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ANALYTICAL DESIGN OF SUPERELASTIC RING SPRINGS FOR HIGH ENERGY DISSIPATION

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Abstract.

Classical ring springs are mechanical elements used in industrial applications and in transport for shock absorption and energy dissipation. They are constituted by a stack of internal and external metal rings (typically high strength steel), with tapered surfaces in contact with one another. Under the action of an axial load these surfaces slide, the rings are deformed circumferentially and energy is dissipated due to friction. The main advantages of these springs are the high specific energy stored and the large damping capacity due to sliding friction. Furthermore, the stiffness and damping are independent on the strain rate and the temperature, which limits or avoids the occurrence of any resonance problems. The superelastic materials, characterized by an almost flat stress plateau and large reversible deformation, can be used to replace traditional steels in ring springs giving a significant performance increase. Compared to the traditional version where energy is dissipated only due to friction, in superelastic ring springs there is an increase of the dissipated energy thanks to the internal hysteresis of the material. This paper studies analytically the ring springs in traditional material and in superelastic material, providing equations to dimension these mechanical elements, which enable the designer to customize this useful structural element.

Introduction

The present work deals with the application of superelastic materials for the ring spring (also known as friction spring) design and manufacturing. This type of spring is particularly interesting because of its peculiar characteristics in terms of compactness, stiffness and energy absorbed. The ring springs are very efficient in case of limited space when there is the need to absorb and dissipate high amounts of energy, such as in the case of rail bumpers [1]. The spring ring is composed of a series of inner and outer rings coupled one to another upon the conical surfaces that slide one over another, as shown in Figure 1. When an axial load is applied to the spring, the inner rings exchange force with the outer rings along the tapered surfaces leading the outer rings to work circumferentially in traction and against the inner ring, forced to work in circumferential compression. These devices typically exploit internal or external tubular guides in order to limit the occurrence of lateral instability of these rings, otherwise too underconstrained and free. The ring springs show several advantages, but also some drawbacks which limit their practical use. Among the advantages we enlist the high work per unit weight or volume, mainly due to the full exploitation of the material, which works circumferentially and takes advantage of all the useful section of the ring. The ring springs show a high dissipative potential, which consume in friction between the rings more or less 2/3 of the energy [2] for a standard design and in good lubrication condition. The behaviour of this particular spring is completely free from resonance issues, thanks to the peculiar ultra-damped characteristic. It is possible to prevent the overloads and possible damages of the spring through the smart rings design which are able to lock when a threshold deflection is overcome. This spring design is often used in seismic applications as energy absorber, in the piping field [2]. Considering a superelastic material for the ring spring construction could lead to several improvements for these devices. The superelastic materials are metallic alloys, typically an alloy made by nickel and titanium (Nitinol). The material exhibits large strain at nearly-constant applied stress, i.e. considerable deformation occurs without a significant increase of the applied stress (stress induced martensite phase). When the stress is removed, the martensite reverts to austenite and the material recovers its original shape. This effect, which makes the alloy appear extremely elastic, is known as superelasticity or pseudoelasticity [3]. Considering the comparative study carried out by Saadat et al. [4] some possible seismic application are envisioned, since the superelastic materials are able to dissipate very high amount of energy. In this paper we analyze the mechanics of the ring spring, considering not only the traditional steel, but also a superelastic alloy. The feasibility of a SMA ringspring is assessed in [5] where an extensive finite element campaign was carried out. The present paper presents the problem from the designer perspective in order to tailor the desired ringspring made in superelastic material under specific constraints. In order to tackle the problem analytically we adopt a simplified but conservative assumption for the superelastic material, which is a rigid-perfectly plastic assumption. This approach grants and immediate and simple comparison between a spring made in steel and the same spring made in superelastic alloy, making possible the application of the model to a real case. This grants a great advantage in terms of weight and dimensions being equal the force, the deflection and the energy absorbed by the two systems.

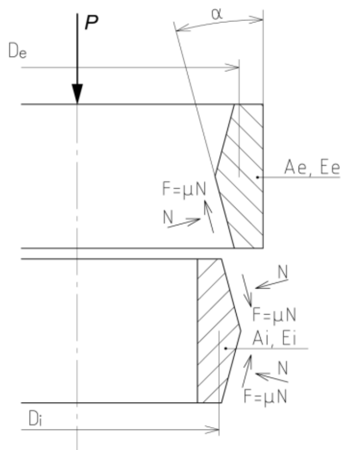


Figure 1 – Sketch of a portion of ring spring with indication of the forces.

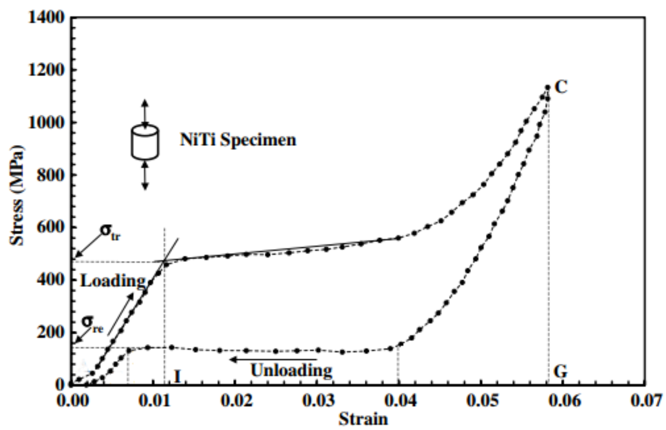


Figure 2 – Stress vs strain diagram of a superelastic Ni-Ti alloy, [11].

MATERIALS AND METHOD

Elastic ring spring: analytical approach. The ring springs (also called friction springs) were for the first time introduced in 1924 [6]. Taking as a reference the scheme in Figure 1, we consider only two rings of the entire springs, which is sufficient to analyze its behaviour. The two rings are depicted during the axial loading and we focus on the internal ring, which has an average diameter D_i, meridian section of the internal ring A_i, and elastic modulus E_i whilst the parameters of the external ring are: D_e, A_e, E_e. According to Figure 1 we name α the angle of the conical surfaces with respect to spring axis, call the friction angle ϕ, with $\mu = \tan\phi$ and consider z stacked rings.

The main design equations [1] regards the hoop stresses for the internal and external ring, during the loading phase under P and the deflection, during the loading (δ) and unloading (δ') phase:

$$\sigma_i = \frac{P}{\pi A_i \tan(\alpha + \phi)} \qquad (1)$$

$$\sigma_e = \frac{P}{\pi A_e \tan(\alpha + \phi)} \quad (2)$$

$$\delta = z\,\delta_1 = \frac{z\,P}{2\pi \tan\alpha \tan(\alpha + \phi)} \left(\frac{D_i}{A_i E_i} + \frac{D_e}{A_e E_e} \right) \quad (3)$$

$$\delta' = \frac{z\,P'}{2\pi \tan\alpha \tan(\alpha - \phi)} \left(\frac{D_i}{A_i E_i} + \frac{D_e}{A_e E_e} \right) \quad (4)$$

The stiffness of the spring in the loading phase, *k*, is reported in eq. (5) while in the unloading phase is given by the force during unloading, *P'* divided by the deflection δ', as in eq. (6).

$$k = \frac{P}{\delta} = \frac{2\pi \tan\alpha \tan(\alpha + \phi)}{z\left(\frac{D_i}{A_i E_i} + \frac{D_e}{A_e E_e} \right)} \quad (5)$$

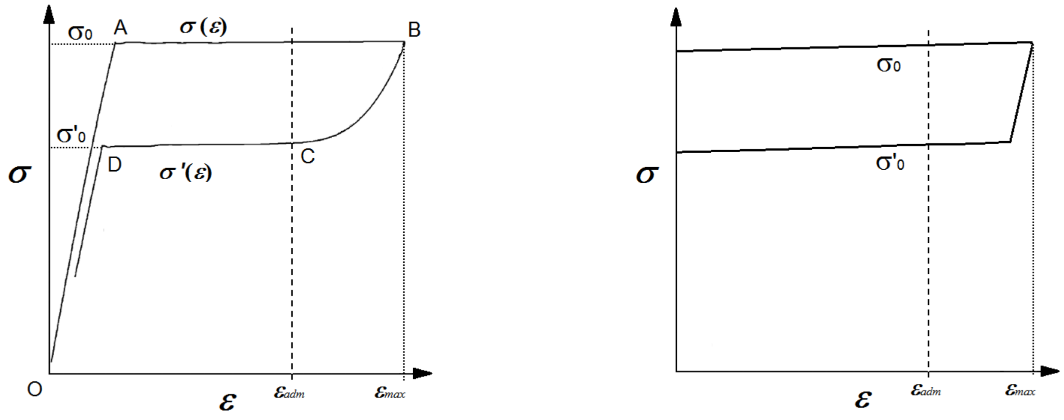
$$k' = \frac{P'}{\delta'} = \frac{2\pi \tan\alpha \tan(\alpha - \phi)}{z\left(\frac{D_i}{A_i E_i} + \frac{D_e}{A_e E_e} \right)} \quad (6)$$

By considering the loading and unloading stiffnesses, the complete loading–unloading cycle of the spring can be analyzed. During the loading phase the force follows the line defined by *k*, than, during the loading phase, the force follows a vertical line since there is a decompression at constant deflection, than it goes down proportionally to the deflection along the *k'* line. The difference between the area under the loading phase and the unloading phase is the absorbed energy, dissipated due to the friction between the rings. The figure of merit of the spring, called coefficient of restitution, ρ_E, is defined as the ratio between the energy released in the unloading phase and the energy stored during the loading phase. Moreover, we have the damping coefficient, m_E, defined as the ratio between the dissipated energy and the absorbed energy:

$$\rho_E = \frac{0.5 \cdot P'\delta}{0.5 \cdot P\delta} = \frac{P'}{P} = \frac{N\sin\alpha - F\cos\alpha}{N\sin\alpha + F\cos\alpha} = \frac{N\sin\alpha - \mu N\cos\alpha}{N\sin\alpha + \mu N\cos\alpha} = \frac{\tan(\alpha - \phi)}{\tan(\alpha + \phi)} \quad (7)$$

$$m_E = \frac{0.5(P - P')\delta}{0.5P\delta} == 1 - \frac{P'}{P} = 1 - \rho_E = 1 - \frac{\tan(\alpha - \phi)}{\tan(\alpha + \phi)} \quad (8)$$

Superelastic ring spring: analytical approach. Figure 3a shows a typical stress-strain curve of a superelastic material, where the loading path (upper OAB curve) and the unloading path (lower BDCO curve) are highlighted. This graph has three main features. First, the maximum strain, which is impressive for a metallic material (ε_{max} = 5 - 8 %), is completely recovered after the unloading. Second, a cyclic deformation leads to internal dissipation, due to the hysteresis in the stress strain curve. Third, the loading and unloading phase, after an initial linear behaviour are almost completely carried out at constant stress (σ_0, σ_0' in Figure 3a). Starting from the early literature works [7], many other complex and precise models for superelastic materials were developed [8]. Considering the aim of this paper and the need of finding simple mathematical description of the system, as in [9], [10], we decided to simplify the stress-strain curve with rigid constant behaviour reported in Figure 3b [11], in which the stress is exactly σ_0 during the loading phase and σ_0' during the unloading phase. This assumption is quite realistic when the deformations are large and the initial linear elastic part is almost negligible. In order to envision a more realistic application, we make another assumption: only the external ring is made of superelastic material, while the internal ring is perfectly elastic, (i.e made of steel) with a Young's modulus much larger than the superelastic one, in accordance with literature [12].



(a) (b)

Figure 3 – Experimental stress vs strain curve of a superelastic material (a) and rigid-plastic approximation used in the analytical model (b).

Therefore, during the loading phase, given that $\sigma_e = \sigma(\varepsilon)$, the equation developed for the traditional spring still holds and the axial force in loading (P), the axial force in unloading (P') phase, the restitution coefficient (ρ_{SE}) and the damping coefficient (m_{SE}) can be obtained:

$$P = \sigma_0 \cdot \pi A_e \tan(\alpha + \phi) \tag{9}$$

$$P' = \sigma'_0 \cdot \pi A_e \tan(\alpha - \phi) \tag{10}$$

$$\rho_{SE} = \frac{P'}{P} = \frac{\sigma'_0}{\sigma_0} \cdot \frac{\tan(\alpha - \phi)}{\tan(\alpha + \phi)} \tag{11}$$

$$m_{SE} = 1 - \rho_{SE} = 1 - \frac{\sigma'_0}{\sigma_0} \cdot \frac{\tan(\alpha - \phi)}{\tan(\alpha + \phi)} \tag{12}$$

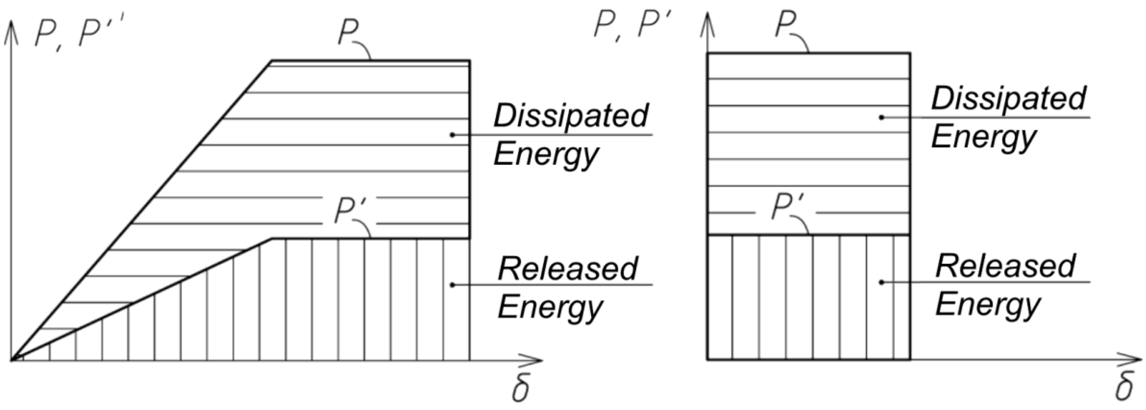
Considering the typical metallic superelastic material (e.g. Nitinol) we have $\sigma_0' \approx 0.3–0.5\sigma_0$ and by comparing equation (12) and (8) it is shown that the damping of the superelastic ring spring is much higher than the traditional ring spring.

Considering the deflection, the compenetration of a single couple of rings in contact is anyhow given by equation (3) with $\Delta D_i \approx 0$ and $\Delta D_e = D_e\, \varepsilon_{adm}$, where ε_{adm} is the maximum working deformation of the superelastic material (Figure 3). This value can be decided by considering the properties of the materials and the fatigue constraint in the problem. The overall deflection is thus given by the deformation of z conical coupling and its expression is:

$$\delta = z\,\delta_1 = \frac{1}{2} z(\Delta D_i + \Delta D_e)\frac{1}{\tan\alpha} = \frac{z\,D_e \varepsilon_{adm}}{2\tan\alpha} \tag{13}$$

Figure 4a shows a more precise diagram where the initial elastic regime of the material is considered, while Figure 4b shows the simplified graph where the initial elastic part is neglected.

The mechanical design of the spring starts from the desired load and energy to be dissipated. By means of eq. (9), (12-13) the geometry of the spring and the material can be selected, aiming at having high $\varepsilon_{adm,}$ and a large σ_0'/σ_0 to improve the damping coefficient. Then, the desired displacement is obtained to by selecting the right amount of stacked ring, z.



(a) (b)

Figure 4 - Force vs displacement (*P*- *δ*) diagram of a superelastic (Ni-Ti) steel ring spring considering the initial ramp, not to scale (a) or simplified by using the rigid-plastic assumption (b).

Discussion

Using a superelastic material for the ring spring manufacturing dramatically increase the specific damping capacity, thanks to the superposition of the intrinsic dissipative behaviour of the material and the friction between the rings [12]. By comparing the two equations that represent the coefficient of restitution, ρ, it is quite simple to quantitatively estimate the difference. When the external ring is made of superelastic material the coefficient of restitution is lowered by a factor σ_0'/σ_0 with respect to the traditional solution (see Figure 3). Considering the typical Nitinol superelastic alloy we have $0.3 < \sigma_0'/\sigma_0 < 0.5$. A lower ρ implies a high energy dissipated by the ring spring. Some major advantages for the superelastic solution are reported:
- high energy absorbed being equal the maximum force and deflection
- constant force response (lower reaction on the frame structure and on mobile elements)
- higher specific damping
- smaller and lighter devices
- intrinsic material hysteresis
-low stiffness

The higher specific damping (energy dissipated / mass), being equal the force and the deflection, is very high because the superelastic ring spring dissipate energy also through the material, the maximum admissible strain is much larger and the mass is lower.

The reaction provided at almost constant force is another very interesting feature, since it provides a uniform deceleration of the structures and makes the spring independent on the load speed.

This quasi-constant force behaviour occurs only using the oversimplified rigid plastic model, described by Figure 3b [11] , but in real application this condition is met only when the superelastic material stress strain curve presents very low “hardening” in the plateau. It is possible to improve the force deflection response of the superelastic spring and deleting the initial slope by adding a small preload to the superelastic spring rings. This preload deforms the superelastic rings up to a stress level near to the austenite→martensite stress transformation level (σ_0 in Figure 3). Using a superelastic material may open some space for the industrial applications of the ring springs, which are often not applicable due to too high stiffness, limiting the practical use of these devices to heavy industries (railways) or civil applications. By comparing the equations, it is easy to understand that there is a very strong mechanical advantage in adopting the superelastic solution. By applying the described procedure to a practical case study it is simple to demonstrate that the axial dimension would be several times lower, while the weight would be an order of magnitude lower. The drawback of course is given by the cost of the superelastic material, which is still very high for a bulk application of this material.

Conclusion

This paper analyzes the behaviour of the ring spring (also known as friction spring) made of superelastic materials, for example nickel-titanium alloys. The ring springs are mechanical components typically used in heavy industry machineries, in railway applications and in civil engineering field as energy absorbers. The work describes analytically the traditional ring spring made in elastic material and also in superelastic material, by applying a simplified material models to describe the superelastic alloy. The work provides the design equations that describe the behaviour of the spring and allow the comparison between the two materials. Considering a fair comparison between the traditional and the superelastic solution, being equal the rings spring specification in terms of dissipated energy and maximum external load, the equations show that there is always a strong advantage for the superelastic solution. Its main highlights are: higher specific dissipation, constant force response, smaller dimensions, lower stiffness, and lower weight. The main drawback of the superelastic solution is the cost of the material, which is an order of magnitude higher than the steel, traditionally used for the commercial ring springs. The application of the superelastic ring springs in industrial fields can be envisioned only when the cost would be overcome.

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