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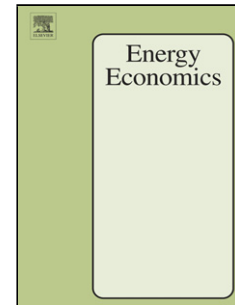
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How do normalization schemes affect net spillovers? A replication of the Diebold and Yilmaz (2012) study

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Highlights

- This paper replicates the Diebold and Yilmaz, DY, (2012) study on financial markets connectedness
- The markets are the commodity and the stock, bond, FX for the US.
- Similar to DY, we use the Generalized Forecast Error Variance Decomposition, GEFVD
- We compare normalization schemes to GEVD
- We show that a scalar-based normalization is preferable to the row normalization suggested by DY

Abstract

This paper replicates the Diebold and Yilmaz (2012) study on the connectedness of the commodity market and three other financial markets: the stock market, the bond market, and the FX market, based on the Generalized Forecast Error Variance Decomposition, GEFVD. We show that the net spillover indices (of directional connectedness), used to assess the net contribution of one market to overall risk in the system, are sensitive to the normalization scheme applied to the GEFVD. We show that, considering data generating processes characterized by different degrees of persistence and covariance, a scalar-based normalization of the Generalized Forecast Error Variance Decomposition is preferable to the row normalization suggested by Diebold and Yilmaz since it yields net spillovers free of sign and ranking errors.

Keywords: causality; normalization schemes; generalized forecast error variance decomposition; spillover; simulation; Vector Autoregression Models.

JEL classification: C15; C53; C58; G17.

1 Introduction

A normalization scheme is a set of one or more constraints to be imposed on a matrix such that the resulting scaled version will satisfy certain conditions. Scaling a matrix such that its rows or columns sum to one is one of the most common normalization schemes. A normalization scheme is adopted either for estimation purposes or simply for interpretative purposes.

The aim of this paper is to show how the choice of a normalization scheme affects the sign and the magnitude of net spillovers in the Diebold and Yilmaz (2012) framework. To this end, we first provide a review of the most common normalization schemes used in different financial applications, with a particular focus on forecast error variance decomposition. In fact, the implementation of the generalized forecast error variance decomposition yields a variance decomposition table that has to be normalized for interpretative purposes. We show that the net spillover indices (of directional connectedness) used to assess the net contribution of one market to systemic risk are sensitive to the normalization scheme of the GEFVD. Moreover, we also show that the choice of the normalization scheme affects the ranking of variables in terms of their relative contribution to the system and the assessment of the total index of connectedness.

Second, in line with Caloia et al. (2018), we suggest a scalar-based normalization scheme overcoming the limits of the traditional row-normalization scheme, used in Diebold and Yilmaz (2012). In this paper however, the advantages and disadvantages of the most popular normalization schemes are assessed through examples and simulations. In particular, we study the extent to which a wrong normalization choice can lead to biased spillover measures, when the data generating process is characterized by different degrees of persistence and covariance.

Finally, we replicate the analysis in Diebold and Yilmaz (2012) in order to show how their results change in terms of net spillovers as the normalization scheme changes. We analyse in particular the row-normalization, the alternative column normalization also suggested in Diebold and Yilmaz (2012) and the maximum row sum scalar normalization proposed in Caloia et al. (2018). We show that the bond market turns out to have received less volatility spillovers than the commodity market if the row-normalization scheme (or the column-normalization) is used, as in Diebold and Yilmaz (2012). However, if we use scalar normalization, it is evident that the result is the opposite: the bond market turns out to have received more volatility spillovers than the commodity market. As a result, the paper warns about the use of the popular row-normalization or column-normalization schemes if the aim of the research is to assess net spillovers.

The results of the paper are intended to be useful not only for deriving spillover measures, but also in any other field where a matrix normalization scheme is adopted, such as network analysis or spatial econometrics.

The structure of the paper is as follows. In Section 2 we provide an overview of the methodology suggested by Diebold and Yilmaz (2012) to construct a network graph and, in particular, indices of connectedness. In Section 3 we review the most common normalization schemes used in various fields. Section 4 highlights how persistence and covariance among the series affect the results of the spillover analysis based on different normalization schemes. Section 5 replicates the study by Diebold and Yilmaz (2012). The final section concludes.

2. Networks and connectedness

Networks are usually represented in graphs, where nodes and edges are graphically displayed. A weighted network is a network that allows for weights on the edges in order to represent stronger or weaker connections between nodes, while direct networks are networks that allow for asymmetries. One example of a weighted and direct network (also varying across forecast horizons) is the forecast error variance decomposition (FEVD) (Diebold, Yilmaz (2014)).

Forecast error variance decomposition is a standard econometric tool used in multivariate time series analysis to assess the contribution in terms of forecast error variance of each variable due to a shock to any of the other variables. Consider a covariance stationary VAR(p) with k endogenous variables:

$$x_t = c + A_1 x_{t-1} + \dots + A_p x_{t-p} + \varepsilon_t \quad (1)$$

where ε_t are i.i.d. disturbances with contemporaneous covariance matrix Σ . In order to derive the moving average representation of the VAR(p), we rewrite (1) as a first-order system:

$$\begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & & A_p \\ I & 0 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

which can be written in compact form:

$$\bar{x}_t = \bar{A} \bar{x}_{t-1} + \bar{\varepsilon}_t \quad (3)$$

where \bar{A} is the $kp \times kp$ companion matrix. Then, using the selection matrix $\bar{e} = \begin{bmatrix} I & \bar{0} & \dots & \bar{0} \end{bmatrix}$ which has k rows and kp columns (and the first k rows and columns are given by the identity matrix I),

we obtain the $k \times k$ moving average coefficient matrix Ψ_h :

$$\Psi_h = \bar{e} \bar{A}^h \bar{e}' \quad (4)$$

Given a set of k endogenous variables, Diebold and Yilmaz (2012; 2014) use the Pesaran and Shin (1998) “generalized” approach (GFEVD) that allows shocks to be correlated (which is insensitive to variable ordering,) to obtain the element in row i and column j of the connectedness matrix proxied by FEVD. More specifically, the contribution of the j -th shock to the h step ahead forecast error variance of the i -th endogenous series is computed as follows:

$$\theta_{ij}^g = \frac{\sigma_{jj}^{-1} \sum_{l=0}^{h-1} (e_i' \Psi_l \Sigma e_j)^2}{\sum_{l=0}^{h-1} (e_i' \Psi_l \Sigma \Psi_l' e_i)} \quad (5)$$

where σ_{jj} the standard deviation of the disturbance of the j th equation, and e_i is the selection vector with one as the i th element and zeros otherwise.

Due to the non-orthogonality of shocks, the sum of the contributions to the forecast error variance (i.e. the row sum) is not equal to one¹. The authors therefore propose a row-normalization of the values of the variance decomposition in equation (5) to interpret its elements as variance shares:

$$\tilde{\theta}_{ij}^g = \frac{\theta_{ij}^g}{\sum_{j=1}^K \theta_{ij}^g} \quad (6)$$

They also mention the equivalence between this row-normalization scheme and the alternative column-normalization scheme (see Section 3).

Lanne and Nyberg (2016) propose a new generalized forecast error variance decomposition, based on the generalized impulse response function, which, in the case of a linear VAR, is given by²:

$$\theta_{ij}^g = \frac{\sigma_{jj}^{-1} \sum_{l=0}^{h-1} (e_i' \Psi_l \Sigma e_j)^2}{\sum_{j=1}^K [\sigma_{jj}^{-1} \sum_{l=0}^{h-1} (e_i' \Psi_l \Sigma e_j)^2]} \quad (6')$$

Equations (5) and (6') share the same numerator (the cumulative effect of the j -th shock), but they have a different denominator: the total forecast error variance of the i -th endogenous variable (eq. 5) and the aggregate cumulative effects of all the shocks (in eq. (6')). This implies that both (6) and (6') are identical and they share the property that the proportions of the impact accounted for by innovations in each variable sum to unity (e.g. they both lead to a row-normalized spillover matrix). Diebold and Yilmaz (2014) rely on the absolute value of net pairwise spillovers when drawing a graph of the network, which is obtained from generalized forecast error variance decomposition. Therefore in this case, the network graph depends on the chosen normalization scheme for the GFEVD.

Moreover, as shown by Diebold and Yilmaz (2014), given the $k \times k$ connectedness row-normalized matrix

¹ The moving average coefficients necessary to compute the spillover indices are obtained by means of the estimation of a traditional VAR in Diebold and Yilmaz (2012). Cipollini et al. (2017) and Fengler et al. (2018) obtain the moving average coefficients through the estimation of a fractionally integrated VAR and of a multivariate GARCH, respectively.

² Lanne and Nyberg (2016) suggest a residual bootstrap approach to compute the row normalized GEFVD in case of a non-linear VAR model.

(with the row i , column j element measured by $\tilde{\theta}_{ij}^g$ described in (6)), the summary descriptive statistics of the network are given by the following directional connectedness indices:

$$DS_{\rightarrow \bullet}^g(h) = \sum_{\substack{j=1 \\ j \neq i}}^K \tilde{\theta}_{ij}^g \quad ; \quad DS_{\bullet \rightarrow}^g(h) = \sum_{\substack{i=1 \\ i \neq j}}^K \tilde{\theta}_{ij}^g \quad (7)$$

The directional spillover measuring the vulnerability of each market to shocks arising in all the other markets (FROM others, $DS_{\rightarrow \bullet}^g$) is computed as the sum of the off-diagonal elements of each row of the variance decomposition matrix. The spillover index measuring the transmission of a shock to one market to all the other markets (TO others, $DS_{\bullet \rightarrow}^g$) is computed as the sum of the off-diagonal elements of each column of the connectedness matrix. A measure of net spillover (NET) of each market is obtained as the difference between the directional spillovers TO others and FROM others:

$$NET^g(h) = DS_{\bullet \rightarrow}^g(h) - DS_{\rightarrow \bullet}^g(h) \quad (8)$$

In this way, in line with Diebold and Yilmaz (2009, 2012, 2014) we are able to identify the markets that are net donors and those that are net receivers in terms of risk transmission.

Consequently, not only the network graph, but also the summary statistics described by the directional connectedness indices depend on the chosen normalization scheme for the GFEVD. Given the critical role of normalization, in the next section we review the different schemes proposed in the literature.

3. Normalization schemes

In this section, we review the most commonly used normalization schemes in GFEVD and spatial regression models. As an example, in financial econometrics the row normalization of the GFEVD yields a direct weighted graph. In spatial econometrics, on the other hand, undirected weighted graphs are mostly used, in order to allow for symmetric linkages (e.g. borders). More specifically, consider a standard generalised spatial autoregressive model of order p , or simply SAR(p) model:

$$u = \sum_{h=1}^p \phi_h W_h u + \varepsilon \quad (9)$$

where $W = (w_{i,j})$ for $i, j = 1, \dots, k$ is the spatial weight matrix and ϕ_h are autoregressive parameters. Equivalently, we can rewrite equation (9) as follows:

$$u = \left(I_N - \sum_{h=1}^p \phi_h W_h \right)^{-1} \varepsilon \quad (10)$$

Mention should be made of the different normalization schemes of the spatial weight matrix W used in the literature to make $(I_N - \sum_{h=1}^p \phi_h W_h)$ non-singular. These schemes can be applied also to GFEVD.

3.1 Row normalization

Given a $(k \times k)$ unscaled matrix $W^* = (w_{ij}^*)$, we can obtain the corresponding row-stochastic matrix $W = (w_{ij})$ by row-normalizing W^* such that:

$$w_{ij} = \frac{w_{ij}^*}{\sum_{j=1}^k w_{ij}^*} \quad (11)$$

The resulting matrix W has row sums equal to one. However, row normalization is not a restrictive task since the same result can be achieved by constraining the parameter space of the autoregressive parameters ϕ_h (Caporin and Paruolo (2015)). As a result, the normalization task would be absorbed by the AR parameter through scaling.

Moreover, this normalization is useful in interpreting spatial weight matrices, the elements of which can be thought of as a fraction of all spatial influence. This interpretative advantage also applies for a forecast error variance decomposition that does not rely on a Cholesky factorization (or any other identifying scheme of structural VAR models) so that the matrix coefficients can be interpreted as variance shares. This is the normalization scheme proposed by Diebold and Yilmaz (2012) when using the generalized forecast error variance decomposition (see eq. 5 in Section 2) and this scheme coincides with the Lanne and Nyberg (2016) method to compute the GEFVD in case of a linear VAR model (see eq. 6' in Section 2).

This scheme has an interpretative advantage. The directional spillovers received FROM others including own sum to one and, as a result, each element of the forecast error variance decomposition matrix can be interpreted as a variance share (by row), i.e. it measures the contribution of each variable to the 100-percentage point forecast error variance of every other variable. Since Diebold and Yilmaz (2012) also aim to measure the contribution of each variable to the forecast error variances of other variables, row normalization serves their purpose quite well.

However, this scheme also has certain drawbacks: by scaling the elements of each row by the corresponding row sum, the order of magnitude among the elements of the matrix is preserved only by row. Therefore, the NET spillover indices are obtained by subtracting the off-diagonal sum of each row from the corresponding sum in each column, that is, by subtracting two quantities measured in different terms (the row elements sum to one, while the column elements do not). This implies that NET spillovers indices may be misspecified in sign and magnitude.

3.2 Column normalization

This scheme mirrors the row-normalization scheme described above. The only difference is that the normalization is by column: in this case only the columns sum to one. In the case of the column normalization, the focus is on the extent to which one variable affects the system: each element of the

forecast error variance decomposition matrix can be interpreted by column as the fraction of total variance transmitted by each variable to the forecast error variance of the other variables.

The issues concerning row-normalization also apply in this case: the elements are normalized only along one dimension (the column) and the order of magnitude is preserved only by column. Note that for the variance decomposition Diebold and Yilmaz (2012) suggest this normalization scheme as an alternative to row normalization.

3.3 Max row normalization

In this normalization scheme, the normalization factor is a scalar equal to the maximum row sum of the unscaled matrix W^* , then the scaled matrix is obtained as $W = W^*/k$ where:

$$k = \max(r_1, \dots, r_k) \quad (12)$$

and:

$$r_i = \sum_{j=1}^k w_{ij}^* \quad (13)$$

where w_{ij}^* is the element in row i and column j of the unscaled matrix W^* . This scheme is characterized by a single normalization factor instead of the k factors of the row normalization scheme (one for each row). As a result, it preserves the magnitude relation among the elements of rows and columns, meaning that column and row values can be safely compared. Moreover, it allows for a comparison between different rows and column sums, making it possible to distinguish between stronger or weaker influences. Third, taking the variable that has the maximum row sum as the reference one, the other directional spillovers by row (FROM others) represent the degree of vulnerability of each variable relative to the reference one. Last, and most importantly, since NET spillovers indices are correctly specified in sign and magnitude, the max row normalization scheme can improve the interpretation of the results to a significant extent.

As argued by Billio et al. (2016) it is also possible to normalize by the maximum row sum over time in order to compare spatial weight matrices in different time periods while preserving a reasonable magnitude of autoregressive parameters.

3.4 Max column normalization

This scheme mirrors the max row normalization described above: the only difference is that the scalar is equal to the maximum column sum of the unscaled matrix W^* . The same advantages of the max row normalization apply.

3.5 Spectral radius normalization

Let W^* be the $(k \times k)$ positive unscaled matrix and let $\{\lambda_1, \dots, \lambda_k\}$ be the eigenvalues of W^* . The spectral radius is the maximum eigenvalue (in modul), formally:

$$\tau = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_k|\} \quad (14)$$

The scalar normalization factor is set equal to the spectral radius and the scaled matrix W is therefore obtained as follows:

$$W = W^* / \tau \quad (15)$$

Under the Perron and Frobenius theorem, the spectral radius satisfies the following inequalities:

$$\min_i \sum_{j=1}^N w_{ij} \leq \tau \leq \max_i \sum_{j=1}^N w_{ij} \quad (16)$$

As a result, some row sums and column sums exceed unity, while others can be less than one. This normalization scheme therefore has one main drawback: the elements can no longer be interpreted as fractions of the overall influence (e.g. the sum by row and by column).

Nevertheless, this normalization scheme is widely used in spatial econometrics: in fact, following LeSage and Pace (2010) a matrix W^* can be transformed to have maximum eigenvalue equal to one using $W = W^* \max(\lambda_{W^*})$, and this is a desirable property because it constrains the autoregressive parameter to have maximum possible value equal to one. In particular, Kelejian and Prucha (2010) show that $(I_N - \sum_{h=1}^p \phi_h W_h)$ is non-singular for all the values of the parameter space in the interval $(-1; 1)$.

4 Comparison of the normalization schemes in GFEVD

The review of different normalization schemes in section 3 shows that row normalization has both interpretative advantages and limits and, in this framework, leads to misspecified spillover measures. In particular:

- If the normalization is carried out by row, the column sum is not necessarily equal to one. As a result, while FROM directional spillovers can be interpreted as a fraction of the total variance received via spillovers, TO directional spillovers lack this kind of interpretation (some column sums are above unity, while some others are below unity).
- Normalization by row implies that the order of magnitude of the entries of the variance decomposition table is preserved only row by row. As a result, it is not possible to make comparisons across rows in order to determine which variable is affected the least (or the most) FROM others.
- NET spillovers are obtained as the difference between two that are non-comparable in magnitude, and as a result they may be misspecified in sign and magnitude.
- The total connectedness index changes if normalization is carried by row or by column.

In this section, we show how the normalization schemes reviewed in section 3 affect the GFEVD, by using data generating processes characterized by different degrees of persistence and covariance. In fact, as shown by Pesaran and Shin (1998), the Impulse Response Function underlying the GFEVD, assuming a multivariate Gaussian distribution for the shocks ε_t and a linear VAR model, depends on the persistence and the covariance structure of the multivariate process. Here, we show how the spillover values are sensitive to the normalization choice, thus leading to misspecified measures of net contribution (NET), and we show how the likelihood of obtaining misspecified NET spillovers depends on i) the forecast horizon h ii) the degree of persistence iii) the covariance structure of the endogenous variables.

We consider four cases: a) LL (Low Persistence; Low Covariance); b) LH (Low Persistence; High Covariance); c) HL (High Persistence; Low Covariance); d) HH (High Persistence; High Covariance), according to the different setups of the VAR model described in eq. (1). For each scenario, we produce a two-days-ahead ($h = 2$) and a ten-days-ahead ($h = 10$) forecast.

We set the number of endogenous variables k , equal to five. The model configurations differ for the coefficient matrices in the lag operator $A(L)$ and of the covariance matrix $\Sigma = P P'$. In particular, the Low Covariance case is defined by using a lower triangular matrix P set as follows:

$$P_0 = \begin{bmatrix} 0.10 & 0 & 0 & 0 & 0 \\ 0.15 & 0.15 & 0 & 0 & 0 \\ 0.20 & 0.20 & 0.20 & 0 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 \\ 0.30 & 0.30 & 0.30 & 0.30 & 0.30 \end{bmatrix} \quad (17)$$

while for the High Covariance case, we define two subcases to disentangle the role of correlation-driven comovements. In particular, the High Covariance case is defined by the following two lower triangular matrices:

$$P_1 = \begin{bmatrix} 0.40 & 0 & 0 & 0 & 0 \\ 0.45 & 0.45 & 0 & 0 & 0 \\ 0.50 & 0.50 & 0.50 & 0 & 0 \\ 0.55 & 0.55 & 0.55 & 0.55 & 0 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.60 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0.40 & 0 & 0 & 0 & 0 \\ 0.55 & 0.45 & 0 & 0 & 0 \\ 0.70 & 0.70 & 0.50 & 0 & 0 \\ 0.75 & 0.75 & 0.75 & 0.55 & 0 \\ 0.80 & 0.80 & 0.80 & 0.80 & 0.60 \end{bmatrix} \quad (18)$$

It is evident that the matrix P_1 keeps the same correlation structure as matrix P_0 , that is: $\rho_{1,i,j} =$

$$\frac{\sigma_{1,i,j}}{\sqrt{\sigma_{1,i,i} \sigma_{1,j,j}}} = \rho_{0,i,j} \quad \forall i \neq j; \text{ where } \sigma_{k,i,j} \text{ is the } ij \text{ --th element of the matrix } \Sigma_k = P_k P_k' \text{ with } k = \{0, 1\}^3.$$

³ The covariance and correlation matrices corresponding to P_0, P_1 and P_2 are respectively:

On the contrary, matrix P_2 induces a higher correlation structure among the endogenous variables with respect to both the matrices P_0 and P_1 .

To ensure a stationary VAR(p) (e.g. with roots of the characteristic polynomial $A(L)$ outside the unit circle) characterized by Low Persistence, we consider a VAR(2) with coefficient matrices A_1 and A_2 with values equal to 0.05. A stationary VAR(p) characterized by High Persistence is a restricted VAR(22) given by the parsimonious Vector HAR representation with coefficient matrices $A^{(d)}, A^{(w)}, A^{(m)}$ described as follows: $A^{(d)}$ with values equal to 0.05, $A^{(w)}$ with values equal to -0.02 and $A^{(m)}$ with values equal to 0.01⁴.

Consequently, we compute the generalized forecast error variance decomposition as defined by equation (5) and we obtain the measures of NET contribution. Formally, the non-normalized NET spillovers for the forecast horizon h , which are taken as benchmark, are obtained as follows:

$$NET_i^g(h) = DS_{\bullet \rightarrow i}^g(h) - DS_{i \rightarrow \bullet}^g(h) \quad (19)$$

where:

$$DS_{\bullet \rightarrow i}^g(h) = \sum_{\substack{j=1 \\ j \neq i}}^K \theta_{ij}^g \quad ; \quad DS_{i \rightarrow \bullet}^g(h) = \sum_{\substack{j=1 \\ j \neq i}}^K \theta_{ij}^g \quad (20)$$

where $DS_{\bullet \rightarrow i}^g$ denotes the non-normalized directional spillover transmitted by the market i to all other markets j (named TO others), while $DS_{i \rightarrow \bullet}^g$ denotes the non-normalized directional spillover received by market i from all the other markets j (named FROM others). Second, we compute the \overline{NET} spillovers obtained from the forecast error variance decomposition normalized by the different schemes:

$$\overline{NET}_i^g(h) = \overline{DS}_{\bullet \rightarrow i}^g(h) - \overline{DS}_{i \rightarrow \bullet}^g(h) \quad (21)$$

$$\Sigma_0 = \begin{bmatrix} 0.01 & 0.03 & 0.02 & 0.02 & 0.03 \\ 0.015 & 0.045 & 0.06 & 0.07 & 0.09 \\ 0.02 & 0.06 & 0.12 & 0.15 & 0.18 \\ 0.025 & 0.075 & 0.15 & 0.25 & 0.30 \\ 0.03 & 0.09 & 0.18 & 0.3 & 0.45 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 0.16 & 0.18 & 0.20 & 0.22 & 0.24 \\ 0.18 & 0.405 & 0.45 & 0.495 & 0.54 \\ 0.20 & 0.45 & 0.75 & 0.825 & 0.90 \\ 0.22 & 0.495 & 0.825 & 1.21 & 1.32 \\ 0.24 & 0.54 & 0.90 & 1.32 & 1.80 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 0.16 & 0.26 & 0.28 & 0.30 & 0.32 \\ 0.26 & 0.63 & 0.77 & 0.83 & 0.88 \\ 0.28 & 0.77 & 1.23 & 1.43 & 1.52 \\ 0.30 & 0.83 & 1.43 & 1.99 & 2.24 \\ 0.32 & 0.88 & 1.52 & 2.24 & 2.92 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 1 & 0.71 & 0.58 & 0.50 & 0.45 \\ 0.71 & 1 & 0.82 & 0 & 0.63 \\ 0.58 & 0.82 & 1 & 0.87 & 0.77 \\ 0.50 & 0.71 & 0.87 & 1 & 0.89 \\ 0.45 & 0.63 & 0.77 & 0.89 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0.71 & 0.58 & 0.50 & 0.45 \\ 0.71 & 1 & 0.82 & 0 & 0.63 \\ 0.58 & 0.82 & 1 & 0.87 & 0.77 \\ 0.50 & 0.71 & 0.87 & 1 & 0.89 \\ 0.45 & 0.63 & 0.77 & 0.89 & 1 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0.82 & 0.63 & 0.53 & 0.47 \\ 0.82 & 1 & 0.88 & 0.74 & 0.65 \\ 0.63 & 0.88 & 1 & 0.91 & 0.80 \\ 0.53 & 0.74 & 0.91 & 1 & 0.93 \\ 0.47 & 0.65 & 0.80 & 0.93 & 1 \end{bmatrix}$$

⁴ In the Vector HAR model the matrices $A^{(d)}, A^{(w)}$ and $A^{(m)}$ are coefficient matrices associated with the three terms of daily, weekly and monthly partial volatility components, respectively. In particular, the Vector HAR model can be written as follows:

$$x_t^{(d)} = c + \phi^{(d)} x_{t-1}^{(d)} + \phi^{(w)} x_{t-1}^{(w)} + \phi^{(m)} x_{t-1}^{(m)} + \varepsilon_t$$

where x_t are daily volatilities, while the terms representing the weekly and monthly volatilities are obtained as the arithmetic average of the daily volatilities recorded in the last week and the last month, respectively.

where the over bar denotes the normalized spillovers. These normalized measures are compared to the benchmark spillovers in equation (19). The comparison is intended to assess the reliability of the different normalization schemes both in terms of order of ranking (to assess which market is the largest net contributor to the total connectedness) and in terms of sign (to distinguish net donors from net receivers).⁵

4.1 Results based on population parameters

In this section we cast light on how the choice of the normalization scheme can affect the ranking and the sign of the NET spillovers, by means of an introductory example. Moreover, in order to show how the spillover tables change for different forecast horizons, two different horizons are reported: the two-day horizon is reported in the upper panel of every Table, while the lower panel contains the ten-day forecast horizon.

For this introductory example we report the results based on the population parameters for the “High Persistence, High Covariance” scenario, which is the most illuminating one. Table 1 shows the spillover table based on the non-normalized forecast error variance decomposition that is taken as a benchmark. Tables 2 to 6 show the same spillover table after applying the different normalization schemes outlined in Section 3 (Table 2 for row normalization, Table 3 for column normalization, Table 4 for normalization by spectral radius, Table 5 for normalization by maximum row sum, Table 6 for normalization by maximum column sum). These Tables show the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others including own), the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others including own), and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for each variable V_i for $i = 1, \dots, 5$. The tables also show the sign of the NET spillover (NET sign): negative if the market is the net receiver and positive if the market is the net donor, and the ranking of the NET spillover from the highest to the lowest (NET ranking). The tables also show the total spillover index computed as the ratio between the sum of cross-variance shares and the sum of cross and own variance shares.

In Table 2 we show the standard row-normalization scheme proposed by Diebold and Yilmaz (2012) which has the interpretative advantage that the directional spillovers received FROM others including own sum to one, and as a result each element of the forecast error variance decomposition matrix can be interpreted as a variance share (by row). For example, in the upper panel variable 2 receives the

⁵ In order to measure the contribution of a variable to the system, Greenwood-Nimmo et al. (2015) propose the computation of an Influence Index for each variable. The Influence Index for variable i is defined as the ratio between the NET index of variable i and the sum of the FROM and TO indices of variable i . The Influence Index is bounded between -1 and 1 and measures the extent to which variable i influences (if the index is positive) or is influenced by the system (if the index is negative). We highlight that the ranking and sign errors associated with the row normalization or to the column normalization would affect the computation of the Influence Index.

most from variable 3 ($0.219 = 21.9\%$), and the least from variable 5 ($0.140 = 14\%$). Moreover, variable 1 represents the variable least affected by the others ($\text{FROM others}=0.653$), while variable 3 is the most affected by the others ($\text{FROM others}=0.705$). In Table 2 Panel B, variable 1 is the market most affected by the others ($\text{FROM others}=0.729$), while variable 5 is the least affected by the others ($\text{FROM others}=0.674$).

On the contrary in Table 3 all the columns (TO others including own) sum to one: each element of the forecast error variance decomposition matrix can be interpreted as the fraction of total variance transmitted. For example, in the upper panel of Table 3 variable 2 gives the least to variable 5 (0.131), and the most to variable 3 (0.216). Moreover, variable 3 transmits the most to the others ($\text{TO others}=0.714$), while variable 1 transmits the least to others ($\text{TO others}=0.592$). In Table 3 Panel B, variable 4 transmits the most to the others ($\text{TO others}=0.728$) and variable 1 transmits the least to the others ($\text{TO others}=0.627$). Variable 3 represents the variable most affected by the others ($\text{FROM others}=0.778$), while variable 1 is the least affected by the others ($\text{FROM others}=0.608$).

In the case of the column normalization, the focus is on how much one variable (market or country) affects the system. Despite the neat interpretation, it may be seen from the example above (comparison of Tables 2 and 3 Panel B) that if row-normalization is used, the variable most affected by the others is the first ($\text{FROM others}=0.729$), while variable 5 is the least affected by the others ($\text{FROM others}=0.674$). If column normalization is used, the variable most affected by the others is the third ($\text{FROM others}=0.778$), while variable 1 is the least affected by the others ($\text{FROM others}=0.608$). Therefore the use of row-normalization or column-normalization provides different results in terms of the variable that affects the system the most (the least), or is affected by it the most (the least). If the results are compared with the original non-normalized matrix (see Table 1, Panel B), we see that the correct ordering is variable 3 as the one that is affected the most from the system ($\text{FROM others}=2.413$) and variable 5 as the one that is affected the least from the system ($\text{FROM others}=2.048$). Most importantly, the row normalization or column normalization schemes affect the NET spillovers, which may have the opposite sign and the incorrect ranking if compared to the non-normalized ones. In fact, for both forecast horizons, the first variable in the column normalization scheme (Table 3) is misconceived as a net donor, while it is a net receiver in the non-normalized case (Table 1), whereas variables 3 and 4 are mistakenly considered as net receivers (see Table 3) instead of net donors, as apparent in Table 1. The same happens in the row normalization scheme (Table 2): for example, variable 2 is misconceived as net donor in the two-day forecast horizon, while it is a net receiver in the non-normalized case (Table 1). As a result, there is also a change in the ranking of the variables (ranging from the one giving the most to the system, that is the major net donor and has rank 1, to the variable receiving the most from the system, which is the major net receiver and has rank 5). For example, in the non-normalized case the variable transmitting the most to the system in net terms is variable 5 (for both forecast horizons), but in the row-normalized case it emerges that the variable

transmitting the most in net terms to the system is variable 3 for the two-day forecast horizon and variable 4 for the ten-day horizon.

Tables 4 to 6 show the scalar-normalization cases. The scalar factors applied are: the spectral radius (Table 4), the maximum row sum (Table 5) and the maximum column sum (Table 6). In the spectral radius normalization, it is not possible to interpret each element of the forecast error variance decomposition matrix as variance shares by column, or by row. In fact, the sum by row and by column (FROM others including own and TO others including own) can attain values higher or lower than 1, given the mathematical property of the maximum eigenvalue described in eq. (16). Despite the lack of interpretability in terms of variance shares, all the net spillovers maintain the correct sign after normalization and the correct ranking as in the non-normalized case.

It may be noted that in the maximum row sum normalization scheme in Table 5 and in the maximum column sum normalization scheme in Table 6, the only values which sum to one are those in the row with the maximum sum (the third row in both Panels of Table 5) and those in the column with the maximum sum (column 3 for Panel A and column 4 for Panel B of in Table 6), respectively. Only for these values is it possible to give a percentage interpretation: in Table 5 it may be seen that for $h=2$ variable 3 receives 70.5% FROM others, while variable 3 in Table 6 Panel A transmits 71.4% TO others.

Moreover, the scalar-based normalization allows for a comparison between different row and column sums, making it possible to distinguish between stronger or weaker influences. For example, in Table 5 Panel B, variable 3 represents the market most affected by the others (FROM others=0.710), while variable 5 represents the market least affected by the others (FROM others=0.603). The same ordering is preserved if the maximum column normalization is used (see Table 6 Panel B).

Furthermore, in Table 5 Panel A, the entry $b_{21}=0.146$ indicates that the strength of the spillover from variable 1 to variable 2 is 14.6% of the total maximum spillover in the system (represented by variable 3, whose total spillover is normalized to one). The spillover from variable 2 to variable 1 (16.0%) is 1.4% greater than the spillover from variable 1 to variable 2. It should be evident that in Table 2 (row normalized) the same comparison is not possible, since each row element is divided by the total of each row, so that the elements that belong to different rows are not comparable in magnitude.

In the end, considering the variable that has the maximum row sum as the reference one, the total spillovers (FROM+own) received by all other variables show the smaller degree of vulnerability of these markets with respect to the reference one. For example, in Table 5 Panel A, variable 3 is the reference variable and we can say that the total (FROM+own) spillover received by variable 1 is 76% of the total spillover (FROM+own) received by variable 3. This can improve the interpretation of the results to a significant extent.

In conclusion, it may be stated that the max row sum and max col sum normalization are slightly better than the spectral radius since they can preserve the ranking and the sign of the spillovers and, at least for one variable, they can preserve the interpretation as variance share.

Scalar-based normalization schemes also have advantages in computing the total connectedness index, if compared with row- or column-normalization schemes. As the total connectedness index is obtained by dividing the sum of cross-variance shares (off-diagonal elements) with the sum of the total of cross and own variance shares, a scalar-normalization scheme ensures that comparable quantities across rows and columns are summed, while the popular row-normalization and column-normalization schemes imply that quantities that are not comparable across row and columns are summed together. In fact, the index of total connectedness changes when using row-normalization (equal to 0.683 for $h=2$, see Table 2) or column-normalization (equal to 0.678 for $h=2$, see Table 3). On the other hand, any scalar-based normalization provides the same value for the index of total connectedness (equal to 0.685, for $h=2$ see Tables 1, 4, 5, 6). This is a clear advantage of using a scalar-normalization scheme over the popular row-normalization or column-normalization schemes.

As Tables 2 to 6 focus on the normalization issue for only the high persistence, high covariance scenario, with same correlation (corresponding to matrix P1, and case H.H.1), in Tables 7 and 8 we show the results based on population parameters for all the other scenarios. By looking at the sign of the net spillovers (Table 7) it is clear that the row-normalization scheme performs fairly well with no errors in sign for the horizon $h=10$ and only one error in sign when covariance or correlation is higher for the horizon $h=2$. On the contrary, in each scenario, and for both horizons, the column normalization produces from 1 to 3 errors in sign. By looking at the ranking errors in Table 8, what emerges is that both the row normalization and the column normalization scheme affect the ranking in most cases. On the other hand, any scalar normalization scheme does not affect the ranking of net spillovers.

4.2 Results based on simulation

In order to account for the role played by parameter estimation on the rank and sign of net spillovers, we simulate a multivariate dynamic system, using the DGP given by eq. (1). The shocks ε_t are given by $P \eta_t$, where η_t are iid Gaussian and orthogonal innovations with unit variance. In order to assess the reliability of the different normalization schemes in preserving the order of magnitude and the sign of net contributions (NET spillovers) obtained from the generalized forecast error variance decomposition, the simulation experiment involves the following steps:

- 1) Five artificial data series (where the time series dimension is equal to 500) are obtained by simulating either the VAR(2) (in the case of Low Persistence) or the restricted VAR(22) (in the case of High Persistence) with Gaussian innovations. The coefficient matrices for the lags and the lower triangular matrices P aiming at capturing the different covariance structure are those used in section 5.

2) For each of the 1000 replications, we estimate the model parameters by OLS, obtaining the impulse-responses for the forecast horizons $h = 2$, $h = 10$ and computing the corresponding generalized forecast error variance decomposition as defined in eq. (5).

After obtaining the simulated datasets, we compare the non-normalized matrix W^* (e.g. the non-normalized variance decomposition table for a given forecast horizon) and the five normalized matrices W (e.g. the normalized variance decomposition table for a given forecast horizon) in terms of sign and ranking errors.

First, we measure the number of errors in the sign of the net spillovers. Errors are counted when the net spillover obtained from the normalized matrix has a sign opposite to that of the net spillover obtained from the non-normalized matrix. The total number of possible errors is 5000 for each scenario (5 variables times 1000 replications).

Second, we measure the errors in the ranking. Errors are counted when the ranking of the net spillovers obtained from the normalized matrix is different from that of the non-normalized matrix. The total number of possible errors is 1000 for each scenario (one ranking times 1000 replications). Results are shown in Table 9 for sign errors and in Table 10 for ranking errors.

Table 9 shows that over a total number of 5000 possible errors for each scenario (5 variables times 1000 replications), the row normalization performs much better than the column normalization for each scenario: in fact, for $h=2$ ($h=10$) the average number of errors in sign is about 439 (217) for the row-normalization scheme and about 2439 (2042) for the column normalization scheme. This result is surprising since the row normalization and column normalization schemes should theoretically be equal. In both normalization schemes, the number of errors increases with the degree of covariance, in both cases in which correlation is kept fixed or increases.

Moreover, as shown in Table 10, the row-normalization proposed by Diebold and Yilmaz (2012) and the alternative column normalization schemes affect the ranking of the spillovers more than 870 times out of 1000 for $h=2$ and more than 743 times out of 1000 for $h=10$ (with the sole exception of the row normalization scheme in the high persistence scenario H.L.).

To conclude, even if the row normalization scheme and the column normalization scheme allow for a useful interpretation of some directional spillover indices obtained from the generalized forecast error variance decomposition, there is a need to be cautious in comparing net spillover indices to distinguish between markets which are net donors from those which are net receivers. On the contrary, any scalar normalization scheme (by maximum eigenvalue, maximum row sum or maximum column sum) will outperform the traditional normalization schemes, preserving the ranking and the sign of the NET spillovers. As a result, we suggest using a scalar normalization scheme to derive the correct measures of net contribution. Among the scalar normalization schemes, the maximum row sum or the maximum column sum are preferred to the spectral radius since they allow for a better interpretation of how much one variable receives or transmits in terms of percentage values.

5 Replication of Diebold and Yilmaz (2012)

In order to shed further light on the normalization issue in obtaining reliable spillover measures, we replicate the original paper of Diebold and Yilmaz (2012) to see whether results would have changed if scalar normalization were applied. Therefore, we compute the 10-days-ahead generalized forecast error variance decomposition, on a VAR of order 4, by using the same daily data as in Diebold and Yilmaz (2012). The data consists of range based volatilities of the S&P 500 (stock market), the 10-year Treasury bond yield (bond market), the New York Board of Trade US dollar index futures (FX market) and the Dow Jones UBS Commodity index (commodity market), recorded from January 25, 1999 to January 29, 2010. Full sample results are presented in Table 11.

Panel I corresponds to Table 2 in the Diebold and Yilmaz (2012) paper (variance decomposition by using row normalization), Panel III corresponds to the variance decomposition by using the alternative column normalization scheme suggested in Diebold and Yilmaz (2012), while Panel II displays the variance decomposition by using scalar normalization (the max row sum). By comparing the NET spillovers in Panels I and II, it may be seen that the stock market turns out to be a net donor while all the others variables are net receivers of volatility spillovers during the entire period. As a result, there are no sign errors in Diebold and Yilmaz (2012), even if the row-normalization scheme is used in place of the max row sum one. However, we can detect some ranking errors. If we concentrate on the bond market, it turns out to have received less volatility spillovers than the commodity market if the row-normalization scheme (or the column-normalization) is used, as in Diebold and Yilmaz (2012). However, if we use scalar normalization, as in Panel II, we can see that the result is the opposite: the bond market turns out to have received more volatility spillovers than the commodity market. As a result, we have a ranking error if the row-normalization scheme is used. This result is not surprising: since Diebold and Yilmaz (2012) consider four distinct asset classes and use a short-order VAR(p) model, the results of their application can be “classified” in our “low-persistence, low-covariance” case that, in fact, recorded the lowest number of sign errors. On the contrary, from our simulation results, it is evident that ranking errors are frequent in data with any degree of persistence and covariance.

Furthermore we notice that also pairwise net spillovers can be safely computed only in the max-row normalized matrix. The net pairwise spillover between stock market and bond market cannot be computed by $= 0.0729 - 0.1021$ (row-normalized matrix Panel I Table 11), since the two quantities refer to different total values by row. The net pairwise spillover is not either $0.0730 - 0.0970$ (column normalized matrix, Panel III Table 11). The correct spillover is $0.0694 - 0.1021$ (max-row normalized matrix, Panel II Table 11).

Finally, we notice that also the index of total connectedness changes if we use row-normalization (equal to 0,126, see Panel I) or column normalization (equal to 0,125, see Panel III), whereas the correct index is equal to 0,121 (see Panel II).

6 Concluding remarks

In this study we replicate the full sample results of Diebold and Yilmaz (2012) applied to range based volatilities of the Dow Jones UBS Commodity index and of three other financial markets: the S&P 500 (stock market), the 10-year Treasury bond yield (bond market), the New York Board of Trade US dollar index futures (FX market). Moreover, we show that the net spillover indices used to assess the net contribution of one market to systemic risk are sensitive to the normalization scheme.

In particular, the row normalization scheme of the Generalized Forecast Error Variance Decomposition suggested by Diebold and Yilmaz (2012) is the best scheme for interpreting each row of the FEV as variance shares. However, it may fail to identify a) the market most (or least) affected by the system b) whether the market is a net risk transmitter or net risk receiver; c) the degree to which a single market influences all the others in net absolute terms d) the exact degree of total connectedness. The column normalization scheme suggested by Diebold and Yilmaz (2012) is the best scheme for interpreting each column of the FEV as variance shares. However, it suffers from the same drawbacks as the row-normalization scheme. As a result, we suggest a scalar normalization scheme (as in Caloia et al. 2018). In particular, the maximum row sum or the maximum column sum schemes are preferable to the spectral radius since they allow for a better interpretation of how much one variable receives or transmits in terms of percentage values. Finally, scalar-based normalization schemes yield a consistent total spillover index, while the total spillover index changes if row-normalization or column-normalization schemes are used.

The results of the paper are strongly prescriptive as regards the use of normalization for the computation of TOTAL and NET spillovers. Since these two indices are able (i) to describe the overall level of risk transmission and (ii) to distinguish between net donors and net receivers markets in terms of risk spillovers, we argue that our results can be particularly useful for scholars using the Diebold Yilmaz methodology to study risk transmission, as well as for macro-prudential regulators to supervise connectedness in the banking and insurance sector and for practitioners and portfolio managers interested in setting up hedging strategies and portfolio allocations. The evidence on the relation between volatility spillovers and portfolio/hedging weights is discussed in Antonakakis et al. (2018) and Arouri et al (2012), while connectedness in the banking sector has been studied in Diebold, Yilmaz (2014).

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Table 1. Spillover Table based on the non-normalized variance decomposition table (VDT).

Panel A: h=2							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.889	0.539	0.435	0.376	0.324	2.561	1.672
V2	0.493	0.975	0.683	0.535	0.438	3.124	2.149
V3	0.337	0.668	0.994	0.758	0.612	3.369	2.375
V4	0.255	0.506	0.753	0.997	0.801	3.313	2.316
V5	0.204	0.406	0.605	0.802	0.997	3.015	2.018
TO others including own	2.178	3.094	3.470	3.467	3.172		TOTAL
TO others	1.289	2.119	2.477	2.470	2.175		0.685
NET	-0.383	-0.030	0.102	0.154	0.157		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.773	0.579	0.540	0.506	0.453	2.850	2.077
V2	0.483	0.940	0.706	0.582	0.490	3.202	2.262
V3	0.343	0.671	0.984	0.769	0.630	3.397	2.413
V4	0.263	0.516	0.759	0.993	0.804	3.334	2.341
V5	0.211	0.416	0.614	0.806	0.992	3.040	2.048
TO others including own	2.073	3.122	3.603	3.656	3.368		TOTAL
TO others	1.300	2.181	2.619	2.663	2.377		0.704
NET	-0.777	-0.080	0.206	0.322	0.329		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the non-normalized generalized forecast error variance decomposition, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-covariance series) and to the forecast horizon $h=2$ (Panel A) and $h=10$ (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable V_i , $i=1, \dots, 5$. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the value of the NET spillover from the highest to the lowest (NET ranking).

Table 2. Spillover Table based on the row-normalized variance decomposition table (VDT).

Panel A: h=2							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.347	0.210	0.170	0.147	0.126	1	0.653
V2	0.158	0.312	0.219	0.171	0.140	1	0.688
V3	0.100	0.198	0.295	0.225	0.182	1	0.705
V4	0.077	0.153	0.227	0.301	0.242	1	0.699
V5	0.068	0.135	0.201	0.266	0.331	1	0.669
TO others including own	0.750	1.008	1.112	1.110	1.021		TOTAL
TO others	0.403	0.696	0.817	0.809	0.690		0.683
NET	-0.250	0.008	0.112	0.110	0.021		
NET sign	-	+	+	+	+		
NET ranking	5	4	1	2	3		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.271	0.203	0.189	0.178	0.159	1	0.729
V2	0.151	0.294	0.221	0.182	0.153	1	0.706
V3	0.101	0.198	0.290	0.226	0.185	1	0.710
V4	0.079	0.155	0.228	0.298	0.241	1	0.702
V5	0.070	0.137	0.202	0.265	0.326	1	0.674
TO others including own	0.671	0.986	1.129	1.149	1.065		TOTAL
TO others	0.400	0.692	0.840	0.851	0.738		0.704
NET	-0.329	-0.014	0.129	0.149	0.065		
NET sign	-	-	+	+	+		
NET ranking	5	4	2	1	3		

Note. This figure shows the spillover Table based on the row-normalized generalized forecast error variance decomposition, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon $h=2$ (Panel A) and $h=10$ (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable V_i , $i=1, \dots, 5$. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the value of the NET spillover from the highest to the lowest (NET ranking).

Table 3: Spillover Table based on the column-normalized variance decomposition table (VDT).

Panel A: h=2							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.408	0.174	0.125	0.108	0.102	0.918	0.510
V2	0.226	0.315	0.197	0.154	0.138	1.031	0.716
V3	0.155	0.216	0.286	0.218	0.193	1.069	0.782
V4	0.117	0.164	0.217	0.288	0.253	1.038	0.750
V5	0.094	0.131	0.174	0.231	0.314	0.945	0.631
TO others including own	1	1	1	1	1		TOTAL
TO others	0.592	0.685	0.714	0.712	0.686		0.678
NET	0.082	-0.031	-0.069	-0.038	0.055		
NET sign	+	-	-	-	+		
NET ranking	1	3	5	4	2		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.373	0.185	0.150	0.138	0.134	0.981	0.608
V2	0.233	0.301	0.196	0.159	0.146	1.035	0.734
V3	0.166	0.215	0.273	0.210	0.187	1.051	0.778
V4	0.127	0.165	0.211	0.272	0.239	1.013	0.741
V5	0.102	0.133	0.170	0.220	0.294	0.921	0.626
TO others including own	1	1	1	1	1		TOTAL
TO others	0.627	0.699	0.727	0.728	0.706		0.697
NET	0.019	-0.035	-0.051	-0.013	0.079		
NET sign	+	-	-	-	+		
NET ranking	2	4	5	3	1		

Note. This figure shows the spillover Table based on the column-normalized generalized forecast error variance decomposition, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon $h=2$ (Panel A) and $h=10$ (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable V_i , $i=1, \dots, 5$. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the value of the NET spillover from the highest to the lowest (NET ranking).

Table 4: Spillover Table based on the variance decomposition table (VDT) normalized by the spectral radius.

Panel A: h=2							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.284	0.172	0.139	0.120	0.103	0.818	0.534
V2	0.157	0.311	0.218	0.171	0.140	0.998	0.686
V3	0.108	0.213	0.317	0.242	0.195	1.076	0.758
V4	0.081	0.162	0.241	0.318	0.256	1.058	0.740
V5	0.065	0.130	0.193	0.256	0.318	0.963	0.644
TO others including own	0.695	0.988	1.108	1.107	1.013		TOTAL
TO others	0.412	0.677	0.791	0.789	0.695		0.685
NET	-0.122	-0.010	0.032	0.049	0.050		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.241	0.181	0.169	0.158	0.141	0.890	0.649
V2	0.151	0.294	0.221	0.182	0.153	1.000	0.706
V3	0.107	0.210	0.307	0.240	0.197	1.061	0.754
V4	0.082	0.161	0.237	0.310	0.251	1.041	0.731
V5	0.066	0.130	0.192	0.252	0.310	0.949	0.639
TO others including own	0.647	0.975	1.125	1.142	1.052		TOTAL
TO others	0.406	0.681	0.818	0.832	0.742		0.704
NET	-0.243	-0.025	0.064	0.100	0.103		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the generalized forecast error variance decomposition normalized by the spectral radius, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon $h=2$ (Panel A) and $h=10$ (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable V_i , $i=1, \dots, 5$. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the value of the NET spillover from the highest to the lowest (NET ranking).

Table 5: Spillover Table based on the variance decomposition table (VDT) normalized by the maximum row sum.

Panel A: h=2							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.264	0.160	0.129	0.111	0.096	0.760	0.496
V2	0.146	0.289	0.203	0.159	0.130	0.927	0.638
V3	0.100	0.198	0.295	0.225	0.182	1	0.705
V4	0.076	0.150	0.224	0.296	0.238	0.984	0.688
V5	0.061	0.121	0.180	0.238	0.296	0.895	0.599
TO others including own	0.647	0.918	1.030	1.029	0.942		TOTAL
TO others	0.383	0.629	0.735	0.733	0.646		0.685
NET	-0.114	-0.009	0.030	0.046	0.047		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.227	0.170	0.159	0.149	0.133	0.839	0.612
V2	0.142	0.277	0.208	0.171	0.144	0.943	0.666
V3	0.101	0.198	0.290	0.226	0.185	1	0.710
V4	0.077	0.152	0.223	0.292	0.237	0.982	0.689
V5	0.062	0.123	0.181	0.237	0.292	0.895	0.603
TO others including own	0.610	0.919	1.061	1.076	0.992		TOTAL
TO others	0.383	0.642	0.771	0.784	0.700		0.704
NET	-0.229	-0.024	0.061	0.095	0.097		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the generalized forecast error variance decomposition normalized by the maximum row sum, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon $h=2$ (Panel A) and $h=10$ (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable V_i , $i=1, \dots, 5$. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the value of the NET spillover from the highest to the lowest (NET ranking).

Table 6: Spillover Table based on the variance decomposition table (VDT) normalized by the maximum column sum.

Panel A: h=2							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.256	0.155	0.125	0.108	0.093	0.738	0.482
V2	0.142	0.281	0.197	0.154	0.126	0.900	0.619
V3	0.097	0.193	0.286	0.218	0.176	0.971	0.684
V4	0.073	0.146	0.217	0.287	0.231	0.955	0.667
V5	0.059	0.117	0.174	0.231	0.287	0.869	0.582
TO others including own	0.628	0.892	1	0.999	0.914		TOTAL
TO others	0.372	0.611	0.714	0.712	0.627		0.685
NET	-0.110	-0.009	0.029	0.044	0.045		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.211	0.158	0.148	0.138	0.124	0.779	0.568
V2	0.132	0.257	0.193	0.159	0.134	0.876	0.619
V3	0.094	0.184	0.269	0.210	0.172	0.929	0.660
V4	0.072	0.141	0.208	0.272	0.220	0.912	0.640
V5	0.058	0.114	0.168	0.220	0.271	0.831	0.560
TO others including own	0.567	0.854	0.986	1	0.921		TOTAL
TO others	0.356	0.597	0.716	0.728	0.650		0.704
NET	-0.212	-0.022	0.056	0.088	0.090		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the generalized forecast error variance decomposition normalized by the maximum column sum, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon $h=2$ (Panel A) and $h=10$ (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable V_i , $i=1, \dots, 5$. The bottom lines report for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the value of the NET spillover from the highest to the lowest (NET ranking).

Table 7: Errors in sign (using population parameters).

Panel A: h=2						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	0	1	1	0	1	1
normalization by column	3	3	2	3	3	2
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0
Panel B: h=10						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	0	0	0	0	0	0
normalization by column	1	3	1	1	3	1
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0

Note. The Table shows the number of errors in sign for each DGP (L.L., L.H.1, L.H.2, H.L., H.H.1, H.H.2) where L.L. = low persistence low covariance, L.H.=low persistence high covariance, H.L.=high persistence low covariance, H.H.=high persistence high covariance). The numbers 1 and 2 in the “High Covariance” DGPs refer to the use of the covariance matrices Σ_1 and Σ_2 , obtained for the matrices P_1 and P_2 respectively: Case 1 keeps correlation fixed, Case 2 corresponds to higher correlation. Errors are counted when the net spillover obtained from the normalized matrix has a sign opposite to the one of the net spillover obtained from the non-normalized matrix. The total number of errors can range from 0 to 5 in each scenario.

Table 8: Errors in ranking (using population parameters).

Panel A: h=2						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	1	1	1	1	1	1
normalization by column	1	1	1	1	1	1
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0
Panel B: h=10						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	1	1	1	1	1	1
normalization by column	0	1	1	0	1	1
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0

Note. The Table shows the number of errors in ranking for each DGP (L.L., L.H.1, L.H.2, H.L., H.H.1, H.H.2) where L.L. = low persistence low covariance, L.H.=low persistence high covariance, H.L.=high persistence low covariance, H.H.=high persistence high covariance). The numbers 1 and 2 in the “High Covariance” DGPs refer to the use of the covariance matrices Σ_1 and Σ_2 , obtained for the matrices P_1 and P_2 respectively Case 1 keeps correlation fixed, Case 2 corresponds to higher correlation. Results refer to the forecast horizon h=2 (panel A) and h=10 (Panel B). Errors are counted when the ranking of the net spillovers obtained from the normalized matrix is different from that of the non-normalized matrix. The number of errors can be either 0 (correct ranking) or 1 (incorrect ranking).

Table 9: Errors in sign (using simulations).

Panel A: h=2						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	118	607	1101	198	492	357
normalization by column	2802	3226	3165	1746	2324	1899
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0
Panel B: h=10						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	28	293	565	111	243	126
normalization by column	1368	3143	2929	1644	1833	1602
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0

Note. The Table shows the number of errors in sign for each DGP (L.L., L.H.1, L.H.2, H.L., H.H.1, H.H.2) where L.L. = low persistence low covariance, L.H.=low persistence high covariance, H.L.=high persistence low covariance, H.H.=high persistence high covariance). The numbers 1 and 2 in the “High Covariance” DGPs refer to the use of the covariance matrices Σ_1 and Σ_2 , obtained for the matrices P_1 and P_2 respectively: Case 1 keeps correlation fixed, Case 2 corresponds to higher correlation. Results refer to the forecast horizon H=2 (panel A) and H=10 (Panel B). Errors are counted when the net spillover obtained from the normalized matrix has a sign opposite to the one of the net spillover obtained from the non-normalized matrix. The total number of errors can range from 0 to 5000 in each scenario (5 variables times 1000 simulations for each scenario).

Table 10: Errors in ranking (using simulations).

Panel A: h=2						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	1000	990	1000	870	989	999
normalization by column	1000	1000	1000	934	949	972
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0
Panel B: h=10						
	L. L.	L.H.1	L.H.2	H.L.	H.H.1	H.H.2
normalization by row	1000	991	999	481	743	850
normalization by column	966	1000	1000	981	977	979
normalization by spectral radius	0	0	0	0	0	0
normalization by max row sum	0	0	0	0	0	0
normalization by max col sum	0	0	0	0	0	0

Note. The Table shows the number of errors in ranking for each DGP (L.L., L.H.1, L.H.2, H.L., H.H.1, H.H.2) where L.L. = low persistence low covariance, L.H.=low persistence high covariance, H.L.=high persistence low covariance, H.H.=high persistence high covariance). The numbers 1 and 2 in the “High Covariance” DGPs refer to the use of the covariance matrices Σ_1 and Σ_2 , obtained for the matrices P_1 and P_2 respectively: Case 1 keeps correlation fixed, Case 2 corresponds to higher correlation. Results refer to the forecast horizon H=2 (panel A) and H=10 (Panel B). Errors are counted when the ranking of the net spillovers obtained from the normalized matrix is different from the one of the non-normalized matrix. The number of errors can range from 0 (correct ranking in each of the 1000 simulations) to 1000 (incorrect ranking in each of the 1000 simulations).

Table 11: Comparison with the results in Diebold and Yilmaz (2012)

Panel I: Row Normalization						
	Stocks	Bonds	Commodities	FX	FROM others (including own)	FROM others
Stocks	0.8876	0.0729	0.0035	0.0361	1	0.112
Bonds	0.1021	0.8145	0.0273	0.0561	1	0.186
Commodities	0.0047	0.037	0.9369	0.0214	1	0.063
FX	0.0569	0.0703	0.0155	0.8573	1	0.143
TO others (including own)	1.051	0.995	0.983	0.971		
TO others	0.164	0.18	0.046	0.114		TOTAL
NET	0.051	-0.005	-0.017	-0.029		0.126
Panel II: Max row Normalization						
	Stocks	Bonds	Commodities	FX	FROM others (including own)	FROM others
Stocks	0.8444	0.0694	0.0033	0.0343	0.951	0.107
Bonds	0.1021	0.8145	0.0273	0.0561	1	0.186
Commodities	0.0041	0.0323	0.8194	0.0187	0.875	0.055
FX	0.0547	0.0675	0.0149	0.8238	0.961	0.137
TO others (including own)	1.005	0.984	0.865	0.933		
TO others	0.161	0.169	0.045	0.109		TOTAL
NET	0.054	-0.016	-0.01	-0.028		0.121
Panel III: Column normalization						
	Stocks	Bonds	Commodities	FX	FROM others (including own)	FROM others
Stocks	0.845	0.073	0.004	0.037	0.959	0.114
Bonds	0.097	0.819	0.028	0.058	1.001	0.183
Commodities	0.004	0.037	0.953	0.022	1.017	0.064
FX	0.054	0.071	0.016	0.883	1.023	0.141
TO others (including own)	1	1	1	1		
TO others	0.156	0.181	0.047	0.117		TOTAL
NET	0.042	-0.002	-0.017	-0.024		0.125

Note. This Table shows the spillover based on the generalized forecast error variance decomposition normalized by row (Panel I) as in Diebold and Yilmaz (2012), the alternative column normalization suggested in Diebold and Yilmaz (2012) (Panel III), and the normalization by maximum row sum (Panel II). Results refer to the 10-days-ahead forecast horizon.