DEMB Working Paper Series

N. 92

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Francesco Giuseppe Caloia*, Andrea Cipollini**, Silvia Muzzioli***

August 2016

*Cà Foscari University of Venice, Department of Economics
Address: 873 Cannaregio, Fondamenta San Giobbe, 30121 Venezia
email: francesco.caloia@unive.it

** University of Palermo, Department of Economics, Accounting and Statistics
Address: V.le delle Scienze, 90128 Palermo
CEFIN, RECent at University of Modena and Reggio Emilia
email: andrea.cipollini@unipa.it

*** University of Modena and Reggio Emilia
CEFIN (Centro Studi Banca e Finanza)
Address: Viale Berengario 51, 41121 Modena, Italy
email: silvia.muzzioli@unimore.it

ISSN: 2281-440X online

Dipartimento di Economia Marco Biagi
Università degli studi di Modena e Reggio Emilia
Via Berengario 51 | 41121 Modena
tel. 059 2056711 | fax. 059 2056937
info.economia@unimore.it | www.economia.unimore.it

A note on normalization schemes: The case of generalized forecast error variance decompositions

Francesco Giuseppe Caloia *
Department of Economics
Cà Foscari University of Venice

Andrea Cipollini **
Department of Economics, Accounting and Statistics
University of Palermo

Silvia Muzzioli ***
Department of Economics
University of Modena and Reggio Emilia

Abstract:

The aim of this paper is to propose new normalization schemes for the values obtained from the generalized forecast error variance decomposition, in order to obtain more reliable net spillover measures. We provide a review of various matrix normalization schemes used in different application domains. The intention is to contribute to the financial econometrics literature aimed at building a bridge between different approaches able to detect spillover effects, such as spatial regressions and network analyses. Considering DGPs characterized by different degrees of correlation and persistence, we show that the popular row normalization scheme proposed by Diebold and Yilmaz (2012), as well as the alternative column normalization scheme, may lead to inaccurate measures of net contributions (NET spillovers) in terms of risk transmission. Results are based on simulations and show that the number of errors increases as the correlation between the variable increases. The normalization schemes we suggest overcome these limits.

Keywords: normalization schemes, forecast error variance decomposition, spillover, networks, spatial econometrics, VAR.

JEL classification: C15; C53; C58; G17.

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* Department of Economics, Cà Foscari University of Venice, 873 Cannaregio, Fondamenta San Giobbe, 30121 Venezia. E-mail: francesco.caloia@unive.it.
** Department of Economics, Accounting and Statistics, University of Palermo, V.le delle Scienze, 90128 Palermo. E-mail: andrea.cipollini@unipa.it, *CEFIN, RECent* Modena;
*** Department of Economics, University of Modena and Reggio Emilia; *CEFIN*; Viale J. Berengario, 51, 41121 Modena. E-mail: silvia.muzzioli@unimore.it

1 Introduction

A normalization scheme is a set of one or more constraints to be imposed on a matrix such that the resulting scaled version will satisfy certain conditions. Equilibration, i.e. scaling a matrix such that its rows or columns sum to one is one of the most common normalization schemes. A normalization scheme is adopted either for estimation purposes or simply for interpretative purposes. The aim of this paper is to provide a review of the most common normalization schemes used in different financial applications, with a particular focus on forecast error variance decomposition. In fact, the implementation of the generalized forecast error variance decomposition yields a variance decomposition table that has to be normalized for interpretative purposes. In this case, we suggest alternative normalization schemes that are aimed at overcoming the limits of the traditional row-normalization scheme, used in Diebold and Yilmaz (2012). The advantages and disadvantages of the normalization schemes are assessed through simulation, using data characterized by different degrees of correlation and persistence. The results of the paper are intended to be useful not only for deriving spillovers measures, but also in any other field where a matrix normalization scheme is adopted, such as network analysis or spatial econometrics. The structure of the paper is as follows. In Section 2 we provide an overview of various econometric fields where a normalization scheme is somehow necessary, highlighting the parallels and differences between the different application domains. In Section 3 we review the most common normalization schemes used in the various fields. Section 4 concentrates on the spillover analysis based on the forecast error variance decomposition and proposes a new normalization scheme for the case of the generalized approach. Section 5 highlights how persistence and correlation among the series affect the results of the spillover analysis. The final section concludes.

2 Proximity, networks and variance decomposition: a bridge based on normalization

In this section we provide an overview of various fields in which a normalization scheme is needed: spatial econometrics, networks and forecast error variance decomposition in order to build a bridge between them, and in the next section we review the different schemes used in the various fields.
Spatial econometrics is a strand of econometrics used when the underlying data-generating process displays a spatial dependence, i.e. when the observations depend on the values of the neighbouring observations. In particular the distance between variables or regions is represented with the so-called contiguity matrices. Here is an example of a contiguity matrix describing a first-order neighbouring relation:

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (1)$$

This matrix is constructed by placing a value of one if two regions are neighbours, zero otherwise. In the example above, the first and the third regions are neighbours of order one of the second region (represented in the second row), and as a result a value of one is placed on the entries c_{21} and c_{23}. The third region (represented in the third row) is neighbour of the second and the fourth regions, so the entries c_{32} and c_{34} take a value of one. On the contrary the first and the last regions (rows) have only one neighbour so a value of one is placed in the entries c_{12} and c_{43}. While the contiguity matrix describes the geographical distances across all the regions, in the model equation what usually appears is the normalized version of the contiguity matrix, named spatial weight matrix. The most common version of spatial weight matrix $W = (w_{i,j})$ for $i, j = 1, \dots, k$ is the one that makes the proximity matrix row-stochastic[1]:

$$W = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (2)$$

For example, given a standard generalised spatial autoregressive model of order p, or simply SAR(p) model:

$$u = \sum_{h=1}^{p} \phi_h W_h u + \varepsilon \quad (3)$$

where ϕ_h are autoregressive parameters. Equivalently, we can rewrite equation (3) as follows:

$$u = \left(I_N - \sum_{h=1}^{p} \phi_h W_h \right)^{-1} \varepsilon \quad (4)$$

The row-normalization of the proximity matrix is the easiest way to make the matrix $\left(I_N - \sum_{h=1}^{p} \phi_h W_h\right)$ non-singular for all the possible values of the parameters ϕ_h, therefore in this case the normalization scheme is necessary for estimation purposes. First-order spatial weight matrices, i.e. the matrices describing first-order neighbouring relations, are symmetric since if A is a neighbour of B, then the reverse is always true, and they usually have zeros on the main diagonal. On the contrary second-order contiguity matrices, i.e. the ones describing second-order neighbouring relations, have one on the main diagonal because every region is a second-order neighbour of itself.

According to Billio et al. (2016) contiguity matrices are not flexible enough to deal with financial linkages because they are unable to describe the asymmetry and the strength of the relations between the variables, but they can be better represented with networks. Contiguity matrices have a number of similarities with the so-called adjacency matrices, i.e. the companion representation of networks.

In networks, the adjacency matrix is a $k \times k$ symmetric matrix such that, for $i = 1, \dots, k$:

[1] A row stochastic matrix is a non-negative square matrix having row sums normalized (i.e. they equal one). Note that the term stochastic here has nothing to do with the usual statistic meaning.

$$a_{ij} = \begin{cases} 1 & if\ there\ is\ an\ edge\ between\ i\ and\ j \\ 0 & otherwise \end{cases} \quad (5)$$

where the edge is a line connecting two nodes, for example friendship between individuals, or credit exposures between banks. Networks are usually represented in graphs, where nodes and edges are graphically displayed. In this formulation, adjacency matrices are similar to proximity (contiguity) matrices. However, it is possible to define a more complicated structure that is better able to proxy the real phenomenon of interest: a weighted network is a network that allows for weights on the edges in order to represent stronger or weaker connections between nodes, while direct networks are networks that allow for asymmetries, i.e. $a_{ij} \neq a_{ji}$. One example of a weighted and direct network is the forecast error variance decomposition (FEVD) (Diebold and Yilmaz, 2014). Forecast error variance decomposition is a standard econometric tool used in multivariate time series analysis to assess the contribution in terms of forecast error variance of each variable due to a shock to any of the other variables. If the shocks are orthogonalized, then the formula of the forecast error variance decomposition (FEVD) is as follows:

$$\theta_{ij}^{o} = \frac{\sum_{l=0}^{h-1} \left(e_i' A_l \mathrm{P}\, e_j\right)^2}{\sum_{l=0}^{h-1} e_i' A_l \Sigma A_l\, e_i} \quad i, j = 1, \dots, N \quad (6)$$

Where o stands for orthogonalized, A_l are coefficient matrices of the moving average representation of a stationary VAR (that captures the impulse responses over any forecast horizons h), Σ is the contemporaneous variance-covariance matrix of the vector of shocks and P is the lower triangular matrix obtained from the Cholesky decomposition of the covariance matrix. θ_{ij}^{o} denotes the fraction of the h-step ahead forecast error variance of x_i due to a shock to x_j. When $i = j$ we have own effects, while when $i \neq j$ we have spillover effects.

Pesaran and Shin (1998) have developed a "generalized" approach that allows shocks to be correlated. The generalized approach, that is insensitive to variable ordering, is generally preferred over the traditional approach. However, due to the non-orthogonality of the shocks, the sum of the contributions is not equal to one. As a result, a normalization scheme is needed in order to interpret the results: the suggested approach is once again to constrain the row sums to be equal to one, so that they can represent variance shares (Diebold, Yilmaz (2014)).

As shown, the parallels between the fields of spatial econometrics, network and variance decomposition are numerous and recent research in finance, which this paper aims to further develop, is attempting to build a bridge between these strands (Billio et al. (2016), Diebold and Yilmaz (2014), Keiler and Eder (2013)). In the next section different normalization schemes are reviewed, focusing on advantages and disadvantages and on the applications to the different research fields.

3. Normalization schemes

In this section we review the most commonly used normalization schemes in the various application domains.

3.1 Row normalization

Given a $(k \times k)$ unscaled matrix $W^* = (w_{ij}^*)$, we can obtain the corresponding row-stochastic matrix $W = (w_{ij})$ by row-normalizing W^* such that:

$$w_{ij} = \frac{w_{ij}^*}{\sum_{j=1}^{k} w_{ij}^*} \tag{7}$$

The resulting matrix W has row sums equal to one. In applications, row normalization is the most common normalization scheme. In some spatial regressions models (e.g. SAR(p) models) it represents the easiest way to make the matrix $\left(I_N - \sum_{h=1}^{p} \phi_h W_h\right)$ in equation (4) non-singular. However, row normalization is not a restrictive task since the same result can be achieved by constraining the parameter space of the autoregressive parameters ϕ_h (Caporin and Paruolo (2015)); as a result, the normalization task would be absorbed by the AR parameter through scaling.

Moreover, this normalization is useful in interpreting spatial weight matrices, whose elements can be thought of as a fraction of all spatial influence. This interpretative advantage also applies for a forecast error variance decomposition that does not rely on Cholesky factorization (or any other identifying scheme of structural VAR models) so that the matrix coefficients can be interpreted as variance shares. This is the normalization scheme proposed by Diebold and Yilmaz (2012) when using the generalized forecast error variance decomposition. However, this scheme also has certain drawbacks: by scaling the elements of each row by the corresponding row sum, the order of magnitude is preserved only by row.

3.2 Column normalization

This scheme is specular to the row-normalization scheme described above. The only difference is that the normalization is done by column: in this case only the columns sum to one. The critical issues concerning the row-normalization scheme apply also in this case. Note that for the variance decomposition Diebold and Yilmaz (2012) suggest this normalization scheme as an alternative to row normalization.

3.3 Max row normalization

In this normalization scheme, the normalization factor is a scalar equal to the maximum row sum of the unscaled matrix W*, then the scaled matrix is obtained as $W = W^*/k$ where:

$$k = \max(r_1, \dots, r_k) \tag{8}$$

and:

$$r_i = \sum\nolimits_{j=1}^{k} w_{ij}^* \tag{9}$$

where w_{ij}^* is the element in row i and column j of the unscaled matrix W^*. This scheme is characterized by a single normalization factor instead of the k factors of the row normalization scheme (one for each row). As a result, it preserves the magnitude relation among the elements of rows and columns and column and row values can therefore be safely compared. Moreover, it allows for a comparison between different rows and column sums, making it possible to distinguish between stronger or weaker influences. As argued by Billio et al. (2016) it is also possible to normalize by the maximum row sum over time in order to compare spatial weight matrices in different time periods while preserving a reasonable magnitude of autoregressive parameters.

3.4 Max column normalization

This scheme is specular to the max row normalization described above: the only difference is that the scalar is equal to the maximum column sum of the unscaled matrix W*. The same advantages of the max row normalization apply.

3.5 Spectral radius normalization

Let W* be the $(k \times k)$ positive unscaled matrix and let $\{\lambda_1, \dots, \lambda_k\}$ be the eigenvalues of W*. The spectral radius is the maximum eigenvalue (in modul), formally:

$$\tau = max\{|\lambda_1|, |\lambda|_2, \dots, |\lambda|_k\} \tag{10}$$

The scalar normalization factor is set equal to the spectral radius and the scaled matrix W is therefore obtained as follows:

$$W = W^*/\tau \tag{11}$$

Under the Perron and Frobenius theorem, the spectral radius satisfies the following inequalities:

$$min_i \sum\nolimits_{j=1}^{N} w_{ij} \le \tau \le max_i \sum\nolimits_{j=1}^{N} w_{ij} \tag{12}$$

As a result, some row sums and column sums exceed unity, while others can be less than one. This normalization scheme therefore has one main drawback: the elements can no longer be interpreted as fractions of the overall influence (e.g. the sum by row and by column)

Nevertheless, this normalization scheme is widely used in spatial econometrics: in fact, following LeSage and Pace (2010) a matrix W^* can be transformed to have maximum eigenvalue equal to one using $W = W^* \, max(\lambda_{W^*})$, and this is a desirable property because it constrains the autoregressive parameter to have maximum possible value equal to one. In particular, Keleijan and Prucha (2010) show that $\left(I_N - \sum_{h=1}^{p} \phi_h W_h\right)$ is non-singular for all the values of the parameter space in the interval $(-1\,;\,1)$.

4 Spillover analysis based on forecast error variance decomposition: problems and solutions

Consider a covariance stationary VAR(p) with k endogenous variables:

$$x_t = c + A_1 x_{t-1} + \cdots + A_p x_{t-p} + \varepsilon_t \tag{13}$$

where x_t is a $(k \times 1)$ vector containing the values at time t of the endogenous variables, the equation above can be written in compact form:

$$A(L)x_t = \varepsilon_t \tag{14}$$

where $A(L)$ are coefficient matrices and ε_t are i.i.d. disturbances with covariance matrix Σ. In order to derive the moving average representation of the VAR model we multiply both sides of equation (14) by $A(L)^{-1} = \left(I - A_1 L - \cdots - A_p L_p\right)^{-1} = \Psi(L)$, then:

$$x_t = \Psi(L)\varepsilon_t \tag{15}$$

where:

$$\Psi(L) = \sum_{i=0}^{\infty} \Psi_i L^i \quad ; \qquad \Psi_0 = I \tag{16}$$

We obtain the impulse responses at the forecast horizon h by exploiting the following recursive relation:

$$A_{t+h} = \Psi_0 A_{t+h} + \Psi_1 A_{t+h-1} + \Psi_2 A_{t+h-2} + \cdots + \Psi_p A_{t+h-p} \tag{17}$$

with $\mathrm{A}_0 = I, \mathrm{A}_i = 0$ for $i < 0$. The traditional approach relies on the Cholesky factorization of the variance covariance matrix:

$$\Sigma = P\,P' \tag{18}$$

where P is lower triangular. By substituting equation (18) into the infinite moving average representation of the VAR(p) the shocks ε_t become orthogonal, formally: $\xi_t = P^{-1}\varepsilon_t$ and $E(\xi_t'\,\xi_t) = I$. The resulting forecast error variance decomposition in equation (6) would be sensitive to variable ordering. To overcome this limit, the generalized approach allows shocks to be correlated and accounts for them by using an assumed or an historical distribution of the errors. In this framework, the generalized forecast error variance decomposition proposed by Pesaran and Shin (1998) is computed as follows:

$$\theta_{ij}^{g} = \frac{\sigma_{ii}^{-1} \sum_{l=0}^{h-1} (e_i' A_l \Sigma\, e_i)^2}{\sum_{l=0}^{h-1} \left(e_i' A_l \Sigma\, A_l e_j\right)} \tag{19}$$

where g stands for generalized. The resulting variance decomposition table $\Theta = \left(\theta_{ij}^{g}\right)$ for $i, j = 1, \dots, k$ is a $(k \times k)$ matrix containing all the variance shares. By using a VAR model on different volatility series, Diebold and Yilmaz (2012) exploited the generalized forecast error variance decomposition framework developed by Pesaran and Shin (1998) in order to derive measures of volatility spillovers.

However, due to the non-orthogonality of shocks, the sum of the contributions to the forecast error variance (i.e. the row sum) is not equal to one. They therefore propose a row-normalization of the values of the variance decomposition in equation (19) in order to interpret its elements as variance shares:

$$\tilde{\theta}_{ij}^{g} = \frac{\theta_{ij}^{g}}{\sum_{j=1}^{K} \theta_{ij}^{g}} \tag{20}$$

The directional spillover received by each market from all the other markets (FROM others) is computed as the off-diagonal row sum; the spillover transmitted by each market to all the other markets (TO others) is computed as the off-diagonal column sum. A measure of net contribution (NET) of each market is obtained as the difference between the directional spillovers TO others and FROM others. In this way we are able to distinguish markets that are net donors from those that are net receivers in terms of risk transmission.

However, as shown in the previous section, row normalization has interpretative limits and, in this framework, leads to misspecified spillover measures. In particular:

- If the normalization is carried out by row, the column sum is not necessarily equal to one. As a result, while FROM directional spillovers can be interpreted as a fraction of the total variance received via spillovers, TO directional spillovers lack this kind of interpretation (some column sums are above unity, while some others are beyond unity).
- Normalization by row implies that the order of magnitude of the entries of the variance decomposition table is preserved only by row. As a result, NET spillovers are obtained as the difference between two values incomparable in magnitude.

The reasoning underlying the choice of Diebold and Yilmaz (2012) to row-normalize the variance decomposition table is that by constraining the row sum to unity, the elements can represent variance shares. However, in their paper they also state that one can alternatively normalize by column but, as we show in the next section, the values of the spillover measures are sensitive to this normalization choice, leading to misspecifed measures of net contribution (NET).

5 Comparison of the normalization schemes

In this section first of all we show by means of an introductory example how the net spillover values are sensitive to the different normalization schemes. Second, we provide simulation results based on different degrees of correlation and persistence. We consider four cases: a) LL (Low Persistence; Low Correlation); b) LH (Low Persistence; High Correlation); c) HL (High Persistence; Low Correlation); d) HH (High Persistence; High Correlation), according to the different setup for the coefficient matrices in the lag operator A(L) and of the covariance matrix $\Sigma = P\,P'$. In particular the Low Correlation case is defined by using a lower triangular matrix P in eq. (18) set as follows:

$$P = \begin{bmatrix} 0.10 & 0 & 0 & 0 & 0 \\ 0.15 & 0.15 & 0 & 0 & 0 \\ 0.20 & 0.20 & 0.20 & 0 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 \\ 0.30 & 0.30 & 0.30 & 0.30 & 0.30 \end{bmatrix} \tag{21}$$

while the High Correlation case is defined by using the following lower triangular matrix P:

$$P = \begin{bmatrix} 0.40 & 0 & 0 & 0 & 0 \\ 0.45 & 0.45 & 0 & 0 & 0 \\ 0.50 & 0.50 & 0.50 & 0 & 0 \\ 0.55 & 0.55 & 0.55 & 0.55 & 0 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.60 \end{bmatrix} \tag{22}$$

The P matrices have dimension $(k \times k)$ with $k = 5$, which is the number of variables included in the multivariate system. To ensure a stationary VAR(p) (e.g. with roots of the characteristic polynomial A(L) outside the unit circle) characterised by Low Persistence, we consider a VAR(2) with coefficient matrices A_1 and A_2 with values equal to 0.05. A stationary VAR(p) characterised by High Persistence is a restricted VAR(22) given by the parsimonious Vector HAR representation with coefficient matrices $A^{(d)}, A^{(w)}, A^{(m)}$ described as follows: $A^{(d)}$ with values equal to 0.05, $A^{(w)}$ with values equal to -0.02 and $A^{(m)}$ with values equal to 0.01[2].

Consequently, we compute the generalized forecast error variance decomposition as defined by equation (19) and we obtain the measures of NET contribution. Formally, the non-normalized NET spillovers for the forecast horizon h, which are taken as benchmark, are obtained as follows:

$$NET_i(h) = DS^{g}_{\bullet\to}(h) - DS^{g}_{\to\bullet}(h) \tag{23}$$

where:

$$DS^{g}_{\bullet\to}(h) = \sum_{\substack{i=1 \\ i\neq j}}^{K} \theta^{g}_{ij} \quad ; \quad DS^{g}_{\to\bullet}(h) = \sum_{\substack{j=1 \\ j\neq i}}^{K} \theta^{g}_{ij} \tag{24}$$

where $DS^{g}_{\bullet\to}$ denotes the non-normalized directional spillover transmitted by the market i to all other markets j (named TO others), while $DS^{g}_{\to\bullet}$ denotes the non-normalized directional spillover received by market i from all the other markets j (named FROM others). Second, we compute the $\overline{NET}$

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2 In the Vector HAR model the matrices $A^{(d)}, A^{(w)}$ and $A^{(m)}$ are coefficient matrices associated with the three terms of daily, weekly and monthly partial volatility components, respectively. In particular, the Vector HAR model can be written as follows:

$$x^{(d)}_t = c + \phi^{(d)} x^{(d)}_{t-1} + \phi^{(w)} x^{(w)}_{t-1} + \phi^{(m)} x^{(m)}_{t-1} + \varepsilon_t$$

where x_t are daily volatilities, while the terms representing the weekly and monthly volatilities are obtained as the arithmetic average of the daily volatilities recorded in the last week and the last month, respectively.

spillovers obtained from the forecast error variance decomposition normalized by the different schemes:

$$\overline{NET_i}(h) = \overline{DS^g_{\bullet\to}}(h) - \overline{DS^g_{\to\bullet}}(h) \quad (25)$$

where the over bar denotes the normalized spillovers. These normalized measures are compared to the benchmark spillovers in equation (23). The comparison is intended to assess the reliability of the different normalization schemes both in terms of order of ranking (to assess which market is the largest net contributor to the total connectedness) and in terms of sign (to distinguish net donors from net receivers).

5.1 Results based on population parameters

In this section we cast light on how the choice of the normalization scheme can affect the ranking and the sign of the NET spillovers, by means of an introductory example. Moreover, in order to show how the spillover tables change for different forecast horizons, two different horizons are reported: the two-day horizon is reported in the upper panel of every Table, while the lower panel contains the ten-day forecast horizon.

For this introductory example we report the results based on the population parameters for the "High Persistence, High Correlation" scenario, which is the most illuminating one. Table 1 shows the spillover table based on the non-normalized forecast error variance decomposition which is taken as a benchmark. Tables 2 to 6 show the same spillover table after applying the different normalization schemes outlined in Section 3 (Table 2 for row normalization, Table 3 for column normalization, Table 4 for normalization by spectral radius, Table 5 for normalization by maximum row sum, Table 6 for normalization by maximum column sum). These Tables show the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others including own), the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others including own), and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for each variable V_i for $i = 1, \dots, 5$. The tables also show the sign of the NET spillover (NET sign): negative if the market is the net receiver and positive if the market is the net donor, and the ranking of the NET spillover from the highest to the lowest (NET ranking).

In Table 2 we show the standard row-normalization scheme proposed by Diebold and Yilmaz (2012) which has the interpretative advantage that the directional spillovers received FROM others including own sum to one, and as a result each element of the forecast error variance decomposition matrix can be interpreted as a variance share (by row). For example, in the upper panel variable 2 receives the most from variable 3 (0.219), and the least from variable 5 (0.140). Moreover, variable 1 represents the market least affected by the others (FROM others=0.653), while variable 3 represents the market most affected by the others (FROM others=0.705).

On the contrary in Table 3 all the columns (TO others including own) sum to one: each element of the forecast error variance decomposition matrix can be interpreted as the fraction of total variance transmitted. For example, in the upper panel of Table 3 variable 2 gives the least to variable 5 (0.131), and the most to variable 3 (0.216). Moreover, variable 3 represents the market that transmits the most to the others (TO others=0.714), while variable 1 represents the market that transmits the least to others (TO others=0.592). In the case of the column normalization, the focus is on how much one variable (market or country) affects the system. Despite the neat interpretation, the row normalization or column normalization schemes affect the NET spillovers, which may have the opposite sign and the wrong ranking if compared to the non-normalized ones. In fact, the first variable in the column normalization scheme (Table 3) is misconceived as the net donor, while it is a net receiver in the non-normalized case (Table 1), whereas variables 3 and 4 are mistakenly considered as net receivers instead of net donors, as apparent in Table 1. The same happens in the row normalization scheme, but only for the ten-day horizon. As a result, also there is a change in the ranking of the variables (ranging from the one giving the most to the system, that is the major net donor and has rank 1, to the variable receiving the most from the system, which is the major net receiver and has rank 5). For example in the non-normalized case the variable transmitting the most to the system is variable 5 (for both forecast horizons), but in the row-normalized case it emerges that the variable transmitting the most to the system is variable 3 for the two-day forecast horizon and variable 4 for the ten-day horizon.

Tables 4 to 6 show the scalar-normalization cases. The scalar factors applied are: the spectral radius (Table 4), the maximum row sum (Table 5) and the maximum column sum (Table 6). In the spectral radius normalization it is not possible to interpret each element of the forecast error variance decomposition matrix as variance shares by column, or by row. In fact, the sum by row and by column (FROM others including own and TO others including own) can attain values higher or lower than 1, given the mathematical property of the maximum eigenvalue described in eq. (12). Despite the lack of interpretability in terms of variance shares, all the net spillovers maintain the correct sign after normalization and the correct ranking as in the non-normalized case.

It may be noted that in the maximum row sum normalization scheme in Table 5 and in the maximum column sum normalization scheme in Table 6, the only values which sum to one are those in the row with the maximum sum (the third row in both Panels of Table 5) and those in the column with the maximum sum (column 3 for Panel A and column 4 for Panel B of in Table 6), respectively. Only for these values is it possible to give a percentage interpretation: in Table 5 it may be seen that for h=2 variable 3 receives 70.5% FROM others, while variable 4 in Table 6 transmits 71.2% TO others.

In conclusion, it may be stated that the max row sum and max col sum normalization are slightly better than the spectral radius since they can preserve the ranking and the sign of the spillovers and, at least for one variable, they can preserve their interpretation as variance share.

As Tables 2 to 6 focus on the normalization issue for only the high correlated and high persistence scenario, in Table 7 and Table 8 we show the results based on population parameters for all the other scenarios: by looking at the sign of the net spillovers (Table 7) it is clear that the row-normalization scheme performs fairly well with no errors in sign for the horizon H=2 and only one error in sign in each of the high-correlated scenarios: on the contrary in each scenario the column normalization produces from 1 to 3 errors in sign. By looking at the ranking errors in Table 8, what emerges is that both the row normalization and the column normalization scheme affect the ranking in most cases. On the other hand, any scalar normalization scheme does not affect the ranking.

5.2 Results based on simulation

In order to account for the role played by parameter estimation on the rank and sign of net spillovers, we simulate a multivariate dynamic system, using the DGP given by eq. (13). The shocks ε_t are given by $P\,\eta_t$, where η_t are iid Gaussian and orthogonal innovations. In order to assess the reliability of the different normalization schemes in preserving the order of magnitude and the sign of net contributions (NET spillovers) obtained from the generalized forecast error variance decomposition, the simulation experiment involves the following steps:

1) Five artificial data series (where the time series dimension is equal to 500) are obtained by simulating either the VAR(2) (in the case of Low Persistence) or the restricted VAR(22) (in the case of High Persistence) with Gaussian innovations. The coefficient matrices for the lags and the lower triangular matrices P aiming at capturing the different degrees of contemporaneous correlation are those used in section 5.

2) For each replication, we estimate the model parameters by OLS, obtaining the impulse-responses for the forecast horizons $h = 2$, $h = 10$ and computing the corresponding generalized forecast error variance decomposition as defined in eq. (19).

After obtaining the simulated datasets, we compare the non-normalized matrix W^* (e.g. the non-normalized variance decomposition table for a given forecast horizon) and the five normalized matrices W (e.g. the normalized variance decomposition table for a given forecast horizon) in terms of sign and ranking errors.

First, we measure the number of errors in the sign of the net spillovers. Errors are counted when the net spillover obtained from the normalized matrix has a sign opposite to that of the net spillover obtained from the non-normalized matrix. The total number of possible errors is 5000 for each scenario (5 variables times 1000 simulations for each scenario).

Second, we measure the errors in the ranking. Errors are counted when the ranking of the net spillovers obtained from the normalized matrix is different from that of the non-normalized matrix. The total number of possible errors is 1000 for each scenario (one ranking times 1000 simulations for each scenario). Results are shown in Table 9 for sign errors and in Table 10 for ranking errors.

Table 9 shows that over a total number of 5000 possible errors for each scenario (5 variables times 1000 simulations for each scenario), the row normalization performs much better than the column normalization for each scenario: in fact, for H=2 (H=10) the average number of errors in sign is about 354 (169) for the row-normalization scheme and about 2525 (1997) for the column normalization scheme. This result is surprising since the row normalization and column normalization schemes should theoretically be equal. In both normalization schemes, the number of errors increases with the degree of correlation. On the contrary, the sum of errors in sign in the low persistence scenarios is slightly higher than the sum of the same errors in the high persistence scenarios for both forecast horizons.

Moreover, as shown in Table 10, the row-normalization proposed by Diebold and Yilmaz (2012) and the alternative column normalization schemes affect the ranking of the spillovers more than 850 times out of 1000 for H=2 and more than 950 times out of 1000 for H=10 (with the sole exception of the row normalization scheme in the high persistence scenarios).

To conclude, even if the row normalization scheme and the column normalization scheme allow for a better interpretation of the values of the generalized forecast error variance decomposition, there is a need to be cautious in interpreting the resulting net spillovers that should discriminate markets which are net donors from those which are net receivers. On the contrary, any scalar normalization scheme (by maximum eigenvalue, maximum row sum or maximum column sum) will outperform the traditional normalization schemes, preserving the ranking and the sign of the NET spillovers. As a result, we suggest using a scalar normalization scheme to derive the correct measures of net contribution. Among the scalar normalization schemes, the maximum row sum or the maximum column sum are preferred to the spectral radius since they allow for a better interpretation of how much one variable receives or transmits in terms of percentage values.

6 Concluding remarks

The focus of this paper was on the variance decomposition to assess how the normalization choice can affect the computation and the interpretation of the spillover measures obtained. With respect to normalization, the intention was to contribute to the financial literature aiming to build a bridge between the strands of spatial econometrics, network analysis and variance decomposition. These are the main approaches used to quantify risk-transmission through spillover analyses, and recent research efforts are intended to make them converge. We reviewed the main normalization schemes used in these strands of literature and in their applications, highlighting the reasons underlying the choice of a normalization scheme, as well as the advantages and disadvantages of each method. Finally, we showed that the standard row normalization scheme suggested by Diebold and Yilmaz (2012) and commonly used in all the applications, as well as the equivalent column normalization scheme, produce numerous errors both in the ranking and in the sign of the resulting NET spillovers. These normalization schemes, although allowing for a better interpretation (as variance shares) of the results, may fail to establish whether the market is a net risk transmitter or net risk receiver. Moreover, they are also unable to assess the degree to which a single market influences all the others in net absolute terms. As a result, we suggest using a scalar normalization scheme to avoid the misspecification of results.

Among the scalar normalization schemes, the maximum row sum or the maximum column sum schemes are preferable to the spectral radius since they allow for a better interpretation of how much one variable receives or transmits in terms of percentage values.

Acknowledgements. The authors gratefully acknowledge financial support from Fondazione Cassa di Risparmio di Modena, for the project "Volatility and higher order moments: new measures and indices of financial connectedness" and from the FAR2015 project "A SKEWness index for Europe (EU-SKEW)". The usual disclaimer applies.

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Table 1. Spillover Table based on the non-normalized variance decomposition table (VDT).

	V1	V2	V3	V4	V5	FROM others including own	FROM others
Panel A: h=2							
V1	0.889	0.539	0.435	0.376	0.324	2.561	1.672
V2	0.493	0.975	0.683	0.535	0.438	3.124	2.149
V3	0.337	0.668	0.994	0.758	0.612	3.369	2.375
V4	0.255	0.506	0.753	0.997	0.801	3.313	2.316
V5	0.204	0.406	0.605	0.802	0.997	3.015	2.018
TO others including own	2.178	3.094	3.470	3.467	3.172		
TO others	1.289	2.119	2.477	2.470	2.175		
NET	-0.383	-0.030	0.102	0.154	0.157		
NET sign	+	-	-	-	+		
NET ranking	2	4	5	3	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.773	0.579	0.540	0.506	0.453	2.850	2.077
V2	0.483	0.940	0.706	0.582	0.490	3.202	2.262
V3	0.343	0.671	0.984	0.769	0.630	3.397	2.413
V4	0.263	0.516	0.759	0.993	0.804	3.334	2.341
V5	0.211	0.416	0.614	0.806	0.992	3.040	2.048
TO others including own	2.073	3.122	3.603	3.656	3.368		
TO others	1.300	2.181	2.619	2.663	2.377		
NET	-0.777	-0.080	0.206	0.322	0.329		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the non-normalized forecast error variance decomposition, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon h=2 (Panel A) and h=10 (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable Vi, i=1,..., 5. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the absolute value of the NET spillover from the highest to the lowest (NET ranking).

Table 2. Spillover Table based on the row-normalized variance decomposition table (VDT).

	V1	V2	V3	V4	V5	FROM others including own	FROM others
Panel A: h=2							
V1	0.347	0.210	0.170	0.147	0.126	1	0.653
V2	0.158	0.312	0.219	0.171	0.140	1	0.688
V3	0.100	0.198	0.295	0.225	0.182	1	0.705
V4	0.077	0.153	0.227	0.301	0.242	1	0.699
V5	0.068	0.135	0.201	0.266	0.331	1	0.669
TO others including own	0.750	1.008	1.112	1.110	1.021		
TO others	0.403	0.696	0.817	0.809	0.690		
NET	-0.250	0.008	0.112	0.110	0.021		
NET sign	+	-	-	-	+		
NET ranking	2	4	5	3	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.271	0.203	0.189	0.178	0.159	1	0.729
V2	0.151	0.294	0.221	0.182	0.153	1	0.706
V3	0.101	0.198	0.290	0.226	0.185	1	0.710
V4	0.079	0.155	0.228	0.298	0.241	1	0.702
V5	0.070	0.137	0.202	0.265	0.326	1	0.674
TO others including own	0.671	0.986	1.129	1.149	1.065		
TO others	0.400	0.692	0.840	0.851	0.738		
NET	-0.329	-0.014	0.129	0.149	0.065		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the row-normalized forecast error variance decomposition, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon h=2 (Panel A) and h=10 (Panel B). The Table reports the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable Vi, i=1,..., 5. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the absolute value of the NET spillover from the highest to the lowest (NET ranking).

Table 3: **Spillover Table based on the column-normalized variance decomposition table (VDT).**

	V1	V2	V3	V4	V5	FROM others including own	FROM others
Panel A: h=2							
V1	0.408	0.174	0.125	0.108	0.102	0.918	0.510
V2	0.226	0.315	0.197	0.154	0.138	1.031	0.716
V3	0.155	0.216	0.286	0.218	0.193	1.069	0.782
V4	0.117	0.164	0.217	0.288	0.253	1.038	0.750
V5	0.094	0.131	0.174	0.231	0.314	0.945	0.631
TO others including own	1	1	1	1	1		
TO others	0.592	0.685	0.714	0.712	0.686		
NET	0.082	-0.031	-0.069	-0.038	0.055		
NET sign	+	-	-	-	+		
NET ranking	2	4	5	3	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.373	0.185	0.150	0.138	0.134	0.981	0.608
V2	0.233	0.301	0.196	0.159	0.146	1.035	0.734
V3	0.166	0.215	0.273	0.210	0.187	1.051	0.778
V4	0.127	0.165	0.211	0.272	0.239	1.013	0.741
V5	0.102	0.133	0.170	0.220	0.294	0.921	0.626
TO others including own	1	1	1	1	1		
TO others	0.627	0.699	0.727	0.728	0.706		
NET	0.019	-0.035	-0.051	-0.013	0.079		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the column-normalized forecast error variance decomposition, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon h=2 (Panel A) and h=10 (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable Vi, i=1,..., 5. The bottom lines report for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the absolute value of the NET spillover from the highest to the lowest (NET ranking).

Table 4: **Spillover Table based on the variance decomposition table (VDT) normalized by the spectral radius.**

	V1	V2	V3	V4	V5	FROM others including own	FROM others
Panel A: h=2							
V1	0.284	0.172	0.139	0.120	0.103	0.818	0.534
V2	0.157	0.311	0.218	0.171	0.140	0.998	0.686
V3	0.108	0.213	0.317	0.242	0.195	1.076	0.758
V4	0.081	0.162	0.241	0.318	0.256	1.058	0.740
V5	0.065	0.130	0.193	0.256	0.318	0.963	0.644
TO others including own	0.695	0.988	1.108	1.107	1.013		
TO others	0.412	0.677	0.791	0.789	0.695		
NET	-0.122	-0.010	0.032	0.049	0.050		
NET sign	+	-	-	-	+		
NET ranking	2	4	5	3	1		
Panel B: h=10							
	V1	V2	V3	V4	V5	FROM others including own	FROM others
V1	0.241	0.181	0.169	0.158	0.141	0.890	0.649
V2	0.151	0.294	0.221	0.182	0.153	1.000	0.706
V3	0.107	0.210	0.307	0.240	0.197	1.061	0.754
V4	0.082	0.161	0.237	0.310	0.251	1.041	0.731
V5	0.066	0.130	0.192	0.252	0.310	0.949	0.639
TO others including own	0.647	0.975	1.125	1.142	1.052		
TO others	0.406	0.681	0.818	0.832	0.742		
NET	-0.243	-0.025	0.064	0.100	0.103		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the forecast error variance decomposition normalized by the spectral radius, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon h=2 (Panel A) and h=10 (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable Vi, i=1,..., 5. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the absolute value of the NET spillover from the highest to the lowest (NET ranking).

Table 5: **Spillover Table based on the variance decomposition table (VDT) normalized by the maximum row sum.**

	V1	V2	V3	V4	V5	FROM others including own	FROM others
Panel A: h=2							
V1	0.264	0.160	0.129	0.111	0.096	0.760	0.496
V2	0.146	0.289	0.203	0.159	0.130	0.927	0.638
V3	0.100	0.198	0.295	0.225	0.182	1	0.705
V4	0.076	0.150	0.224	0.296	0.238	0.984	0.688
V5	0.061	0.121	0.180	0.238	0.296	0.895	0.599
TO others including own	0.647	0.918	1.030	1.029	0.942		
TO others	0.383	0.629	0.735	0.733	0.646		
NET	-0.114	-0.009	0.030	0.046	0.047		
NET sign	+	-	-	-	+		
NET ranking	2	4	5	3	1		
Panel B: h=10							
V1	0.227	0.170	0.159	0.149	0.133	0.839	0.612
V2	0.142	0.277	0.208	0.171	0.144	0.943	0.666
V3	0.101	0.198	0.290	0.226	0.185	1	0.710
V4	0.077	0.152	0.223	0.292	0.237	0.982	0.689
V5	0.062	0.123	0.181	0.237	0.292	0.895	0.603
TO others including own	0.610	0.919	1.061	1.076	0.992		
TO others	0.383	0.642	0.771	0.784	0.700		
NET	-0.229	-0.024	0.061	0.095	0.097		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the forecast error variance decomposition normalized by the maximum row sum, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon h=2 (Panel A) and h=10 (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable Vi, i=1,..., 5. The bottom lines show for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the absolute value of the NET spillover from the highest to the lowest (NET ranking).

Table 6: **Spillover Table based on the variance decomposition table (VDT) normalized by the maximum column sum.**

	V1	V2	V3	V4	V5	FROM others including own	FROM others
Panel A: h=2							
V1	0.256	0.155	0.125	0.108	0.093	0.738	0.482
V2	0.142	0.281	0.197	0.154	0.126	0.900	0.619
V3	0.097	0.193	0.286	0.218	0.176	0.971	0.684
V4	0.073	0.146	0.217	0.287	0.231	0.955	0.667
V5	0.059	0.117	0.174	0.231	0.287	0.869	0.582
TO others including own	0.628	0.892	1	0.999	0.914		
TO others	0.372	0.611	0.714	0.712	0.627		
NET	-0.110	-0.009	0.029	0.044	0.045		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		
Panel B: h=10							
V1	0.211	0.158	0.148	0.138	0.124	0.779	0.568
V2	0.132	0.257	0.193	0.159	0.134	0.876	0.619
V3	0.094	0.184	0.269	0.210	0.172	0.929	0.660
V4	0.072	0.141	0.208	0.272	0.220	0.912	0.640
V5	0.058	0.114	0.168	0.220	0.271	0.831	0.560
TO others including own	0.567	0.854	0.986	1	0.921		
TO others	0.356	0.597	0.716	0.728	0.650		
NET	-0.212	-0.022	0.056	0.088	0.090		
NET sign	-	-	+	+	+		
NET ranking	5	4	3	2	1		

Note. This figure shows the spillover Table based on the forecast error variance decomposition normalized by the maximum column sum, which is displayed in the central frame. Results refer to the HH scenario (high-persistent and high-correlated series) and to the forecast horizon h=2 (Panel A) and h=10 (Panel B). The Table shows the directional spillover received from others (FROM others), the directional spillover received from others including own (FROM others (including own)) the directional spillover transmitted to others (TO others), the directional spillover transmitted to others including own (TO others (including own)) and the net contribution (NET) defined as the difference between the directional spillover transmitted TO others and the directional spillover received FROM others for variable Vi, i=1,…, 5. The bottom lines report for each variable the sign of the NET spillover (NET sign): negative if the variable is a net receiver and positive if the variable is a net donor, and the ranking based on the absolute value of the NET spillover from the highest to the lowest (NET ranking).

Table 7: Errors in sign (using population parameters).

Panel A: h=2				
	L. L.	L.H.	H.L.	H.H.
normalization by row	0	1	0	1
normalization by column	3	3	3	3
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0
Panel B: h=10				
	L. L.	L.H.	H.L.	H.H.
normalization by row	0	0	0	0
normalization by column	1	3	1	3
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0

Note. The Table shows the number of errors in sign for each scenario (L.L. , L.H. , H.L. , H.H. , where L.L. = low persistence low correlation, L.H.=low persistence high correlation, H.L.=high persistence low correlation, H.H.=high persistence high correlation). Results refer to the forecast horizon h=2 (panel A) and h=10 (Panel B). Errors are counted when the net spillover obtained from the normalized matrix has a sign opposite to the one of the net spillover obtained from the non-normalized matrix. The total number of possible errors is 5 for each scenario.

Table 8: Errors in ranking (using population parameters).

Panel A: h=2				
	L. L.	L.H.	H.L.	H.H.
normalization by row	1	1	1	1
normalization by column	1	1	1	1
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0
Panel B: h=10				
	L. L.	L.H.	H.L.	H.H.
normalization by row	1	1	1	1
normalization by column	0	1	0	1
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0

Note. The Table shows the number of errors in ranking for each scenario (L.L. , L.H. , H.L. , H.H. , where L.L. = low persistence low correlation, L.H.=low persistence high correlation, H.L.=high persistence low correlation, H.H.=high persistence high correlation). Results refer to the forecast horizon h=2 (panel A) and h=10 (Panel B). Errors are counted when the ranking of the net spillovers obtained from the normalized matrix is different from that of the non-normalized matrix. The total number of possible errors is 1 for each scenario.

Table 9: Errors in sign (using simulations).

	L. L.	L.H.	H.L.	H.H.
Panel A: h=2				
normalization by row	118	607	198	492
normalization by column	2802	3226	1746	2324
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0
Panel B: h=10				
	L. L.	L.H.	H.L.	H.H.
normalization by row	28	293	111	243
normalization by column	1368	3143	1644	1833
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0

Note. The Table shows the number of errors in sign for each scenario (L.L. , L.H. , H.L. , H.H. , where L.L. = low persistence low correlation, L.H.=low persistence high correlation, H.L.=high persistence low correlation, H.H.=high persistence high correlation). Results refer to the forecast horizon H=2 (panel A) and H=10 (Panel B). Errors are counted when the net spillover obtained from the normalized matrix has a sign opposite to the one of the net spillover obtained from the non-normalized matrix. The total number of possible errors is 5000 for each scenario (5 variables times 1000 simulations for each scenario).

Table 10: Errors in ranking (using simulations).

	L. L.	L.H.	H.L.	H.H.
Panel A: h=2				
normalization by row	1000	990	870	989
normalization by column	1000	1000	934	949
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0
Panel B: h=10				
	L. L.	L.H.	H.L.	H.H.
normalization by row	1000	991	481	743
normalization by column	966	1000	981	977
normalization by spectral radius	0	0	0	0
normalization by max row sum	0	0	0	0
normalization by max col sum	0	0	0	0

Note. The Table shows the number of errors in ranking for each scenario (L.L. , L.H. , H.L. , H.H. , where L.L. = low persistence low correlation, L.H.=low persistence high correlation, H.L.=high persistence low correlation, H.H.=high persistence high correlation). Results refer to the forecast horizon H=2 (panel A) and H=10 (Panel B). Errors are counted when the ranking of the net spillovers obtained from the normalized matrix is different from the one of the non-normalized matrix. The total number of possible errors is 1000 for each scenario (one ranking times 1000 simulations for each scenario).