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Pullout modelling of viscoelastic synthetic fibres for cementitious composites

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Abstract

The problem of the pullout of a viscoelastic synthetic fibre embedded in a cementitious matrix and subjected to an external time-dependent axial load is considered in the present work. A 1D phenomenological model able to simulate the contribution of viscoelastic relaxation as well as the hardening behavior due to abrasion phenomena during slippage is developed. The cement matrix compliance is neglected with respect to the fibre elongation. The interfacial shear stress between the fibre and the surrounding matrix is assumed to depend on the slippage distance through a second degree polynomial law, thus involving three constitutive parameters. Two distinct phases are recognized: An earlier debonding stage followed by the effective fibre pullout process. Two different creep functions have been assumed for modelling the viscous response of polymeric fibres: A function based on the fraction-exponential Rabotnov operator and a classical exponential model. Identification of the governing constitutive parameters allows obtaining the relation between the external strain and the axial displacement, which has been compared with experimental results provided by pullout tests both on plain and treated fibres, finding a good agreement. It is shown that the proposed approach can predict the whole pullout process of discrete synthetic macrofibres.

Keywords: Pull-out, Fibre reinforced concrete, Synthetic fibres, Creep, Rabotnov operator, Analytical modelling.

1. Introduction

Concrete is widely used in civil engineering because its versatility and cheapness as compared with other building materials like steel and masonry. During the last decades, concrete technology has known a rapid improvement in order to obtain cementitious-based materials with specific physical properties, like

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6 lightweight concrete, ultra-high-performance concrete (UHPC), self-compacting
7 concrete and foamed concrete (e.g. Scerrato et al. [1]). Despite these advan-
8 tageous properties, concrete is a brittle material undergoing various damaging
9 phenomena like crack initiation and growth, especially under the action of ten-
10 sile stresses and impact loads (aging effects in concrete structures have been
11 discussed in [2], [3]). This detrimental aspect can be mitigated by inserting
12 proper reinforcements during the cast of the mixture, like traditional steel bars,
13 brackets ribbed, wire meshes etc., or by introducing discrete fibres in the con-
14 crete mixture at the mixing stage [4]. Indeed, fibres provide a uniform reinforce-
15 ment as they are randomly distributed in the concrete cast, allowing increasing
16 the tensile resistance, ductility and, in turn, its lifespan [5, 6]. In particular,
17 synthetic polypropylene (PP) macrofibres have proved to impart a significant
18 toughness to the concrete, increasing its durability and mechanical performances
19 in time [7]. Moreover synthetic macrofibres offer many advantages in terms of
20 lightness, cheapness, magnetic permeability and chemical stability in aggressive
21 environments as compared with the metallic ones. Recent studies about the me-
22 chanical performances of fabric-reinforced cementitious matrix composites can
23 be found in [8, 9, 10, 11].

24 The strengthening contribution of macrofibres embedded in a brittle concrete
25 matrix is mainly due to their capacity of transferring stresses across the crack
26 surfaces, thus mitigating the tendency of stress to concentrate under increasing
27 loads by creating a crack bridging mechanism. Such a mechanism is the working
28 principle of FRC [12, 13]. Under proper conditions, it allows FRC to display
29 hardening post-cracking behavior, thus increasing both the ultimate bearing
30 capacity and toughness of FRC structural elements [5, 14, 15]. These beneficial
31 effects strongly depend on the nature of the fibre-matrix interface, namely the
32 non-homogeneous region of the cement matrix which originates just around the
33 fibre, known as interface transition zone (ITZ) [16].

34 Recently, Di Maida et al. [17] performed pull-out and flexural experimental
35 tests on FRC based on macro-synthetic fibres, both for plain fibres and for
36 fibres treated with nanosilica on their surface in order to improve the adhesion
37 to the cementitious matrix. They observed slip hardening behavior, namely an
38 increase in the frictional stress acting on the fibre surface as the fibre is pulled
39 out of the cement matrix caused by the progressive wearing of the fibre surfaces
40 and the accumulation of wear debris due to abrasion phenomena. The pull-
41 out hardening behavior is responsible, in turn, of the increase in the residual
42 strength during the post-cracking phase and, thus, of the ductile behavior of
43 FRC [18].

44 A number of analytical and numerical studies have been devoted to simulate
45 the pullout response of various kind of fibres focussing on the mechanism of
46 stress transfer between the fibrous reinforcements and the cement matrix. A
47 simple frictional analytical model for the characterization of the stress-slipage
48 relationship of steel fibres embedded in cementitious matrices was proposed by
49 Naaman et al. [19]. Later, Cunha et al. [20] proposed some bond-slip relation-
50 ships reproducing the experimental pullout behavior of both straight and hooked
51 steel fibres. A numerical approach has been adopted also by Choi et al. [21], in

52 order to characterize the interface between the fibre and the cementitious ma-
53 trix for three kinds of fibres: carbon fibre, polypropylene (PP) fibre and twisted
54 wire strand steel cord. Radi et al. [22] developed a 1D phenomenological model
55 for simulating the pullout behavior of synthetic fibres. The latter Authors ne-
56 glected the Poisson effect on the fibre pullout because it would provide softening
57 behavior, whereas the main effect highlighted by the experimental results was
58 hardening frictional behavior. They also neglected the matrix compliance as
59 compared to the fibre elongation, but assumed *large* deformations of the poly-
60 meric fibre and imposed the balance conditions in the deformed configuration.
61 Moreover, they assumed the interfacial shear stress as a second degree piecewise
62 function of the slippage.

63 It must be remarked that the viscous behavior of the fibre has been neglected
64 in all the aforementioned references. However, the overall mechanical behavior
65 of FRC is expected to be strongly influenced by the rheological properties of
66 synthetic fibres, as pointed out in [5, 23]. Indeed, the viscous relaxation typical
67 of polymeric materials may provide a significant contribution in the relation
68 between the applied tensile load and displacement of the actuated fibre cross
69 section measured by pullout tests. In particular, the pullout response is ex-
70 pected to be strongly affected by viscous effects taking place in the outer part
71 of fibre, namely between the cement sample and the actuator, whereas the vis-
72 cous deformation of the embedded part of the fibre can be neglected due to
73 the constraint provided by the surrounding rigid matrix. Therefore, in order
74 to validate the analytical model for the pull-out response of polymeric fibres, it
75 becomes necessary to take into considerations also rheological properties of the
76 free part of the fibre.

77 In the present work, the approach proposed by Radi et al. [22] for the
78 simulation of the hardening behavior exhibited in pullout tests of synthetic
79 fibres has been extended to account for the viscous behavior of the outer part
80 of the fibre, within the framework of hereditary linear viscoelasticity. A general
81 creep function compliant with the Rabotnov viscoelastic operator as well as a
82 simplified creep function following to the idealized Zener viscoelastic scheme,
83 called Standard Linear Solid (SLS), have been considered. The use of fraction-
84 exponential operators, like the Rabotnov one, allows describing experimental
85 data of real materials with sufficient accuracy and, at the same time, allows
86 finding explicit analytical results.

87 The interfacial shear stress between the fibre and the surrounding matrix is
88 assumed to depend on the slippage distance through a second degree polynomial
89 law, thus involving three constitutive parameters. The balance condition for
90 the fibre leads to a nonlinear ordinary differential equation, which is solved
91 through a numerical procedure. The constitutive parameters characterizing
92 the frictional interface behavior as well as the rheological response of the fibre
93 have been determined by comparing the relations between the pull-out load
94 and the displacement of the actuated fibre cross section provided by theoretical
95 simulations with those obtained from the test performed by Di Maida et al.
96 [17]. The theoretical pull-out curves are then found to agree well with the
97 experimental results, both for plain fibres and for fibres treated with nano-

98 silica. The present model can thus simulate the pull-out response of various
99 kinds of polymeric or steel fibres, once the constitutive parameters occurring in
100 the definition of the interfacial shear stress and viscoelastic behavior of the fibre
101 material have been properly set.

102 The paper is organized as follows. The governing equations are reported
103 and discussed in Section 2. The viscous effects are considered in Section 3 and
104 the main results are reported and discussed in Section 4. Finally, Section 5
105 addresses the concluding remarks.

106 1.1. Nomenclature

107 A, B, C constant parameters

108 d diameter of the fibre cross section

109 E elastic Young modulus

110 E_0 elastic Young modulus at $t = 0$

111 E_∞ elastic Young modulus at $t \rightarrow \infty$

112 F axial load applied at the outer end of the fibre

113 L fibre length embedded in the matrix

114 L_e outer fibre length

115 m rate of the actuator

116 $s(x)$ displacement of the fibre cross section at x

117 s_0 displacement of the fibre cross section at $x = 0$

118 s_L displacement of the fibre cross section initially at $x = L$

119 t time

120 $u(t)$ displacement at time t of the fibre cross section loaded by the actuator
121 with the force $F(t)$

122 x axial abscissa

123 ε axial strain of the fibre

124 λ parameter describing the debonding phase, $\lambda \in [0, 1]$

125 α, β, ν parameter of the creep function

126 σ tensile stress in the fibre

127 τ shear stress at the interface

128 τ_0, a, b parameters of shear stress law of the interface

129 $\psi(t)$ creep function at time t
 130 ψ_0 creep function at time $t = 0$
 131 ψ_∞ creep function at time $t \rightarrow \infty$
 132 k_1, k_2, c constant parameters

133 2. Governing equations

134 2.1. Constitutive model for the shear stress interface

In order to simulate the pull-out process of polymeric fibres from the cement matrix a 1D analytical model is developed here by neglecting the deformation of the cement matrix and imposing the equilibrium of the fibre in the undeformed configuration. The fibre is assumed to display linear visco-elastic behavior under small strains. An abscissa x measured along the fibre is taken, with the origin at the embedded end of the fibre and moving with it. A suitable law for the frictional shear stress arising between the fibre and the surrounding matrix is considered. In particular, by denoting with $s(x)$ the slippage distance of the fibre section placed at a generic abscissa x , according to Radi et al. [22] the shear stress is assumed as a non-linear function of the slippage, namely

$$\tau(s) = \tau_0 + as + bs^2. \quad (1)$$

135 This constitutive relation for the interface is plotted in Fig. 1. Since the cement
 136 matrix is assumed as rigid, then the slippage $s(x)$ coincides with the axial dis-
 137 placement of the fibre cross section at abscissa x . The elastic deformation of
 138 the interface considered in Radi et al. [22] has been neglected here, because its
 139 effect on the whole process of the fibre extraction was found negligible. Thus,
 140 according to eqn (1), $\tau(0) = \tau_0 \neq 0$.

141 Reference is made to a fibre embedded in a matrix for a length L and sub-
 142 jected to the tensile load F . Let s_L be the displacement of the fibre cross section
 143 initially at $x = L$, namely on the surface of the cement sample, as illustrated
 144 in Fig. 2. As the load F increases, two distinct phases occur during the pullout
 145 process. At first, a debonding stage originates in the embedded part of the fibre
 146 of length $(1 - \lambda)L$, with $0 \leq \lambda \leq 1$, starting from the outer side, as reported
 147 in Fig. 2(a). As the load increases, the bonded part λL of the fibre decreases
 148 till $\lambda = 0$. At the end of the debonding phase the effective pullout stage takes
 149 place. As a consequence, a rigid body motion s_0 occurs at $x = 0$, starting from
 150 $s_0 = 0$ (corresponding to $\lambda = 0$) till the complete extraction of the fibre, which
 151 is reached as soon as $s_0 = L$ (see Fig. 2(b)).

152 2.2. Strain-displacement relation of the fibre

Let d denotes the diameter of the fibre cross section. Making reference to Fig. 2 and supposing that no axial load acts on the fibre cross section at $x = 0$,

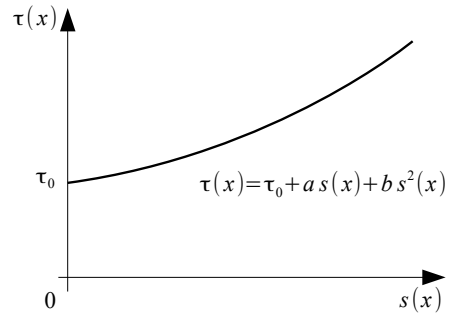


Figure 1: Constitutive model between interface shear stress $\tau(x)$ and slippage distance $s(x)$ at abscissa x .

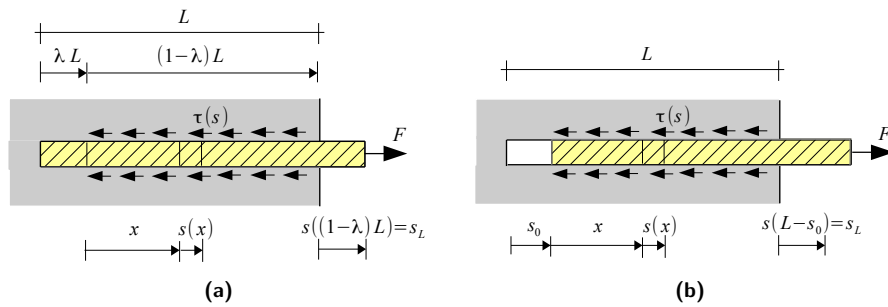


Figure 2: (a) Debonding phase. (b) Pullout phase.

the tensile stress $\sigma(x)$ acting on the cross section of the fibre at abscissa x is given by balance condition as

$$\sigma(x) = \frac{4}{d} \int_0^x [\tau_0 + as(\chi) + bs^2(\chi)] d\chi, \quad (2)$$

where $0 \leq x \leq L(1-\lambda)$ in debonding stage and $0 \leq x \leq L-s_0$ in pullout stage. Let

$$\varepsilon(x) = \frac{\sigma(x)}{E} = s'(x), \quad (3)$$

denotes the axial strain of the fibre, where the apex denotes derivative with respect to the function argument. By deriving $\varepsilon(x)$ with respect the spatial coordinate x , eqns (2) and (3) provide

$$s''(x) - Bs^2(x) - As(x) = C, \quad (4)$$

where the prime denotes differentiation with respect to the argument function and

$$A = \frac{4a}{Ed}, \quad B = \frac{4b}{Ed}, \quad C = \frac{4\tau_0}{Ed}. \quad (5)$$

By using the definition (3) of the axial strain, eqn (4) becomes a nonlinear ordinary differential equation for the function ε of the displacement s , namely

$$\varepsilon \frac{d\varepsilon}{ds} - Bs^2 - As = C. \quad (6)$$

By integrating eqn (6) with respect to s and then with respect to x one finds the following nonlinear relation between ε and s

$$\varepsilon(x) = \sqrt{2Cs(x) + \frac{2}{3}Bs^3(x) + As^2(x) + 2C_0}, \quad (7)$$

that can be solved for $x(s)$ after integrating between x_0 and $x(s)$, namely

$$\int_{x_0}^x dx = \int_{s(x_0)}^{s(x)} \frac{ds}{\sqrt{2Cs + \frac{2}{3}Bs^3 + As^2 + 2C_0}}. \quad (8)$$

Proper boundary conditions are imposed for each phase. In the debonding phase the boundary conditions at $x = 0$ require $s(0) = 0$ and $\varepsilon(0) = 0$, thus from eqn (7) one obtains $C_0 = 0$, and from eqn (8), for $x_0 = 0$ and $x = L - \lambda L$, one gets

$$\lambda = 1 - \frac{1}{L} \int_0^{sL} \frac{ds}{\sqrt{2Cs + \frac{2}{3}Bs^3 + As^2}}, \quad (9)$$

153 where $s_L = s(L - \lambda L)$. By varying λ in the range $[0, 1]$, the corresponding values
 154 of the displacement s_L are assessed from eqn (9). Then, the corresponding axial
 155 strain $\varepsilon_L = \varepsilon(L - \lambda L)$ is found from eqn (7).

In the pullout phase the boundary conditions at $x = 0$ require $s(0) = s_0$ and
 $\varepsilon(0) = 0$, thus from eqn (7) one obtains

$$C_0 = -(Cs_0 + \frac{B}{3}s_0^3 + \frac{A}{2}s_0^2), \quad (10)$$

and from eqn (8), for $x_0 = 0$ and $x = L - s_0$, one has

$$s_0 = L - \int_{s_0}^{s(L-s_0)} \frac{ds}{\sqrt{2Cs + \frac{2}{3}Bs^3 + As^2 + C_0}}. \quad (11)$$

156 Then, the axial displacement $s_L = s(L - s_0)$ is determined from eqn (11) and
 157 the axial strain $\varepsilon_L = \varepsilon(L - s_0)$ follows from eqn (7).

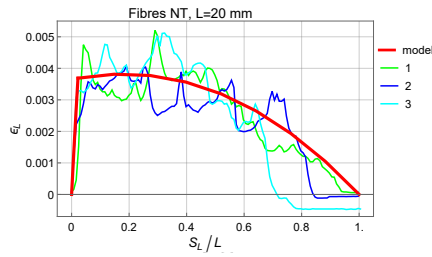
158 The constitutive parameters τ_0, a, b of the interface frictional model introduced
 159 in eqn (1) have been calibrated by fitting the pullout curves provided
 160 by the experimental tests performed by Di Maida et al. [17] (Fig. 3) with the
 161 theoretical strain-displacement curve obtained from eqns (7), (9) and (11) for
 162 the debonding and pullout phases, respectively, and they are listed in Tab. 2.
 163 These parameters are found to depend on the embedded length of the fibre. This
 164 unexpected behavior is probably due to the progressive abrasion phenomenon
 165 occurring at the fibre surface as the pullout process grows. It follows that the
 166 pullout behavior depends not only on the nature of the fibre and the matrix but
 167 also on the length of the embedded part of the fibre.

168 It is remarked that in the present Section and in Sec. 2.3, the viscous deformation
 169 has not been addressed. However, the experimental tests above mentioned display
 170 both the elastic and viscous elongation of the fibre. Therefore, the viscous effects
 171 will be considered in Sec. 3.

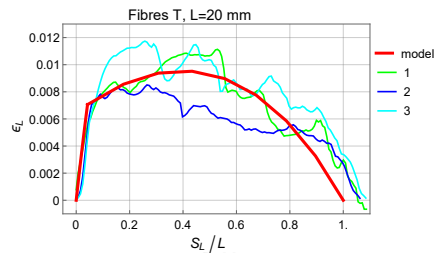
172 2.3. Applied time-dependent load $F(t)$

173 The aforementioned experimental results show the load (in terms of axial
 174 strain) vs the displacement during the pullout of the fibre. Such tests have been
 175 performed at constant displacement rate $m = 1$ mm/min and thus the time t
 176 corresponding to a specific value of the displacement s_L can be inferred from
 177 the relation $t = s/m$. Based on such a relationship between the external load
 178 and time, a piecewise analytical function $F(t)$ of time for the external load can
 179 be obtained. In particular, the time-dependent load $F(t)$ has been assumed as a
 180 polynomial function that fits adequately the whole averaged load-displacement
 181 curve provided by the experimental results reported in [17].

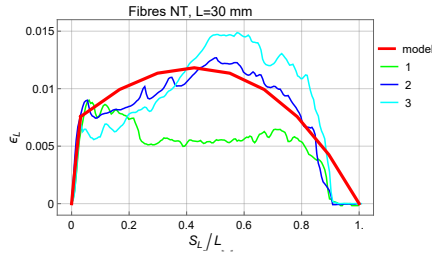
182 Both the experimental and theoretical curves of the axial strain $\varepsilon_L(t)$ vs time
 183 t are shown in Fig. 4. Note that the theoretical interpolating piecewise function
 184 $\varepsilon_L(t)$ consists of two parts. A first one, almost linear and increasing for $t \in [0, t_1]$,
 185 reproduces the debonding phase, and a second one, at first increasing and then



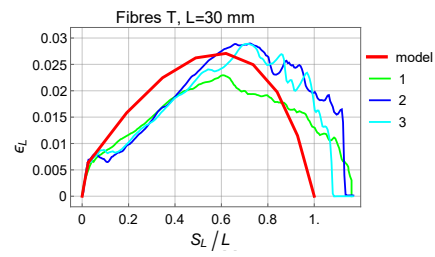
(a) Untreated fibres (NT) with embedded fibre length $L = 20$ mm.



(b) Treated fibres (T) with embedded fibre length $L = 20$ mm.



(c) Untreated fibres (NT) with embedded fibre length $L = 30$ mm.



(d) Treated fibres (T) with embedded fibre length $L = 30$ mm.

Figure 3: Variation of axial strain of the outer part of the fibre ε_L with the normalized displacement s_L/L of the actuated fibre cross section: Comparison between the theoretical curve (model) and three experimental curves (1, 2, 3) for untreated (a), (c) and treated (b), (d) fibres with embedded fibre length $L = 20$ mm and $L = 30$ mm (from [17]).

Table 1: Time at the end of the debonding phase t_1 and at the end of the pullout phase t_2 .
(NT: not treated fibres; T: treated fibres; 20 and 30 mm: embedded length)

Time	NT 20 mm	T 20 mm	NT 30 mm	T 30 mm
t_1 [s]	58.8	87.0	79.2	58.8
t_2 [s]	1046.4	1261.8	1634.4	2097.4

186 decreasing for $t \in [t_1, t_2]$, represents the pullout phase, being t_1 the time at the
187 end of the debonding and t_2 the time at the end of the pullout.

188 The end of the debonding phase and, in turn, the beginning of the pullout
189 process has been assessed numerically from eqn (9) by setting $\lambda = 0$, as listed
190 in Tab. 1.

The use of the piecewise analytical function $F(t)$ allows reproducing the whole pullout test accounting for the elastic displacement as well as the viscous counterpart of the fibre, as detailed in Sec. 3. Let

$$\sigma(t) = \frac{4F(t)}{\pi d^2}, \quad (12)$$

denotes the tensile stress in the outer part of the fibre at time t . Then, the axial strain at $x = L - \lambda L$ in the debonding phase follows from eqn (7) as

$$\varepsilon_L(t) = \sqrt{2Cs_L(t) + \frac{2}{3}Bs_L^3(t) + As_L^2(t)} = \frac{\sigma(t)}{E}, \quad (13)$$

being $F(t)$ the applied load at time t . So, for $t \in [0, t_1]$, $s_L(t)$ is determined by means of eqn (13), where $\lambda = 1$ at time $t = 0$ and $\lambda = 0$ at $t = t_1$. Similarly, for the pullout phase, eqn (7) for $x = L - s_0$ can be rewritten in the form

$$\varepsilon_L(t) = \sqrt{2Cs_L(t) + \frac{2}{3}Bs_L^3(t) + As_L^2(t) + 2C_0} = \frac{\sigma(t)}{E}, \quad (14)$$

The introduction of eqn (10) for C_0 into eqn (14) then provides the following relation between s_L and s_0

$$2C[s_L(t) - s_0(t)] + \frac{2}{3}B[s_L(t) - s_0(t)]^3 + A[s_L(t) - s_0(t)]^2 = \left[\frac{\sigma(t)}{E} \right]^2, \quad (15)$$

191 that gives $s_L(t)$ as a function of $s_0(t)$ for $t \in [t_1, t_2]$ by solving a cubic equation,
192 being $s_0 = 0$ at time $t = t_1$ and $s_0 = L$ at $t = t_2$. Note that the displacement
193 $s_L(t)$ takes into account for the rigid body motion $s_0(t)$ of the fibre during the
194 pullout stage.

195 3. Contribution of viscosity in the outer part of the fibre

196 The viscous counterparts of the strain of the fibre during the debonding and
197 the pullout phases are here considered by adopting two different creep functions
198 [24].

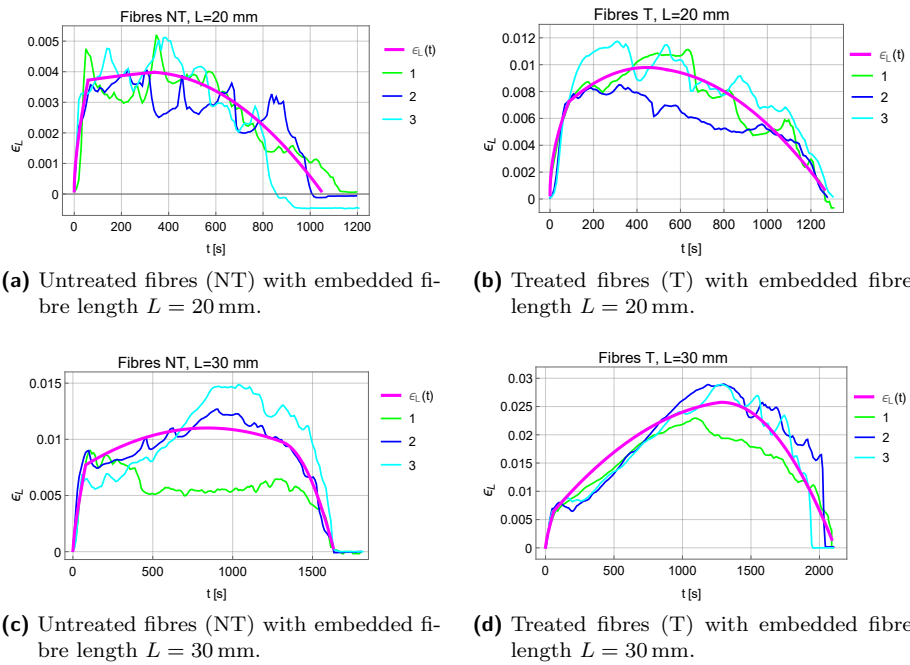


Figure 4: Interpolating function of the axial strain $\varepsilon_L(t)$ of the outer part of the fibre vs time t and experimental axial strain vs t of three experimental tests (1, 2, 3) for untreated (a), (c) and treated (b), (d) fibres with embedded fibre length $L = 20$ mm and $L = 30$ mm (from [17]).

199 In particular, a creep function based on a fraction-exponential kernel accord-
 200 ing to the Rabotnov model [25] is considered. Furthermore, in order to compare
 201 the results with simplified formulations, an alternative viscoelastic scheme fol-
 202 lowing the SLS model is considered too.

203 Since the embedded part of the fibre is practically undeformed owing to the
 204 constraint offered by the surrounding rigid matrix, then the viscous effects on
 205 that part of the fibre have been neglected.

206 3.1. Rabotnov model

A general fraction-exponential function $\psi(t)$ accounting for the viscous de-
 formation of the fibre is considered here. The Rabotnov operator turns out to
 be

$$R_\alpha(\beta - \tilde{\lambda}, t) = t^\alpha \sum_{n=0}^{\infty} \frac{(\beta - \tilde{\lambda})^n t^{n(1+\alpha)}}{\Gamma[(n+1)(1+\alpha)]}. \quad (16)$$

Then, the corresponding creep function reads

$$\psi(t) = \frac{1}{E_0} \left[1 - \frac{\tilde{\lambda}}{\beta - \tilde{\lambda}} \left\{ M_{1+\alpha}[(\beta - \tilde{\lambda})t^{1+\alpha}] - 1 \right\} \right], \quad (17)$$

where $M_a(z)$ is the well-known Mittag-Leffler function [23, 26]

$$M_a(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(m a + 1)}, \quad (18)$$

207 where $\Gamma(m) = (m-1)!$ is the Gamma function, $\tilde{\lambda} = \beta(E_0 - E_\infty)/E_0$ and
 208 α and β are two parameters which have been assessed by fitting properly the
 209 results provided by a creep test on PP macrofibres performed by Sorzia [27].
 210 Such parameters are listed in Tab. 3. As known, ψ_0 denotes the inverse of the
 211 instantaneous elastic Young modulus, whereas ψ_∞ stands for the inverse of the
 212 Young modulus for $t \rightarrow \infty$.

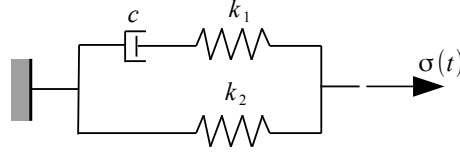
213 3.2. SLS model

A simplified creep function $\psi(t)$ accounting for the viscous deformation of
 the fibre follows an exponential map according to the classical Zener model

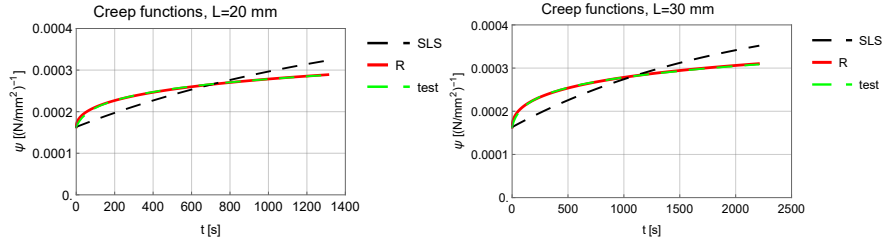
$$\psi(t) = \psi_\infty - (\psi_\infty - \psi_0)e^{-\nu t}. \quad (19)$$

Similarly to the parameters involved in the Rabotnov model, the parameters ψ_0 ,
 ψ_∞ and ν have been set on the basis of creep tests performed on PP macrofibres
 and reported in Sorzia [27]. The values of such parameters are shown in Tab. 3.
 The creep function described by eqn (19) corresponds to a Maxwell scheme
 connected in parallel with an elastic spring, as sketched in Fig. 5(a). It is easy
 to show that, for this model, the following relationships hold true

$$\psi_0 = \frac{1}{k_1 + k_2}, \quad \psi_\infty = \frac{1}{k_2}, \quad \nu = \frac{k_1 + k_2}{c}, \quad (20)$$



(a) SLS model.



(b) Exponential creep function, Mittag-Leffler creep function and creep test until a time over 1200 s. (c) Exponential creep function, Mittag-Leffler creep function and creep test until a time over 2000 s.

Figure 5: (a) Sketch of the Standard Linear Solid model (SLS). (b), (c) Comparison between the SLS creep function (SLS), the Rabotnov creep function (R) and the behavior of the PP fibre during the creep test for the time of pullout of embedded fibre length $L = 20$ mm and $L = 30$ mm (from [27]).

214 where k_1 and k_2 denote the stiffness of the elastic springs in the SLS scheme and
 215 c accounts for the dumping effect. Fig. 5(b) displays the comparison between
 216 the Rabotnov creep function, the SLS creep function and the experimental creep
 217 test in the time range $t \in [0, 1200]$ s, whereas Fig. 5(c) refers to a time range
 218 over 2000 s. Note that the creep function related to the Rabotnov operator fits
 219 the experimental results better than the SLS scheme.

220 3.3. Viscoelastic strain field of the fibre

Let L_e denote the initial length of the outer part of the fibre between the
 sample and the actuator and let z be a new abscissa taken from the surface of
 the sample in the outward direction, namely $z = x - L + s_0$, so that $0 \leq z \leq$
 $L_e + s_0$. Based on the superposition principle (i.e. the Boltzmann integral) in
 the framework of hereditary linear viscoelasticity, the axial strain in the outer
 part of the fibre turns out to be

$$\varepsilon_L(t) = \sigma(t)\psi_0 - \int_{t_z}^t \sigma(\tau)\dot{\psi}(t - \tau)d\tau, \quad \text{for } 0 \leq z \leq s_L(t), \quad (21)$$

$$\varepsilon_L(t) = \sigma(t)\psi_0 - \int_0^t \sigma(\tau)\dot{\psi}(t - \tau)d\tau, \quad \text{for } s_L(t) \leq z \leq s_L(t) + L_e, \quad (22)$$

where $\sigma(t)$ is the tensile stress in the outer part of the fibre at time t defined
 in eqn (12), t_z denotes the time instant when the fibre cross section placed at

abscissa z at time t came out of the cement matrix at $z = 0$ and started to behave viscously, as sketched in Fig. 6(a). Therefore, the viscous deformation at cross section z occurs during the time interval ranging from t_z up to the actual time t , as computed in the integral in eqn (21). The displacement u of the actuated fibre cross section at time t in the debonding and pullout phases is then given by

$$u(t) = s_L(t) + \int_0^{s_L(t)} \varepsilon_L(t) dz + L_e \varepsilon_L(t). \quad (23)$$

Then, by using the previous results (21) and (22) for $\varepsilon_L(t)$ one gets

$$u(t) = s_L(t) + \int_0^{s_L(t)} \left[\sigma(t)\psi_0 - \int_{t_z}^t \sigma(\tau)\dot{\psi}(t-\tau)d\tau \right] dz + \left[\sigma(t)\psi_0 - \int_0^t \sigma(\tau)\dot{\psi}(t-\tau)d\tau \right] L_e. \quad (24)$$

In order to evaluate the first integral in eqn (24) for a fixed value of t , let us introduce the displacement $s_L(t_z)$ of the fibre cross section that was placed at $z = 0$ at time t_z and is placed at z at time t (see Fig. 6(b)), which is given by the difference

$$s_L(t_z) = s_L(t) - z, \quad \text{for} \quad 0 \leq t_z \leq t \quad \text{and} \quad 0 \leq z \leq s_L(t). \quad (25)$$

Then, being the time t fixed, from differentiation of eqn (25) one has

$$dz = -\dot{s}_L(t_z)dt_z, \quad \text{for} \quad 0 \leq t_z \leq t, \quad (26)$$

where the over dot denotes the derivative with respect to t_z . The result (26) allows for the substitution of dz into eqn (24), by considering that $t_z = 0$ for $z = s_L(t)$ and $t_z = t$ for $z = 0$, thus giving

$$u(t) = s_L(t) + \int_0^t \left[\sigma(t)\psi_0 - \int_{t_z}^t \sigma(\tau)\dot{\psi}(t-\tau)d\tau \right] \dot{s}_L(t_z)dt_z + \left[\sigma(t)\psi_0 - \int_0^t \sigma(\tau)\dot{\psi}(t-\tau)d\tau \right] L_e, \quad (27)$$

221 where the first integral can be evaluated as shown in Appendix.

222 Note that $s_L(t_z)$ is known once the history of $s_L(t)$ has been evaluated up to
 223 time $t \geq t_z$, whereas the time t_z can be evaluated as a function of t by inverting
 224 the map (25), see Fig. 6(b). The displacement $s_L(t)$ is evaluated through the
 225 model reported in Sec. 2.2 based on the eqns (7), (9) and (11) in debonding and
 226 pullout phase, respectively, using the applied load $F(t)$ as found in Sec. 2.3.

227 The model of Sec. 2.2 accounts for the elastic contribution of the displace-
 228 ment s_L in debonding and pullout phase, but it does not account for the elastic
 229 contribution of the rigid motion s_0 at the pullout stage, which has been added
 230 apart. Eqn (27) allows evaluating straightforwardly the displacement of the end
 231 of the fibre based on the map $s_L(t_z)$.

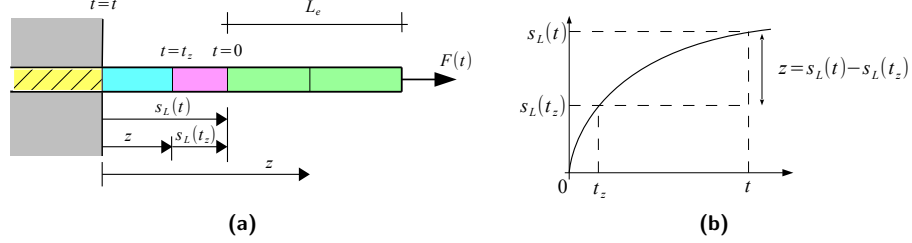


Figure 6: (a) Sketch of the elongation of the outer part of the fibre due to viscoelastic effects.
(b) Qualitative variation of displacement $s_L(t_z)$ with time t_z for $z = s_L(t) - s_L(t_z)$.

Table 2: Parameters of the constitutive law involved in eqn (1).
(NT: not treated fibres; T: treated fibres; 20 and 30 mm: embedded length)

Parameter	NT 20 mm	T 20 mm	NT 30 mm	T 30 mm
τ_0 [N/mm ²]	0.22	0.42	0.3	0.25
a [N/mm ³]	0.016	0.065	0.04	0.122
b [N/mm ⁴]	1.0×10^{-5}	1.0×10^{-5}	1.0×10^{-5}	0.003

Table 3: Property of PP fibre and relevant data concerning the pullout test.

Property	Value
Diameter d [mm]	0.78
Tensile strength [N/mm ²]	273
Elastic Young modulus E [N/mm ²]	4.591×10^3
Elastic Young modulus E_0 at $t = 0$ [N/mm ²]	4.591×10^3
Elastic Young modulus E_∞ at $t \rightarrow \infty$ [N/mm ²]	2.246×10^3
Creep modulus ψ_0 at $t = 0$ [(N/mm ²) ⁻¹]	$1/(4.591 \times 10^3)$
Creep modulus ψ_∞ at $t \rightarrow \infty$ [(N/mm ²) ⁻¹]	$1/(2.246 \times 10^3)$
α [-]	-1/3
β [s ^{-(1+\alpha)}]	-0.00608
ν [s ⁻¹]	0.00037
Rate of the actuator in pullout tests m [mm/min]	1
Embedded fibre length L [mm]	20 and 30
Outer fibre length L_e [mm]	110

232 **4. Results**

233 The strain vs displacement curves provided by the proposed theoretical
 234 model have been compared with the experimental curves provided by Di Maida
 235 et al. [17]. Such tests were carried out both on plain fibres and fibres treated
 236 with nano-silica embedded in a cementitious matrix for a total length $L = 20$ mm
 237 and $L = 30$ mm. The length of the outer part of the fibres was $L_e = 110$ mm,
 238 which coincides with the distance between the surface of the cement sample and
 239 the actuator grip. The geometric and mechanical parameters of the PP fibres
 240 and some relevant setup details are listed in Tab. 3.

241 The curves plotted in Fig. 7 show that the theoretical predictions closely
 242 fit the experimental results provided by the pullout test performed both on un-
 243 treated and treated PP macrofibres. It is worth noting that both creep functions
 244 considered in the present investigation allows reproducing closely the experimen-
 245 tal results in terms of axial strain $\varepsilon_L(t)$ of the outer part of the fibre vs its axial
 246 displacement u/L . Note also that the experimental curves shown in Fig. 7(a),(c)
 247 exhibit a small plateau with vanishing pullout resistance before the complete
 248 pullout. This is probably due to the fact that the load cell cannot measure ac-
 249 curately the sudden loss of the external load occurring for the untreated fibres
 250 near the end of the test.

251 The curves reported in Fig. 7(b),(d) for treated fibres show that the external
 252 load driving the pullout phase exhibits a remarkable increasing with respect to
 253 untreated fibres, owing to the improvement in the bond strength. This is due
 254 to abrasion phenomena occurring on the fibre surface during the pullout test.
 255 Indeed, as shown in Figs. 7, the peak value of the treated fibres is more than
 256 twice than that recorded for untreated ones. It is worth noting that the predicted
 257 response of the untreated fibres exhibits a plateau just after the debonding
 258 phase. Conversely, the theoretical prediction of the treated fibres exhibits a
 259 peak value in correspondance of the dimensionless axial displacement $u/L \simeq 0.4$
 260 for $L = 20$ mm and $u/L \simeq 0.8$ for $L = 30$ mm.

261 Furthermore, the computed axial displacement $u(t)$ has been compared with
 262 the displacement recorded during the experimental tests. This comparison is
 263 shown in Fig. 8. Note that the constant displacement rate of the actuator during
 264 the test was $m = 1$ mm/min, so that the imposed displacement at the end section
 265 of the fibre during time was $m \cdot t$ and the tests continued for 1200 s and 1800 s,
 266 being $L = 20$ mm and $L = 30$ mm the embedded fibre length respectively.

267 As shown in Figs. 8(a),(c) for untreated fibres, there is a gap between the
 268 theoretical and experimental curves that increases with time until a time of
 269 about 1046 s and 1634 s for $L = 20$ mm and $L = 30$ mm of embedded fibre
 270 length respectively, namely at a time shorter than the effective duration of the
 271 test. This difference is due to the fact that the interpolating function of the
 272 load $F(t)$ drops at that time (see Figs. 4(a),(c)), when the fibre does not exhibit
 273 any more pullout strength, even though the complete extraction of the fibre
 274 has not been achieved. This justifies the fact that, at the time of 1046 s and
 275 1634 s, the theoretical model predicts a displacement longer than 20 mm and
 276 30 mm whereas, at the same time, the actuator reached a displacement of about

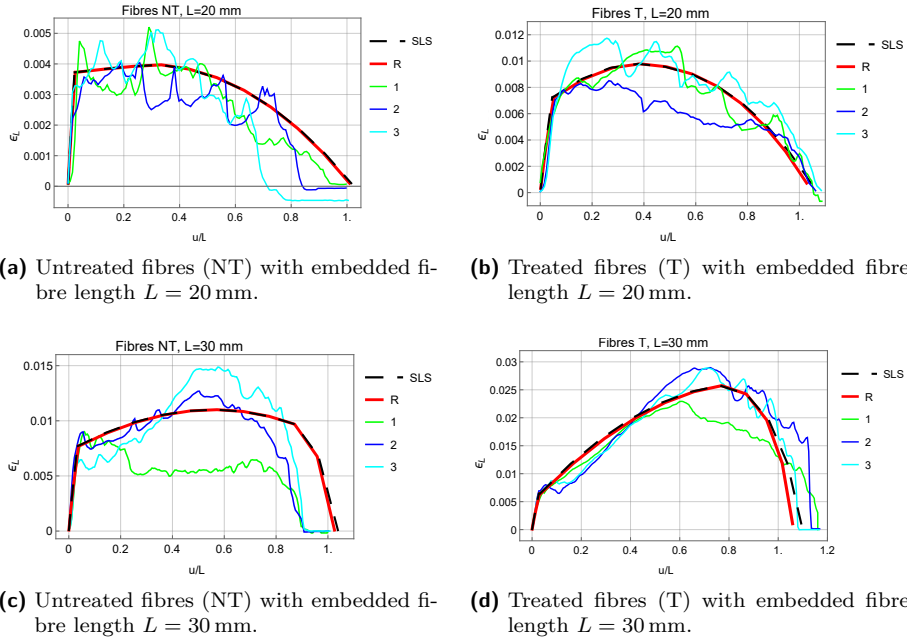


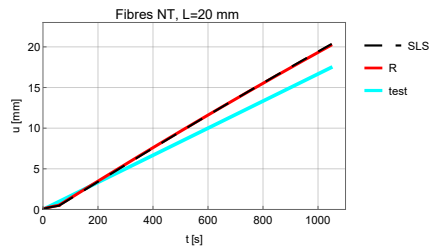
Figure 7: Variation of axial strain ϵ_L of the outer part of the fibre vs u/L : Comparison between the theoretical model (SLS with SLS creep function, R with Rabotnov creep function) and three experimental curves (1, 2, 3) for untreated (a), (c) and treated (b), (d) fibres with embedded fibre length $L = 20$ mm and $L = 30$ mm (from [17]).

277 17.5 mm and 27.5 mm respectively.

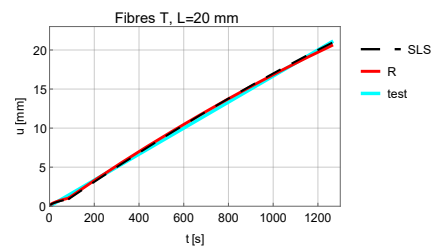
278 Concerning the treated fibres with $L = 20$ mm (Fig. 8(b)) embedded fibre
 279 length, there is a very small gap between the theoretical and experimental curves
 280 just at the beginning of the test, namely at the debonding phase. This can
 281 be ascribed to the fact that the recorded data are not sufficiently accurated
 282 during the initial stage of the test, in which the fibre detaches from the matrix.
 283 Conversely, as shown in Fig. 8(d) for treated fibres with $L = 30$ mm, there is
 284 a certain gap between predictions and experimental results near the end of the
 285 test.

286 5. Concluding remarks

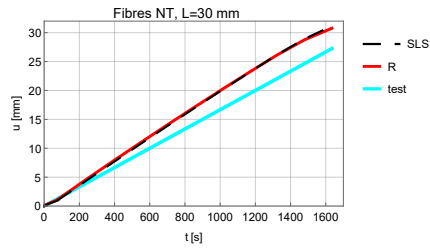
287 An analytical formulation of the pullout problem of a viscoelastic fibre embed-
 288 ded in a cementitious matrix has been proposed in the present work. Based
 289 on proper creep functions, the proposed model allows evaluating the whole pull-
 290 out process of synthetic macrofibres accounting for the viscoelastic strain of the
 291 outer part of the fibre.



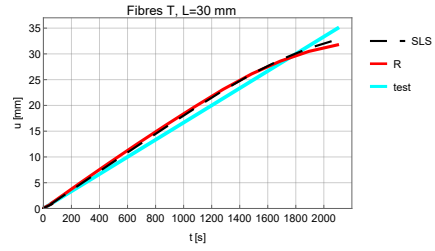
(a) Untreated fibres (NT) with embedded fibre length $L = 20$ mm.



(b) Treated fibres (T) with embedded fibre length $L = 20$ mm.



(c) Untreated fibres (NT) with embedded fibre length $L = 30$ mm.



(d) Treated fibres (T) with embedded fibre length $L = 30$ mm.

Figure 8: Displacement $u(t)$ of the actuated fibre cross section during time: Comparison between the proposed model (SLS with SLS creep function, R with Rabotnov creep function) and experimental tests (1, 2, 3) for untreated (a), (c) and treated (b), (d) fibres with embedded fibre length $L = 20$ mm and $L = 30$ mm (from [17]).

292 Three parameters τ_0 , a , b fully characterize the interfacial frictional stress
 293 as a monotonic function of the slippage, which reveals adequate for properly
 294 describing the adhesion properties of both treated and untreated fibres. The
 295 same model can be used for modelling the pullout behavior of other kinds of
 296 synthetic fibres as well as the softening pullout response of steel fibres, provided
 297 that the constitutive parameters of the interface are selected properly. It is
 298 remarked that the model is able to evaluate not only the elastic elongation, but
 299 also the viscous effects of the fibre during the debonding and pullout phases.

300 The proposed analytical model can be used as a valid tool for predicting the
 301 post-cracking behavior of FRC structures. In particular, by assuming a uniform
 302 distribution of the fibre on the cross section of a sample, accordingly, by defin-
 303 ing a proper moment-curvature relationship starting from the proposed pullout
 304 model, the equivalent flexural strength and, in turn, the mechanical behavior of
 305 FRC structures near their limit state can be properly assessed, with particular
 306 reference to bent beams [28, 29] and Kirchhoff plates [30, 31, 32, 33, 34]. Indeed,
 307 an experimental evidence of the benefits induced by the fibre treatment with
 308 nanosilica on the post cracking residual strength of FRC elements has been in-
 309 vestigated by Di Maida et al.[18] through three-point loading bending tests on
 310 beam-like samples. As a further example, the proposed approach could be used
 311 to predict time-dependent crack mouth opening displacement of FRC beam-like
 312 notched samples subjected to three-point loading bending tests and to simulate
 313 the creep behavior of bent beams in post-cracking state [35]. In these cases in-
 314 deed, the present model can capture the delayed deformation due to the viscous
 315 relaxation of the fibres bridging the crack surfaces.

316 The evaluation of the viscous strain in the embedded part of the fibre dur-
 317 ing its pullout represents a complex challenge as it involves moving boundary
 318 conditions, that the Authors wish to address in a forthcoming work.

319 Appendix

The first integral in eqn (27) can be evaluated by considering finite time
 instants. Let $I(t)$ denote the first integral in eqn (27), namely

$$I(t) = \int_0^t \left[\sigma(t)\psi_0 - \int_{t_z}^t \sigma(\tau)\dot{\psi}(t-\tau)d\tau \right] \dot{s}_L(t_z)dt_z. \quad (28)$$

Integrating by parts the integral in $d\tau$ in eqn (28) one obtains

$$I(t) = \int_0^t \left[\sigma(t_z)\psi(t-t_z) + \int_{t_z}^t \dot{\sigma}(\tau)\psi(t-\tau)d\tau \right] \dot{s}_L(t_z)dt_z, \quad (29)$$

namely

$$I(t) = \int_0^t \left[\int_0^{t_z} \dot{\sigma}(\tau)\psi(t-t_z)d\tau + \int_{t_z}^t \dot{\sigma}(\tau)\psi(t-\tau)d\tau \right] \dot{s}_L(t_z)dt_z, \quad (30)$$

or equivalently

$$I(t) = \int_{s(0)}^{s_L(t)} \left[\int_{\sigma(0)}^{\sigma(t_z)} d\sigma(\tau)\psi(t - t_z) + \int_{\sigma(t_z)}^{\sigma(t)} d\sigma(\tau)\psi(t - \tau) \right] ds_L(t_z). \quad (31)$$

By considering a finite number of time instants t_i for $i = 0, 1, \dots, n$, with $t_0 = 0$ and $t_n = t$, and the corresponding increments of the axial displacement $\Delta s_i = s_L(t_i) - s_L(t_{i-1})$ and tensile stress $\Delta\sigma_i = \sigma(t_i) - \sigma(t_{i-1})$, then the integral $I(t)$ at time t can be evaluated by the following sum

$$I(t) = \sum_{j=1}^n \Delta s_j \left[\sum_{i=1}^j \Delta\sigma_i \psi(t - t_j) + \sum_{i=j+1}^n \Delta\sigma_i \psi(t - t_i) \right], \quad \text{for } t_n \leq t. \quad (32)$$

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323 COI statement

324 The authors declare that the present work has been realized in compliance
325 with the Ethical Standards.

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327 Conflict of Interest: The authors declare that they have no conflict of interest.

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