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Human-structure interaction effects on the maximum dynamic response based on an equivalent spectral model for pedestrian-induced loading

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Abstract. This paper investigates the effects of the human-structure interaction (HSI) on the dynamic response based on a spectral model for vertical pedestrian-induced forces. The spectral load model proposed in literature can be applied for the vibration serviceability analysis of footbridges subjected to unrestricted pedestrian traffic as well as in crowded conditions, however, in absence of HSI phenomena. To allow for a more accurate prediction of the maximum structural response, the present study in addition accounts for the vertical mechanical interaction between pedestrians, represented by simple lumped parameter models, and the supporting structure. By applying the classic methods of linear random dynamics, the maximum dynamic response is evaluated based on the analytical expression of the spectral model of the loading and the frequency response function (FRF) of the coupled system. The most significant HSI-effect is in the increase of the effective damping ratio of the coupled system that leads to a reduction of the structural response. However, in some cases the effect of the change in the frequency of the coupled system is more significant, whereby this results into a higher structural response when the HSI-effects are accounted for.

1. Introduction

Due to the upcoming of advanced design methods and high strength materials, modern footbridges have become increasingly slender and lively structures, prone to human-induced vibrations. Hence, vibration serviceability has become a topical issue in the design of modern slender footbridges with large span. Pedestrian load models proposed by existing design guidelines for footbridges are typically based on measurements of single footfalls replicated at precise intervals (e.g. [1, 2]), without addressing adequately essential features of the walking process, i.e. the fact that it is stochastic and narrow-band [3]. Based on the analysis of real continuous walking forces obtained from an instrumented treadmill and the effect of their random imperfections through time simulations of structural response, Brownjohn et al. [4] show that there are significant differences between responses due to the imperfect real walking forces and the equivalent perfectly periodic simulation. Similar results are obtained from other researchers, such as [5, 6, 7]. Van Nimmen et al. [7] show that in contrast to perfectly periodic forces that are exclusively composed of the harmonics of the step frequency, imperfect real walking results in a



distribution of forces around the dominant harmonics. While the perfectly periodic model, which is expanded in Fourier coefficients with a precise frequency, leads itself to deterministic time domain analysis, stochastic analysis is more suited for representing the random nature of human walking. In the frequency domain this can be performed by representing the walking forces as power spectral densities (PSDs) [4]. Application of PSDs, with the Gaussian form of probability distribution of mean pacing rates, leads to a framework for more realistic frequency domain representation for large number of pedestrians. Finally, the consideration of correlation among pedestrians at different locations on a structure suggests an approach similar to estimating dynamic response of structures to turbulent buffeting by wind. The approach can be used for single pedestrians as well as crowd loading, based on the definition of correlation among pedestrians and statistics of their pacing rates [4, 8]. The frequency domain approach proposed by Brownjohn et al. [4] for a realistic treatment of vertical forces induced by groups of normally walking pedestrians accounts for the imperfection of individual walking excitation as well as the statistical distribution of pacing rates in the walking crowd. However, the model can only be adopted to assess the effect of low level pedestrian traffic, as the correlation among pedestrians is not accounted for.

With the same objective, Krenk [9] formulates a model for the pedestrian load in the form of a stochastic process, based on a rational function representation of the spectral density of the process. Starting from the representation of the pedestrian load via the frequency spectral density, Krenk proposes a simple equation for the evaluation of the dynamic amplification for resonant harmonic excitation by introducing an effective damping ratio. The latter is obtained combining the damping ratio of the structure and the bandwidth parameter of the load process. In addition, the dynamic amplification at non-resonant loading is described by a closed-form extension of the resonant loading result, as an alternative to the numerical solution based on the use of the Lyapunov equations.

The present paper focuses on the spectral model proposed by Piccardo and Tubino [10] and Ferrarotti and Tubino [8] for the modal force induced by pedestrian groups modeled as a stationary random process. The model relies on an analytical definition of the PSD of the modal force that can be adopted in unrestricted traffic conditions, as formulated in [10]. In addition, it can also be adopted in crowded conditions, but in absence of human-structure interaction, thanks to the generalized formulation proposed in [8]. The latter relies on a physically-based expression of the coherence function, which accounts for the increased synchronization among pedestrians with increasing pedestrian density. Starting from the analytical spectral model of the load, the maximum dynamic response is evaluated through simple closed-form expressions with the methods of linear random dynamics. However, the presence of a crowd of pedestrians can cause significant changes in the modal characteristics of the coupled crowd-structure system with respect to those of the empty footbridge. Therefore, to allow for a more accurate prediction of the maximum structural response, the present study in addition accounts for the vertical mechanical interaction between pedestrians, represented by simple lumped parameter models, and the supporting structure, as proposed by Van Nimmen et al. in [11, 12, 13].

The aim of this paper is to evaluate the impact of HSI on the structural response to pedestrian excitation. The maximum response of the coupled crowd-structure system is evaluated based on the equivalent spectral load model. A parametric study is performed involving a wide range of pedestrian and footbridge parameters.

The outline of the paper is as follows. First, the procedure for the evaluation of the maximum response based on the equivalent spectral load model is presented and discussed. Second, the coupled human-structure model is introduced and the effects of the interaction are examined. Finally, the parametric study is performed to evaluate the structural response in the presence of HSI-effects.

2. Evaluation of the maximum dynamic response based on the equivalent spectral model for pedestrian-induced loading

The procedure proposed by Piccardo and Tubino [10] and Ferrarotti and Tubino [8] for the serviceability analysis of footbridges, is based on the estimation of the maximum dynamic response through the methods of linear random dynamics (e.g., [14]). In particular, the acceleration response of a system subjected to a random excitation, such as the pedestrian groups modeled as stationary random process, is statistically characterized. When a lightly-damped linear system is subjected to random excitation, the characteristics of the excitation are modified by the system, which behaves as a filter, and the resulting response is a narrow band random process [15, 16]. The mean value of the distribution of maxima can be obtained from the standard deviation of the output acceleration $\sigma_{\ddot{p}_j}$ and a peak coefficient $g_{\ddot{p}_j}$ [17, 18, 19]:

$$\ddot{p}_{jmax} = g_{\ddot{p}_j} \sigma_{\ddot{p}_j} \quad (1)$$

where \ddot{p}_{jmax} is the maximum accelerations of the principal coordinates. The standard deviation of the output acceleration is estimated from the acceleration variance, which in turn is calculated as [15, 16]:

$$\sigma_{\ddot{p}_j}^2 = \int_0^\infty |H_B(\omega)|^2 S_{F_j}(\omega) d\omega \quad (2)$$

where $S_{F_j}(\omega)$ is the one-sided PSD of the modal force induced by pedestrians and $H_B(\omega)$ is the FRF of the footbridge relating the harmonic input excitation to the acceleration response. For lightly-damped systems and when the response may be assumed mainly resonant, the acceleration variance can be calculated as [14]:

$$\sigma_{\ddot{p}_j}^2 = \frac{\pi \omega_j}{4 \xi_j m_j^2} S_{F_j}(\omega_j) \quad (3)$$

where ω_j , ξ_j and m_j are the structural frequency in rad/s, the damping ratio [%] and the modal mass [kg], respectively. The PSD of the modal force is discussed in section 2.1, while the peak coefficient is described in section 2.2.

2.1. Spectral model of the modal force

The spectral model proposed by Piccardo and Tubino [10] and Ferrarotti and Tubino [8] relies on an analytical definition of the PSD of the modal force. A summary of the procedure for the derivation of the analytical expression of the spectral load is given here. The reader is referred to [8, 10] for the detailed derivation. The modal force is obtained as:

$$F_j(t) = \int_0^L f(x, t) \varphi_j(x) dx \quad (4)$$

where L [m] is the footbridge length and $f(x, t)$ is the force induced by N_P pedestrians. The latter, considering only the contribution of the first harmonic of the walking force, can be expressed as:

$$f(x, t) = \sum_{z=1}^{N_P} \alpha_z G_z \sin(\Omega_z(t - \tau_z) + \Psi_z) \delta[x - c_z(t - \tau_z)] \left[H(t - \tau_z) - H\left(t - \tau_z - \frac{L}{c_z}\right) \right] \quad (5)$$

where, $\alpha_z G_z$ [N], Ω_z [rad/s], Ψ_z [rad], c_z [m/s] and τ_z [s] are, respectively, the force amplitude, the step circular frequency, the phase angle, the velocity and the arrival time of the z th pedestrian,

while δ and H represent the Dirac and Heaviside functions. The procedure is then reformulated based on the following non-dimensional parameters:

$$\tilde{t} = \omega_j t, \quad \tilde{x} = \frac{x}{L}, \quad \tilde{\alpha}_z = \frac{\alpha_z}{\alpha_m}, \quad \tilde{G}_z = \frac{G_z}{G_m}, \quad f(\tilde{x}, \tilde{t}) = \frac{f(x, t)L}{\alpha_m G_m}, \quad \tilde{\Omega}_z = \frac{\Omega_z}{\omega_j}, \quad \tilde{\Omega}_{cz} = \frac{c_z}{\omega_j L}, \quad \tilde{\tau}_z = \omega_j \tau_z \quad (6)$$

where $\alpha_m G_m$ is the mean amplitude of the fundamental harmonic of the vertical force from a single pedestrian. Randomness of force amplitudes, pedestrian velocities and pedestrian arrivals are neglected and these variables assumed equal to their mean value. Hence, the randomness of the pedestrian excitation is given by the step frequency, represented by a random variable following a normal distribution. According to Matsumoto et al. [20], the distribution of the step frequency can be characterized with a standard deviation equal to 0.173 Hz. Moreover, in [8, 10] it is assumed that the structural mode shape $\varphi_j(\tilde{x})$ is sinusoidal and that the time required for a pedestrian to cross the footbridge is long compared to the time required to take a step, i.e. $\tilde{\Omega}_{cm} = \varepsilon \tilde{\Omega}_z$ with $\varepsilon \rightarrow 0$. Under these assumptions and after some mathematical manipulations (see [10]), the non-dimensional modal force can be expressed depending on the pedestrian number N_P and the probability density function (PDF) of the non-dimensional step frequency $p_{\tilde{\Omega}}(\tilde{\Omega})$. According to Piccardo and Tubino [10], the one-sided PSD of the modal force can be analytically defined as:

$$S_{\tilde{F}_j}(\tilde{\Omega}) \simeq \frac{N_P}{4} p_{\tilde{\Omega}}(\tilde{\Omega}) \quad (7)$$

Then, by introducing the admittance function $\chi_j(\tilde{\varepsilon}, C)$, a formulation of the PSD that can take into account the correlation among pedestrians and the structural mode shape is obtained [8]:

$$S_{\tilde{F}_j}(\tilde{\Omega}) = \frac{N_P}{4\tilde{\varepsilon}} p_{\tilde{\Omega}}(\tilde{\Omega}) \chi_j(\tilde{\varepsilon}, C) \quad (8)$$

where:

$$\chi_j(\tilde{\varepsilon}, C) = \int_0^1 \int_0^1 \text{Coh}_{\tilde{f}\tilde{f}}(\tilde{x}, \tilde{x}', \tilde{\varepsilon}, C) \varphi_j(\tilde{x}) \varphi_j(\tilde{x}') d\tilde{x} d\tilde{x}' \quad (9)$$

Ferrarotti and Tubino [8] propose a physically-based expression for the coherence function $\text{Coh}_{\tilde{f}\tilde{f}}(\tilde{x}, \tilde{x}', \tilde{\varepsilon}, C)$, representing the correlation among pedestrians. It depends on the pedestrian positions (\tilde{x} and \tilde{x}'), the width of the initial unitary step function $\tilde{\varepsilon}$, representing the distance that each pedestrian interposes to others in order to avoid contact, and a correlation coefficient C that accounts for the increase in synchronization with increasing pedestrian density.

2.2. Peak coefficient

The peak coefficient, adopted to calculate the maximum response from eq.(1), is given by [10, 17, 18, 19]:

$$g_{\tilde{p}_j} = \left(\sqrt{2 \ln 2 \tilde{v}_e \tilde{T}} + \frac{0.5772}{\sqrt{2 \ln 2 \tilde{v}_e \tilde{T}}} \right) \quad (10)$$

where \tilde{v}_e is the modified non-dimensional expected frequency of the response and \tilde{T} is the conventional non-dimensional time interval in which the maximum response is evaluated:

$$\tilde{T} = \omega_j N \frac{L}{c_m} = \frac{N}{\tilde{\Omega}_{cm}} \quad (11)$$

In eq.(11), c_m is the mean velocity, L/c_m is the mean duration of the footbridge crossing and N represents the number of groups of N_P pedestrians that subsequently cross the footbridge in order to make the process stationary. In [10], it is stated that good results are obtained for

a value N of about 10. Although the latter is also applied in this study, it is highlighted that concerning this parameter further validations are required. The modified expected frequency \tilde{v}_e is expressed as [19]:

$$\tilde{v}_e = (1.63q^{0.45} - 0.38)\tilde{v}_0 \quad (12)$$

where \tilde{v}_0 is the non-dimensional average frequency of the system $\tilde{v}_0 = \omega_j/(2\pi\omega_j) = 1/(2\pi)$, and q is the spectral bandwidth parameter, given by [18]:

$$q = \left(1 - \frac{\lambda_1^2}{\lambda_0\lambda_1}\right)^{1/2} \quad (13)$$

with $\lambda_n = \int_0^\infty \tilde{\Omega}^n S(\tilde{\Omega}) d\tilde{\Omega}$. For a lightly-damped system, the spectral bandwidth parameter q can be approximated by the following expression [18]:

$$q \approx 2\left(\frac{\xi}{\pi}\right)^{1/2} \quad (14)$$

According to [19], eqs. (10) and (12) are applicable in the range $0.1 \leq q \leq 1$ and $5 \leq v_0T \leq 1000$.

2.3. Outline of the procedure

The present section shows the main stages of the procedure for the estimation of the maximum dynamic response, presented in the flow chart of figure 1. For each value of pedestrian density ρ , the mean pedestrian velocity c_m is obtained from the relation proposed by Venuti and Bruno [21]:

$$c_m(\rho) = c_u \left\{ 1 - \exp \left[-\gamma \rho_{jam} \left(\frac{1}{\rho} - \frac{1}{\rho_{jam}} \right) \right] \right\} \quad (15)$$

where c_u is the free-flow speed, γ is a parameter depending on the travel purpose and ρ_{jam} is the jam density. The free-flow speed c_u , i.e. the mean pedestrian velocity in unrestricted traffic conditions, is a random variable that follows a Gaussian distribution characterized by the mean value $c_{um} = 1.34$ m/s and the standard deviation $\sigma_{c_u} = 0.26$ m/s [22]. Eq.(15) is a generic form of the velocity-density relation originally proposed by Weidmann, the so called *Kladek formula* [23]. By assuming $\gamma = 0.354$, $\rho_{jam} = 5.4$ ped/m² and $c_u = 1.34$ m/s, eq.(15) matches the original one [8]. Starting from the mean pedestrian velocity, Bruno and Venuti [24] propose a relation, based on a cubic curve fitting of the experimental data presented in [25], to evaluate the mean step frequency f_m :

$$f_m = 0.35c_m^3 - 1.59c_m^2 + 2.93c_m \quad (16)$$

The applicability limits for this empirical relation can be assumed either as [0.2; 2.5] m/s or [0.23; 2.2] m/s, in accordance with [24] or [25], respectively. Then, from the mean step frequency and its standard deviation (assumed equal to 0.173 Hz [20] independent on the pedestrian density), the PDF of the step frequency $p_{\tilde{\Omega}}(\tilde{\Omega})$ and the correlation coefficient C are defined. Consequently, the PSD of the modal force is obtained from eq.(8). The model of Piccardo and Tubino [10] and Ferrarotti and Tubino [8] is proposed for the fundamental harmonic of the walking force. In the present study, the contribution of the resonant harmonic is accounted for by: (1) defining a probability distribution of the frequency related to the first three harmonics (i.e. the k th harmonic is characterized by mean value and standard deviation which are k times the ones of the first harmonic), (2) calculating the response for all the harmonics and (3) estimating the total response as the envelope of the responses given by the three harmonics. The DLFs of the first three harmonics are calculated, according to Kerr [26], as follows:

$$\alpha_1 = -0.2649f_m^3 + 1.3206f_m^2 - 1.7597f_m + 0.7613, \alpha_2 = 0.07, \alpha_3 = 0.06 \quad (17)$$

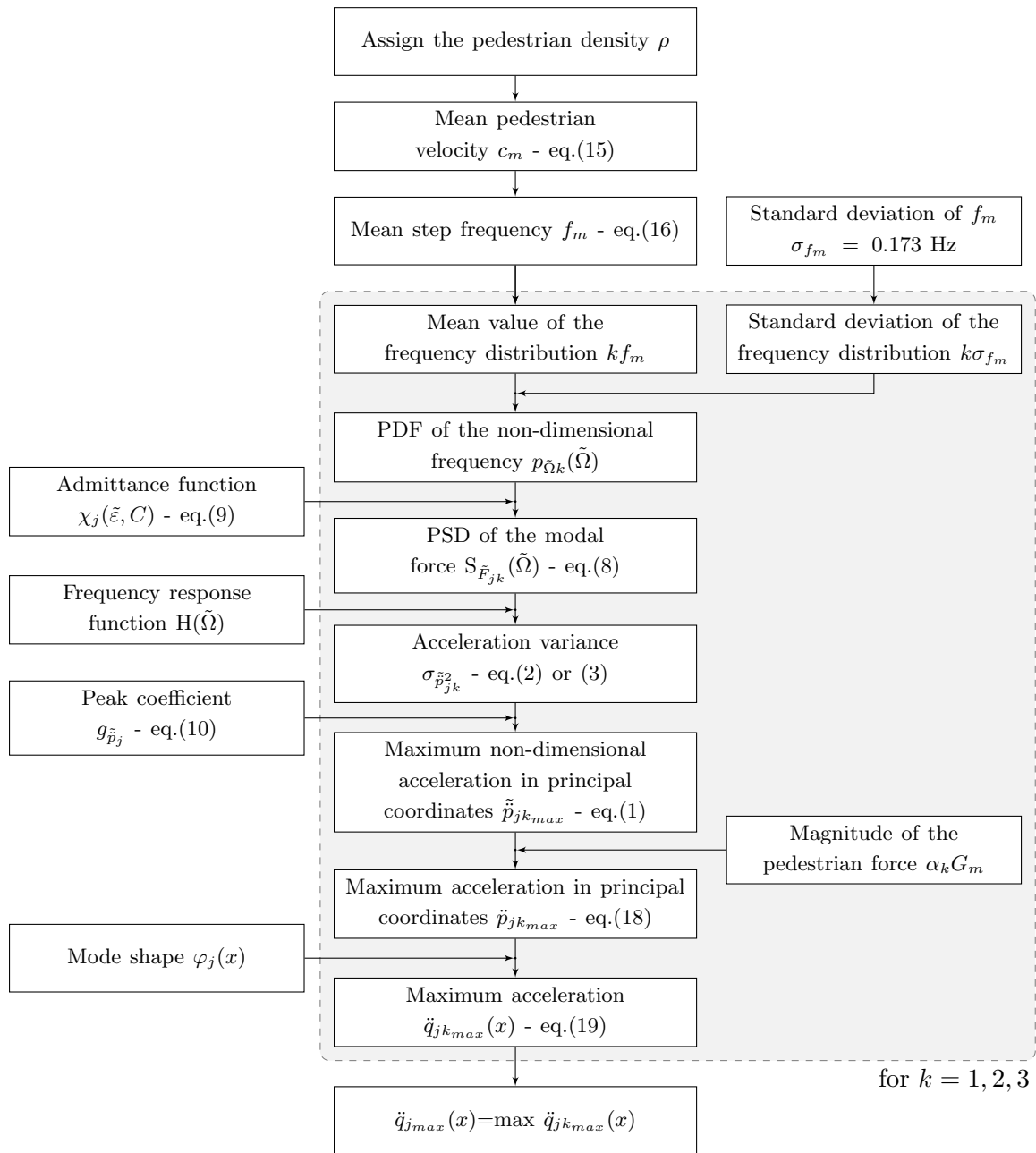


Figure 1. Flow chart of the procedure for estimating the maximum expected acceleration

valid when $1 \leq f_m \leq 2.65$ Hz.

The maximum non-dimensional accelerations of the principal coordinates $\ddot{p}_{jk_{max}}$ given by the contribution of the j th mode and the k th harmonic is evaluated from eq.(1) (note that eq.(1) is defined for the dimensional parameters, but it can be applied considering the non-dimensional acceleration \ddot{p}_j instead of \ddot{p}_j). By considering the force amplitude $\alpha_k G_m$, being α_k the DLF of the k th harmonic, the maximum acceleration of the principal coordinate $\ddot{p}_{jk_{max}}$ can be rewritten in dimensional form as follows:

$$\ddot{p}_{jk_{max}} = \alpha_k G_m \ddot{p}_{jk_{max}} \quad (18)$$

Then, considering that the structural response is dominated by the j th natural mode shape, the maximum acceleration in physical coordinates given by the k th harmonic is estimated as:

$$\ddot{q}_{kmax}(x) = \varphi_j(x) \ddot{p}_{jkmax} \quad (19)$$

Finally, the maximum acceleration $\ddot{q}_{max}(x)$ is obtained as the maximum among the responses due to the three harmonics.

3. Human-structure interaction effects

Pedestrians are mechanical systems which interact with the structure that is supporting them. Hence, in some cases the modal characteristics of the coupled crowd-structure system can significantly differ from those of the empty footbridge. The degree to which the dynamic behavior is modified is expected to increase with an increasing crowd to structural mass ratio, making these effects non-negligible for lightweight footbridges [11]. HSI-phenomena can be taken into account by characterizing the dynamic behavior of the crowd-structure coupled system. The reader is referred to [13] for a detailed description of the coupled system and corresponding system matrices. The low-frequency (0 – 10 Hz) behavior of the human body in the vertical direction can be represented by a highly-damped single degree of freedom (SDOF) system [11, 27]. As regards the parameters of the SDOF system representing the human body, extensive experimental investigations have been performed by Van Nimmen et al. [12, 13] to derive a reasonable range of human body model parameter values. The mechanical properties of the SDOF system ($f_{H1,z}$ and $\xi_{H1,z}$) largely depend on the posture. In the present study, stationary human body models representative of a body posture with one or two legs slightly bent are considered as an approximation of the postures assumed during the walking cycle. As the people are assumed to be stationary, the coupled system is time invariant [11]. In addition, the intra-subject variability can be accounted for by assuming a Gaussian distribution of the natural frequency and modal damping ratio of the human body model, while the mass is considered the same for each pedestrian, equal to the mean value of 70 kg. The mean values and coefficient of variation are based on the results reported in [27] and the results of experimental studies performed in [12, 13]:

$$f_{H1,z} \sim \mathcal{N}(3.25, 0.32) \text{ [Hz]} \quad (20)$$

$$\xi_{H1,z} \sim \mathcal{N}(0.30, 0.05) \text{ [-]} \quad (21)$$

In the present study, all pedestrians are given the same model parameters, equal to their mean value. The latter is in line with the fact that the spectral load model ([8, 10]) is for the evaluation of the mean value of the maximum response. The influence of inter-subject variability of human body model parameters remains to be investigated. According to the parametric study developed in [12, 13], the level of interaction between the two subsystems, i.e. the crowd and the footbridge, depends on the ratio of the natural frequency of the human body f_{H1} to that of one of the relevant mode of vibration of the empty footbridge f_B and can be summarized as follows:

- *For low frequency ratio* ($f_{H1}/f_B < 0.5$), the crowd is a much more flexible system than the supporting structure and the movements of the two subsystems remains strongly uncoupled;
- *For intermediate frequency ratios* ($0.5 < f_{H1}/f_B < 1.5$), the coupled system is characterized by a high level of interaction between the two subsystems, which is reflected in an increasing damping ratio and the change in the peak position of the coupled system FRF;
- *For high frequency ratios* ($f_{H1}/f_B > 1.5$), the crowd is much stiffer than the supporting structure and, for the first mode of vibration of the coupled system, the crowd moves in phase and along with the footbridge.

The HSI-effects can be characterized through the effective parameters of the coupled system, namely the effective frequency and damping ratio. These are quantified based on the comparison between the acceleration FRF of the empty footbridge ($H_B(\omega)$) and that of the coupled system ($H_{HB}(\omega)$). In particular, the effective frequency of the coupled system f_{eff} is given by:

$$f_{\text{eff}} = \frac{\omega_{HB}}{2\pi} \quad (22)$$

where ω_{HB} is the abscissa of the peak value of $H_{HB}(\omega)$. The effective damping ratio ξ_{eff} is defined as a measure for the change in maximum steady-state acceleration response $\ddot{u}_{HB,max}$ in relation to that of the empty footbridge $\ddot{u}_{B,max}$:

$$\ddot{u}_{HB,max} \propto \frac{1}{2\xi_{\text{eff}}m_B} \rightarrow \xi_{\text{eff}} = \frac{\ddot{u}_{B,max}}{\ddot{u}_{HB,max}} \xi_B = \frac{|H_B(\omega_B)|}{|H_{HB}(\omega_{HB})|} \xi_B \quad (23)$$

Finally, to account for the HSI-effects in the evaluation of the maximum response based on the spectral load model, the variance of the acceleration response is calculated from eq.(2) or (3) considering the FRF of the coupled system $H_{HB}(\omega)$ instead of that of the empty footbridge $H_B(\omega)$.

4. Parametric study

In the final section, the maximum structural response is predicted considering various footbridge parameters and a wide range of pedestrian density ρ , for simulations in sparse ($\rho < 1 \text{ ped/m}^2$) and dense ($\rho \geq 1 \text{ ped/m}^2$) crowd conditions. First, the relevant properties of the footbridge, the crowd and the corresponding parameter ranges of interest are defined. Second, the output quantities of interest are determined and finally the predicted structural response is evaluated in section 4.3.

4.1. Input parameters

4.1.1. Footbridge parameters The footbridge considered here is a simply supported beam with a span of 50 m and a bridge deck width of 3 m, for which only the contribution of the fundamental mode is considered. The corresponding mode shape is sinusoidal with a modal damping ratio ξ_B of 0.4% and a modal mass m_B of $25 \times 10^3 \text{ kg}$. The simulations are performed considering the natural frequency f_B of 2, 4 and 5 Hz, chosen as resonant with the first three harmonics of the walking force.

4.1.2. Crowd parameters Pedestrian densities ranging between 0.1 ped/m^2 and 2.5 ped/m^2 are considered, corresponding to a total number of pedestrians from 15 to 375. Analyses are performed considering the mean value of the free-flow speed, $c_u = c_{um} = 1.34 \text{ m/s}$ that leads to a mean step frequency f_m decreasing from 1.91 Hz to 1 Hz with increasing pedestrian density. However, to have a complete overview of the maximum response, the variability of the free-flow speed c_u has to be considered, in order to account for all the possible ranges of mean step frequency. The standard deviation of the step frequency is assumed equal to 0.173 Hz, as suggested by Matsumoto et al. [20]. The maximum response is predicted as the envelope of the responses given by the first three harmonics of the walking force, namely for each pedestrian density the contribution of the resonant harmonic is accounted for (figure 1).

4.2. Output quantities of interest

The parametric study aims to determine the maximum structural response for various footbridge and crowd parameters. The maximum response is evaluated through the procedure summarized in figure 1. In particular, the variance of the acceleration is calculated from eq.(2), to take into

account that the response can not be considered always as resonant and that, due to HSI-effects, the system could be no longer lightly-damped. Finally, the spectral bandwidth parameter q is evaluated from eq.(14).

4.3. Results

As a starting point, the behavior of the empty footbridge, i.e. when the HSI-effects are not taken into account, is discussed. Figure 2(a) presents the comparison between the center frequency of the Gaussian distribution for each harmonic and the considered fundamental natural frequencies of the footbridge. With reference to the footbridge with natural frequency of 2 Hz, it is observed that for pedestrian density lower than almost 0.7 ped/m² the mean step frequency (i.e. the mean frequency of the first harmonic) is basically (near-)resonant, and then, it decreases on increasing pedestrian density. For high pedestrian densities ($\rho > 1.7$ ped/m²), the second harmonic of the walking force becomes the resonant one. This behavior is reflected in the trend of the maximum response predicted without the HSI-effects, represented in figure 2(b). At the beginning, the maximum midspan acceleration is caused by the contribution of the first harmonic and increases on increasing pedestrian number. Then, when the mean step frequency moves far from the natural frequency, the acceleration decreases and finally grows up again due to the contribution of the second harmonic.

As regards the natural frequency of 4 Hz, the (near-)resonant contribution is given by the second harmonic for pedestrian densities lower than about 1.3 ped/m² and by the third one for higher densities, resulting in the maximum acceleration without HSI-effects shown in figure 2(c).

Finally, for the footbridge with natural frequency of 5 Hz, the contribution of the third harmonic is always the resonant one. Note that, for the natural frequency resonant with the third harmonic of the walking force, the value of 5 Hz is chosen instead of 6 Hz. In fact, considering the natural frequency of 6 Hz, the near-resonance is caused by low pedestrian densities, implying structural acceleration with negligible amplitude with respect to the other cases.

Then, the HSI-effects are accounted for by defining the coupled crowd-footbridge system (see section 3) characterized by the effective frequencies and damping ratios shown in figure 3. In particular, due to increasing number of pedestrians, the effective frequency of the coupled system decreases for the structure with natural frequency of 2 Hz, lower than the mean natural frequency of the human body ($f_{H1} = 3.25$ Hz). On the contrary, the effective frequency of the coupled system increases on increasing pedestrian density for the natural frequencies of 4 Hz and 5 Hz, higher than f_{H1} . The effective damping ratio increases monotonically with the pedestrian density but is highly dependent on the natural frequency of the footbridge. Increases of effective damping ratio up to 25% and 17% are obtained for natural frequencies of 4 and 5 Hz, respectively, while it reaches maximum value of 3% for the natural frequency of 2 Hz.

Indeed, maximum levels of interaction, in terms of increasing effective damping ratio, are reached when the frequency ratio f_{H1}/f_B is in the range [0.5; 1.5]. This happens for the natural frequencies of 4 and 5 Hz, characterized by a frequency ratio of 0.8125 and 0.65, respectively. On the contrary, for the footbridge with natural frequency of 2 Hz, the frequency ratio is higher than 1.5 ($f_{H1}/f_B = 1.625$), implying a lower increase in the effective damping ratio.

First, the effects of the effective frequency and damping ratio are analyzed separately. The effective frequency causes a change in the peak position due to the change in the density causing resonance between the pedestrian action and the coupled system.

For the natural frequency of 2 Hz, the effective frequency decreases on increasing pedestrian density and (near-)resonant conditions are obtained for higher values of density, so that the maximum acceleration is obtained for a pedestrian density of about 1.3 ped/m². Moreover, the acceleration reaches higher amplitudes because the resonance is caused by an higher number of pedestrians. On the contrary, for the cases of 4 and 5 Hz, the effective frequency increases on increasing pedestrian density and the peak is obtained for lower densities and characterized by

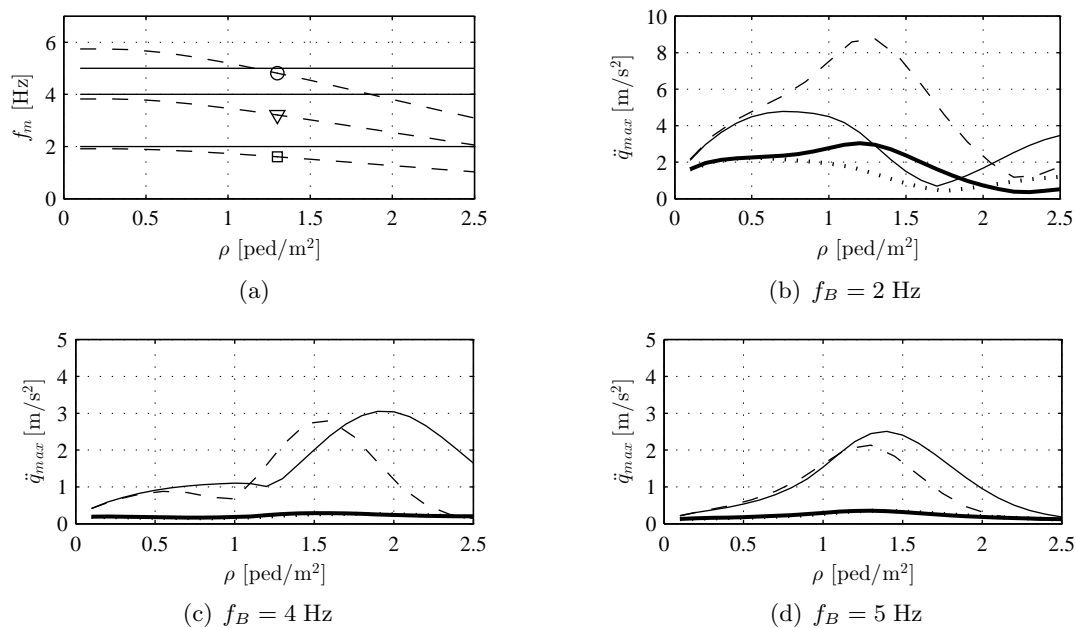


Figure 2. (a) Center frequency of the Gaussian distribution for the first (dashed line \square), second (dashed line ∇) and third (dashed line \circ) harmonic of the walking force and fundamental frequencies (solid black lines) of the footbridge (2, 4 and 5 Hz). (b, c, d) Maximum midspan acceleration obtained without the HSI-effects (solid line), with the HSI-effects (bold line) and with the HSI-effects accounting only for the effective frequency (dashed line) or the effective damping (dotted line).

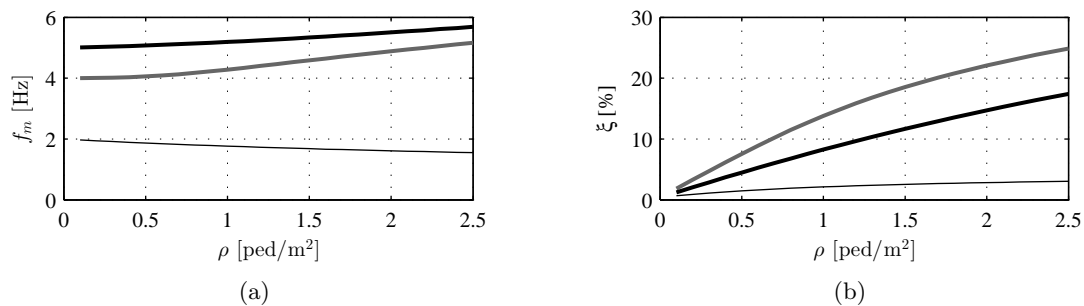


Figure 3. Effective (a) frequency and (b) damping ratio for the fundamental frequencies: 2 Hz (solid black line), 4 Hz (bold grey line) and 5 Hz (bold black line).

a slightly lower amplitude. The increasing effective damping ratio causes a general reduction of the acceleration amplitude, as expected.

Finally, the maximum response is predicted considering the interaction effects in terms of both effective frequency and damping ratio of the coupled system. For natural frequencies of the empty structure of 4 and 5 Hz, the main effect comes from the significant increase in the effective damping ratio. The latter causes a decrease of the maximum response of the coupled system in comparison to that of the empty footbridge from values of the order of 3 m/s² up to about 0.3 m/s². On the contrary, for the case of 2 Hz the increase of the damping ratio is not as high as in previous cases and the drop of the maximum acceleration is less sharp, decreasing from about 5 m/s² up to 3 m/s². Moreover, it is observed that in the pedestrian

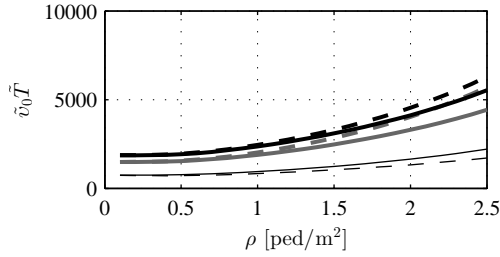


Figure 4. Product $\tilde{v}_0 \tilde{T}$ for the different natural frequencies of the footbridge: 2 Hz (solid black line), 4 Hz (bold grey line) and 5 Hz (bold black line), obtained considering (dashed lines) or not (solid lines) the HSI-effects.

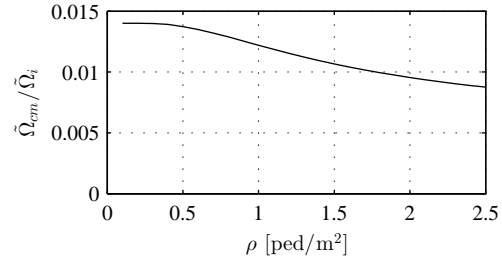


Figure 5. Trend of $\tilde{\Omega}_{cm}/\tilde{\Omega}_i$ with increasing pedestrian density.

density range $[1.2; 1.8]$ ped/m², the maximum acceleration of the coupled system is higher than the one obtained without the HSI-effects, as in that range the effect of the effective frequency prevails on the effect of the effective damping ratio.

As mentioned before, results are obtained evaluating the spectral bandwidth parameter from eq.(14). Note that if the spectral bandwidth parameter is evaluated from eq.(13) enforcing the conditions $0.1 \leq q \leq 1$, results (not reported here) are basically the same. On the contrary, if the spectral bandwidth parameter is evaluated from eq.(13) without enforcing the applicability conditions, in some cases it reaches values significantly lower than 0.1. This may cause unreliable estimation of the modified expected frequency of the response \tilde{v}_e and consequently of the peak coefficient $g_{\tilde{p}_j}$. Furthermore, figure 4 shows that the product $\tilde{v}_0 \tilde{T}$ reaches values significantly outside the applicability range $[5; 1000]$ especially for high values of pedestrian density. In fact, when the pedestrian density increases, the mean velocity decreases and the time interval \tilde{T} in which the maximum response is evaluated increases as well. The latter can be reduced by reducing the number N of footbridge crossings in order to make the process stationary. Considering that the maximum value reached by the product $\tilde{v}_0 \tilde{T}$ is about 6000, the number N of footbridge crossings should be reduced by 6 times to obtain $\tilde{v}_0 \tilde{T} \sim 1000$. However, as the number N of footbridge crossings is assumed equal to 10, to reduce it by 6 times implies to consider 1.6 pedestrian groups crossing the footbridge, which does not ensure the stationarity of the process. The above mentioned applicability ranges as well as the reliability of the peak coefficient estimated in presence of HSI (i.e. when the system is no longer lightly-damped) are to be addressed in the future research.

The spectral load model ([8, 10]) is based on the assumption that $\tilde{\Omega}_{cm} = \varepsilon \tilde{\Omega}_i$, with $\varepsilon \rightarrow 0$. For the presented simulations the ratio $\tilde{\Omega}_{cm}/\tilde{\Omega}_i$ ranges from 1.4% to 0.8% with increasing pedestrian density, as shown in figure 5, validating the hypothesis of ε being a small parameter.

5. Conclusions

A parametric study is performed to investigate the influence of the HSI on the maximum response evaluated based on an equivalent spectral model for the pedestrian-induced loading. The spectral load model proposed in literature allows to predict the maximum response for both unrestricted and crowded pedestrian traffic conditions. The present study in addition accounts for the HSI-effects through the dynamic characterization of the coupled system composed of the structure and pedestrians modeled as simple lumped parameter models. The parametric study is performed considering a wide range of crowd and footbridge parameters. To consider

resonance with the first, second and third harmonic of the walking force, the natural frequency of the empty footbridge is set to 2, 4 and 5 Hz.

Results show that effects of the HSI are highly dependent on the natural frequency of the footbridge. In fact, for the natural frequencies of 4 and 5 Hz, characterized by frequency ratios f_{H1}/f_B within the range of maximum interaction [0.5; 1.5], the increase in the effective damping ratio of the coupled system is such that the maximum response is drastically reduced. On the contrary, for the footbridge with natural frequency of 2 Hz there is a range of pedestrian density in which the maximum acceleration obtained with the HSI-effects is higher than that of the empty footbridge. This is due to the effect of the change in the frequency of the coupled system which is, in that range, more significant than the effect of the increasing damping ratio.

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