Dipartimento di Economia Marco Biagi

DEMB Working Paper Series

N. 179

Investment in Early Education and Job Market Signaling

Luigi Brighi[1], Marcello D'Amato[2]

October 2020

[1] University of Modena and Reggio Emilia
RECent, Research Centre for Economics
Address: Viale Berengario 51, 41121, Modena, Italy
Email: luigi.brighi@unimore.it

[2] University of Naples Suor Orsola Benincasa
Address: corso Vittorio Emanuele 334-ter, naples, Italy
Email: marcello.damato@unisa.it

ISSN: 2281-440X online

Investment in Early Education and Job Market Signaling

Luigi Brighi[a] Marcello D'Amato[b]

October 2020

Abstract

We consider a signaling model of the job market in which workers, before choosing their level of education, have the opportunity to undertake an unobservable investment in activities aimed at saving on future education costs. Sufficiently high levels of investments allow a low productivity worker to cut the marginal costs of signaling below the high productivity worker's. In contrast to standard results, we find that the equilibrium outcome will depend on the relative magnitude of workers' average productivity. If average productivity exceeds a certain threshold the most plausible solution is a refined pooling equilibrium in which all workers attain the same level of over-education and are paid the same wage. Otherwise, the most plausible outcome is the standard least cost separating equilibrium in which only high ability workers are over-educated.

JEL Classification Numbers: C72, D82, J24.

Keywords: Signaling, Pooling Equilibrium, Single Crossing, Early Education

[a]Dipartimento di Economia Marco Biagi, Università di Modena e Reggio Emilia, viale Berengario 51, 41121 Modena, Italy, RECent, e-mail: luigi.brighi@unimore.it.

[b]Dipartimento di Scienze Giuridiche, Università di Napoli Suor Orsola Benincasa, corso Vittorio Emanuele 334-ter, 80135 Napoli, Italy, Celpe, Csef, e-mail: marcello.damato@unisa.it.

1 Introduction

Parents usually undertake costly investments in activities aimed at improving children performance in education and working activities in adulthood. These investments may consists of effort, time and money spent in more qualified pre-school and school activities, in extra teaching and more generally in all those activities, which will be referred to as *early education*, aimed at promoting the social and cognitive development of the child. Investments in early education are usually pursued by parents in order to allow their children to achieve higher levels of education in adulthood and secure better job opportunities. The aim of this paper is to examine how the private choice to invest in early education may affect the working of job markets in which education acts as a signal of the unobserved productive abilities of workers.

In the well-known job market signaling model put forward by Spence (1973, 1974), workers are privately informed about their productivity and use their education choice to convey information to the employer. If worker's ability and education costs are negatively correlated, two key equilibrium outcomes arise from the analysis of the model. A separating equilibrium in which high ability workers signal themselves to the employer by attaining a higher level of education than that of low ability workers and a pooling equilibrium where workers of high and low ability are not screened by the employer because they obtain the same level of education. The application of standard refinements yields as the most plausible outcome the separating equilibrium in which the high ability workers signal their productivity with the least cost level of education and the low ability workers attain the compulsory level.

There is empirical evidence, however, suggesting that education does not seem to be an effective signal of worker's productivity. For example, Clark, Jouber and Maurel (2014) show that in the US a large fraction of workers hold occupations that do not require as much schooling as they have acquired. Over-education is not a transitory phenomenon, possibly due to frictions in the labour market, but it persists for a number of years and only a small fraction of overeducated workers exit from this status. Moreover, over-educated workers earn more than less educated workers, but earn less than similarly educated workers in occupations which require their level of schooling. The existing evidence seems to suggest that the operation of some sectors of the labour market is more consistent with the outcome of a pooling equilibrium in which the level of education is above the compulsory level rather than with the outcome of a separating one.

Although evidence is not easily reconciled with the results of the standard Spence's model, we think that it may nevertheless be partly explained by the analysis of a signaling model in which the costs of education is endogenously determined by workers through non observable investments in early education. In fact, there is some evidence that early education may have a significant impact on the academic performance of prospective workers [see, for example, Andrews, Jargowsky and Kuhne (2012)]. By undertaking investments in early education prospective workers will be able to lower their future education costs so that they will have better opportunities to mimic the education level of the most productive workers. This may explain why education seems to be seen by employers as a weak signal of worker's ability in a number of labour market sectors. In this paper we are interested in understanding how the endogeneity of education costs through unobservable investments affects the strategic interaction between workers and employers and, particularly, under what conditions unobservable investment, which enables savings on future education costs, alters the operation of the job market.

To address these issues we consider a simple job market signaling model where the productive ability of a worker applying for a job is not observable by employers. Workers are privately informed about their productivity which can be either high or low. As the low ability worker's education costs are higher than the high ability's, the choice of the level of education may be used by the worker as a signal to convey information to the employer, who makes a wage offer based on the observation of the actual education level attained by the worker. We depart from the standard model because we assume that the worker, before an education choice is made, has the opportunity to decide how much to invest in activities aimed at lowering future costs of education. Moreover, we assume that the investment decision is not observed by the employer.

The new problem arising in the context of the modified signaling model is due to the fact that the cost advantage in acquiring education by the high ability worker over the low type, can be completely eroded by investment in early eduction by the low ability worker. This makes the signaling role of education more doubtful as high education signals can now originate more easily from low ability workers. The question then arises as to whether or not and under what conditions the presence of unobservable investment radically modifies the incentive of the high ability worker to convey credible information about his productivity.

We derive conditions under which the presence of unobservable investment alters the standard prediction of the equilibrium outcome in job market signaling models. These conditions refers to the average productivity of workers or more generally to the distribution of productivity over the labour force. We find that when the average productivity is sufficiently high, the most plausible outcome is a pooling equilibrium satisfying the Divinity Criterion in which all types of worker are over-educated. Specifically, our result is that workers will never pool at the lowest level of education, because, as in the standard signaling model, this pooling equilibrium outcome fails to be intuitive. On the other hand, if average productivity is relatively low, a separating outcome obtains in which the high ability workers are required to choose a still greater level of education, as compared to the level needed if investment were absent.

Other contributions to the literature on job market signaling have proposed models in which refined pooling equilibria are possible. Alos-Ferrer and Prat (2012) and Daley and Green (2014) deal with job-market signaling models in wich employers observe both costly education as well as a noisy signal which is correlated to the worker's productivity. In Alos-Ferrer and Prat (2012), after the signaling stage, employers have the opportunity to update their beliefs about the worker's productivity based on noisy observations of the worker's performance on the job. In Daley and Green (2014) the noisy signal is provided by 'grades' or scores on a test that the employer observes prior to making a wage offer. In both kinds of models the precision of the noisy signal reduces the incentives for the high ability worker to signal with costly education. In these models, pooling equilibria may turn out to be 'stable' because deviations to higher levels of education by the high ability worker are less profitable than relying on the information conveyed to the employer by the noisy signal. In the present work we deal with another kind of phenomenon and specifically the opportunity that a low type agent might have to cut the costs of the education signal by investing in early education. Although our focus is on a quite different phenomenon, the results of our analysis are somewhat related to those of Alos-Ferrer and Prat (2012) and Daley and Green (2014). In fact, we identify conditions under which only pooling equilibria survive stability based refinements.

The paper is organized as follows. Section 2 sets up the signaling model introducing assumptions and notation. The analysis of equilibria is carried out in Section 3 and further refinements are applied in Section 4. Summary and conclusions are in Section 5. All the proofs are collected in the appendix.

2 The signaling game. Notation and assumptions

In this model of the job market workers have either high or low productive abilities. High ability workers, or type H, have productivity π_H, while workers with low ability, type L, have productivity π_L, with $\pi_H > \pi_L > 0$. Employers do not directly observe workers' productivity. Nevertheless, they know the prior probability that a worker has high productivity, i.e. $\mu = \Pr(H)$.

Before entering the labour market, workers make a choice about the level of education to attain, which is denoted by the real variable $e \geq 0$. The worker's level of education is observed by employers. Education is costly both in terms of resources and utility and, to simplify matters and isolate its signaling effect, it is assumed to have no effects on productivity. We also assume that education is more expensive for the low ability than for the high ability worker. Specifically, marginal costs of education are assumed to be constant and to differ across types. Marginal costs of education will be denoted by θ_H and θ_L, respectively for type H and type L, with $\theta_L > \theta_H > 0$.

A key departure from Spence's model is the assumption that the low ability worker has the opportunity to undertake *unobserved investments* in early education, i.e. in activities aimed at reducing the future cost of education. The investment choice is denoted by the real variable $x \geq 0$ and it is not observed by employers. We assume that the L type worker's marginal cost of education is a strictly decreasing and convex function of investment which is denoted by $\theta_L(x)$, with $\theta_L(0) = \theta_L$. To further simplify the analysis we also assume that no reduction in the marginal cost of education can be obtained by the high ability worker. Formally, the marginal cost of education for type H is held fixed at θ_H.[1]

The utility of a low ability worker is a linear function of the level of education, the amount of investment and the labour income, denoted by $w \geq 0$, and is given by

$$V(e, x, w) = w - x - \theta_L(x)e. \tag{1}$$

As high ability workers can not lower education costs, investment is dropped as a choice variable in the utility function of the H type, which is simply given by

$$U_H(e, w) = w - \theta_H e. \tag{2}$$

Labour income of a worker with no education is equal to the lowest productivity level, therefore the *reservation utility* is equal to π_L for both high and low ability workers.

The worker's indifference curves at any given level of investment are represented by positively sloped straight lines in the plane of coordinates e and w, with education measured on the horizontal axis and wages on the vertical axis. Provided that $\theta_H < \theta_L(x)$, the high ability worker's indifference curves are flatter than the low ability worker's.

Based on the observation of the worker's level of education, the employer makes a wage offer. Competition among firms forces employers to offer the same wage at every given level of education and firms' profits to vanish. As productivity is not observable, firms expected profits are equal to zero if the wage paid is equal to the worker's expected productivity. Hence, after observing the worker's level of education, the employer makes an inference about the type of worker and evaluates the worker's expected productivity, which is given by

$$\pi(e) = (1 - \mu(e))\pi_L + \mu(e)\pi_H, \tag{3}$$

where $\mu(e)$ is the employer's *posterior belief* that the worker has high ability upon observing the level of education e, i.e. $\mu(e) = \Pr(H|e)$.[2] The wage offered by the employer is equal to $\pi(e)$ and is

[1]This assumption could be relaxed and replaced by the assumption that the decline in marginal cost of education due to investment is smaller for type H than for type L.

[2]As usual, it is assumed that all the employers share the same beliefs upon having observed the same level of education.

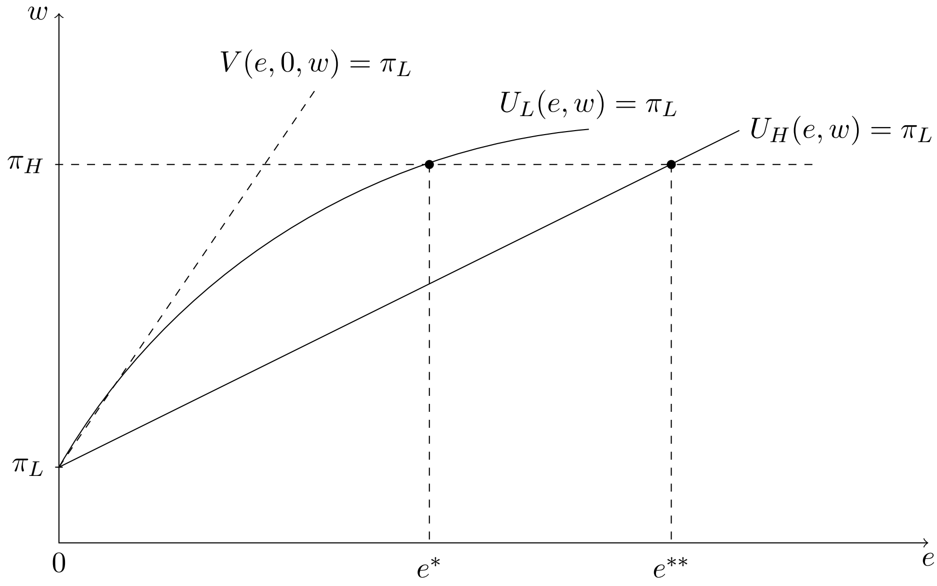

Figure 1: Indifference curves at reservation utility

bounded from below by π_L and from above by π_H.

The key element in the present signaling game, as compared to Spence's model, is that the cost of signaling is endogenously determined by the choice of investment and thereby the worker's incentive to signal must be carefully evaluated by the employer. For example, the L type may want to invest in activities which lower the marginal cost of education because this makes mimicking the H type cheaper and the chances of being offered a higher wage greater. The employer should realize that a higher level of education may mean that either the worker has high ability or he is a L type who can afford to send a high signal because he has previously invested in reducing education costs. Therefore, in the process of forming his *posterior* beliefs, the employer will have to take into account the endogeneity of signaling costs and make rational conjectures about the amount of investment chosen by the low ability worker.

Let us then focus on rational investment behaviour by the L type's worker. For any given level of wage and education, the L type will choose the amount of investment which maximizes utility.

The solution to the worker's optimization problem is the *investment function*,[3]

$$x(e) = \text{argmax}_x V(e, x, w). \tag{4}$$

The optimum value of utility,

$$U_L(e, w) = w - x(e) - \theta_L(x(e))e, \tag{5}$$

is the *best utility function* of the low ability worker. As long as investment is chosen optimally, the low ability worker's payoff is given by $U_L(e, w)$. The indifference curves, when the optimal investment decision is built into the utility function, have a concave shape and lie below the indifference curves that would result if investment were absent. Indeed, at any given level of education the same utility is obtained by means of a lower wage, because education costs can be conveniently lowered by a suitable choice of investment.[4]

Figure 1 compares the new indifference curve at reservation utility, $U_L(e, w) = \pi_L$, with the indifference curves of the two types of workers in the absence of investment, respectively $V(e, 0, w) = \pi_L$ and $U_H(e, w) = \pi_L$. In the absence of investment the slopes are constant and are respectively given by θ_L and θ_H. Education is always relatively more expensive for the low ability worker and indifference curves satisfy the *single crossing property.* On the other hand, in the model where investment is optimally chosen by the worker, the indifference curve of $U_L(e, w)$ changes its slope. In particular, the effect of investment on the marginal cost of education can be strong enough that a violation of the single crossing property may occur. In fact, it will be assumed that at sufficiently high levels of education the marginal costs of education across types are reversed as a result of the optimal choice of investment.

In order to specify this assumption more clearly, let us introduce some more notation. The levels

[3]Notice that, by linearity of V, the investment function does not depend on w for sufficiently high levels of the wage. Investments are equal to zero at lower levels of w.

[4]Properties of $x(e)$, $U_L(e, w)$ and its indifference curves are in Lemma 1 in the Appendix.

of education e^* and e^{**}, defined by the equations

$$U_L(e^*, \pi_H) = \pi_L \tag{6}$$

$$U_H(e^{**}, \pi_H) = \pi_L, \tag{7}$$

are respectively the highest education levels for type L and for type H which are consistent with participation constraints. For higher levels of education no type of worker would enter the labour market even if the highest market wage is paid. We will assume that the high ability worker is willing to acquire a higher level of education than the low ability, therefore we require that $e^* < e^{**}$, as depicted in Figure 1.[5]

Let us consider the education wage pair (e^*, π_H). As will be seen in the next section, e^* is the least cost separating equilibrium level of education and π_H is the equilibrium wage of the high ability worker. We shall assume that the low ability worker's indifference curve through the point (e^*, π_H) is flatter than the high ability worker's. Specifically, we require that the low ability worker's marginal cost of education at e^* be lower than the high ability worker's, i.e.

$$\theta_L(x(e^*)) < \theta_H. \tag{8}$$

Condition (8) implies that the single-crossing property does not hold, because the indifference curves of different types can cross each other twice. In particular, take both types' indifference curves through the point (e^*, π_H) and notice that, as in Figure 2, they have another intersection at the point of coordinates (e_0, w_0) formally defined as follows:

$$(e_0, w_0) \neq (e^*, \pi_H) \quad \text{such that} \quad U_L(e_0, w_0) = U_L(e^*, \pi_H) \quad \text{and} \quad U_H(e_0, w_0) = U_H(e^*, \pi_H) \tag{9}$$

In other words, (e_0, w_0) is the education-wage pair which yields to both types of workers the same payoff as in the least cost separating equilibrium. The threshold value of the wage w_0, which by (8) exists and is well defined,[6] will play an important role in the following analysis. Notice also that (8) implies that, as depicted in Figure 2, there exists a level of education, e_b, which falls within e_0 and

[5]This is a natural assumption to make if one wants education to play a role as a signal of the worker's ability.

[6]See Lemma 1 in the Appendix.

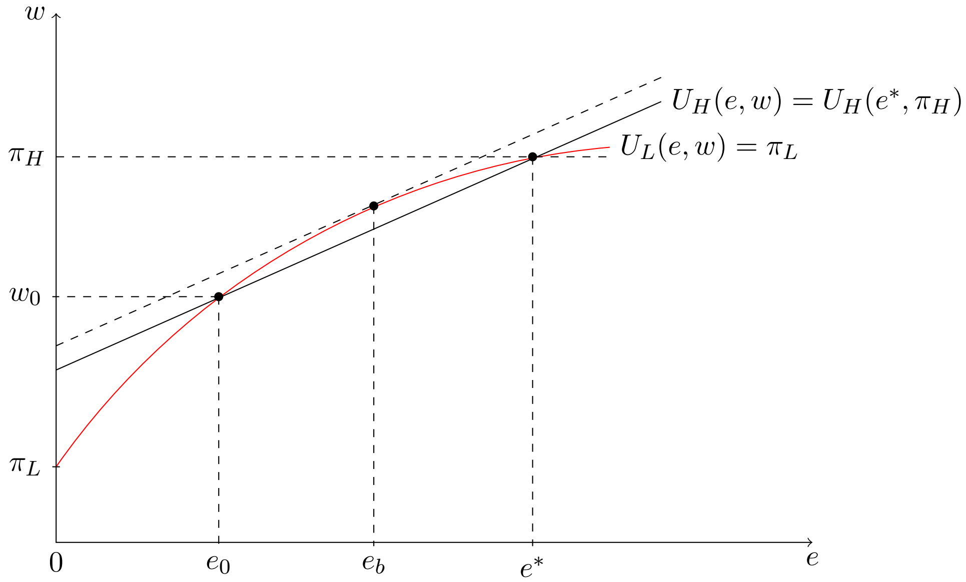

Figure 2: Threshold levels of education and wage

e^*, at which the marginal cost of education for the two types is the same, i.e. there exists e_b such that

$$\theta_L(x(e_b)) = \theta_H. \tag{10}$$

The problem described above can be formalized as a signaling game between a worker and an employer. First, a random move of nature determines whether a worker is of type H or type L. The worker privately learns her/his own productivity, decides how much to invest in order to reduce the marginal costs of education and finally chooses a level of the education signal to attain. Employers only observes the worker's education level and make a wage offer. Finally, the worker accepts or rejects the job and payoffs are paid. A pure strategy for the worker sets out a level of education and an amount of investment for type L, (e_L, x_L), and a level of education for type H, e_H. A strategy for the employer is a function $w(e)$ providing a wage offer for any observed level of education. Workers' payoffs are given by (1) and (2), while the employer's payoff is given by $\pi_t - w(e)$, where $t = L, H$. The marginal costs of education $\theta_L(\cdot)$ and θ_H, productivities π_L and π_H and the prior belief μ are common knowledge.

3 Signaling equilibria with unobservable investment

The solution concept we employ in the analysis of the signaling game is the Perfect Bayesian Equilibrium.

Definition 1. *A pure strategy Perfect Bayesian Equilibrium (PBE) consists of a profile of strategies e_L, x_L, e_H, $w(e)$ and a system of beliefs $\mu(e)$ such that*

i) Worker's strategy is optimal, i.e.

$$e_t = \text{argmax}_e\, U_t(e, w(e)) \quad \text{with} \quad t = H, L \qquad \text{and} \qquad x_L = x(e_L)$$

ii) Employer's strategy is optimal, i.e. $w(e) = (1 - \mu(e))\pi_L + \mu(e)\pi_H$ for all e.

iii) Beliefs and worker's strategy are consistent with Bayes' rule.

To cope with multiplicity, the set of equilibria is refined by restricting off equilibrium beliefs. We apply the Intuitive Criterion amended to make allowance for endogeneity of signaling costs, hence, following the spirit of forward induction we assume that built in the employer posterior beliefs is the conjecture that investment is optimally chosen by the worker at all education levels *on and off the equilibrium path.* By using the best utility function $U_L(e, w)$, we conveniently state the modified version of the Intuitive Criterion employed in our analysis. Fix a PBE with strategies denoted by e_L, x_L, e_H, $w(e)$. A deviation $\tilde{e}$ is *equilibrium dominated* for type L if $U_L(\tilde{e}, \pi_H) < U_L(e_L, w(e_L))$, the equilibrium payoff of type L. A PBE survives the *Intuitive Criterion* if there exists no deviation which is equilibrium dominated for the low ability worker and strictly preferred by the high ability type.

Definition 2. *A PBE with the profile of strategies e_L, x_L, e_H and $w(e)$ satisfies the Intuitive Criterion if there exists no deviation $\tilde{e}$ such that $U_L(\tilde{e}, \pi_H) < U_L(e_L, w(e_L))$ and $U_H(\tilde{e}, \pi_H) > U_H(e_H, w(e_H))$.*

Before proceeding to the analysis of equilibrium let us briefly recall, as a benchmark, the solution to the standard job market signaling game. Figure 3 illustrates separating and pooling equilibria

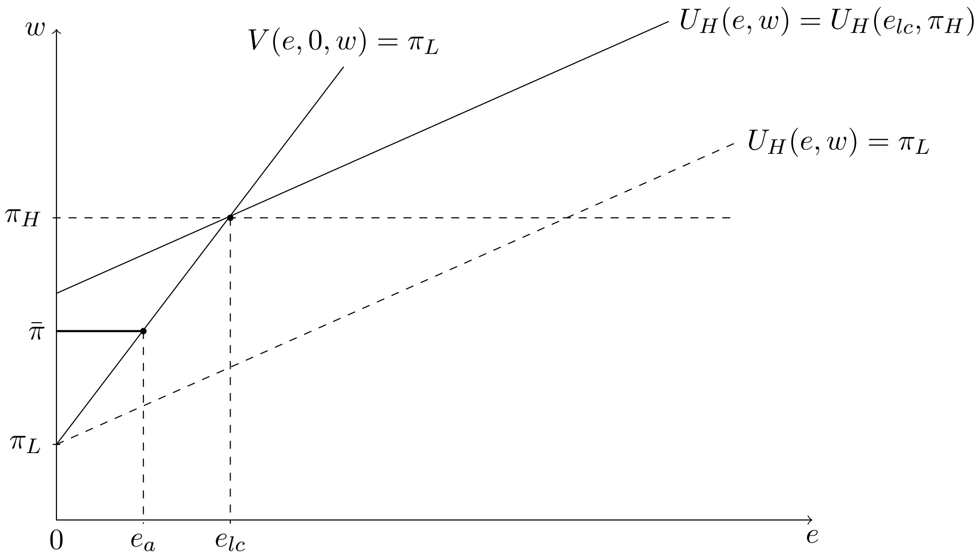

Figure 3: The benchmark case. Separating and pooling equilibria without investment.

in the case where investment is absent, or x is held fixed at zero. Denote by e_{lc} the 'least cost' education level, which is defined by the condition $U_L(e_{lc}, 0, \pi_H) = \pi_L$. By $\bar{\pi} = \mu\pi_H + (1-\mu)\pi_L$ denote the average productivity evaluated at the prior belief μ, and by e_a the level of education defined by the equation $U_L(e_a, 0, \bar{\pi}) = \pi_L$. The separating equilibrium strategies are characterized by $e_L = 0$, $e_H \geq e_{lc}$ and $w(e) = \pi_L$ if $e < e_{lc}$ and $w(e) = \pi_H$ if $e \geq e_{lc}$. The worker's pooling strategies are $e_L = e_H = e_P$ with $0 \leq e_P \leq e_a$, whereas $w(e_P) = \bar{\pi}$ and $w(e) = \pi_L$ for $e \neq e_P$ can be taken as the employer's strategy. As is well know (see Cho and Kreps (1987), Cho and Sobel (1990)) only the least cost separating equilibrium, where $e_L = e_{lc}$, survives the Intuitive Criterion. This is the most plausible outcome of Spence's model, which involves the high ability worker choosing a high education level to distinguish her/himself from a low ability type and each type of worker being paid according to her/his own productivity.

Let us turn to the analysis of equilibrium in the model with unobservable investment and, for simplicity, only consider pure strategies equilibria. Separating and pooling equilibria will be examined in turn.

3.1 Separating equilibrium

In a separating equilibrium different types choose different levels of education, so that the employer correctly identifies the worker's productivity. A low type is offered the lowest wage, π_L, while a high type is given the highest wage, π_H. As a low type has no incentive to invest and to choose positive levels of education, her/his equilibrium choice is $e_L = 0$ and $x_L = 0$. The equilibrium education choice of the H type, e_H, must satisfy an incentive compatibility condition requiring that the low productivity worker is better off by revealing his type and obtaining the wage π_L, rather than by mimicking the H type and receiving the wage π_H, i.e.

$$U_L(e_H, \pi_H) \leq U_L(0, \pi_L). \tag{11}$$

Moreover, the equilibrium level e_H must satisfy the participation condition for type H,

$$U_H(e_H, \pi_H) \geq \pi_L, \tag{12}$$

requiring that the welfare of the high ability worker be greater than the reservation utility. From the point of view of the employer, a separating equilibrium strategy may consist of the wage offer $w(e) = \pi_L$ for $e < e_H$ and $w(e) = \pi_H$ for $e \geq e_H$. As can be shown, any worker's strategy with $e_L = 0$, $x_L = 0$ and with e_H satisfying (11) and (12) supports a separating equilibrium. Moreover, only one separating equilibrium surviving the Intuitive Criterion exists in which the high ability worker acquires the 'least cost' education level e^*.

Proposition 1. *There exists a unique separating equilibrium satisfying the Intuitive Criterion, called the Least Cost Separating Equilibrium (LCSE), which is supported by the worker's strategy* $e_L = 0$, $x_L = 0$ *and* $e_H = e^*$, *where* e^* *is the LCSE education level defined by (6).*

Compared to the benchmark case where investment is not available, we have $e^* > e_{lc}$. Here, we notice that, as in Figure 4, a higher level of education at equilibrium is needed by the high ability worker in order to distinguish her/himself from a low ability one. The mere fact that investment makes mimicking cheaper for the low ability worker forces the high type to pursue an even greater distortion in the level of education in order to signal her/his productivity to the employer. As a

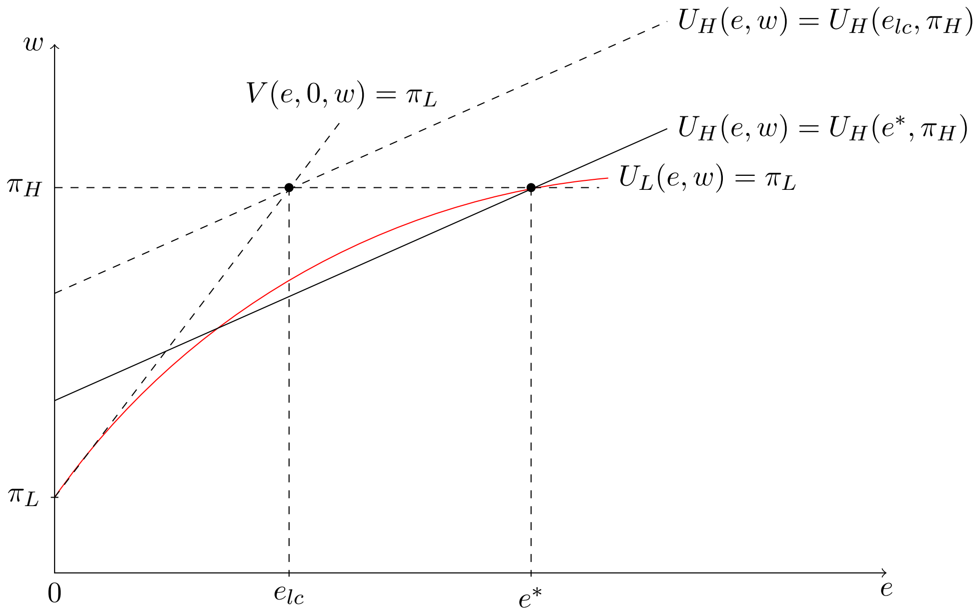

Figure 4: Comparison between least cost separating equilibria

result, at equilibrium, the high ability worker is worse off than in the benchmark case, while the welfare of the low ability worker remains unchanged. In Figure 4, the least cost separating equilibria in the benchmark case (dashed lines) and in the model with investment (solid lines) are depicted. As pointed out, the education level of the high ability worker is shifted to the right as compared to the benchmark case. Moreover, the indifference curve at the LCSE level of education, e^*, lies below the indifference curve at the least cost equilibrium level, e_{lc}, in the benchmark.

3.2 Pooling equilibrium

In a pooling equilibrium both types of workers choose the same level of education so that the employer does not learn any new piece of information. At equilibrium, workers are paid the same wage which is equal to the average productivity, $\bar{\pi} = (1-\mu)\pi_L + \mu\pi_H$. A pooling equilibrium level of education, denoted by e_P, must satisfy the participation conditions for both types of workers, respectively

$$U_L(e_P, \bar{\pi}) \geq \pi_L \tag{13}$$

and

$$U_H(e_P, \bar{\pi}) \geq \pi_L. \tag{14}$$

It can be noticed that, as $U_L(e, w)$ is decreasing in education, (13) implies that education can not exceed the level $\bar{e}$ defined by

$$U_L(\bar{e}, \bar{\pi}) = \pi_L. \tag{15}$$

As shown in the Appendix (Lemma 2), pooling equilibria exist and are characterized by the worker's strategies, e_L, x_L, e_H such that $x_L = x(e_L)$, $e_L = e_H = e_P$, with $0 \leq e_P \leq \bar{e}$. The employer's strategy can be taken as $w(e) = \bar{\pi}$ for $e \geq e_P$ and $w(e) = \pi_L$ for $e < e_P$.

While in a standard Spence's model no pooling equilibrium survives the Intuitive Criterion, in this model an intuitive pooling equilibrium may exist in which both types of worker exhibit a positive level of education. The existence condition of intuitive equilibria refers to the average productivity of workers. In fact, one of our key results is that, as long as the average productivity exceeds the threshold level of wage w_0, defined by (9), there exists at least an intuitive pooling equilibrium.

Figure 5, which depicts a case where $\bar{\pi} > w_0$, helps illustrate this result. The set of pooling equilibria is associated with points lying on the solid segment delimited by A and B. Take the pooling equilibrium associated with B and supported by the level of education $\bar{e}$. Any deviation $\tilde{e}$ which lies below the education level e' is preferred by both types and any deviation above e^* is equilibrium dominated for all types of worker. Moreover, any deviation in the interval (e', e^*) is equilibrium dominated for the H type and strictly preferred by the L type. As a result, if the low ability worker deviated by choosing a level of education in this interval, he would reveal his type to the employer who will accordingly offer a low wage to the worker. Hence, we conclude that all the deviations pass the Intuitive Criterion test and the education level $\bar{e}$ supports an intuitive pooling equilibrium. It may also be noticed that at such an equilibrium the L type is not worse off whereas the H type is strictly better off than he is at the intuitive LCSE characterized in Proposition 1.

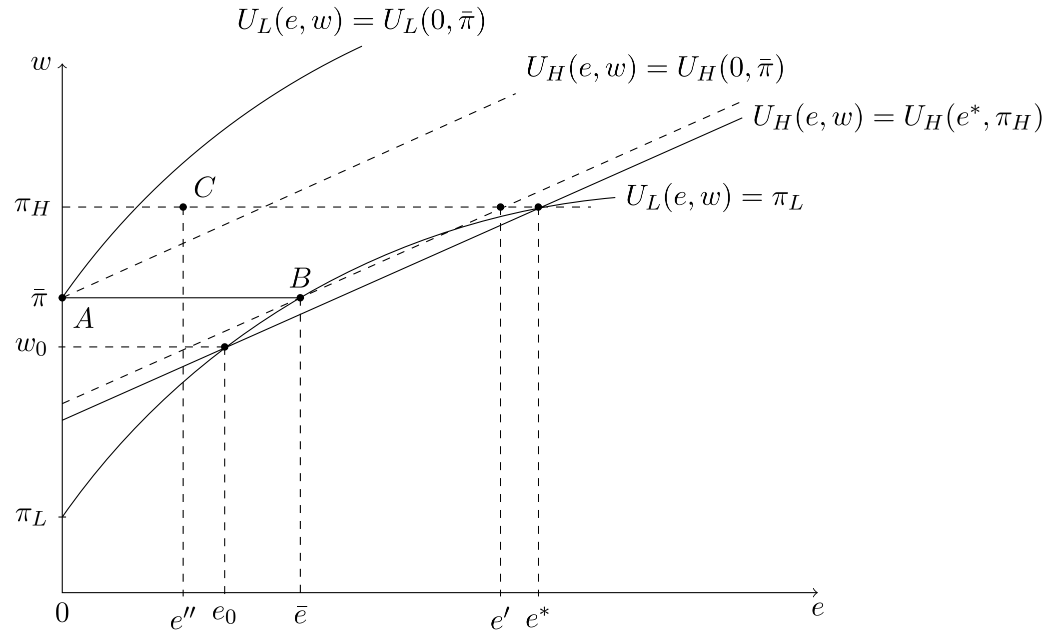

Figure 5: Intuitive pooling equilibrium

Interestingly, the pooling equilibrium associated with point A in Figure 5 and supported by a zero level of education (with $w = \bar{\pi}$) is easily seen to be non intuitive. In fact, the deviation $\tilde{e} = e''$ associated with point C is strictly preferred by H and it is equilibrium dominated for type L. Hence, a pooling equilibrium supported by no education can not be intuitive. In fact, it will be shown that intuitive pooling equilibria can only be supported by strictly positive education levels.

It can also be shown that no pooling equilibrium satisfying the Intuitive Criterion exists if the average productivity lies below the threshold level of wage, w_0. This result can be easily seen from Figure 6, which depicts a case where $\bar{\pi} < w_0$. Indeed, no pooling equilibrium supported by a level of education between e' and $\bar{e}$ can satisfy the Intuitive Criterion, because a deviation in the interval $]e''', e''[$ is easily found which is strictly dominated for L and strictly preferred by the H type of worker. Next, take the pooling equilibrium associated with point A' and supported by the level of education e'. It is straightforward to see that any deviation $\tilde{e}$ in the interval $]e''', e^*[$ is equilibrium dominated for L and strictly preferred by H and thereby the equilibrium supported by e' fails on

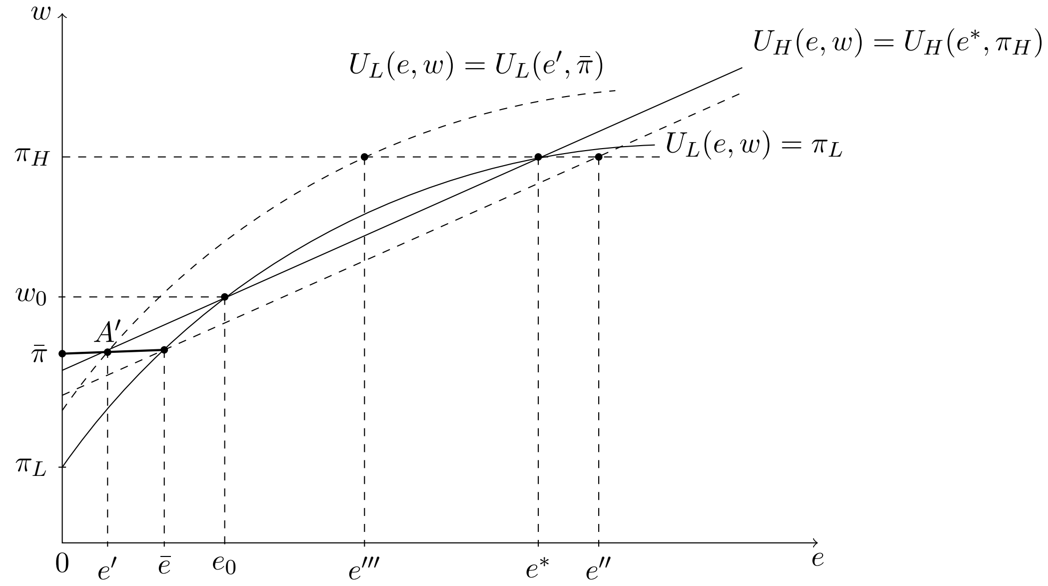

Figure 6: No intuitive pooling equilibria

the Intuitive Criterion. The same argument allows us to discard all pooling equilibria supported by education levels lying to the left of e'. This graphical analysis agrees with the conclusion that no intuitive pooling equilibrium exists if the average productivity of labour is relatively low.

In order to provide a formal characterization of the set of intuitive pooling equilibria let us introduce a few definitions. For any education-wage pair (e, w) with $w \in [\pi_L, \pi_H]$ consider the *highest profitable deviation for type t*, as the function $\delta_t(e, w) = \delta_t$ implicitly defined by the condition $U_t(\delta_t, \pi_H) = U_t(e, w)$, with $t = H, L$.[7] In other words, these functions provide the maximal education level which makes a worker's type who earns the highest wage indifferent to the given education-wage pair. Notice that $\delta_H(0, \pi_L) = e^{**}$ and, by (9), $\delta_H(e_0, w_0) = e^*$. Moreover, at all wage-education pairs along the indifference curve of the low ability worker at the reservation utility the function $\delta_L(e, w)$ is constant and specifically $\delta_L(0, \pi_L) = \delta_L(e_0, w_0) = e^*$.

[7] Given our assumptions on utility we have $\delta_H(e, w) = e + (\pi_H - w)/\theta_H$, while the function $\delta_L(e, w)$ is implicitly defined as the solution to $\pi_H - w - x(\delta_L) + x(e) - \theta_L(x(\delta_L))\delta_L + \theta_L(x(e))e = 0$.

Next, let us define the *profitable deviation gap* as the function

$$g(e, w) = \delta_H(e, w) - \delta_L(e, w)$$

This construction is used to identify intuitive equilibria. If the profitable deviation gap is positive at the equilibrium supported by the education-wage pair (e, w), i.e. if $g(e, w) > 0$, then $\delta_H(e, w) > \delta_L(e, w)$ and there exists a deviation $\tilde{e}$ with

$$\delta_L(e, w) < \tilde{e} < \delta_H(e, w)$$

which is strictly preferred by the high ability worker and equilibrium dominated by the low ability. As a result, the given equilibrium does not satisfy the Intuitive Criterion, according to Definition 2. By contrast, if $g(e, w) \leq 0$ then the given equilibrium passes the Intuitive Criterion test.

As will be seen, if workers' average productivity is greater than the wage threshold w_0, then there exists a minimum education level which makes the profitable deviation gap function to vanish at wage $w = \bar{\pi}$, i.e. there exists $\underline{e}$ such that

$$g(\underline{e}, \bar{\pi}) = 0 \tag{16}$$

This level of education is the lowest level of education consistent with an intuitive pooling equilibrium and can be seen to lie between e_0 and e_b.

The main results of this section are summarized in the following

Proposition 2. *Let w_0, e_0 , $\bar{e}$ and $\underline{e}$ be as respectively defined by* (9), (15) *and* (16).

i) If $\bar{\pi} < w_0$, no pooling equilibrium exists that satisfy the Intuitive Criterion.

ii) If $\bar{\pi} \geq w_0$, there exist pooling equilibria satisfying the Intuitive Criterion and, specifically, any level of education in the interval $[\underline{e}, \bar{e}]$ supports an intuitive pooling equilibrium. The minimum value of education $\underline{e}$ lies in the interval $]e_0, e_b[$. The intuitive pooling equilibrium with $e_P = \underline{e}$ is called the Least Cost Pooling Equilibrium (LCPE).

The above results can also be illustrated by means of a simple numerical example. Suppose that $\theta_L = 1$, $\theta_H = 1/4$, $\pi_L = 1$, $\pi_H = 4$ and $\theta_L(x) = \exp(-x)$. The low ability worker best utility function is $U_L(e, w) = w - e$ if $e \leq 1$ and $U_L(e, w) = w - 1 - \ln(e)$ if $e > 1$. The LCSE level of education is $e^* = 7.39$, while the threshold values of e and w are $e_0 = 1.85$ and $w_0 = 2.62$. If the high ability workers are relatively more numerous, for example $\mu = 2/3$, then the average productivity is $\bar{\pi} = 3 > w_0$ and we have $\underline{e} = 2.33$ and $\bar{e} = 2.72$. Thus, the LCPE is supported by a level of education equal to 2.33, while the LCSE requires the high ability worker to obtain a level of education equal to 7.39.

4 Equilibrium selection

In order to arrive at a sharper prediction about the outcome of the game, a further selection of the set of intuitive equilibria can be accomplished by applying a slightly more stringent requirement on off equilibrium beliefs which is called the Divinity Criterion and was first introduced by Banks and Sobel (1987) (see also Cho and Kreps, 1987). Let us introduce the Divinity Criterion with the help of some additional notation.

For any given level of education e an employer's best response to e is a wage offer which belongs to the closed interval delimited by worker's productivities, i.e. $w \in [\pi_L, \pi_H]$, and depends on the employer's beliefs about types. Let us fix a PBE and denote type t worker's equilibrium payoff by U_t^*. If an education level $\tilde{e}$ is not on the equilibrium path, the set of employer's best responses to $\tilde{e}$ that cause type t to defect from the equilibrium strategy is given by

$$D(t \mid \tilde{e}) = \{w \in [\pi_L, \pi_H] \mid U_t^* < U_t(\tilde{e}, w)\}. \tag{17}$$

By $D^0(t \mid \tilde{e})$ we denote the set of employer's best responses to $\tilde{e}$ which yield to type t exactly the same utility as in equilibrium.

If the set $D^0(t \mid \tilde{e}) \cup D(t \mid \tilde{e})$ is not empty, type t has a preference to deviate from the equilibrium strategy if this deviation is met with some best response from the employer. If the above set is empty but there exists a type $t' \neq t$ such that $D(t' \mid \tilde{e}) \neq \emptyset$, then following $\tilde{e}$ the employer should have beliefs which place probability zero on type t as required by the Intuitive Criterion. Indeed, if $t = L$ and $t' = H$ the deviation $\tilde{e}$ is equilibrium dominated for type L and strictly preferred for type H

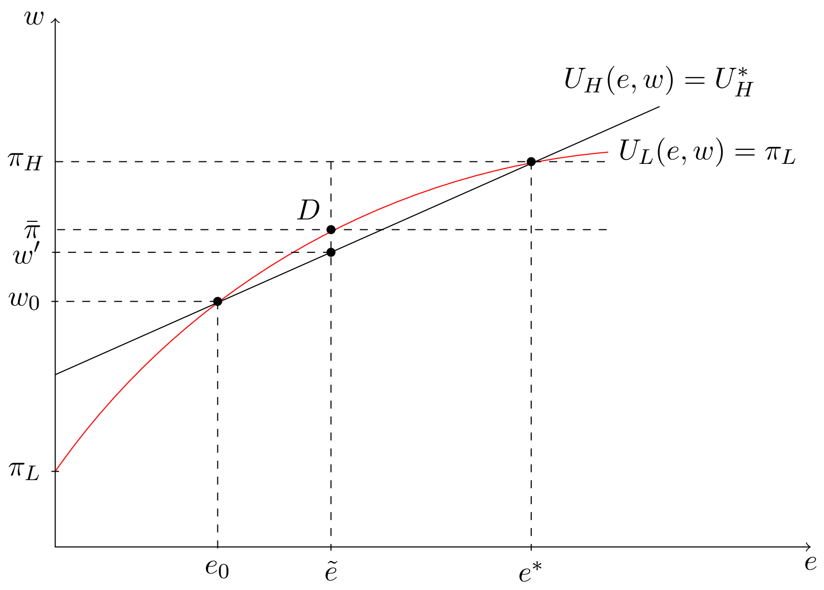

Figure 7: LCSE and the Divinity Criterion

and it must hold $\mu(\tilde{e}) = 1$. The Divinity Criterion adds to the Intuitive Criterion a slightly more stringent requirement which applies when both types of worker have an incentive to deviate.

Definition 3. *(Divinity Criterion) Let $\tilde{e}$ be a deviation from a PBE. If both sets $D^0(t \mid \tilde{e}) \cup D(t \mid \tilde{e})$ and $D(t' \mid \tilde{e})$ are not empty and $D^0(t \mid \tilde{e}) \cup D(t \mid \tilde{e}) \subset D(t' \mid \tilde{e})$, then $\mu(\tilde{e}) \geq \mu$, if $t = L$ and $t' = H$, and $\mu(\tilde{e}) \leq \mu$, if $t = H$ and $t' = L$. Moreover, if $D^0(t \mid \tilde{e}) \cup D(t \mid \tilde{e}) = \emptyset$ and $D(t' \mid \tilde{e}) \neq \emptyset$, then $\mu(\tilde{e}) = 1$, if $t = L$ and $t' = H$, and $\mu(\tilde{e}) = 0$, if $t = H$ and $t' = L$.*

Beside the restrictions imposed by the Intuitive Criterion, Divinity requires that if type t' has 'more opportunities' to deviate than type t, the employer's beliefs following the deviation should be updated by increasing the prior probability of type t'.

Let us now apply the Divinity Criterion to refine the set of intuitive equilibria. Our main result is that if the workers' average productivity is relatively low, the predicted outcome is the LCSE characterized in Proposition 1. By contrast, if the average productivity is sufficiently large the predicted outcome is the LCPE characterized in Proposition 2.

Proposition 3. *Let w_0, $\underline{e}$, $\bar{e}$ and e_b be as respectively defined by* (9), (16), (15) *and* (10).

i) If $\bar{\pi} < w_0$ the LCSE is the unique equilibrium satisfying the Divinity Criterion.

ii) If $\bar{\pi} > w_0$ only pooling equilibria survive the Divinity Criterion. Specifically, divine pooling equilibria are supported by education levels in the interval $[\underline{e}, \bar{e}]$ if $w_0 < \bar{\pi} < w_b$ and by education levels $[\underline{e}, e_b]$ if $\bar{\pi} > w_b$, where w_b is determined by $U_L(e_b, w_b) = \pi_L$.

A formal proof of Proposition 3 is found in the Appendix. Figure 7 provides an illustration of why the LCSE fails to pass the Divinity Criterion test when workers' average productivity is relatively high. In fact, take the deviation $\tilde{e}$ from the equilibrium level of education e^*. It is easily seen that $D^0(L \mid \tilde{e}) \cup D(L \mid \tilde{e}) = [\bar{\pi}, \pi_H]$ and $D(H \mid \tilde{e}) =]w', \pi_H]$. Since $D^0(L \mid \tilde{e}) \cup D(L \mid \tilde{e}) \subset D(H \mid \tilde{e})$ and the sets are not empty, the Divinity Criterion requires the beliefs to be $\mu(\tilde{e}) \geq \mu$ and consequently the wage offer $w(\tilde{e}) \geq \bar{\pi}$. The point D in Figure 7 lies above the indifference curve of the high ability worker so that the equilibrium choice of e^* is not any more optimal for the high ability worker, which means that the LCSE can not be supported by beliefs satisfying the Divinity Criterion.

Divinity rules out pooling equilibria supported by levels of education in excess of e_b, the level which triggers investments in early education large enough to equalize the marginal costs of education of both types. The intuition for this result is that for $e_P > e_b$ the marginal cost of education of the low ability is lower than that of the high ability worker so that a deviation to a lower education signal is more profitable for the H type. As a result, the employer's belief can not put less weight than that at the prior on the worker being of high type. This in turn means that the wage offer can not fall below the average productivity, but this makes the deviation to a lover level of education more profitable than the equilibrium choice.

According to Proposition 3, there still remains a multiplicity of equilibria when workers' average productivity is relatively high. However, among all the pooling equilibria satisfying the Divinity Criterion the LCPE defined in Proposition 2 is Pareto undominated, as it involves the lowest level of education, and thereby can be singled-out as the most plausible solution.

5 Conclusions

In a labour market in which the average productivity is relatively low, the most plausible outcome is a separating equilibrium where highly productive workers are overeducated and all workers are paid according to their productivity. By contrast, if the average productivity is relatively high, the most plausible outcome is a pooling equilibrium where both types of workers are overeducated and are paid a wage equal to the average productivity. Both types of workers are better off than they would be if education revealed worker's productivity to the employer, although all workers are forced to achieve extra education. This last result seems quite consistent with empirical evidence. In fact, it is not uncommon to see workers holding occupations that do not require as much schooling as they have acquired and this does not seem to be a temporary or transitory phenomenon, as the bulk of overeducated workers is seen to persists after many years. Moreover, overeducated workers tend to earn more than less educated workers, but tend to earn less than equally educated workers in occupations requiring their level of schooling.

An intuitive explanation of our findings and in particular of the result that the pooling solution requires a strictly positive level of education can be given as follows. Investment in early education, which reduces the future cost of schooling, makes mimicking of the high type cheaper for the low ability worker. In order to separate from a low type, a high ability worker is forced to send an even higher education signal at a separating equilibrium, which in turn implies that the H type worker will incur in higher costs of education and will be worse off. This makes pooling equilibria even more appealing to the H type especially when the average productivity and the employer's wage offers are relatively high. However, a pooling equilibrium can be a plausible solution provided that it survives stability based refinements, i.e. if there are no other education signals that can make the H type better off and the low type worse off. In other words, a pooling outcome can be a plausible solution only if whenever a deviation is profitable for the high ability worker it also results profitable for the low ability. This will occur if, at equilibrium, the investment in early education by the low ability worker is high enough to annihilate the advantage in marginal costs of the high ability over the low ability worker. This can not happen in a pooling equilibrium where both worker's types acquire zero education. In fact, as a low ability worker will not invest, the gap in the marginal cost of education between types will remain large, so that the high ability worker will profitably use the education signal to induce the employer to offer a higher wage. For this reason, a pooling equilibrium with zero education can not be a stable equilibrium and should not be considered as a plausible solution.

A few empirical implications stem from this analysis. In the first place our findings are not consistent with the observation of a pooling equilibrium where workers have zero (the compulsory level of) education and earn an average wage. According to our results average wages can only be observed in association with positive levels of education, i.e. our results seem to be quite consistent with the phenomenon of over-education.

Secondly, our analysis implies that the marginal cost of education of low and high ability workers should be quite similar at equilibrium. Although marginal costs are not directly observable, their difference might have observable implications that could be tested. For example, low ability workers who invested in early education should have similar skills as high ability workers as to their school performance. As a result, it could be tested whether workers with the same average education and the same average wages exhibit similar test scores or transcripts.

Another implication of our analysis is that in economic sectors or geographical regions where for various reasons the average productivity of workers is relatively low, the education-wage pattern that should be observed is similar to that resulting from a separating equilibrium, i.e. low wage and low education on one side and high wage and high education on the other. The same type of patterns should also be observed in contexts where a large majority of households are financially constrained and can not invest in early education. In such a case the conclusions of Spence's model should apply.

In contrast, in more affluent socio economic contexts where parents have higher average level of schooling, investment in the early education of children is not constrained and the results of our analysis should be more appropriate.

Appendix

Let us first establish the main properties of investment and best utility functions.

Lemma 1. *Let $x(e)$ and $U_L(e,w)$ be as respectively defined by (4) and (5), and $e_c = -1/\theta'_L(0)$. The following properties hold:*

(i) The investment function $x(e)$ is well defined with $x(e) = 0$ for $0 \leq e \leq e_c$ and it is strictly increasing for $e > e_c$.

(ii) The function $U_L(e,w)$ is strictly increasing in w and strictly decreasing in e, with

$$\frac{\partial U_L(e,w)}{\partial e} = -\theta_L(x(e))$$

(iii) The indifference curve of $U_L(e,w)$ at the utility level u, $w_L(e) = u + x(e) + \theta_L(x(e))e$, is strictly increasing with slope given by $\theta_L(x(e))$ and it is strictly concave for $e \geq e_c$.

(iv) Let e^* be as defined by (6). There exists a unique education level $e_b \in]e_c, e^*[$ such that $\theta_L(x(e_b)) = \theta_H$.

(v) There exists a unique pair of values $(e_0, w_0) \neq (e^*, \pi_H)$ satisfying (9), i.e. such that

$$\begin{aligned} U_L(e_0,w_0) &= U_L(e^*,\pi_H) \\ U_H(e_0,w_0) &= U_H(e^*,\pi_H). \end{aligned}$$

Moreover, $e_0 < e_b$.

Proof of Lemma 1.
(i) As $V(e,x,w)$ depends linearly and negatively on x, an optimal level of investment must be bounded from above. Thus we can restrict the analysis of the best value of x to a compact interval with 0 as the left end point. By continuity of V a solution to the maximization problem (4) exists. Moreover, as $\theta_L(x)$ is strictly decreasing and convex the solution is unique and $x(e)$ is a well defined

function.[8] Next, notice that as $V(e, x, w)$ is strictly concave in x, the first order condition are necessary and sufficient for a maximum and $x = 0$ satisfies the FOC for any $0 \leq e < e_c$.

In order to show that $x(e)$ is strictly increasing, let e' and e'' be such that $e'' > e' > e_c$ and denote respectively by x' and x'' the optimal levels of investment. From the conditions $V(e', x', w) > V(e', x'', w)$ and $V(e'', x'', w) > V(e'', x', w)$, we obtain

$$\left[\theta_L(x') - \theta_L(x'')\right](e'' - e') > 0$$

which means $\theta_L(x') > \theta_L(x'')$, which in turns implies $x' < x''$ as $\theta_L(\cdot)$ is strictly decreasing.

(ii) By (5), $U_L(e, w)$ is strictly increasing in w. By the Envelope Theorem

$$\frac{\partial U_L(e, w)}{\partial e} = \frac{\partial V(e, x, w)}{\partial e} |_{x=x(e)} = -\theta_L(x(e)) < 0.$$

(iii) By the Implicit Function Theorem, the slope of the indifference curve of $U_L(e, w)$ is given by

$$-\frac{\partial U_L(e, w)}{\partial e} = \theta_L(x(e)) > 0$$

where we used the result in Lemma 1.(ii). Hence, indifference curves are strictly increasing and their second derivative is given by

$$\theta_L'(x(e))x'(e) \leq 0$$

which, by Lemma 1.(i), is equal to zero if $0 \leq e \leq e_c$ and it is strictly negative for $e > e_c$.

(iv) By (8) we have

$$\theta_L(x(e^*)) < \theta_H < \theta_L = \theta_L(x(e_c))$$

and $e_c < e^*$, since $\theta_L(x(e))$ is strictly decreasing by Lemma 1.(ii). As $\theta_L(x(e))$ is the derivative of type L 's indifference curve, Darboux Theorem implies that there exists $e_b \in]e_c, e^*[$ such that $\theta_L(x(e_b)) = \theta_H$. Moreover, by strict monotonicity of $\theta_L(\cdot)$, e_b is unique.

[8] Arguing by contradiction to prove uniqueness, let x' and $x'' \neq x'$ be two solutions so that $V(e, x', w) = V(e, x'', w)$. By strict convexity of $\theta_L(\cdot)$, $\theta_L(\hat{x}) < (1/2(\theta_L(x') + \theta_L(x''))$, where $\hat{x} = (x' + x'')/2$. Hence $w - \hat{x} - \theta_L(\hat{x})e > (1/2)(w - x' - \theta_L(x')e + w - x'' - \theta_L(x'')e)$ implying $V(e, \hat{x}, w) > V(e, x', w)$, which is impossible.

(v) Take the indifference curves of the two types passing through the point (e^*, π_H), respectively

$$w_L(e) = \pi_L + x(e) + \theta_L(x(e))e \qquad (18)$$

$$w_H(e) = (\pi_H - \theta_H e^*) + \theta_H e \qquad (19)$$

and define the function

$$f(e) = w_L(e) - w_H(e) = \pi_L - \pi_H + \theta_H e^* + x(e) + (\theta_L(x(e)) - \theta_H)e$$

We have to show that $f(e)$ vanishes for some $e \in]0, e^*[$. By construction, $f(e^*) = 0$. Moreover, as $f'(e^*) = \theta_L(x(e^*)) - \theta_H < 0$ by (8), a standard argument shows that for $e' < e^*$ and sufficiently close to e^* it holds $f(e') > 0$. As $f(0) = \pi_L - \pi_H + \theta_H e^* < 0$, by the assumption[9] $e^* < e^{**}$, continuity of f implies that there exists $e_0 \in]0, e'[$ such that $f(e_0) = 0$. It can also be shown that e_0 is unique, i.e. f does not vanish anywhere else. Now let $w_0 = w_L(e_0) = w_H(e_0)$, then from (18) we obtain $U_L(e_0, w_0) = U_L(e^*, \pi_H)$ and from (19), $w_0 - \theta_H e_0 = \pi_H - \theta_H e^*$ or $U_H(e_0, w_0) = U_H(e^*, \pi_H)$.

Finally, as $\theta_L(x(e))$ is strictly decreasing and $\theta_L(x(e_0)) > \theta_H = \theta_L(x(e_b))$ we have $e_0 < e_b$.

Q.E.D.

Lemma 2. *The worker's strategy (e_L, x_L), e_H with $e_L = e_H = e_P$ supports a pooling equilibrium if and only if it satisfies (13), (14) and $x_L = x(e_L)$.*

Proof of Lemma 2.

Let the worker's strategy satisfy (13), (14) and $x_L = x(e_L)$ and take the beliefs $\mu(e) = \mu$ if $e > e_P$ and $\mu(e) = 0$ otherwise. Accordingly, take the wage offer $w(e) = \bar{\pi}$ if $e > e_P$ and $w(e) = \pi_L$ if $e < e_P$. Hence, $w(e)$ and $\mu(e)$ satisfy Definition 1.(ii). Next, as U_L is decreasing in e, $U_L(e_P, \bar{\pi}) > U_L(e, \bar{\pi})$ for $e \geq e_P$ and $U_L(e_P, \bar{\pi}) > U_L(e, \pi_L)$ for $e < e_P$, because by (13), $U_L(e_P, \bar{\pi}) \geq U_L(0, \pi_L) > U_L(e, \pi_L)$. Therefore e_P is an optimal choice for type L given $w(e)$ and so is the choice of x since $x_L = x(e_L)$. Similarly, for type H, $U_H(e_P, \bar{\pi}) > U_H(e, \bar{\pi})$ for $e > e_P$, because U_H is decreasing in e. Moreover, for $e < e_P$ we have $U_H(e, \pi_L) < U_H(0, \pi_L) = \pi_L \leq U_H(e_P, \bar{\pi})$ by (14). Thus, also the choice of type H is optimal. Finally, it is easily seen that the worker's strategy and beliefs obey the Bayes's rule.

[9]Note that the assumption $e^* < e^{**}$ can be stated as $e^* < (\pi_H - \pi_L)/\theta_H$.

The converse is easily seen to hold.

Q.E.D.

Let us see a few important properties of the profitable deviation gap function. Recall that for all education-wage pairs along the indifference curve of type L at reservation utility, we have $g(e,w) = \delta_H(e,w) - e^*$, and specifically

$$g(e,w) = \frac{\pi_H - \pi_L}{\theta_H} - \frac{x(e)}{\theta_H} + \frac{\theta_H - \theta_L(x(e))}{\theta_H} e - e^*$$

for all (e,w) such that $U_L(e,w) = \pi_L$. Moreover, $g(0,\pi_L) = e^{**} - e^* > 0$ by the assumption $e^* < e^{**}$, and $g(e_0,w_0) = 0$ by (9).

Lemma 3. *Let w_0, e_0 and e_b as respectively defined by* (9) *and* (10)*.*

i) The function $g(e,w)$ is decreasing in e; moreover $g(e,w)$ is increasing in w if $\delta_L > e_b$ where $\delta_L = \delta_L(e,w)$.

ii) Let (e,w) be such that $U_L(e,w) = \pi_L$; then $g(e,w) > 0$ iff $w < w_0$ (which in turn implies $g(e,w) < 0$ iff $w_0 < w < \pi_H$).

iii) $g(e_0,w) > 0$ for all $w_0 < w < \pi_H$; moreover, $g(e_b,w) < 0$ for all $w_b < w < \pi_H$, where w_b is such that $U_L(e_b,w_b) = \pi_L$.

Proof of Lemma 3.

(i) The derivative of $g(e,w)$ with respect to e for any given $w \in (\pi_L,\pi_H)$ is given by

$$\frac{\partial g(e,w)}{\partial e} = 1 - \frac{\partial \delta_L(e,w)}{\partial e}$$

In order to compute the derivative of $\delta_L(e,w)$ w.r.t. e, apply the Implicit Function Theorem to the equality defining δ_L, i.e. $U_L(\delta_L,\pi_H) - U_L(e,w) = 0$, which yields

$$\frac{\partial \delta_L(e;w)}{\partial e} = \frac{\partial_e U_L(e,w)}{\partial_{\delta_L} U_L(\delta_L,\pi_H)} = \frac{\theta_L(x(e))}{\theta_L(x(\delta_L))} > 1$$

because $\delta_L > e$. Therefore,

$$\frac{\partial g(e;w)}{\partial e} = 1 - \frac{\theta_L(x(e))}{\theta_L(x(\delta_L))} < 0.$$

and the function g is strictly decreasing in e.

Next, consider the derivative of $g(e,w)$ w.r.t. w,

$$\frac{\partial g(e;w)}{\partial w} = \frac{\partial \delta_H(e;w)}{\partial w} - \frac{\partial \delta_L(e;w)}{\partial w}.$$

The derivative of δ_L is

$$\frac{\partial \delta_L(e;w)}{\partial w} = \frac{\partial_w U_L(e,w)}{\partial_{\delta_L} U_L(\delta_L, \pi_H)} = -\frac{1}{\theta_L(x(\delta_L))}$$

while the derivative of δ_H is

$$\frac{\partial \delta_H(e,w)}{\partial w} = -\frac{1}{\theta_H}.$$

Finally, we have

$$\frac{\partial g(e,w)}{\partial w} = \frac{1}{\theta_L(x(\delta_L))} - \frac{1}{\theta_H}$$

which is positive if $\delta_L > e_b$ where $\delta_L = \delta_L(e,w)$.

(ii) Let us first show that $w < w_0$ implies $g(e,w) > 0$. Let $w_L(e)$ be the indifference curve implicitly defined by $U_L(e,w) = \pi_L$. Given $w_1 < w_0$, let e_1 be determined by $w_1 = w_L(e_1)$, so that, by monotonicity of $w_L(\cdot)$, $e_1 < e_0$. Consider

$$g(e_1, w_1) = g(e_1, w_1) - g(e_0, w_0) = \frac{w_L(e_0) - w_L(e_1)}{\theta_H} - (e_0 - e_1) \tag{20}$$

By the Mean Value Theorem, there exists $\xi \in]e_1, e_0[$ such that

$$w(e_0) - w(e_1) = w'(\xi)(e_0 - e_1) \tag{21}$$

By strict concavity of $w_L(\cdot)$, the derivative $w_L'(\cdot)$ is strictly decreasing so that $w_L'(e_1) < w_L'(\xi) < w_L'(e_0)$ and since by Lemma 1, $w_L'(e) = \theta_L(x(e))$, we have

$$\theta_L(x(\xi) > \theta_H. \tag{22}$$

From (21) and (22) we obtain $w_L(e_0) - w_L(e_1) > \theta_H(e_0 - e_1)$ and finally

$$\frac{w_L(e_0) - w_L(e_1)}{\theta_H} - (e_0 - e_1) > 0. \tag{23}$$

Hence, (20) and (23) yield $g(e_1, w_1) > 0$ and the first part of the proof is complete.

Let us show that $g(e, w) > 0$ implies $w < w_0$. In negation to this suppose that $g(e_1, w_1) > 0$ but $w_1 > w_0$, where $e_1 (> e_0)$ is defined by $w_1 = w_L(e_1)$.[10] By using concavity of $w_L(\cdot)$ (Lemma 1) it can be shown that

$$\frac{w_L(e_1) - w_L(e_0)}{e_1 - e_0} > \theta_H.$$

Hence, rearranging terms we get

$$\frac{w_L(e_0) - w_L(e_1)}{\theta_H} - (e_0 - e_1) < 0$$

which, by (20), yields the contradiction $g(e_1, w_1) < 0$, which establishes the result.

(iii) Take $w' \in]w_0, \pi_H[$ and consider the indifference curve of the low ability worker passing through the education-wage pair (e_0, w'), i.e. the function $w = \tilde{w}_L(e)$ implicitly defined by the equation $U_L(e, w) = U_L(e_0, w')$. As can be easily checked, the indifference curve also passes through the pair $(e^*, \pi_H + (w' - w_0)$. It can also be checked that the indifference curve of the high type worker through the education-wage pair (e_0, w'), i.e. $w = w' + \theta_H(e - e_0)$, passes through the point $(e^*, \pi_H + (w' - w_0)$. Let us take $\delta'_H = \delta_H(e_0, w')$ which is known to lie between e_0 and e^*. By concavity of $\tilde{w}_L(e)$, it is easily seen that $\tilde{w}_L(\delta'_H) > \pi_H$. Moreover, by definition, $\delta'_L = \delta_L(e_0, w')$ is given by the equation $\pi_H = \tilde{w}_L(\delta'_L)$. Hence, by the above inequality we have $\tilde{w}_L(\delta'_H) > \tilde{w}_L(\delta'_L)$. Since $\tilde{w}_L(e)$ is increasing it follows that $\delta'_H > \delta'_L$ so that $g(e_0, w') = \delta'_H - \delta'_L > 0$.

Let us turn to the second point and let $\hat{w}_L(e)$ be the indifference curve of the low ability worker passing through the point (e_b, w'). By definition, the slope of the indifference curve at e_b is equal to the slope of the indifference curve of the high type, i.e. $\mathrm{d}\, \hat{w}_L(e_b)/\mathrm{d}\, e = \theta_H$. Therefore, by concavity, the graph of $\hat{w}_L(e)$ lies below the indifference curve of the high type passing through (e_b, w'). The high type 's indifference curve also crosses the point (δ'_H, π_H) where $\delta'_H = \delta_H(e_b, w')$, so that by concavity of $\hat{w}_L(e)$ we have $\hat{w}_L(\delta'_H) < \pi_H$. Noting that $\hat{w}_L(\delta'_L) = \pi_H$ we obtain $\hat{w}_L(\delta'_H) < \hat{w}_L(\delta'_L)$. As $\hat{w}_L(e)$ is increasing we have $\delta'_H < \delta'_L$, so that $g(e_b, w') = \delta'_H - \delta'_L < 0$ and this completes the proof.

[10]The case $w_1 = w_0$ leads immediately to a contradiction given strict monotonicity of $w_L(\cdot)$.

Q.E.D.

Proof of Proposition 2.

Notice first, that by using Lemma 2, it is easily seen that the education level $\bar{e}$ defined by the equation $U_L(\bar{e}, \bar{\pi}) = \pi_L$ supports a pooling equilibrium at wage $w = \bar{\pi}$. Moreover, any level of education between 0 and $\bar{e}$ supports a pooling equilibrium.

(i) Let $\bar{\pi} < w_0$. By Lemma 3.(ii) we have $g(\bar{e}, \bar{\pi}) > 0$; moreover, as g is decreasing in e (Lemma 3.(i)) we have $g(e, \bar{\pi}) > 0$ for all $e \in [0, \bar{e}]$. Therefore, for any education level $e_P \in [0, \bar{e}]$ supporting a pooling equilibrium, it must hold $g(e_P, \bar{\pi}) > 0$, i.e. $\delta_H(e_P, \bar{\pi}) > \delta_L(e_P, \bar{\pi})$, so that a deviation $\tilde{e}$ such that $\delta_L(e_P, \bar{\pi}) < \tilde{e} < \delta_H(e_P, \bar{\pi})$ exists which violates the conditions in Definition 2 for an intuitive equilibrium. Therefore, there is no intuitive pooling equilibrium if $\pi < w_0$.

(ii) Let $\bar{\pi} > w_0$. By Lemma 3.(ii) and 3.(iii), $g(\bar{e}, \bar{\pi}) < 0$ and $g(e_0, \bar{\pi}) > 0$. Given continuity and monotonicity of g in e, there exists and is unique the level of education $\underline{e}$ defined by (16) with $e_0 < \underline{e} < \bar{e}$. Noting that $g(e, \bar{\pi}) < 0$ for all $e \in]\underline{e}, \bar{e}]$, it is easily seen that any pooling equilibrium supported by an education level in this interval survives the Intuitive Criterion.

Q.E.D.

Proof of Proposition 3.

(i) By Proposition 2, there are no intuitive pooling equilibria, therefore only the LCSE could satisfy the Divinity Criterion when $\bar{\pi} < w_0$. Let us take the following beliefs: $\mu(e) = 1$ if $e \geq e^*$, $\mu(e) = \mu$ if $e_0 \leq e < e^*$ and $\mu(e) = 0$ if $e < e_0$. These beliefs are easily seen to support the LCSE strategy. We will show now that they obey the Divinity Criterion. Consider the deviation $e_0 \leq \tilde{e} < e^*$ and notice that the indifference curves of both types of worker cross at e_0 and at e^*. By applying (17), we have $D^0(L \mid \tilde{e}) \cup D(L \mid \tilde{e}) = [w_L(\tilde{e}), \pi_H]$ and $D(H \mid \tilde{e}) =]w_H(\tilde{e}), \pi_H]$, where $w_L(e)$ and $w_H(e)$ denote the indifference curves of the two types of worker. By concavity of the low ability worker indifference curve we have $w_L(\tilde{e}) > w_H(\tilde{e})$; therefore, $D^0(L \mid \tilde{e}) \cup D(L \mid \tilde{e}) \subset D(H \mid \tilde{e})$ and Definition 3 requires $\mu(\tilde{e}) \geq \mu$, which is satisfied by the above beliefs. Now take $\tilde{e} < e_0$. By concavity of the indifference curves of the L type, we have $w_L(\tilde{e}) < w_H(\tilde{e})$ so that $D^0(H \mid \tilde{e}) \cup D(H \mid \tilde{e}) \subset D(L \mid \tilde{e})$ and Definition 3 requires $\mu(\tilde{e}) \leq \mu$, which is satisfied by the above beliefs. Hence the LCSE passes the Divinity criterion test.

(ii) Let us first show that if $\bar{\pi} > w_0$ then the LCSE fails the Divinity Criterion. Consider the deviation $\tilde{e} = \bar{e}$, where $\bar{e}$ is given by $U_L(\bar{e}, \bar{\pi}) = \pi_L$. As we know, $e_0 < \bar{e} < e^*$. By concavity of the L type indifference curve, $\bar{\pi} > w'$, where $U_H(\tilde{e}, w') = U_H(e^*, \pi_H)$. As a result, $D^0(L \mid \tilde{e}) \cup D(L \mid \tilde{e}) = [\bar{\pi}, \pi_H]$ and $D(H \mid \tilde{e}) =]w', \pi_H]$. Since $D^0(L \mid \tilde{e}) \cup D(L \mid \tilde{e}) \subset D(H \mid \tilde{e})$ and the sets are not empty, Definition 3 requires that $\mu(\tilde{e}) \geq \mu$ and consequently the wage offered by the employer must be $w(\tilde{e}) \geq \bar{\pi}$. Since $U_H(e^*, \pi_H) = U_H(\bar{e}, w') < U_H(\bar{e}, \bar{\pi})$, we conclude that the separating equilibrium level of education e^* is not an optimal choice for the high ability worker if beliefs obey the Divinity Criterion. Hence, the LCSE can not be supported by beliefs satisfying the Divinity Criterion.

Let us turn to pooling equilibria. When $w_0 < \bar{\pi} < w_b$, the set of intuitive pooling equilibria is supported by $e \in [\underline{e}, \bar{e}]$ as stated in Proposition 2.(ii). It can be shown, by using similar arguments to those in (i), that for any $e_P \in [\underline{e}, \bar{e}]$, the beliefs $\mu(\tilde{e}) = \mu$ if $\tilde{e} > e_P$ and $\mu(\tilde{e}) = 0$ if $\tilde{e} < e_P$ support the pooling equilibrium and satisfy the Divinity Criterion.

Next, suppose that $\bar{\pi} > w_b$ and take a pooling equilibrium supported by $e_P \in]e_b, \bar{e}]$. Consider the deviation $\tilde{e} \in]e_b, e_P[$. The Divinity Criterion requires that $\mu(\tilde{e}) \geq \mu$, so that the wage offer is $w(e) \geq \mu$, which makes this deviation more profitable than the equilibrium choice e_P. Thus, no pooling equilibrium supported by $e_P > e_b$ can survive the Divinity Criterion.

Q.E.D.

References

Alós-Ferrer, C. and J. Prat (2012) Job Market Signaling and Employer Learning, *Journal of Economic Theory*, 147, 1787-1817.

Andrew, R. J., P. Jargowsky and K. Kuhne (2012) The Effects of Texas's Targeted Pre-Kindergarten Program on Academic Performance, *NBER Working paper No. 18598*

Banks J. S. and J. Sobel (1987), Equilibrium Selection in Signaling games, *Econometrica*, 55, 647-661.

Cho, I-K. and D. M. Kreps (1987), Signaling Games and Stable Equilibria, *Quarterly Journal of Economics*, 102, 179-221.

Cho, I-K. and J. Sobel (1990), Strategic Stability and Uniqueness in signaling games, *Journal of Economic Theory*, 50, 381-413.

Clark, B., Joubert, C. and A. Maurel (2014) The Career Prospects of Overeducated Americans *NBER Working Paper No. 20167*

Daley B. and B. Green (2014), Market Signaling with Grades, *Journal of Economic Theory*, 151, 114-145.

Fudenberg D. and J. Tirole, (1991), *Game Theory*, The MIT Press.

Spence, M.A. (1973), Job Market signaling, *Quarterly Journal of Economics*, 90, 225-243.

Spence, M.A. (1974), *Market Signaling*, Harvard University Press, Cambridge, MA.