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Cultural transposition of Chinese lesson study to Italy

An exploratory study on fractions in a fourth-grade classroom

Maria Giuseppina Bartolini Bussi, Chiara Bertolini and Alessandro Ramploud
Department of Education and Human Studies, Universita degli Studi di Modena e Reggio Emilia, Reggio Emilia, Italy, and
Xuhua Sun
University of Macau, Taipa, Macao

Abstract

Purpose – The purpose of this paper is to explore the early implementation of a model of mathematics teacher development in Italian schools, inspired by Chinese lesson study (CLS), focusing on similarities and differences.
Design/methodology/approach – A research lesson study on fractions in the fourth grade was conducted. The approach was designed based on the theory of semiotic mediation (TSM) enriched by means of quaternary analysis and the variation pedagogy of CLS. In this study, qualitative methods were employed involving the collection of data including lesson plans, observations and post-lesson analyses.
Findings – The purpose of this study is to determine what works and what does not work in the Italian context. Answers to the following research questions are provided: How did LS incorporate quaternary analysis and variation pedagogy in the TSM? How and why were changes introduced in the structure of the lesson plan with respect to the CLS? How did members of the Italian Mathematics Teaching Research Group increase their knowledge of teaching methods and content?
Research limitations/implications – The issues to be considered in further studies include the possible conflicts emerging between the cultures of teaching in China and Italy and the way to overcome them.
Practical implications – The main ideas of CLS are consistent with the general indications of the Italian Ministry of Education for the three-year program (2016-2019) of mandatory teacher development.
Originality/value – Reporting the findings of an Italian LS is aimed at exploring the differences and similarities among the different forms of LS, under the influence of cultural and institutional constraints.

Keywords Chinese lesson study, Cultural transposition, Fractions, Italian school system, Primary school mathematics, Semiotic mediation

Paper type Research paper

Introduction

There are various approaches to teacher development (OECD, 2013), including coaching, lesson study, mentoring, workshop, conference and reflective supervision. Lesson study is a teaching improvement model widely employed in many countries having their origin in Japanese elementary education (Isoda *et al.*, 2007).

Chinese lesson study (CLS) is less known as compared to Japanese lesson study (JLS); it has a similar structure of collective lesson preparation (lesson plan), observation and post-lesson analysis which is taught by a demonstrating teacher. Chen and Yang (2013) reconstructed the social-cultural history of CLS, which dates back to the beginning of the twentieth century. The Mathematics Teachers Research Group in schools was organized by means of an open-class approach (Sun *et al.*, 2015). An open class is taught by one teacher and open for observation to a group of teachers and administrators from inside a school or outside a school. Open classes are now a routine activity and have played a role in securing incremental, accumulative and sustainable improvements of mathematics teaching in China (Huang *et al.*, 2011).



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CLS has been developed, in large part, as a robust system of policies and practices; hence, it is not easy to find a comprehensive theoretical framework for it. Huang and Shimizu (2016) characterized CLS in comparison with JLS, by pointing out three major aspects:

> First, CLS is content-focused and strategies-oriented. This means that the essential goal of CLS is to develop an exemplary lesson to demonstrate the effective teaching of specific content or using specific teaching strategies. Second, rehearsal teaching is repeated multiple times until teachers feel satisfied with the goals they set out to achieve. Third, knowledgeable others could be involved throughout the entire process of lesson study (Huang and Shimizu, 2016, p. 395).

Sun *et al.* (2015) stated that there are several kinds of CLS and distinguished them between routine lesson studies for schools and lesson studies for outside audience.

According to Yang and Ricks (2012), Chinese mathematics educators have developed a quaternary analysis to be exploited in both the preparation and observation of a lesson. The authors described the quaternary analysis of a lesson by highlighting the following points:

(1) the lesson's key point (i.e. the central objective of the lesson for which it was designed);

(2) the lesson's difficult point (i.e. the cognitive difficulty that the students might encounter as they try to learn the mathematical key point);

(3) the lesson's critical point (i.e. the heart of the lesson which sharpens the teaching method); and

(4) the learning effect (i.e. the effectiveness of these three points by judging the students' understanding and mastery).

This quaternary analysis is consistent with the instructional coherence of the lesson (Mok, 2013; Wang *et al.*, 2015), which highlights the (implicit or explicit) interrelation of all mathematical components of the lesson. In particular, Mok (2013) has described five strategies for studying instructional coherence: the what-why-how in a set of lessons; the use of review; the use of summary; the what-why-how in classroom discourse (about a specific task); and the consolidation with variation. Variation pedagogy (Huang and Li, 2017; Sun, 2011) is enacted in CLS. According to Sun (2011), "variation practices are so widely shared and familiar that they became nearly invisible to members within the Chinese culture" (p. 67).

In Italy, no systematic form of LS has been implemented until now. Recently, for the first time in all schools, a mandatory three-year program (2016-2019) of teacher development has been issued (MIUR, 2015). The general guidelines highlight issues like peer-to-peer tutoring, practical workshops, team teaching and self-evaluation which evoke the practice of LS. Hence, it was considered appropriate to explore the implementation of LS as a model of teacher development. The Italian authors were familiar with both the CLS and the JLS. They chose to explore the CLS as they were interested in a content-focused and strategy-oriented model. Moreover, CLS provides a practical and economical research model in terms of organization, budgeting and scheduling (Sun *et al.*, 2015).

The general purpose of this exploratory study is: "the learning from, and about the process of importing cultural routines" (Stigler and Hiebert, 2016, p. 581). More specifically, the authors believe that the difference between the Chinese and the Italian culture of teaching requires a careful "cultural transposition where the different cultural backgrounds generate possibilities of meaning and of mathematics education perspectives, which in turn, organize the contexts and school mathematics practices in different ways" (Mellone and Ramploud, 2015, p. 571). The collaboration with a Chinese colleague (the fourth author) was essential to capture the subtleties of CLS and to look at the Italian situation from the perspective of an outsider.

In the following, first, the theoretical framework is introduced, i.e. the theory of semiotic mediation (TSM) (Bartolini Bussi and Mariotti, 2008), and some research questions are considered. Thereafter, a research LS implemented in Italy is reported. The need for coping with the crucial elements of both the CLS and the local culture and some open problems are addressed.

Theoretical framework

TSM

The TSM, after a Vygotskian approach, aims to describe and explain the process that starts when students use an artifact to solve a given task and then leads to the students' appropriation of a particular piece of mathematical knowledge. The TSM was introduced by Bartolini Bussi and Mariotti (2008), by drawing on several preliminary teaching experiments. By the semiotic potential of an artifact, they mean the double semiotic link that may be established between an artifact, the students' personal meanings (emerging from its use when they try to accomplish a task) and the mathematical meanings evoked by its use and recognized as mathematics by an expert (p. 754). The process of semiotic mediation consists of development from the initial situated signs (evoking the artifact) to mathematical signs (referring to mathematical knowledge). In this process, pivot signs hinting at both the artifact and the mathematical knowledge appear and are exploited by the teacher. This process is shown in Figure 1(a). On the left is the triangle of the semiotic potential of the artifact. The teacher plays two different roles in this scheme: the role of task design (on the left); and the guidance role (on the right) in the development from artifact signs to mathematical signs. This development is promoted through the iteration of didactical cycles (Figure 1(b)), where different categories of activities take place: individual or small group activities with the artifact to solve a given task; individual production of signs of different kinds (e.g. drawing, written or oral wording and gesturing); collective production of signs, where the individual productions are shared and the semiotic potential of the artifact is unfolded. The collective phases are called mathematical discussions which are polyphonies of voices articulated on a mathematical object, i.e. one of the motives of the activity of teaching and learning.

Research questions

With regards to the above introduction, the following research questions were considered:

RQ1. How did the LS incorporate quaternary analysis and variation pedagogy in the TSM?

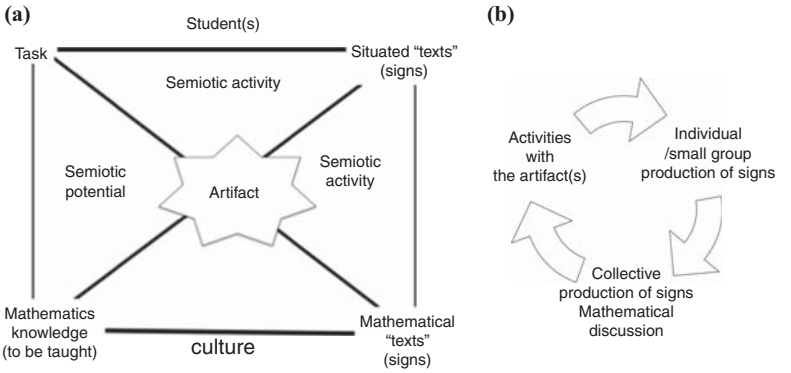


Figure 1.
XXX

Notes: (a) Semiotic mediation; (b) didactical cycle

RQ2. How and why were changes introduced in the structure of the lesson plan with respect to the CLS?

RQ3. How did members of the Italian Mathematics Teaching Research Group (IMTRG) increase their knowledge of teaching method and content?

An exploratory study in the Italian context

Background: the Italian school system

The school system includes pre-primary school for children between three and six years of age and the first cycle of education (eight years) divided into primary school (five years) and lower secondary school (three years). Since 1971, the education system has been totally inclusive by law "acknowledging the fact that all students, including those with disabilities, have the right to education, and that all schools have the responsibility to teach every child, and that it is the responsibility of the school to make the adjustments that may be necessary to make sure that all students can learn" (UNESCO, 2009, p. 51). Italian teachers are very much in favor of inclusion, which is considered a moral issue; thus, it is more important than a legal mandate (Ferri, 2008).

Mathematics teachers in primary schools are not specialists but generalists. In primary schools, the same teacher usually follows the same class from the first to the fifth grade. On a daily basis, every Italian teacher spends the school hours in the same classroom without any interruption between periods.

The need for an active involvement of students in mathematical explorations and manipulation (not limited to direct teaching) is highlighted in the Italian standards for mathematics (MIUR, 2012) and in the research tradition of mathematics education (Bartolini Bussi and Martignone, 2013). Drawing on national (Zan, 2016) and international (Verschaffel *et al.*, 2009) studies, Italian teachers are sensitive to the introduction of problems in a narrative way, by means of storytelling.

Design

In 2012, Italian authors started a large program of voluntary teacher development carried out cooperatively by the University of Modena and Reggio Emilia, some local networks of primary and junior secondary schools and municipal agencies for school support. Some IMTRGs were created in primary and junior secondary schools. Each IMTRG includes teachers, prospective teachers and general educators.

Each LS consists of 5-7 hours: two hours for a collective lesson plan; about one hour for a lesson; two hours for post-lesson analysis (and, if needed, the construction of an updated lesson plan). Two additional hours are planned in order to watch the lesson videos and to deepen the crucial incident analysis: this usually occurs in research lesson studies, like the one reported in this manuscript.

The participants in this LS are the following (full names are given in the Acknowledgments): Roberta (the Demonstrating Teacher) is a Teacher-Researcher, with many years of experience in teaching mathematics in primary school. Since she teaches two parallel fourth-grade classes in the same school, she is capable of teaching a lesson twice with different groups of students, hence entering into a double cycle of LS. The other members of the IMTRG are: Laura and Loretta (Teacher-Researchers, temporarily appointed by the Ministry of Education at the university as part-time internship Supervisors with LS experience); Silvia (Doctoral Student in Mathematics Education); Gabriele (Educator with LS experience); Maria G. (University Professor of Mathematics Education); Chiara (University Professor of General Didactics and Special Education); Alessandro (Teacher-Researcher, temporarily appointed by the Ministry of Education at the University as a full-time Internship Supervisor). The last four act as knowledgeable others, as a result of their knowledge of the content and teaching methods, as well as their experience with lesson study.

Methodology

A lesson was delivered by Roberta in the first class, and followed by a revision in the second class with a modified lesson plan. This exploratory study focused on the implementation of the revised lesson plan in the second class. The data of the current study were obtained from the following sources:

(1) the lesson plans created during the whole process;

(2) the videos of the design session;

(3) the videos of the observed lesson;

(4) the videos of the revision session; and

(5) students' individual protocols in post-lesson tasks.

The data were collected for two months during the Fall of 2016.

The qualitative analysis of sessions, transcribed from videos if necessary, and of lesson plans and students' protocols was exploited in order to detect elements of the quaternary analysis and variation pedagogy in the lesson plan and of teacher's strategies in classroom implementation, focusing on the orchestration of the mathematical discussion to shift from artifact signs to mathematical signs.

The structure of LS on the approach to fractions in the fourth grade

The revised lesson plan (Table I) adopted some features of the standard lesson plan in CLS (rows 3-5) but introduced other elements (rows 1, 2, 6, 7 and 8) in order to take into account the different teaching traditions. Table II presents the design of fine grain timing.

The contextualization of the lesson in the whole sequence (row 2 in Table I). The lesson plan took into account the Italian teaching tradition and challenges. In Italian classrooms, fractional units are usually introduced by means of round cakes or pizzas, cut in equal parts. This approach has been criticized in the literature (e.g. Fandiño Pinilla, 2007) as potentially misleading, when fractions greater than 1 are to be introduced, because of the ambiguity of the interpretation of symbols such as 5/4 and of the risk of putting in shade the property that unites all real numbers: "they possess magnitudes that can be ordered on a number line" (Siegler *et al.*, 2016, p. 13). Hence, in this LS, the demonstrating teacher chose to follow a supplemental teacher's guide on describing a long-term sequence (Robotti *et al.*, 2016), aiming at the representation of fractions on a number line. Drawing on this teacher's guide, the lesson plan was collectively prepared by the IMTRG. The whole sequence of more than 30 lessons illustrated in the teacher's guide was designed, tested and published with inclusive intention (Robotti, 2017), according to the Italian school system. The first step was the partitioning of an A4 paper sheet. In the LS reported here, only a part of this step was implemented, consisting of two lessons. In the first lesson (not examined in this paper), a task was introduced by means of a story (according to the tradition of storytelling) about a pizza maker and the tablecloths for his new pizza shop. The pizza maker wished to design beautiful colored tablecloths by dividing the whole tablecloth (represented by an A4 paper sheet) into many shapes with the same size. In the first lesson, the students produced prototypes by exploring the division of the tablecloth into two equal parts, using concrete materials. Most students (16, i.e. 89 percent) were able to build at least two different solutions to the problem. 25 percent were able to find four different solutions, by folding the sheet in different ways before cutting it. These shapes were later used in the task of the second lesson.

Artifact analysis: the semiotic potential of the artifact (row 6 in Table I). The mathematical meanings to be mediated are connected to the relationships between a whole (an A4 paper sheet)

Heading	Explanation
1. Class context analysis	4th grade. 27 school hours per week (6 for Mathematics). 18 students: 5 boys and 13 girls 3 second-generation students are from other cultures/languages still having some understanding and writing difficulties 4 students have attention deficit disorder (difficulty in sustaining attention for a satisfactory time) due to different reasons: difficulty in understanding, hyperactivity, immaturity. For the most complex case, after meetings at a neuropsychiatry center, a personalized teaching plan was prepared, with personal approaches and possible compensatory tools and dispensatory measures
2. Contextualization of the lesson in the sequence of lessons	See after Table I (see section heading "The contextualization of the lesson in the whole sequence" in text)
3. Lesson topic/content	Fractions: relationship between a half and the whole
4. Lesson goal	Consolidate the concept of division into equal parts in the case of shapes; compare the same fractional units, highlighting the fact that they are likely to be non-congruent, but equivalent
5. Lesson plan (fine grain)	See Table II
6. Artifact analysis: the semiotic potential of the artifact	See after Table I (see section heading "Artefact analysis: the semiotic potential of the artefact" in text)
7. Observation goals	According to the TSM, the observation was planned to focus on the strategies used by the small groups to solve the problem. In particular, the observation was centered on the relationships (if any) between gestures in the activity and the report of the activity; and then the pertinence of the students' wording and explanation during mathematical discussion. The observation was planned also to focus on the teacher's role in the mathematical discussion
8. Observation tools	Observations were planned through video cameras and observers' paper and pencil report

Table I.
The lesson plan: a global view

and fractional unit (e.g. halves), as both the starting point (a whole to be cut) and the point of arrival (a whole to be covered):

(1) construction of fractional units starting from a given unit of measurement (the A4 paper sheet) by folding and cutting out an A4 paper sheet;

(2) equivalence between fractional units, by folding or cutting the fractional units in order to show the equivalence between the areas; and

(3) sum of fractional units, in order to obtain the given unit of measure by covering the A4 paper sheet with different fractional units (Robotti, 2017).

In the second lesson (reported in this paper), the division into two equivalent parts was addressed with a specific task: small groups with similar cognitive levels (three students each) were given five A4 sheets of white paper, a tube of glue, a pair of scissors and a collection of pairs of shapes on red paper (see Figure 2, where the numbers are added for reference purpose only): four pairs (1-2-3-4) consisted of congruent shapes (right triangles, trapezoids and two pairs of rectangles), while a fifth pair consisted of two equivalent but non-congruent rectangles. The reconstruction of tablecloths may be done by simple juxtaposition for the first four pairs, while the fifth pair of non-congruent rectangles requires a cut and paste strategy. Students' learning happens on two different levels: the practical level (cut and paste) and the conceptual level (construct the meaning of mathematical signs: one half, a whole; with both verbal and symbolic expressions: 1/2 and 1).

Heading	Description of the teacher's actions and interventions as planned	Educational goal	Planned time (mins)	Actual time (mins)
Introduction	Teacher: do you remember the pizza maker tablecloths? You tried to divide them into two equal parts	To review the past activity	1	1
Activity on the task in small groups	The teacher is expected to split the students into 6 small groups of 3 (students at the same cognitive level) with different roles: reporter, writer and sizer, as decided by the students. Each group is expected to receive the artifact (Figure 2)	To organize small group work	5	2
Presentation of the situation	Teacher: the pizza maker has given us the tablecloths cut into two equal parts. They are in these folders. He told me that each part is half of the whole tablecloth	To use fractions in everyday situation by means of storytelling	1	1
Task of the day	Teacher: the pizza maker has asked for your help to reconstruct the whole tablecloths. Get them together and paste them on the sheet you have on the desk, to reconstruct the whole tablecloth	To compare the same fractional units and link them with the whole, starting from the congruent pairs	3	3
Start of the small group work		To assemble the first four pairs (congruent parts)	–	6
Presentation/ explanation of the sub-task	Students are expected to ask questions on the two non-congruent parts of the tablecloth (Figure 2, fifth pair)	To compare the fractional units	2	2
Work on a sub-task	Teacher: the pizza maker told us that they are all halves. Find ways to recompose the whole tablecloth. Eventually, you have to explain to other groups your solution and how you found it	To compare the shapes of the fifth pair that are not congruent but equivalent, to be checked by means of cutting, folding and so on	3	18
Activity on the task and sub-task, either individual or in small groups	Each group has to answer, in writing, the following questions, preparing a report: Describe the pieces of tablecloths given by the pizza maker What have you done to assemble the tablecloth with the pairs of equal pieces? What have you done to assemble the tablecloth with the pair of different pieces? What have you got in both cases? Try to explain why you can assemble the tablecloth with either equal pieces or different pieces	To organize the report focusing on what-why-how the activity was implemented	20	10

Table II.
The lesson plan: fine grain timing

(*continued*)

Heading	Description of the teacher's actions and interventions as planned	Educational goal	Planned time (mins)	Actual time (mins)
Presentation of work by students	The class presentation is planned as follows. Students (reporters) verbally explain to the other groups how they have formed the whole, helping themselves by reading answers to the questions. The teacher groups similar solutions and hangs them on the board (by means of clips). The teacher tries to identify in the student texts and discourses, some mathematical signs like: half, whole, big, equal, form, part, all, etc.	To foster argumentation, to increase students' awareness of what-why-how they implemented the activity (metacognition)	10	31
Mathematical discussion of the various solution (strategies)	The teacher plans to guide the discussion by saying: let us try to make a synthesis, writing on the blackboard of what we found. Describe how you formed the whole tablecloth. Why did you cut it in this way? How many cuts di you make?	As in row 9	10	3
Teacher's summary and highlighting of key point in the lesson	Planned summary (an outline). Today we discovered that two halves always form a whole tablecloth. All halves of the tablecloth are of the same size. If the shapes are the same, the activity is easier; if the shapes are different, the activity will be more difficult	To figure out the abstract halves beyond the concrete shapes. To shift the acquired knowledge and activities toward a formal setting or first formalization (according to national standards)	2	
Homework (assignment)	The children will receive one half of an A4 paper sheet (shape: triangle) and another half (shape: trapezoid). The task is to reconstruct the whole tablecloth by means of these two different halves	To push the students to practice, on the lesson focus, to achieve a self-assessment and to reinforce the acquired skills	3	–
Total time			60	77

Table II.

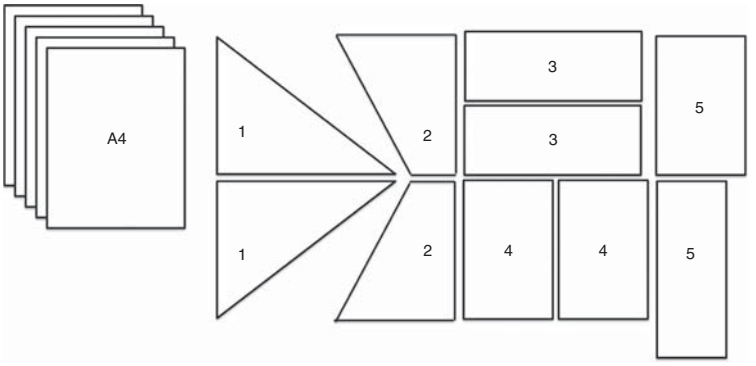


Figure 2.
The artifact

The lesson plan: fine grain timing (row 5 in Table I)

The fine grain lesson plan was revised after the first testing. The revised lesson plan is shown in Table II. The total time was planned to be 60 minutes (the standard class period in Italy). Each part of the fine grain lesson plan is detailed in Table II. In the last column, the actual time observed is shown.

Some observation data

The small group work. The six groups worked intensely for about half an hour. The teacher walking around in the classroom noticed that the first four pairs of congruent shapes (Figure 2) were easy to be glued to the A4 paper sheets, as expected. Students spent some time to adjust the shapes carefully before gluing them in order to have no white part visible on the red side and vice versa. In some cases, after gluing, they trimmed the narrow protruding strips, in order to achieve precision. The teacher suggested students draw in the report if the name of some shape had been forgotten. Only the fifth pair of rectangles (Figure 2) required a more complex strategy as the rectangles were not congruent. This case is analyzed in more details below.

The groups' report and the mathematical discussion. When all the groups had finished, the teacher started a mathematical discussion and called them to report about their activity. The different solutions (sol.) proposed by the six small groups for the fifth pair of rectangles (Figure 3) are summarized below, in the order of the classroom presentation:

- Sol.1: we pasted easily the first rectangle (1), then we placed the second one (2) and cut the protruding part.
- Sol.2: we pasted easily the first rectangle (1) like the other groups, but then we noticed that the other rectangle (2) did not fit. We tried superimposing the second one in different ways, but it did not work. Then, we noticed that cutting in two pieces, the long one was possible to exactly fill the white part.
- Sol.3: we worked like the first group, testing the solution with a white sheet of paper, before cutting the red one.
- Sol.4: we started with the "long one" (1) and then we cut the "fat" one (2) into halves.
- Sol.5: we worked like the first group, with much trouble about the cutting precision.
- Sol.6: we measured this one (2) and divided by 2: we got 10.5 and 10.5. Then, we cut. Then, we noticed that we could also cut the long one (1). We could have found four small pieces.

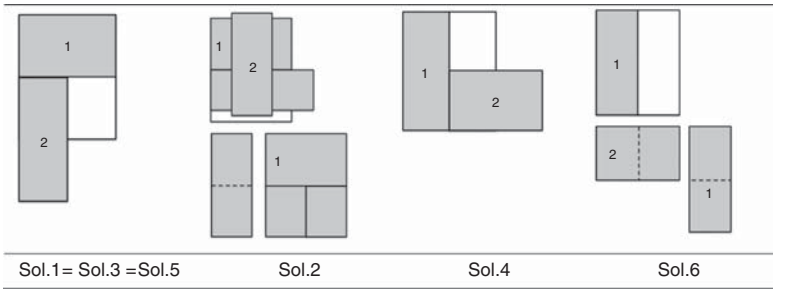


Figure 3.
The different solutions for the fifth pair

The Sol.6 strategy raised an interesting exchange between two students.

- Sa (low voice to a schoolfellow, gesturing at the four pieces): it is mathematical. You get a piece, a piece, a piece, a piece.
- Sb: you may cut both the "fat" rectangle and the "long" rectangle, so you have four parts.
- Sa: the four pieces are likely to be equal.

In every report, the reporter used many deictic gestures (i.e. pointing at "this piece" "that piece;" McNeill, 1992) and artifact signs such as "the whole tablecloth." The teacher always mirrored the expressions introducing, whenever possible, the mathematical sign "the whole." Yet, the students continued to talk about "the whole tablecloth" whilst they accepted the mathematical sign "half." No student seemed ready to use the mathematical sign "the whole."

The teacher started (1) thanking the students who highlighted the possible division into four equal pieces:

(1) Teacher: we may thank the students who gave us something to think over for the next lesson. What did we discover today? A lot of things. First, we know that in each case, the pizza maker has given us two halves. Two halves, even if they have not the same shape, what can we do?

(2) S1: we can divide them into equal parts.

(3) Teacher: and the halves, all the halves what did they form?

(4) Students (voices): a tablecloth.

(5) Teacher: a tablecloth. A whole. A whole tablecloth. Have we always formed the whole tablecloth?

(6) S2 (low achiever): many tablecloths.

(7) Teacher: yes many tablecloths. When the halves are equal, it is easy. And when the halves are different in shape, was it easy or difficult?

(8) Students (voices): difficult.

(9) Teacher (smiling): yes more difficult but finer, as usual.

(10) S3: looking at the different ones we thought, may be the pizza maker was wrong.

(11) Teacher (smiling): everybody thought he was wrong. But he was right!

(12) S4: I asked myself. Was he or I wrong?

(13) Teacher: all the halves eventually form the whole. Do you agree? Very good. You now deserve a break!

The teacher's intention was to realize the shift to and from the whole and the fraction unit (one half), introducing simultaneously the mathematical sign "whole." But she was cautious, paying attention to the low achievers' needs (the line 5 in the mathematical discussion), who were likely to understand and use the first and the third expression whilst the second (the whole) was out of reach. She refused direct teaching, according to the lesson plan. The teacher intentionally used "whole tablecloth" as a pivot sign, hinting at both the artifact and the mathematical knowledge world. Mathematical signs are not intended to be taught directly at the beginning, to suddenly substitute artifact signs; in fact, the latter may survive for a short or a long time, as, according to the TSM, formal mathematical definitions and the reasoning of the corresponding concepts require longer (and individual) processes to be achieved.

Learning effect: an individual task the day after. The day after, the teacher, as usual, asked the students to summarize the activity individually in their notebooks. All the students prepared a good written report: 13 students (72 percent) correctly used the mathematical sign "whole"; five students (28 percent), including some of the low achievers, still used an artifact sign (the "whole tablecloth" or the "whole sheet"). This is the text produced by a low achiever, who succeeded in using the mathematical sign (the whole) helping herself with drawings: We have done. We had two halves with different shapes. Then, we cut them to assemble the tablecloths. Also, with two different halves, it is possible to obtain the whole.

Interviews (excerpts) during the post-lesson analysis. Some members of the IMTRG were interviewed by the first author of this paper during the post-lesson analysis.

Roberta (the demonstrating ~~T~~eacher-~~R~~esearcher): I wished to design a lesson on fractions. I was not comfortable with the standard approach in Italy by means of cakes or pizzas. Hence, I studied the approach designed in the book suggested by the university researcher and the more expert colleague. But the need for designing so carefully, all the minutes of a single lesson with the other members of the IMTRG, fostered a very deep reflection of each step. Now, I feel much more competent in fraction teaching.

Laura (observing ~~T~~eacher-~~R~~esearcher): at the beginning of the experiments, we were a bit puzzled by the presence of observers in our classroom, but quickly we realized that their attitude was very constructive and that the focus was not on the demonstrating teacher but on the lesson design being collectively prepared. I am looking forward to being observed myself!

Loretta (observing ~~T~~eacher-~~R~~esearcher): the most difficult thing seems to be the time in the whole lesson!

Gabriele (observing ~~E~~ducator): at the beginning, the lesson design seemed very rigid. Actually, this very careful design allows more attention to the learners' processes.

Findings

The findings are reported here with reference to each research question, by drawing on Tables I and II and on the observation data. Tables I and II, in particular, highlight both the similarities and differences of the Italian lesson study and the CLS at the design level.

RQ1. How did the LS incorporate quaternary analysis and variation pedagogy in the TSM? The topic of the lesson (row 3 in Table I) is the key point: it is related to the system of mathematical meanings to be mediated (a-b-c in the artifact analysis) for the specific case of a whole divided into two halves.

The difficult points of the lesson are considered in Table II (rows 6 and 7) and concern the interpretation of equal parts of the whole, not only as congruent fractional units but also as equivalent fractional units. The fifth pair of shapes in the artifact (Figure 2) proactively molds the activity, in order to shift from the former interpretation to the latter.

The critical point (row 8 in Table II) reports the revision, introduced after the first testing of the LS, in order to further structure the small group work:

- by giving each student a specific role (row 2 in Table II), according to the standards suggested in cooperative learning (Johnson and Johnson, 1994); and
- by adding an explicit list of questions (rows 8 and 9 in Table II), to guide both the problem solving and the report.

The what-how-why technique is introduced in the planning (row 9 in Table II).

Variation pedagogy is exploited in the design of the task of the day (row 4 in Table II), where a combination of variation problems (Sun, 2011) is introduced in order to cope with

the difficult point of the lesson: the first four pairs indicate one problem multiple changes, while the fifth pair introduces a new variation for rectangles (non-congruent rectangles with the same area).

The learning effect was observed during the post-lesson tasks, while paying attention to low achievers. The example reported showed the level of mathematical signs attained by a ~~lower~~ achiever.

RQ2. How and why were changes introduced in the structure of the lesson plan with respect to the CLS?

In this section, the main differences (and/or additions) between the CLS and the Italian implementation are highlighted with reference to Tables I and II.

The class context analysis (row 1 in Table I) is introduced because of the inclusiveness of the Italian system that requires planning the lessons for students with different cognitive levels (including students with special needs), by taking into account both the constraints and resources to be considered.

The contextualization of the lesson in the sequence of lessons (row 2 in Table I) focuses on long-term processes highlighted in the Italian school system.

The semiotic potential of the artifact (row 6 in Table I) defines the main observational goals as participants' semiotic productions (row 7 in Table I); this analysis guides the orchestration of the discussion by the teacher. In the mathematical discussion (row 10 in Table II), different voices are articulated to unfold the semiotic potential of an artifact.

Storytelling (rows 1, 3 and 4 in Table II) and cooperative learning (row 8 in Table II) are introduced as consistent with the values shared by the members of the IMTRG.

RQ3. How did members of the Italian MTRG increase their knowledge of teaching methods and content?

Evidence comes mainly from interviews within the post-lesson analysis, when both the videos of the lesson and the individual reports of the day after were focused. The demonstrating ~~T~~teacher (Roberta) expressed her knowledge about the content (fractional units) and teaching method, highlighting the importance of the careful design (all the minutes of a single lesson) in this process. The observing teacher-researcher Loretta focused on the issue of time. The mismatch between the allotted time (60 min) and the observed time (77 min) in the lesson was debated in the post-lesson analysis and linked by the members of the IMTRG to their own experiences in other LS. Time management was considered an issue to be better focused on a revised lesson plan. The initial resistance to be observed during a lesson was reported by Laura (observing ~~T~~teacher-~~R~~researcher) together with the reasons for change: hence, her learning attitude was increased. The observing ~~E~~ducator (Gabriele) highlighted the importance of a detailed design, in order to focus better on the learners' processes.

In general, all the IMTRG members observed that the individual reports prepared the day after showed that most students had moved from the artifact sign ("the whole tablecloth") to the mathematical sign (the whole), hence were analyzed as having learnt and being ready to understand and use a symbolic relation like:

$$\frac{1}{2}+\frac{1}{2}=1.$$

Actually, the demonstrating teacher Roberta started to introduce mathematical signs in the following weeks in a more systematic way.

Discussion and conclusion

In the Italian context, this observation study showed the effects of teacher's learning on both the teaching method and content, as reported in the original CLS (Huang *et al.*, 2014). The detailed analysis of the content needed to prepare the lesson plan forced awareness of the crucial elements of the quaternary analysis when coordinated with the elements of the TSM.

But there are also differences between the original CLS and the Italian implementation. They concern the changes and additions in the structure of the lesson plan. The structure of the lesson plan shows differences that are likely to be unavoidable in the transposition from the Chinese to the Italian culture of teaching (Mellone and Ramploud, 2015; Stigler and Hiebert, 2016).

Other cultural aspects deserve some comment.

The first one is the initial resistance to be observed in one's own classroom, consistent with reports about the USA teachers. Ferreras *et al.* (2010) highlighted the difficulties of the USA teachers to accept activities which are perceived as contrasting teachers' individualistic and heroic images, where autonomy and privacy are the most important values, in contrast with the Confucian heritage culture. In Italy, there is an additional cultural constraint, which does not show in other countries. Teaching autonomy and freedom is stated in the Constitution of the Italian Republic, issued in 1947 at the end of the Second World War. The Constitution reads: "Art. 33. The Republic guarantees the freedom of the arts and sciences, which may be freely taught." (Senato della Repubblica, 1947, p. 33) The constituent assembly reacted against the 20 years of Fascist dictatorship, when all teachers were forced to have the Fascist Party card and were strongly controlled with regards to their political ideas. Later, this right was mistaken for the demand of teaching without any control from the outside and without any observers in the classroom. This historical bias explains why Italian teachers are not usually willing to take part in academic projects, if they suspect that the aim is to assess teaching (with possible wage cuts) rather than to improve teaching. This cultural bias was overcome by members of the IMTRG, whose early negative attitude quickly changed, such that they are now the most enthusiastic supporters of the LS. This fact may be related to school reform with compulsory teacher development (MIUR, 2015), where some strategies consistent with the general idea of the LS, such as peer-to-peer observation, are mentioned.

The second aspect worthy of comment is the lesson length. The actual lesson lasted for 77 minutes (as shown in the last column in Table II) instead of the allotted 60 minutes. This lengthening was possible because the teacher had additional time in the same classroom, due to the flexible school schedule. Group work was longer than planned, in spite of the introduction of the more detailed structure for the small group work. The report was faster due to the structured list of questions. Every student was involved, including low achievers. Yet, the presentation was longer than planned. The average length of each presentation (six groups) was 5 minutes, with reports of different solutions. Each reporter was encouraged to carefully explain his/her group's strategies. The fifth pair of different rectangles (Figure 2) was reported as difficult as it required both cutting and paste operations, which hint at the mathematical concept of polygon equidecomposibility. In most cases, the reporter used not only words, but also gestures miming the actual actions to explain the solution. The teacher always mirrored (Rogers, 1965) the relevant expressions (as planned, see row 7 in Table I) and paraphrased gestures with appropriate words. After each report, the other two members of the same small group were asked to add something. Then, all the students were invited to comment on it. Even in the case of similar solutions, different wordings or gesturings were used, showing a rich process of problem solving. This format, where individual interventions are solicited and mirrored by the teacher, who refuses direct teaching, is consistent with the requirements of the mathematical discussion orchestrated by the teacher (Bartolini Bussi and Mariotti, 2008) where the focus

is on the construction of meaning for all students, including low achievers. In order to reduce the lesson length, a possible alternative choice (as suggested by Fernandez and Yoshida, 2004) is to design a new lesson plan, by splitting the lesson into two separate ones: the small group work; and the class presentation. In the study reported in this paper, such a separation was discussed but considered as ineffective by the IMTRG. If both activities are realized in the same lesson, the students, in a natural way, may exploit memory as well as the written notes they have prepared. Future lesson plans will allot a duration of about 75 minutes from the beginning, in order to meet this constraint. It is likely that this duration depends on the culture of teaching. The length of lessons was analyzed in the TIMSS (1999) showing a large variation of length of lesson across seven countries, from 26 to 119 minutes (p. 36).

This study suggests that to effectively promote CLS in a different education system, the subtleties of cultural transposition should be taken into consideration. For instance, cooperative learning is known and appreciated by Italian teachers but is not so common in the Confucian heritage culture (Phuong-Mai *et al.*, 2005). In the process of constructing a shared theoretical framework for CLS, it is likely that cooperative learning raises a cultural conflict. Another open problem concerns the way of coordinating the focus on short-term processes emerging in just one lesson with the focus on long-term processes typical of the Italian culture of teaching where schools have the responsibility of the annual plan, unlike in China, where such plans are determined by more central authorities.

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References

Bartolini Bussi, M.G. and Mariotti, M.A. (2008), "Semiotic mediation in the mathematics classroom: artifacts and signs after a Vygotskian perspective", in English, L., Bartolini Bussi, M.G., Jones, G., Lesh, R., Sriraman, B. and Tirosh, D. (Eds), *Handbook of International Research in Mathematics Education*, 2nd ed., Routledge and Taylor and Francis Group, New York, NY, pp. 746-783.

Q1

Bartolini Bussi, M.G. and Martignone, F. (2013), "Cultural issues in the communication of research on mathematics education", *For the Learning of Mathematics*, Vol. 33, pp. 2-8.

Chen, X. and Yang, F. (2013), "Chinese teachers' reconstruction of the curriculum reform through lesson study", *International Journal for Lesson and Learning Studies*, Vol. 2 No. 3, pp. 218-236.

Fandiño Pinilla, M.I. (2007), "Fractions: conceptual and didactic aspects", *Acta Didactica Universitatis Comenianae*, ~~Vol. 7, pp. 23-45.~~

Fernandez, C. and Yoshida, M. (2004), *Lesson Study. A Japanese Approach to Improving Mathematics Teaching and Learning*, Lawrence Erlbaum Publishers, Mahwah, NJ.

Ferreras, A., Olson, S. and Szlein, A.E. (Eds), (2010), *The Teacher Development Continuum in the United States and China. Summary of a Workshop*, The National Academic Press, Washington, DC.

Ferri, B. (2008), "Inclusion in Italy: what happens when everyone belongs", in Gabel, S.L. and Danforth, S. (Eds), *Disability and the Politics of Education: An International Reader*, Peter Lang Publishers, New York, NY, pp. 41-52.

Huang, R. and Li, Y. (2017), *Teaching and Learning Mathematics Through Variation. Confucian Heritage Meets Western Theories*, Sense Publishers, Rotterdam.

Huang, R. and Shimizu, Y. (2016), "Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: an international perspective", *ZDM – The International Journal on Mathematics Education*, Vol. 48, pp. 393-409.

Huang, R., Li, Y., Zhang, J. and Li, X. (2011), "Developing teachers' expertise in teaching through exemplary lesson development and collaboration", *ZDM – The International Journal on Mathematics Education*, Vol. 43, pp. 805-817.

Huang, R., Su, H. and Xu, S. (2014), "Developing teachers' and teaching researchers' professional competence in mathematics through Chinese, lesson study", *ZDM – The International Journal on Mathematics Education*, Vol. 46, pp. 239-251.

Isoda, M., Stephens, M., Ohara, Y. and Miyakawa, T. (2007), *Japanese Lesson Study in Mathematics: Its Impact, Diversity and Potential for Educational Improvement*, World Scientific.

Johnson, D.W. and Johnson, R.T. (1994), "Learning together", in Sharan, S. (Ed.), *Handbook of Cooperative Learning Methods*, Greenwood Press, Westport, CT, pp. 51-65.

McNeill, D. (1992), *Hand and Mind: What Gestures Reveal about Thought*, University of Chicago Press, Chicago, IL.

Mellone, M. and Ramploud, A. (2015), "Additive structure: an educational experience of cultural transposition", in Sun, X., Kaur, B. and Novotná, J. (Eds), *Proceedings of the 23rd ICMI Study Primary Mathematics Study on Whole Numbers*, University of Macau, pp. 567-574.

MIUR (2012), "Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione (National Standards for pre-school, primary and junior secondary school)", available at: www.indicazioninazionali.it/documenti_Indicazioni_nazionali/indicazioni_nazionali_infanzia_primo_ciclo.pdf (accessed December 10, 2016).

MIUR (2015), "Labuonascuola (The good school)" (in Italian), available at: https://labuonascuola.gov.it (accessed December 10, 2016).

Mok, I.A.C. (2013), "Five strategies for coherence: lessons from a Shanghai teacher", in Li, Y. and Huang, R. (Eds), *How Chinese Teach Mathematics and Improve Teaching*, Routledge, New York, NY, pp. 120-133.

OECD (2013), "Teachers' professional development: Europe in international comparison. An analysis of teachers' professional development based on the OECD's Teaching and Learning International Survey (TALIS)", available at: www.oecd.org/edu/school/talis.htm (accessed March 20, 2017).

Phuong-Mai, N., Terlouw, C. and Pilot, A. (2005), "Cooperative learning vs Confucian heritage culture's collectivism: confrontation to reveal some cultural conflicts and mismatch", *Asia Europe Journal*, Vol. 3, pp. 403-419.

Robotti, E. (2017), "How the representations take on a key role in an inclusive educational sequence concerning fractions", available at: https://keynote.conference-services.net/resources/444/5118/pdf/CERME10_0616.pdf (accessed May 6, 2017).

Robotti, E., Censi, A., Segor, I. and Peraillon, L. (2016), *Frazioni sul filo. Strumenti e strategie per la scuola primaria*, Centro Studi Erickson, Trento.

Rogers, C. (1965), *Client-Centered Therapy: Its Current Practice, Implications and Theory*, Houghton Mifflin, Boston, MA.

Senato della Repubblica (1947), "Constitution of the Italian Republic", available at: www.senato.it/documenti/repository/istituzione/costituzione_inglese.pdf (accessed December 10, 2016).

Siegler, R.S., Fazio, L.K., Bailey, D.H. and Zhou, X. (2016), "Fractions: the new frontier for theories of numerical development", *Trends in Cognitive Sciences*, Vol. 17, pp. 13-19.

Stigler, J.W. and Hiebert, J. (2016), "Lesson study, improvement and the importing of cultural routines", *ZDM – The International Journal on Mathematics Education*, Vol. 48, pp. 581-587.

Sun, X. (2011), "Variation problems and their roles in the topic of fraction division in Chinese mathematics textbook examples", *Educational Studies in Mathematics*, Vol. 76, pp. 65-85.

Sun, X., Teo, T. and Chan, T.C. (2015), "Application of the open-class approach to pre-service teacher training in Macao: a qualitative assessment", *Research Papers in Education*, Vol. 30, pp. 567-584.

TIMSS (1999), *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study*, Diane Publishing, Darby, PA.

UNESCO (2009), *Towards Inclusive Education for Students with Disabilities: A Guideline*, UNESCO Asia and Pacific Regional Bureau for Education, available at: www.uis.unesco.org/Library/Documents/disabchild09-en.pdf (accessed April 25, 2017).

Verschaffel, L., Greer, B., van Dooren, W. and Mukhopadhyay, S. (Eds) (2009), *Words and Worlds: Modelling Verbal Descriptions of Situations*, Sense Publishers, Rotterdam.

Wang, T., Cai, J. and Hwang, S. (2015), "Achieving coherence in the mathematics classroom: toward a framework for examining instructional coherence", in Wong, N.Y., Cai, J. and Li, S. (Eds), *How Chinese Teach Mathematics: Perspective from Insiders*, World Scientific Publishing, pp. 111-148.

Yang, Y. and Ricks, T.E. (2012), "How crucial incidents analysis support Chinese lesson study", *International Journal for Lesson and Learning Studies*, Vol. 1, pp. 41-48.

Zan, R. (2016), *I problemi di matematica. Difficoltà di comprensione e formulazione del testo*, Carocci Editore, Roma.

Corresponding author
Maria Giuseppina Bartolini Bussi can be contacted at: mariagiuseppina.bartolini@unimore.it

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