Towards a skewness index for the Italian stock market

Elyas Elyasiani*, Luca Gambarelli, Silvia Muzzioli**

* Fox School of Business and Management
Temple University, Philadelphia, PA, USA
Goldsmiths - University of London
email: elyas@temple.edu
** University of Modena and Reggio Emilia
CEFIN, Centro Studi Banca e Finanza
email: silvia.muzzioli@unimore.it

Towards a skewness index for the Italian stock market

Elyas Elyasiani[a], Luca Gambarelli, Silvia Muzzioli[b][*]

[a] *Fox School of Business and Management, Temple University, Philadelphia, PA, USA, elyas@temple.edu.*

[b] *Department of Economics and CEFIN, University of Modena and Reggio Emilia, Italy, silvia.muzzioli@unimore.it*

Abstract

The present paper is a first attempt of computing a skewness index for the Italian stock market. We compare and contrast different measures of asymmetry of the distribution: an index computed with the *CBOE SKEW* index formula and two other asymmetry indexes, the *SIX* indexes, as proposed in Faff and Liu (2014). We analyze the properties of the skewness indexes, by investigating their relationship with model-free implied volatility and the returns on the underlying stock index. Moreover, we assess the profitability of skewness trades and disentangle the contribution of the left and the right part of the risk neutral distribution to the profitability of the latter strategies. The data set consists of FTSE MIB index options data and covers the years 2011-2014, allowing us to address the behavior of skewness measures both in bullish and bearish market periods.

We find that the Italian *SKEW* index presents many advantages with respect to other asymmetry measures: it has a significant contemporaneous relation with both returns, model-free implied volatility and has explanatory power on returns, after controlling for volatility. We find a negative relation between volatility changes and changes in the Italian *SKEW* index: an increase in model-free implied volatility is associated with a decrease in the Italian *SKEW* index. Moreover, the *SKEW* index acts as a measure of market greed, since returns react more negatively to a decrease in the *SKEW* index (increase in risk neutral skewness) than they react positively to an increase of the latter (decrease in risk neutral skewness).

The results of the paper point to the existence of a skewness risk premium in the Italian market. This emerges both from the fact that implied skewness is more negative than physical one in the sample period and from the profitability of skewness trading strategies. In addition, the higher performance of the portfolio composed by only put options indicates that the mispricing of options is mainly focused on the left part of the distribution.

---

[*] Corresponding author: Department of Economics and CEFIN, University of Modena and Reggio Emilia, Viale Berengario 51, 41121 Modena (I), Tel. +390592056771 Fax +390592056947, ORCID iD: 0000-0003-0738-6690.

1. Introduction

Measuring the asymmetry of a distribution has gained an increasingly important role in finance in the recent decades. A symmetric volatility specification, precludes the disentanglement of positive and negative extreme stock price movements. The third order moment of a distribution (skewness) captures the asymmetry of the distribution. Hence, accounting for skewness allows one to model risk-neutral probability distributions with different shapes (more skewed to the left or to the right). Skewness can be measured with two alternative methods: first, using historical realizations of the underlying asset returns (called physical skewness) or second, by using options traded on the underlying asset (called implied skewness). While the first methodology is backward looking, the latter is forward looking in nature, since option prices reflect the investors' expectations about the underlying asset distribution at the maturity date. Many studies find that the option-implied information is superior to the historical approach (see e.g. Giamouridis and Skiadopoulos (2012) for a literature review) in forecasting future realized moments.

The most important signal of the importance of measuring the skewness of the financial markets is the listing on February 2011 at the Chicago Board Options Exchange (CBOE) of the $CBOE\ SKEW$ index. As explained in the CBOE white paper, $CBOE\ SKEW$ measures the risk-neutral skewness of the distribution of S&P500 log-returns at a 30-day constant maturity date and it complements the $CBOE\ VIX$ volatility index with an additional piece of information. In fact, while the $CBOE\ VIX$ measures the expected standard deviation of 30-day S&P500 log-returns, the $SKEW$ index describes the tail risk of the S&P500 distribution. Both are risk-neutral measures and therefore embed the investors' sentiment about the next-30-days volatility and skewness of the S&P500 log-returns. If the volatility index VIX measures the overall risk in the 30-day S&P500 log-returns, without disentangling the probability attached to positive and negative returns; the skewness index $SKEW$ measures the perceived tail risk, i.e. the probability that investors attach to extreme negative returns (if the $SKEW$ index is high, which points to a negative skewness and a distribution which is skewed to the left, extreme negative returns are more often expected than positive ones). The (negatively) skewed risk-neutral distribution points to the presence of sizable risk premiums in order to be hedged against negative realizations of the underlying asset (tail risk).

The existence of a skewness risk premium charged by the market, i.e. the difference between physical and risk-neutral skewness, is investigated in a few papers and the sign of the latter is debated. Lin et al. (2008) in the English market, find a positive relation between physical and risk-neutral skewness: the discrepancy between the two suggests that the market charges a high risk premium on downside index movements. Kozhan et al. (2013) generalize the notion of variance swap (Carr and Wu, 2009) to higher order moments: the fixed leg is the option-implied moment and the floating leg is the realized moment. The average profit from the strategy can be interpreted as the premium for being exposed to the moment's risk. In the S&P500 equity index options market they find that the average realized skew is negative and substantially smaller, in absolute terms, than the average implied skew. Elyasiani et al. (2014), in the time-period January 2005-December 2009, find that implied risk-neutral skewness is less negative than the subsequently realized one in the Italian index options market.

Another strand of literature investigates the skewness risk premium by using portfolio strategies consisting of positions in options and in the underlying asset. Javaheri (2005), based on the assumption that the option implied distribution is in general more negatively skewed than the historical one, finds mixed evidence on the profitability of skewness trades in the American market. Liu (2007) implements vega and delta neutral strategies by using FTSE 100 index options data and finds that portfolios with long positions in put options and short positions in call options achieve significant negative returns. Bali and Murray (2013) investigate the pricing of risk-neutral skewness by using options on individual stocks in the American market and find results consistent with a negative skewness risk premium and an investor's preference for positive skewness. Similar findings are obtained by Conrad et al. (2013) on a sample of individual stock options in the American market.

The predictive power of risk-neutral skewness on future realized returns is debated in the literature. In fact, if Bali and Murray (2013) and Conrad et al. (2013) find a negative relation, many other papers find a positive relation. Xing et al. (2010) find that stocks with the steepest smirks in the options market underperform stocks with a less pronounced smirk. Yan (2011), finds that low slope portfolios earn higher returns than high slope portfolios. Cremers and Weinbaum (2010) find that stocks with relatively expensive calls outperform stocks with relatively expensive puts. Rehman and Vilkov (2012) find that option-implied ex ante skewness is positively related to future stock returns. Last, Faff and Liu (2014), find that the more negatively skewed is the risk-neutral distribution, the lower the future returns in the SPX market. Stilger et al. (2015) argue that the underperformance of the portfolios with the lowest risk-neutral skewness is driven by those stocks that are perceived as overpriced by investors but hard to sell short.

To sum up, a positive relation between risk-neutral and physical skewness is generally found. The skewness risk premium is generally found to be significant but the evidence on the sign is mixed. The relationship between skewness and subsequently realized returns is debated: some papers find a positive relation, others a negative one. The majority of the papers have investigated the American market, and have used single stocks, very little is the evidence on market indexes, in particular, European ones.

In this setting, the present paper is a first attempt of filling the gap, in order to delineate a skewness index for the Italian stock market. In fact, in the Italian index-options market, while implied volatility is currently measured by the implied volatility index, called the IVI index (which is computed similarly to the VIX index), a measure of the asymmetry and tail risk (such as a skewness index) has yet to be introduced. We compute both an index similar to the $CBOE\ SKEW$ index and investigate also other asymmetry indexes, the SIX indexes, as proposed in Faff and Liu (2014). We analyze the properties of the skewness indexes, by investigating the relationship between the skewness measures, implied volatility and the returns on the underlying stock index. Moreover, we assess the profitability of skewness trades and disentangle the contribution of the left and the right part of the risk neutral distribution to the profitability of the latter strategies. The data set consists of FTSE MIB index options data and covers the years 2011-2014, allowing us to address the behavior of skewness measures both in bullish and bearish market periods.

The results show that in the Italian market, the risk-neutral distribution of the stock market index presents a negative asymmetry which is higher in absolute terms than the one of the physical distribution. This implies that there exist a negative skewness risk premium, which is supported by the empirical evidence that selling out-of-the-money puts and buying out-of-the-money calls is on average profitable. In addition, the higher performance of the portfolio composed by only put options indicates that the mispricing of options is mainly focused on the left part of the distribution. We find a negative relation between volatility changes and changes in the Italian $SKEW$ index: an increase in model-free implied volatility is associated with a decrease in the Italian $SKEW$ index (less negative risk neutral distribution). We do not find any significant relation between model-free implied volatility and the other asymmetry SIX_{mf} indexes.

By investigating the relation between the skewness indexes and market returns, we find that an increase in the $SKEW$ index (i.e. the risk neutral distribution becomes more negatively skewed), is associated with an increase in the returns. We also detect an asymmetric effect: a decrease in the $SKEW$ index is associated with a strong decrease in the returns, while an increase in the $SKEW$ index is associated with a less pronounced increase in the returns. The market reacts more negatively to decreases in the $SKEW$ index than it reacts positively to increases in the $SKEW$ index.

Therefore in this setting the $SKEW$ index acts as a measure of market greed and the opposite of the $SKEW$ index (risk neutral skewness) acts as a measure of market fear, since returns react more negatively to a decrease in the $SKEW$ index (increase in risk neutral skewness) than they react positively to an increase of the latter (decrease in risk neutral skewness). When skewness is proxied by the SIX_{mf} indexes, the slope coefficients are non-significant, pointing to the uselessness of the SIX_{mf} indexes as indicators of current risk. Therefore, we find that the $SKEW$ index presents many advantages with respect the SIX_{mf} indexes: it has a significant contemporaneous relation with both returns, model-free implied volatility and is still significant in the explanation of returns, even after having controlled for volatility. We also find weak evidence that positive changes in the $SKEW$ index are reflected in a negative return the following day, and that a positive return is reflected in an increase of the $SKEW$ index. This is in line with Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative, consistently with the so-called "bubble theory": if past returns have been high, this means that the bubble has been inflating and therefore a large drop can be expected when the bubble is going to burst. Given the possibility to use the Italian $SKEW$ index for settling portfolio strategies and to forecast future returns, and the properties of the $SKEW$ index as indicator of market greed, we believe that the results of the paper can be of importance for both investors and regulators.

The plan of the paper is as follows: in Section 2 we review the existing literature about skewness measuring and forecasting, in Section 3 we present the different skewness measures, in Section 4 we describe the data-set and the methodology in order to compute the skewness measures. In Section 5 we analyze the properties of the skewness indexes obtained and finally in Section 6 we investigate the profitability of skewness trades in line with Bali and Murray (2013), where three different portfolios (a $PUTCALL\ asset$, a $PUT\ asset$ and a $CALL\ asset$ are created in order to disentangle the contribution to the profitability of differences in the left or in the right part of the distribution or in both. The last section concludes.

2. Literature review

After the October 1987 crash, many authors recognize that the implied volatility of index options varies with a pre-specified pattern: out-of-the-money put options are more expensive than out-of-the-money call options (the so called skew or smirk). This phenomenon has been called (Rubinstein 1994) the "crash-o-phobia", since put options are deemed to be more expensive than call options because they provide protection against stock market crashes. Jackwerth and Rubinstein (1996)

investigate S&P 500 index option prices over an eight-year period from April, 1986 through December, 1993. They find that risk-neutral skewness and kurtosis show a discontinuity across the 1987 market crash: the risk-neutral probability of another significant decline in the S&P 500 index is increased after the crash. Aït-Sahalia and Lo (1998) propose a non-parametric technique for the estimation of the state price density implicit in option prices which is able to account for the skewness and the kurtosis of the risk-neutral density. Dennis and Mayhew (2002) investigate the volatility skew observed in S&P 500 index option from April, 1986 through December, 1996. The find that risk-neutral density tends to be more negatively skewed for stocks with higher betas, in periods of higher market volatility, and in periods when the implied density of the index is more negatively skewed. They also find evidence that some firm specific factors including liquidity and firm size are important in explaining the variation in the skew for individual firms, while they do not find a robust relationship between risk-neutral skewness and the underlying stock's leverage ratio.

Even though the skew pattern of implied volatilities has been widely documented in the literature, only recently it has attracted the attention of researchers from the modelling perspective. The skew is reflected in a (negatively) skewed risk-neutral distribution and this points to the presence of sizable risk premiums in order to be hedged against negative realizations of the underlying asset (tail risk). Some papers investigate the relation between the risk-neutral skewness of the index and the skewness of the individual stocks which are part of the index. Overall, they find that the risk-neutral distribution of the index is more negatively skewed than that of individual stocks. Bakshi et al. (2003) propose a formula (BKM formula from now on) to extract implied moments from a cross-section of option prices. The formula is model-free because it is not based on any option pricing model and it is consistent with many different asset price dynamics. The authors, by using options prices on the S&P 100 index and on the 30 largest stocks of the S&P 100 index in the period between January, 1991 and December, 1995, find that the risk-neutral distribution of the market index is more negatively skewed than that of the individual stocks. Similar results are obtained by Lin et al. (2008) who investigate the structure of the implied volatility smile in the English options market by using prices from 79 individual stock options and the FTSE 100 index options recorded from March 1992 through December 2002. They find that the slope of the implied volatility curve is significantly negative for both individual stock options and stock index options, however the risk-neutral skewness of individual stocks is less negative than that of the market index. Compared to the American market, the English market displays a flatter implied volatility skew. Moreover, they find a humped shape relationship between the underlying asset's skewness and the options' time-to-maturity: when the latter increases, the slope of the skew increases up to a

point above which it decreases for longer-term maturity options. They also find a significantly positive relation between the physical and the risk-neutral moment.

Other papers investigate the skewness risk premium i.e. the difference between physical and risk-neutral skewness. The first paper that points to the existence on a premium charged by the market on downside index movements is Foresi and Wu (2005), by analyzing twelve major equity indexes on ten years of data (May 1995-May 2005). Lin et al. (2008), on a data set made of individual stock options traded in the LIFFE find a positive relation between physical and risk-neutral skewness: the discrepancy between the two suggests that the market charges a high risk premium on downside index movements. More recently, Neuberger (2012) finds that implied skewness predicts future realized skewness, computed from high-frequency returns, in the S&P500 index options market from December 1997 to September 2009. By providing an unbiased estimate of the third moment, it is found that the realized second and third moments are highly correlated. Realized skewness increases with the horizon: investors with long horizons require a higher risk premium. Kozhan et al. (2013) generalize the notion of variance swap (Carr and Wu, 2009) to higher order moments: the fixed leg is the option-implied moment and the floating leg is the realized moment. The average profit from the strategy can be interpreted as the premium for being exposed to the moment's risk. In the S&P500 equity index options market in the period between January, 1996 and January, 2012, it is found that the average realized skew is negative and substantially smaller, in absolute terms, than the average implied skew. In addition, they show that the skew risk premium is closely related to the variance risk premium: they both vary over time and are driven by a common factor (strategies to capture one and hedge out exposure to the other earn insignificant trading profits). Elyasiani et al. (2014), in the time-period January 2005-December 2009 find that implied risk-neutral skewness is less negative than the subsequently realized one in the Italian index options market. Chang et al. (2013), by analyzing various stocks in the American market in the time period January 1996-January 2012, point to a negative skewness risk premium, which is economically significant and not explained by other common risk factors. Bali and Murray (2013), by investigating individual stocks in the American market in the time period January 1996-October 2010, find a negative skewness risk premium, reflected in a preference of the investors for assets with positive skewness.

Another strand of literature investigates the skewness risk premium by using portfolio strategies consisting of positions in options and in the underlying asset. Javaheri (2005) looks for profit opportunities arising from the mispricing of options. Based on the assumption that the option implied distribution is in general more negatively skewed than historical one, the author suggests to buy out-of-the-money calls and sell out-of-the-money put. This portfolio can be interpreted as an

insurance selling strategy. In fact the trade generates consistent profits if no crash happens but in case of a sudden downward large movement, it yields a significant loss. By using S&P500 options from January, 2002 to January, 2003, the author finds mixed evidence on the profitability of skewness trades. Liu (2007) implements vega and delta neutral strategies by using FTSE 100 index options data from January 1996 to April 2000. Portfolios with long positions in put options and short positions in call options achieve significant negative returns. The evidence suggests that out-of-the-money put options are overpriced compared to out-of-the-money call options. However, the profitability of the opposite strategy is unlikely to materialize because arbitrage profits are eroded by bid-ask spreads. Bali and Murray (2013) investigate the pricing of risk-neutral skewness by using options on individual stocks in the American market from January, 1996 to October, 2010. The portfolios are delta and vega neutral, isolating a position in skewness (hence the portfolios are called skewness assets). They find a strong and robust relationship between risk-neutral skewness (measured with BKM methodology) and the skewness asset returns which represent a long skewness position. They argue that this results are consistent with a negative skewness risk premium and an investor's preference for positive skewness. Similar results are obtained by Conrad et al. (2013) on a sample of individual stock options in the American market from January, 1996 to December, 2005. They find a strong and negative relationship between the third order moment and the subsequent returns: firms with less negative or positive skewness are associated with lower returns over the next month. This means that investors seem to prefer assets with positive skewness. The relationship between skewness and returns is both economically and statistically significant and persists even after various controls. Moreover, they find that risk-neutral skewness can be considered as a market-based forward looking prediction of physical skewness.

On this latter point, i.e. the relationship between skewness and subsequently realized returns the evidence in the literature is mixed. In fact, if Bali and Murray (2013) and Conrad et al. (2013) find a negative relation, many other papers find a positive relation. Xing et al. (2010) investigate the relationship between the shape of the volatility smirk and the cross-section of future equity returns, by using options on individual stocks in the time-period from January 1996 to December 2005. They find that stocks with the steepest smirks in the options market underperform stocks with a less pronounced smirk. Yan (2011), in the Option Metrics database from January 1996-January 2005, finds that low slope portfolios earn higher returns than high slope portfolios, where the average stock jump size is proxied by the slope of option implied volatility smile. Cremers and Weinbaum (2010), in the Option Metrics database from January 1996-January 2005 find that stocks with relatively expensive calls outperform stocks with relatively expensive puts. Rehman and Vilkov (2012) in the Option Metrics database from January 1996 to June 2007 find that option-implied ex

ante skewness is positively related to future stock returns. Last, Faff and Liu (2014) in the S&P index options market on the time-period from January 1996 to August 2013, find that the more negatively skewed is the risk-neutral distribution, the lower the future returns in the SPX market. Stilger et al. (2015) investigate the relationship between risk-neutral skewness of individual stocks and future realized stock returns over the period January, 1996 and December, 2012. By using a strategy that is long the quintile portfolio with the highest risk-neutral skewness stocks and short the quintile portfolio with the lowest risk-neutral skewness stocks, they find that the relationship is significant and positive. They also argue that the underperformance of the portfolios with the lowest risk-neutral skewness is driven by those stocks that are perceived as overpriced by investors but hard to sell short.

To sum up, the majority of the papers find a positive relation between risk-neutral and physical skewness, individual stocks are generally more negatively skewed than the market indexes. The skewness risk premium is generally found to be significant but the evidence on the sign is mixed (even if we can say that most of the studies find that risk-neutral skewness is generally greater in absolute value than physical skewness). The relationship between skewness and subsequently realized returns is debated: some papers find a positive relation, others a negative one. The majority of the papers have investigated the American market, and individual stocks, very little is the evidence on European markets and market indexes.

3. Skewness measures

Bakshi et al. (2003) develop a model-free method in order to extract volatility, skewness and kurtosis of the risk-neutral distribution from a cross section of option prices. Their methodology is called model-free, since it does not rely on any option pricing model, being consistent with many underlying asset price dynamics. Model-free skewness is obtained from the following equation:

$$SK(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3e^{r\tau}\mu(t,\tau)V(t,\tau) + 2\mu(t,\tau)^3}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^{3/2}} \tag{1}$$

with

$$\mu(t,\tau) \equiv E^q \ln[S(t+\tau)/S(t)] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau) \tag{2}$$

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2(1 - ln[K/S(t)])}{K^2} C(t,\tau;\, K)\, dK + \int_{0}^{S(t)} \frac{2(1 + ln[S(t)/K]}{K^2} P(t,\tau;\, K) dK \qquad (3)$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\, ln[K/S(t)] - 3\, ln[K/S(t)]^2}{K^2} C(t,\tau;\, K) dK$$

$$- \int_{0}^{S(t)} \frac{6\, ln\,[S(t)/K] + 3\, ln\,[S(t)/K]^2}{K^2} P(t,\tau;\, K) dK \qquad (4)$$

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12\, ln[K/S(t)]^2 - 4\, ln[K/S(t)]^3}{K^2} C(t,\tau;\, K) dK$$

$$+ \int_{0}^{S(t)} \frac{12\, ln\,[S(t)/K]^2 + 4\, ln\,[S(t)/K]^3}{K^2} P(t,\tau;\, K) dK \qquad (5)$$

where $C(t,\tau;\, K)$ and $P(t,\tau;\, K)$ are the prices of a call and a put option at time t with maturity τ and strike K, respectively, $S(t)$, is the underlying asset price at time t.

Equation (1) is used in the computation of the skewness index called $SKEW$, which measures the investors' perceived skewness of the Chicago Board Option Exchange. Since the risk-neutral skewness attains typically negative values for equity indices, in order to enhance the interpretation, CBOE defines $SKEW$ as:

$$SKEW = 100 - 10 \times SK \qquad (6)$$

where SK is the 30-day measure of risk-neutral skewness. Therefore, with a negative risk-neutral skewness, $CBOE\ SKEW$ attains positive values bigger than 100. $CBOE\ SKEW$ measures the slope of the implied volatility curve: the more the curve is steep, the higher the $SKEW$ index. Therefore, $CBOE\ SKEW$ can be also considered as a measure of the perceived tail risk of S&P500 log-returns at a 30-day horizon. Tail risk is the risk associated with an increase in the probability of extreme negative returns: returns two or more standard deviations below the mean (market crash, black swan). The probability of this type of events may be negligible for a normal distribution, but it could be significant for a skewed one with fat tails. This is the case of the distribution of S&P500 log-returns which have a sizeable left tail and it is therefore riskier than a normal distribution with the same mean and variance: $CBOE\ SKEW$ quantifies this additional risk. Historically, $CBOE\ SKEW$ has varied in a range of about 50 points around an average value of 115. Its maximum value is 146.88 reached on October 16, 1998 during the Russian crisis and after a surprising rate cut by the Fed. $CBOE\ SKEW$ reached its all-time low of 101.09 on March 21, 1991 at the end of the recession that started in July 1990. This means that the implied distribution of S&P500 log-returns has been historically always left-skewed.

It is worth noting that, in order to apply formula (1) in the financial market, where a continuum of option prices in strikes is not traded, both truncation and discretization errors occur. In the computation of the $CBOE\ SKEW$ index, only at-the-money and out-of-the money options with maturity of at least one week are considered. Furthermore, the interval of strike prices used is cut once two consecutive options with zero bid prices are found. As a result, if a change in volatility occurs then the number of options considered in the computation may change. Other critical issues in the CBOE methodology concern the linear interpolation between near and next term maturities, which may induce a bias if model-free skewness is not a linear function of maturity. Furthermore the use of the average between the lowest ask price and the highest bid price as a proxy of the option price may lead to errors when the bid-ask spreads are wide. Nonetheless, the $CBOE\ SKEW$ index formula is the market standard for the computation of skewness indexes nowadays.

In order to overcome some of the limits of the $CBOE\ SKEW$ methodology when only a few strike prices are traded, Faff and Liu (2014) propose a model-based methodology to compute skewness in a Black-Scholes framework, by using a state-preference pricing approach. They use Black-Scholes implied volatilities instead of the model-free formula extracted from a few options: only two at-the-money call and put options with maturity closest to a 30-day period.

They define the skewness index SIX, which is computed as the ratio of the lower partial moment volatility to the upper partial moment volatility of market returns as follows:

$$SIX = \frac{BEX}{BUX} \tag{7}$$

where BEX (the bear index) and BUX (the bull index) are the lower and upper partial moment volatility indexes of market returns. Liu (2014) define BEX^2 as a financial asset that pays a dollar amount of $ln(S_T/S_t)^2$ at some future date T, for every future index level S_T and spot price level S_t if $S_T \leq S_t$, or \$0 otherwise. BEX^2 can be obtained as:

$$BEX^2 = \sum_{s=1}^{S} \Phi_s \left[ln\left(\frac{S_T}{S_t}\right) - h \right]^2 I_{\ln(S_T/S_t) \leq h} \tag{8}$$

with S_t that is the current stock market index, S_T is the index value at time T, Φ_s is the risk-neutral probability of reaching S_T (or equivalently the state price density) and h is the threshold level that can be set to any arbitrary value (e.g. 0, the risk-free rate, or the expected return $E(R)$). Symmetrically BUX^2 is computed as follows:

$$BUX^2 = \sum_{s=1}^{S} \Phi_s \left[ln\left(\frac{S_T}{S_t}\right) - h \right]^2 I_{\ln(S_T/S_t) > h} \tag{9}$$

i.e. it pays a dollar amount of $ln(S_T/S_t)^2$ if $S_T > S_t$, or \$0 otherwise. The argument underlying the SIX formula is the following: if the risk-neutral distribution is symmetric, SIX in equation (7) is equal to 1; if the risk-neutral distribution is left (right) skewed, SIX is greater (lower) than 1.

Liu (2014) estimated the state prices as:

$$\Phi(K_i, K_{i+1}) = e^{-rT}\{N[d_2(K_i)]\} - \{N[d_2(K_{i+1})]\} \tag{10}$$

where $K_i < K_{i+1}$

$$d_2(K) = \frac{\ln(S_t/K) + (r - \partial - \sigma^2/2)T}{\sigma\sqrt{T}} \tag{11}$$

where ∂ is the dividend yield and σ is estimated as the average of four Black-Scholes implied volatilities from two at-the-money calls and two at-the-money puts with maturities the closest to a 30-day period. In order to discretize the state price density, Liu (2014) choose a grid of states spanning from 0.1 to 9999, with a 0.10 increments.

It is worth recalling that in order to capture the asymmetry of the distribution, many authors in the literature use the difference between the implied volatilities of two options with different moneyness as a proxy of risk-neutral skewness. Bali et al. (2014) use the difference in implied volatilities between an out-of-the-money call option and an out-of-the-money put option with delta equal to -0.25 and 0.25 respectively. Yan (2011) uses the difference in implied volatilities between a near-the-money put option and a near-the-money call option with delta equal to -0.5 and 0.5 respectively. Xing, Zhang and Zhao (2010) measure the slope of the volatility smile as the difference between the implied volatility of an out-of-the-money put option whose moneyness is between 0.80 and 0.95 and the implied volatility of an at-the-money call option with moneyness between 0.95 and 1.05. Similar proxies are used in the present paper in order to define portfolio trading strategies.

4. The data-set and the methodology for the computation of the skewness indexes

The data set consists of closing prices on FTSE MIB-index options (MIBO), recorded from 3 January 2011 to 28 November 2014. MIBO are European options on the FTSE MIB, which is a capital weighted index composed of 40 major stocks quoted on the Italian market. As for the underlying asset, closing prices of the FTSE MIB-index recorded in the same time period are used. The FTSE MIB is adjusted for dividends as follows:

$$\hat{S}_t = S_t e^{-\delta_t \Delta t} \tag{12}$$

where S_t is the FTSE MIB index value at time t, δ_t is the dividend yield at time t and Δt is the time to maturity of the option. As a proxy for the risk-free rate, Euribor rates with maturities one week, one, two and three months are used: the appropriate yield to maturity is computed by linear interpolation. The data-set for the MIBO is kindly provided by Borsa Italiana S.p.A; the time series of the FTSE MIB index, the dividend yield and the Euribor rates are obtained from Datastream.

Several filters are applied to the option data set. First, consistently with the computational methodology of other indexes (such as the $CBOE\ SKEW$), we eliminate options near to expiry which may suffer from pricing anomalies that might occur close to expiration (options with time to maturity of less than eight days). Second, following Ait-Sahalia and Lo (1998) only at-the-money option and out-of-the-money options are retained (put options with moneyness lower than 1.03 and call options with moneyness higher 0.97). Last, option prices violating the standard no-arbitrage constraints are eliminated.

In order to compute risk-neutral skewness, in this paper we follow two different model-free methods: we use both the methodology adopted by the CBOE that relies on the Bakshi et al. (2003) formula and the Faff and Liu (2014) formula with some modifications. The Faff and Liu (2014) formula provides an appealing intuition on the possibility to measure skewness as the ratio between the right and left part of the distribution of the asset return, however, it suffers from the following drawbacks. First, it is a model-based approach, since it relies on the Black-Scholes formula. Many papers in the literature have highlighted the inconsistency of the assumption of a constant volatility as supposed in the Black-Scholes model with the empirical evidence in the financial market. Second, it considers only four around-the-money options in the estimation of the implied volatility to plug in the Black-Scholes formula, discarding the other options traded, this results in a considerable loss of information. In order to overcome this limits we propose to compute the asymmetry index, which we indicate as $\overline{SIX}_{mf}$, in a model-free setting.

We stick to the Faff and Liu (2014) intuition of computing the ratio between the volatility of the left and the right part of the distribution, however, in order to have a model-free measure of the upside and downside volatility, we use the enhanced Derman and Kani method (Moriggia et al. 2009) in order to derive the risk-neutral distribution of the underlying asset at the maturity date T. The implied tree has uniformly spaced levels Δt apart. Let $j = 0, \ldots, n$ be the number of levels of the tree, that are spaced by $\Delta t = T/n$. As the tree recombines, $i = 1, \ldots, j + 1$ is the number of nodes at level j. The use of the Enhanced Derman and Kani method ensures the absence of no-arbitrage violations in the implied tree and it is motivated by its simplicity and the good replication of the smile pattern, as documented e.g. in Muzzioli 2013a (2013b) and Elyasiani et al. (2015).

The volatilities in the upper and lower part of the tree are computed as:

$$VOL_{UP}(t,T) = \sqrt{\sum_{i=1}^{j+1} \Phi_i [ln(S_i/S_t) - h]^2 \, I_{ln(S_i/S_t)>h}} \qquad (13)$$

$$VOL_{DW}(t,T) = \sqrt{\sum_{i=1}^{j+1} \Phi_i [ln(S_i/S_t) - h]^2 \, I_{ln(S_i/S_t)\leq h}} \qquad (14)$$

where Φ_i is the state price density and corresponds to the risk-neutral probability of reaching the ending node i at time T, with $i = 1, ..., j + 1$; $ln(S_i/S_t)$ is the log-return of the underlying asset at node i; S_i is the underlying asset price at the ending node i; S_t is the underlying spot price at time 0 and h is threshold level. In particular, following Faff and Liu (2014) we use two values for h:

- $h = 0$ to compute SIX_{mf0};
- $h = E(R)$ to calculate SIX_{mfR}.

The model-free skewness index $\overline{SIX}_{mf}$ is computed (following the Faff and Liu (2014) intuition) as follows:

$$\overline{SIX}_{mf} = \frac{VOL_{UP}}{VOL_{DW}} \qquad (15)$$

In order to have a constant 30-days measure for the implied skewness, we derive the skewness indexes by using a linear interpolation with the same formula which is adopted for the computation of the $CBOE\ SKEW$ index:

$$SK = w\ SK_{near} + (1 - w)\ SK_{next} \qquad (16)$$

with $w = \frac{T_{next}-30}{T_{next}-T_{near}}$, and T_{near} (T_{next}) is the time to expiration of the near (next) term options, SK_{near} (SK_{next}) is the skewness measure which refers to the near (next) term options, respectively. Physical moments are obtained from daily FTSE MIB log-returns by using a rolling window of 30 calendar days. In this way the physical measures refer to the same time-period covered by the risk-neutral counterparts.

Following the methodology adopted by the CBOE, to facilitate the interpretation we compute the skewness indexes as in equation (6). For the SIX_{mf} index $SK = (1 - \overline{SIX}_{mf0})$ or $(1 - \overline{SIX}_{mfR})$ in equation (6) in order to have the same interpretation: values above the threshold level 100 suggest that the distribution displays negative skewness and vice versa. Physical skewness is computed as in equation (6) for ease of comparison.

5. The results for the skewness indexes

Skewness indexes are depicted in Figure 1. We can observe that SIX_{mf0} and SIX_{mfR} show the same pattern, but SIX_{mf0} is shifted upward and its variation range is slightly narrower. This is due to the different barrier level h used in the two measures ($h = 0$ and $h = E(R)$ for SIX_{mf0} and SIX_{mfR} respectively). Compared to the latter measures, $SKEW$ displays a higher standard deviation. Moreover, SIX_{mf} indexes present a few number of peaks than $SKEW$.

Table 1 provides the summary statistics for the FTSE-MIB index returns, the model-free implied volatility (which is computed using the model-free methodology as in Muzzioli (2013) (with an extrapolation outside the existing domain of strike prices with a constant volatility function)), the skewness indexes (physical and risk-neutral), the daily changes in the model-free implied volatility and the daily changes in the risk-neutral skewness indexes. The physical returns display negative skewness and excess kurtosis and they are far from the normality assumption. For model-free implied volatility the hypothesis of a normal distribution is strongly rejected, indicating the presence of extreme movements in volatility. We can observe that all the skewness indexes are on average higher than the threshold level of 100. This suggests that in general, both physical and risk-neutral skewness are negative in the sample period, with the physical distribution less negatively skewed than the risk-neutral one (as measured by both the $SKEW$ and the SIX_{mf} indexes). Unlike previous evidence (Elyasiani et al. 2014) on a different time-period (2005-2009), we find that extreme price decreases are more likely than extreme price rises and that they are more often expected (under the risk-neutral distribution) than subsequently realized (similar findings are in Conrad et al. (2013)). All the skewness measures (both physical and risk neutral) display positive skewness and excess kurtosis and the hypothesis of a normal distribution is strongly rejected, indicating the presence of extreme movements also in the skewness measures. Physical skewness is the most symmetric one among skewness measures, followed by the $SKEW$ index. The SIX_{mf} indexes are the most far from the normal distribution. The SIX_{mf} indexes are not directly comparable in the levels to the other indexes. They are less volatile than $SKEW$ and on average SIX_{mf0} points to a more negative skewed distribution than SIX_{mfR}, being the distribution sliced slightly left of zero. This suggests that $E(R)$ implied in option prices is slightly less than zero.

In Table 2 we report the correlation coefficients between the skewness measures and other moments of the return distribution, both in the levels and in the daily changes. We can observe that the $SKEW$ index displays the highest correlation (0.156) with realized skewness, while the SIX_{mf} indexes are almost unrelated with physical skewness. $SKEW$ presents a positive correlation (0.21)

with daily returns, the highest in absolute value, while the SIX_{mf} indexes are almost unrelated to daily returns. Interestingly, while the $SKEW$ index has a negative correlation with the model-free implied volatility, the SIX_{mf} indexes show a positive correlation. Therefore, according to the $SKEW$ (SIX_{mf}) index, the risk-neutral distribution of the FTSE-MIB index returns is less (more) negatively skewed when model-free implied volatility is high. The value of the correlation between $SKEW$ and model-free implied volatility (-0.284) is similar to the value of the correlation between $CBOE\ SKEW$ index and $CBOE\ VIX$ index computed over the same period (-0.291). $SKEW$ also shows an average value of about 104 over the period 2011-2014, far lower than the corresponding average value of $CBOE\ SKEW$ index (123.68). This means that in general the risk-neutral distribution of log-returns in the Italian market is less asymmetric in the sample period than the one of the S&P 500 index. The correlation between the daily changes of the $SKEW$ index and the daily changes in model-free implied volatility is negative suggesting that a positive change in model-free implied volatility is associated to a negative change in the $SKEW$ index. Daily changes in the SIX_{mf} indexes are almost unrelated to volatility changes. The correlation between the daily changes of the $SKEW$ index and the returns is positive suggesting that a positive return is associated to a positive change in the $SKEW$ index. The SIX_{mf} indexes are almost unrelated to returns.

We report in Figure 2 the graphs of the $SKEW$ index and of the SIX_{mf0} index along with the FTSE MIB index (the graphical comparison with SIX_{mfR} is not reported, since it shares the same pattern of SIX_{mf0}). In order to investigate the relation between changes in the skewness measures and changes in model-free implied volatility, we estimate the following regression:

$$\Delta skewness_t = \alpha + \beta \Delta IV_t + \varepsilon_t \tag{17}$$

where $\Delta skewness_t$ is proxied by $\Delta SKEW_t$, ΔSIX_{mf0_t}, ΔSIX_{mfR_t} and provide the results in Table 3. The results point to a negative relation between volatility changes and changes in the $SKEW$ index (β statistically different from zero): an increase in model-free implied volatility is associated with a decrease in the $SKEW$ index (less negative risk neutral distribution). We do not find any significant relation between model-free implied volatility and the SIX_{mf} indexes. The results for the $SKEW$ index are consistent with the findings in Chang et al. (2013) in the S&P500 index options market. Moreover, Neuberger (2012) finds also a positive correlation coefficient (equal to 0.297 in the time period 1997-2009, on S&P500 index options market) between model-free variance and skewness, implying that the higher the variance, the less skewed the distribution. Recall that the correlation between volatility and returns is negative (leverage effect). A possible explanation is that when volatility is high, returns are low (for example in a stressed market or after a market

crash) and a repeat crash (as indicated by the *SKEW* index) may not be viewed as that likely. On the other hand, when volatility is low, returns are high (tranquil market) and the possibility of a crash is viewed as more probable.

In order to investigate the relation between changes in the skewness measures and the returns of the FTSE-MIB index, we estimate the following regression:

$$R_t = \alpha + \beta \Delta skewness_t + \varepsilon_t \qquad (18)$$

where $\Delta skewness_t$ is proxied by $\Delta SKEW_t$, ΔSIX_{mf0_t}, ΔSIX_{mfR_t} and provide the results in Table 4. We want to assess if the skewness measures can be considered as indicators of market stress or market greed. The slope coefficient of changes in the *SKEW* index is positive and significant, this means that an increase in the *SKEW* index (i.e. the risk neutral distribution becomes more negatively skewed), is associated with an increase in the returns. Therefore positive peaks in *SKEW* can be considered as indicators of investors' greed, negative peaks in *SKEW* can be considered as indicators of investors' fear (market stress). When skewness is proxied by the SIX_{mf} indexes, the slope coefficients are non-significant, pointing to the uselessness of the SIX_{mf} indexes as indicators of current risk.

In order to disentangle the effect of positive and negative changes in the *SKEW* index on FTSE-MIB index returns, we divide the changes in the *SKEW* index into positive ones:

$$\Delta SKEW_t^+ = \Delta SKEW_t \text{ if } \Delta SKEW_t > 0, \text{ otherwise } \Delta SKEW_t^+ = 0 \qquad (19)$$

and negative ones:

$$\Delta SKEW_t^- = \Delta SKEW_t \text{ if } \Delta SKEW_t < 0, \text{ otherwise } \Delta SKEW_t^- = 0 \qquad (20)$$

and estimate the following regression:

$$R_t = \alpha + \beta_1 \Delta SKEW_t^+ + \beta_2 \Delta SKEW_t^- + \varepsilon_t \qquad (21)$$

Table 5 reports the regression results. Both positive and negative changes in the *SKEW* index are highly significant. Both slope coefficients are positive, however, the slope coefficient of negative changes in the *SKEW* index is more than twice the slope of positive changes in the *SKEW* index. This indicates an asymmetric effect: a decrease in the *SKEW* index is associated with a strong decrease in the returns, while an increase in the *SKEW* index is associated with a less pronounced increase in the returns. The market reacts more negatively to decreases in the *SKEW* index than it reacts positively to increases in the *SKEW* index. Therefore in this setting the *SKEW* index acts as a measure of market greed and the opposite of the *SKEW* index (risk neutral skewness) acts as a measure of market fear, since returns react more negatively to a decrease in the *SKEW* index

(increase in risk neutral skewness) than they react positively to an increase of the latter (decrease in risk neutral skewness). An increase (decrease) in the $SKEW$ index means that the risk-neutral distribution becomes more (less) negatively skewed, and this is seen as good (bad) news from the return side.

As a last step, in order to assess the relation among returns, changes in model-free implied volatility and changes in the skewness measures, we estimate the following regression:

$$R_t = \alpha + \beta_1 \Delta IV_t + \beta_2 \Delta skewness_t + \varepsilon_t \tag{22}$$

where $\Delta skewness_t$ is proxied by $\Delta SKEW_t$, ΔSIX_{mf0_t} , ΔSIX_{mfR_t}, ΔIV_t is the change in model-free implied volatility, and provide the results in Table 6. We can see that the beta coefficient of changes in model-free implied volatility is highly significant in every regression. While changes in the $SKEW$ index are significant, changes in the SIX_{mf} indexes are not significant. Therefore, the $SKEW$ index presents many advantages with respect the SIX_{mf} indexes: it has a significant contemporaneous relation with both returns, model-free implied volatility and is still significant in the explanation of returns, even after having controlled for volatility.

As a second goal of the study, we want to assess if the $SKEW$ index can be used in order to forecast future market returns. In a previous study, Muzzioli (2013) find that changes in implied volatility (as measured by both Black-Scholes implied volatility and model-free implied volatility) can be used as an early-warning of market stress, and that the returns have explanatory power in forecasting future implied volatility. Pan and Poteshman (2006) find that publicly observable option signals are able to predict stock returns for only the next one or two trading days, before stock prices subsequently reverse. Other papers (Xing et al. 2010) find that the predictability from volatility smirks persists for a period much longer (six months).

To this end, we estimate a vector autoregression (VAR) model as follows:

$$R_t = c + \sum_{l=1}^{K} a_l \Delta SKEW_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t \tag{23}$$

$$\Delta SKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta SKEW_{t-l} + u_t \tag{24}$$

with $k = 2$, chosen according to both the Schwarz and Hannan-Quinn information criterions, in order to keep the model as parsimonious as possible, the estimation output is reported in Table 7. We perform a Granger causality test, the null hypothesis is $a_l = 0$, $l = 1,2$ in order to see if $\Delta SKEW$ does not Granger cause R in the first regression and R does not Granger cause $\Delta SKEW$ in the second regression. The results are reported in Table 8. The VAR estimates, show that changes in

$SKEW$ index can be explained by past returns (one lag) and by past changes in the $SKEW$ index (lags one and two). On the other hand, returns cannot be explained by past returns, but by past changes in the $SKEW$ index (one lag). The Granger causality test shows that the null hypothesis that returns (changes in the $SKEW$ index) do not Granger cause changes in the $SKEW$ index (returns) is marginally rejected (at the 5% level). Therefore there is weak evidence that positive changes in the $SKEW$ index are reflected in a negative return the following day, on the other hand, a positive return is reflected in an increase of the $SKEW$ index in the following day. This is in line with Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative, consistently with the so-called "bubble theory": if past returns have been high, this means that the bubble has been inflating and therefore a large drop can be expected when the bubble is going to burst.

To investigate further this issue, in Figure 3 it is proposed a comparison between the FTSE MIB index and model-free implied volatility. Looking at the graph we can observe two different medium term trends: a negative one (bearish market) characterized by a slightly higher volatility between February 2011 and the end of July 2012 and a positive one (bullish market) in the second part of the sample period, which is characterized by a lower volatility. We may address the inversion in trend to the positive effect of the "whatever it takes" London Speech of the ECB President Mario Draghi (July 26, 2012) that put an end to the acute phase of the European sovereign debt crisis. Therefore, in order to assess the behavior of the skewness indexes in high and low volatility periods, we split the data set accordingly and report in Table 9 the descriptive statistics of the skewness indexes in the two sub-periods. Physical skewness is negative in the first time period characterized by a bearish market and slightly positive in the second time period characterized by a bullish market. Risk-neutral skewness indexes attain in both sub-periods a value higher than 100, pointing to an overall negative skewness. The $SKEW$ index is high in the tranquil period and low in the turmoil period pointing to a more negatively skewed distribution in the low volatility period, consistently with the findings in Neuberger (2012), in the S&P500 options market: in the period when index volatility was very low (2003-2007) skewness was relatively high, whereas skewness was rather low in the volatility spike of 2008. When the market returns are positive (bullish market) risk-neutral skewness tends to be more negative. On the other hand, in periods of bearish market, risk-neutral skewness tends to be more positive, since investors are expecting an inversion of the tendency. In fact, when the market is bearish, investors may purchase out-of-the-money calls instead of the underlying asset, shifting the risk-neutral distribution to the right. The information we obtain from the SIX_{mf} is the opposite. SIX_{mf} is more consistent with physical skewness which is less negative (more positive) in the low volatility period. The SIX_{mf} index is consistent with Dennis

and Mayhew (2002) who find that risk-neutral density tends to be more negatively skewed for stocks in periods of higher market volatility.

The difference between physical and risk-neutral skewness (as measured by the $SKEW$ index) is higher in the bullish market period. This indicates the presence of a skewness risk premium in the Italian market, which is higher in the low volatility period. This means that in bullish market periods investors expect a more negatively skewed risk-neutral distribution than it is subsequently realized. This is investigated further by means of portfolio trading strategies in the following section.

6. Trading strategies

The difference between risk-neutral and physical skewness may be exploited by skewness trades which allow to profit from an overvalued or undervalued third moment. Javaheri (2005) suggests to buy out-of-the-money calls and sell out-of-the-money puts when the implied third moment is undervalued with respect to the physical one. This strategy is exploited also in Bali and Murray (2013) where three different skewness assets are used to test mispricing in different portions of the risk-neutral density of returns. Therefore, in order to assess if it is possible to exploit the difference between risk-neutral and physical skewness, in line with Bali and Murray (2013), we create three different portfolios: a $PUTCALL\ asset$ (a short position in out-of-the-money puts and a long position in out-of-the-money calls) a $PUT\ asset$ (a short position in out-of-the-money puts and a long position in at-the-money puts) and a $CALL\ asset$ (a long position in out-of-the-money calls and a short position in at-the money calls). In order to isolate the effect of skewness, the exposure to changes in the underlying asset (delta-neutral) and volatility (vega-neutral) is removed.

The $PUTCALL\ asset$ is designed to change value if there is a change in the skewness of the risk-neutral return density coming either from a change in the left tail or from a change in the right tail, or from both:

$$PUTCALL\ asset = C_{OTM} - \frac{V_{C_{OTM}}}{V_{P_{OTM}}}\ P_{OTM} - \left(\Delta_{C_{OTM}} - \frac{V_{C_{OTM}}}{V_{P_{OTM}}}\Delta_{P_{OTM}}\right)S \qquad (25)$$

where C_{OTM} and P_{OTM} indicate the price of out-of-the-money call and put respectively, Δ is the delta of the option, V is the vega of the option and S is the position in the underlying asset. The return of the $PUTCALL\ asset$ is expected to be positive if OTM calls are undervalued relative to OTM puts. This condition is consistent with an implied distribution more negatively-skewed than the physical one.

The *PUT asset* is designed to change value if there is a change in the skewness of the underlying asset coming from a change in the left tail of the risk-neutral density:

$$PUT\ asset = -P_{OTM} + \frac{V_{P_{OTM}}}{V_{P_{ATM}}} P_{ATM} - \left(-\Delta_{P_{OTM}} + \frac{V_{P_{OTM}}}{V_{P_{ATM}}} \Delta_{P_{ATM}} \right) S \qquad (26)$$

where P_{OTM} and P_{ATM} indicate the price of out-of-the-money put and at-the-money put respectively, Δ is the delta of the option, V is the vega of the option and S is the position in the underlying asset. The return of the *PUT asset* is expected to be positive if OTM puts are overvalued relative to ATM puts.

The *CALL asset* is designed to change value if there is a change in the skewness of the underlying asset arising from a change in the right tail of the risk-neutral density.

$$CALL\ asset = C_{OTM} - \frac{V_{C_{OTM}}}{V_{C_{ATM}}} C_{ATM} - \left(\Delta_{C_{OTM}} - \frac{V_{C_{OTM}}}{V_{C_{ATM}}} \Delta_{C_{ATM}} \right) S \qquad (27)$$

where C_{OTM} and C_{ATM} indicate the price of out-of-the-money put and at-the-money put respectively, Δ is the delta of the option, V is the vega of the option and S is the position in the underlying asset. The return of the *CALL asset* is expected to be positive if OTM calls are undervalued relative to ATM calls.

We create skewness assets by using next-term option prices, that usually have a maturity between 30 and 70 days. The options with the closest strike price to the underlying asset value are taken to be the at-the-money options. Out-of-the-money options are taken to be the ones whose strike price to underlying asset price ratio is the closest to 0.90 for puts and 1.10 for call options respectively. In order to have delta and vega neutral portfolios, trades are set at t and are closed at day $t + 1$. Daily profits and losses are computed as the difference between the value of the portfolios in $t + 1$ and in t and represent the daily risk premium for being exposed to skewness. Daily return is computed as:

$$r = \frac{P_{t+1} - P_t}{|P_t|} \qquad (28)$$

where P_{t+1} and P_t are the prices of the skewness asset at day $t + 1$ and t respectively. In line with Bali and Murray (2013) we use the absolute value of the skewness asset price at time t because skewness asset prices are not guaranteed to be positive.

The cumulative return of the three skewness assets is reported in Figure 4. We can observe that the cumulative return of all skewness assets is positive during the sample. This means that the implied distribution is in general more negatively-skewed with respect to the physical one. This result is consistent with the literature that documents the overvaluation of out-of-the-money put options with respect to out-of-the-money call options (see e.g. Javaheri (2005) and Liu (2007)). The descriptive

statistics of the skewness assets' returns are reported in Table 10. Average daily returns are ascertained to be statistically different from zero, by using Newey West. The $PUTCALL$ $asset$ achieves the best performance with a cumulative return of 56.43%, the average daily return is statistically different from zero, pointing to a heavy overvaluation of out-of-the-money put options and a symmetrical undervaluation of out-of-the-money call options. The PUT $asset$ achieves a cumulative return of 49.16% and the average daily returns are statistically different from zero. This result suggests that out-of-the-money put options are highly overvalued with respect to at-the-money ones. The $CALL$ $asset$ realizes a cumulative return of 10.64%, however the average daily return is not statistically different from zero. Therefore the undervaluation of out-of-the-money call options with respect to at-the-money ones is not statistically significant. We can conclude that the mispricing of options is concentrated in the left tail of the distribution.

7. Conclusions

In this paper we have analyzed different skewness measures in the Italian equity index options market. We have investigated both the CBOE methodology to compute a skewness index ($SKEW$) and two model-free measure based on the ratio between the volatility in the left and in the right part of the risk-neutral distribution (SIX_{mf} indexes). The CBOE methodology yields a skewness index which is negatively related to the model-free implied volatility, consistently with the results obtained by Han (2008) and Faff and Liu (2014) in the S&P500 options market. Unlike $SKEW$, the SIX_{mf} indexes have a weak positive correlation with model-free implied volatility. We find a negative relation between volatility changes and changes in the Italian $SKEW$ index: an increase in model-free implied volatility is associated with a decrease in the Italian $SKEW$ index (less negative risk neutral distribution). We do not find any significant relation between model-free implied volatility and the other asymmetry indexes (SIX_{mf}). Unlike the SIX_{mf} indexes, $SKEW$ index can also be considered as a predictor of future realized skewness, thanks to the good correlation with future realized skewness.

By investigating the relation between the skewness indexes and market returns, we find that an increase in the $SKEW$ index (i.e. the risk neutral distribution becomes more negatively skewed), is associated with an increase in the returns. We also detect an asymmetric effect: a decrease in the $SKEW$ index is associated with a strong decrease in the returns, while an increase in the $SKEW$ index is associated with a less pronounced increase in the returns. Therefore in this setting the $SKEW$ index acts as a measure of market greed and the opposite of the $SKEW$ index (risk neutral skewness) acts as a measure of market fear, since returns react more negatively to a decrease in the

$SKEW$ index (increase in risk neutral skewness) than they react positively to an increase of the latter (decrease in risk neutral skewness). When skewness is proxied by the SIX_{mf} indexes, the slope coefficients are non-significant, pointing to the uselessness of the SIX_{mf} indexes as indicators of current risk. Therefore, we find that the $SKEW$ index presents many advantages with respect the SIX_{mf} indexes: it has a significant contemporaneous relation with both returns, model-free implied volatility and is still significant in the explanation of returns, even after having controlled for volatility. We also find weak evidence that positive changes in the $SKEW$ index are reflected in a negative return the following day, and that a positive return is reflected in an increase of the $SKEW$ index. This is in line with Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative, consistently with the so-called "bubble theory": if past returns have been high, this means that the bubble has been inflating and therefore a large drop can be expected when the bubble is going to burst.

The results of the paper point to the existence of a skewness risk premium in the Italian market. This emerges both from the fact that implied skewness is more negative than physical one in the sample period and from the profitability of skewness trading strategies. The positive returns of the three portfolios (a short position in out-of-the-money puts and a long position in out-of-the-money calls; a short position in out-of-the-money puts and a long position in at-the-money puts; a long position in out-of-the-money calls and a short position in at-the money calls) confirm that the implied distribution of log-returns is more skewed than the physical one. In addition, the higher performance of the portfolio composed by only put options indicates that the mispricing of options is mainly focused on the left part of the distribution.

Given the properties of the Italian $SKEW$ index, we believe that the results of the paper can be of importance for both investors and regulators. Investors may take advantage of the discrepancy between physical and risk neutral skewness by creating skewness assets and use skewness in order to forecast future returns. Regulators may view the $SKEW$ index as indicator of market greed and settle the appropriate actions.

This analysis is preliminary and should be extended in many directions. Further research is needed in order to assess the relationship among implied moments and the study of other asymmetry measures which, similarly to the portfolio strategies, are able to capture changes in the implied distribution coming from the different tails. Moreover, being the skewness coefficient a normalized measure which is divided by variance, the study of non-normalized measures which react only to asymmetry and not to both asymmetry and variance will be useful to better understand the properties of the skewness indexes.

References

Ait-Sahalia, Y., & Lo, A.W. (1998). Nonparametric estimation of state-price densities implicit in financial asset prices. *Journal of Finance,* 53(2), 499-547.

Ait-Sahalia, Y., & Lo, A.W. (2000). Nonparametric risk management and implied risk aversion. *Journal of Econometrics,* 94, 9-51.

Amaya, D., Christoffersen P., Jacobs K., & Vasquez, A. (2015). Does Realized Skewness Predict the Cross-Section of Equity Returns? *Journal of Financial Economics, forthcoming.*

Arrow, K. J. (1964). The role of securities in the optimal allocation of risk-bearing. *Review of Economic Studies*, 31, 91-96.

Arrow, K. J., & Debreu, G. (1954). Existence of an Equilibrium for a Competitive Economy. *Econometrica*, 22 , 265-290.

Back, K., (1993). Asymmetric information and options. *Review of Financial Studies,*6, 435–472.

Bakshi, G., & Kapadia, N. (2003). Delta-hedged gains and the negative volatility risk premium. *Review of Financial Studies,* 16, 527-566.

Bakshi, G., Kapadia, N., & Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies,* 16, 101-143.

Bakshi, G., & Madan, D. (2000). Spanning and Derivative Security Valuation. *Journal of Financial Economics*, 55, 205–238.

Bali, T. G., Hu, J., & Murray, S., (2014). Option Implied Volatility, Skewness, and Kurtosis and the Cross-Section of Expected Stock Returns. *Georgetown McDonough School of Business Research Paper. Available at http://dx.doi.org/10.2139/ssrn.2322945.*

Bali, T. G., & Murray, S. (2013). Does Risk-Neutral Skewness Predict the Cross-Section of Equity Option Portfolio Returns? *Journal of Financial and Quantitative Analysis,* 48(4), 1145-1171.

Black, F. (1975). Fact and fantasy in the use of option. *Financial Analysts Journal,* 31, 36–41, 61–72.

Black, F., & Scholes, M. (1973). Pricing of Options and Corporate Liabilities. *Journal of Political Economy,* 81(3), 637-654.

Bollen, N. P., & Whaley, R. E. (2004). Does Net Buying Pressure Affect the Shape of Implied Volatility Functions? *Journal of Finance,* 59, 711-753.

Breeden, D. T., & Litzenberger, R. H. (1978). Prices of state-contingent claims implicit in option prices. *Journal of Business,* 51, 621-651.

Britten-Jones, M., & Neuberger, A. (2000). Option prices, implied price processes, and stochastic volatility. *Journal of Finance,* 55(2), 839-866.

Carr, P., & Madan, D. (1998). Towards a theory of volatility trading. In Jarrow R. (Ed.), *Volatility: New Estimation Techniques for Pricing Derivatives,* 417-427, Vol. Risk Books.

Carr, P., & Madan, D. (2001). Optimal Positioning in Derivative Securities. *Quantitative Finance*, 1, 19–37.

Carr, P., & Wu, L. R. (2006). A Tale of Two Indices. *Journal of Derivatives,* 13(3), 13-29.

Carr, P., & Wu, L. R. (2009). Variance Risk Premia. *Review of Financial Studies,* 22(3), 1311-1341.

CBOE (2009). The CBOE Volatility index: VIX. *CBOE White Paper.*

CBOE (2010). The CBOE Skew Index. *CBOE White Paper.*

Chang, E. C., Zhao, H., & Zhang, J. E. (2013). The Relation between Physical and Risk-neutral Cumulants. *International Review of Finance,* 13(2).

Conrad, J., Dittmar, R. F., & Ghysels, E. (2013). Ex Ante Skewness and Expected Stock Returns. *Journal of Finance*, 68, 85-12.

Conrad, J., Kapadia, N., & Xing, Y. (2014). Death and jackpot: Why do individual investors hold overpriced stocks? *Journal of Financial Economics*, 113(3), 455-475

Coval, J. D., & Shumway, T. (2001). Expected options returns. *Journal of Finance*, 56, 983-1009.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics,* 7, 229-263.

Cremers, M., & Weinbaum, D. (2010). Deviations from Put-Call Parity and Stock Return Predictability. *Journal of Financial and Quantitative Analysis*, 45(2), 335-367.

Debreu, G. (1959). Theory of Value. *New York: Wiley*.

Demeterfi, K., Derman, E., Kamal, M., & Zou, J. (1999). More than you ever wanted to know about volatility swaps. *Goldman Sachs Quantitative Strategies Research Notes*.

Dennis, P., & Mayhew, S. (2002). Risk-neutral skewness: evidence from stock options. *Journal of Financial and Quantitative Analysis,* 37, 471-493.

Derman, E., & Kani, I. (1994). Riding on a smile. *Risk,* 7(2), 32-39.

Diavatopoulos, D., Doran, J. S., Fodor, A. & Peterson, D. R. (2012). The information content of implied skewness and kurtosis changes prior to earnings announcements for stock and option returns. *Journal of Banking & Finance Volume,* 36(3), 786–802.

Elyasiani, E., Muzzioli, S., & Ruggieri, A. (2014). Pricing and forecasting performance of option implied trees and implied moments. *Proceedings of the Annual Meeting of the Financial Management Association, 2015 Orlando.*

Faff, R. W., & Liu, Z. F. (2014). Hitting SKEW for SIX. 27th *Australasian Finance and Banking Conference 2014 Paper. Available at http://dx.doi.org/10.2139/ssrn.2190724*

Foresi, S., & Wu, L. (2005). Crash-O-Phobia: A Domestic Fear or a Worldwide Concern? *The Journal of Derivatives*, 13(2) 8–21.

Fu, X., Arisoy, Y. E., Shackleton, M. B., & Umutlu, M. (2013). Option-Implied Volatility Measures and Stock Return Predictability. *Available at http://dx.doi.org/10.2139/ssrn.2378969.*

Garleanu, N., Pederson, L. H., & Poteshman, A. (2009). Demand-Based Option Pricing. *Review of Financial Studies,*22, 4259-4299.

Giamouridis, D., & Skiadopoulos, G. (2012). The Informational Content of Financial Options for Quantitative Asset Management: A Review. In Scherer B., & Winston K. (Eds.) *Handbook of Quantitative Asset Management*, 243-265, Oxford University Press.

Giot, P. (2005). Relationships Between Implied Volatility Indexes and Stock Index Returns. *The Journal of Portfolio* Management, 31(3), 92-100

Han, B. (2008). Investor Sentiment and Option Prices. *The Review of Financial Studies*, 21(1).

Harvey, C. R., & Siddique, A. (2000). Conditional skewness in asset pricing tests. *Journal of Finance,* 55, 1263–1295.

Jackwerth, J. C. (2000). Recovering Risk Aversion From Option Prices and Realized Returns. *Review of Financial Studies,* 13, 433–451.

Jackwerth, J. C., & Rubinstein, M. (1996). Recovering Probability Distributions from Option Prices. *Journal of Finance,* 51, 1611–31.

Javaheri, A. (2005). *Inside Volatility Arbitrage : The Secrets of Skewness.* John Wiley & Sons, Hoboken, New Jersey.

Jiang, G. J., & Tian, Y. S. (2005). The model-free implied volatility and its information content. *Review of Financial Studies,* 18(4), 1305-1342.

Jiang, G. J., & Tian, Y. S. (2007). Extracting model-free volatility from option prices: An examination of the VIX index. *Journal of Derivatives,* 14(3), 35-60.

Kozhan, R., Neuberger, A., & Schneider, P. (2013). The Skew Risk Premium in the Equity Index Market. *Review of Financial Studies,* 26, 2174-2203.

Lin, B-H., Chang, I-J., & Paxon, D. A. (2008). Smiling less at LIFFE. *Journal of Futures Markets,* 28 (1), 57–81

Liu, X. (2007). Returns to trading portfolios of FTSE 100 index options. *Applied Financial Economics,* 17(15), 1211-1225.

Liu, Z. F. (2014). Partial Moment Volatility Indices. *Available at http://dx.doi.org/10.2139/ssrn.2401543.*

Manaster, S., & Rendleman, R. J. (1982). Option Prices as Predictors of Equilibrium Stock Prices. *Journal of Finance,* 37, 1043-1057.

Merton, R. (1980). On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics,* 8, 323-361.

Moriggia, V., Muzzioli, S., & Torricelli, C. (2009). On the no-arbitrage condition in option implied trees. *European Journal of Operational Research,* 193(1), 212-221.

Muzzioli, S. (2010). Option-based forecasts of volatility: an empirical study in the DAX-index options market. *The European Journal of Finance,* 16(6), 561-586.

Muzzioli, S. (2010). The relation between implied and realised volatility in the DAX index options market. *Mathematical and Statistical Methods for Actuarial Sciences and Finance,* 215-224,

Springer Milan.

Muzzioli, S. (2013a). The forecasting performance of corridor implied volatility in the Italian market. *Computational Economics*, 41 (3), 359-386.

Muzzioli, S. (2013b). The Information Content of Option-Based Forecasts of Volatility: Evidence from the Italian Stock Market. *The Quarterly Journal of Finance*, 3(1).

Muzzioli, S. (2015). The optimal corridor for implied volatility: from calm to turmoil periods. *Journal of Economics and Business*, 81, 77-94.

Muzzioli, S., Ruggieri, A., & De Baets, B. (2015). A comparison of fuzzy regression methods for the estimation of the implied volatility smile function. *Fuzzy Sets and Systems*, 2015, 131-143.

Muzzioli, S., & Torricelli, C. (2000). Combining the theory of evidence withy fuzzy sets for binomial option pricing. *Working paper n. 312, Dep. of Economics, University of Modena and Reggio Emilia.*

Muzzioli, S., & Torricelli, C. (2002). Implied trees in illiquid markets: a Choquet pricing approach. *International Journal of Intelligent Systems,* 17(6), 577-594.

Muzzioli, S., & Torricelli, C. (2005). The pricing of options on an interval binomial tree: an application to the DAX-index option market. *European Journal of Operational Research*, 163, 192-200.

Neuberger, A. (2012). Realized Skewness. *The Review of Financial Studies*, 25.

Neumann, M., & Skiadopoulos, G. (2013). Predictable Dynamics in Higher Order Risk-Neutral Moments: Evidence from the S&P 500 options. *Journal of Financial and Quantitative Analysis,* 48.

Pan, J., & Poteshman, A. (2006). The information in option volume for future stock prices. *Review of Financial Studies* 19, 871-908.

Panigirtzoglou, N., & Skiadopoulos, G. (2004). A new approach to modeling the dynamics of implied distributions: Theory and evidence from the S&P 500 options. *Journal of Banking and Finance*, 28, 1499-1520.

Rehman, Z., & Vilkov, G. (2012). Risk-Neutral Skewness: Return Predictability and Its Sources. *Available at http://dx.doi.org/10.2139/ssrn.1301648.*

Rouah, F. D., & Vainberg, G. (2007). *Option Pricing Models and Volatility Using Excel-VBA*. John Wiley & Sons, Hoboken, New Jersey.

Rubinstein, M. (1985). Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978. *The Journal of Finance,* 40(2), 455–480.

Rubinstein, M. (1994). Implied Binomial Trees. *Journal of Finance*, 49, 771-818.

Skiadopoulos, G. (2004). The Greek Implied Volatility Index: Construction and Properties, *Applied Financial Economics,* 14(16), 1187–1196.

Stilger, P. S., Kostakis, A., & Poon, S-H. (2015). What Does Risk-Neutral Skewness Tell Us About Future Stock Returns? *Available at http://dx.doi.org/10.2139/ssrn.2417021*.

Whaley, R. E. (2000) The Investor Fear Gauge. *Journal of Portfolio Management* 26(3), 12–17.

Xing, Y., Zhang, X., & Zhao, R. (2010). What Does the Individual Option Volatility Smirk Tell us About Future Equity Returns? *Journal of Financial and Quantitative Analysis,* 45, 641–662.

Yan, S. (2011). Jump Risk, Stock Returns, and Slope of Implied Volatility Smile. *Journal of Financial Economics,* 99, 216–233.

Table 1 – Descriptive statistics

	$SKEW_{PH}$	$SKEW$	SIX_{mf0}	SIX_{mfR}	IV	R	ΔIV	$\Delta SKEW$	$\Delta SKEW^+$	$\Delta SKEW^-$	ΔSIX_{mf0}	ΔSIX_{mfR}
Mean	100.13	103.78	103.11	101.44	0.10	0.00	0.00	0.00	0.01	-0.01	0.00	0.00
Median	100.08	103.84	103.27	101.62	0.09	0.00	0.00	0.00	0.00	-0.00	-0.00	0.00
Maximum	103.52	126.36	115.99	112.24	0.22	0.06	0.30	0.15	0.15	0.00	0.13	0.11
Minimum	95.97	89.11	96.31	95.27	0.04	-0.07	-0.52	-0.20	0.00	-0.20	-0.10	-0.09
Std. Dev.	1.11	4.56	2.03	1.77	0.03	0.02	0.08	0.03	0.02	0.02	0.02	0.02
Skewness	-0.00	0.39	0.65	0.49	1.29	-0.24	-0.81	0.31	3.13	-3.10	0.22	0.21
Kurtosis	4.37	4.92	5.97	5.61	4.51	4.44	6.82	7.70	15.39	20.13	4.74	4.44
Jarque-Bera	76.58	174.53	428.85	319.10	362.52	92.98	700.75	913.73	7833.07	13493.71	130.34	91.87
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: The table reports the descriptive statistics of physical and risk-neutral skewness indexes, the model-free implied volatility, FTSE MIB returns and daily changes in volatility and skewness measures. We indicate as $SKEW_{PH}$ the index of subsequently realized skewness in the next 30 days, $SKEW$ is the index we compute using the CBOE methodology, SIX_{mf0} and SIX_{mfR} refer to the SIX_{mf} indexes computed as the ratio between upside and downside corridor implied volatilities with barriers equal to 0 and R respectively, where R is the expected return; IV is the model-free implied volatility on a monthly basis; R is the FTSE MIB daily return (continuously compounded); $\Delta SKEW^+$ and $\Delta SKEW^-$ are the positive and negative changes in the $SKEW$ index respectively.

Table 2 – Correlation table

	$SKEW_{PH}$	$SKEW$	SIX_{mf0}	SIX_{mfR}	IV	R	ΔIV	$\Delta SKEW$	$\Delta SKEW^+$	$\Delta SKEW^-$	ΔSIX_{mf0}	ΔSIX_{mfR}
$SKEW_{PH}$	1.000											
$SKEW$	0.156	1.000										
SIX_{mf0}	0.030	0.063	1.000									
SIX_{mfR}	0.047	0.090	0.991	1.000								
IV	0.008	-0.284	0.209	0.142	1.000							
R	-0.017	0.208	-0.038	-0.032	-0.117	1.000						
ΔIV	0.057	-0.145	0.031	0.023	0.134	-0.573	1.000					
$\Delta SKEW$	0.003	0.354	-0.061	-0.036	-0.059	0.439	-0.435	1.000				
$\Delta SKEW^+$	-0.026	0.352	-0.063	-0.063	-0.060	0.286	-0.432	0.830	1.000			
$\Delta SKEW^-$	0.034	0.214	-0.062	-0.039	-0.034	0.431	-0.264	0.788	0.310	1.000		
ΔSIX_{mf0}	0.004	0.009	0.565	0.565	0.006	-0.026	0.031	-0.061	-0.038	-0.0620	1.000	
ΔSIX_{mfR}	0.005	0.020	0.561	0.564	0.004	-0.015	0.023	-0.036	-0.0206	-0.0390	0.997	1.000

Note: The table reports the correlation coefficients between the measures used in the study. For the definition of the measures see Table 1.

Table 3 - Regression output for the changes in the skewness measures and the changes in model-free implied volatility

	α	β	R^2
$\Delta\, SKEW_t$	-0.000 (0.766)	-0.177 (0.000)	0.189
$\Delta\, SIX_{mf0_t}$	-0.000 (0.976)	0.009 (0.351)	0.003
$\Delta\, SIX_{mfR_t}$	-0.000 (0.975)	0.006 (0.487)	0.001

Note: The table presents the estimated output of the regression: $\Delta skewness_t = \alpha + \beta\Delta IV_t + \varepsilon_t$, where for $\Delta skewness_t$ we use changes in the three risk-neutral skewness measures: $SKEW$, SIX_{mf0}, SIX_{mfR}; p-values in parentheses.

Table 4 - Regression output for the changes in the skewness measures and daily returns on the FTSE-MIB.

	α	β	R^2
$\Delta\, SKEW_t$	-0.000 (0.922)	0.237 (0.000)	0.192
$\Delta\, SIX_{mf0_t}$	-0.000 (0.986)	-0.019 (0.414)	0.001
$\Delta\, SIX_{mfR_t}$	-0.000 (0.987)	-0.013 (0.618)	0.000

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta\Delta skewness_t + \varepsilon_t$, where for $\Delta skewness_t$ we use changes in the three risk-neutral skewness measures: $SKEW$, SIX_{mf0}, SIX_{mfR}; p-values in parentheses.

Table 5 - Regression output for positive and negative changes in the $SKEW$ index and daily returns on the FTSE-MIB.

α	β_1	β_2	R^2
-0.002 (0.000)	0.140 (0.000)	0.349 (0.000)	0.212

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta_1\Delta SKEW_t^+ + \beta_2\Delta SKEW_t^- + \varepsilon_t$; p-values in parentheses.

Table 6 - Regression output for the changes in the skewness measures, changes in model-free implied volatility and daily returns on the FTSE-MIB.

	α	β_1	β_2	R^2
$\Delta\,SKEW_t$	-0.000 (0.944)	0.126 (0.000)	-0.103 (0.000)	0.373
$\Delta\,SIX_{mf0_t}$	-0.000 (0.886)	-0.006 (0.760)	-0.126 (0.000)	0.328
$\Delta\,SIX_{mfR_t}$	-0.000 (0.885)	-0.002 (0.925)	-0.126 (0.000)	0.328

Note: The table presents the estimated output of the regression: $R_t = \alpha + \beta_1\Delta skewness_t + \beta_2\Delta IV_t + \varepsilon_t$, where for $\Delta skewness_t$ we use changes in the three risk-neutral skewness measures: $SKEW$, SIX_{mf0}, SIX_{mfR}; p-values in parentheses.

Table 7 - VAR Estimation output

	R_t	$\Delta\,SKEW_t$
R_{t-1}	0.009831 (0.27549)	0.182693 (2.86033)
R_{t-2}	-0.019276 (-0.53848)	0.066370 (1.03584)
$\Delta\,SKEW_{t-1}$	-0.042144 (-2.11985)	-0.306903 (-8.62454)
$\Delta\,SKEW_{t-2}$	0.001620 (0.08129)	-0.085880 (-2.40804)
c	-2.93E-06 (-0.00544)	-0.000221 (-0.22926)

Note: The table reports the estimation output (t-stat in parentheses) of the VAR model:

$$R_t = c + \sum_{l=1}^{K} a_l\Delta\,SKEW_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t$$

$$\Delta\,SKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l\Delta\,SKEW_{t-l} + u_t$$

Table 8 - Granger causality test between daily returns on the FTSE-MIB and daily changes in $SKEW\ index$.

Null Hp.	X^2	p-value
$\Delta SKEW$ does not Granger cause R	7.12	0.029
R does not Granger cause $\Delta SKEW$	8.68	0.013

Note: The table reports the Granger causality test for the VAR model as defined in note to Table 7.

Table 9 – Descriptive statistics of skewness measures in the two sub-periods.

03/01/2011 – 25/07/2012: *Bearish market*				
	$SKEW_{PH}$	$SKEW$	SIX_{mf0}	SIX_{mfR}
Mean	100.43	103.35	103.47	101.68
Median	100.43	103.65	103.50	101.77
Maximum	103.09	114.77	115.99	112.24
Minimum	97.95	93.68	96.31	95.27
Std. Dev.	0.93	3.70	2.41	2.08
Skewness	0.25	-0.09	0.65	0.57
Kurtosis	0.17	0.00	2.82	2.83
26/07/2012 – 28/11/2014: *Bullish market*				
	$SKEW_{PH}$	$SKEW$	SIX_{mf0}	SIX_{mfR}
Mean	99.93	104.16	102.86	101.28
Median	99.82	104.10	103.12	101.57
Maximum	103.52	126.36	109.22	107.05
Minimum	95.97	89.11	99.46	98.15
Std. Dev.	1.18	5.03	1.67	1.51
Skewness	0.13	0.41	0.15	0.03
Kurtosis	1.59	1.77	0.17	-0.01

Note: for the definition of the measures see Table 1.

Table 10 - Skewness assets returns for the entire sample period

	PUTCALL asset	*PUT asset*	*CALL asset*
Cumulative return	56.43%	49.16%	10.64%
Average daily return	0.05%	0.04%	0.01%
t-statistic	5.49	4.05	0.95
p-value	0.000	0.000	0.343
Average ann. return	11.44%	10.28%	2.73%
Annualized volatility	4.14%	5.03%	5.70%

Note: The table reports the descriptive statistics for the Skewness assets returns used in the study in order to disentangle the contribution to the profitability of differences between the physical and the risk-neutral distribution in the left (*PUT asset*) or in the right (*CALL asset*) parts of the distribution or in both (*PUTCALL asset*) .

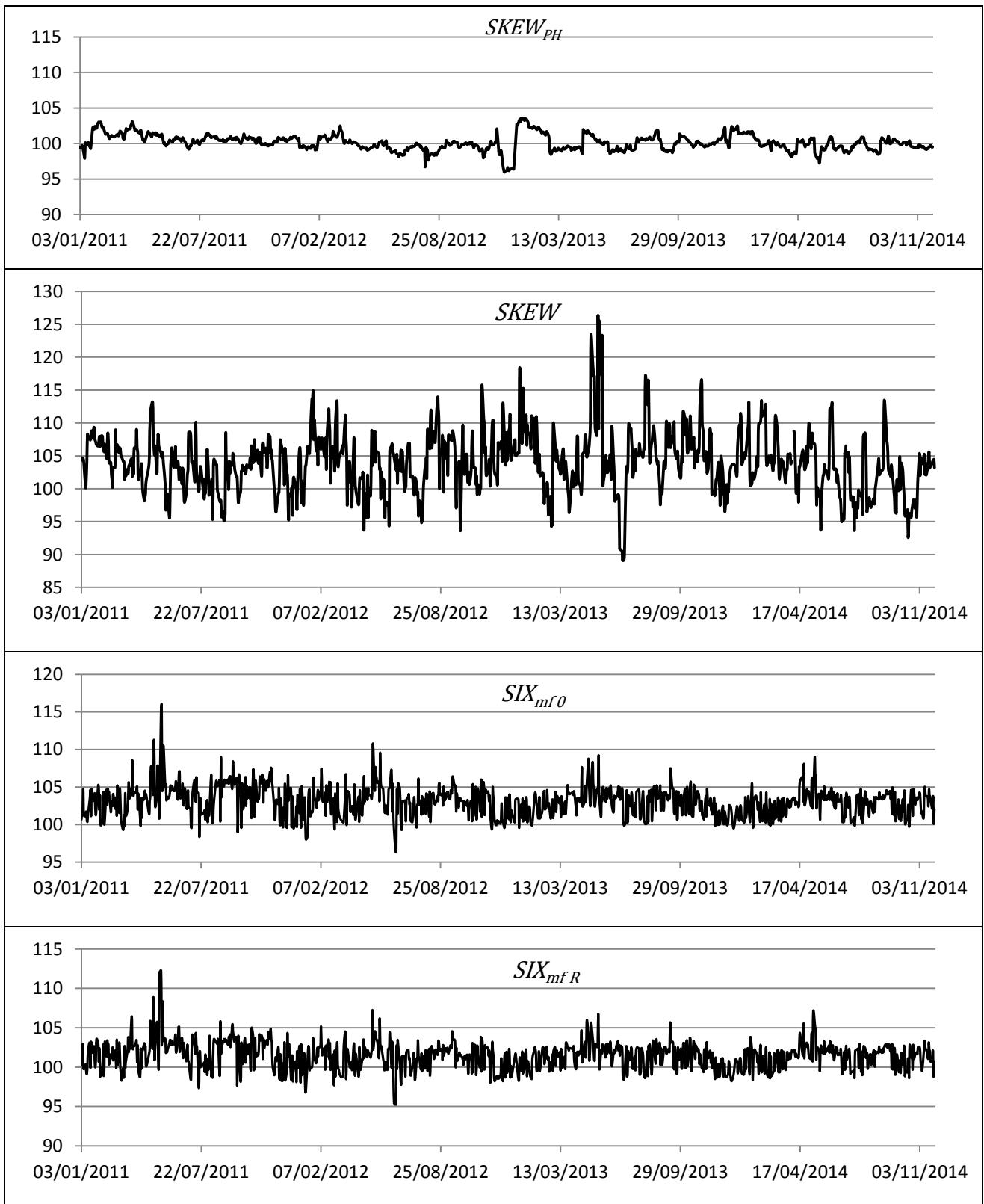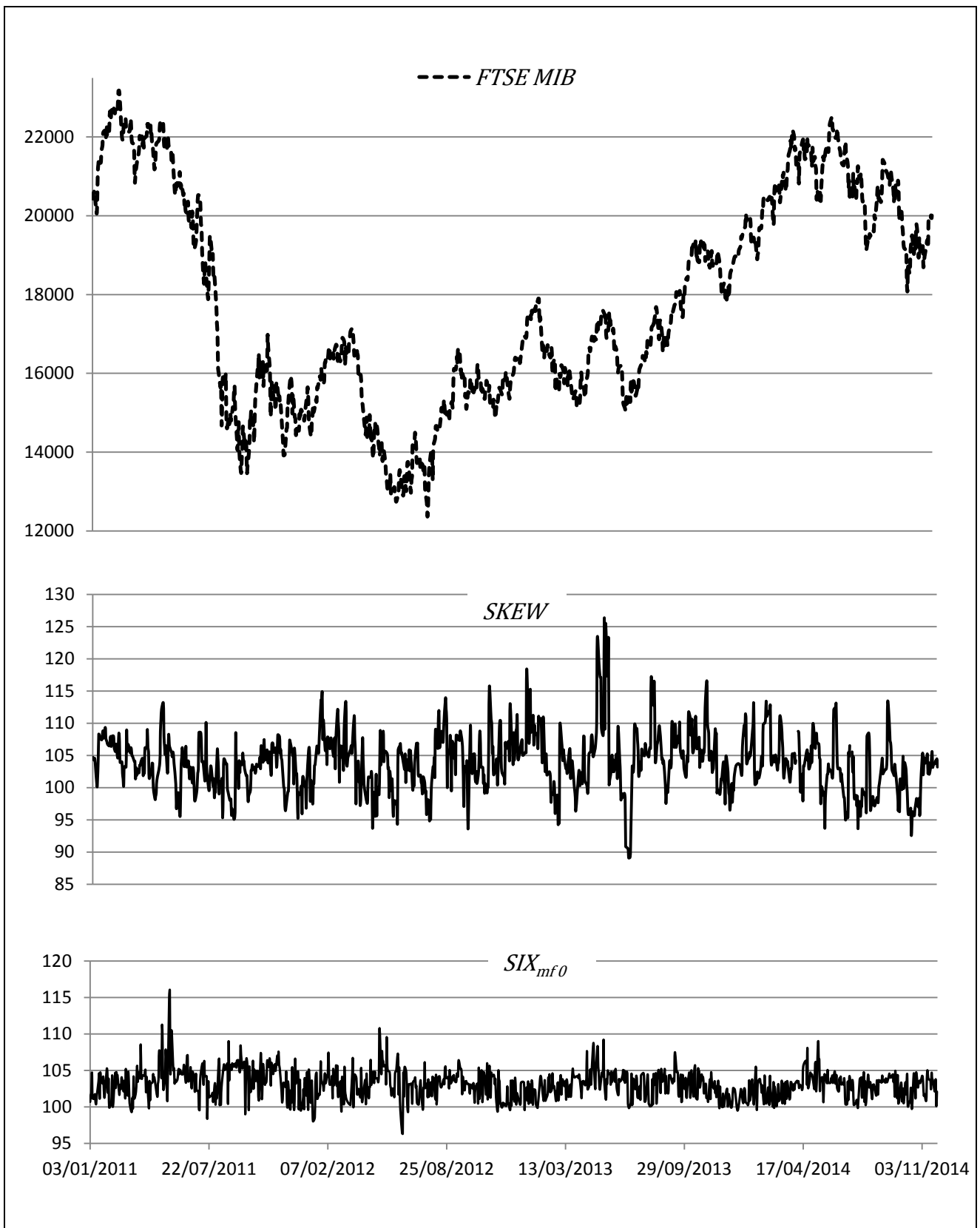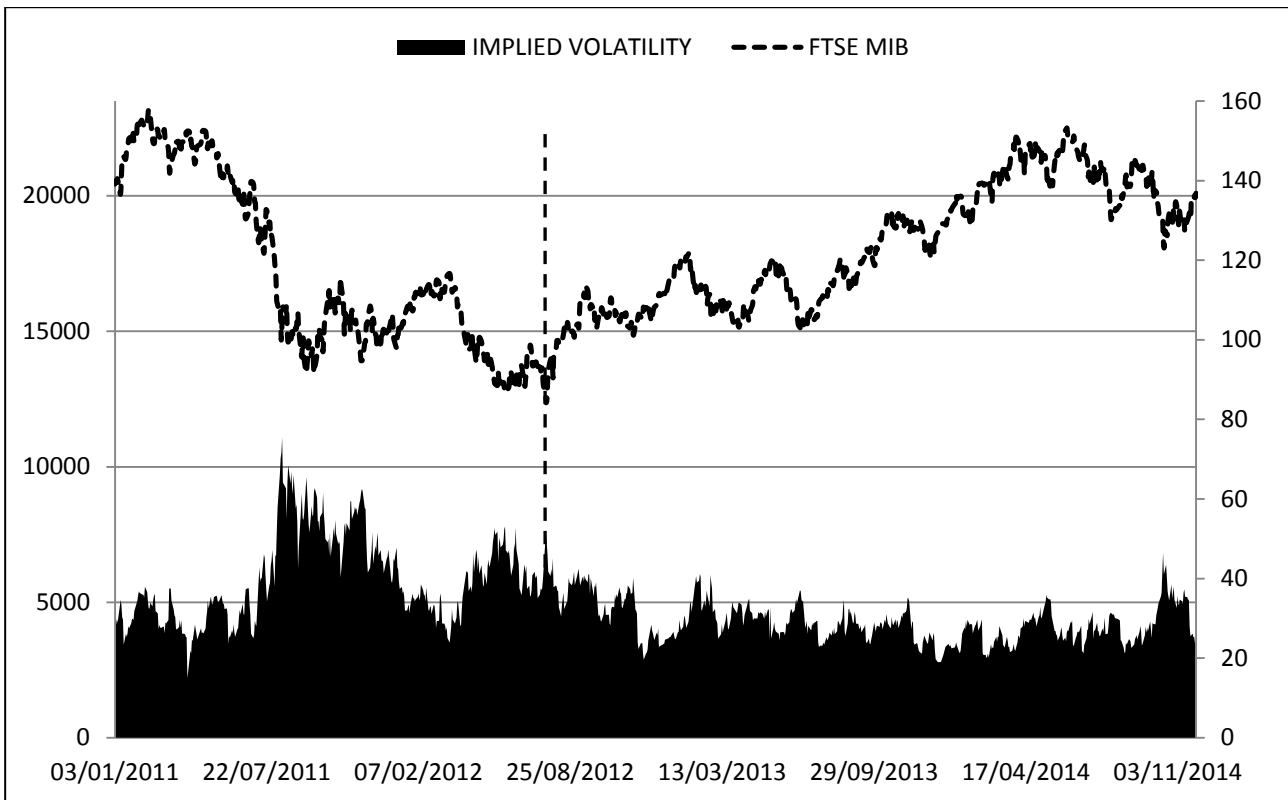Figure 1 - Graphical comparison of Skewness measures

Figure 2 – Comparison between the FTSE MIB index, the *SKEW* index and the SIX_{mf0}

Note: The Figure reports the closing values of the Italian market index FTSE MIB and the skewness indexes ($SKEW$ and SIX_{mf0})

Figure 3 – Comparison between FTSE MIB index and model-free implied volatility

Note: FTSE MIB index refers to the values on the left and implied volatility refers to the values on the right. Implied volatility values are obtained as the model-free implied volatility multiplied by 100 (*VIX* methodology).

Figure 4 – Skewness assets cumulative performance during the sample period