ISVCS10, 11th International Symposium on Vibrations of Continuous Systems
Royal Victoria Hotel, Llanberis, Snowdonia, Wales, UK, July 16-22, 2017

Vibration Localization of Imperfect Circular Cylindrical Shells

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Abstract

The goal of the present paper is the analysis of the effect of geometric imperfections in circular cylindrical shells. Perfect circular shells are characterized by the presence of double shell-like modes, i.e., modes having the same frequency with modal shape shifted of a quarter of wavelength in the circumferential direction. In presence of geometric imperfections, the double natural frequencies split into a pair of distinct frequencies, the splitting is proportional to the level of imperfection. In some cases, the imperfections cause an interesting phenomenon on the modal shapes, which present a strong localization in the circumferential direction. This study is carried out by means of a semi-analytical approach compared with standard finite element analyses.

1. Introduction

Circular cylindrical shells have been proven to be sensitive to initial geometric imperfections. Kubenko and Koval'chuk [1] reviewed many studies on the influence of the initial imperfections on the natural frequencies and mode shapes of elastic shells. They analysed the splitting of the natural frequencies of two conjugate modes as a measure of the initial geometric imperfections imposed. Katawala and Nash [2] studied the influence of the initial geometric imperfections on the vibrations of thin circular cylindrical shells. They found that the natural frequency of free vibration of the imperfect modes increases with increasing the amplitude of imperfection on the modal shape. However, the effect of imperfections on mode shapes appears not yet well understood, in particular localization phenomena have been not evidenced in the past.

In the present paper, the linear vibrations and modal localization in circular cylindrical shells are analysed in the framework of the Sanders-Koiter theory (see e.g. [3, 4]). The shell deformation is described in terms of longitudinal, circumferential and radial displacement fields. Clamped-clamped boundary conditions are investigated. The three displacement fields are expanded by means of a double mixed series based on Chebyshev polynomials for the longitudinal variable and harmonic functions for the circumferential variable; the Rayleigh-Ritz method is used to get approximated natural frequencies (eigenvalues) and mode shapes (eigenfunctions). The three displacement fields are re-expanded by using the previous approximated eigenfunctions; conjugate mode shapes are used; geometric imperfections are imposed on the modes in order to investigate the modal localization of the shells.

The semi-analytical approach proposed in the paper is validated in linear field by means of comparisons with finite element analyses.

2. Shell theory

In Figure 1, a circular cylindrical shell having radius R, length L and thickness h is represented; a cylindrical coordinate system (0; x, θ, z) is considered to take advantage from the axial symmetry; the origin 0 of the reference system is located at the centre of one end of the shell. Three displacement fields are represented: longitudinal u (x, θ, t), circumferential v (x, θ, t) and radial w (x, θ, t).

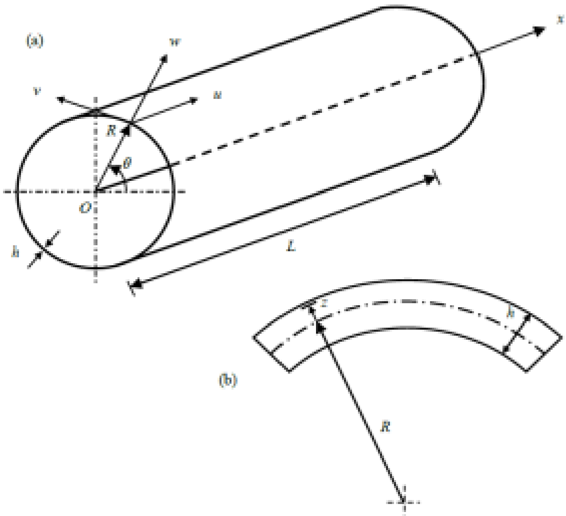

Figure 1: Geometry of the functionally graded shell. (a) Complete shell; (b) cross-section of the shell surface.

The Sanders-Koiter shell theory is based on the Love's first approximation [3, 4]. The strain components (ε_x, ε_θ, $\gamma_{x\theta}$) are related to the middle surface strains ($\varepsilon_{x,0}$, $\varepsilon_{\theta,0}$, $\gamma_{x\theta,0}$) and to the changes in curvature and torsion (k_x, k_θ, $k_{x\theta}$) of the middle surface of the shell by the following relationships [3,4]. Isotropic shells are considered.

The elastic strain energy and the kinetic energy of a cylindrical shell are given by [3,4]

$$U_s = \frac{1}{2} LR \int_0^1 \int_0^{2\pi} \int_{-h/2}^{h/2} (\sigma_x \varepsilon_x + \sigma_\theta \varepsilon_\theta + \tau_{x\theta} \gamma_{x\theta}) d\eta d\theta dz \qquad T_s = \frac{1}{2} \rho h LR \int_0^1 \int_0^{2\pi} (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) d\eta d\theta \tag{4}$$

where h, R and L are the thickness, radius and length of the shell, respectively.

Discretization approach

In order to carry out the linear analysis of the shell, a two-step procedure is applied [3, 4]: i) the three displacement fields are expanded using a double mixed series and the Rayleigh-Ritz method is applied to obtain approximated eigenfunctions; ii) the displacement fields are re-expanded by using the approximated eigenfunctions and modal geometric imperfections are imposed on the modes.

3. Numerical results

The mechanical properties of the circular cylindrical shell considered are reported in Table 1; these data are referred to a PET shell, which are generally used for our lab experiments. In order to validate the semi-analytical approach proposed in this paper, the natural frequencies of the clamped-clamped cylindrical shell of Table 1 obtained by applying the Sanders-Koiter theory are compared with those obtained by FE analyses, see Table 2. These comparisons show that the present semi-analytical method gives results close to the FEM, the differences being less than 2%.

Table 1: Mechanical parameters of the clamped-clamped circular cylindrical shell.

Young's modulus E [GPa]	27.58
Poisson's ratio ν	0.42
Mass density ρ [kg/m3]	1541
Thickness h [mm]	0.25
Radius R [mm]	43.65
Length L [mm]	95.50

Table 2: Natural frequencies of the clamped-clamped circular cylindrical shell of Table 1. Comparisons between Sanders-Koiter shell theory (SKT) and finite element method (FEM).

Mode	Natural frequency (Hz)		Difference %
(j,n)	SKT	FEM	
(1,7)	1850.49	1834.56	0.86
(1,6)	1850.97	1828.80	1.20
(1,12)	4099.48	4087.13	0.30
(1,0)	15303.9	15315.8	0.08

The analysis is carried out considering a clamped clamped shell with small geometric modal imperfections (modes (1,6) and (1,7)) having magnitude 20%h; Figure 2 shows the effect of imperfections on modes (1,6) and (1,7,c), where “c” means the “conjugate mode”, a macroscopic localization on mode (1,6) and a slight asymmetry on mode (1,7c) are visible. Results are compared with FEM analyses, see also Table 2. The localization appears when the imperfection is a combination of two distinct modes and the magnitude exceeds the threshold of 10%. Indeed, if the imperfection is considered on mode (1,6) (or (1,7)) only, no localization appears, even if the imperfection magnitude is greatly increased. Moreover, the boundary conditions play a role, e.g. free-free shells do not display localization.

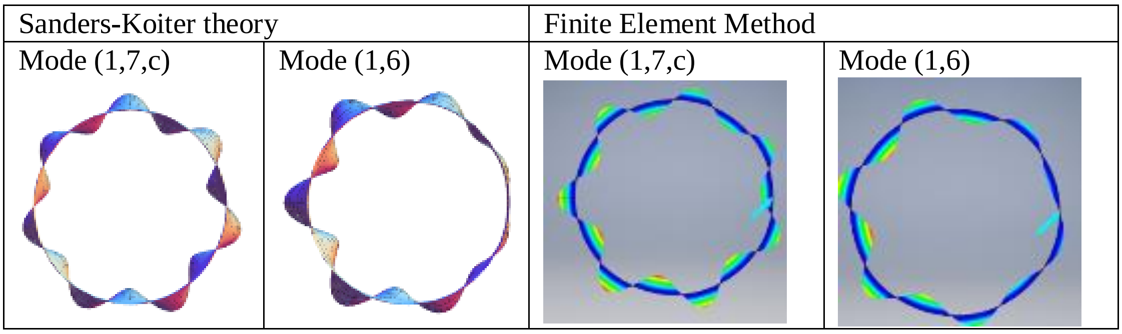

Figure 2: Geometric imperfections wp(1,6)=wp(1,7)=0.2h, wp(1,6,c)=wp(1,7,c)= 0. Localization.

4. Conclusions

In this paper, the effect of the geometric imperfections on the modal shapes of cylindrical shells is analysed. The Sanders-Koiter shell theory is used. The shell deformation is described in terms of longitudinal, circumferential and radial displacement fields. Clamped-clamped boundary conditions are applied. Geometric imperfections are imposed to investigate the modal localization of the shells.

The semi-analytical approach proposed in this paper is validated in linear field by means of comparisons with finite element analyses.

It is found that small initial geometric imperfections imposed on single modes or conjugate pairs of modes gives no localization on the modal shape. Conversely, it is seen that small initial geometric imperfections imposed on two non-conjugate modes give modal localization on all the modes; this localization is maximum on the modal shape corresponding to the initial modal imperfection.

References

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