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Fuzzy logic and the capability approach

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Abstract

The definition of well being in Sen's capability approach (Sen, 1985, 1993) implies the evaluation of unobservables in a context of complexity and interaction amongst the different capabilities. The issue of measurement of well being in the capability approach is interested by problems related to the difficulties in observing directly the capabilities (a set of opportunities that the individual can convert into observables functionings) behind the achieved functionings and in the very definition of the different dimensions of well being not closed by Sen in a given list. Different techniques have been proposed in the literature to measure well being in the capability approach (see Kuklys, 2005, Robeyns, 2006, Chiappero-Martinetti, 2008, Comim, 2008). Here we aim at showing how, in the field of fuzzy logic, fuzzy expert system can be used to measure well being in the capability approach by focusing on the methods and by referring to its implementation in different areas of the evaluation of well being. The use of fuzzy expert system to measure well being has been proposed in Addabbo, Di Tommaso and Facchinetti (2004) and applied for the evaluation of children well being (Addabbo, Facchinetti and Mastroleo, 2007), of the capability of living an healthy life (Addabbo, Chiarolanza, Fuscaldo, and Pirotti, 2010) while the measurement of the quality of work by using fuzzy expert system has been pursued in Addabbo, Facchinetti, Mastroleo and Solinas (2006). In Section 1 we discuss the mathematical framework of fuzzy logic and the transition from classical logic to fuzzy logic. In Section 2 we present the phases of implementation of fuzzy expert system and in Section 3 we discuss cases of its implementation in the measurement of different areas of well being. Section 4 concludes.

Key words: fuzzy logic, capabilities, well being.
JEL codes: C6, I31

Introduction[1]

The definition of well being in Sen's capability approach (Sen, 1985, 1993) implies the evaluation of unobservables in a context of complexity and interaction amongst the different capabilities.

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Different techniques have been proposed in the literature to measure well being in the capability approach Kuklys (2005), Robeyns (2006), Chiappero-Martinetti (2008), Comim (2008).

Structural equation modelling and factor analysis have been experimented (Di Tommaso, 2007; Addabbo and Di Tommaso, 2009; Addabbo, Di Tommaso, Maccagnan, 2013, Kuklys, 2005; Krishnakumar, 2005, 2008). Fuzzy set theory has been already used to measure functionings (Chiappero Martinetti 1996, 2000, Lelli 2001). Here we aim at showing how, in the field of fuzzy logic, fuzzy expert systems can be used to measure well being in the capability approach by focusing on the methods and by referring to its implementation in different areas of the evaluation of well being. The use of fuzzy expert system to measure well being has been proposed in Addabbo, Di Tommaso and Facchinetti (2004) and applied for the evaluation of children well being (Addabbo, Facchinetti and Mastroleo, 2007) and of the capability of living an healthy life (Addabbo, Chiarolanza, Fuscaldo, and Pirotti, 2010). The measurement of the quality of work has been pursued in Addabbo, Facchinetti, Mastroleo and Solinas (2006) and in Facchinetti, Solinas & Pirotti (2013).

Conventional mathematics enables processing of precise information. However, in the reality, we very often meet with imprecise information such as: sufficient well-being, good level of quality of life, etc. People have used imprecise information for thousands of years. However, until quite recently it has not been used at all in methods based on conventional mathematics. Therefore it has been lost. Because of this, the efficiency of many control, modeling, forecasting and decision- making methods was considerably limited all the more, as in some systems imprecise information is the only accessible one. In Section 1 we discuss the mathematical framework of

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fuzzy logic and the transition from classical logic to fuzzy logic. In Section 2 we present the phases of implementation of fuzzy expert system and in Section 3 we discuss cases of its implementation in the measurement of different areas of well being. Section 4 concludes.

Section 1. From Classical Logic to Fuzzy Logic

As is well known, classical logic is based on the assumptions that there are exactly two truth-values, *false* and *true*, and that the truth-value of any logical formula is uniquely defined by the truth-values of its components. These assumptions are usually called *bivalence* and *truth functionality*, respectively. The various many-valued logics, which have been of interest and under investigation since the beginning of the twentieth century (Rescher 1969; Gottwald 2000), abandon bivalence while adhering to truth functionality. This means that additional truth-values are recognized in each many-valued logic. Even though it is not obvious how to interpret these additional truth-values, they are usually viewed as intermediary truth-values between *false* and *true* and interpreted as degrees of truth. Many-valued logics differ from one another in the sets of truth-values they employ and in the definitions they use for basic logical operations, that is, negation, conjunction, disjunction, implication, and equivalence.

Classical logic is closely connected with classical set theory. Each predicate is uniquely associated with a classical set. In other words, for any given object, a proposition formed by the predicate is true for this object if and only if the object is a member of the associated set. The associated set plays the role of the extension of the predicate.

When the assumption of bivalence was abandoned in the various proposed many-valued logics, the connection between predicates and sets was lost. Classical sets were simply not able to play the role of extensions of many-valued predicates, that is, predicates that apply to objects to intermediary degrees. The connection was eventually renewed when Lotfi Zadeh introduced the concept of a fuzzy set in his seminal paper (Zadeh 1965).

Zadeh returned to the connection between fuzzy sets and many-valued logics ten years later after his seminal paper, and began to use the term *fuzzy logic* in the following sense (Zadeh 1975, 409) "A fuzzy Logic FL, may be viewed, in part, as a fuzzy extension of a multivalued logic which constitutes a base for FL". However, he also attempted to expand the notion of fuzzy logics in this sense (usually referred to as fuzzy logics in the narrow sense) with the aim of developing approximate reasoning that would ultimately be able to emulate commonsense human reasoning in natural language. To this end, he introduced appropriate fuzzy sets for representing certain types of linguistic terms employed in human reasoning. For example, fuzzy truth-values are fuzzy sets defined on the set of recognized truth-values (usually the inter- val [0,1]) that represent linguistic terms such as *true*, *false*, *very true*, *more or less true*, *very false*. Fuzzy quantifiers are fuzzy sets defined on appropriate sets of number that represent linguistic terms such as , *many*, *most*, *almost most*, *very few* and so forth. Then , he begins his seminal paper as follows:

"More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1. Clearly, the "class of all real numbers that are much greater than 1," or "the class of beautiful women," or "the class of tall men" do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking. . . . The purpose of this note is to explore in a preliminary way some of the basic properties and implications of a concept which may be of use in dealing with "classes" of the type cited above. The concept in question is that of a *fuzzy set*, that is a "class" with a continuum of grades of membership. "(Zadeh 1965, 338)

To represent and deal with classes of objects that are not precisely defined was thus the principal motivation for introducing fuzzy sets. Since such classes are pervasive in all human activities involving natural language, fuzzy sets opened new and potentially useful ways of looking at human cognition, reasoning, communication, decision making, and the like. Perhaps the most important of these was a new way of looking at knowledge expressed by statements in natural language. Such knowledge assumed a new significance owing to the possibility of representing it and dealing with it in a mathematically rigorous way. Its utility in science, engineering, and other areas of human affairs has been increasingly recognized, especially since the early 1990s,

Shortly after Zadeh introduced fuzzy sets, Joseph Goguen, a mathematician and computer scientist, published an important paper entitled "The Logic of Inexact Concepts," where he writes:

The "hard" sciences, such as physics and chemistry, construct exact mathematical models of empirical phenomena, and then use these models to make predictions. Certain aspects of reality always escape such models, and we look hopefully to future refinements. But sometimes there is an elusive fuzziness, a readjustment to context, or an effect of observer upon observed. These phenomena are particularly indigenous to natural language, and are common in the "soft" sciences, such as biology and psychology. . . . "Exact concepts" are the sort envisaged in pure mathematics, while "inexact concepts" are rampant in everyday life. .

. . Ordinary logic is much used in mathematics, but applications to everyday life have been criticized because our normal language habits seem so different. Various modifications of orthodox logic have been suggested as remedies. . . . Without a semantic representation for inexact concepts it is hard to see that one modification of traditional logic really provides a more satisfactory syntactic theory of inexact concepts than another. How-ever, such a representation is now available (Zadeh 1965). (Goguen 1968–69, 325)

The idea of fuzzy logic was first advanced by Dr. Lotfi Zadeh of the University of California at Berkeley in the 1960s. Dr. Zadeh was working on the problem of computer understanding of natural language. Natural language (like most other activities in life and indeed the universe) is not easily translated into the absolute terms of 0 and 1. Whether everything is ultimately describable in binary terms is a philosophical question worth pursuing, but in practice much data we might want to feed a computer is in some state in between and so, frequently, are the results of computing. **Fuzzy logic** is an infinite-values logic. In contrast with traditional logic, called binary or two values logic in which only two realities are present true or false, Fuzzy Logic can have infinite values, with a truth value that ranges in degree between 0 and 1. Fuzzy logic has been extended to handle the concept of partial truth, where the truth value may range between completely true and completely false. While variables in mathematics usually take numerical values, in fuzzy logic applications, the non-numeric *linguistic variables* are often used to describe the expression of rules and facts and may be managed by specific functions. It is born to work with imprecise information, imprecise concepts like I am old, the weather is good, or warm and so on.

Fuzzy logic seems closer to the way our brains work. We aggregate data and form a number of partial truths which we aggregate further into higher truths which in turn, when certain thresholds are exceeded, cause certain further results such as motor reaction. A similar kind of process is used in artificial computer neural network and expert systems..

Fuzzy logic and probabilistic logic are mathematically similar – both have truth values ranging between 0 and 1, but they are conceptually distinct, due to different interpretations. Fuzzy logic corresponds to "degrees of truth", while probabilistic logic corresponds to "probability, likelihood"; as these differ, fuzzy logic and probabilistic logic yield different models of the same real-world situations. It is essential to realize that fuzzy logic uses truth degrees as a mathematical model of the vagueness phenomenon while probability is a mathematical model of ignorance.

The mathematical structure of fuzzy sets, that derives from the introduction of fuzzy logic, is complex as the presence of a logic with more than two values produces the collapse of the two Aristotelian logic postulates: the low of non-contradiction and the law of excluded middle.

Starting from this point, every concept present in mathematical literature is to be reinvented. The numbers, the operations, the functions and so on are concepts completely different from what is present in classical literature.

Fuzzy sets are defined on any given universal set as functions that are analogous to characteristic functions of classical sets. Each of these functions assigns to each object in the universal set a truth degree. If the truth degrees are real numbers in the unit interval [0,1], the defined fuzzy set is called a *standard fuzzy set*. Formally, a standard fuzzy set A in universe U is a function

$$A : U \rightarrow [0,1]$$

The number A(x) is called degree or grade of membership of x in A that is the truth degree of proposition "x is a member of A"

Difference between average and fuzzy inference (that is difference between linear and non linear model)

In a fuzzy system the aggregation operators that translate the impute variables into one intermediate or final variable is built by an inference operation h using f If-Then rules.

The properties we require to them are: order preserving in every arguments, idempotency that is $h(a,a,a...,a) = a$, continuity, and usually symmetricity. It is known that

$$\min(a_1,a_2,...a_n) \leq h(a_1,a_2,...a_n) \leq max(a_1,a_2,...a_n)$$

This result show as an aggregation operation fill the gap between min and max operation that are the fuzzy translation of intersection and union operation.

One class of aggregation operators is $h_\lambda : [0,1]^n \rightarrow R$ so defined

$$\forall \lambda \neq 0 \ \ h_\lambda(a_1,a_2,...a_n) = \left(\frac{a_1^\lambda + a_2^\lambda + ... + a_n^\lambda}{n} \right)^{\frac{1}{\lambda}}$$

For $\lambda = 1, h_\lambda$ clearly is the arithmetic average. For $\lambda \rightarrow 0, h_\lambda$ converges to geometric average, For $\lambda \rightarrow -\infty$ and $\lambda \rightarrow +\infty, h_\lambda$ converges to the minimum and maximum respectively.

This fact show that the aggregation we propose are usually not linear.

Section 2. Fuzzy expert system versus fuzzy set theory

A Fuzzy Expert System utilizes fuzzy sets and fuzzy logic to overcome some of the problems that occur when the data provided by the user are vague or incomplete. The power of fuzzy set theory comes from the ability to describe linguistically a particular phenomenon or process, and then to represent that description with a small number of very flexible rules. In a Fuzzy Expert System, the knowledge is contained both in its rules and in fuzzy sets, which hold general description of the properties of the phenomenon under consideration. One of the major differences between a Fuzzy Expert System and another Expert System is that the first can infer multiple conclusions. In fact it provides all possible solutions whose truth is above a certain threshold, and the user or the application program can then choose the appropriate solution depending on the particular situation. This fact adds flexibility to the system and makes it more powerful. Fuzzy Expert Systems use fuzzy data, fuzzy rules, and fuzzy inference, in addition to the standard ones implemented in the ordinary Expert Systems.

Functionally a fuzzy system can be described as a function approximator. More specifically it aims at performing an approximate implementation of an unknown mapping $\mathrm{f} : \mathrm{A} \subseteq \mathrm{R}^{\mathrm{n}} \to R^{\mathrm{m}}$ where A is a compact of R^n. By means of variable knowledge relevant to the unknown mapping [Kosko, 1992] and [Wang, 1992] independently proved that fuzzy systems are dense in the space of continuous functions on a compact domain and therefore can approximate arbitrarily well any continuous function on a compact domain. The following are the main phases of a Fuzzy Expert System design:

1. Identification of the problem and choice of the type of Fuzzy Expert System, which best suits the problem requirement. A modular system can be designed. It consists of several fuzzy modules linked together. A modular approach may greatly simplify the design of the whole system, dramatically reducing its complexity and making it more comprehensible.
2. Definition of input and output variables, their linguistic attributes (fuzzy values) and their membership function (fuzzification of input and output).
3. Definition of the set of heuristic fuzzy rules. (IF-THEN rules).
4. Choice of the fuzzy inference method (selection of aggregation operators for precondition and conclusion).
5. Translation of the fuzzy output in a crisp value (defuzzification methods).
6. Test of the fuzzy system prototype, drawing of the goal function between input and output fuzzy variables, change of membership functions and fuzzy rules if necessary, tuning of the fuzzy system, validation of results.

In building Fuzzy Expert System, the crucial steps are the fuzzification and the construction of blocks of fuzzy rules. These steps can be handled in two different ways. The first is accomplished by using information obtained through interviews to the experts of the problem. The second is accomplished by using methods of machine-learning, neural networks and genetic algorithms to learn membership functions and fuzzy rules. The two approaches are quite different. The first does not use the past history of the problem, but it relies on the experience of experts who have worked in the field for years. The second is based on past data and project into the future the same structure of the past.

We can formalize the steps in the following manner. For each linguistic variable, input x_i $(i=1\dots m)$ and output y, we have to fix its own range of variability U_i and V. $\forall\ i$, $(i=1\dots m)$, if n_i is the number of the linguistic attribute of the variable x_i and $\hat{n} = \max_{i\in[1,m]} n_i$, we define the sets

$$A^i = \{A_1^i, A_2^i, .., A_{j_i}^i, .., A_{n_i}^i\},\ B = \{B_1, B_2, ..., B_k, ..., B_r\}$$

where $\forall\ j_i \in [1,\ n_i]$, $\forall\ n_i\ \in [1, \hat{n}]$ $A_{j_i}^i$ are the fuzzy numbers describing the linguistic attributes of the input variable x_i, and $\forall k \in [1,r]$, B_k are the fuzzy numbers describing the linguistic attributes of the output variable y.

At every elements of A^i and B a membership function is associated such that

$$\mu_{A_{j_i}^i}(x) : U_i \to [0,1] \qquad \text{and} \qquad \mu_{B_k} : V \to [0,1]$$

The elements of A^i and B overlap in some "grey" zone, which cannot be characterised precisely. Many phenomena in the world do not fall clearly into one crisp category or another. Experts, that use abstraction as a way of simplifying the problem, can contribute to identify these "grey" zones.

The choice of the slopes of the elements of A^i and B is a mathematical translation of what the experts think about the single terms.

The second step is the block-rules construction.

We define the set of L fuzzy rules, where

$$L \le \prod_1^m n_i,\ \forall\ j_i \in [1,\ n_i],\ \forall\ n_i\ \in [1, \hat{n}]\ \forall k \in [1,r]$$

IF (x_1 is $A_{j_1}^1$) $\otimes$ (x_2 is $A_{j_2}^2$) $\otimes \dots \otimes$ (x_m is $A_{j_m}^m$) (2.1-1)

THEN (y is B_k), (2.1-2)

The relation (2.1-1) is called "precondition" and the symbol $\otimes$ represents one of the possible aggregation operators. In practical applications, the MIN and MAX operators, or a convex combination of them, are widely used and so a "negative" or "positive" compensation will occur between them.

Instead of Min and Max, it is also possible to use other t-norms and s-norms, which represent different ways of linking the "and" with the "or".

The relation (2.1-2) is called conclusion. The aggregation of precondition and conclusion can be made in several ways. The most used are the MAX and the BSUM methods. The choice depends on the type of application. The MAX has the meaning of keeping as "winner" the strongest rule, in the sense that if a rule is "firing" (activated) more than one time, the result is the maximum level of firing. In the BSUM case, all the firing degree is considered and the final result is the sum of the different level of activation (not over one). In any case, the two methods produce a fuzzy set, which has membership function $\mu_{agg}(y)$.

Now we have a result of the fuzzy inference system, which is a fuzzy replay. We need to return to a "crisp" value, and this step is called "defuzzification". This operation produces a "crisp" action $\overline{y}$ that adequately represents the membership function $\mu_{agg}(y)$. There is no unique way to perform this operation. To select the proper method, it is necessary to understand the linguistic meaning that underlies the defuzzification process. Two of these different linguistic meanings are of practical importance: the "*best compromise*" and the "*most plausible result*". A method of the first type is the Centre of Area (CoA) that produces the abscissa of the centre of gravity of the fuzzy output set

$$\overline{y} = \frac{\int_V y\mu_{agg}(y)dy}{\int_V \mu_{agg}(y)dy}$$

A method of the second type is the "Mean of Maximum" (MoM). Rather then balancing out the different inference results, this method selects the typical value of the terms that is most valid [ ].

Several authors (Chiappero, 2000, Lelli, 2001, Cheli-Lemmi, 1995) have faced several economic problems using methodological tools based on fuzzy set theory. Their approach is really different from fuzzy expert system. The unique concept the two approaches have in common is the necessity to use fuzzy logic and not Boolean logic. The more relevant differences are two. One is due to the starting point.

These authors start from data they have to pass from a crisp definition to a fuzzy one of the several concepts they study (functioning, capability, personal and social characteristics, etc.). Starting with distribution functions they built the membership functions they need. Next they propose different ways to aggregate these results to reach the final evaluation.

Here we propose a completely different method. The basic idea is that this problem is configurable as a multicriteria-problem. The last ones are faceable with the techniques of Knowledge-Based Systems that is starting from knowledge that is: experts. The experts describe which are the initial variables (Var.), how they may be aggregated to have intermediate variables (Int.) and then how to aggregate the last ones to reach the final evaluation (output). At the several levels, they use only linguistic attributes and linguistic rules to aggregate the starting points. They use imprecise information and the process is done without the knowledge of data. It is a procedure completely torn off data. The reasoning followed is peculiar of the problem and has not to be affected by disposable data. They propose clear linguistic attributes and rules. They may be criticized, but we cannot say that the choices they make are not transparent. When the system is ready, a sensitive analysis can be done to control if there are some incongruities. The data are used only at the final step. The data file is put in the system and the output is ready in a second. The choice of experts depends on the type of problem we face.

Another important difference is from the mathematical point of view. The unknown function which connects variables with output is, usually, not linear. Even in case of high non-linearity, these methods are able to approximate it very well. The mathematical method other authors propose are average, weighted average and so on. These methods produce linear functions, but, unfortunately, the real world is too much complex to be linear.

Section 3. How fuzzy expert system can be used to measure well being

When faced with the problem of measuring well being in the capability approach we meet:

- complexity and need of interdisciplinarity in the definition of the different dimensions of well being and of the factors affecting them in a transparent way.

- vagueness. The very list of capabilities in Sen's thought is not closed and the functionings connected to each capability are not identified.

- interdependence. Each capability or the set of functionings can be interconnected and interact in a complex way that can be hardly approximated by linear functions.

- data. Primary source of data to get to capability and their construction are rarely available to measure well being. Secondary source of data are therefore used for this purpose and observables indicators can be an incomplete picture even of the set of achieved functionings.

- policy suggestions and evaluation. The stratification in trees allows to go back to measure the single components of the final crisp value of well being to measure the intermediate variables that lead to the observed result. This can be used for policy suggestions.

Complexity & interdisciplinarity in the definition of well being

The definition of well being in its many dimensions can produce a complex and not defined problem where different views from different disciplines would cast light to achieve higher understanding.

A typical example of this complexity coupled with the need of taking into account its interdisciplinarity can be considered the implementation of a fuzzy expert system to measure the extent of the balance between paid working environment an child care needs by assessing the degree of compliance of firms. This analysis has been performed in Addabbo, Facchinetti & Mastroleo (2009) within a wider project of evaluation of firms' compliance to gender equity (Cardinali, 2009). The research group was faced with the need of detecting the indicators showing the extent of balance between working time/organization and employees' life with special regards to the presence of children. Different disciplines could provide insights on this issue from labour organization (working time schedule, diversity management), labour law (norms on working time, parental leaves, firing and hirings), labour relations (including how different level of bargaining could be used to achieve better work-life balance), labour economics (discrimination, compensating differentials, interaction of firm's welfare policies with the local institutions and welfare system). The first discussion in the research group showed the difficulties of taking into account the different disciplines and the different theoretical expectations within each discipline to advance in the knowledge of the firm's environment in its effect on a crucial dimension of well being of employees and their families. On this regard the ability of fuzzy expert system to translate in rules the different 'judgements' and to provide a full representation of the implications of different linguistic rules and selection of relevant variables were essential elements guiding the discussion amongst different experts. Each expert could make explicit the point of view in the selection of the indicators to be used, in the link with the intermediate variables and the final outcomes discussing in an interdisciplinary setting the rules behind the outcomes. Fuzzy logic can use the non-numeric *linguistic variables* to describe the expression of rules and facts in a fuzzy expert system.

The fuzzy expert design was then tested to provide results on the different dimensions leading to the final outcomes and each expert could see the implication of her contribution to the design of the whole system and how sensitive the final outcome was to different rules. Moreover the rules can be made explicit to the public leading to a better understanding of their implications on the observed final value.

Vagueness and interdependence

The very definition of well being in Sen's approach is vague and incomplete as well as the definition of indicators that can be considered as each capability achievements. As discussed in Section 1 Fuzzy Logic has been extended to handle the concept of partial truth, vagueness and imprecise concepts. Therefore it can be fit to measure well being in the capability approach.

The final output of a fuzzy expert system can be considered as a value of the unobserved capability whose value can be related to different indicators. Moreover interdependence can occur in the very construction or the capabilities since each indicator can be related to more than one capability/dimension.

For instance in the analysis of the production of well being at work (Addabbo, Facchinetti, Mastroleo & Solinas, 2007) the final output is the outcome of different dimensions (control, economic, ergonomic, complexity, social and work life balance) where the level of training at work is an observable indicator that enters the fuzzy expert system affecting both the economic and the complexity dimensions with different rules on its contribution to different elements of well being at work.

Data.

Fuzzy expert systems do not require data to be designed as discussed in the previous section. A Fuzzy Expert System can indeed be designed without taking into account the available data. Its construction can be done with the interaction of experts, their choice of variables and the verbal expression of rules will be translated into the system. This is in line with Robeyns (2003) formulation of a two-levels list of capabilities and on the need of including all important elements in a first step. Moreover its design that is independent on the available data allows the results not to be affected by the specific data/context.
This advantage is evident when comparing the outcome in terms of health by using fuzzy expert system (Addabbo, Facchinetti & Pirotti, 2011) if compared to SF-12. The latter is constructed on weights that have been computed by means of regression analyses on the American population the former is the outcome of a fuzzy expert system built by using health experts' knowledge in a specific context and provides more variability in the obtained results.

The system built in the first step - not requiring any data - can then be reduced to an implementable fuzzy expert system taking into account the available data but being aware of the limits within the available resources in terms of lack of indicators.

The stratification of the tree can allow to disentangle different steps of analysis of the construction of well being being aware of the existing interaction with other dimensions to highlight the limits of the current implementation of the system when forced with the need of measuring having only limited information.

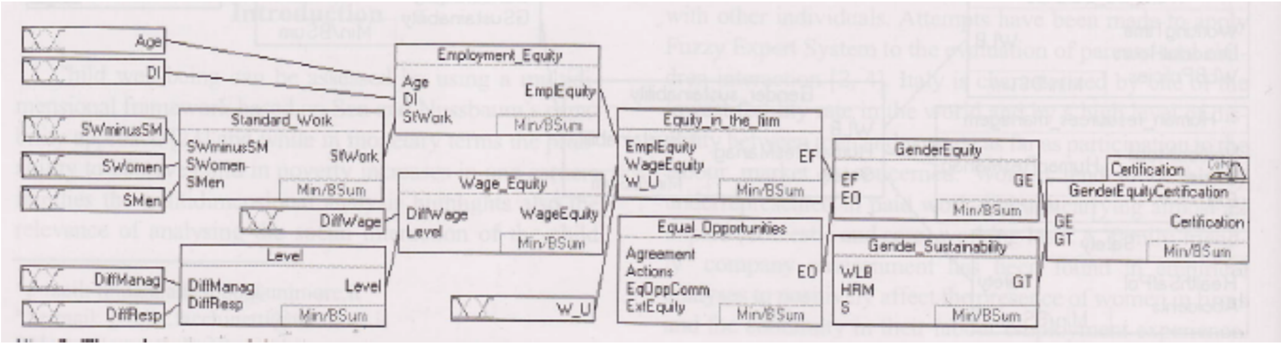
Its implementation does not require a large set of data it can be applied even with less than 50 observations.

Policy suggestions and evaluation.

The stratification in a tree allows to go back the branches to measure the contribution of intermediate variables to the final crisp value that summarizes the development of a given capability. For policy purposes this allows to detect the different degree of development of intermediate variables contributing to the final assessement of a given dimension of well being. This allows in the attempt to get a better development of well being to focus on those items whose contribution to the final outcome is lower and to proceed in the backward anlysis to other branches to understand the causes of the observed synthetic crisp value without losing the complexity of its design.

When faced with the problem of measuring gender equity in firms (Cardinali, 2009) the available system (Figure 1) could allow each firm to detect the dimensions leading to the broader assessment of the firm in terms of gender equity and suggest the firm, wishing to achieve a higher degree of gender equity, to focus on those dimensions whose contribution to the whole index of gender equity was critically lower and design strategies to improve their position on this regards. Monitoring of the effect of a given policy can then be tested by estimating the same model on the new available data to analyse the path towards more gender equity.

Figure 1 - Fuzzy expert system of gender equity



Source: Addabbo, Facchinetti & Mastroleo (2009) .

Section 4. Conclusions

Measuring well being in the capability approach has to do with vagueness, complexity with different capabilities interacting and with a limited number of indicators that can be reflected in an imprecise evaluation of well being.

Rather than looking for a reduction in complexity or a linearization of the existent interactions amongst the different dimensions, fuzzy logic applied to the measurement of well being in the capability approach allows to keep complexity and to trace the results obtained from the final output through intermediate variables back to the very elementary indicators of well being.

This paper provides a presentation of fuzzy logic and examples of its application from the literature that can allow to perceive the potentiality of fuzzy expert system applied to well being evaluation. A reconstruction of the procedure is presented to show how this type of analysis can be used also for policy purposes and to trace the origin of a given bad score and design a strategy to achieve better outocomes.

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