_j=0} c_j^- x_j \leq \sum_{j:\,\hat{x}_j=1} c_j^+ - 1 \qquad \forall \hat{\boldsymbol{x}} \in \hat{X}.$$

Given sufficient computation time and memory space, the iDS algorithm using best-scenario constraints is theoretically an exact approach for the MMR-BIP, because the newly excluded search space is non-empty in each iteration, and all excluded solutions are guaranteed to be no better than the candidate solution according to Lemma 4.2.

4.4 Local Exact Subroutine

Note that if we set $d = 1$, the iDS approach using Hamming-distance constraints at every iteration only excludes from the search space those $\hat{\boldsymbol{x}}$ solutions that have already been checked during the search. When $d \geq 2$, the iDS approach also removes from the search space unchecked solutions around $\hat{\boldsymbol{x}}$. The unchecked space around a solution $\hat{\boldsymbol{x}}$ can be defined as

$$1 \leq \sum_{j:\,\hat{x}_j=0} x_j + \sum_{j:\,\hat{x}_j=1} (1 - x_j) \leq d - 1. \tag{38}$$

When $d \geq 2$, we consider an option that exactly solves, whenever a solution $\hat{\boldsymbol{x}}$ is obtained by solving D-MMR-BIP($\hat{X}$), the MMR-BIP described in (8) with the additional constraint (38) only for the newly obtained $\hat{\boldsymbol{x}}$, which we call the MMR-BIP($\hat{\boldsymbol{x}}$). Taking this option, the iDS algorithm using Hamming-distance constraints is guaranteed to find an exact optimal solution for the MMR-BIP when computation time and memory space are sufficient to run until the D-MMR-BIP($\hat{X}$) search space becomes empty.

The branch-and-cut approach described in Section 3.2 can be used to solve the MMR-BIP($\hat{\boldsymbol{x}}$).

5 Test Problems

To test the performance of the proposed algorithms for binary integer programming problems, we selected four representative combinatorial optimization problems: two maximization problems (the knapsack problem and the multidimensional knapsack problem) and two minimization problems (the set covering problem and the generalized assignment problem). All these problems are known to be NP hard, and the corresponding problems under min–max regret criteria are Σ_2^p hard. We describe the original problems in this section. The corresponding problems under min–max regret criteria can be defined using the definitions in Sections 2.1–2.2.

5.1 Knapsack Problem

The *knapsack problem* (KP) is defined as follows. Given a set J of n items $\{1, 2, \ldots, n\}$, each item j having weight a_j and value c_j, choose items maximizing the total value such that the total weight is less than or equal to the knapsack capacity b.

The KP can be formulated over binary variables x_j, indicating that item j is chosen if and only if $x_j = 1$:

$$\max \sum_{j=1}^{n} c_j x_j \tag{39}$$

$$\text{s.t.} \sum_{j=1}^{n} a_j x_j \leq b \tag{40}$$

$$x_j \in \{0, 1\} \qquad \forall j \in J. \tag{41}$$

The KP has been thoroughly studied due to its simple structure and important practical applications (see Kellerer et al. 2004, and Martello and Toth 1990).

5.2 Multidimensional Knapsack Problem

The *multidimensional knapsack problem* (MKP) is an extension of the KP. Given a set J of n items $\{1, 2, \dots, n\}$ and a set I of m multiple-resource restraints $\{1, 2, \dots, m\}$, choose items with maximum total value, subject to satisfying every resource restraint. Choosing item j provides value c_j and consumes an amount a_{ij} of resource in dimension i, with total resource availability (capacity) along dimension i being b_i.

The MKP can be formulated over binary variables x_j, indicating that item j is chosen if and only if $x_j = 1$:

$$\max \sum_{j=1}^{n} c_j x_j \tag{42}$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad \forall i \in I, \tag{43}$$

$$x_j \in \{0, 1\} \qquad \forall j \in J. \tag{44}$$

The MKP is known to be computationally much harder than the KP, even for $m = 2$ (see Kulik and Shachnai 2010). The problem has been thoroughly studied over many decades due to its theoretical interest and its broad applications in several fields, including cargo loading, cutting stock, bin-packing, finance, and management issues (see Laabadi et al. 2018 and Puchinger et al. 2010).

5.3 Set Covering Problem

The *set covering problem* (SCP) is defined as follows. Given a set I of n items $\{1, 2, \dots, n\}$ and m subsets of I, each subset j having a cost c_j, select subsets with minimum cost such that each item is covered by at least one selected subset. Letting a_{ij} be a binary value that equals 1 when subset j covers item i and 0 otherwise, the SCP can be formulated over binary variables x_j, indicating that subset j is chosen if and only if $x_j = 1$:

$$\min \sum_{j=1}^{n} c_j x_j \tag{45}$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \ge 1 \qquad \forall i \in I, \tag{46}$$

$$x_j \in \{0, 1\} \qquad \forall j \in J. \tag{47}$$

The decision version of the SCP is one of Karp's 21 NP-complete problems (see Karp 1972). There are many real-world applications involving this type of problem, including facility location (see Farahani et al. 2012), airline crew scheduling (see Wu et al. 2016a), and vehicle routing (see Cacchiani et al. 2014).

5.4 Generalized Assignment Problem

The *generalized assignment problem* (GAP) is defined as follows. Given a set J of n jobs $\{1, 2, \dots, n\}$ and a set I of m agents $\{1, 2, \dots, m\}$, look for a minimum cost assignment, subject to assigning each job to exactly one agent and satisfying a resource constraint for each agent. Assigning job j to agent i incurs a cost c_{ij} and consumes an amount a_{ij} of resource, where the total amount of the resource available for agent i (the agent capacity) is b_i.

A natural formulation of the GAP is defined over a two-dimensional binary variable x_{ij}, indicating that job j is assigned to agent i if and only if $x_{ij} = 1$:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{48}$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_{ij} \le b_i \qquad \forall i \in I, \tag{49}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad \forall j \in J, \tag{50}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in I, \ \forall j \in J. \tag{51}$$

The GAP has many real-world applications, including production planning, scheduling, timetabling, telecommunication, investment, and transportation (see Fisher and Jaikumar 1981, Ruland 1999, and Wu et al. 2018b).

6 Computational Experiments

Let MMR-KP, MMR-MKP, MMR-SCP, and MMR-GAP denote respectively the KP, the MKP, the SCP, and the GAP under the min–max regret criterion. The following sections present computational results for these four resulting problems.

6.1 Implementation Details

All experiments were carried out on a PC with a Xeon E-2144G CPU running at 3.60 GHz, with 64 GB memory. All computations were performed on a single CPU core. We used Gurobi version 8.1 to solve the mixed integer linear programming problems. Detailed results for the four considered problems are presented in four appendices and are available online at `http://www.co.mi.i.nagoya-u.ac.jp/~yagiura/ids/`.

Concerning iDS using Hamming-distance constraints, we used the constraints (33) with $d = 1$ for all tested problems, because preliminary experiments indicated that this value gives better results than do larger values of d. All algorithms were given a time limit of 1 CPU hour per instance.

6.2 Min–Max Regret Multidimensional Knapsack Problem

To the best of our knowledge, this is the first study on the MMR-MKP. To generate MMR-MKP instances, we used the classical MKP benchmarks of Chu and Beasley (1998) with $(m, n) \in \{(5, 100), (10, 100), (5, 250)\}$. There are 30 instances for each (m, n) pair. For each MKP instance, we generated the extremes of the value intervals c_j^- and c_j^+ as random integers uniformly distributed in $[(1-\delta)c_j, c_j]$ and $[c_j, (1+\delta)c_j]$, respectively, with $\delta \in \{0.1, 0.2, 0.3\}$. The 270 generated instances are available at `http://www.co.mi.i.nagoya-u.ac.jp/~yagiura/mmr-mkp/`.

Table 1: MMR-MKP results

	B&C				Fix			DS			iDS-H				iDS-B			
instances	time	%gap	#opt	#best	time	%gap	#best	time	%gap	#best	time	iter	%gap	#best	time	iter	%gap	#best
05-100-10	109.40	**0.00**	30	**30**	1.17	3.92	13	1.68	1.55	18	3.71	1.77	**0.00**	**30**	3.74	1.77	**0.00**	**30**
05-100-20	755.31	2.32	26	28	0.92	3.49	18	3.58	2.37	25	5.40	1.27	**2.18**	**30**	5.32	1.27	**2.18**	**30**
05-100-30	1415.65	11.71	7	18	0.86	11.85	15	13.14	10.84	19	24.63	1.53	**10.62**	**30**	24.95	1.53	**10.62**	**30**
10-100-10	1603.19	9.79	24	26	8.88	12.61	14	11.09	12.20	23	33.56	1.77	**8.53**	**30**	32.26	1.77	**8.53**	**30**
10-100-20	2609.02	34.80	6	7	7.70	31.39	9	21.12	31.54	20	63.05	1.57	**29.83**	**30**	61.93	1.57	**29.83**	**30**
10-100-30	1897.73	37.69	2	6	7.67	34.45	13	113.41	33.63	20	343.83	1.70	**33.42**	**29**	350.37	1.70	**33.42**	**29**
05-250-10	2752.21	58.59	0	1	24.16	45.10	14	181.55	46.94	21	238.91	1.47	**44.40**	**30**	238.16	1.47	**44.40**	**30**
05-250-20	2379.45	59.58	0	0	17.98	43.43	13	2197.58	43.33	24	2258.67	1.17	**43.21**	**26**	2255.03	1.17	**43.21**	**26**
05-250-30	2550.39	53.29	0	0	17.66	39.94	16	3353.68	**39.82**	**23**	3353.68	1.00	**39.82**	**23**	3353.68	1.00	**39.82**	**23**
	1785.82	29.75	95	116	9.67	25.13	125	655.20	24.69	216	702.83	1.47	**23.56**	**258**	702.83	1.47	**23.56**	**258**

Table 1 shows the results of the algorithms described in Sections 3 and 4: the branch-and-cut algorithm ("B&C"), the fixed-scenario algorithm ("Fix"), the DS algorithm ("DS"), the iDS algorithm with Hamming-distance constraints ("iDS-H"), and iDS with the best-scenario constraints ("iDS-B"). A row "ww-xxx-yy" refers to the 30 instances generated with $m = ww$, $n = xxx$, and $100\delta = yy$. For the branch-and-cut algorithm, the entries show the average CPU time in seconds required to obtain the best solution ("time"), the average percentage gap between the solution value and the best lower-bound value ("%gap"), the number of instances solved to proven optimality ("#opt"), and the number of instances for which the solution value obtained was the best among the tested algorithms ("#best"). Since the fixed-scenario approach under the median-value scenario is a 2-approximation algorithm (see Lemma 3.1), half of its solution value is a valid lower bound. The lower-bound value used to compute the average percentage optimality gap was the best between such value and the lower bound produced by the branch-and-cut. For iDS-H and iDS-B, we provide the average number of iterations ("iter") required to obtain the best solution. Values in bold indicate that the corresponding algorithm(s) obtained the best results among the tested algorithms. The final line of the table reports the overall average and total values over the 270 instances.

The branch-and-cut algorithm exactly solved 63 instances with $m = 5$ and $n = 100$, 32 with $m = 10$ and $n = 100$, and 0 with $m = 5$ and $n = 250$. In total, it only solved 95 out of 270 instances to proven optimality, and failed to obtain solutions with low optimality gaps for all instances with $m = 5$ and $n = 250$. Focusing on instances with same m and n values, we observe that it becomes hard to obtain optimal solutions or even good lower bounds for instances with large δ values, resulting in large “gap" column values. This indicates that an MMR-MKP instance with the tested size should be preferably handled through heuristic algorithms.

The fixed-scenario approach under the median-value scenario obtained solutions within 25 seconds on average even for the most hard instances with $m = 5$ and $n = 250$. It produced solutions with good optimality gaps for instances with $m = 5$ and $n = 100$.

The DS algorithm obtained all best solutions within 20 minutes on average. However, for 37 out of 60 instances with $(m, n) = (5, 250)$ and $\delta \geq 0.2$, the DS algorithm failed to complete within the time limit (see appendix A).

By comparing the five heuristics (the branch-and-cut algorithm can be also used as a heuristic by imposing a time limit), we can observe that the fixed-scenario approach under the median-value scenario obtained solutions within 10 seconds on average for all tested instances. Concerning the solution quality, the two iDS algorithms obtained the best solution values for 258 out of 270 instances, while branch-and-cut, fixed-scenario approach, and the DS algorithm obtained the best solution values for 116, 125, and 216 instances, respectively. The iDS algorithms obtained optimal values for all instances with proven optimality, and they dominated the other algorithms for all instances with $m = 5$ and $n = 100$. For instances where the branch-and-cut and iDS algorithms obtained solutions with the same value, the iDS algorithms used smaller CPU times. However, the two iDS algorithms failed to obtain the best solution for 11 instances with $(m, n) = (5, 250)$ and $\delta \geq 0.2$. These failures occurred because the computation of each iteration was very expensive, and even the first iteration was not completed within the time limit in most cases.

We finally observe that there was no clear winner between iDS-B and iDS-H. They always obtained the best solutions at the same iteration with similar CPU times.

We refer to appendix A for more details on the computational results for the MMR-MKP.

6.3 Min–Max Regret Knapsack Problem

For the MMR-KP, we used the benchmark instances generated by Furini et al. (2015) (available at `http://www.or.deis.unibo.it/research_pages/ORinstances/MRKP_instances.zip`). They first generated 18 KP instances of nine types using a parameter $\bar{R} \in \{1000, 10000\}$ as follows (u.d. stands for “uniformly distributed”):

1. **Uncorrelated**: a_j and c_j are random integers u.d. in $[1, \bar{R}]$.
2. **Weakly correlated**: a_j is a random integer u.d. in $[1, \bar{R}]$, c_j is a random integer u.d. in $[a_j - \bar{R}/10, a_j + \bar{R}/10]$ so that $c_j \geq 1$.
3. **Strongly correlated**: a_j is a random integer u.d. in $[1, \bar{R}]$, $c_j = a_j + \bar{R}/10$.
4. **Inverse strongly correlated**: c_j is a random integer u.d. in $[1, \bar{R}]$, $a_j = c_j + \bar{R}/10$.
5. **Almost strongly correlated**: a_j is a random integer u.d. in $[1, \bar{R}]$, c_j is a random integer u.d. in $[a_j + \bar{R}/10 - \bar{R}/500, a_j + \bar{R}/10 + \bar{R}/500]$.
6. **Subset sum**: a_j is a random integer u.d. in $[1, \bar{R}]$, $c_j = a_j$.
7. **Even–odd subset sum**: a_j is an even random integer u.d. in $[1, \bar{R}]$, $c_j = a_j$, b is odd.
8. **Even–odd strongly correlated**: a_j is an even random integer u.d. in $[1, \bar{R}]$, $c_j = a_j + \bar{R}/10$, b is odd.
9. **Uncorrelated with similar weights**: a_j is a random integer u.d. in $[100\bar{R}, 100\bar{R} + \bar{R}/10]$, c_j is a random integer u.d. in $[1, \bar{R}]$.

Then, for each KP instance, 27 MMR-KP instances were generated by considering all combinations of

- $n \in \{50, 60, 70\}$;
- $b = \lfloor \gamma \sum_{j=1}^{n} a_j \rfloor$, with tightness ratio $\gamma \in \{0.45, 0.5, 0.55\}$ (b was increased by 1 if its value was even for classes 7 and 8);
- c_j^- and c_j^+ set as random integers u.d. in $[(1 - \delta)c_j, c_j]$ and $[c_j, (1 + \delta)c_j]$, respectively, with $\delta \in \{0.1, 0.2, 0.3\}$,

obtaining in total 486 MMR-KP instances.

Table 2: MMR-KP results

	Best Known		DS			iDS-H				iDS-B			
instances	%gap	#opt	time	%gap	#b(w)	time	iter	%gap	#b(w)	time	iter	%gap	#b(w)
Type 1	0.000	54	0.01	0.262	0 (5)	0.01	1.17	0.000	0 (0)	0.01	1.15	0.000	0 (0)
Type 2	1.307	48	0.05	1.604	0 (10)	0.06	1.22	1.423	0 (1)	0.06	1.19	1.423	0 (1)
Type 3	11.239	18	0.27	11.276	0 (9)	0.42	1.28	11.237	1 (0)	0.40	1.28	11.237	1 (0)
Type 4	9.895	18	0.21	9.964	0 (10)	0.27	1.24	9.895	0 (0)	0.26	1.24	9.895	0 (0)
Type 5	8.151	24	0.19	8.407	0 (16)	0.30	1.44	8.201	1 (1)	0.30	1.44	8.202	0 (1)
Type 6	22.144	0	0.44	22.182	0 (15)	5.19	6.43	22.140	3 (0)	2.73	3.89	22.140	3 (0)
Type 7	22.268	0	0.50	22.293	4 (17)	5.40	4.65	22.253	14 (5)	3.62	3.17	22.253	14 (5)
Type 8	9.627	20	0.25	9.801	0 (20)	0.37	1.39	9.723	0 (9)	0.36	1.39	9.723	0 (9)
Type 9	0.000	54	0.01	0.005	0 (1)	0.01	1.02	0.000	0 (0)	0.01	1.02	0.000	0 (0)
overall	9.4033	236	0.22	9.5328	4 (103)	1.34	2.20	9.4303	19 (16)	0.86	1.75	9.4304	18 (16)

Table 2 shows the results obtained by DS, iDS-H and iDS-B. Each row of the table refers to the 54 instances of the same type. The “Best Known” values (optimality gaps and number of instances solved to proven optimality) are the values obtained from the three heuristic algorithms and the three exact algorithms by Furini et al. (2015).

The entries “%gap,” “#opt,” “time,” and “iter” provide the same information as in Table 1. In columns “#b(w),” the value outside (resp. inside) parentheses shows the total number of instances for which the objective function value was better (resp. worse) than the best known solution value. The final line reports the overall average and total values over the 486 instances.

For all 108 instances of Types 1 and 9, both iDS-H and iDS-B obtained optimal solutions within 0.01 seconds on average. For the 162 instances of Types 3, 4, and 6, both iDS-H and iDS-B obtained solutions of value equal to or better than the best known solution value, improving the best known solution values for 4 instances. For instances of Types 2 and 5, both iDS algorithms very quickly obtained solutions with objective values equal to the best known solution values for most instances (106 out of 108), with iDS-H updating one best known objective value. For Type 7 instances, the iDS algorithms updated the best known objective values for 14 out of 54 instances, although they obtained worse solution values for 4 instances. For no Type 8 instance the iDS algorithms improved the best known solution value. We next evaluate the iDS algorithms by comparing them with the results of the DS algorithm. The DS updated 4 best known objective values, but obtained worse solutions for 103 instances. Both iDS-H and iDS-B improved most of these worse solutions (86 out of 103), and iDS-H (resp. iDS-B) further updated the best known objective values for 15 (resp. 14) more instances.

In total, the iDS algorithms updated the best known objective values for 19 instances, and obtained worse solutions for 16 instances. They obtained optimal solutions for most instances (235 out of 236, see appendix B) whose optimal solutions were known. These results suggest that the iDS algorithms can be used as a heuristic for quickly obtaining high-quality solutions for the MMR-KP, except for Type 8 instances.

Comparing the two iDS algorithms, on the basis of the detailed results in appendix B, it turns out that iDS-B was the winner (better solution or solution of the same value in fewer iterations) in 25 instances, while iDS-H won only once. For Type 6 and 7 instances in particular, iDS-B won in 23 instances.

6.4 Min–Max Regret Set Covering Problem

For the MMR-SCP, we used the benchmark instances of Pereira and Averbakh (2013), which were generated from 25 benchmark instances for the classical SCP, namely instance sets 4, 5, and 6 in the OR-Library (see Beasley 1990). Instance set 4 contains 10 instances with $n = 1000$ and $m = 200$, instance set 5 contains 10 instances with $n = 2000$ and $m = 200$, and instance set 6 contains 5 instances with $n = 1000$ and $m = 200$. In all cases, the original cost values range between 1 and 100. The total number of classical SCP instances is 25. The extremes of the cost intervals c_j^- and c_j^+ for each instance family of the MMR-SCP were set according to the following three types:

B: the extremes of cost intervals c_j^- and c_j^+ were generated as random integers u.d. in $[(1-\delta)c_j, c_j]$ and $[c_j, (1+\delta)c_j]$, respectively, with $\delta \in \{0.1, 0.3, 0.5\}$, where c_j are the original costs of the corresponding SCP instances from the OR-Library. The total number of Type-B instances is thus 75;

M: for every $j \in N$, c_j^+ was generated as a random integer u.d. in $[0, 1000]$, and c_j^- as a random integer u.d. in $[0, c_j^+]$. Three instances were generated from each original SCP instance, resulting in a total of 75 Type-M instances;

K: for every $j \in N$, c_j^- was generated as a random integer u.d. in $[0, 1000]$, and c_j^+ as a random integer u.d. in $[c_j^-, c_j^+ + 1000]$. Three instances were generated from each original SCP instance, producing in total 75 Type-K instances.

Table 3: MMR-SCP results

	Best Known		DS			iDS-H				iDS-B			
instances	%gap	#opt	time	%gap	#b(w)	time	iter	%gap	#b(w)	time	iter	%gap	#b(w)
B4	0.000	30	0.4	0.834	0 (13)	8.2	9.03	0.000	0 (0)	4.8	4.83	0.000	0 (0)
B5	0.000	30	0.4	1.752	0 (14)	1.7	3.23	0.000	0 (0)	1.5	2.33	0.000	0 (0)
B6	0.000	15	0.6	1.499	0 (6)	5.8	7.47	0.000	0 (0)	2.0	2.67	0.000	0 (0)
M4	0.000	30	0.5	0.021	0 (4)	1.5	2.20	0.000	0 (0)	1.2	1.53	0.000	0 (0)
M5	0.000	30	0.4	0.008	0 (1)	0.5	1.13	0.000	0 (0)	0.4	1.07	0.000	0 (0)
M6	0.000	15	0.6	0.063	0 (3)	1.2	1.60	0.000	0 (0)	1.0	1.40	0.000	0 (0)
K4	11.583	0	150.1	11.531	6 (6)	241.8	1.63	11.490	8 (0)	352.1	1.73	11.489	8 (0)
K5	6.036	1	46.9	6.040	3 (8)	71.2	1.70	6.004	3 (0)	66.5	1.70	6.004	3 (0)
K6	1.349	8	134.1	1.410	0 (3)	146.8	1.20	1.349	0 (0)	144.6	1.20	1.349	0 (0)
overall	2.4392	159	35.5	2.8896	9 (58)	53.6	3.21	2.4225	11 (0)	66.7	2.11	2.4223	11 (0)

Table 3 shows the results obtained by the DS and iDS algorithms for each instance group. A row "Tx" refers to all Type-T instances whose corresponding SCP instance is in family x from the OR-Library. The "Best Known" values come from the algorithms by Pereira and Averbakh (2013). All notations are the same as in Table 2.

For all 150 Type B and M instances, both iDS-H and iDS-B required less than 10 seconds on average. For all 75 Type K instances, both algorithms obtained solutions with objective function value equal to or better than the best known solution value, and they updated the best known solution values for 11 instances. Most best solutions obtained by the iDS algorithms were found in early iterations. For the 159 instances whose optimal values are known, both algorithms provided all optimal solutions.

For 58 instances for which the best solution values obtained by the DS algorithm were worse than the best known solution values, the iDS algorithms improved all the solutions in or after the second iteration. The detailed results in appendix C show that iDS-B is the winner for 26 out of 64 instances whose best solutions were obtained in or after the second iteration,while both iDS algorithms show the same performance for the other instances.

6.5 Min–Max Regret Generalized Assignment Problem

For the MMR-GAP, we used the benchmark instances by Wu et al. (2018a), available at `http://www.co.mi.i.nagoya-u.ac.jp/~yagiura/mmr-gap/`. The instances were generated from the classical GAP benchmark instances (see Chu and Beasley 1997, and Laguna et al. 1995) of the following types:

A: $\forall i, j$, a_{ij} is a random integer u.d. in $[5, 25]$, c_{ij} is a random integer u.d. in $[10, 50]$, and $b_i = 0.6(n/m)15 + 0.4\gamma$, where $\gamma = \max_{i \in I} \sum_{j \in J,\, \theta_j = i} a_{ij}$ and $\theta_j = \min\{i \mid c_{ij} \le c_{kj}, \forall k \in I\}$.

B: a_{ij} and c_{ij} as for Type A; b_i is set to 70% of the Type A value.

C: a_{ij} and c_{ij} as for Type A; $b_i = 0.8 \sum_{j=1}^{n} a_{ij}/m$.

E: $\forall i, j$, $a_{ij} = 1 - 10 \ln e_2$ (e_2 is a random number u.d. in $(0, 1]$), $c_{ij} = 1000/a_{ij} - 10e_3$ (e_3 is a random number u.d. in $[0, 1]$); $b_i = 0.8 \sum_{j=1}^{n} a_{ij}/m$.

The GAP instances were generated with a number of agents $m \in \{5, 10\}$ and a number of jobs $n \in \{40, 80\}$. The cost interval extremes c_{ij}^- and c_{ij}^+ were generated as random integers u.d. in $[(1-\delta)c_{ij}, c_{ij}]$ and $[c_{ij}, (1+\delta)c_{ij}]$, respectively, with $\delta \in \{0.10, 0.25, 0.50\}$. By generating 5 instances for each δ, we obtained 240 instances in total.

Table 4 shows the results for the MMR-GAP for each instance group. A row denoted "Txx" includes 30 Type-T instances whose number of agents is xx. The "Best Known" values are taken from Wu et al. (2018a). All notations are the same as in Table 2.

For most instances of Types A, B, and C (179 out of 180), both iDS-H and iDS-B obtained solutions whose objective values were equal to or better than the best known objective values. Both algorithms updated 17 best known objective values out of 180 instances. For the 60 instances of Type E, iDS-B updated 9 best known objective values, while iDS-H updated 8, but both obtained worse solutions for 19 instances. Note that in most of these 19 instances, the iDS

Table 4: MMR-GAP results

	Best Known		DS			iDS-H				iDS-B			
instances	%gap	#opt	time	%gap	#b(w)	time	iter	%gap	#b(w)	time	iter	%gap	#b(w)
A05	5.26	23	0.2	5.54	0 (3)	0.2	1.13	5.26	0 (0)	0.2	1.10	5.26	0 (0)
A10	9.29	14	0.5	9.57	0 (2)	15.6	11.50	9.24	4 (0)	23.1	11.53	9.21	5 (0)
B05	6.93	20	11.4	8.21	0 (3)	15.0	1.67	6.90	2 (0)	14.8	1.53	6.92	1 (0)
B10	8.51	15	514.5	10.54	0 (12)	559.2	2.47	8.41	5 (1)	552.9	2.47	8.41	5 (1)
C05	7.31	19	9.1	7.32	0 (1)	12.5	1.13	7.21	2 (0)	13.2	1.13	7.21	2 (0)
C10	8.02	17	380.1	9.83	0 (13)	481.1	6.10	7.92	4 (0)	478.2	5.97	7.92	4 (0)
E05	18.18	11	872.3	18.35	1 (7)	894.5	1.93	18.06	4 (3)	893.0	1.93	18.06	4 (3)
E10	24.50	3	2143.2	25.74	2 (17)	2294.8	1.70	25.44	4 (16)	2391.0	1.80	25.44	5 (16)
overall	11.00	122	491.4	11.89	3 (58)	534.1	3.45	11.06	25 (20)	545.8	3.43	11.05	26 (20)

algorithms failed to complete the first iteration (see appendix D, Table D.4), indicating that the performance of iDS algorithms depends on how fast the classical DS can exactly solve the D-MMR-BIP.

The iDS algorithms improved solutions for 38 out of 58 instances for which the best solution values obtained by the DS were worse than the best known solution values, and iDS-H (resp. iDS-B) further updated the best known objective values for 22 (resp. 23) more instances than the DS.

A similar observation can be made for the MMR-GAP, namely that most best solutions obtained by the iDS algorithms were found in early iterations. For the 122 instances whose optimal values are known, the iDS algorithms provided all optimal solutions. Thus, similarly to what happens for the MMR-MKP, MMR-KP, and MMR-SCP cases, the iDS algorithms provide a good heuristic for quickly obtaining optimal or near-optimal solutions for MMR-GAP, especially for Type A, B, and C instances.

By comparing iDS-B and iDS-H through the results in appendix D, focusing on the 60 instances whose best solutions were obtained in or after the second iteration, it turns out that iDS-B outperformed iDS-H for 9 instances, while iDS-H was superior for only 2 instances, suggesting that iDS-B is a better choice than iDS-H for the MMR-GAP.

7 Conclusions

We studied a robust version of the binary integer programming problem (BIP) called the min–max regret BIP. We proposed a new heuristic framework, the iterated dual substitution (iDS) algorithm. In the iDS approach, we considered two kinds of constraints to exclude from the search space already checked solutions: constraints pruning checked solutions based on Hamming distance and constraints using a lemma on dominance between solutionsbased on the best scenario. We tested the two resulting heuristics (iDS-H and iDS-B) on four representative problems under the min–max regret criterion: the knapsack problem, the multidimensional knapsack problem, the generalized assignment problem, and the set covering problem.

For the min–max regret multidimensional knapsack problem, we compared the proposed algorithms with two classical approaches: a branch-and-cut algorithm and a fixed-scenario algorithm. Computational results showed that the iDS algorithms outperformed in terms of solution quality the other algorithms for most tested instances. No clear winner emerged between iDS-H and iDS-B.

For the min–max regret knapsack problem, the min–max regret generalized assignment problem, and the min–max regret set covering problem, we compared the iDS algorithms with state-of-the-art results. For all these problems, the iDS algorithms successfully updated records for a number of benchmark instances, and iDS-B exhibited better performance than iDS-H.

Overall, the computational results for all tested problems suggest that the iDS approach is an effective heuristic for binary integer programming problems.

Acknowledgement

This work was supported by JSPS KAKENHI [Grant No.15K12460, 15H02969, 19K11843, 20H02388], and by the Air Force Office of Scientific Research under Award number FA8655-20-1-7012.

References

H. Aissi, C. Bazgan, and D. Vanderpooten. Min–max and min–max regret versions of combinatorial optimization problems: A survey. *European journal of operational research*, 197(2):427–438, 2009.

N. Ayvaz-Cavdaroglu, S. Kachani, and C. Maglaras. Revenue management with minimax regret negotiations. *Omega*, 63:11–22, 2016.

J. E. Beasley. Or-library: distributing test problems by electronic mail. *Journal of the operational research society*, 41(11):1069–1072, 1990.

V. Cacchiani, V. C. Hemmelmayr, and F. Tricoire. A set-covering based heuristic algorithm for the periodic vehicle routing problem. *Discrete Applied Mathematics*, 163:53–64, 2014.

A. Candia-Véjar, E. Álvarez-Miranda, and N. Maculan. Minmax regret combinatorial optimization problems: an algorithmic perspective. *RAIRO-Operations Research-Recherche Opérationnelle*, 45(2):101–129, 2011.

C.-F. Chien and J.-N. Zheng. Mini–max regret strategy for robust capacity expansion decisions in semiconductor manufacturing. *Journal of Intelligent Manufacturing*, 23(6):2151–2159, 2012.

P. C. Chu and J. E. Beasley. A genetic algorithm for the generalised assignment problem. *Computers & Operations Research*, 24 (1):17–23, 1997.

P. C. Chu and J. E. Beasley. A genetic algorithm for the multidimensional knapsack problem. *Journal of heuristics*, 4(1):63–86, 1998.

V. G. Deineko and G. J. Woeginger. Pinpointing the complexity of the interval min–max regret knapsack problem. *Discrete Optimization*, 7(4):191–196, 2010.

C. Dong, G. Huang, Y. Cai, and Y. Xu. An interval-parameter minimax regret programming approach for power management systems planning under uncertainty. *Applied Energy*, 88(8):2835–2845, 2011.

R. Z. Farahani, N. Asgari, N. Heidari, M. Hosseininia, and M. Goh. Covering problems in facility location: A review. *Computers & Industrial Engineering*, 62(1):368–407, 2012.

M. L. Fisher and R. Jaikumar. A generalized assignment heuristic for vehicle routing. *Networks*, 11(2):109–124, 1981.

F. Furini, M. Iori, S. Martello, and M. Yagiura. Heuristic and exact algorithms for the interval min–max regret knapsack problem. *INFORMS Journal on Computing*, 27(2):392–405, 2015.

M. R. Garey and D. S. Johnson. *Computers and intractability: A Guide to the Theory of NP-completeness*, volume 174. W.H. Freeman, San Francisco, 1979.

O. E. Karaşan, M. c. Pınar, and H. Yaman. The robust shortest path problem with interval data. Technical report, Bilkent University, Ankara, Turkey, 2001. available from the web site of Optimization Online (http://www.optimization-online.org/DB_HTML/2001/08/361.html, accesed on June 1, 2019).

R. M. Karp. Reducibility among combinatorial problems. In *Complexity of computer computations*, pages 85–103. Springer, 1972.

A. Kasperski. *Discrete Optimization with Interval Data*. Springer, Berlin, 2008.

A. Kasperski and P. Zieliński. An approximation algorithm for interval data minmax regret combinatorial optimization problems. *Information Processing Letters*, 97(5):177–180, 2006.

H. Kellerer, U. Pferschy, and D. Pisinger. *Knapsack problems*. Springer, Berlin, 2004.

P. Kouvelis and G. Yu. *Robust discrete optimization and its applications*, volume 14. Springer Science & Business Media, Dordrecht, 2013.

A. Kulik and H. Shachnai. There is no eptas for two-dimensional knapsack. *Information Processing Letters*, 110(16):707–710, 2010.

S. Laabadi, M. Naimi, H. El Amri, B. Achchab, et al. The 0/1 multidimensional knapsack problem and its variants: a survey of practical models and heuristic approaches. *American Journal of Operations Research*, 8(05):395, 2018.

M. Laguna, J. P. Kelly, J. González-Velarde, and F. Glover. Tabu search for the multilevel generalized assignment problem. *European Journal of Operational Research*, 82:176–189, 1995.

R. Loulou and A. Kanudia. Minimax regret strategies for greenhouse gas abatement: methodology and application. *Operations Research Letters*, 25(5):219–230, 1999.

S. Martello and P. Toth. *Knapsack problems: Algorithms and computer implementations*. John Wiley & Sons, Chichester, 1990.

R. Montemanni. A benders decomposition approach for the robust spanning tree problem with interval data. *European Journal of Operational Research*, 174(3):1479–1490, 2006.

R. Montemanni and L. M. Gambardella. A branch and bound algorithm for the robust spanning tree problem with interval data. *European Journal of Operational Research*, 161(3):771–779, 2005.

R. Montemanni, J. Barta, M. Mastrolilli, and L. M. Gambardella. The robust traveling salesman problem with interval data. *Transportation Science*, 41(3):366–381, 2007.

J. Pereira. The robust (minmax regret) assembly line worker assignment and balancing problem. *Computers & Operations Research*, 93:27–40, 2018.

J. Pereira and I. Averbakh. Exact and heuristic algorithms for the interval data robust assignment problem. *Computers & Operations Research*, 38(8):1153–1163, 2011.

J. Pereira and I. Averbakh. The robust set covering problem with interval data. *Annals of Operations Research*, 207(1):217–235, 2013.

H. Poorsepahy-Samian, R. Kerachian, and M. R. Nikoo. Water and pollution discharge permit allocation to agricultural zones: application of game theory and min-max regret analysis. *Water resources management*, 26(14):4241–4257, 2012.

J. Puchinger, G. R. Raidl, and U. Pferschy. The multidimensional knapsack problem: Structure and algorithms. *INFORMS Journal on Computing*, 22(2):250–265, 2010.

K. S. Ruland. A model for aeromedical routing and scheduling. *International Transactions in Operational Research*, 6(1):57–73, 1999.

W. Wu, Y. Hu, H. Hashimoto, T. Ando, T. Shiraki, and M. Yagiura. A column generation approach to the airline crew pairing problem to minimize the total person-days. *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, 10: JAMDSM0040, 2016a.

W. Wu, M. Iori, S. Martello, and M. Yagiura. An iterated dual substitution approach for the min-max regret multidimensional knapsack problem. In *2016 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pages 726–730, 2016b.

W. Wu, M. Iori, S. Martello, and M. Yagiura. Exact and heuristic algorithms for the interval min-max regret generalized assignment problem. *Computers & Industrial Engineering*, 125:98–110, 2018a.

W. Wu, M. Yagiura, and T. Ibaraki. Generalized assignment problem. In T. F. Gonzalez, editor, *Handbook of Approximation Algorithms and Metaheuristics: Methologies and Traditional Applications*, volume 1, chapter 40, pages 713–736. Chapman & Hall/CRC, 2 edition, 2018b.

P. Xidonas, G. Mavrotas, C. Hassapis, and C. Zopounidis. Robust multiobjective portfolio optimization: A minimax regret approach. *European Journal of Operational Research*, 262(1):299–305, 2017.

H. Yaman, O. E. Karaşan, and M. c. Pınar. The robust spanning tree problem with interval data. *Operations research letters*, 29(1): 31–40, 2001.

Appendices

A Detailed results for the Min–Max Regret Multidimensional Knapsack Problem

Tables A.1–A.3 show the results of the algorithms including the branch-and-cut algorithm (“B&C”), the fixed-scenario algorithm (“Fix”), the DS algorithm (“DS”), iDS using Hamming-distance constraints (“iDS-H”), and iDS using best-scenario constraints (“iDS-B”). An MMR-MKP instance denoted by “$wwxxxyy$-zz” is generated from the zzth MKP instance with $m = ww$, $n = xxx$, $100\delta = yy$. Concerning the branch-and-cut algorithm, each entry shows the best obtained solution value (“obj”), the CPU time in seconds required to obtain that solution (“time”), the lower-bound value obtained within the time limit (“LB”), and the optimality gap as a percentage (“%gap”), that is, the gap between the solution value and the best lower-bound values. Since the fixed-scenario approach under the median-value scenario is a 2-approximation algorithm, half of its solution value is a valid lower bound. The lower-bound value used to compute the percentage optimality gap was the best between such value and the lower bound produced by branch-and-cut. For iDS-H and iDS-B, we provide the number of iterations (“iter”) required to obtain the best solution. Values in bold signify that the corresponding algorithm(s) obtained the smallest percentage optimality gap among the tested algorithms.

B Detailed results for the Min–Max Regret Knapsack Problem

Tables B.1–B.9 show the results of the DS algorithm (“DS”) and iDS algorithms using Hamming-distance constraints (“iDS-H”) and best-scenario constraints (“iDS-B”) for the MMR-SCP for each instance type. The MMR-KP instances are denoted by “v-ww-xx-yy-zz,” where type $= f$, $n = ww$, $\bar{R}/1000 = xx$, $100\gamma = yy$, and $100\delta = zz$. Best known lower-bound values (“LB”) and solution values (“UB”) are the best results as obtained from three heuristic algorithms and three exact algorithms by Furini et al. (2015). Concerning the best solution obtained by DS and iDS for each instance, the table shows its objective value (“obj”), required CPU time in seconds (“time”), and optimality gap as a percentage (“%gap”), that is, the gap between the solution value and the best known lower-bound value (“LB”). For the best solution obtained by iDS, the table shows iteration index (“iter”) other than “obj,” “time” and “gap.” Values in “obj” columns marked by “↓” (or “↑”) indicate instances whose objective function obtained by the proposed algorithm were better (or worse) than the best known solution values in “UB” columns. Bold values in “obj” columns indicate better objective values obtained within the time limit (3600 seconds) between iDS-H and iDS-B. Bold values in “iter” columns signify that fewer iterations were needed when iDS-H and iDS-B obtained solutions with the same objective value in “obj” columns.

C Detailed results for the Min–Max Regret Set Covering Problem

Tables C.1–C.3 show the results of the DS algorithm (“DS”) and iDS algorithms using Hamming-distance constraints (“iDS-H”) and best-scenario constraints (“iDS-B”) for MMR-SCP for each instance type. An MMR-SCP instance denoted by “B$xyyzz$” indicates a Type-B instance whose corresponding SCP instance is the yyth instance in family x from the OR-Library with $zz = 100\delta$, while “Mxyy-z” (or “Kxyy-z”) stands for the zth Type-M (or Type-K) instance whose corresponding SCP instance is the yyth instance in family x. The best known lower-bound value (“LB”) and solution value (“UB”) are the results obtained from three heuristic algorithms and three exact algorithms by Pereira and Averbakh (2013). The notations “obj,” “time,” “ite,” “gap,” “LB,” and “↓,” as well as the bold values in columns “obj” and columns “ite,” are the same as in Tables B.1–B.9 for the MMR-KP.

D Detailed results for the Min–Max Regret Generalized Assignment Problem

Tables D.1–D.4 show the results of the DS algorithm (“DS”) and iDS algorithms using Hamming-distance constraints (“iDS-H”) and best-scenario constraints (“iDS-B”) for the MMR-GAP for each instance type. An MMR-GAP instance denoted by “$Txxyyzz$-i” indicates the ith instance of the (T, xx, yy, zz) combination, where type $= T$, $m = xx$, $n = yy$, and $100\delta = zz$. The best known lower-bound value (“LB”) and solution value (“UB”) are the results obtained from two heuristic algorithms and two exact algorithms by Wu et al. (2018a). The notations “obj,” “time,” “ite,” “gap,” “LB,” “↓,” and “↑,” as well as the bold values in columns “obj” and “ite,” are the same as those in Tables B.1–B.9 for the MMR-KP.

Table A.1: MMR-MKP results with $m = 5, n = 100$

	B&C				Fix			DS			iDS-H				iDS-B			
instance	obj	time	LB	%gap	obj	time	%gap	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
0510010-01	**662**	73.7	662	0.0	**662**	0.5	0.0	**662**	1.2	0.0	**662**	1.2	1	0.0	**662**	1.2	1	0.0
0510010-02	**663**	94.9	663	0.0	**663**	1.8	0.0	676	0.7	1.9	**663**	2.1	2	0.0	**663**	2.6	2	0.0
0510010-03	**426**	51.7	426	0.0	457	2.1	6.8	441	1.3	3.4	**426**	6.7	5	0.0	**426**	6.3	5	0.0
0510010-04	**809**	783.7	809	0.0	**809**	1.9	0.0	**809**	11.3	0.0	**809**	11.3	1	0.0	**809**	11.3	1	0.0
0510010-05	**737**	85.9	737	0.0	755	3.5	2.4	**737**	1.7	0.0	**737**	1.7	1	0.0	**737**	1.7	1	0.0
0510010-06	**482**	121.8	482	0.0	**482**	0.4	0.0	541	1.7	10.9	**482**	3.7	2	0.0	**482**	3.3	2	0.0
0510010-07	**603**	34.0	603	0.0	**603**	0.5	0.0	**603**	0.8	0.0	**603**	0.8	1	0.0	**603**	0.8	1	0.0
0510010-08	**406**	341.7	406	0.0	482	1.2	15.8	**406**	0.8	0.0	**406**	0.8	1	0.0	**406**	0.8	1	0.0
0510010-09	**469**	110.4	469	0.0	499	1.9	6.0	499	1.3	6.0	**469**	3.1	2	0.0	**469**	3.2	2	0.0
0510010-10	**587**	362.3	587	0.0	644	2.0	8.9	604	3.0	2.8	**587**	8.6	2	0.0	**587**	8.2	2	0.0
0510010-11	**422**	62.2	422	0.0	**422**	0.1	0.0	**422**	0.4	0.0	**422**	0.4	1	0.0	**422**	0.4	1	0.0
0510010-12	**667**	47.3	667	0.0	696	0.8	4.2	**667**	2.5	0.0	**667**	2.5	1	0.0	**667**	2.5	1	0.0
0510010-13	**618**	204.2	618	0.0	633	1.9	2.4	633	6.9	2.4	**618**	22.3	3	0.0	**618**	22.4	3	0.0
0510010-14	**442**	78.7	442	0.0	466	1.9	5.2	476	1.8	7.1	**442**	10.8	4	0.0	**442**	11.3	4	0.0
0510010-15	**409**	62.4	409	0.0	**409**	0.8	0.0	**409**	1.0	0.0	**409**	1.0	1	0.0	**409**	1.0	1	0.0
0510010-16	**507**	53.0	507	0.0	603	0.6	15.9	**507**	1.0	0.0	**507**	1.0	1	0.0	**507**	1.0	1	0.0
0510010-17	**413**	38.1	413	0.0	**413**	0.3	0.0	417	0.6	1.0	**413**	1.6	2	0.0	**413**	1.5	2	0.0
0510010-18	**579**	85.3	579	0.0	638	3.0	9.2	**579**	1.6	0.0	**579**	1.6	1	0.0	**579**	1.6	1	0.0
0510010-19	**646**	187.7	646	0.0	647	0.7	0.2	647	0.9	0.2	**646**	5.2	3	0.0	**646**	4.4	3	0.0
0510010-20	**525**	46.8	525	0.0	554	0.5	5.2	554	0.9	5.2	**525**	3.2	3	0.0	**525**	3.3	3	0.0
0510010-21	**326**	19.6	326	0.0	377	0.4	13.5	**326**	0.2	0.0	**326**	0.2	1	0.0	**326**	0.2	1	0.0
0510010-22	**413**	33.0	413	0.0	**413**	0.2	0.0	**413**	1.4	0.0	**413**	1.4	1	0.0	**413**	1.4	1	0.0
0510010-23	**488**	71.6	488	0.0	494	1.1	1.2	494	1.9	1.2	**488**	12.2	4	0.0	**488**	13.9	4	0.0
0510010-24	**417**	54.8	417	0.0	**417**	0.6	0.0	**417**	1.2	0.0	**417**	1.2	1	0.0	**417**	1.2	1	0.0
0510010-25	**363**	22.2	363	0.0	366	3.0	0.8	380	0.6	4.5	**363**	3.2	3	0.0	**363**	3.1	3	0.0
0510010-26	**497**	21.9	497	0.0	544	1.0	8.6	**497**	1.4	0.0	**497**	1.4	1	0.0	**497**	1.4	1	0.0
0510010-27	**350**	42.3	350	0.0	**350**	0.3	0.0	**350**	0.4	0.0	**350**	0.4	1	0.0	**350**	0.4	1	0.0
0510010-28	**363**	9.5	363	0.0	**363**	0.3	0.0	**363**	0.4	0.0	**363**	0.4	1	0.0	**363**	0.4	1	0.0
0510010-29	**381**	32.0	381	0.0	**381**	0.2	0.0	**381**	0.4	0.0	**381**	0.4	1	0.0	**381**	0.4	1	0.0
0510010-30	**340**	49.7	340	0.0	383	1.6	11.2	**340**	1.0	0.0	**340**	1.0	1	0.0	**340**	1.0	1	0.0
0510020-01	**1963**	764.6	1963	0.0	1968	1.7	0.3	1984	7.5	1.1	**1963**	45.3	4	0.0	**1963**	44.0	4	0.0
0510020-02	**1434**	465.5	1434	0.0	**1434**	1.0	0.0	1457	2.3	1.6	**1434**	8.6	3	0.0	**1434**	8.7	3	0.0
0510020-03	**1363**	81.7	1363	0.0	1474	2.4	7.5	**1363**	1.5	0.0	**1363**	1.5	1	0.0	**1363**	1.5	1	0.0
0510020-04	**2190**	3418.2	1987	9.3	2240	0.4	11.3	**2190**	11.1	9.3	**2190**	11.1	1	9.3	**2190**	11.1	1	9.3
0510020-05	**1732**	1906.1	1732	0.0	**1732**	2.6	0.0	**1732**	3.0	0.0	**1732**	3.0	1	0.0	**1732**	3.0	1	0.0
0510020-06	**1364**	178.8	1364	0.0	**1364**	0.5	0.0	**1364**	0.4	0.0	**1364**	0.4	1	0.0	**1364**	0.4	1	0.0
0510020-07	**1679**	882.1	1679	0.0	1742	1.8	3.6	**1679**	5.6	0.0	**1679**	5.6	1	0.0	**1679**	5.6	1	0.0
0510020-08	**1577**	274.9	1577	0.0	**1577**	0.5	0.0	**1577**	2.2	0.0	**1577**	2.2	1	0.0	**1577**	2.2	1	0.0
0510020-09	**1393**	457.5	1393	0.0	1424	0.5	2.2	1424	1.3	2.2	**1393**	2.6	2	0.0	**1393**	2.7	2	0.0
0510020-10	**1707**	940.0	1707	0.0	**1707**	0.5	0.0	1717	2.9	0.6	**1707**	5.8	2	0.0	**1707**	5.7	2	0.0
0510020-11	1615	2785.1	1301	19.4	1602	2.2	18.8	**1574**	8.1	17.3	**1574**	8.1	1	17.3	**1574**	8.1	1	17.3
0510020-12	**1224**	214.7	1224	0.0	**1224**	0.3	0.0	**1224**	1.8	0.0	**1224**	1.8	1	0.0	**1224**	1.8	1	0.0
0510020-13	**1767**	3286.1	1493	15.5	**1767**	0.5	15.5	**1767**	7.8	15.5	**1767**	7.8	1	15.5	**1767**	7.8	1	15.5
0510020-14	**1109**	484.6	1109	0.0	1123	1.2	1.2	**1109**	1.5	0.0	**1109**	1.5	1	0.0	**1109**	1.5	1	0.0
0510020-15	**1272**	990.9	1272	0.0	**1272**	0.8	0.0	**1272**	2.2	0.0	**1272**	2.2	1	0.0	**1272**	2.2	1	0.0
0510020-16	**1229**	191.3	1229	0.0	1346	0.7	8.7	**1229**	1.4	0.0	**1229**	1.4	1	0.0	**1229**	1.4	1	0.0
0510020-17	**1420**	100.8	1420	0.0	1446	0.2	1.8	**1420**	0.8	0.0	**1420**	0.8	1	0.0	**1420**	0.8	1	0.0
0510020-18	**1743**	791.5	1743	0.0	1761	1.1	1.0	**1743**	5.7	0.0	**1743**	5.7	1	0.0	**1743**	5.7	1	0.0
0510020-19	2175	299.1	1626	25.2	2135	3.6	23.8	**2122**	21.7	23.4	**2122**	21.7	1	23.4	**2122**	21.7	1	23.4
0510020-20	**1592**	746.0	1592	0.0	**1592**	0.4	0.0	**1592**	1.9	0.0	**1592**	1.9	1	0.0	**1592**	1.9	1	0.0
0510020-21	**1055**	137.0	1055	0.0	**1055**	0.3	0.0	**1055**	0.8	0.0	**1055**	0.8	1	0.0	**1055**	0.8	1	0.0
0510020-22	**809**	235.5	809	0.0	**809**	0.4	0.0	**809**	0.5	0.0	**809**	0.5	1	0.0	**809**	0.5	1	0.0
0510020-23	**1143**	245.5	1143	0.0	**1143**	0.4	0.0	**1143**	2.3	0.0	**1143**	2.3	1	0.0	**1143**	2.3	1	0.0
0510020-24	**1011**	287.9	1011	0.0	**1011**	0.1	0.0	**1011**	0.2	0.0	**1011**	0.2	1	0.0	**1011**	0.2	1	0.0
0510020-25	**1232**	79.2	1232	0.0	1354	0.8	9.0	**1232**	1.0	0.0	**1232**	1.0	1	0.0	**1232**	1.0	1	0.0
0510020-26	**1428**	343.6	1428	0.0	**1428**	0.4	0.0	1433	3.6	0.3	**1428**	10.2	2	0.0	**1428**	8.7	2	0.0
0510020-27	**1122**	1232.4	1122	0.0	**1122**	0.6	0.0	**1122**	4.8	0.0	**1122**	4.8	1	0.0	**1122**	4.8	1	0.0
0510020-28	**1003**	132.6	1003	0.0	**1003**	0.3	0.0	**1003**	1.0	0.0	**1003**	1.0	1	0.0	**1003**	1.0	1	0.0
0510020-29	**1214**	479.6	1214	0.0	**1214**	0.9	0.0	**1214**	1.1	0.0	**1214**	1.1	1	0.0	**1214**	1.1	1	0.0
0510020-30	**1101**	226.3	1101	0.0	**1101**	0.4	0.0	**1101**	1.0	0.0	**1101**	1.0	1	0.0	**1101**	1.0	1	0.0
0510030-01	2774	2476.9	2134	23.1	2672	0.6	20.1	2672	9.8	20.1	**2666**	21.9	2	20.0	**2666**	22.2	2	20.0
0510030-02	**3187**	784.4	2752	13.6	3327	2.4	17.3	**3187**	25.1	13.6	**3187**	25.1	1	13.6	**3187**	25.1	1	13.6
0510030-03	**2912**	1075.5	2912	0.0	**2912**	0.4	0.0	2915	2.8	0.1	**2912**	6.2	2	0.0	**2912**	5.8	2	0.0
0510030-04	4122	2400.7	3111	24.5	**3927**	0.9	20.8	3979	41.7	21.8	**3927**	89.1	2	20.8	**3927**	85.1	2	20.8
0510030-05	**3046**	2193.2	2659	12.7	3109	2.0	14.5	**3046**	10.1	12.7	**3046**	10.1	1	12.7	**3046**	10.1	1	12.7
0510030-06	**3122**	2503.7	2915	6.6	**3122**	0.3	6.6	**3122**	10.6	6.6	**3122**	10.6	1	6.6	**3122**	10.6	1	6.6
0510030-07	**3445**	1112.6	3244	5.8	3500	0.6	7.3	3500	5.0	7.3	**3445**	13.4	2	5.8	**3445**	13.0	2	5.8
0510030-08	**2556**	3161.6	2375	7.1	2644	1.5	10.2	**2556**	2.6	7.1	**2556**	2.6	1	7.1	**2556**	2.6	1	7.1
0510030-09	**2350**	1641.1	2350	0.0	2423	0.8	3.0	**2350**	1.6	0.0	**2350**	1.6	1	0.0	**2350**	1.6	1	0.0
0510030-10	3172	1849.5	2467	22.2	**3022**	1.3	18.4	**3022**	14.9	18.4	**3022**	14.9	1	18.4	**3022**	14.9	1	18.4
0510030-11	3591	2424.5	2778	22.6	3508	0.6	20.8	3521	15.6	21.1	**3488**	31.5	2	20.4	**3488**	31.6	2	20.4
0510030-12	3226	2388.1	2461	23.7	**3055**	0.3	19.4	**3055**	14.0	19.4	**3055**	14.0	1	19.4	**3055**	14.0	1	19.4
0510030-13	3814	112.2	2767	27.5	3697	1.6	25.2	3677	94.8	24.7	**3660**	220.8	2	24.4	**3660**	217.1	2	24.4
0510030-14	3873	775.4	3054	21.1	**3771**	0.7	19.0	**3771**	13.1	19.0	**3771**	13.1	1	19.0	**3771**	13.1	1	19.0
0510030-15	3096	199.2	2373	23.4	**3001**	1.1	20.9	**3001**	7.6	20.9	**3001**	7.6	1	20.9	**3001**	7.6	1	20.9
0510030-16	**2996**	283.9	2523	15.8	3058	3.0	17.5	2997	14.5	15.8	**2996**	31.0	2	15.8	**2996**	27.8	2	15.8
0510030-17	2801	137.7	2347	16.2	2878	1.4	18.5	2801	6.8	16.2	**2795**	17.8	2	16.0	**2795**	18.0	2	16.0
0510030-18	3294	3476.8	2688	18.4	3320	0.5	19.0	**3249**	10.8	17.3	**3249**	10.8	1	17.3	**3249**	10.8	1	17.3
0510030-19	3170	1328.2	2406	24.1	**3018**	0.2	20.3	**3018**	11.6	20.3	**3018**	11.6	1	20.3	**3018**	11.6	1	20.3
0510030-20	**2933**	112.3	2821	3.8	**2933**	0.2	3.8	**2933**	2.0	3.8	**2933**	2.0	1	3.8	**2933**	2.0	1	3.8
0510030-21	**2450**	1193.4	2334	4.7	**2450**	0.5	4.7	**2450**	9.4	4.7	**2450**	9.4	1	4.7	**2450**	9.4	1	4.7
0510030-22	**2453**	1543.9	2453	0.0	**2453**	0.3	0.0	**2453**	6.7	0.0	**2453**	6.7	1	0.0	**2453**	6.7	1	0.0
0510030-23	**2257**	1082.6	2140	5.2	**2257**	0.3	5.2	**2257**	6.9	5.2	**2257**	6.9	1	5.2	**2257**	6.9	1	5.2
0510030-24	**2206**	2703.0	2080	5.7	2265	0.5	8.2	2208	4.4	5.8	**2206**	10.5	2	5.7	**2206**	10.3	2	5.7
0510030-25	**2357**	70.8	2357	0.0	**2357**	0.6	0.0	**2357**	5.8	0.0	**2357**	5.8	1	0.0	**2357**	5.8	1	0.0
0510030-26	**2168**	1205.5	2168	0.0	**2168**	0.3	0.0	**2168**	3.7	0.0	**2168**	3.7	1	0.0	**2168**	3.7	1	0.0
0510030-27	3202	1013.5	2685	16.1	**3103**	0.4	13.5	3142	26.6	14.5	**3103**	52.5	2	13.5	**3103**	71.2	2	13.5
0510030-28	**2902**	1462.3	2691	7.3	3006	0.3	10.5	2943	6.7	8.6	**2902**	78.7	7	7.3	**2902**	81.1	7	7.3
0510030-29	**2200**	1290.5	2200	0.0	2260	0.7	2.7	**2200**	6.4	0.0	**2200**	6.4	1	0.0	**2200**	6.4	1	0.0
0510030-30	**2194**	466.5	2194	0.0	2394	1.5	8.4	**2194**	2.4	0.0	**2194**	2.4	1	0.0	**2194**	2.4	1	0.0

Table A.2: MMR-MKP results with $m = 10, n = 100$

	B&C				Fix			DS			iDS-H				iDS-B			
instance	obj	time	LB	%gap	obj	time	%gap	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
1010010-01	**798**	872.3	798	0.0	847	15.1	5.8	**798**	21.1	0.0	**798**	21.1	1	0.0	**798**	21.1	1	0.0
1010010-02	**474**	1170.3	474	0.0	**474**	3.7	0.0	**474**	3.5	0.0	**474**	3.5	1	0.0	**474**	3.5	1	0.0
1010010-03	**759**	3600.0	492	35.2	789	9.4	37.6	**759**	15.3	35.2	**759**	15.3	1	35.2	**759**	15.3	1	35.2
1010010-04	1195	2648.4	650	45.6	1148	47.4	43.4	**1093**	69.6	40.5	**1093**	69.6	1	40.5	**1093**	69.6	1	40.5
1010010-05	**753**	3217.5	753	0.0	881	11.7	14.5	805	20.9	6.5	**753**	44.7	2	0.0	**753**	47.9	2	0.0
1010010-06	**807**	1633.0	451	44.1	**807**	10.9	44.1	**807**	33.7	44.1	**807**	33.7	1	44.1	**807**	33.7	1	44.1
1010010-07	**640**	515.2	640	0.0	699	7.5	8.4	666	10.5	3.9	**640**	117.0	8	0.0	**640**	113.4	8	0.0
1010010-08	**684**	1085.3	684	0.0	731	6.7	6.4	**684**	6.8	0.0	**684**	6.8	1	0.0	**684**	6.8	1	0.0
1010010-09	**611**	570.1	611	0.0	613	3.2	0.3	613	6.2	0.3	**611**	15.6	2	0.0	**611**	13.5	2	0.0
1010010-10	**593**	1057.0	593	0.0	**593**	3.5	0.0	**593**	5.9	0.0	**593**	5.9	1	0.0	**593**	5.9	1	0.0
1010010-11	**552**	1912.1	552	0.0	**552**	3.3	0.0	**552**	4.1	0.0	**552**	4.1	1	0.0	**552**	4.1	1	0.0
1010010-12	**661**	3115.1	661	0.0	**661**	1.6	0.0	**661**	3.0	0.0	**661**	3.0	1	0.0	**661**	3.0	1	0.0
1010010-13	**523**	1485.7	523	0.0	**523**	6.0	0.0	**523**	2.7	0.0	**523**	2.7	1	0.0	**523**	2.7	1	0.0
1010010-14	936	3445.7	305	61.0	**729**	9.8	49.9	**729**	10.6	58.2	**729**	10.6	1	49.9	**729**	10.6	1	49.9
1010010-15	**724**	3071.9	724	0.0	**724**	7.9	0.0	**724**	9.7	0.0	**724**	9.7	1	0.0	**724**	9.7	1	0.0
1010010-16	**721**	2072.1	721	0.0	774	10.5	6.8	722	11.2	0.1	**721**	23.4	2	0.0	**721**	22.5	2	0.0
1010010-17	694	3546.0	112	47.6	728	25.8	50.0	**608**	17.6	81.6	**608**	17.6	1	40.1	**608**	17.6	1	40.1
1010010-18	**447**	2487.3	447	0.0	**447**	9.1	0.0	**447**	5.6	0.0	**447**	5.6	1	0.0	**447**	5.6	1	0.0
1010010-19	**553**	531.1	553	0.0	651	8.8	15.1	**553**	10.0	0.0	**553**	10.0	1	0.0	**553**	10.0	1	0.0
1010010-20	768	3600.0	59	60.2	611	30.5	49.9	577	24.4	89.8	**568**	477.7	10	46.1	**568**	443.2	10	46.1
1010010-21	**365**	78.9	365	0.0	386	1.0	5.4	386	1.1	5.4	**365**	3.7	2	0.0	**365**	4.5	2	0.0
1010010-22	**569**	1834.2	569	0.0	572	5.4	0.5	572	13.2	0.5	**569**	79.5	4	0.0	**569**	77.5	4	0.0
1010010-23	**371**	714.5	371	0.0	**371**	3.7	0.0	**371**	3.0	0.0	**371**	3.0	1	0.0	**371**	3.0	1	0.0
1010010-24	**449**	527.1	449	0.0	**449**	2.7	0.0	**449**	3.9	0.0	**449**	3.9	1	0.0	**449**	3.9	1	0.0
1010010-25	**280**	183.7	280	0.0	337	1.6	16.9	**280**	1.1	0.0	**280**	1.1	1	0.0	**280**	1.1	1	0.0
1010010-26	**466**	1497.9	466	0.0	552	9.2	15.6	**466**	6.8	0.0	**466**	6.8	1	0.0	**466**	6.8	1	0.0
1010010-27	**512**	949.7	512	0.0	554	6.9	7.6	**512**	7.4	0.0	**512**	7.4	1	0.0	**512**	7.4	1	0.0
1010010-28	**335**	156.9	335	0.0	**335**	0.7	0.0	**335**	1.0	0.0	**335**	1.0	1	0.0	**335**	1.0	1	0.0
1010010-29	**396**	394.4	396	0.0	**396**	1.8	0.0	**396**	0.9	0.0	**396**	0.9	1	0.0	**396**	0.9	1	0.0
1010010-30	**310**	122.5	310	0.0	**310**	1.2	0.0	**310**	2.0	0.0	**310**	2.0	1	0.0	**310**	2.0	1	0.0
1010020-01	1990	2696.6	1230	38.2	1877	7.0	34.5	1883	36.4	34.7	**1827**	76.2	2	32.7	**1827**	76.3	2	32.7
1010020-02	1798	3240.1	1126	37.4	1746	4.9	35.5	**1666**	14.3	32.4	**1666**	14.3	1	32.4	**1666**	14.3	1	32.4
1010020-03	**1671**	1065.1	1671	0.0	**1671**	7.9	0.0	**1671**	5.6	0.0	**1671**	5.6	1	0.0	**1671**	5.6	1	0.0
1010020-04	2052	1063.6	1198	41.6	2074	31.3	42.2	2082	92.8	42.5	**2023**	461.8	4	40.8	**2023**	465.2	4	40.8
1010020-05	2213	1174.4	1209	45.4	**1870**	0.7	35.3	**1870**	4.9	35.3	**1870**	4.9	1	35.3	**1870**	4.9	1	35.3
1010020-06	2514	3600.0	1361	45.9	2282	7.5	40.4	**2232**	44.6	39.0	**2232**	44.6	1	39.0	**2232**	44.6	1	39.0
1010020-07	2019	2043.1	1481	26.6	2019	3.8	26.6	2019	7.9	26.6	**1969**	22.9	2	24.8	**1969**	21.9	2	24.8
1010020-08	2243	2544.7	1490	33.6	**1997**	5.0	25.4	**1997**	18.4	25.4	**1997**	18.4	1	25.4	**1997**	18.4	1	25.4
1010020-09	**1652**	2510.8	1652	0.0	1687	1.2	2.1	1687	2.8	2.1	**1652**	10.4	2	0.0	**1652**	10.1	2	0.0
1010020-10	2424	3276.4	1279	47.2	2318	8.8	44.8	**2230**	27.8	42.6	**2230**	27.8	1	42.6	**2230**	27.8	1	42.6
1010020-11	2042	2556.9	810	54.3	1866	23.3	50.0	1795	74.9	54.9	**1753**	450.4	5	46.8	**1753**	437.2	5	46.8
1010020-12	1916	3088.7	804	56.3	1675	7.8	50.0	1643	63.3	51.1	**1636**	198.4	3	48.8	**1636**	203.0	3	48.8
1010020-13	1519	3171.3	565	60.2	**1210**	2.3	50.0	**1210**	2.9	53.3	**1210**	2.9	1	50.0	**1210**	2.9	1	50.0
1010020-14	2659	3098.9	976	59.6	**2148**	21.6	50.0	2152	71.8	54.6	**2148**	287.7	3	50.0	**2148**	261.4	3	50.0
1010020-15	1902	1672.3	1041	45.3	1793	7.9	41.9	**1656**	11.0	37.1	**1656**	11.0	1	37.1	**1656**	11.0	1	37.1
1010020-16	1892	3510.8	759	54.4	1725	9.9	50.0	**1595**	4.8	52.4	**1595**	4.8	1	45.9	**1595**	4.8	1	45.9
1010020-17	2583	5901.5	1063	58.8	2005	2.2	47.0	**1954**	6.7	45.6	**1954**	6.7	1	45.6	**1954**	6.7	1	45.6
1010020-18	1866	1591.0	1133	39.3	1826	6.5	38.0	**1718**	6.6	34.1	**1718**	6.6	1	34.1	**1718**	6.6	1	34.1
1010020-19	1519	4972.5	564	56.4	1324	6.9	50.0	**1293**	7.8	56.4	**1293**	7.8	1	48.8	**1293**	7.8	1	48.8
1010020-20	1628	5876.1	562	57.0	1399	19.7	50.0	**1389**	10.2	59.5	**1389**	10.2	1	49.6	**1389**	10.2	1	49.6
1010020-21	**942**	2277.9	789	16.2	949	2.6	16.9	**942**	3.6	16.2	**942**	3.6	1	16.2	**942**	3.6	1	16.2
1010020-22	1625	2786.9	861	47.0	1609	23.6	46.5	1540	77.6	44.1	**1528**	169.8	2	43.7	**1528**	169.6	2	43.7
1010020-23	1045	2962.4	779	25.5	1042	3.0	25.2	**988**	3.9	21.2	**988**	3.9	1	21.2	**988**	3.9	1	21.2
1010020-24	1563	3515.6	1081	30.8	**1489**	1.0	27.4	**1489**	12.9	27.4	**1489**	12.9	1	27.4	**1489**	12.9	1	27.4
1010020-25	**901**	522.9	901	0.0	**901**	0.7	0.0	**901**	1.1	0.0	**901**	1.1	1	0.0	**901**	1.1	1	0.0
1010020-26	1408	3237.9	1039	26.2	**1352**	4.9	23.2	1354	4.7	23.3	**1352**	11.3	2	23.2	**1352**	10.5	2	23.2
1010020-27	1376	1086.5	813	40.9	1235	4.0	34.2	**1227**	8.6	33.7	**1227**	8.6	1	33.7	**1227**	8.6	1	33.7
1010020-28	**924**	1032.3	924	0.0	963	1.8	4.0	**924**	2.0	0.0	**924**	2.0	1	0.0	**924**	2.0	1	0.0
1010020-29	**1396**	1786.9	1396	0.0	**1396**	1.6	0.0	**1396**	2.9	0.0	**1396**	2.9	1	0.0	**1396**	2.9	1	0.0
1010020-30	**981**	406.6	981	0.0	988	1.6	0.7	988	0.9	0.7	**981**	2.2	2	0.0	**981**	2.1	2	0.0
1010030-01	3218	2668.4	2137	33.6	**3027**	0.8	29.4	**3027**	16.1	29.4	**3027**	16.1	1	29.4	**3027**	16.1	1	29.4
1010030-02	3437	3582.6	2071	39.7	3306	7.6	37.4	**3219**	26.7	35.7	**3219**	26.7	1	35.7	**3219**	26.7	1	35.7
1010030-03	**3200**	660.3	2267	29.2	**3200**	5.1	29.2	**3200**	8.5	29.2	**3200**	8.5	1	29.2	**3200**	8.5	1	29.2
1010030-04	4224	340.3	2623	37.9	4069	24.0	35.5	4068	348.7	35.5	**4057**	2325.0	5	35.3	**4057**	2534.7	5	35.3
1010030-05	3590	1434.4	2107	41.3	3530	7.0	40.3	**3336**	22.7	36.8	**3336**	22.7	1	36.8	**3336**	22.7	1	36.8
1010030-06	4557	408.8	2667	41.5	4415	16.8	39.6	4328	286.0	38.4	**4284**	883.6	2	37.7	**4284**	864.1	2	37.7
1010030-07	2790	2027.7	1660	40.5	2845	4.6	41.7	**2628**	6.5	36.8	**2628**	6.5	1	36.8	**2628**	6.5	1	36.8
1010030-08	**3183**	2124.8	2263	28.9	3268	7.9	30.8	3229	18.3	29.9	**3183**	39.0	2	28.9	**3183**	37.2	2	28.9
1010030-09	3255	2539.4	2174	33.2	**2937**	0.4	26.0	**2937**	8.3	26.0	**2937**	8.3	1	26.0	**2937**	8.3	1	26.0
1010030-10	3773	848.0	2253	40.3	**3601**	2.8	37.4	**3601**	33.5	37.4	**3601**	33.5	1	37.4	**3601**	33.5	1	37.4
1010030-11	3375	1677.0	1861	44.9	3326	8.6	44.0	3278	130.4	43.2	**3263**	427.1	3	43.0	**3263**	424.0	3	43.0
1010030-12	3965	973.2	2152	45.7	3622	9.2	40.6	3618	137.7	40.5	**3608**	297.6	2	40.4	**3608**	300.7	2	40.4
1010030-13	3584	2495.2	1914	46.6	3623	5.3	47.2	**3479**	173.1	45.0	**3479**	172.9	1	45.0	**3479**	173.1	1	45.0
1010030-14	4587	475.4	2077	54.7	**3937**	32.8	47.2	**3937**	825.0	47.2	**3937**	823.1	1	47.2	**3937**	825.0	1	47.2
1010030-15	3650	1745.7	1987	45.6	**3337**	2.7	40.5	**3337**	127.5	40.5	**3337**	127.7	1	40.5	**3337**	127.5	1	40.5
1010030-16	3386	1511.6	2054	39.3	3364	7.8	38.9	3294	47.8	37.6	**3273**	111.7	2	37.2	**3273**	109.2	2	37.2
1010030-17	3956	1830.9	1913	51.6	**3384**	18.6	43.5	**3384**	137.2	43.5	**3384**	137.7	1	43.5	**3384**	137.2	1	43.5
1010030-18	3600	729.0	1228	65.0	**2522**	11.2	50.0	**2522**	17.3	51.3	**2522**	17.5	1	50.0	**2522**	17.3	1	50.0
1010030-19	3931	3600.0	1746	55.6	3472	8.2	49.7	3471	207.1	49.7	**3461**	902.6	4	49.6	**3461**	910.2	4	49.6
1010030-20	3422	2848.7	1611	52.9	3190	11.0	49.5	**3142**	93.1	48.7	**3142**	93.8	1	48.7	**3142**	93.1	1	48.7
1010030-21	2327	1393.7	1575	32.3	2264	1.5	30.4	2264	10.8	30.4	**2251**	48.5	3	30.0	**2251**	48.9	3	30.0
1010030-22	**3200**	3600.0	2178	31.9	**3200**	7.0	31.9	3243	565.3	32.8	3214	3600.0	6	32.2	3214	3600.0	6	32.2
1010030-23	2610	2990.7	1703	34.8	2473	3.3	31.1	**2380**	10.7	28.4	**2380**	10.7	1	28.4	**2380**	10.7	1	28.4
1010030-24	2638	618.9	1790	32.1	**2385**	8.1	24.9	**2385**	23.4	24.9	**2385**	23.4	1	24.9	**2385**	23.4	1	24.9
1010030-25	**2059**	3069.8	2059	0.0	**2059**	0.5	0.0	**2059**	2.8	0.0	**2059**	2.8	1	0.0	**2059**	2.8	1	0.0
1010030-26	2871	1071.0	1687	41.2	2677	8.3	37.0	2644	26.0	36.2	**2589**	55.5	2	34.8	**2589**	57.9	2	34.8
1010030-27	2970	937.0	1636	44.9	2673	5.7	38.8	**2659**	85.6	38.5	**2659**	86.2	1	38.5	**2659**	85.6	1	38.5
1010030-28	2420	3131.4	1596	34.0	2266	2.0	29.6	**2099**	1.2	24.0	**2099**	1.2	1	24.0	**2099**	1.2	1	24.0
1010030-29	**2092**	3600.0	1855	11.3	**2092**	1.1	11.3	**2092**	3.7	11.3	**2092**	3.7	1	11.3	**2092**	3.7	1	11.3
1010030-30	**2134**	1998.1	2134	0.0	**2134**	0.4	0.0	**2134**	1.5	0.0	**2134**	1.5	1	0.0	**2134**	1.5	1	0.0

Table A.3: MMR-MKP results with $m = 5, n = 250$

	B&C				Fix			DS			iDS-H				iDS-B			
instance	obj	time	LB	%gap	obj	time	%gap	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
0525010-01	1176	3284.8	481	58.6	973	9.4	49.9	976	71.8	50.7	**940**	145.4	2	48.2	**940**	144.1	2	48.2
0525010-02	1378	3600.0	501	63.6	993	25.4	49.5	993	59.1	49.5	**978**	123.6	2	48.8	**978**	129.8	2	48.8
0525010-03	1288	1073.5	353	62.7	959	8.3	49.9	959	110.9	63.2	**956**	530.2	4	49.8	**956**	529.3	4	49.8
0525010-04	1943	574.9	600	66.0	**1321**	121.2	50.0	**1321**	575.0	54.6	**1321**	575.0	1	50.0	**1321**	575.0	1	50.0
0525010-05	1203	3133.8	311	61.0	937	17.0	49.9	**919**	65.7	66.2	**919**	65.7	1	49.0	**919**	65.7	1	49.0
0525010-06	1554	466.3	645	58.5	**1209**	18.1	46.7	**1209**	202.1	46.7	**1209**	202.1	1	46.7	**1209**	202.1	1	46.7
0525010-07	1472	1959.1	696	52.7	1263	63.8	44.9	**1218**	298.1	42.9	**1218**	298.1	1	42.9	**1218**	298.1	1	42.9
0525010-08	1580	3221.4	534	66.0	1073	10.5	50.0	1073	88.3	50.2	**1067**	302.9	3	49.7	**1067**	282.4	3	49.7
0525010-09	1480	2924.7	661	55.3	**1154**	7.6	42.7	**1154**	97.8	42.7	**1154**	97.8	1	42.7	**1154**	97.8	1	42.7
0525010-10	1342	3600.0	635	52.7	**1086**	11.2	41.5	**1086**	84.9	41.5	**1086**	84.9	1	41.5	**1086**	84.9	1	41.5
0525010-11	1750	3600.0	644	63.2	1240	42.7	48.1	**1233**	347.3	47.8	**1233**	347.3	1	47.8	**1233**	347.3	1	47.8
0525010-12	1046	2894.1	573	45.2	**947**	7.3	39.5	**947**	79.7	39.5	**947**	79.7	1	39.5	**947**	79.7	1	39.5
0525010-13	1854	3600.0	504	72.8	**1004**	9.2	49.8	**1004**	110.9	49.8	**1004**	110.9	1	49.8	**1004**	110.9	1	49.8
0525010-14	1658	3600.0	535	67.7	**1021**	7.9	47.6	**1021**	83.8	47.6	**1021**	83.8	1	47.6	**1021**	83.8	1	47.6
0525010-15	2845	1111.0	531	79.6	1162	19.3	50.0	**1135**	671.2	53.2	**1135**	671.2	1	48.8	**1135**	671.2	1	48.8
0525010-16	1653	1389.0	581	64.9	1072	50.8	45.8	1072	481.9	45.8	**1061**	983.6	2	45.2	**1061**	1003.1	2	45.2
0525010-17	1397	3285.8	573	59.0	1103	17.1	48.1	**1099**	155.0	47.9	**1099**	155.0	1	47.9	**1099**	155.0	1	47.9
0525010-18	2622	942.6	648	71.5	1496	82.3	50.0	**1476**	1028.9	56.1	**1476**	1028.9	1	49.3	**1476**	1028.9	1	49.3
0525010-19	2819	3600.0	311	85.3	**828**	4.3	50.0	**828**	33.6	62.4	**828**	33.6	1	50.0	**828**	33.6	1	50.0
0525010-20	1355	3600.0	448	66.3	911	15.7	49.9	921	84.2	51.4	**904**	311.0	3	49.6	**904**	302.8	3	49.6
0525010-21	1407	3600.0	487	64.5	999	79.1	49.9	**968**	315.2	49.7	**968**	315.2	1	48.3	**968**	315.2	1	48.3
0525010-22	579	3455.3	376	35.1	**543**	1.2	30.8	**543**	10.5	30.8	**543**	10.5	1	30.8	**543**	10.5	1	30.8
0525010-23	1169	2469.9	511	56.3	**919**	36.6	44.4	**919**	226.3	44.4	**919**	226.3	1	44.4	**919**	226.3	1	44.4
0525010-24	908	3169.6	345	61.1	**706**	18.5	50.0	708	41.6	51.3	**706**	208.8	3	50.0	**706**	197.2	3	50.0
0525010-25	1123	5206.2	387	65.5	**749**	13.3	48.3	**749**	27.4	48.3	**749**	27.4	1	48.3	**749**	27.4	1	48.3
0525010-26	959	2131.1	437	54.4	**712**	3.3	38.6	**712**	14.2	38.6	**712**	14.2	1	38.6	**712**	14.2	1	38.6
0525010-27	667	3167.1	242	60.0	533	5.2	49.9	**532**	34.4	54.5	**532**	34.4	1	49.8	**532**	34.4	1	49.8
0525010-28	790	3103.4	550	30.4	**766**	3.4	28.2	790	25.4	30.4	**766**	62.7	2	28.2	**766**	58.0	2	28.2
0525010-29	771	3050.9	442	42.7	703	7.4	37.1	685	9.0	35.5	**669**	24.7	2	33.9	**669**	23.8	2	33.9
0525010-30	**639**	1752.0	543	15.0	696	7.5	22.0	**639**	12.4	15.0	**639**	12.4	1	15.0	**639**	12.4	1	15.0
0525020-01	4758	2158.3	2057	56.8	**3404**	3.6	39.6	**3404**	818.9	39.6	**3404**	818.9	1	39.6	**3404**	818.9	1	39.6
0525020-02	4977	1013.2	1728	64.9	**3492**	7.7	50.0	**3492**	1509.9	50.5	**3492**	1509.9	1	50.0	**3492**	1509.9	1	50.0
0525020-03	5207	1686.4	2445	53.0	4253	17.6	42.5	**4205**	2243.5	41.9	**4205**	2243.5	1	41.9	**4205**	2243.5	1	41.9
0525020-04	6016	2642.2	3034	49.6	**5025**	50.5	39.6	5043	3600.0	39.8	5043	3600.0	1	39.8	5043	3600.0	1	39.8
0525020-05	6697	468.7	1720	74.3	3317	7.0	48.1	3317	709.1	48.1	**3302**	1258.0	2	47.9	**3302**	1181.1	2	47.9
0525020-06	5263	2611.0	2316	56.0	4020	83.7	42.4	**3962**	3600.0	41.5	**3962**	3600.0	1	41.5	**3962**	3600.0	1	41.5
0525020-07	4810	842.4	1605	65.5	3318	12.0	50.0	**3289**	1888.1	51.2	**3289**	1888.1	1	49.6	**3289**	1888.1	1	49.6
0525020-08	4773	2593.0	2237	53.1	3837	59.4	41.7	**3813**	3600.0	41.3	**3813**	3600.0	1	41.3	**3813**	3600.0	1	41.3
0525020-09	4079	3600.0	1808	55.7	3324	6.7	45.6	**3288**	637.7	45.0	**3288**	637.7	1	45.0	**3288**	637.7	1	45.0
0525020-10	3805	3600.0	1770	53.5	**3104**	9.1	43.0	**3104**	1457.5	43.0	**3104**	1457.5	1	43.0	**3104**	1457.5	1	43.0
0525020-11	7673	1684.9	2155	71.3	**4408**	20.3	50.0	4467	3600.0	51.8	4467	3600.0	1	50.7	4467	3600.0	1	50.7
0525020-12	4635	387.8	2045	55.9	3477	14.5	41.2	**3427**	2635.2	40.3	**3427**	2635.2	1	40.3	**3427**	2635.2	1	40.3
0525020-13	4583	3600.0	1937	57.7	**3323**	5.6	41.7	3346	3600.0	42.1	3346	3600.0	1	42.1	3346	3600.0	1	42.1
0525020-14	4671	3379.2	2406	48.5	4134	16.1	41.8	**4093**	3600.0	41.2	**4093**	3600.0	1	41.2	**4093**	3600.0	1	41.2
0525020-15	5615	3600.0	2386	57.5	4390	42.0	45.6	**4365**	3600.0	45.3	**4365**	3600.0	1	45.3	**4365**	3600.0	1	45.3
0525020-16	4905	3310.5	2220	54.7	3816	7.5	41.8	**3774**	3600.0	41.2	**3774**	3600.0	1	41.2	**3774**	3600.0	1	41.2
0525020-17	6666	615.2	1651	75.2	**3161**	16.0	47.8	**3161**	778.0	47.8	**3161**	778.0	1	47.8	**3161**	778.0	1	47.8
0525020-18	5722	3600.0	2398	58.1	4061	18.5	41.0	**4034**	3600.0	40.6	**4034**	3600.0	1	40.6	**4034**	3600.0	1	40.6
0525020-19	7501	449.4	1770	76.4	3371	4.2	47.5	**3362**	2077.0	47.4	**3362**	2077.0	1	47.4	**3362**	2077.0	1	47.4
0525020-20	6733	817.7	1661	75.3	**3315**	2.1	49.9	**3315**	2196.7	49.9	**3315**	2196.7	1	49.9	**3315**	2196.7	1	49.9
0525020-21	4950	2804.3	2088	57.8	**3516**	21.1	40.6	3543	3600.0	41.1	3543	3600.0	1	41.1	3543	3600.0	1	41.1
0525020-22	3559	2303.4	1366	61.6	2435	13.2	43.9	**2411**	179.0	43.3	**2411**	179.0	1	43.3	**2411**	179.0	1	43.3
0525020-23	3593	2388.0	1708	52.5	2786	7.3	38.7	**2776**	1204.1	38.5	**2776**	1204.1	1	38.5	**2776**	1204.1	1	38.5
0525020-24	3556	3390.4	1420	60.1	**2405**	7.5	41.0	**2405**	1607.5	41.0	**2405**	1607.5	1	41.0	**2405**	1607.5	1	41.0
0525020-25	4129	3001.2	1795	56.5	**3023**	17.2	40.6	**3023**	3600.0	40.6	**3023**	3600.0	1	40.6	**3023**	3600.0	1	40.6
0525020-26	4075	3046.0	1578	61.3	2785	50.4	43.3	**2772**	3600.0	43.1	**2772**	3600.0	1	43.1	**2772**	3600.0	1	43.1
0525020-27	3005	3057.1	1328	55.8	2254	2.6	41.1	2252	338.5	41.0	**2247**	1622.3	5	40.9	**2247**	1589.9	5	40.9
0525020-28	2999	3541.9	1594	46.8	2550	11.3	37.5	**2514**	2314.7	36.6	**2514**	2314.7	1	36.6	**2514**	2314.7	1	36.6
0525020-29	2959	1209.4	1230	58.4	**2190**	1.0	43.8	**2190**	35.3	43.8	**2190**	35.3	1	43.8	**2190**	35.3	1	43.8
0525020-30	3547	3981.8	1292	63.6	**2207**	3.4	41.5	**2207**	96.7	41.5	**2207**	96.7	1	41.5	**2207**	96.7	1	41.5
0525030-01	9578	2959.9	4616	51.8	7806	39.4	40.9	**7793**	3600.0	40.8	**7793**	3600.0	1	40.8	**7793**	3600.0	1	40.8
0525030-02	9933	3469.1	4083	58.9	**7011**	3.0	41.8	**7011**	2047.9	41.8	**7011**	2047.9	1	41.8	**7011**	2047.9	1	41.8
0525030-03	10528	3003.9	4956	52.9	8202	19.0	39.6	**8176**	3600.0	39.4	**8176**	3600.0	1	39.4	**8176**	3600.0	1	39.4
0525030-04	8941	2415.6	4486	49.8	**7601**	13.5	41.0	7610	3600.0	41.1	7610	3600.0	1	41.1	7610	3600.0	1	41.1
0525030-05	8061	2872.0	4077	49.4	6678	14.6	38.9	**6668**	3600.0	38.9	**6668**	3600.0	1	38.9	**6668**	3600.0	1	38.9
0525030-06	9247	3874.1	4283	53.7	**7055**	7.9	39.3	**7055**	3600.0	39.3	**7055**	3600.0	1	39.3	**7055**	3600.0	1	39.3
0525030-07	9285	2391.7	3319	64.3	6754	16.7	50.9	**6706**	3179.0	50.5	**6706**	3179.0	1	50.5	**6706**	3179.0	1	50.5
0525030-08	10985	2492.1	5174	52.9	**8441**	6.3	38.7	**8441**	3600.0	38.7	**8441**	3600.0	1	38.7	**8441**	3600.0	1	38.7
0525030-09	8741	847.9	4568	47.7	**7172**	2.8	36.3	**7172**	2562.5	36.3	**7172**	2562.5	1	36.3	**7172**	2562.5	1	36.3
0525030-10	8110	2533.9	4172	48.6	6642	11.2	37.2	**6641**	3600.0	37.2	**6641**	3600.0	1	37.2	**6641**	3600.0	1	37.2
0525030-11	12736	2330.6	4754	62.7	**8338**	46.4	43.0	8341	3600.0	43.0	8341	3600.0	1	43.0	8341	3600.0	1	43.0
0525030-12	10717	3391.2	4760	55.6	7947	6.5	40.1	**7885**	3600.0	39.6	**7885**	3600.0	1	39.6	**7885**	3600.0	1	39.6
0525030-13	8479	4933.2	3926	53.7	6888	32.7	43.0	**6820**	3600.0	42.4	**6820**	3600.0	1	42.4	**6820**	3600.0	1	42.4
0525030-14	10423	3844.6	4590	56.0	7616	33.9	39.7	**7544**	3600.0	39.2	**7544**	3600.0	1	39.2	**7544**	3600.0	1	39.2
0525030-15	18185	1740.9	4542	75.0	**9067**	24.7	49.9	9093	3600.0	50.0	9093	3600.0	1	50.0	9093	3600.0	1	50.0
0525030-16	12987	838.5	4120	68.3	7148	12.5	42.4	**7114**	3600.0	42.1	**7114**	3600.0	1	42.1	**7114**	3600.0	1	42.1
0525030-17	9429	2795.1	4584	51.4	**7471**	7.7	38.6	**7471**	3600.0	38.6	**7471**	3600.0	1	38.6	**7471**	3600.0	1	38.6
0525030-18	9985	2914.9	5068	49.2	**8644**	28.7	41.4	**8644**	3600.0	41.4	**8644**	3600.0	1	41.4	**8644**	3600.0	1	41.4
0525030-19	10166	2852.4	5293	47.9	8891	6.6	40.5	**8851**	3600.0	40.2	**8851**	3600.0	1	40.2	**8851**	3600.0	1	40.2
0525030-20	11712	3187.5	5262	55.1	**8774**	79.3	40.0	8798	3600.0	40.2	8798	3600.0	1	40.2	8798	3600.0	1	40.2
0525030-21	6545	2734.0	3239	50.5	5418	8.8	40.2	**5369**	679.3	39.7	**5369**	679.3	1	39.7	**5369**	679.3	1	39.7
0525030-22	6471	2002.1	3179	50.9	**5204**	4.5	38.9	**5204**	3600.0	38.9	**5204**	3600.0	1	38.9	**5204**	3600.0	1	38.9
0525030-23	8702	2665.0	3833	56.0	6359	22.7	39.7	**6312**	3600.0	39.3	**6312**	3600.0	1	39.3	**6312**	3600.0	1	39.3
0525030-24	6274	3412.4	3330	46.9	**5246**	10.9	36.5	**5246**	3600.0	36.5	**5246**	3600.0	1	36.5	**5246**	3600.0	1	36.5
0525030-25	8802	1918.8	3815	56.7	**6070**	15.8	37.1	6080	3600.0	37.3	6080	3600.0	1	37.3	6080	3600.0	1	37.3
0525030-26	6501	1458.4	3618	44.3	5728	12.2	36.8	**5726**	3600.0	36.8	**5726**	3600.0	1	36.8	**5726**	3600.0	1	36.8
0525030-27	6429	1461.1	2993	53.4	4813	15.9	37.8	**4776**	3600.0	37.3	**4776**	3600.0	1	37.3	**4776**	3600.0	1	37.3
0525030-28	5973	2471.4	3322	44.4	**5130**	12.6	35.2	5134	3600.0	35.3	5134	3600.0	1	35.3	5134	3600.0	1	35.3
0525030-29	6226	1665.4	3359	46.0	**5273**	8.7	36.3	**5273**	2141.8	36.3	**5273**	2141.8	1	36.3	**5273**	2141.8	1	36.3
0525030-30	6611	1034.3	3658	44.7	**5747**	4.4	36.3	5768	3600.0	36.6	5768	3600.0	1	36.6	5768	3600.0	1	36.6

Table B.1: MMR-KP results for type-1 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
1-50-01-45-10	15	15	15	0.01	0.0	15	0.01	1	0.0	15	0.01	1	0.0
1-50-01-50-10	0	0	0	0.01	0.0	0	0.01	1	0.0	0	0.01	1	0.0
1-50-01-55-10	114	114	114	0.01	0.0	114	0.01	1	0.0	114	0.01	1	0.0
1-60-01-45-10	91	91	91	0.01	0.0	91	0.01	1	0.0	91	0.01	1	0.0
1-60-01-50-10	111	111	111	0.01	0.0	111	0.01	1	0.0	111	0.01	1	0.0
1-60-01-55-10	64	64	64	0.01	0.0	64	0.01	1	0.0	64	0.01	1	0.0
1-70-01-45-10	32	32	32	0.01	0.0	32	0.01	1	0.0	32	0.01	1	0.0
1-70-01-50-10	44	44	44	0.01	0.0	44	0.01	1	0.0	44	0.01	1	0.0
1-70-01-55-10	118	118	127 ↑	0.01	7.1	118	0.08	5	0.0	118	0.04	**4**	0.0
1-50-01-45-20	198	198	198	0.01	0.0	198	0.01	1	0.0	198	0.01	1	0.0
1-50-01-50-20	129	129	129	0.01	0.0	129	0.01	1	0.0	129	0.01	1	0.0
1-50-01-55-20	239	239	242 ↑	0.01	1.2	239	0.03	3	0.0	239	0.03	3	0.0
1-60-01-45-20	273	273	273	0.01	0.0	273	0.01	1	0.0	273	0.01	1	0.0
1-60-01-50-20	211	211	211	0.01	0.0	211	0.01	1	0.0	211	0.01	1	0.0
1-60-01-55-20	244	244	244	0.01	0.0	244	0.01	1	0.0	244	0.01	1	0.0
1-70-01-45-20	109	109	109	0.01	0.0	109	0.01	1	0.0	109	0.01	1	0.0
1-70-01-50-20	239	239	239	0.01	0.0	239	0.01	1	0.0	239	0.01	1	0.0
1-70-01-55-20	297	297	297	0.02	0.0	297	0.02	1	0.0	297	0.02	1	0.0
1-50-01-45-30	445	445	445	0.02	0.0	445	0.02	1	0.0	445	0.02	1	0.0
1-50-01-50-30	442	442	442	0.01	0.0	442	0.01	1	0.0	442	0.01	1	0.0
1-50-01-55-30	605	605	606 ↑	0.01	0.2	605	0.03	2	0.0	605	0.03	2	0.0
1-60-01-45-30	596	596	596	0.02	0.0	596	0.02	1	0.0	596	0.02	1	0.0
1-60-01-50-30	604	604	604	0.02	0.0	604	0.02	1	0.0	604	0.02	1	0.0
1-60-01-55-30	668	668	668	0.02	0.0	668	0.02	1	0.0	668	0.02	1	0.0
1-70-01-45-30	434	434	434	0.01	0.0	434	0.01	1	0.0	434	0.01	1	0.0
1-70-01-50-30	606	606	606	0.01	0.0	606	0.01	1	0.0	606	0.01	1	0.0
1-70-01-55-30	770	770	770	0.03	0.0	770	0.03	1	0.0	770	0.03	1	0.0
1-50-10-45-10	225	225	225	0.01	0.0	225	0.01	1	0.0	225	0.01	1	0.0
1-50-10-50-10	0	0	0	0.01	0.0	0	0.01	1	0.0	0	0.01	1	0.0
1-50-10-55-10	0	0	0	0.01	0.0	0	0.01	1	0.0	0	0.01	1	0.0
1-60-10-45-10	159	159	159	0.01	0.0	159	0.01	1	0.0	159	0.01	1	0.0
1-60-10-50-10	0	0	0	0.01	0.0	0	0.01	1	0.0	0	0.01	1	0.0
1-60-10-55-10	216	216	216	0.01	0.0	216	0.01	1	0.0	216	0.01	1	0.0
1-70-10-45-10	54	54	54	0.01	0.0	54	0.01	1	0.0	54	0.01	1	0.0
1-70-10-50-10	112	112	112	0.01	0.0	112	0.01	1	0.0	112	0.01	1	0.0
1-70-10-55-10	275	275	275	0.01	0.0	275	0.01	1	0.0	275	0.01	1	0.0
1-50-10-45-20	1902	1902	1902	0.01	0.0	1902	0.01	1	0.0	1902	0.01	1	0.0
1-50-10-50-20	1392	1392	1392	0.01	0.0	1392	0.01	1	0.0	1392	0.01	1	0.0
1-50-10-55-20	0	0	0	0.01	0.0	0	0.01	1	0.0	0	0.01	1	0.0
1-60-10-45-20	2106	2106	2106	0.01	0.0	2106	0.01	1	0.0	2106	0.01	1	0.0
1-60-10-50-20	1165	1165	1165	0.01	0.0	1165	0.01	1	0.0	1165	0.01	1	0.0
1-60-10-55-20	324	324	324	0.01	0.0	324	0.01	1	0.0	324	0.01	1	0.0
1-70-10-45-20	2561	2561	2561	0.01	0.0	2561	0.01	1	0.0	2561	0.01	1	0.0
1-70-10-50-20	147	147	147	0.01	0.0	147	0.01	1	0.0	147	0.01	1	0.0
1-70-10-55-20	954	954	954	0.01	0.0	954	0.01	1	0.0	954	0.01	1	0.0
1-50-10-45-30	2778	2778	2778	0.01	0.0	2778	0.01	1	0.0	2778	0.01	1	0.0
1-50-10-50-30	3284	3284	3284	0.01	0.0	3284	0.01	1	0.0	3284	0.01	1	0.0
1-50-10-55-30	1043	1043	1043	0.01	0.0	1043	0.01	1	0.0	1043	0.01	1	0.0
1-60-10-45-30	3393	3393	3393	0.01	0.0	3393	0.01	1	0.0	3393	0.01	1	0.0
1-60-10-50-30	2636	2636	2636	0.01	0.0	2636	0.01	1	0.0	2636	0.01	1	0.0
1-60-10-55-30	2090	2090	2116 ↑	0.01	1.2	2090	0.01	2	0.0	2090	0.01	2	0.0
1-70-10-45-30	4103	4103	4103	0.01	0.0	4103	0.01	1	0.0	4103	0.01	1	0.0
1-70-10-50-30	2632	2632	2754 ↑	0.01	4.4	2632	0.01	2	0.0	2632	0.01	2	0.0
1-70-10-55-30	2176	2176	2176	0.01	0.0	2176	0.01	1	0.0	2176	0.01	1	0.0

Table B.2: MMR-KP results for type-2 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
2-50-01-45-10	231	231	231	0.01	0.0	231	0.01	1	0.0	231	0.01	1	0.0
2-50-01-50-10	164	164	164	0.01	0.0	164	0.01	1	0.0	164	0.01	1	0.0
2-50-01-55-10	194	194	194	0.01	0.0	194	0.01	1	0.0	194	0.01	1	0.0
2-60-01-45-10	272	272	278 ↑	0.03	2.2	272	0.05	2	0.0	272	0.05	2	0.0
2-60-01-50-10	240	240	243 ↑	0.02	1.2	240	0.05	2	0.0	240	0.04	2	0.0
2-60-01-55-10	237	237	237	0.01	0.0	237	0.01	1	0.0	237	0.01	1	0.0
2-70-01-45-10	272	272	274 ↑	0.02	0.7	272	0.08	4	0.0	272	0.04	**2**	0.0
2-70-01-50-10	242	242	242	0.01	0.0	242	0.01	1	0.0	242	0.01	1	0.0
2-70-01-55-10	289	289	289	0.04	0.0	289	0.04	1	0.0	289	0.04	1	0.0
2-50-01-45-20	886	886	886	0.04	0.0	886	0.04	1	0.0	886	0.04	1	0.0
2-50-01-50-20	870	870	870	0.03	0.0	870	0.03	1	0.0	870	0.03	1	0.0
2-50-01-55-20	880	880	880	0.06	0.0	880	0.06	1	0.0	880	0.06	1	0.0
2-60-01-45-20	1151	1151	1151	0.08	0.0	1151	0.08	1	0.0	1151	0.08	1	0.0
2-60-01-50-20	1128	1128	1128	0.06	0.0	1128	0.06	1	0.0	1128	0.06	1	0.0
2-60-01-55-20	1131	1131	1131	0.10	0.0	1131	0.10	1	0.0	1131	0.10	1	0.0
2-70-01-45-20	1272	1272	1272	0.09	0.0	1272	0.09	1	0.0	1272	0.09	1	0.0
2-70-01-50-20	1274	1274	1274	0.10	0.0	1274	0.10	1	0.0	1274	0.10	1	0.0
2-70-01-55-20	1243	1243	1243	0.09	0.0	1243	0.09	1	0.0	1243	0.09	1	0.0
2-50-01-45-30	2050	2050	2050	0.06	0.0	2050	0.06	1	0.0	2050	0.06	1	0.0
2-50-01-50-30	2090	2090	2090	0.07	0.0	2090	0.07	1	0.0	2090	0.07	1	0.0
2-50-01-55-30	2041	2041	2041	0.06	0.0	2041	0.06	1	0.0	2041	0.06	1	0.0
2-60-01-45-30	2185	2467	2467	0.15	11.4	2467	0.15	1	11.4	2467	0.15	1	11.4
2-60-01-50-30	2222	2505	2505	0.27	11.3	2505	0.27	1	11.3	2505	0.27	1	11.3
2-60-01-55-30	2241	2270	2425 ↑	0.07	7.6	2425 ↑	0.07	1	7.6	2425 ↑	0.07	1	7.6
2-70-01-45-30	2311	2801	2801	0.11	17.5	2801	0.11	1	17.5	2801	0.11	1	17.5
2-70-01-50-30	2350	2814	2827 ↑	0.13	16.9	2814	0.22	2	16.5	2814	0.22	2	16.5
2-70-01-55-30	2359	2698	2698	0.08	12.6	2698	0.08	1	12.6	2698	0.08	1	12.6
2-50-10-45-10	2678	2678	2678	0.02	0.0	2678	0.02	1	0.0	2678	0.02	1	0.0
2-50-10-50-10	2716	2716	2716	0.03	0.0	2716	0.03	1	0.0	2716	0.03	1	0.0
2-50-10-55-10	2444	2444	2444	0.02	0.0	2444	0.02	1	0.0	2444	0.02	1	0.0
2-60-10-45-10	2694	2694	2694	0.04	0.0	2694	0.04	1	0.0	2694	0.04	1	0.0
2-60-10-50-10	2604	2604	2652 ↑	0.04	1.8	2604	0.08	2	0.0	2604	0.08	2	0.0
2-60-10-55-10	2125	2125	2125	0.03	0.0	2125	0.03	1	0.0	2125	0.03	1	0.0
2-70-10-45-10	3984	3984	4062 ↑	0.07	1.9	3984	0.13	2	0.0	3984	0.11	2	0.0
2-70-10-50-10	3558	3558	3558	0.04	0.0	3558	0.04	1	0.0	3558	0.04	1	0.0
2-70-10-55-10	2699	2699	2699	0.01	0.0	2699	0.01	1	0.0	2699	0.01	1	0.0
2-50-10-45-20	8684	8684	8684	0.07	0.0	8684	0.07	1	0.0	8684	0.07	1	0.0
2-50-10-50-20	8070	8070	8155 ↑	0.05	1.0	8070	0.16	3	0.0	8070	0.17	3	0.0
2-50-10-55-20	7172	7172	7172	0.02	0.0	7172	0.02	1	0.0	7172	0.02	1	0.0
2-60-10-45-20	8921	8921	8958 ↑	0.05	0.4	8921	0.11	2	0.0	8921	0.11	2	0.0
2-60-10-50-20	8228	8228	8228	0.05	0.0	8228	0.05	1	0.0	8228	0.05	1	0.0
2-60-10-55-20	7668	7668	7668	0.05	0.0	7668	0.05	1	0.0	7668	0.05	1	0.0
2-70-10-45-20	12238	12238	12238	0.06	0.0	12238	0.06	1	0.0	12238	0.06	1	0.0
2-70-10-50-20	11941	11941	11941	0.05	0.0	11941	0.05	1	0.0	11941	0.05	1	0.0
2-70-10-55-20	11428	11428	11428	0.08	0.0	11428	0.08	1	0.0	11428	0.08	1	0.0
2-50-10-45-30	9924	9924	9924	0.02	0.0	9924	0.02	1	0.0	9924	0.02	1	0.0
2-50-10-50-30	9242	9242	9242	0.02	0.0	9242	0.02	1	0.0	9242	0.02	1	0.0
2-50-10-55-30	8763	8763	8763	0.02	0.0	8763	0.02	1	0.0	8763	0.02	1	0.0
2-60-10-45-30	11486	11486	11486	0.03	0.0	11486	0.03	1	0.0	11486	0.03	1	0.0
2-60-10-50-30	11324	11324	11324	0.05	0.0	11324	0.05	1	0.0	11324	0.05	1	0.0
2-60-10-55-30	10490	10490	10496 ↑	0.03	0.1	10490	0.06	2	0.0	10490	0.05	2	0.0
2-70-10-45-30	17726	17726	17726	0.06	0.0	17726	0.06	1	0.0	17726	0.06	1	0.0
2-70-10-50-30	17300	17300	17300	0.06	0.0	17300	0.06	1	0.0	17300	0.06	1	0.0
2-70-10-55-30	16239	16239	16239	0.05	0.0	16239	0.05	1	0.0	16239	0.05	1	0.0

Table B.3: MMR-KP results for type-3 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
3-50-01-45-10	726	726	726	0.07	0.0	726	0.07	1	0.0	726	0.07	1	0.0
3-50-01-50-10	719	719	719	0.06	0.0	719	0.06	1	0.0	719	0.06	1	0.0
3-50-01-55-10	714	714	714	0.05	0.0	714	0.05	1	0.0	714	0.05	1	0.0
3-60-01-45-10	868	868	868	0.07	0.0	868	0.07	1	0.0	868	0.07	1	0.0
3-60-01-50-10	809	909	909	0.24	11.0	909	0.24	1	11.0	909	0.24	1	11.0
3-60-01-55-10	846	909	909	0.24	6.9	909	0.24	1	6.9	909	0.24	1	6.9
3-70-01-45-10	850	977	977	0.36	13.0	977	0.36	1	13.0	977	0.36	1	13.0
3-70-01-50-10	867	1008	1008	0.27	14.0	1008	0.27	1	14.0	1008	0.27	1	14.0
3-70-01-55-10	877	1027	1027	0.25	14.6	1027	0.25	1	14.6	1027	0.25	1	14.6
3-50-01-45-20	1793	2112	2124 ↑	0.29	15.6	2112	2.09	5	15.1	2112	2.15	5	15.1
3-50-01-50-20	1782	2135	2146 ↑	0.31	17.0	2135	0.67	2	16.5	2135	0.65	2	16.5
3-50-01-55-20	1696	2098	2098	0.23	19.2	2098	0.23	1	19.2	2098	0.23	1	19.2
3-60-01-45-20	1961	2517	2525 ↑	0.52	22.3	2517	1.83	3	22.1	2517	1.52	3	22.1
3-60-01-50-20	1932	2530	2530	0.40	23.6	2530	0.40	1	23.6	2530	0.40	1	23.6
3-60-01-55-20	1919	2441	2441	0.11	21.4	2441	0.11	1	21.4	2441	0.11	1	21.4
3-70-01-45-20	2073	2760	2760	0.70	24.9	2756 ↓	1.89	2	24.8	2756 ↓	1.48	2	24.8
3-70-01-50-20	2081	2786	2786	0.47	25.3	2786	0.47	1	25.3	2786	0.47	1	25.3
3-70-01-55-20	2044	2715	2715	0.38	24.7	2715	0.38	1	24.7	2715	0.38	1	24.7
3-50-01-45-30	2455	2640	2651 ↑	0.19	7.4	2640	0.46	2	7.0	2640	0.44	2	7.0
3-50-01-50-30	2480	2866	2866	0.50	13.5	2866	0.50	1	13.5	2866	0.50	1	13.5
3-50-01-55-30	2465	2974	2974	0.43	17.1	2974	0.43	1	17.1	2974	0.43	1	17.1
3-60-01-45-30	2836	3361	3361	0.33	15.6	3361	0.33	1	15.6	3361	0.33	1	15.6
3-60-01-50-30	2871	3668	3670 ↑	0.75	21.8	3668	2.40	2	21.7	3668	1.97	2	21.7
3-60-01-55-30	2851	3790	3790	0.93	24.8	3790	0.93	1	24.8	3790	0.93	1	24.8
3-70-01-45-30	3002	3628	3632 ↑	0.31	17.3	3628	1.07	3	17.3	3628	0.96	3	17.3
3-70-01-50-30	3013	3958	3958	0.89	23.9	3958	0.89	1	23.9	3958	0.89	1	23.9
3-70-01-55-30	3023	4111	4111	1.17	26.5	4111	1.17	1	26.5	4111	1.17	1	26.5
3-50-10-45-10	3656	3656	3656	0.04	0.0	3656	0.04	1	0.0	3656	0.04	1	0.0
3-50-10-50-10	4167	4167	4167	0.03	0.0	4167	0.03	1	0.0	4167	0.03	1	0.0
3-50-10-55-10	4132	4132	4132	0.04	0.0	4132	0.04	1	0.0	4132	0.04	1	0.0
3-60-10-45-10	4500	4500	4500	0.04	0.0	4500	0.04	1	0.0	4500	0.04	1	0.0
3-60-10-50-10	5172	5172	5172	0.06	0.0	5172	0.06	1	0.0	5172	0.06	1	0.0
3-60-10-55-10	5568	5568	5568	0.16	0.0	5568	0.16	1	0.0	5568	0.16	1	0.0
3-70-10-45-10	5816	5816	5816	0.06	0.0	5816	0.06	1	0.0	5816	0.06	1	0.0
3-70-10-50-10	6128	6128	6128	0.06	0.0	6128	0.06	1	0.0	6128	0.06	1	0.0
3-70-10-55-10	6623	6623	6623	0.07	0.0	6623	0.07	1	0.0	6623	0.07	1	0.0
3-50-10-45-20	11130	11130	11130	0.04	0.0	11130	0.04	1	0.0	11130	0.04	1	0.0
3-50-10-50-20	12264	12264	12264	0.08	0.0	12264	0.08	1	0.0	12264	0.08	1	0.0
3-50-10-55-20	12427	12427	12427	0.12	0.0	12427	0.12	1	0.0	12427	0.12	1	0.0
3-60-10-45-20	13482	13482	13482	0.07	0.0	13482	0.07	1	0.0	13482	0.07	1	0.0
3-60-10-50-20	13484	14384	14384	0.18	6.3	14384	0.18	1	6.3	14384	0.18	1	6.3
3-60-10-55-20	13602	14975	14975	0.27	9.2	14975	0.27	1	9.2	14975	0.27	1	9.2
3-70-10-45-20	14344	16027	16027	0.09	10.5	16027	0.09	1	10.5	16027	0.09	1	10.5
3-70-10-50-20	14723	17352	17352	0.23	15.2	17352	0.23	1	15.2	17352	0.23	1	15.2
3-70-10-55-20	14742	18054	18054	0.55	18.3	18054	0.55	1	18.3	18054	0.55	1	18.3
3-50-10-45-30	17258	17258	17271 ↑	0.06	0.1	17258	0.13	2	0.0	17258	0.14	2	0.0
3-50-10-50-30	17716	18954	18954	0.13	6.5	18954	0.13	1	6.5	18954	0.13	1	6.5
3-50-10-55-30	18053	19774	19798 ↑	0.13	8.8	19774	0.27	2	8.7	19774	0.28	2	8.7
3-60-10-45-30	19160	22057	22057	0.15	13.1	22057	0.15	1	13.1	22057	0.15	1	13.1
3-60-10-50-30	19366	23229	23229	0.28	16.6	23229	0.28	1	16.6	23229	0.28	1	16.6
3-60-10-55-30	19512	24219	24219	0.31	19.4	24219	0.31	1	19.4	24219	0.31	1	19.4
3-70-10-45-30	20524	25142	25142	0.29	18.4	25142	0.29	1	18.4	25142	0.29	1	18.4
3-70-10-50-30	20672	26380	26380	0.33	21.6	26380	0.33	1	21.6	26380	0.33	1	21.6
3-70-10-55-30	20595	26878	26931 ↑	0.29	23.5	26878	0.65	2	23.4	26878	0.66	2	23.4

Table B.4: MMR-KP results for type-4 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
4-50-01-45-10	582	582	582	0.09	0.0	582	0.09	1	0.0	582	0.09	1	0.0
4-50-01-50-10	547	547	547	0.06	0.0	547	0.06	1	0.0	547	0.06	1	0.0
4-50-01-55-10	490	490	490	0.07	0.0	490	0.07	1	0.0	490	0.07	1	0.0
4-60-01-45-10	657	710	710	0.10	7.5	710	0.10	1	7.5	710	0.10	1	7.5
4-60-01-50-10	592	668	668	0.12	11.4	668	0.12	1	11.4	668	0.12	1	11.4
4-60-01-55-10	603	603	603	0.08	0.0	603	0.08	1	0.0	603	0.08	1	0.0
4-70-01-45-10	656	754	754	0.15	13.0	754	0.15	1	13.0	754	0.15	1	13.0
4-70-01-50-10	622	708	709 ↑	0.25	12.3	708	0.61	2	12.1	708	0.53	2	12.1
4-70-01-55-10	634	637	639 ↑	0.13	0.8	637	0.26	2	0.5	637	0.25	2	0.5
4-50-01-45-20	1343	1532	1532	0.33	12.3	1532	0.33	1	12.3	1532	0.33	1	12.3
4-50-01-50-20	1345	1528	1528	0.25	12.0	1528	0.25	1	12.0	1528	0.25	1	12.0
4-50-01-55-20	1324	1456	1474 ↑	0.21	10.2	1456	0.57	2	9.1	1456	0.46	2	9.1
4-60-01-45-20	1536	1913	1913	0.47	19.7	1913	0.47	1	19.7	1913	0.47	1	19.7
4-60-01-50-20	1517	1920	1920	0.37	21.0	1920	0.37	1	21.0	1920	0.37	1	21.0
4-60-01-55-20	1495	1859	1859	0.45	19.6	1859	0.45	1	19.6	1859	0.45	1	19.6
4-70-01-45-20	1674	2173	2173	0.54	23.0	2173	0.54	1	23.0	2173	0.54	1	23.0
4-70-01-50-20	1650	2156	2156	0.44	23.5	2156	0.44	1	23.5	2156	0.44	1	23.5
4-70-01-55-20	1639	2065	2065	0.33	20.6	2065	0.33	1	20.6	2065	0.33	1	20.6
4-50-01-45-30	2035	2235	2235	0.28	8.9	2235	0.28	1	8.9	2235	0.28	1	8.9
4-50-01-50-30	2003	2144	2144	0.10	6.6	2144	0.10	1	6.6	2144	0.10	1	6.6
4-50-01-55-30	1993	1993	1993	0.04	0.0	1993	0.04	1	0.0	1993	0.04	1	0.0
4-60-01-45-30	2261	2749	2749	0.40	17.8	2749	0.40	1	17.8	2749	0.40	1	17.8
4-60-01-50-30	2242	2696	2696	0.28	16.8	2696	0.28	1	16.8	2696	0.28	1	16.8
4-60-01-55-30	2217	2546	2556 ↑	0.25	13.3	2546	0.50	2	12.9	2546	0.50	2	12.9
4-70-01-45-30	2386	2992	2992	0.41	20.3	2992	0.41	1	20.3	2992	0.41	1	20.3
4-70-01-50-30	2369	2930	2930	0.72	19.1	2930	0.72	1	19.1	2930	0.72	1	19.1
4-70-01-55-30	2331	2734	2734	0.08	14.7	2734	0.08	1	14.7	2734	0.08	1	14.7
4-50-10-45-10	3623	3623	3655 ↑	0.03	0.9	3623	0.09	3	0.0	3623	0.08	3	0.0
4-50-10-50-10	3503	3503	3503	0.03	0.0	3503	0.03	1	0.0	3503	0.03	1	0.0
4-50-10-55-10	3363	3363	3363	0.05	0.0	3363	0.05	1	0.0	3363	0.05	1	0.0
4-60-10-45-10	4587	4587	4587	0.07	0.0	4587	0.07	1	0.0	4587	0.07	1	0.0
4-60-10-50-10	4016	4016	4016	0.03	0.0	4016	0.03	1	0.0	4016	0.03	1	0.0
4-60-10-55-10	3536	3536	3536	0.05	0.0	3536	0.05	1	0.0	3536	0.05	1	0.0
4-70-10-45-10	5627	5627	5627	0.11	0.0	5627	0.11	1	0.0	5627	0.11	1	0.0
4-70-10-50-10	4672	4672	4672	0.04	0.0	4672	0.04	1	0.0	4672	0.04	1	0.0
4-70-10-55-10	4261	4261	4268 ↑	0.09	0.2	4261	0.17	2	0.0	4261	0.18	2	0.0
4-50-10-45-20	9450	9450	9450	0.09	0.0	9450	0.09	1	0.0	9450	0.09	1	0.0
4-50-10-50-20	9274	9274	9286 ↑	0.09	0.1	9274	0.26	3	0.0	9274	0.29	3	0.0
4-50-10-55-20	8098	8098	8098	0.04	0.0	8098	0.04	1	0.0	8098	0.04	1	0.0
4-60-10-45-20	12075	13415	13415	0.18	10.0	13415	0.18	1	10.0	13415	0.18	1	10.0
4-60-10-50-20	11982	12942	13009 ↑	0.30	7.9	12942	0.61	2	7.4	12942	0.61	2	7.4
4-60-10-55-20	11131	11131	11131	0.07	0.0	11131	0.07	1	0.0	11131	0.07	1	0.0
4-70-10-45-20	11881	14202	14202	0.10	16.3	14202	0.10	1	16.3	14202	0.10	1	16.3
4-70-10-50-20	11877	13696	13696	0.28	13.3	13696	0.28	1	13.3	13696	0.28	1	13.3
4-70-10-55-20	11902	12570	12581 ↑	0.12	5.4	12570	0.26	2	5.3	12570	0.26	2	5.3
4-50-10-45-30	19337	22100	22100	0.31	12.5	22100	0.31	1	12.5	22100	0.31	1	12.5
4-50-10-50-30	19052	21724	21724	0.10	12.3	21724	0.10	1	12.3	21724	0.10	1	12.3
4-50-10-55-30	18601	20216	20216	0.09	8.0	20216	0.09	1	8.0	20216	0.09	1	8.0
4-60-10-45-30	20197	25186	25186	0.27	19.8	25186	0.27	1	19.8	25186	0.27	1	19.8
4-60-10-50-30	20254	25408	25441 ↑	0.49	20.4	25408	1.63	3	20.3	25408	1.62	3	20.3
4-60-10-55-30	19827	23982	23982	0.26	17.3	23982	0.26	1	17.3	23982	0.26	1	17.3
4-70-10-45-30	21474	28220	28220	0.35	23.9	28220	0.35	1	23.9	28220	0.35	1	23.9
4-70-10-50-30	21347	28298	28298	0.45	24.6	28298	0.45	1	24.6	28298	0.45	1	24.6
4-70-10-55-30	21056	26642	26642	0.38	21.0	26642	0.38	1	21.0	26642	0.38	1	21.0

Table B.5: MMR-KP results for type-5 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
5-50-01-45-10	731	731	736 ↑	0.11	0.7	731	0.49	3	0.0	731	0.43	3	0.0
5-50-01-50-10	770	770	770	0.29	0.0	770	0.29	1	0.0	770	0.29	1	0.0
5-50-01-55-10	777	777	777	0.15	0.0	777	0.15	1	0.0	777	0.15	1	0.0
5-60-01-45-10	786	874	874	0.15	10.1	874	0.15	1	10.1	874	0.15	1	10.1
5-60-01-50-10	827	924	932 ↑	0.59	11.3	924	1.58	3	10.5	924	1.61	3	10.5
5-60-01-55-10	828	931	931	0.25	11.1	931	0.25	1	11.1	931	0.25	1	11.1
5-70-01-45-10	845	986	993 ↑	0.66	14.9	986	1.24	2	14.3	986	1.33	2	14.3
5-70-01-50-10	881	1012	1012	0.14	12.9	1012	0.14	1	12.9	1012	0.14	1	12.9
5-70-01-55-10	888	1034	1034	0.29	14.1	1034	0.29	1	14.1	1034	0.29	1	14.1
5-50-01-45-20	1566	1566	1566	0.06	0.0	1566	0.06	1	0.0	1566	0.06	1	0.0
5-50-01-50-20	1521	1688	1705 ↑	0.26	10.8	1688	1.49	5	9.9	1688	1.51	5	9.9
5-50-01-55-20	1517	1726	1726	0.23	12.1	1726	0.23	1	12.1	1726	0.23	1	12.1
5-60-01-45-20	1737	1951	1951	0.38	11.0	1951	0.38	1	11.0	1951	0.38	1	11.0
5-60-01-50-20	1748	2073	2086 ↑	0.36	16.2	2073	0.70	2	15.7	2073	0.71	2	15.7
5-60-01-55-20	1756	2142	2142	0.39	18.0	2142	0.39	1	18.0	2142	0.39	1	18.0
5-70-01-45-20	1905	2225	2225	0.17	14.4	2225	0.17	1	14.4	2225	0.17	1	14.4
5-70-01-50-20	1951	2385	2399 ↑	0.34	18.7	**2384** ↓	0.71	2	18.2	2385	0.69	2	18.2
5-70-01-55-20	1917	2524	2524	0.78	24.0	2524	0.78	1	24.0	2524	0.78	1	24.0
5-50-01-45-30	1926	2251	2251	0.25	14.4	2251	0.25	1	14.4	2251	0.25	1	14.4
5-50-01-50-30	1882	2200	2200	0.08	14.5	2200	0.08	1	14.5	2200	0.08	1	14.5
5-50-01-55-30	1847	2087	2087	0.06	11.5	2087	0.06	1	11.5	2087	0.06	1	11.5
5-60-01-45-30	2216	2768	2768	0.29	19.9	2768	0.29	1	19.9	2768	0.29	1	19.9
5-60-01-50-30	2197	2752	2752	0.34	20.2	2752	0.34	1	20.2	2752	0.34	1	20.2
5-60-01-55-30	2182	2657	2657	0.21	17.9	2657	0.21	1	17.9	2657	0.21	1	17.9
5-70-01-45-30	2621	3450	3450	0.52	24.0	3450	0.52	1	24.0	3450	0.52	1	24.0
5-70-01-50-30	2587	3287	3415 ↑	0.38	24.2	3406 ↑	0.76	2	24.0	3406 ↑	0.75	2	24.0
5-70-01-55-30	2590	3303	3303	0.23	21.6	3303	0.23	1	21.6	3303	0.23	1	21.6
5-50-10-45-10	3322	3322	3322	0.06	0.0	3322	0.06	1	0.0	3322	0.06	1	0.0
5-50-10-50-10	3358	3358	3374 ↑	0.04	0.5	3358	0.07	2	0.0	3358	0.08	2	0.0
5-50-10-55-10	3494	3494	3494	0.05	0.0	3494	0.05	1	0.0	3494	0.05	1	0.0
5-60-10-45-10	3458	3458	3461 ↑	0.03	0.1	3458	0.09	2	0.0	3458	0.09	2	0.0
5-60-10-50-10	3685	3685	3815 ↑	0.09	3.4	3685	0.27	3	0.0	3685	0.29	3	0.0
5-60-10-55-10	3797	3797	3851 ↑	0.07	1.4	3797	0.13	2	0.0	3797	0.13	2	0.0
5-70-10-45-10	4431	4431	4431	0.05	0.0	4431	0.05	1	0.0	4431	0.05	1	0.0
5-70-10-50-10	4575	4575	4575	0.06	0.0	4575	0.06	1	0.0	4575	0.06	1	0.0
5-70-10-55-10	5061	5061	5071 ↑	0.08	0.2	5061	0.25	3	0.0	5061	0.28	3	0.0
5-50-10-45-20	7664	7664	7664	0.04	0.0	7664	0.04	1	0.0	7664	0.04	1	0.0
5-50-10-50-20	8764	8764	8764	0.05	0.0	8764	0.05	1	0.0	8764	0.05	1	0.0
5-50-10-55-20	9369	9369	9369	0.06	0.0	9369	0.06	1	0.0	9369	0.06	1	0.0
5-60-10-45-20	8020	8020	8020	0.03	0.0	8020	0.03	1	0.0	8020	0.03	1	0.0
5-60-10-50-20	8730	8730	8730	0.03	0.0	8730	0.03	1	0.0	8730	0.03	1	0.0
5-60-10-55-20	9873	9873	9932 ↑	0.07	0.6	9873	0.15	2	0.0	9873	0.15	2	0.0
5-70-10-45-20	11891	11891	11891	0.06	0.0	11891	0.06	1	0.0	11891	0.06	1	0.0
5-70-10-50-20	13582	13582	13582	0.06	0.0	13582	0.06	1	0.0	13582	0.06	1	0.0
5-70-10-55-20	13900	14862	14897 ↑	0.28	6.7	14862	0.98	3	6.5	14862	1.07	3	6.5
5-50-10-45-30	16984	16984	16984	0.08	0.0	16984	0.08	1	0.0	16984	0.08	1	0.0
5-50-10-50-30	17680	17680	17680	0.08	0.0	17680	0.08	1	0.0	17680	0.08	1	0.0
5-50-10-55-30	17894	17894	17985 ↑	0.05	0.5	17894	0.11	2	0.0	17894	0.12	2	0.0
5-60-10-45-30	17521	18381	18396 ↑	0.08	4.8	18381	0.19	2	4.7	18381	0.17	2	4.7
5-60-10-50-30	17593	19414	19414	0.07	9.4	19414	0.07	1	9.4	19414	0.07	1	9.4
5-60-10-55-30	17909	19884	19884	0.07	9.9	19884	0.07	1	9.9	19884	0.07	1	9.9
5-70-10-45-30	21641	24536	24536	0.14	11.8	24536	0.14	1	11.8	24536	0.14	1	11.8
5-70-10-50-30	22103	26382	26382	0.28	16.2	26382	0.28	1	16.2	26382	0.28	1	16.2
5-70-10-55-30	22134	27692	27692	0.27	20.1	27692	0.27	1	20.1	27692	0.27	1	20.1

Table B.6: MMR-KP results for type-6 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
6-50-01-45-10	646	760	765 ↑	0.09	15.6	760	0.16	2	15.0	760	0.17	2	15.0
6-50-01-50-10	644	740	740	0.07	13.0	740	0.07	1	13.0	740	0.07	1	13.0
6-50-01-55-10	644	743	743	0.07	13.3	743	0.07	1	13.3	743	0.07	1	13.3
6-60-01-45-10	775	991	991	0.14	21.8	991	0.14	1	21.8	991	0.14	1	21.8
6-60-01-50-10	775	998	999 ↑	0.13	22.4	998	0.53	3	22.3	998	0.48	3	22.3
6-60-01-55-10	780	1005	1005	0.16	22.4	1005	0.16	1	22.4	1005	0.16	1	22.4
6-70-01-45-10	828	1090	1090	0.43	24.0	1090	0.43	1	24.0	1090	0.43	1	24.0
6-70-01-50-10	829	1109	1109	0.38	25.2	1109	0.38	1	25.2	1109	0.38	1	25.2
6-70-01-55-10	828	1117	1117	0.40	25.9	1117	0.40	1	25.9	1117	0.40	1	25.9
6-50-01-45-20	1358	1691	1698 ↑	0.36	20.0	1691	2.49	9	19.7	1691	1.31	**5**	19.7
6-50-01-50-20	1361	1705	1705	0.15	20.2	1705	0.15	1	20.2	1705	0.15	1	20.2
6-50-01-55-20	1368	1706	1706	0.28	19.8	1706	0.28	1	19.8	1706	0.28	1	19.8
6-60-01-45-20	1626	2213	2213	0.45	26.5	2213	0.45	1	26.5	2213	0.45	1	26.5
6-60-01-50-20	1638	2240	2240	0.51	26.9	2240	0.51	1	26.9	2240	0.51	1	26.9
6-60-01-55-20	1634	2245	2245	0.51	27.2	2245	0.51	1	27.2	2245	0.51	1	27.2
6-70-01-45-20	1795	2556	2556	0.84	29.8	2554 ↓	1.70	2	29.7	2554 ↓	1.78	2	29.7
6-70-01-50-20	1818	2597	2597	0.82	30.0	2597	0.82	1	30.0	2597	0.82	1	30.0
6-70-01-55-20	1818	2601	2601	0.87	30.1	2601	0.87	1	30.1	2601	0.87	1	30.1
6-50-01-45-30	2134	2559	2559	0.20	16.6	2559	0.20	1	16.6	2559	0.20	1	16.6
6-50-01-50-30	2135	2560	2560	0.12	16.6	2560	0.12	1	16.6	2560	0.12	1	16.6
6-50-01-55-30	2124	2535	2535	0.26	16.2	2535	0.26	1	16.2	2535	0.26	1	16.2
6-60-01-45-30	2434	3157	3159 ↑	0.52	23.0	3157	1.07	2	22.9	3157	1.08	2	22.9
6-60-01-50-30	2437	3175	3175	0.30	23.2	3175	0.30	1	23.2	3175	0.30	1	23.2
6-60-01-55-30	2426	3163	3163	0.38	23.3	3163	0.38	1	23.3	3163	0.38	1	23.3
6-70-01-45-30	2587	3481	3481	0.66	25.7	3481	0.66	1	25.7	3481	0.66	1	25.7
6-70-01-50-30	2598	3501	3501	0.42	25.8	3501	0.42	1	25.8	3501	0.42	1	25.8
6-70-01-55-30	2577	3477	3478 ↑	0.55	25.9	3475 ↓	1.64	3	25.8	3475 ↓	1.10	**2**	25.8
6-50-10-45-10	6504	7851	7851	0.33	17.2	7851	0.33	1	17.2	7851	0.33	1	17.2
6-50-10-50-10	6449	7904	7904	0.74	18.4	7904	0.74	1	18.4	7904	0.74	1	18.4
6-50-10-55-10	6389	7589	7597 ↑	0.22	15.9	7589	2.60	9	15.8	7589	1.56	**5**	15.8
6-60-10-45-10	7604	9667	9667	0.40	21.3	9667	0.40	1	21.3	9667	0.40	1	21.3
6-60-10-50-10	7574	9774	9774	0.60	22.5	9774	0.60	1	22.5	9774	0.60	1	22.5
6-60-10-55-10	7513	9441	9462 ↑	0.59	20.6	9441	1.07	2	20.4	9441	1.12	2	20.4
6-70-10-45-10	8217	11060	11060	1.89	25.7	11060	1.89	1	25.7	11060	1.89	1	25.7
6-70-10-50-10	8197	11054	11054	1.00	25.8	11054	1.00	1	25.8	11054	1.00	1	25.8
6-70-10-55-10	8081	10722	10722	0.90	24.6	10722	0.90	1	24.6	10722	0.90	1	24.6
6-50-10-45-20	11106	12921	12921	0.08	14.0	12921	0.08	1	14.0	12921	0.08	1	14.0
6-50-10-50-20	11244	13240	13250 ↑	0.20	15.1	13240	1.03	4	15.1	13240	0.81	**3**	15.1
6-50-10-55-20	11173	13070	13112 ↑	0.22	14.8	13070	197.26	190	14.5	13070	89.64	**98**	14.5
6-60-10-45-20	13361	16849	16878 ↑	0.36	20.8	16849	3.23	9	20.7	16849	1.76	**5**	20.7
6-60-10-50-20	13556	17333	17348 ↑	0.49	21.9	17333	8.52	15	21.8	17333	4.31	**8**	21.8
6-60-10-55-20	13487	17123	17148 ↑	0.35	21.3	17123	1.10	3	21.2	17123	0.70	**2**	21.2
6-70-10-45-20	14104	18673	18686 ↑	0.58	24.5	18673	6.71	10	24.5	18673	3.94	**6**	24.5
6-70-10-50-20	14151	18890	18890	0.58	25.1	18890	0.58	1	25.1	18890	0.58	1	25.1
6-70-10-55-20	14070	18668	18671 ↑	0.34	24.6	18652 ↓	32.35	44	24.6	18652 ↓	18.66	**25**	24.6
6-50-10-45-30	18876	23190	23190	0.19	18.6	23190	0.19	1	18.6	23190	0.19	1	18.6
6-50-10-50-30	19376	24190	24190	0.31	19.9	24190	0.31	1	19.9	24190	0.31	1	19.9
6-50-10-55-30	19530	24324	24324	0.34	19.7	24324	0.34	1	19.7	24324	0.34	1	19.7
6-60-10-45-30	21122	27417	27417	0.36	23.0	27417	0.36	1	23.0	27417	0.36	1	23.0
6-60-10-50-30	21364	28524	28524	0.52	25.1	28524	0.52	1	25.1	28524	0.52	1	25.1
6-60-10-55-30	21371	28476	28476	0.47	25.0	28476	0.47	1	25.0	28476	0.47	1	25.0
6-70-10-45-30	22925	31130	31130	0.52	26.4	31130	0.52	1	26.4	31130	0.52	1	26.4
6-70-10-50-30	23305	32207	32230 ↑	0.62	27.7	32207	1.60	2	27.6	32207	1.80	2	27.6
6-70-10-55-30	23164	32067	32067	0.46	27.8	32067	0.46	1	27.8	32067	0.46	1	27.8

Table B.7: MMR-KP results for type-7 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
7-50-01-45-10	689	833	833	0.21	17.3	833	0.21	1	17.3	833	0.21	1	17.3
7-50-01-50-10	688	810	810	0.07	15.1	810	0.07	1	15.1	810	0.07	1	15.1
7-50-01-55-10	691	803	803	0.08	13.9	803	0.08	1	13.9	803	0.08	1	13.9
7-60-01-45-10	820	1061	1061	0.38	22.7	1061	0.38	1	22.7	1061	0.38	1	22.7
7-60-01-50-10	820	1073	1069 ↓	0.29	23.3	1069 ↓	0.29	1	23.3	1069 ↓	0.29	1	23.3
7-60-01-55-10	816	1072	1075 ↑	0.32	24.1	1075 ↑	0.32	1	24.1	1075 ↑	0.32	1	24.1
7-70-01-45-10	868	1157	1157	0.52	25.0	1157	0.52	1	25.0	1157	0.52	1	25.0
7-70-01-50-10	869	1179	1179	0.38	26.3	1178 ↓	0.74	2	26.2	1178 ↓	0.78	2	26.2
7-70-01-55-10	868	1183	1185 ↑	0.38	26.8	1183	1.30	3	26.6	1183	1.23	3	26.6
7-50-01-45-20	1366	1661	1669 ↑	0.31	18.2	1661	0.76	3	17.8	1661	0.77	3	17.8
7-50-01-50-20	1367	1679	1679	0.19	18.6	1679	0.19	1	18.6	1679	0.19	1	18.6
7-50-01-55-20	1374	1674	1674	0.25	17.9	1674	0.25	1	17.9	1674	0.25	1	17.9
7-60-01-45-20	1635	2193	2193	0.54	25.4	2193	0.54	1	25.4	2193	0.54	1	25.4
7-60-01-50-20	1642	2222	2214 ↓	0.41	25.8	2214 ↓	0.41	1	25.8	2214 ↓	0.41	1	25.8
7-60-01-55-20	1647	2220	2220	0.50	25.8	2219 ↓	2.58	5	25.8	2219 ↓	2.66	5	25.8
7-70-01-45-20	1809	2532	2530 ↓	0.75	28.5	2530 ↓	0.75	1	28.5	2530 ↓	0.75	1	28.5
7-70-01-50-20	1827	2562	2565 ↑	0.39	28.8	2565 ↑	0.39	1	28.8	2565 ↑	0.39	1	28.8
7-70-01-55-20	1823	2578	2582 ↑	0.67	29.4	2578	2.04	3	29.3	2578	2.10	3	29.3
7-50-01-45-30	1998	2346	2346	0.10	14.8	2346	0.10	1	14.8	2346	0.10	1	14.8
7-50-01-50-30	1997	2382	2382	0.24	16.2	2382	0.24	1	16.2	2382	0.24	1	16.2
7-50-01-55-30	1986	2385	2389 ↑	0.12	16.9	2385	0.56	3	16.7	2385	0.31	**2**	16.7
7-60-01-45-30	2360	3059	3059	0.37	22.9	3059	0.37	1	22.9	3059	0.37	1	22.9
7-60-01-50-30	2371	3117	3118 ↑	0.38	24.0	3116 ↓	1.43	4	23.9	3116 ↓	1.54	4	23.9
7-60-01-55-30	2368	3120	3120	0.31	24.1	3120	0.31	1	24.1	3120	0.31	1	24.1
7-70-01-45-30	2486	3326	3328 ↑	0.44	25.3	3328 ↑	0.44	1	25.3	3328 ↑	0.44	1	25.3
7-70-01-50-30	2503	3399	3401 ↑	0.46	26.4	3400 ↑	1.91	4	26.4	3400 ↑	1.40	**3**	26.4
7-70-01-55-30	2496	3413	3413	0.51	26.9	3412 ↓	0.94	**2**	26.8	3412 ↓	2.00	4	26.8
7-50-10-45-10	6333	7519	7529 ↑	0.32	15.9	7519	2.61	9	15.8	7519	1.99	**6**	15.8
7-50-10-50-10	6334	7600	7600	0.30	16.7	7600	0.30	1	16.7	7600	0.30	1	16.7
7-50-10-55-10	6317	7372	7377 ↑	0.27	14.4	7372	0.92	3	14.3	7372	0.60	**2**	14.3
7-60-10-45-10	7456	9335	9335	0.37	20.1	9318 ↓	1.17	3	20.0	9318 ↓	0.76	**2**	20.0
7-60-10-50-10	7493	9499	9499	0.65	21.1	9499	0.65	1	21.1	9499	0.65	1	21.1
7-60-10-55-10	7456	9252	9259 ↑	0.37	19.5	9252	1.85	5	19.4	9252	1.13	**3**	19.4
7-70-10-45-10	8082	10700	10700	0.56	24.5	10698 ↓	2.44	3	24.5	10698 ↓	1.67	**2**	24.5
7-70-10-50-10	8115	10765	10765	0.57	24.6	10747 ↓	1.41	2	24.5	10747 ↓	1.41	2	24.5
7-70-10-55-10	8031	10475	10475	0.68	23.3	10466 ↓	2.17	3	23.3	10466 ↓	1.34	**2**	23.3
7-50-10-45-20	11315	13464	13485 ↑	0.22	16.1	13464	1.35	5	16.0	13464	0.77	**3**	16.0
7-50-10-50-20	11416	13920	13920	0.45	18.0	13920	0.45	1	18.0	13920	0.45	1	18.0
7-50-10-55-20	11420	13728	13768 ↑	0.28	17.1	13726 ↓	2.91	8	16.8	13726 ↓	1.88	**5**	16.8
7-60-10-45-20	13573	17262	17287 ↑	0.43	21.5	17264 ↑	1.44	3	21.4	17264 ↑	0.86	**2**	21.4
7-60-10-50-20	13789	17964	17964	0.74	23.2	17964	0.74	1	23.2	17964	0.74	1	23.2
7-60-10-55-20	13726	17776	17798 ↑	0.43	22.9	17776	3.87	9	22.8	17776	2.55	**6**	22.8
7-70-10-45-20	14306	19238	19238	0.83	25.6	19238	0.83	1	25.6	19238	0.83	1	25.6
7-70-10-50-20	14419	19557	19557	0.74	26.3	19557	0.74	1	26.3	19557	0.74	1	26.3
7-70-10-55-20	14359	19284	19284	0.67	25.5	19273 ↓	237.99	135	25.5	19273 ↓	146.92	**73**	25.5
7-50-10-45-30	20595	25515	25515	0.47	19.3	25515	0.47	1	19.3	25515	0.47	1	19.3
7-50-10-50-30	20758	26163	26163	0.56	20.7	26163	0.56	1	20.7	26163	0.56	1	20.7
7-50-10-55-30	20630	25509	25507 ↓	0.65	19.1	25507 ↓	0.65	1	19.1	25507 ↓	0.65	1	19.1
7-60-10-45-30	22821	30203	30203	0.65	24.4	30203	0.65	1	24.4	30203	0.65	1	24.4
7-60-10-50-30	23026	30978	30978	0.81	25.7	30978	0.81	1	25.7	30978	0.81	1	25.7
7-60-10-55-30	22939	30456	30456	1.07	24.7	30456	1.07	1	24.7	30456	1.07	1	24.7
7-70-10-45-30	24479	33427	33427	0.92	26.8	33427	0.92	1	26.8	33427	0.92	1	26.8
7-70-10-50-30	24621	34350	34357 ↑	0.86	28.3	34350	1.92	2	28.3	34350	3.55	2	28.3
7-70-10-55-30	24364	34054	34054	2.48	28.5	34054	2.48	1	28.5	34054	2.48	1	28.5

Table B.8: MMR-KP results for type-8 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
8-50-01-45-10	620	620	620	0.05	0.0	620	0.05	1	0.0	620	0.05	1	0.0
8-50-01-50-10	644	644	646 ↑	0.07	0.3	644	0.22	3	0.0	644	0.20	3	0.0
8-50-01-55-10	632	632	634 ↑	0.03	0.3	632	0.05	2	0.0	632	0.05	2	0.0
8-60-01-45-10	782	782	782	0.05	0.0	782	0.05	1	0.0	782	0.05	1	0.0
8-60-01-50-10	815	815	826 ↑	0.09	1.3	826 ↑	0.09	1	1.3	826 ↑	0.09	1	1.3
8-60-01-55-10	783	847	848 ↑	0.24	7.7	848 ↑	0.24	1	7.7	848 ↑	0.24	1	7.7
8-70-01-45-10	825	884	885 ↑	0.18	6.8	885 ↑	0.18	1	6.8	885 ↑	0.18	1	6.8
8-70-01-50-10	838	927	930 ↑	0.32	9.9	930 ↑	0.32	1	9.9	930 ↑	0.32	1	9.9
8-70-01-55-10	848	927	927	0.07	8.5	927	0.07	1	8.5	927	0.07	1	8.5
8-50-01-45-20	1784	2098	2098	0.39	15.0	2098	0.39	1	15.0	2098	0.39	1	15.0
8-50-01-50-20	1795	2112	2112	0.26	15.0	2112	0.26	1	15.0	2112	0.26	1	15.0
8-50-01-55-20	1778	2054	2054	0.08	13.4	2054	0.08	1	13.4	2054	0.08	1	13.4
8-60-01-45-20	1962	2492	2492	0.41	21.3	2492	0.41	1	21.3	2492	0.41	1	21.3
8-60-01-50-20	1937	2538	2541 ↑	2.76	23.8	2538	4.29	3	23.7	2538	4.33	3	23.7
8-60-01-55-20	1935	2417	2501 ↑	0.40	22.6	2484 ↑	0.99	2	22.1	2484 ↑	0.92	2	22.1
8-70-01-45-20	2071	2732	2740 ↑	0.62	24.4	2740 ↑	0.62	1	24.4	2740 ↑	0.62	1	24.4
8-70-01-50-20	2070	2785	2785	0.51	25.7	2785	0.51	1	25.7	2785	0.51	1	25.7
8-70-01-55-20	2059	2729	2729	0.50	24.6	2729	0.50	1	24.6	2729	0.50	1	24.6
8-50-01-45-30	2490	2542	2542	0.12	2.0	2542	0.12	1	2.0	2542	0.12	1	2.0
8-50-01-50-30	2402	2720	2720	0.32	11.7	2720	0.32	1	11.7	2720	0.32	1	11.7
8-50-01-55-30	2391	2759	2759	0.22	13.3	2759	0.22	1	13.3	2759	0.22	1	13.3
8-60-01-45-30	2696	3074	3074	0.13	12.3	3074	0.13	1	12.3	3074	0.13	1	12.3
8-60-01-50-30	2699	3255	3284 ↑	0.55	17.8	3284 ↑	0.55	1	17.8	3284 ↑	0.55	1	17.8
8-60-01-55-30	2656	3363	3372 ↑	0.39	21.2	3364 ↑	0.83	2	21.0	3364 ↑	0.90	2	21.0
8-70-01-45-30	2804	3417	3417	0.34	17.9	3417	0.34	1	17.9	3417	0.34	1	17.9
8-70-01-50-30	2866	3619	3636 ↑	0.50	21.2	3629 ↑	2.18	4	21.0	3629 ↑	2.18	4	21.0
8-70-01-55-30	2842	3724	3724	0.57	23.7	3724	0.57	1	23.7	3724	0.57	1	23.7
8-50-10-45-10	4205	4205	4205	0.03	0.0	4205	0.03	1	0.0	4205	0.03	1	0.0
8-50-10-50-10	4592	4592	4592	0.04	0.0	4592	0.04	1	0.0	4592	0.04	1	0.0
8-50-10-55-10	4911	4911	4911	0.05	0.0	4911	0.05	1	0.0	4911	0.05	1	0.0
8-60-10-45-10	4990	4990	5026 ↑	0.04	0.7	4990	0.09	2	0.0	4990	0.09	2	0.0
8-60-10-50-10	5651	5651	5675 ↑	0.06	0.4	5651	0.16	2	0.0	5651	0.14	2	0.0
8-60-10-55-10	6271	6271	6271	0.10	0.0	6271	0.10	1	0.0	6271	0.10	1	0.0
8-70-10-45-10	6203	6203	6203	0.05	0.0	6203	0.05	1	0.0	6203	0.05	1	0.0
8-70-10-50-10	6915	6915	6915	0.15	0.0	6915	0.15	1	0.0	6915	0.15	1	0.0
8-70-10-55-10	7513	7513	7513	0.09	0.0	7513	0.09	1	0.0	7513	0.09	1	0.0
8-50-10-45-20	11220	11220	11220	0.05	0.0	11220	0.05	1	0.0	11220	0.05	1	0.0
8-50-10-50-20	11900	11900	11900	0.05	0.0	11900	0.05	1	0.0	11900	0.05	1	0.0
8-50-10-55-20	12756	12756	12756	0.11	0.0	12756	0.11	1	0.0	12756	0.11	1	0.0
8-60-10-45-20	12892	13681	13719 ↑	0.11	6.0	13681	0.36	3	5.8	13681	0.39	3	5.8
8-60-10-50-20	13656	14419	14419	0.07	5.3	14419	0.07	1	5.3	14419	0.07	1	5.3
8-60-10-55-20	13577	15147	15147	0.25	10.4	15147	0.25	1	10.4	15147	0.25	1	10.4
8-70-10-45-20	14611	16111	16111	0.14	9.3	16111	0.14	1	9.3	16111	0.14	1	9.3
8-70-10-50-20	14883	17460	17460	0.29	14.8	17460	0.29	1	14.8	17460	0.29	1	14.8
8-70-10-55-20	14834	18295	18295	0.42	18.9	18295	0.42	1	18.9	18295	0.42	1	18.9
8-50-10-45-30	17390	17390	17390	0.06	0.0	17390	0.06	1	0.0	17390	0.06	1	0.0
8-50-10-50-30	18060	18060	18136 ↑	0.06	0.4	18060	0.28	4	0.0	18060	0.27	4	0.0
8-50-10-55-30	18830	18830	18830	0.12	0.0	18830	0.12	1	0.0	18830	0.12	1	0.0
8-60-10-45-30	19189	21541	21580 ↑	0.10	11.1	21541	0.26	2	10.9	21541	0.25	2	10.9
8-60-10-50-30	19436	22592	22592	0.13	14.0	22592	0.13	1	14.0	22592	0.13	1	14.0
8-60-10-55-30	19275	23212	23337 ↑	0.31	17.4	23212	0.65	2	17.0	23212	0.66	2	17.0
8-70-10-45-30	20743	25429	25447 ↑	0.26	18.5	25429	0.54	2	18.4	25429	0.42	2	18.4
8-70-10-50-30	21157	26486	26539 ↑	0.25	20.3	26486	0.57	2	20.1	26486	0.51	2	20.1
8-70-10-55-30	21303	26652	26652	0.08	20.1	26652	0.08	1	20.1	26652	0.08	1	20.1

Table B.9: MMR-KP results for type-9 instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
9-50-01-45-10	46	46	46	0.01	0.0	46	0.01	1	0.0	46	0.01	1	0.0
9-50-01-50-10	31	31	31	0.01	0.0	31	0.01	1	0.0	31	0.01	1	0.0
9-50-01-55-10	51	51	51	0.01	0.0	51	0.01	1	0.0	51	0.01	1	0.0
9-60-01-45-10	46	46	46	0.01	0.0	46	0.01	1	0.0	46	0.01	1	0.0
9-60-01-50-10	40	40	40	0.01	0.0	40	0.01	1	0.0	40	0.01	1	0.0
9-60-01-55-10	71	71	71	0.01	0.0	71	0.01	1	0.0	71	0.01	1	0.0
9-70-01-45-10	92	92	92	0.01	0.0	92	0.01	1	0.0	92	0.01	1	0.0
9-70-01-50-10	34	34	34	0.01	0.0	34	0.01	1	0.0	34	0.01	1	0.0
9-70-01-55-10	71	71	71	0.01	0.0	71	0.01	1	0.0	71	0.01	1	0.0
9-50-01-45-20	335	335	335	0.01	0.0	335	0.01	1	0.0	335	0.01	1	0.0
9-50-01-50-20	242	242	242	0.01	0.0	242	0.01	1	0.0	242	0.01	1	0.0
9-50-01-55-20	168	168	168	0.01	0.0	168	0.01	1	0.0	168	0.01	1	0.0
9-60-01-45-20	335	335	335	0.01	0.0	335	0.01	1	0.0	335	0.01	1	0.0
9-60-01-50-20	249	249	249	0.01	0.0	249	0.01	1	0.0	249	0.01	1	0.0
9-60-01-55-20	247	247	247	0.01	0.0	247	0.01	1	0.0	247	0.01	1	0.0
9-70-01-45-20	421	421	421	0.01	0.0	421	0.01	1	0.0	421	0.01	1	0.0
9-70-01-50-20	286	286	286	0.01	0.0	286	0.01	1	0.0	286	0.01	1	0.0
9-70-01-55-20	247	247	247	0.01	0.0	247	0.01	1	0.0	247	0.01	1	0.0
9-50-01-45-30	637	637	637	0.01	0.0	637	0.01	1	0.0	637	0.01	1	0.0
9-50-01-50-30	581	581	581	0.01	0.0	581	0.01	1	0.0	581	0.01	1	0.0
9-50-01-55-30	551	551	551	0.01	0.0	551	0.01	1	0.0	551	0.01	1	0.0
9-60-01-45-30	749	749	749	0.01	0.0	749	0.01	1	0.0	749	0.01	1	0.0
9-60-01-50-30	748	748	748	0.01	0.0	748	0.01	1	0.0	748	0.01	1	0.0
9-60-01-55-30	685	685	687 ↑	0.01	0.3	685	0.02	2	0.0	685	0.02	2	0.0
9-70-01-45-30	864	864	864	0.01	0.0	864	0.01	1	0.0	864	0.01	1	0.0
9-70-01-50-30	819	819	819	0.02	0.0	819	0.02	1	0.0	819	0.02	1	0.0
9-70-01-55-30	701	701	701	0.01	0.0	701	0.01	1	0.0	701	0.01	1	0.0
9-50-10-45-10	124	124	124	0.01	0.0	124	0.01	1	0.0	124	0.01	1	0.0
9-50-10-50-10	298	298	298	0.01	0.0	298	0.01	1	0.0	298	0.01	1	0.0
9-50-10-55-10	259	259	259	0.01	0.0	259	0.01	1	0.0	259	0.01	1	0.0
9-60-10-45-10	245	245	245	0.01	0.0	245	0.01	1	0.0	245	0.01	1	0.0
9-60-10-50-10	0	0	0	0.01	0.0	0	0.01	1	0.0	0	0.01	1	0.0
9-60-10-55-10	181	181	181	0.01	0.0	181	0.01	1	0.0	181	0.01	1	0.0
9-70-10-45-10	274	274	274	0.01	0.0	274	0.01	1	0.0	274	0.01	1	0.0
9-70-10-50-10	245	245	245	0.01	0.0	245	0.01	1	0.0	245	0.01	1	0.0
9-70-10-55-10	121	121	121	0.01	0.0	121	0.01	1	0.0	121	0.01	1	0.0
9-50-10-45-20	1349	1349	1349	0.01	0.0	1349	0.01	1	0.0	1349	0.01	1	0.0
9-50-10-50-20	907	907	907	0.01	0.0	907	0.01	1	0.0	907	0.01	1	0.0
9-50-10-55-20	927	927	927	0.01	0.0	927	0.01	1	0.0	927	0.01	1	0.0
9-60-10-45-20	905	905	905	0.01	0.0	905	0.01	1	0.0	905	0.01	1	0.0
9-60-10-50-20	562	562	562	0.01	0.0	562	0.01	1	0.0	562	0.01	1	0.0
9-60-10-55-20	959	959	959	0.01	0.0	959	0.01	1	0.0	959	0.01	1	0.0
9-70-10-45-20	1720	1720	1720	0.01	0.0	1720	0.01	1	0.0	1720	0.01	1	0.0
9-70-10-50-20	905	905	905	0.01	0.0	905	0.01	1	0.0	905	0.01	1	0.0
9-70-10-55-20	784	784	784	0.01	0.0	784	0.01	1	0.0	784	0.01	1	0.0
9-50-10-45-30	2376	2376	2376	0.01	0.0	2376	0.01	1	0.0	2376	0.01	1	0.0
9-50-10-50-30	1600	1600	1600	0.01	0.0	1600	0.01	1	0.0	1600	0.01	1	0.0
9-50-10-55-30	590	590	590	0.01	0.0	590	0.01	1	0.0	590	0.01	1	0.0
9-60-10-45-30	2673	2673	2673	0.01	0.0	2673	0.01	1	0.0	2673	0.01	1	0.0
9-60-10-50-30	1492	1492	1492	0.01	0.0	1492	0.01	1	0.0	1492	0.01	1	0.0
9-60-10-55-30	1218	1218	1218	0.01	0.0	1218	0.01	1	0.0	1218	0.01	1	0.0
9-70-10-45-30	3248	3248	3248	0.01	0.0	3248	0.01	1	0.0	3248	0.01	1	0.0
9-70-10-50-30	2673	2673	2673	0.01	0.0	2673	0.01	1	0.0	2673	0.01	1	0.0
9-70-10-55-30	995	995	995	0.01	0.0	995	0.01	1	0.0	995	0.01	1	0.0

Table C.1: MMR-SCP results for type-B instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
B40110	21	21	21	0.493	0.0	21	0.05	1	0.0	21	0.05	1	0.0
B40130	67	67	67	0.516	0.0	67	0.26	1	0.0	67	0.26	1	0.0
B40150	124	124	126 ↑	0.124	1.6	124	3.78	8	0.0	124	1.60	**4**	0.0
B40210	23	23	23	0.273	0.0	23	0.12	1	0.0	23	0.12	1	0.0
B40230	93	93	94 ↑	0.422	1.1	93	3.88	9	0.0	93	2.58	**5**	0.0
B40250	163	163	164 ↑	0.085	0.6	163	1.71	3	0.0	163	2.11	3	0.0
B40310	12	12	12	0.268	0.0	12	0.03	1	0.0	12	0.03	1	0.0
B40330	86	86	86	0.538	0.0	86	0.31	1	0.0	86	0.31	1	0.0
B40350	190	190	191 ↑	0.212	0.5	190	5.22	7	0.0	190	5.69	7	0.0
B40410	20	20	20	0.456	0.0	20	0.07	1	0.0	20	0.07	1	0.0
B40430	85	85	85	0.557	0.0	85	0.55	1	0.0	85	0.55	1	0.0
B40450	160	160	160	0.334	0.0	160	0.94	1	0.0	160	0.94	1	0.0
B40510	13	13	13	0.349	0.0	13	0.04	1	0.0	13	0.04	1	0.0
B40530	73	73	74 ↑	0.868	1.4	73	5.66	15	0.0	73	2.65	**7**	0.0
B40550	165	165	165	0.110	0.0	165	0.56	1	0.0	165	0.56	1	0.0
B40610	16	16	17 ↑	0.444	5.9	16	0.27	3	0.0	16	0.36	3	0.0
B40630	84	84	85 ↑	0.615	1.2	84	3.32	8	0.0	84	3.97	**7**	0.0
B40650	175	175	181 ↑	0.045	3.3	175	199.30	165	0.0	175	113.01	**73**	0.0
B40710	17	17	17	0.181	0.0	17	0.09	1	0.0	17	0.09	1	0.0
B40730	59	59	60 ↑	0.519	1.7	59	0.82	3	0.0	59	0.59	**2**	0.0
B40750	104	104	106 ↑	0.415	1.9	104	1.13	5	0.0	104	0.40	**2**	0.0
B40810	24	24	24	0.835	0.0	24	0.21	1	0.0	24	0.21	1	0.0
B40830	54	54	56 ↑	0.864	3.6	54	0.97	4	0.0	54	1.03	**3**	0.0
B40850	155	155	155	0.493	0.0	155	0.79	1	0.0	155	0.79	1	0.0
B40910	33	33	33	0.546	0.0	33	0.36	1	0.0	33	0.36	1	0.0
B40930	99	99	100 ↑	0.894	1.0	99	12.93	16	0.0	99	2.09	**4**	0.0
B40950	263	263	263	0.081	0.0	263	1.14	1	0.0	263	1.14	1	0.0
B41010	17	17	17	0.568	0.0	17	0.03	1	0.0	17	0.03	1	0.0
B41030	63	63	63	0.592	0.0	63	0.21	1	0.0	63	0.21	1	0.0
B41050	141	141	143 ↑	0.157	1.4	141	2.01	8	0.0	141	2.09	8	0.0
B50110	24	24	24	0.262	0.0	24	0.55	1	0.0	24	0.55	1	0.0
B50130	53	53	54 ↑	0.567	1.9	53	16.80	20	0.0	53	17.42	**16**	0.0
B50150	93	93	95 ↑	0.496	2.1	93	4.77	5	0.0	93	4.66	5	0.0
B50210	19	19	19	0.902	0.0	19	0.48	1	0.0	19	0.48	1	0.0
B50230	69	69	72 ↑	1.219	4.2	69	2.24	2	0.0	69	2.33	2	0.0
B50250	121	121	122 ↑	0.12	0.8	121	2.54	2	0.0	121	2.48	2	0.0
B50310	12	12	13 ↑	0.38	7.7	12	0.75	7	0.0	12	0.41	**3**	0.0
B50330	24	24	24	0.42	0.0	24	0.10	1	0.0	24	0.10	1	0.0
B50350	66	66	67 ↑	0.07	1.5	66	1.43	5	0.0	66	0.54	**2**	0.0
B50410	18	18	19 ↑	0.67	5.3	18	1.12	4	0.0	18	1.47	4	0.0
B50430	50	50	51 ↑	0.10	2.0	50	1.49	3	0.0	50	1.23	**2**	0.0
B50450	84	84	86 ↑	0.17	2.3	84	1.15	2	0.0	84	1.21	2	0.0
B50510	10	10	11 ↑	0.57	9.1	10	0.25	2	0.0	10	0.29	2	0.0
B50530	34	34	37 ↑	0.48	8.1	34	1.22	4	0.0	34	1.51	4	0.0
B50550	57	57	57	0.86	0.0	57	0.42	1	0.0	57	0.42	1	0.0
B50610	13	13	13	0.11	0.0	13	0.09	1	0.0	13	0.09	1	0.0
B50630	29	29	29	0.33	0.0	29	0.27	1	0.0	29	0.27	1	0.0
B50650	51	51	52 ↑	0.44	1.9	51	8.77	17	0.0	51	2.08	**4**	0.0
B50710	17	17	17	0.16	0.0	17	0.21	1	0.0	17	0.21	1	0.0
B50730	54	54	54	0.19	0.0	54	0.46	1	0.0	54	0.46	1	0.0
B50750	88	88	88	1.04	0.0	88	0.56	1	0.0	88	0.56	1	0.0
B50810	24	24	25 ↑	0.32	4.0	24	2.24	5	0.0	24	1.13	**3**	0.0
B50830	57	57	57	0.47	0.0	57	0.35	1	0.0	57	0.35	1	0.0
B50850	105	105	105	0.31	0.0	105	0.87	1	0.0	105	0.87	1	0.0
B50910	21	21	21	0.29	0.0	21	0.11	1	0.0	21	0.11	1	0.0
B50930	56	56	57 ↑	0.53	1.8	56	1.38	3	0.0	56	1.66	3	0.0
B50950	91	91	91	0.19	0.0	91	0.62	1	0.0	91	0.62	1	0.0
B51010	10	10	10	0.13	0.0	10	0.05	1	0.0	10	0.05	1	0.0
B51030	37	37	37	0.49	0.0	37	0.18	1	0.0	37	0.18	1	0.0
B51050	76	76	76	0.25	0.0	76	0.52	1	0.0	76	0.52	1	0.0
B60110	17	17	17	0.55	0.0	17	0.42	1	0.0	17	0.42	1	0.0
B60130	38	38	39 ↑	0.82	2.6	38	71.73	80	0.0	38	17.99	**18**	0.0
B60150	58	58	58	0.90	0.0	58	0.86	1	0.0	58	0.86	1	0.0
B60210	15	15	15	0.74	0.0	15	0.49	1	0.0	15	0.49	1	0.0
B60230	40	40	41 ↑	1.01	2.4	40	1.69	3	0.0	40	1.42	**2**	0.0
B60250	56	56	59 ↑	0.89	5.1	56	1.85	2	0.0	56	1.80	2	0.0
B60310	6	6	6	0.29	0.0	6	0.08	1	0.0	6	0.08	1	0.0
B60330	33	33	33	0.42	0.0	33	0.57	1	0.0	33	0.57	1	0.0
B60350	54	54	54	0.68	0.0	54	0.59	1	0.0	54	0.59	1	0.0
B60410	13	13	14 ↑	0.18	7.1	13	0.57	3	0.0	13	0.49	**2**	0.0
B60430	31	31	32 ↑	0.70	3.1	31	3.99	12	0.0	31	1.66	**4**	0.0
B60450	46	46	47 ↑	0.62	2.1	46	1.73	3	0.0	46	1.47	3	0.0
B60510	13	13	13	0.59	0.0	13	0.50	1	0.0	13	0.50	1	0.0
B60530	43	43	43	0.54	0.0	43	0.90	1	0.0	43	0.90	1	0.0
B60550	71	71	71	0.35	0.0	71	1.22	1	0.0	71	1.22	1	0.0

Table C.2: MMR-SCP results for type-M instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
M401-1	3160	3160	3160	0.68	0.0	3160	0.68	1	0.0	3160	0.68	1	0.0
M401-2	3495	3495	3495	0.32	0.0	3495	0.32	1	0.0	3495	0.32	1	0.0
M401-3	3382	3382	3382	0.93	0.0	3382	0.93	1	0.0	3382	0.93	1	0.0
M402-1	3610	3610	3610	1.00	0.0	3610	1.00	1	0.0	3610	1.00	1	0.0
M402-2	2986	2986	2986	0.36	0.0	2986	0.36	1	0.0	2986	0.36	1	0.0
M402-3	4294	4294	4294	1.12	0.0	4294	1.12	1	0.0	4294	1.12	1	0.0
M403-1	3233	3233	3233	0.24	0.0	3233	0.24	1	0.0	3233	0.24	1	0.0
M403-2	4205	4205	4205	0.60	0.0	4205	0.60	1	0.0	4205	0.60	1	0.0
M403-3	3589	3589	3594 ↑	0.79	0.1	3589	9.20	11	0.0	3589	1.94	**2**	0.0
M404-1	3628	3628	3628	0.28	0.0	3628	0.28	1	0.0	3628	0.28	1	0.0
M404-2	3954	3954	3954	0.80	0.0	3954	0.80	1	0.0	3954	0.80	1	0.0
M404-3	2957	2957	2957	0.19	0.0	2957	0.19	1	0.0	2957	0.19	1	0.0
M405-1	3546	3546	3546	0.70	0.0	3546	0.70	1	0.0	3546	0.70	1	0.0
M405-2	3589	3589	3589	0.25	0.0	3589	0.25	1	0.0	3589	0.25	1	0.0
M405-3	3698	3698	3698	0.66	0.0	3698	0.66	1	0.0	3698	0.66	1	0.0
M406-1	3549	3549	3549	0.98	0.0	3549	0.98	1	0.0	3549	0.98	1	0.0
M406-2	2975	2975	2979 ↑	0.18	0.1	2975	1.89	9	0.0	2975	1.23	**4**	0.0
M406-3	3199	3199	3199	0.26	0.0	3199	0.26	1	0.0	3199	0.26	1	0.0
M407-1	3115	3115	3115	0.61	0.0	3115	0.61	1	0.0	3115	0.61	1	0.0
M407-2	4674	4674	4680 ↑	0.98	0.1	4674	20.78	18	0.0	4674	19.19	**12**	0.0
M407-3	4136	4136	4136	0.41	0.0	4136	0.41	1	0.0	4136	0.41	1	0.0
M408-1	3802	3802	3802	0.63	0.0	3802	0.63	1	0.0	3802	0.63	1	0.0
M408-2	3302	3302	3310 ↑	0.36	0.2	3302	0.61	2	0.0	3302	0.89	2	0.0
M408-3	3147	3147	3147	0.21	0.0	3147	0.21	1	0.0	3147	0.21	1	0.0
M409-1	3602	3602	3602	0.21	0.0	3602	0.21	1	0.0	3602	0.21	1	0.0
M409-2	4222	4222	4222	0.58	0.0	4222	0.58	1	0.0	4222	0.58	1	0.0
M409-3	3365	3365	3365	0.25	0.0	3365	0.25	1	0.0	3365	0.25	1	0.0
M410-1	3000	3000	3000	0.11	0.0	3000	0.11	1	0.0	3000	0.11	1	0.0
M410-2	4340	4340	4340	0.71	0.0	4340	0.71	1	0.0	4340	0.71	1	0.0
M410-3	3235	3235	3235	0.10	0.0	3235	0.10	1	0.0	3235	0.10	1	0.0
M501-1	1864	1864	1864	0.28	0.0	1864	0.28	1	0.0	1864	0.28	1	0.0
M501-2	1683	1683	1683	0.09	0.0	1683	0.09	1	0.0	1683	0.09	1	0.0
M501-3	1708	1708	1708	0.53	0.0	1708	0.53	1	0.0	1708	0.53	1	0.0
M502-1	1972	1972	1972	0.88	0.0	1972	0.88	1	0.0	1972	0.88	1	0.0
M502-2	1805	1805	1805	0.07	0.0	1805	0.07	1	0.0	1805	0.07	1	0.0
M502-3	1930	1930	1930	0.12	0.0	1930	0.12	1	0.0	1930	0.12	1	0.0
M503-1	1889	1889	1889	0.38	0.0	1889	0.38	1	0.0	1889	0.38	1	0.0
M503-2	2220	2220	2220	0.42	0.0	2220	0.42	1	0.0	2220	0.42	1	0.0
M503-3	1571	1571	1571	0.07	0.0	1571	0.07	1	0.0	1571	0.07	1	0.0
M504-1	2179	2179	2184 ↑	0.67	0.2	2179	3.32	5	0.0	2179	2.16	**3**	0.0
M504-2	1902	1902	1902	0.10	0.0	1902	0.10	1	0.0	1902	0.10	1	0.0
M504-3	1870	1870	1870	0.17	0.0	1870	0.17	1	0.0	1870	0.17	1	0.0
M505-1	1998	1998	1998	0.57	0.0	1998	0.57	1	0.0	1998	0.57	1	0.0
M505-2	1781	1781	1781	0.48	0.0	1781	0.48	1	0.0	1781	0.48	1	0.0
M505-3	1869	1869	1869	0.86	0.0	1869	0.86	1	0.0	1869	0.86	1	0.0
M506-1	1803	1803	1803	0.11	0.0	1803	0.11	1	0.0	1803	0.11	1	0.0
M506-2	1943	1943	1943	0.33	0.0	1943	0.33	1	0.0	1943	0.33	1	0.0
M506-3	2001	2001	2001	0.44	0.0	2001	0.44	1	0.0	2001	0.44	1	0.0
M507-1	2075	2075	2075	0.16	0.0	2075	0.16	1	0.0	2075	0.16	1	0.0
M507-2	1878	1878	1878	0.19	0.0	1878	0.19	1	0.0	1878	0.19	1	0.0
M507-3	1791	1791	1791	1.04	0.0	1791	1.04	1	0.0	1791	1.04	1	0.0
M508-1	1656	1656	1656	0.32	0.0	1656	0.32	1	0.0	1656	0.32	1	0.0
M508-2	1570	1570	1570	0.47	0.0	1570	0.47	1	0.0	1570	0.47	1	0.0
M508-3	1765	1765	1765	0.31	0.0	1765	0.31	1	0.0	1765	0.31	1	0.0
M509-1	1710	1710	1710	0.29	0.0	1710	0.29	1	0.0	1710	0.29	1	0.0
M509-2	1769	1769	1769	0.53	0.0	1769	0.53	1	0.0	1769	0.53	1	0.0
M509-3	2003	2003	2003	0.19	0.0	2003	0.19	1	0.0	2003	0.19	1	0.0
M510-1	2217	2217	2217	0.13	0.0	2217	0.13	1	0.0	2217	0.13	1	0.0
M510-2	1937	1937	1937	0.49	0.0	1937	0.49	1	0.0	1937	0.49	1	0.0
M510-3	1838	1838	1838	0.25	0.0	1838	0.25	1	0.0	1838	0.25	1	0.0
M601-1	800	800	800	0.55	0.0	800	0.55	1	0.0	800	0.55	1	0.0
M601-2	1340	1340	1340	0.82	0.0	1340	0.82	1	0.0	1340	0.82	1	0.0
M601-3	1091	1091	1096 ↑	0.90	0.5	1091	2.73	3	0.0	1091	3.18	3	0.0
M602-1	1112	1112	1114 ↑	0.74	0.2	1112	3.55	4	0.0	1112	2.55	**3**	0.0
M602-2	1052	1052	1052	1.01	0.0	1052	1.01	1	0.0	1052	1.01	1	0.0
M602-3	1264	1264	1268 ↑	0.89	0.3	1264	4.46	5	0.0	1264	2.65	**3**	0.0
M603-1	683	683	683	0.29	0.0	683	0.29	1	0.0	683	0.29	1	0.0
M603-2	966	966	966	0.42	0.0	966	0.42	1	0.0	966	0.42	1	0.0
M603-3	1200	1200	1200	0.68	0.0	1200	0.68	1	0.0	1200	0.68	1	0.0
M604-1	1022	1022	1022	0.18	0.0	1022	0.18	1	0.0	1022	0.18	1	0.0
M604-2	1055	1055	1055	0.70	0.0	1055	0.70	1	0.0	1055	0.70	1	0.0
M604-3	971	971	971	0.62	0.0	971	0.62	1	0.0	971	0.62	1	0.0
M605-1	1083	1083	1083	0.59	0.0	1083	0.59	1	0.0	1083	0.59	1	0.0
M605-2	1053	1053	1053	0.54	0.0	1053	0.54	1	0.0	1053	0.54	1	0.0
M605-3	1031	1031	1031	0.35	0.0	1031	0.35	1	0.0	1031	0.35	1	0.0

Table C.3: MMR-SCP results for type-K instances

instance	Best Known		DS			iDS-H				iDS-B			
	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
K401-1	13365	14440	14440	20.403	7.4	14440	20.4	1	7.4	14440	20.4	1	7.4
K401-2	14026	16372	16247 ↓	419.292	13.7	16243 ↓	897.8	2	13.6	16243 ↓	912.1	2	13.6
K401-3	11803	12974	12974	45.133	9.0	12974	45.2	1	9.0	12974	45.1	1	9.0
K402-1	12628	14235	14235	77.207	11.3	14235	77.2	1	11.3	14235	77.2	1	11.3
K402-2	14325	16335	16351 ↑	155.660	12.4	16335	496.7	3	12.3	16335	488.5	3	12.3
K402-3	12445	14306	14306	351.183	13.0	14306	351.2	1	13.0	14306	351.4	1	13.0
K403-1	14031	15139	15139	28.780	7.3	15139	28.8	1	7.3	15139	28.8	1	7.3
K403-2	14349	16523	16523	176.895	13.2	16523	177.3	1	13.2	16523	177.6	1	13.2
K403-3	12436	13613	13613	30.240	8.6	13613	30.2	1	8.6	13613	30.2	1	8.6
K404-1	13046	13472	13472	9.412	3.2	13472	9.4	1	3.2	13472	9.4	1	3.2
K404-2	13291	15871	15820 ↓	580.562	16.0	15820 ↓	580.6	1	16.0	**15813** ↓	4110.3	7	15.9
K404-3	13255	14898	14954 ↑	38.057	11.4	14881 ↓	152.8	4	10.9	14881 ↓	146.6	4	10.9
K405-1	13518	16242	16242	296.749	16.8	16242	297.8	1	16.8	16242	296.7	1	16.8
K405-2	14204	16311	16355 ↑	77.639	13.2	16311	372.7	4	12.9	16311	348.7	4	12.9
K405-3	12933	14549	14549	71.876	11.1	14539 ↓	147.4	2	11.0	14539 ↓	167.1	2	11.0
K406-1	12885	14067	14067	90.892	8.4	14067	91.1	1	8.4	14067	90.9	1	8.4
K406-2	12522	14551	14555 ↑	199.734	14.0	14551	427.5	2	13.9	14551	390.6	2	13.9
K406-3	12312	13941	13941	177.886	11.7	13941	178.4	1	11.7	13941	177.9	1	11.7
K407-1	13055	14928	14908 ↓	123.242	12.4	14908 ↓	123.9	1	12.4	14908 ↓	123.2	1	12.4
K407-2	13509	15875	15901 ↑	380.145	15.0	15875	1166.1	3	14.9	15875	1151.9	3	14.9
K407-3	13465	15262	15268 ↑	41.457	11.8	15262	246.8	5	11.8	15262	108.8	**2**	11.8
K408-1	12193	13752	13654 ↓	91.841	10.7	13654 ↓	91.8	1	10.7	13654 ↓	92.0	1	10.7
K408-2	13907	15843	15843	47.826	12.2	15843	47.9	1	12.2	15843	47.8	1	12.2
K408-3	12430	13566	13566	34.834	8.4	13566	34.8	1	8.4	13566	34.8	1	8.4
K409-1	13283	14872	14872	160.887	10.7	14872	161.2	1	10.7	14872	160.9	1	10.7
K409-2	12285	14020	13971 ↓	117.825	12.1	13971 ↓	117.8	1	12.1	13971 ↓	117.8	1	12.1
K409-3	12949	14414	14414	50.856	10.2	14414	50.9	1	10.2	14414	50.9	1	10.2
K410-1	13049	15367	15287 ↓	93.437	14.6	15254 ↓	316.7	3	14.5	15254 ↓	291.5	3	14.5
K410-2	14265	15903	15903	168.399	10.3	15903	168.4	1	10.3	15903	168.7	1	10.3
K410-3	13983	16635	16635	344.680	15.9	16635	346.0	1	15.9	16635	346.3	1	15.9
K501-1	10925	11743	11743	36.964	7.0	11743	37.0	1	7.0	11743	37.0	1	7.0
K501-2	10997	11899	11899	16.887	7.6	11899	16.9	1	7.6	11899	16.9	1	7.6
K501-3	11328	11631	11631	12.344	2.6	11631	12.3	1	2.6	11631	12.3	1	2.6
K502-1	9891	10144	10152 ↑	15.681	2.6	10144	67.2	3	2.5	10144	58.5	3	2.5
K502-2	11450	12422	12422	125.340	7.8	12422	125.3	1	7.8	12422	125.3	1	7.8
K502-3	10465	11016	11016	18.901	5.0	11016	18.9	1	5.0	11016	18.9	1	5.0
K503-1	10265	10265	10291 ↑	4.596	0.3	10265	9.2	2	0.0	10265	9.5	2	0.0
K503-2	11305	12321	12321	22.899	8.2	12321	22.9	1	8.2	12321	22.9	1	8.2
K503-3	10650	11957	11919 ↓	69.742	10.6	11912 ↓	134.9	2	10.6	11912 ↓	128.8	2	10.6
K504-1	10931	11429	11429	7.445	4.4	11429	7.4	1	4.4	11429	7.4	1	4.4
K504-2	12234	13388	13388	30.963	8.6	13388	31.0	1	8.6	13388	31.0	1	8.6
K504-3	11281	11943	11943	22.659	5.5	11943	22.7	1	5.5	11943	22.7	1	5.5
K505-1	11342	12102	12116 ↑	10.666	6.4	12102	20.6	2	6.3	12102	21.4	2	6.3
K505-2	11847	13663	13663	342.522	13.3	13663	342.5	1	13.3	13663	342.7	1	13.3
K505-3	10815	12159	12159	253.301	11.1	12159	253.3	1	11.1	12159	253.3	1	11.1
K506-1	10047	10232	10232	10.340	1.8	10232	10.3	1	1.8	10232	10.3	1	1.8
K506-2	11645	12236	12237 ↑	31.781	4.8	12236	64.9	2	4.8	12236	63.6	2	4.8
K506-3	9858	10291	10291	10.506	4.2	10291	10.5	1	4.2	10291	10.5	1	4.2
K507-1	10605	10661	10661	3.128	0.5	10661	3.1	1	0.5	10661	3.1	1	0.5
K507-2	11591	12200	12200	27.002	5.0	12200	27.0	1	5.0	12200	27.0	1	5.0
K507-3	10506	11105	11105	16.122	5.4	11105	16.1	1	5.4	11105	16.1	1	5.4
K508-1	10641	11095	11095	4.337	4.1	11095	4.3	1	4.1	11095	4.3	1	4.1
K508-2	11352	12557	12527 ↓	74.643	9.4	12514 ↓	158.0	2	9.3	12514 ↓	138.8	2	9.3
K508-3	11179	11554	11554	9.094	3.2	11554	9.1	1	3.2	11554	9.1	1	3.2
K509-1	11362	12151	12111 ↓	49.222	6.2	12111 ↓	49.2	1	6.2	12111 ↓	49.2	1	6.2
K509-2	12101	13236	13246 ↑	66.195	8.6	13236	197.6	3	8.6	13236	168.0	3	8.6
K509-3	11061	11862	11871 ↑	9.010	6.8	11862	36.8	3	6.8	11862	26.2	3	6.8
K510-1	10443	10969	11006 ↑	23.418	5.1	10969	256.6	9	4.8	10969	201.6	9	4.8
K510-2	11437	12298	12300 ↑	39.650	7.0	12298	128.0	3	7.0	12298	116.1	3	7.0
K510-3	11276	12253	12253	41.684	8.0	12253	41.7	1	8.0	12253	41.7	1	8.0
K601-1	7099	7127	7136 ↑	110.892	0.5	7127	208.5	2	0.4	7127	199.3	2	0.4
K601-2	7190	7242	7242	344.573	0.7	7242	344.6	1	0.7	7242	344.6	1	0.7
K601-3	5834	5834	5834	73.339	0.0	5834	73.3	1	0.0	5834	73.3	1	0.0
K602-1	6640	6640	6640	77.797	0.0	6640	77.8	1	0.0	6640	77.8	1	0.0
K602-2	7146	7146	7165 ↑	41.927	0.3	7146	81.3	2	0.0	7146	74.1	2	0.0
K602-3	6605	6733	6733	72.275	1.9	6733	72.3	1	1.9	6733	72.3	1	1.9
K603-1	6986	6986	7023 ↑	43.893	0.5	6986	98.3	2	0.0	6986	81.9	2	0.0
K603-2	7404	7828	7828	108.521	5.4	7828	108.5	1	5.4	7828	108.5	1	5.4
K603-3	6943	6943	6943	64.947	0.0	6943	64.9	1	0.0	6943	64.9	1	0.0
K604-1	6343	6343	6343	53.900	0.0	6343	53.9	1	0.0	6343	53.9	1	0.0
K604-2	7271	7822	7822	616.776	7.0	7822	616.8	1	7.0	7822	616.8	1	7.0
K604-3	5673	5673	5673	4.224	0.0	5673	4.2	1	0.0	5673	4.2	1	0.0
K605-1	6641	6641	6641	34.390	0.0	6641	34.4	1	0.0	6641	34.4	1	0.0
K605-2	7589	7674	7674	146.931	1.1	7674	146.9	1	1.1	7674	146.9	1	1.1
K605-3	7189	7462	7462	216.769	3.7	7462	216.8	1	3.7	7462	216.8	1	3.7

Table D.1: MMR-GAP results for type-A instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
a0504010-01	16	16	16	0.01	0.0	16	0.01	1	0.0	16	0.01	1	0.0
a0504010-02	18	18	18	0.02	0.0	18	0.02	1	0.0	18	0.02	1	0.0
a0504010-03	5	5	5	0.01	0.0	5	0.01	1	0.0	5	0.01	1	0.0
a0504010-04	18	18	18	0.02	0.0	18	0.02	1	0.0	18	0.02	1	0.0
a0504010-05	12	12	12	0.02	0.0	12	0.02	1	0.0	12	0.02	1	0.0
a0504025-01	80	80	81 $\uparrow$	0.07	1.2	80	0.13	2	0.0	80	0.13	2	0.0
a0504025-02	67	67	67	0.03	0.0	67	0.03	1	0.0	67	0.03	1	0.0
a0504025-03	59	59	59	0.03	0.0	59	0.03	1	0.0	59	0.03	1	0.0
a0504025-04	71	71	71	0.07	0.0	71	0.07	1	0.0	71	0.07	1	0.0
a0504025-05	84	84	86 $\uparrow$	0.21	2.3	84	0.49	2	0.0	84	0.39	2	0.0
a0504050-01	261	261	261	0.21	0.0	261	0.21	1	0.0	261	0.21	1	0.0
a0504050-02	204	204	204	0.12	0.0	204	0.12	1	0.0	204	0.12	1	0.0
a0504050-03	164	164	164	0.05	0.0	164	0.05	1	0.0	164	0.05	1	0.0
a0504050-04	227	227	227	0.13	0.0	227	0.13	1	0.0	227	0.13	1	0.0
a0504050-05	219	219	219	0.22	0.0	219	0.22	1	0.0	219	0.22	1	0.0
a0508010-01	23	23	23	0.03	0.0	23	0.03	1	0.0	23	0.03	1	0.0
a0508010-02	29	29	29	0.03	0.0	29	0.03	1	0.0	29	0.03	1	0.0
a0508010-03	19	19	20 $\uparrow$	0.07	5.0	19	0.18	3	0.0	19	0.14	**2**	0.0
a0508010-04	27	27	27	0.02	0.0	27	0.02	1	0.0	27	0.02	1	0.0
a0508010-05	27	27	27	0.06	0.0	27	0.06	1	0.0	27	0.06	1	0.0
a0508025-01	118	141	141	0.15	16.3	141	0.15	1	16.3	141	0.15	1	16.3
a0508025-02	108	108	108	0.07	0.0	108	0.07	1	0.0	108	0.07	1	0.0
a0508025-03	91	91	91	0.23	0.0	91	0.23	1	0.0	91	0.23	1	0.0
a0508025-04	104	116	116	0.07	10.3	116	0.07	1	10.3	116	0.07	1	10.3
a0508025-05	105	105	105	0.12	0.0	105	0.12	1	0.0	105	0.12	1	0.0
a0508050-01	300	427	427	1.03	29.7	427	1.05	1	29.7	427	1.05	1	29.7
a0508050-02	291	388	388	0.31	25.0	388	0.31	1	25.0	388	0.31	1	25.0
a0508050-03	348	487	487	1.21	28.5	487	1.24	1	28.5	487	1.24	1	28.5
a0508050-04	307	390	390	0.25	21.3	390	0.25	1	21.3	390	0.25	1	21.3
a0508050-05	307	418	418	0.52	26.6	418	0.52	1	26.6	418	0.52	1	26.6
a1004010-01	14	14	14	0.04	0.0	14	0.04	1	0.0	14	0.04	1	0.0
a1004010-02	16	16	16	0.02	0.0	16	0.02	1	0.0	16	0.02	1	0.0
a1004010-03	14	14	14	0.01	0.0	14	0.01	1	0.0	14	0.01	1	0.0
a1004010-04	13	13	13	0.02	0.0	13	0.02	1	0.0	13	0.02	1	0.0
a1004010-05	16	16	17 $\uparrow$	0.05	5.9	16	0.15	3	0.0	16	0.11	**2**	0.0
a1004025-01	78	78	78	0.16	0.0	78	0.16	1	0.0	78	0.16	1	0.0
a1004025-02	54	54	54	0.03	0.0	54	0.03	1	0.0	54	0.03	1	0.0
a1004025-03	64	64	64	0.08	0.0	64	0.08	1	0.0	64	0.08	1	0.0
a1004025-04	45	45	45	0.02	0.0	45	0.02	1	0.0	45	0.02	1	0.0
a1004025-05	73	73	75 $\uparrow$	0.17	2.7	73	0.69	4	0.0	73	0.79	4	0.0
a1004050-01	184	211	211	0.24	12.8	210 $\downarrow$	2.10	9	12.4	210 $\downarrow$	2.40	9	12.4
a1004050-02	165	182	182	0.11	9.3	182	0.11	1	9.3	182	0.11	1	9.3
a1004050-03	181	206	206	0.29	12.1	206	0.29	1	12.1	206	0.30	1	12.1
a1004050-04	161	169	169	0.06	4.7	169	0.06	1	4.7	169	0.06	1	4.7
a1004050-05	178	206	206	0.55	13.6	205 $\downarrow$	1.87	3	13.2	205 $\downarrow$	1.92	3	13.2
a1008010-01	22	22	22	0.02	0.0	22	0.02	1	0.0	22	0.02	1	0.0
a1008010-02	34	34	34	0.10	0.0	34	0.10	1	0.0	34	0.10	1	0.0
a1008010-03	36	39	39	0.20	7.7	39	0.20	1	7.7	39	0.20	1	7.7
a1008010-04	16	16	16	0.04	0.0	16	0.04	1	0.0	16	0.04	1	0.0
a1008010-05	29	29	29	0.12	0.0	29	0.12	1	0.0	29	0.12	1	0.0
a1008025-01	77	87	87	0.05	11.5	87	0.05	1	11.5	**86** $\downarrow$	0.17	3	10.5
a1008025-02	103	117	117	0.11	12.0	117	0.11	1	12.0	117	0.11	1	12.0
a1008025-03	102	126	126	0.69	19.0	126	0.69	1	19.0	126	0.69	1	19.0
a1008025-04	98	113	113	0.08	13.3	113	0.08	1	13.3	113	0.08	1	13.3
a1008025-05	91	109	109	0.12	16.5	109	0.12	1	16.5	109	0.12	1	16.5
a1008050-01	248	367	367	0.26	32.4	366 $\downarrow$	164.77	275	32.2	366 $\downarrow$	459.16	275	32.2
a1008050-02	296	433	433	2.56	31.6	433	2.56	1	31.6	433	2.56	1	31.6
a1008050-03	306	419	419	8.63	27.0	418 $\downarrow$	291.91	27	26.6	418 $\downarrow$	221.88	27	26.8
a1008050-04	240	321	321	0.11	25.2	321	0.11	1	25.2	321	0.11	1	25.2
a1008050-05	264	376	376	0.20	29.8	376	0.20	1	29.8	376	0.20	1	29.8

Table D.2: MMR-GAP results for type-B instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
b0504010-01	18	18	24 ↑	0.12	25.0	18	0.23	2	0.0	18	0.23	2	0.0
b0504010-02	19	19	19	0.11	0.0	19	0.11	1	0.0	19	0.11	1	0.0
b0504010-03	12	12	12	0.06	0.0	12	0.06	1	0.0	12	0.06	1	0.0
b0504010-04	25	25	28 ↑	0.09	10.7	25	0.42	4	0.0	25	0.29	4	0.0
b0504010-05	22	22	22	0.14	0.0	22	0.14	1	0.0	22	0.14	1	0.0
b0504025-01	84	84	84	0.28	0.0	84	0.28	1	0.0	84	0.28	1	0.0
b0504025-02	64	64	64	0.14	0.0	64	0.14	1	0.0	64	0.14	1	0.0
b0504025-03	38	38	38	0.12	0.0	38	0.12	1	0.0	38	0.12	1	0.0
b0504025-04	66	66	68 ↑	0.13	2.9	66	1.89	11	0.0	66	1.62	**10**	0.0
b0504025-05	77	77	77	0.20	0.0	77	0.20	1	0.0	77	0.20	1	0.0
b0504050-01	248	248	248	0.59	0.0	248	0.59	1	0.0	248	0.59	1	0.0
b0504050-02	227	227	227	1.06	0.0	227	1.06	1	0.0	227	1.06	1	0.0
b0504050-03	108	108	108	0.15	0.0	108	0.15	1	0.0	108	0.15	1	0.0
b0504050-04	215	215	215	0.68	0.0	215	0.68	1	0.0	215	0.68	1	0.0
b0504050-05	191	191	191	0.43	0.0	191	0.43	1	0.0	191	0.43	1	0.0
b0508010-01	31	31	31	0.87	0.0	31	0.87	1	0.0	31	0.87	1	0.0
b0508010-02	29	29	29	0.19	0.0	29	0.19	1	0.0	29	0.19	1	0.0
b0508010-03	30	30	30	0.12	0.0	30	0.12	1	0.0	30	0.12	1	0.0
b0508010-04	45	45	45	2.25	0.0	45	2.25	1	0.0	45	2.25	1	0.0
b0508010-05	31	31	31	0.20	0.0	31	0.20	1	0.0	31	0.20	1	0.0
b0508025-01	94	111	111	15.28	15.3	111	33.58	2	15.3	111	34.29	2	15.3
b0508025-02	113	122	122	4.20	7.4	122	4.20	1	7.4	122	4.20	1	7.4
b0508025-03	123	147	147	10.61	16.3	147	10.61	1	16.3	147	10.61	1	16.3
b0508025-04	113	131	131	7.99	13.7	**130** ↓	48.99	4	13.1	131	7.99	1	13.7
b0508025-05	95	107	107	3.37	11.2	107	3.37	1	11.2	107	3.37	1	11.2
b0508050-01	258	346	346	54.15	25.4	345 ↓	99.58	3	25.2	345 ↓	135.25	3	25.2
b0508050-02	298	415	415	49.21	28.2	415	49.21	1	28.2	415	49.21	1	28.2
b0508050-03	292	402	402	63.67	27.4	402	63.67	1	27.4	402	63.67	1	27.4
b0508050-04	293	421	421	70.98	30.4	421	70.98	1	30.4	421	70.98	1	30.4
b0508050-05	269	398	398	55.11	32.4	398	55.13	1	32.4	398	55.13	1	32.4
b1004010-01	21	21	24 ↑	0.17	12.5	21	0.56	4	0.0	21	0.66	4	0.0
b1004010-02	16	16	18 ↑	0.11	11.1	16	0.34	3	0.0	16	0.31	3	0.0
b1004010-03	16	16	16	0.17	0.0	16	0.17	1	0.0	16	0.17	1	0.0
b1004010-04	14	14	14	0.12	0.0	14	0.12	1	0.0	14	0.12	1	0.0
b1004010-05	28	28	31 ↑	0.27	9.7	28	0.86	**3**	0.0	28	1.08	4	0.0
b1004025-01	100	103	103	2.42	2.9	102 ↓	7.80	3	2.0	102 ↓	9.53	3	2.0
b1004025-02	71	71	72 ↑	0.38	1.4	71	1.01	3	0.0	71	1.06	3	0.0
b1004025-03	63	63	69 ↑	0.34	8.7	63	0.78	2	0.0	63	0.72	2	0.0
b1004025-04	58	58	58	0.31	0.0	58	0.31	1	0.0	58	0.31	1	0.0
b1004025-05	76	76	79 ↑	1.18	3.8	76	4.15	3	0.0	76	3.89	3	0.0
b1004050-01	184	194	194	3.55	5.2	193 ↓	6.94	2	4.7	193 ↓	7.17	2	4.7
b1004050-02	189	189	189	0.74	0.0	189	0.74	1	0.0	189	0.74	1	0.0
b1004050-03	186	188	192 ↑	5.46	3.1	187 ↓	39.88	7	0.5	187 ↓	39.95	7	0.5
b1004050-04	190	199	199	4.20	4.5	199	4.20	1	4.5	199	4.20	1	4.5
b1004050-05	185	197	197	9.64	6.1	195 ↓	21.82	2	5.1	195 ↓	18.94	2	5.1
b1008010-01	24	24	24	0.34	0.0	24	0.34	1	0.0	24	0.34	1	0.0
b1008010-02	40	40	41 ↑	4.05	2.4	40	10.21	2	0.0	40	10.14	2	0.0
b1008010-03	42	42	44 ↑	3.65	4.5	42	130.41	19	0.0	42	123.66	**18**	0.0
b1008010-04	25	25	26 ↑	0.60	3.8	25	1.49	2	0.0	25	1.31	2	0.0
b1008010-05	25	25	25	0.72	0.0	25	0.72	1	0.0	25	0.72	1	0.0
b1008025-01	83	91	91	5.04	8.8	91	5.11	1	8.8	91	5.09	1	8.8
b1008025-02	123	171	171	829.66	28.1	170 ↓	1967.46	2	27.6	170 ↓	1785.74	2	27.6
b1008025-03	113	124	125 ↑	5.77	9.6	124	11.90	2	8.9	124	13.02	2	8.9
b1008025-04	97	116	116	23.35	16.4	116	23.35	1	16.4	116	23.35	1	16.4
b1008025-05	96	116	116	29.19	17.2	116	29.19	1	17.2	116	29.19	1	17.2
b1008050-01	236	338	338	1083.19	30.2	338	1083.19	1	30.2	338	1083.19	1	30.2
b1008050-02	293	444	445 ↑	3600.00	34.2	445 ↑	3600.00	1	34.2	445 ↑	3600.00	1	34.2
b1008050-03	291	409	409	2621.89	28.9	409	2621.89	1	28.9	409	2621.89	1	28.9
b1008050-04	259	381	381	3600.00	32.0	381	3600.00	1	32.0	381	3600.00	1	32.0
b1008050-05	238	346	346	3600.00	31.2	346	3600.00	1	31.2	346	3600.00	1	31.2

Table D.3: MMR-GAP results for type-C instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
c0504010-01	18	18	18	0.14	0.0	18	0.14	1	0.0	18	0.14	1	0.0
c0504010-02	16	16	16	0.06	0.0	16	0.06	1	0.0	16	0.06	1	0.0
c0504010-03	14	14	14	0.08	0.0	14	0.08	1	0.0	14	0.08	1	0.0
c0504010-04	17	17	17	0.15	0.0	17	0.15	1	0.0	17	0.15	1	0.0
c0504010-05	23	23	23	0.15	0.0	23	0.15	1	0.0	23	0.15	1	0.0
c0504025-01	47	47	47	0.18	0.0	47	0.18	1	0.0	47	0.18	1	0.0
c0504025-02	60	60	60	0.17	0.0	60	0.17	1	0.0	60	0.17	1	0.0
c0504025-03	103	103	103	0.27	0.0	103	0.27	1	0.0	103	0.27	1	0.0
c0504025-04	83	83	83	0.19	0.0	83	0.19	1	0.0	83	0.19	1	0.0
c0504025-05	85	85	85	0.34	0.0	85	0.34	1	0.0	85	0.34	1	0.0
c0504050-01	160	160	160	1.37	0.0	160	1.37	1	0.0	160	1.37	1	0.0
c0504050-02	202	202	202	2.85	0.0	202	2.85	1	0.0	202	2.85	1	0.0
c0504050-03	217	217	217	2.76	0.0	217	2.76	1	0.0	217	2.76	1	0.0
c0504050-04	173	173	173	0.49	0.0	173	0.49	1	0.0	173	0.49	1	0.0
c0504050-05	244	269	269	3.58	9.3	269	3.59	1	9.3	269	3.59	1	9.3
c0508010-01	25	25	25	0.70	0.0	25	0.70	1	0.0	25	0.70	1	0.0
c0508010-02	28	28	28	0.18	0.0	28	0.18	1	0.0	28	0.18	1	0.0
c0508010-03	35	35	35	0.69	0.0	35	0.69	1	0.0	35	0.69	1	0.0
c0508010-04	39	39	39	0.89	0.0	39	0.89	1	0.0	39	0.89	1	0.0
c0508010-05	28	28	28	0.29	0.0	28	0.29	1	0.0	28	0.29	1	0.0
c0508025-01	118	132	132	0.88	10.6	132	0.88	1	10.6	132	0.88	1	10.6
c0508025-02	107	112	112	2.38	4.5	112	2.38	1	4.5	112	2.38	1	4.5
c0508025-03	114	131	131	5.61	13.0	131	5.66	1	13.0	131	5.66	1	13.0
c0508025-04	119	142	142	20.82	16.2	141 $\downarrow$	62.26	3	15.6	141 $\downarrow$	72.20	3	15.6
c0508025-05	103	116	116	3.78	11.2	113 $\downarrow$	9.39	2	8.8	113 $\downarrow$	8.91	2	8.8
c0508050-01	275	402	402	59.66	31.6	402	59.66	1	31.6	402	59.66	1	31.6
c0508050-02	283	389	389	18.80	27.2	389	18.80	1	27.2	389	18.80	1	27.2
c0508050-03	284	418	419 $\uparrow$	41.25	32.2	418	96.07	2	32.1	418	107.59	2	32.1
c0508050-04	326	503	503	88.67	35.2	503	88.67	1	35.2	503	88.67	1	35.2
c0508050-05	261	365	365	15.01	28.5	365	15.05	1	28.5	365	15.05	1	28.5
c1004010-01	20	20	21 $\uparrow$	0.21	4.8	20	2.12	9	0.0	20	1.84	**8**	0.0
c1004010-02	15	15	15	0.19	0.0	15	0.19	1	0.0	15	0.19	1	0.0
c1004010-03	24	24	28 $\uparrow$	0.23	14.3	24	0.48	2	0.0	24	0.46	2	0.0
c1004010-04	26	26	26	0.11	0.0	26	0.11	1	0.0	26	0.11	1	0.0
c1004010-05	25	25	25	0.11	0.0	25	0.11	1	0.0	25	0.11	1	0.0
c1004025-01	67	67	68 $\uparrow$	0.30	1.5	67	1.84	5	0.0	67	1.77	5	0.0
c1004025-02	58	58	61 $\uparrow$	0.28	4.9	58	1.24	4	0.0	58	1.15	4	0.0
c1004025-03	70	70	70	0.39	0.0	70	0.39	1	0.0	70	0.39	1	0.0
c1004025-04	65	65	66 $\uparrow$	0.35	1.5	65	7.84	17	0.0	65	8.36	17	0.0
c1004025-05	83	83	85 $\uparrow$	0.63	2.4	83	4.64	6	0.0	83	4.60	6	0.0
c1004050-01	202	212	216 $\uparrow$	5.52	6.5	212	318.00	34	4.7	212	343.69	34	4.7
c1004050-02	219	219	219	1.61	0.0	219	1.61	1	0.0	219	1.61	1	0.0
c1004050-03	189	199	201 $\uparrow$	6.23	6.0	197 $\downarrow$	17.99	3	4.1	197 $\downarrow$	17.38	3	4.1
c1004050-04	187	187	187	1.31	0.0	187	1.31	1	0.0	187	1.31	1	0.0
c1004050-05	202	226	230 $\uparrow$	14.72	12.2	225 $\downarrow$	1013.36	48	10.2	225 $\downarrow$	1056.08	**46**	10.2
c1008010-01	29	29	29	0.67	0.0	29	0.67	1	0.0	29	0.67	1	0.0
c1008010-02	40	40	43 $\uparrow$	19.64	7.0	40	913.48	28	0.0	40	889.42	**27**	0.0
c1008010-03	33	33	34 $\uparrow$	0.56	2.9	33	1.28	2	0.0	33	1.31	2	0.0
c1008010-04	37	37	41 $\uparrow$	2.36	9.8	37	11.14	3	0.0	37	7.77	3	0.0
c1008010-05	32	32	32	1.09	0.0	32	1.09	1	0.0	32	1.09	1	0.0
c1008025-01	85	87	87	4.70	2.3	87	4.70	1	2.3	87	4.70	1	2.3
c1008025-02	121	157	159 $\uparrow$	329.82	23.9	155 $\downarrow$	1024.08	3	21.9	155 $\downarrow$	908.12	3	21.9
c1008025-03	111	129	129	52.73	14.0	128 $\downarrow$	146.62	3	13.3	128 $\downarrow$	135.10	3	13.3
c1008025-04	112	120	120	7.15	6.7	120	7.15	1	6.7	120	7.15	1	6.7
c1008025-05	111	142	142	257.09	21.8	142	257.09	1	21.8	142	257.09	1	21.8
c1008050-01	250	346	346	804.54	27.7	346	805.55	1	27.7	346	805.55	1	27.7
c1008050-02	285	436	436	3600.00	34.6	436	3600.00	1	34.6	436	3600.00	1	34.6
c1008050-03	283	393	393	1334.99	28.0	393	1334.99	1	28.0	393	1334.99	1	28.0
c1008050-04	293	418	418	1355.04	29.9	418	1355.04	1	29.9	418	1355.04	1	29.9
c1008050-05	264	390	390	3600.00	32.3	390	3600.00	1	32.3	390	3600.00	1	32.3

Table D.4: MMR-GAP results for type-E instances

	Best Known		DS			iDS-H				iDS-B			
instance	LB	UB	obj	time	%gap	obj	time	iter	%gap	obj	time	iter	%gap
e0504010-01	224	224	224	0.44	0.0	224	0.44	1	0.0	224	0.44	1	0.0
e0504010-02	184	184	184	0.70	0.0	184	0.70	1	0.0	184	0.70	1	0.0
e0504010-03	133	133	133	0.16	0.0	133	0.16	1	0.0	133	0.16	1	0.0
e0504010-04	190	190	190	0.79	0.0	190	0.79	1	0.0	190	0.79	1	0.0
e0504010-05	221	221	225 ↑	2.87	1.8	221	12.36	4	0.0	221	14.77	4	0.0
e0504025-01	826	826	826	11.01	0.0	826	11.01	1	0.0	826	11.01	1	0.0
e0504025-02	706	706	706	6.51	0.0	706	6.51	1	0.0	706	6.51	1	0.0
e0504025-03	659	659	660 ↑	3.61	0.2	659	77.88	13	0.0	659	79.42	13	0.0
e0504025-04	639	639	639	7.37	0.0	639	7.37	1	0.0	639	7.37	1	0.0
e0504025-05	670	670	670	15.90	0.0	670	15.90	1	0.0	670	15.90	1	0.0
e0504050-01	1689	2056	2056	20.71	17.9	2056	20.71	1	17.9	2056	20.71	1	17.9
e0504050-02	1562	2028	2028	129.15	23.0	2028	129.15	1	23.0	2028	129.15	1	23.0
e0504050-03	1312	1519	1519	4.42	13.6	1519	4.42	1	13.6	1519	4.42	1	13.6
e0504050-04	1365	1689	1689	18.42	19.2	1670 ↓	362.72	9	18.3	1670 ↓	358.70	9	18.3
e0504050-05	1420	1731	1731	13.56	18.0	1731	13.56	1	18.0	1731	13.56	1	18.0
e0508010-01	290	331	337 ↑	19.46	13.9	331	124.75	3	12.4	331	105.86	3	12.4
e0508010-02	240	314	318 ↑	29.24	24.5	309 ↓	113.63	3	22.3	309 ↓	79.86	3	22.3
e0508010-03	274	343	343	31.18	20.1	334 ↓	68.43	2	18.0	334 ↓	77.30	2	18.0
e0508010-04	282	400	400	154.01	29.5	400	154.01	1	29.5	400	154.01	1	29.5
e0508010-05	264	264	264	3.44	0.0	264	3.44	1	0.0	264	3.44	1	0.0
e0508025-01	833	1230	1230	1897.28	32.3	1230	1905.65	1	32.3	1230	1905.65	1	32.3
e0508025-02	775	1254	1251 ↓	3600.00	38.0	1251 ↓	3600.00	1	38.0	1251 ↓	3600.00	1	38.0
e0508025-03	828	1271	1271	268.21	34.9	1271	269.27	1	34.9	1271	269.27	1	34.9
e0508025-04	941	1529	1541 ↑	3600.00	38.9	1541 ↑	3600.00	1	38.9	1541 ↑	3600.00	1	38.9
e0508025-05	783	1122	1122	31.56	30.2	1122	31.56	1	30.2	1122	31.56	1	30.2
e0508050-01	2447	3985	4009 ↑	3600.00	39.0	4009 ↑	3600.00	1	39.0	4009 ↑	3600.00	1	39.0
e0508050-02	2173	3596	3603 ↑	3600.00	39.7	3603 ↑	3600.00	1	39.7	3603 ↑	3600.00	1	39.7
e0508050-03	2194	3655	3655	3600.00	40.0	3655	3600.00	1	40.0	3655	3600.00	1	40.0
e0508050-04	2337	3711	3711	3600.00	37.0	3711	3600.00	1	37.0	3711	3600.00	1	37.0
e0508050-05	2103	3448	3448	1899.93	39.0	3448	1899.93	1	39.0	3448	1899.93	1	39.0
e1004010-01	257	266	273 ↑	94.38	5.9	272 ↑	1757.52	14	5.5	272 ↑	1678.94	14	5.5
e1004010-02	223	223	278 ↑	21.43	19.8	278 ↑	21.43	1	19.8	278 ↑	21.43	1	19.8
e1004010-03	259	270	277 ↑	58.88	6.5	270	190.75	3	4.1	270	190.62	3	4.1
e1004010-04	250	250	255 ↑	20.99	2.0	255 ↑	20.99	1	2.0	255 ↑	20.99	1	2.0
e1004010-05	191	191	201 ↑	4.83	5.0	201 ↑	4.83	1	5.0	201 ↑	4.83	1	5.0
e1004025-01	629	778	793 ↑	708.83	20.7	779 ↑	2102.07	2	19.3	779 ↑	2488.24	2	19.3
e1004025-02	573	674	674	113.52	15.0	642 ↓	1030.07	5	10.7	642 ↓	844.25	5	10.7
e1004025-03	566	676	676	1029.30	16.3	676	1029.31	1	16.3	676	1029.31	1	16.3
e1004025-04	690	829	829	515.01	16.8	824 ↓	957.66	2	16.3	824 ↓	1126.31	2	16.3
e1004025-05	562	629	629	187.29	10.7	629	187.29	1	10.7	629	187.29	1	10.7
e1004050-01	1312	1651	1651	1005.86	20.5	1651	1005.86	1	20.5	**1647** ↓	3600.00	4	20.3
e1004050-02	1318	1656	1656	836.70	20.4	1656	836.70	1	20.4	1656	836.70	1	20.4
e1004050-03	1289	1667	1667	1963.44	22.7	1667	1963.44	1	22.7	1667	1963.44	1	22.7
e1004050-04	1470	1988	2006 ↑	3600.00	26.7	2006 ↑	3600.00	1	26.7	2006 ↑	3600.00	1	26.7
e1004050-05	1222	1456	1456	136.91	16.1	1456	136.91	1	16.1	1456	136.91	1	16.1
e1008010-01	391	572	577 ↑	3600.00	32.2	577 ↑	3600.00	1	32.2	577 ↑	3600.00	1	32.2
e1008010-02	362	574	578 ↑	3600.00	37.4	578 ↑	3600.00	1	37.4	578 ↑	3600.00	1	37.4
e1008010-03	312	427	428 ↑	3600.00	27.1	428 ↑	3600.00	1	27.1	428 ↑	3600.00	1	27.1
e1008010-04	396	578	583 ↑	3600.00	32.1	583 ↑	3600.00	1	32.1	583 ↑	3600.00	1	32.1
e1008010-05	370	538	534 ↓	3600.00	30.7	534 ↓	3600.00	1	30.7	534 ↓	3600.00	1	30.7
e1008025-01	1210	1893	1889 ↓	3600.00	35.9	1889 ↓	3600.00	1	35.9	1889 ↓	3600.00	1	35.9
e1008025-02	1107	1867	1895 ↑	3600.00	41.6	1895 ↑	3600.00	1	41.6	1895 ↑	3600.00	1	41.6
e1008025-03	1012	1616	1616	3600.00	37.4	1616	3600.00	1	37.4	1616	3600.00	1	37.4
e1008025-04	1058	1713	1713	3600.00	38.2	1713	3600.00	1	38.2	1713	3600.00	1	38.2
e1008025-05	1090	1726	1726	3600.00	36.8	1726	3600.00	1	36.8	1726	3600.00	1	36.8
e1008050-01	2775	4588	4660 ↑	3600.00	40.5	4660 ↑	3600.00	1	40.5	4660 ↑	3600.00	1	40.5
e1008050-02	2707	4349	4351 ↑	3600.00	37.8	4351 ↑	3600.00	1	37.8	4351 ↑	3600.00	1	37.8
e1008050-03	2581	4233	4239 ↑	3600.00	39.1	4239 ↑	3600.00	1	39.1	4239 ↑	3600.00	1	39.1
e1008050-04	2390	3947	3953 ↑	3600.00	39.5	3953 ↑	3600.00	1	39.5	3953 ↑	3600.00	1	39.5
e1008050-05	2394	4040	4062 ↑	3600.00	41.1	4062 ↑	3600.00	1	41.1	4062 ↑	3600.00	1	41.1