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# Direct numerical simulation of the flow around a rectangular cylinder at a moderately high Reynolds number

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# Abstract

We report a Direct Numerical Simulation (DNS) of the flow around a rectangular cylinder with a chord-to-thickness ratio B/D = 5 and Reynolds number Re = 3000. Global and single-point statistics are analysed with particular attention to those relevant for industrial applications such as the behaviour of the mean pressure coefficient and of its variance. The mean and turbulent flow is also assessed. Three main recirculating regions are found and their dimensions and turbulence levels are characterized. The analysis extends also to the asymptotic recovery of the equilibrium conditions for self-similarity in the fully developed wake. Finally, by means of two-point statistics, the main unsteadinesses and the strong anisotropy of the flow are highlighted. The overall aim is to shed light on the main physical mechanisms driving the complex behaviour of separating and reattaching flows. Furthermore, we provide well-converged statistics not affected by turbulence modelling and mesh resolution issues. Hence, the present results can also be used to quantify the influence of numerical and modelling inaccuracies on relevant statistics for the applications.

*Keywords:* Flow benchmark; Rectangular cylinder; Flow reattachment; Direct Numerical Simulation;

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#### 1 1. Introduction

The flow around bluff bodies with sharp corners is known to be of over-2 whelming interest for several wind engineering applications [1]. The case of 3 a rectangular cylinder encompasses the range of bluff bodies from a flat plate normal to the flow, to a square cylinder, and, finally, to a flat plate parallel to the flow, as its chord-to-thickness ratio is varied from zero to infinity. For these reasons, these kind of flows have been the subject of several numerical and experimental studies. Of particular interest for civil engineering applications is the case of slender bodies typical of buildings and structures. A peculiarity of these shapes resides in the fact that the flow exhibits a large-scale separation at 10 the leading-edge and also a reattachment before the definitive separation at the 11 trailing-edge. Indeed, while the shedding instability in the wake is observed in 12 all bluff bodies, only long bluff bodies present further instabilities which are due 13 to the separating and reattaching leading-edge shear layer. This leads to the 14 formation of an additional shedding of large-scale vortices before the trailing 15 edge. A detailed investigation into the nature of this separating and reattach-16 ing flow is found in Cherry et al. [2]. Despite the fact that these kind of flows 17 have been the subject of several numerical and experimental studies, the topic 18 is still attractive, as highlighted in a recent work by Bruno et al. [3]. From an 19 applicative point of view, the interest is given by the fact that both experimen-20 tal and numerical techniques appear to be unable to tackle the problem in a 21 unequivocal way. Indeed, a large variability of results is found in the literature, 22 even for global or first order statistics, see again the review of Bruno *et al.* [3]. 23 The reason of these discrepancies is the high sensitivity of the flow on the test 24 boundary conditions and measurement accuracy in experiments and on the tur-25 bulence model, numerical schemes and mesh properties in CFD analysis. Here, 26 we focus on the numerical approach. 27

For low Reynolds numbers,  $10^2 < Re < 10^3$  where  $Re = U_{\infty}D/\nu$ ,  $U_{\infty}$ the free-stream velocity, D the rectangular cylinder thickness and  $\nu$  the kinematic viscosity, the flow around rectangular cylinders has been studied in several

works, see e.g. Nakamura et al. [4], Ohya et al. [5], Hourigan et al. [6], and 31 Tan et al. [7]. The main aim of the above mentioned works is the assessment of 32 the main instabilities of the flow and of the self-sustaining mechanisms which 33 generate them. Concerning the high Reynolds number regime,  $Re > 10^4$ , it is 34 worth mentioning the works of Shimada and Ishihara [8] and of Yu and Ka-35 reem [9] where RANS (Reynolds Average Navier-Stokes) and LES (Large Eddy 36 Simulation) techniques are respectively used. In this context, it is important to 37 point out a benchmark activity on the aerodynamics of rectangular cylinders at 38 Reynolds numbers of the order of  $10^4$ , i.e. the BARC project (Benchmark on 39 the Aerodynamics of a Rectangular 5:1 Cylinder) [10]. Within this framework, 40 a series of experiments and simulations have been conducted aiming at estab-41 lishing reliable standards for the simulation and measurement of such a flow 42 configuration, see e.g. Bruno et al. [11], Mannini et al. [12], Ricci et al. [13] 43 and Patruno et al. [14] for its extension to non-null angles of attack. 44

As summarized in Bruno et al. [3], the recent results within the BARC 45 project are still characterized by a large scatter, thus highlighting that a clear 46 picture of the combined influence of mesh resolution, turbulence model and 47 boundary conditions on the flow statistics is still missing. One of the possible 48 reasons is that, up to now, no reference data are available in the literature, i.e. 49 experimental data obtained under well-defined boundary conditions (e.g. free-50 stream turbulence level) and unaffected by measurement errors, or numerical 51 data not influenced by modelling and mesh resolution issues. Indeed, to the 52 best of the authors' knowledge, no Direct Numerical Simulation (DNS) for suf-53 ficiently high Reynolds number has been performed in such a flow configuration. 54 We found only two attempts in the literature. In the first one, Tamura et al. 55 [15] approached the problem by means of a finite difference technique at high 56 Reynolds number,  $Re = 10^4$ . However, the grid resolution adopted was not 57 fine enough to capture the smallest scales of motion and, hence, the simulation 58 reported appears to be more an implicit LES than a DNS. More recently Houri-59 gan et al. [6] proposed a more accurate analysis through a spectral-element 60 method. However, the DNS data reported refer to very low Reynolds numbers, 61

namely from Re = 350 to Re = 500, and a fully developed turbulent state is not achieved.

In the present work we produce, for the first time, high-fidelity data of 64 the flow around a rectangular cylinder with chord-to-thickness ratio B/D =65 5 and Reynolds number Re = 3000. The study is aimed at understanding 66 the main physical mechanisms driving the flow and at providing statistics, not 67 affected by numerical issues, to be used for the validation and calibration of 68 CFD techniques. For obvious computational cost reasons, the Reynolds number 69 considered,  $Re = 3 \cdot 10^3$ , is smaller than the ones considered in the recent 70 literature. However, let us point out that as shown by Sasaki and Kiya [16], the 71 flow develops the main turbulent structures typical of larger Reynolds numbers 72 already for Re > 380. By further increasing Re, it is also found that the bubble 73 length does not increase significantly anymore. It is also worth mentioning that, 74 based on spectral arguments, Nakamura et al. [17] argue that an asymptotic 75 large Reynolds number regime is attained for Re = 3000 since for Re > 300076 the Strouhal number of the spectral peak does not increase anymore. Based on 77 these results, we argue that the considered Reynolds number is sufficiently large 78 to capture the main physical features observed at larger Reynolds numbers. As 79 an example, the two main unsteadinesses observed by Kiya and Sasaki [18, 19] 80 for very large Reynolds numbers and consisting of a shedding of vortices from 81 the separation bubble and of a large scale oscillation encompassing the entire 82 flow field, are found to be reproduced both qualitatively and quantitatively at 83 the present Reynolds number (see section  $\S3$  for the details). 84

The paper is organized as follows. A description of the numerical simulation and of the statistical procedure is reported in section §2. The main statistical properties of the flow, with particular attention to those mostly debated in the BARC project, are shown in section §3. In order to rigorously assess the physical features characterizing the flow, single-point and two-point statistics are analysed in detail in sections §4 and §5. The paper is then closed by final remarks in section §6.

# <sup>92</sup> 2. Direct Numerical Simulation and statistical convergence

A Direct Numerical Simulation has been performed to study the flow around
a rectangular cylinder. The evolution of the flow is governed by the continuity
and momentum equations,

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j x_j}$$
(1)

where  $x = x_1$  ( $u = u_1$ ),  $y = x_2$  ( $v = u_2$ ),  $z = x_3$  ( $w = u_3$ ) are the stream-96 wise, vertical and spanwise directions (velocity components), p is the pressure 97 field and  $Re = U_{\infty}D/\nu$  is the Reynolds number where  $U_{\infty}$  is the free-stream 98 velocity, D is the thickness of the rectangular cylinder and  $\nu$  is the kinematic 99 viscosity. In accordance with the above equations, all the variables presented in 100 the following will be reported in a dimensionless form by using D as length scale 101 and  $D/U_{\infty}$  as time scale. A cell-centered finite volume method has been chosen 102 to discretize the equations by means of the OpenFOAM<sup>®</sup> open source code 103 [20]. Time integration is performed by means of a second-order backward Euler 104 implicit scheme while convective and diffusive fluxes at the volume faces are 105 evaluated through a second-order central difference scheme. Finally, a pressure-106 implicit split-operator algorithm [21] is used to numerically solve the pressure-107 velocity coupling. Given the simple geometry of the problem, a block-structured 108 Cartesian grid is adopted. Inlet-outlet boundary conditions are imposed in the 109 streamwise direction. The inlet condition is a simple unperturbed flat velocity 110 profile. The outlet boundary condition combines a Neumann/Dirichlet condi-111 tion. In particular, a stress-free (zero gradient) condition is enforced when the 112 flow exits the boundary, while a zero velocity vector is imposed when an in-113 ward flow is detected. The same kind of boundary condition is imposed in the 114 vertical direction, the only difference being that in case of inward flow the im-115 posed Dirichlet condition equals the free-stream inlet velocity. Finally, periodic 116 conditions are imposed in the spanwise direction. 117



The flow case consists of a rectangular cylinder whose dimensions are  $(L_x, L_y) =$ 



Figure 1: Configuration of the system.

(5D, D). The Reynolds number considered is Re = 3000. The extent of the nu-119 merical domain is  $(\mathcal{D}_x, \mathcal{D}_y, \mathcal{D}_z) = (112D, 50D, 5D)$  and is found large enough to 120 not interfere with the flow dynamics, see the analysis of the spanwise correla-121 tion function shown in section §5. A sketch of the system configuration and of 122 the reference coordinate system is reported in figure 1. The structured Carte-123 sian grid employed is composed by  $1.5 \cdot 10^7$  volumes. A multi block-structured 124 approach is used by employing 5 main blocks characterized by a stepwise vari-125 ation of the number of volumes in the spanwise direction. In particular, in 126 the inner block, the number of volumes in the (x, z)-plane above the rectangle 127 is (Nx, Nz) = (128, 144) in the streamwise and spanwise direction, respec-128 tively. The volume distribution is homogeneous in the spanwise direction while 129 in the streamwise and vertical directions a geometric progression is adopted, 130  $\Delta x_i = k_x^{i-1} \Delta x_1$  and  $\Delta y_j = k_y^{j-1} \Delta y_1$  with  $k_x = 1.06, k_y = 1.04, \Delta x_1 = 0.004$ 131 and  $\Delta y_1 = 0.004$ . This approach is used to obtain higher resolution levels in the 132 near-wall leading- and trailing-edge regions. Such a practice leads to a mean 133 wall resolution of  $(\overline{\Delta x^+}, \overline{\Delta y^+}, \overline{\Delta z^+}) = (6.1, 0.31, 5.41)$ , where  $\overline{(\cdot)}$  denotes the 134 streamwise average along the rectangle length and the superscript + implies 135 normalization with friction units. The time step varies during simulation to 136 obtain a condition CFL < 1 in each point of the domain, the resulting average 137



Figure 2: Time-behaviour of the friction drag coefficient  $C_f$ . The vertical line denotes the end of the initial transient and the start of the fully developed state used for the computation of the statistics. To note that the plot starts from t = 115. Indeed, from t = 0 to t = 115 a precursory coarser simulation has been used to reduce the computational time for reaching a flow state close to the statistical equilibrium.

138 time step being  $\Delta t = 0.0023$ .

In the present flow case, the computational demand for well-converged statis-139 tics denoted as  $\langle \cdot \rangle$ , is mitigated by the statistical stationarity of the flow field 140 and by the statistical homogeneity in the spanwise direction. Furthermore, the 141 flow exhibits certain statistical symmetries in the vertical direction which are 142 better expressed by shifting the origin of the vertical coordinate to the centre of 143 the prism,  $\tilde{y} = y - D/2$ . Indeed, the transformation  $\tilde{y} \to -\tilde{y}$  leaves quantities 144 like  $U = \langle u \rangle$ ,  $\langle u_i u_i \rangle$  statistically invariant while reversing the sign of quantities 145 like  $V = \langle v \rangle$ ,  $\langle uv \rangle$  and  $\partial \langle \cdot \rangle / \partial \tilde{y}$ . In conclusion, the average of a generic quantity 146  $\beta$  is defined as 147

$$\langle \beta \rangle(x,\tilde{y}) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} \left( \frac{1}{L_z} \int_{-L_z/2}^{L_z/2} \beta(x,\tilde{y},z,t) dz \pm \frac{1}{L_z} \int_{-L_z/2}^{L_z/2} \beta(x,-\tilde{y},z,t) dz \right),$$
(2)

where the sum and difference of the two integrals is given by the symmetric or antisymmetric property of the considered variable, respectively. The number of fields sampled during time is N = 317. These samples are taken once the initial



Figure 3: (a) Time evolution of the lift coefficient  $C_l$ . (b) Frequency spectrum of the lift coefficient,  $\phi_{c_l}^f$ , as a function of the Strouhal number St.

transition of the flow field is washed out, see figure 2, and are taken with a time separation  $\Delta T = T$  where  $T = D/U_{\infty}$  is the characteristic time scale of the flow. In the following, the customary Reynolds decomposition of the velocity field is adopted,  $u_i = U_i + u'_i$ , where  $U_i$  and  $u'_i$  denote the mean and fluctuating velocity.

# <sup>156</sup> 3. Main properties of the flow

<sup>157</sup> We start the analysis by considering the main integral parameters of the flow, <sup>158</sup> i.e. the lift,  $C_l$ , and drag,  $C_d$ , coefficients. Obviously, for symmetry reasons, <sup>159</sup> the average lift coefficient is null,  $\tilde{C}_l = 0$  where  $\tilde{\cdot}$  denotes the time average. <sup>160</sup> As shown in figure 3(a), instantaneously the lift coefficient is not zero, but it <sup>161</sup> fluctuates in time. Different time scales are recognized and can be studied by <sup>162</sup> considering the frequency spectrum defined as

$$\phi_{C_l}^f(St) = \hat{C}_l \hat{C}_l^* \tag{3}$$

where  $\hat{\cdot}$  denotes the Fourier transform, \* the complex conjugate and St the dimensionless frequency, i.e. the so-called Strouhal number. As shown in figure 3(b), the frequency spectrum confirms the presence of different temporal scales. In particular, a clear peak for  $St \approx 0.14$  is present and will be shown in the following to be related with the frequency of the large scale vortex detachment



Figure 4: Streamlines of the mean velocity field (U, V)(x, y). The green lines show the *primary* vortex, the red lines mark the secondary vortex and the black lines denote the wake vortex. The red dots denote the locations of the probes used for the computation of time spectra in section §5.

in the wake. A second peak is recognizable at a lower frequency, namely  $St \approx$ 168 0.042. This very low frequency is responsible for the long term fluctuations of  $C_l$ 169 observed in figure 3(a) and could explain the scatter of the data of  $\widetilde{C}_l$  obtained 170 in different works, see the review of Bruno et al. [3], as a result of statistical 171 convergence problems. The frequencies of the two spectral peaks detected here 172 quantitatively agree with those observed in Kiya and Sasaki [18, 19] for very 173 large Reynolds numbers. In accordance with these works, we argue that the 174 peak at  $St \approx 0.14$  is related with a large scale shedding of vortices from the 175 main recirculating region while the very slow peak at  $St \approx 0.042$  is due to 176 the presence of a low-frequency unsteadiness encompassing the entire flow field. 177 Concerning the drag coefficient, we measure that  $\widetilde{C}_d = 0.96$ . 178

Let us now consider the topology of the mean flow field. As shown in figure 179 4, the streamlines of the mean flow highlight the presence of a large scale recir-180 culation extending from the leading edge up to  $x_r \approx 3.65$ , see the green lines in 181 figure 4. The centre of rotation, defined as the singularity point within the recir-182 culating region where the mean velocity field vanishes (U, V) = (0, 0), is located 183 at  $(x_c^{pv}, y_c^{pv}) = (2.04, 0.35)$ . This separation bubble will be hereafter called the 184 primary vortex. As shown by the isocontours of mean pressure in figure 5, this 185 separated region is associated with a large area of low pressure with a mini-186



Figure 5: Isocontours of the mean pressure field P(x, y). The dashed lines report the location of the *primary vortex*, secondary vortex and wake vortex.

mum located at  $(x_p^{pv}, y_p^{pv}) = (1.81, 0.42)$ . It is worth noting that the location 187 of this minimum does not coincide with the centre of the *primary vortex*, thus 188 highlighting the strongly inhomogeneous non-axisymmetric shape taken by the 189 recirculating flow. Actually, a second recirculating region is present and high-190 lighted with red lines in figure 4. This secondary vortex takes place below the 191 primary vortex. Indeed, the reverse flow induced in the near-wall region by the 192 primary vortex creates a boundary layer moving upstream. As shown by the 193 isocontours of the mean pressure field in figure 5, the induced boundary layer 194 undergoes an adverse pressure gradient, hence it decelerates, becomes thicker 195 and, finally, breaks down leading to a further separation. Hence, the secondary 196 vortex, being induced by the primary vortex, is counter-rotating with respect 197 to the *primary vortex* and its characteristic length and time scales are smaller 198 than those of the *primary vortex*. The secondary vortex is found to extend for 199 0.4 < x < 1.4. The intensity of the mean flow within this region is very low, 200 thus highlighting how this object is difficult to capture from a statistical point 201 of view, see Bruno et al. [3] for a review of the different results in literature. 202 After the reattachment point, for  $x > x_r$ , the flow evolves in a downstream 203 boundary layer and finally detaches at the trailing edge where it develops into 204 the wake. As shown with black lines in figure 4, the separated wake highlights 205

a wake vortex extending from the trailing edge down to  $x \approx 6.2$  and centered at  $(x_c^{wv}, y_c^{wv}) = (5.5, -0.25)$ . As for the *primary vortex*, the wake vortex sets a low pressure region centered at  $(x_p^{wv}, y_p^{wv}) = (5.52, -0.27)$ , see figure 5. The main difference with respect to the leading-edge recirculation is given by the fact that the extent and intensity of the low pressure area are smaller.

Let us now study the streamwise behaviour of the skin friction and pressure 211 coefficients along the horizontal surface of the rectangular cylinder. As shown 212 in figure 6(a), the mean skin friction coefficient  $\langle c_f \rangle(x)$  exhibits strong negative 213 values in the very first part of the rectangular cylinder, close to the leading-edge 214 corner. Then,  $\langle c_f \rangle$  increases and reaches small but positive values in the region 215 0.4 < x < 1.4. This region of positive shear is the near-wall footprint of the 216 secondary vortex. Moving downstream, for 1.4 < x < 3.65, the average skin 217 friction coefficient becomes negative again and shows a minimum at  $x \approx 2.37$ . 218 This region of negative shear is the near-wall footprint of the *primary vortex*. 219 Actually, the *primary vortex* is responsible also for the previously observed 220 strong negative values of  $\langle c_f \rangle(x)$  in the leading-edge region thus indicating that 221 the primary vortex reattaches upstream the secondary vortex. Finally, in the 222 last part of the rectangular cylinder, for x > 3.65, a forward attached boundary 223 layer takes place. Indeed, the skin friction is positive and assumes increasingly 224 large values moving downstream. 225

Let us now considering the streamwise behaviour of the pressure coefficient 226  $\langle c_p \rangle(x)$  and of its variance  $\langle c_p'^2 \rangle(x)$  shown in figure 6(b). These two observables 221 are of paramount relevance for the applications since they carry information 228 about the aerodynamic loads and their fluctuations. However, as pointed out 229 in Bruno *et.* al [3], the prediction of these two statistical observables is very 230 challenging for numerical simulations, as highlighted by the variability of the 231 results obtained within the BARC project. The reason is that the behaviour 232 of  $\langle c_p \rangle(x)$  and  $\langle c'_p \rangle(x)$  is strongly influenced by the shape and extent of the re-233 circulating regions of the flow, which in turn are determined by the turbulence 234 levels therein. Hence, the turbulence modelling and mesh resolution adopted 235 in numerical simulations strongly impacts the resolved turbulent dynamics and, 236



Figure 6: (a) Streamwise behaviour of the average skin friction coefficient  $\langle c_f \rangle(x)$ . (b) Streamwise evolution of the average pressure coefficient  $\langle c_p \rangle(x)$  and of its standard deviation  $\sqrt{\langle c_p'^2 \rangle}(x)$ .

hence, the predicted extent of the recirculation regions. Accordingly, the present
data should help to clarify the behaviour of mean and fluctuating pressure field
at the wall, since they are not affected by turbulence modelling and mesh resolution issues.

Figure 6(b) shows that the mean pressure coefficient  $\langle c_p \rangle(x)$  is always nega-241 tive on the rectangular cylinder. It starts from its minimum at the leading-edge 242 corner and shows a sharp increase in a very small region corresponding to the 243 region of upstream reattachment of the *primary vortex*. Then, for x > 0.13, 244 a weak decrease is observed up to  $x \approx 1.81$ . This streamwise location is the 245 wall footprint of the low-pressure levels associated with the *primary vortex* core. 246 Further downstream, the wall pressure shows a significant increase. This pres-247 sure rise is maintained up to  $x \approx 4.39$  where it forms a maximum, since a slight 248 pressure decrease follows up to the trailing edge corner. 249

As shown again in figure 6(b), the variance of the wall pressure fluctuations is very low in the very first part of the rectangular cylinder. However, a monotonic increase with the streamwise location is observed, leading to a maximum intensity for  $x \approx 2.96$ . Downstream this maximum, an almost equivalent decrease of the intensity of the wall pressure fluctuations is observed and maintained up to the trailing edge corner.

#### **4.** Single-point statistics

In figure 7, the behaviour of the turbulent intensities and of the fluctuating 257 pressure is reported in the (x, y)-plane. These plots show that the separation at 258 the leading-edge gives rise to a free-shear layer which is essentially laminar in its 259 first portion. The instabilities associated with the shear layer are then amplified 260 moving downstream and, for x > 1, a three dimensional turbulent pattern is 261 observed. Indeed, all the three components of turbulence are different from 262 zero. By following the streamline of the *primary vortex*, shown with dashed 263 line in figure 7, it is evident that the most amplified fluctuations in the leading 264 edge free shear layer are the streamwise ones, while the vertical and spanwise 265 fluctuations are still significant but remain weaker. 266

Let us analyze in detail the behaviour of each component of turbulent fluc-267 tuation. By considering first the intensity of the streamwise fluctuations, figure 268 7(a), a well-defined region of maximum turbulent intensity can be identified. 269 This region crosses the external paths of the average recirculating bubble for 270 1.5 < x < 3.5 with a maximum reached at (x, y) = (2.57, 0.44). The iso-levels of 271 streamwise fluctuations are stretched in the streamwise direction. Apparently, 272 no deflection of the isocontours towards the wall is observed, indicating that 273 detached streamwise fluctuations are mostly convected from the *primary vortex* 274 to the wake region without interacting with the wall. A second weaker region 275 of streamwise fluctuations is observed in the wake with a relative maximum lo-276 cated at (x, y) = (6.05, -0.05). The vertical location suggests that this relative 277 maximum is a result of the amplification, throughout the trailing edge shear 278 layer, of fluctuations produced in the attached forward boundary layer. 279

The vertical fluctuations show a substantially different behaviour with respect to the streamwise ones. As already mentioned, the typical intensity of vertical fluctuations is smaller than the streamwise ones. However, as shown in figure 7(b), significant differences are observed also from a topological point of view. The region of maximum intensity is located further downstream and closer to the wall with a maximum at (x, y) = (2.96, 0.31). This aspect could be



Figure 7: Isocontours of the turbulent intensities and of the fluctuating pressure in the (x, y)-plane:  $\sqrt{\langle u'^2 \rangle}$ ,  $\sqrt{\langle v'^2 \rangle}$ ,  $\sqrt{\langle w'^2 \rangle}$  and  $\sqrt{\langle p'^2 \rangle}$  in (a), (b), (c) and (d) respectively. The dashed lines report the location of the *primary vortex*, secondary vortex and wake vortex.

partially ascribed to the mean velocity paths associated with the downstream 286 part of the *primary vortex*. Indeed, downstream the peak of intensity of the 287 streamwise fluctuations, the mean velocity field bends towards the wall before 288 reattaching. Hence, the structures associated with these intense streamwise 289 fluctuations, being advected and stretched by the mean velocity field, undergo 290 a deflection towards the wall thus leading to a partial reorientation of stream-291 wise fluctuations in vertical ones. Also in this case, the shape of the isocontours 292 of vertical fluctuations is found to be essentially elongated in the streamwise 293 direction. Hence, as for the streamwise ones, the produced vertical fluctua-294 tions do not seem to interact with the wall, but rather they appear to be freely 295 advected downstream towards the free flow. The second region of activity of 296 vertical fluctuations occurs in the wake region. Contrarily to what happens for 297 streamwise fluctuations, this region is centered in the symmetry plane of the 298 wake at y = -0.5 for  $x \approx 6.15$ . This is a clear footprint of the periodic shedding 299 of large scale vortices in the separated wake. 300

The most interesting aspect of the spanwise fluctuations shown in figure 7(c)301 is that the region of high turbulent intensity, located at (x, y) = (2.96, 0.28) and 302 associated with the *primary vortex*, extends also to the wall region and forms a 303 thin layer of turbulent activity centered at (x, y) = (2.96, 0.06). A possible ex-304 planation for the high intensity of spanwise fluctuations in the near-wall region is 305 the following. Turbulent fluctuations produced through the *primary vortex* are, 306 on average, transported towards the wall due to the deflection of the streamline 307 paths of the mean velocity field. The consequent impingement gives rise to hor-308 izontal fluctuations, since the impermeability constraint of the wall leads to a 309 dumping of the vertical fluctuations in the near-wall region. Actually, as shown 310 here in quantitative terms, the impingement essentially gives rise to spanwise 311 fluctuations. Indeed, we observe a near-wall layer in the reattachment region 312 characterized by intense spanwise and weak streamwise fluctuations. From the 313 reattachment region, a reverse flow takes origin, transporting turbulent fluctu-314 ations towards the leading-edge shear layer. Since the higher levels of turbulent 315 intensities in the reattachment region are detected in the spanwise direction, 316



Figure 8: (a) Streamwise behaviour of the centerline velocity defect  $1 - U_0(\tilde{x})$  (solid line). The self-similar power law decay (4) is reported in dashed line where A = 0.66 and  $\tilde{x}_0 = 4$  are considered. (b) Vertical profiles of velocity defect  $[U_0(\tilde{x}) - U(\tilde{x}, \tilde{y})]/[U_0(\tilde{x}) - 1]$  evaluated at different streamwise locations and reported as function of  $\tilde{y}/\tilde{y}_{1/2}(\tilde{x})$  where  $\tilde{y}_{1/2}(\tilde{x})$  is the wake half-width defined such that  $U(\tilde{x}, \pm \tilde{y}_{1/2}) = (1 + U_0)/2$ .

these fluctuations result to be the most intense also in the reverse flow region, thus forming the observed near-wall layer of intense spanwise fluctuations.

Let us now analyse the behaviour of the fluctuating pressure field shown in 319 figure 7(d). The most intense region of fluctuations is again the leading-edge 320 shear layer and its consequent evolution along the *primary vortex*. In accordance 321 with the behaviour of the pressure coefficient shown in figure 6(b), the region of 322 reverse flow of the *primary vortex* is essentially unperturbed. Only for x > 2 the 323 high levels of pressure fluctuations above the primary vortex are felt in the near-324 wall region. Interestingly, the variance of the pressure fluctuations associated 325 with the vortex shedding in the separated wake forms a second region of activity, 326 which however appears to be much weaker with respect to the region associated 327 with the leading-edge shear layer. 328

To conclude this section, we consider the evolution of the wake also in the far field. For this analysis it is useful to consider a shifted reference frame  $(\tilde{x}, \tilde{y}) = (x + 5D, y - D/2)$ . In figure 8(a) the streamwise behaviour of the wake centerline velocity defect,  $1 - U_0(\tilde{x})$  with  $U_0(\tilde{x}) = U(\tilde{x}, 0)$ , is shown. As



Figure 9: Vertical profiles of turbulent intensities evaluated at different streamwise locations and scaled in similarity variables. (a) streamwise, (b) vertical and (c) spanwise turbulent intensities.

<sup>333</sup> apparent, the wake centerline velocity approaches the self-similar decay,

$$1 - U_0(\tilde{x}) \approx \frac{A}{\sqrt{\tilde{x} - \tilde{x}_0}} \tag{4}$$

for  $\tilde{x} > 10$ . The self-similarity of the mean streamwise velocity is confirmed in figure 8(b) where the vertical profiles of the velocity defect,

$$\frac{U_0(\tilde{x}) - U(\tilde{x}, \tilde{y})}{U_0(\tilde{x}) - 1} \tag{5}$$

evaluated at different streamwise locations for  $\tilde{x} > 10$  are reported as a function of the similarity variable  $\tilde{y}/\tilde{y}_{1/2}(\tilde{x})$  where  $\tilde{y}_{1/2}(\tilde{x})$  is the wake half-width defined such that,

$$U(\tilde{x}, \pm \tilde{y}_{1/2}) = (1 + U_0)/2 .$$
(6)

<sup>339</sup> It is worth noting that the above self-similar behaviour implies [22] that

$$\frac{1}{1-U_0}\frac{d\tilde{y}_{1/2}}{d\tilde{x}} \approx const\tag{7}$$

and, hence, that the wake spreads as a power law, i.e.  $\tilde{y}_{1/2} \sim \tilde{x}^{1/2}$ .

To complete the analysis of the wake in the far field, let us analyse the 341 behaviour of the turbulent intensities shown in figure 9. Similarity variables 342 are again utilized and a good degree of scaling is observed for the streamwise 343 and spanwise components of turbulent fluctuations. In contrast, the vertical 344 fluctuations do not exhibit self-similarity, at least at the considered streamwise 345 locations. This aspect denotes a slower asymptotic recovery of the equilibrium 346 conditions needed for self-similarity, i.e. the vertical fluctuations are found to 347 maintain memory of the shedding mechanisms of separating and reattaching 348 flow over a longer distance. 349

# <sup>350</sup> 5. The structure of turbulence and two-point statistics

In this section, the main unsteadiness of the flow and the inherent multiscale nature of the turbulent fluctuations are analysed by means of two-point statistics. Before that, we start by addressing the topological pattern taken by the main turbulent fluctuations populating the flow. To this aim we consider the eduction scheme proposed by Jeong *et al.* [23] and based on the second largest eigenvalue  $(\lambda_2)$  of the tensor

$$S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj} \tag{8}$$

357 where

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$
  

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$
(9)

are the symmetric and antisymmetric part of the velocity gradient tensor. This
eduction scheme is known to accurately extract the three-dimensional pattern
of vortical structures, see e.g. Cucitore *et al.* [24] and Dubief *et al.* [25].

In figure 10, the instantaneous pattern taken by  $\lambda_2 = -2$  is shown with 361 iso-surfaces colored by  $\tilde{y}$ . From the perspective and top views, it is evident that 362 in the very first part of the leading-edge shear layer, for x < 0.3, the flow is lam-363 inar, in accordance with the statistical analysis reported so far. Then, almost 364 two-dimensional spanwise rolls appear as a result of the well-known Kelvin-365 Helmholtz instability. Under the effect of the mean shear, perturbations of the 366 flow field lead to the lift up and stretching of these spanwise vortices, thus 367 forming hairpin-like structures [6, 26, 27]. Hence, the flow motion develops 368 streamwise vortices [19, 28, 29], the legs of the hairpin vortices, which in turn 369 induce high- and low-speed streaks in between them. The statistical evidence of 370 the presence of streamwise vortices and streaks is reported in the following anal-371 ysis of the two-point spanwise correlation function. These sparsely distributed 372 structures, grow, burst and decay thus giving rise to a fully turbulent flow for 373 x > 1. Most of this complex multiscale pattern, particularly the large scale 374 structures, is shed downstream in the free flow. On the other hand, a por-375 tion of the structures, in particular small scale fluctuations, are pushed towards 376 the wall. The consequent impingement gives rise to both forward and reverse 377 boundary layers characterized by small scale motions. By following the branch 378 moving downstream, we observe that turbulent fluctuations become more and 379



Figure 10: Instantaneous isosurfaces of  $\lambda_2 = -2$  colored with  $\tilde{y}$ . Perspective, top and lateral views in (a), (b) and (c) plots, respectively.

more aligned in the streamwise direction. Finally, by reaching the trailing edge,
these relatively small fluctuations are encompassed by an oscillatory large scale
motion reminiscent of the laminar von Kármán instability of the separated wake.

The statistical signature of the above mentioned turbulent motion can be 383 highlighted by means of two-point statistics, such as the correlation function in 384 physical space and the energy spectrum in wavenumber/frequency space. While 385 the spatial correlation function is a measure of how the velocity fluctuations 386 are coherent in space, the energy spectrum allows us to analyse how turbulent 387 kinetic energy is distributed across the different scales of motion. Let us start 388 with the spatial correlation function. Due to the symmetry of the flow, the 389 only statistical homogeneous direction where it is possible to define a space 390 of homogeneous scales not affected by the inhomogeneity of the flow, is the 391 spanwise direction (see Cimarelli et al. [30] for an example of the complexity 392 emerging from the study of the space of inhomogeneous scales). In this setting, 393 the spatial spanwise correlation function is rigorously defined and for a generic 394 quantity  $\beta$  can be written as, 395

$$R_{\beta\beta}(x,y,r_z) = \frac{\langle \beta'(x,y,z+r_z/2,t)\beta'(x,y,z-r_z/2,t)\rangle}{\langle \beta'\beta'\rangle(x,y)} .$$
(10)

In figure 11, the spanwise correlation function of the three components of 396 velocity and pressure fluctuations evaluated in the shedding region of the *pri*-397 mary vortex, is shown. This region of the flow is fully turbulent and relatively 398 small spanwise correlation lengths are observed. In particular, the fluctuating 399 velocity field shows a zero value of correlation at  $r_z \approx 0.3$  for the vertical and 400 streamwise fluctuations and at  $r_z \approx 0.5$  for the spanwise ones. Interestingly, 401 for larger values of the spanwise increment  $r_z$ , a degree of negative correlation 402 is observed. The peak of anticorrelation occurs at  $r_z \approx 0.45, 0.5$  and 0.9 for 403 the vertical, streamwise and spanwise fluctuations, respectively. These negative 404 correlation peaks are in accordance with the presence of streamwise vortices 405 and streamwise velocity streaks as a result of motions induced by the hairpin 406 vortices observed in figure 10. Indeed, the peak of negative correlation of ver-407 tical and spanwise fluctuations can be understood as a statistical evidence of 408



Figure 11: Spanwise correlation function of streamwise, vertical, spanwise and pressure fluctuations evaluated in the shedding region of the *primary vortex* at a (x, y)-position corresponding to probe 9. The specific location of the probe within the flow is shown in figure 4.

the presence of counter-rotating pairs of streamwise vortices. In particular, the 409 negative peak of  $R_{vv}$  at  $r_z \approx 0.45$  is a measure of the statistical diameter of 410 streamwise vortices while the negative peak of  $R_{ww}$  at  $r_z \approx 0.9$  is indicative of 411 the mean spacing between counter-rotating vortex pairs. On the other hand, 412 the negative peak of  $R_{uu}$  is an evidence of the presence of streamwise velocity 413 streaks. In particular, the negative peak of  $R_{uu}$  at  $r_z \approx 0.5$  is a measure of 414 the mean spanwise spacing between high and low streamwise velocity streaks. 415 See Kim et al. [31] for well-established and analogous considerations in wall-416 turbulent flows. The pressure fluctuations show a wider correlation length and 417 no negative correlation peak is observed. Finally, it is worth pointing out that 418 the behaviour shown here by the two-point spatial correlation function supports 419 the choice of the numerical domain width,  $\mathcal{D}_z = 5$ . Indeed, figure 11 shows that 420 the velocity and pressure fields are uncorrelated for spanwise lengths that are 421 significantly shorter than the domain width. 422

Let us consider now the scale-space distribution of kinetic energy by means of a spectral analysis. In particular, we consider the multiscale features of the flow both in time and space by means of one-dimensional spectra of turbulent kinetic energy. By taking advantage of the statistical homogeneity of the flow <sup>427</sup> in the spanwise direction and in time, the spectrum of turbulent kinetic energy <sup>428</sup>  $q = u_i u_i/2$  can be defined as

$$\Phi_{qq}(k_z, St, x, y) = \frac{1}{2} \langle \tilde{u}_i(k_z, St, x, y) \tilde{u}_i^*(k_z, St, x, y) \rangle$$
(11)

where  $k_z$  and St are the spanwise wavenumber and frequency, while  $(\tilde{\cdot})$  denotes the two-dimensional Fourier transform with respect to the spanwise direction and time. To simplify the analysis, in the following we consider separately the one-dimensional wavenumber spectrum and the one-dimensional frequency spectrum. The one-dimensional wavenumber spectrum is derived from the twodimensional one by integration with respect to St,

$$\Phi_{qq}^{k_z}(k_z, x, y) = \int \Phi_{qq}(k_z, St, x, y) dSt$$
(12)

and, analogously, the one-dimensional frequency spectrum is computed by integrating with respect to  $k_z$ ,

$$\Phi_{qq}^f(St, x, y) = \int \Phi_{qq}(k_z, St, x, y) dk_z$$
(13)

<sup>437</sup> By using the Taylor's hypothesis of frozen turbulence we also address the puta<sup>438</sup> tive wavenumber spectrum in the streamwise direction defined as

$$\Phi_{qq}^{k_x}(k_x, x, y) = \Phi_{qq}^f(St/U(x, y), x, y) .$$
(14)

The main unsteadinesses of the flow are analysed in figure 12 by means of 439 frequency spectra of turbulent kinetic energy evaluated in the leading-edge shear 440 layer and in the wake. As shown in figure 12(a), the leading-edge shear layer is 441 characterized by well-defined peaks of spectral energy at a relatively large range 442 of frequencies. In particular, moving from probe P1 to P7 along the shear layer, 443 the peaks move from  $St \approx 1.8$  to  $St \approx 0.9$ . This range of frequencies represents 444 the temporal scales of the fluctuations amplified by the transitional mechanisms 445 of the shear layer. By looking at the frequency spectrum in the wake 12(b), a 446 clear separation of scales is evinced between the vortex detachment in the sepa-447 rated wake and the amplified fluctuations through the leading-edge shear layer. 448 In fact, the frequency spectrum in the wake highlights a well-defined peak at 449



Figure 12: Frequency spectrum of turbulent kinetic energy,  $\phi_{qq}^f(St)$ , evaluated along the leading-edge shear layer at probes 1 to 7 (a) and in the wake at probe 10 (b). The arrow in (a) indicates probes from 1 to 7. The specific location of probes within the flow is shown in figure 4.

relatively larger temporal scales for  $St \approx 0.14$ . This peak is a clear statistical 450 evidence of the oscillatory large scale motion reminiscent of the laminar von 451 Kármán instability. Being at much larger temporal scales, the vortex shedding 452 mechanisms in the wake are found to be not directly connected with the for-453 mation processes of fluctuations in the transitional region of the leading-edge 454 shear layer. To appreciate this, one may compare the spectral peak at frequency 455  $St \approx 0.14$  shown in figure 12(b) with the amplified spectral peak frequencies 456 from  $St \approx 1.8$  to  $St \approx 0.9$  in figure 12(a). On the contrary, we find a clear 457 matching of scales between the slow vortex detachment in the separated wake 458 and the fluctuation of the lift coefficient, which appear evidently by comparing 459 the spectral peaks for  $St \approx 0.14$  in figures 3(b) and 12(b). Being driven by 460 the pressure differences between the top and bottom sides of the rectangular 461 cylinder, the temporal fluctuations of the lift coefficient are essentially given by 462 the instantaneous difference in the extent of the low pressure regions settled by 463 the separated recirculating flow in the opposite sides of the rectangle. Hence, 464 we argue that the spectral peak frequency  $St \approx 0.14$  shown by the lift coefficient 465 is representative of the time scale of the large scale phenomenon of alternative 466 enlargment and shrinking of the separation bubble in the top and bottom sides 467 of the rectangle [18, 19]. Accordingly, the clear matching of scales between the 468



Figure 13: Streamwise (a) and spanwise (b) spectrum of turbulent kinetic energy,  $\phi_{qq}^{k_x}(k_x)$ and  $\phi_{qq}^{k_z}(k_z)$ , respectively, evaluated at probe 8. Streamwise (c) and spanwise (d) dissipation spectrum of turbulent kinetic energy,  $k_x^2 \phi_{qq}^{k_x}(k_x)$  and  $k_z^2 \phi_{qq}^{k_z}(k_z)$ , respectively, evaluated again at probe 8.

vortex detachment in the wake and the fluctuation of the lift coefficient suggests
a locking of the vortex shedding phenomena in the wake with the alternative
shedding of large scale vortices from the top and bottom *primary vortices* and,
hence, with their enlargment and shrinking [2].

Let us consider now the spatial scales of the flow. In figure 13(a) and (b), 473 the streamwise and spanwise spectra of turbulent kinetic energy in the fully 474 developed part of the leading-edge shear layer are shown. The two plots re-475 veal a markedly anisotropic behaviour, consisting of turbulent fluctuations elon-476 gated in the streamwise direction. Indeed, the energy-containing fluctuations 477 are found to fill the streamwise spectrum up to  $k_x \approx 1$  while, in the spanwise 478 one, up to  $k_z \approx 10$ . In other words, the most energetic fluctuations are stretched 479 in the streamwise direction and their size is of the order of O(D) and  $O(10^{-1}D)$ 480 in the streamwise and spanwise direction, respectively. This anisotropy is re-481

tained up to the small dissipative scales as shown by the spectra of turbulent 482 dissipation shown in figure 13(c) and (d). Indeed, the maximum rate of dissi-483 pation is achieved for  $k_x \approx 1.4$  and  $k_z \approx 15$ , respectively in the streamwise and 484 spanwise direction. These aspects need to be taken carefully into account when 485 CFD techniques such as LES (Large Eddy Simulation) are applied for the simu-486 lation of the flow. Accordingly with Bruno et al. [32], the spanwise resolution is 487 a central object when setting up a CFD simulation being the site of the smallest 488 but energy containing scales of the flow as shown here in quantitative terms. 489

# 490 6. Conclusions

The flow around rectangular cylinders is recognized to be an extremely in-491 teresting case both for fundamental and applicative studies. Despite the simple 492 geometry, this category of flows contains basic phenomena characterizing the 493 behaviour of more complex flows typical of the applications. In this respect, it 494 is still difficult to achieve reliable results both from a numerical and experimen-495 tal point of view. The reason is the high sensitivity of the different phenomena 496 driving the flow on the experimental conditions from one side and on the tur-497 bulence modelling and mesh properties from the other. Here, we report for 498 the first time a Direct Numerical Simulation of such a flow for a moderately 499 high Reynolds number. The main goal is to shed light on the main physical 500 mechanisms driving the complex behaviour of the flow and to provide well con-501 verged statistics not affected by uncertainties on the boundary conditions and 502 by inaccuracies related to turbulence modelling and mesh resolution. 503

Global and single-point statistics are reported aiming at defining the exact behaviour of relevant statistical quantities. As an example, we characterize the behaviour of the pressure coefficient and of its variance which are known to be very important quantities for civil engineering applications aiming at predicting wind loads over buildings. In this respect, also the behaviour of the lift coefficient in the frequency space is reported, highlighting a well defined peak for  $St \approx 0.14$ . The mean and turbulent flow is then assessed. Three main

recirculating regions are found and their dimensions and turbulence levels are 511 characterized. The first one is a large scale bubble originating from the flow 512 separation at the leading edge corner and it is herein called *primary vortex*. 513 The related shear layer is the locus of the instability and transitional processes 514 giving rise to turbulence as shown here in quantitative terms by the increasing 515 levels of intensity of the fluctuations along its development. This primary vortex 516 is found to shed vortices downstream giving rise to a region of high turbulence 517 levels. The *primary vortex* is responsible for the observed second recirculating 518 region, here called *secondary vortex*. Indeed, it induces a reverse flow at the 519 wall which experiences an adverse pressure gradient and thus separates giving 520 rise to the second near-wall recirculating region. Finally, the third recirculat-521 ing region, here called *wake vortex*, takes place in the separated flow at the 522 trailing edge. As for the *primary vortex*, the trailing-edge shear layer is respon-523 sible for additional instability mechanisms, thus giving rise to a second region 524 of high turbulence intensity. Finally, the behaviour of the fully developed wake 525 is also analysed. A rigorous assessment of the wake flow features is reported 526 in a simple way by making use of self-similarity variables. The analysis reveals 527 a slower asymptotic recovery of the equilibrium conditions for self-similarity of 528 the vertical fluctuations with respect to the streamwise and spanwise ones. 529

To complete the study of the flow, two-point statistics are also computed, 530 namely the spanwise correlation function and the energy spectrum. The study 531 of the two-point spatial correlation function of the fluctuating velocity and pres-532 sure fields allows us first to prove that the width of the numerical domain is large 533 enough to reproduce the main flow features. On the other hand, we found a 534 clear statistical evidence of the presence of streamwise vortices and high- and 535 low- streamwise streaks within the flow. In particular, we found that stream-536 wise vortices statistically occur as pairs of counter-rotating vortices. The spac-537 ing between counter-rotating vortex pairs is 0.9 while the diameter of a single 538 streamwise vortex is 0.45. Regarding the velocity streaks we statistically prove 539 their presence and we measure that the spanwise spacing between postive and 540 negative streamwise velocity streaks is 0.9. We argue that both streamwise 541

vortices and streaks are a result of the flow motion induced by the presence of hairpin-like turbulent structures here detected by analysing the instantaneous pattern taken by  $\lambda_2$ .

The spectral analysis in the frequency space allows us to identify and quan-545 tify the main unsteadinesses of the flow. In particular, a separation of scales be-546 tween the amplified fluctuations in the leading-edge shear layer for  $St = O(10^0)$ 547 and the large scale vortex detachment at the wake for  $St = O(10^{-1})$  is observed. 548 Indeed, this low frequency is found to be actually locked with the shedding of 549 vortices from the *primary vortex* and, in particular, with the alternative enlarg-550 ment and shrinking of the recirculating regions in the top and bottom sides of 551 the rectangle. On the other hand, the analysis in the wavenumber space allows 552 us to study the anisotropy of the flow in the space of scales. It is found that 553 both the energy-containing and dissipative fluctuations are anisotropic, being 554 elongated in the streamwise direction and thin in the spanwise one. This infor-555 mation should be taken carefully into account when designing meshes in CFD 556 applications. 557

It is finally worth pointing out that the present results, besides reporting a detailed statistical analysis of the flow, are also intended to be a reference for CFD studies. Indeed, the statistical objects here reported would be useful to quantify and understand the effects of modelling and mesh resolution issues by means of a comparison with the same statistics obtained with CFD simulations at the same Reynolds number. Hence, they would be useful for developing a *best practice* for CFD.

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