## Analytical bounds for the pull-in voltage of carbon nanotubes

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Carbon nanotubes (CNTs) display a number of attractive electronic and mechanical properties that are currently exploited in a wide variety of industrial applications, such as sensors, nanoactuators, memory devices, switches, high frequncy nanoresonators and nanotweezers. Due to their tiny size they indeed display ultra-low mass and very high resonance frequency as well as the capability to carry huge electrical currents and to sustain high current densities. These properties, in conjunction with the significant progress recently made in the fabrication of carbon nanostructures, allow CNTs to become essential components in the fabrication of enhanced nano-electromechanical systems (NEMS)

An accurate determination of the stable actuating range and the pull-in instability threshold is a crucial issues for designing reliable CNT based NEMS. Despite the amount of numerical or approximated investigations, analytical models and closed form expressions for pull-in instability analysis of CNT still appears to be limited. An analytical methodology for assessing accurate lower and upper bounds to the pull-in parameters of an electrostatically actuated micro- or nanocantilever has been provided in two previous works [1, 2], taking into consideration the contributions of flexible support and compressive axial load.

In the present work, attention is paid to investigate the pull-in phenomenon in CNT with circular cross-section, by considering the proper expressions of the electrostatic and van der Waals forces per unit length acting on a CNT, as well as the significant reduction of the pull-in voltage induced by the charge concentration at the free end [3]. Two-side accurate analytical estimates of the pull-in parameters of a carbon nanotube switch clamped at one end under electrostatic actuation are provided by considering the effects of van der Waals interactions and charge concentration at the free end. The problem is governed by a fourth-order nonlinear boundary value problem, according to the Eulero-Bernoulli beam theory. Two-side estimates on the deflection are first derived, then very accurate lower and upper bounds to the pull-in voltage and deflection are obtained as functions of the geometrical and material parameters. The analytical predictions are then found to agree remarkably well with the numerical results provided by the shooting method.

## References

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