Counting the coils in cylindrical helical compression springs: a clarification note

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Counting the coils in cylindrical helical compression springs: a clarification note

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Abstract

Despite an early paper by Vogt addressing the difference between free and active coils and between end and dead coils, current design recommendations for the coil count in cylindrical helical compression springs do not enter in such details. This results in underestimating the flexibility of the spring and overestimating its solid length. This technical note clarifies the meaning of the relevant coil numbers and provides formal relationships that help the designer improve the prediction of the spring rate and of the solid length. Though the analysis can be applied to any type of spring ends, it is particularly effective for the typical squared and ground end setup, which is most frequently encountered in practice.

Keywords: Design, compression springs, cylindrical helix, number of coils, solid length

Introduction

Properties, design equations and nomenclature for cylindrical helical compression springs are documented in many well-established sources, including textbooks [1,2,3], international standards [4-6] and technical guidelines[7,8]. The shape of the end coils is a key factor influencing the manufacturing, the elastic response and the solid height of the spring. Figure 1 shows the four most commonplace types of end coils [7]: plain (Figure 1a), plain and ground (Figure 1b), squared (Figure 1c), squared and ground (Fig. 1d). Each geometry is characterized by a smaller of greater amount of eccentricity of the force applied by the end coils to the main body of the spring [1,9]. While the plain end in Figure 1a is the easiest to manufacture, it is also affected by the greatest eccentricity. The shape in Figure 1d requires a greater manufacturing care but ensures negligible eccentricity. The geometries in Figures 1 b,c fall in between. The end set-up also affects the number of active coils of the spring (hence its compliance) and its solid length (hence the minimum space allowance required by the spring). The coil count and the solid height recommended in the technical literature [7] for the end arrangements in Figure 1 are listed in Table 1. Though the need to distinguish between free and active coils and between dead and end coils is addressed by some sources [1-3], the values in Table 2 are reproduced by virtually all textbooks of machine design without comments or warnings. This technical note critically revisits the values in Table 1 and proposes new comprehensive relationships, applicable to the end designs in Figure 1, with particular reference to the most widespread type, squared and ground, shown in Figure 1d.

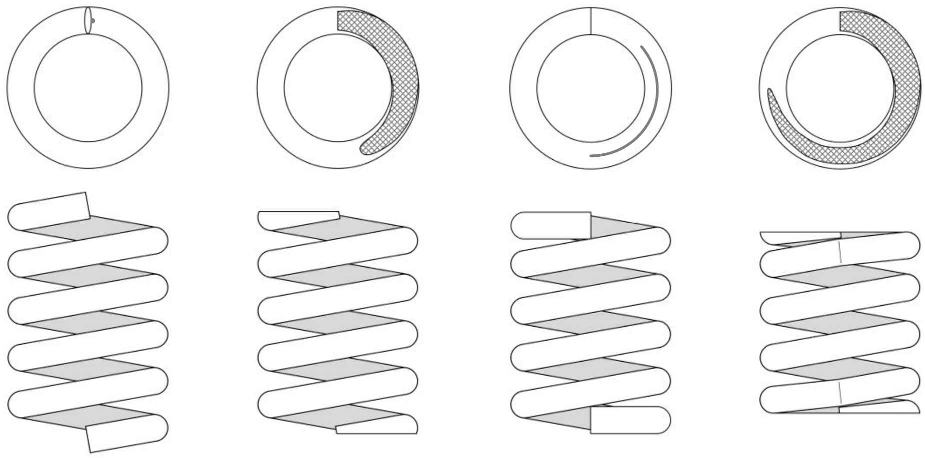


Figure 1. Type of ends for cylindrical helical compression springs: a) plain; b) plain and ground; c) squared; d) squared and ground. Figures on top show the nominal contact area.

Table 1. Textbook relationships [7] for the coil count and the solid length of helical compression springs with end types from Figure 1.

Term	Symbol	Type of end			
		Plain	Plain and ground	Squared	Squared and ground
Total coils	N_T	N_T	N_T	N_T	N_T
End coils	N_E	0	1	2	2
Active coils	N_A	N_T	$N_T - 1$	$N_T - 2$	$N_T - 2$
Solid coils	N_S	$N_T + 1$	N_T	$N_T + 1$	N_T
Solid length	L_S	$N_S\, d$	$N_S\, d$	$N_S\, d$	$N_S\, d$

Method

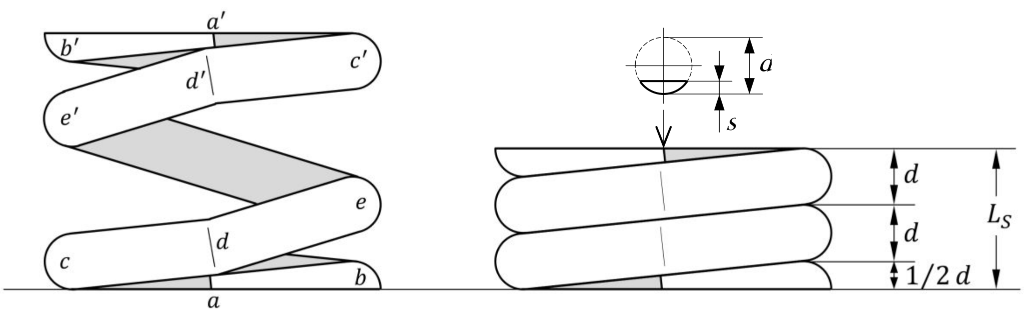
To begin with, the spring with squared and ground ends (Figure 1d) is considered, having a tip thickness (the thickness of the sections at the very ends of the ground coils) of one fourth of the wire diameter (See Figure 2). This is by far the spring most frequently seen in practice and is characterized by the entire set of distinctive features which help identifying the correct number of coils. For this end type, the outcome of a finite element analysis is also provided, which corroborates the findings of an early study [10] in terms of residual compliance. The relevant relationships developed for the coil count in squared and ground end will then be applied to different tip thicknesses and to the other spring ends depicted in Figure 1. The results obtained will be compared with the textbook values in Table 1 to highlight similarities and explain disagreements.

Squared and ground spring ends

The considerations put forward in this paper are facilitated by defining without ambiguity a set of characteristic coil numbers that occur in a spring. Some of these terms are new with respect to those currently used in the spring literature. Other terms, although already in use, are clarified in greater detail than commonly done to eliminate any uncertainty about their meaning. For the sake of example, reference is made to Figure 2, which depicts schematically a spring with squared and ground ends in two limit states: load-free (Figure 2a) and compressed solid (Figure 2b). Deliberately, the spring in Figure 2 exhibits a neat transition between the internal coils, which are wound with the normal pitch, and the end coils, which are closely wound together with the minimal pitch physically compatible with the wire diameter. The contact between internal coils and end coils is assumed closed in the transition zones between normal pitch and closed pitch (sections ad and $a'd'$ in Figure 2a). The presence of a gap in these locations, as well as the gradual transition between closed pitch and normal pitch would origin non-linearities in the load-deflection curve of the spring, which are outside the scope of this technical note. Following these assumptions, the physical properties and the behaviour of the spring are univocally determined by the coil counts explained below.

Number of ***total coils*** (N_T) – This is the easiest number to identify. The coils are counted starting from one tip of the wire (e.g. section a in Figure 2) and moving through the spring up to the tip at the opposite end (section a'). The spring in Figure 2 has $N_T = 3$ total coils ($abcd - dee'd' - d'c'b'a'$).

Number of ***free coils*** (N_F) – Represents the number of coils counted from the first contact point between end coils and fully formed coils (interaction da in Figure 2) to the corresponding point at the other spring end ($d'a'$ in Figure 2). Practically speaking, the free coils are those free from contact for their entire extension, being supported only at the sections where they join the end coils. These coils contribute fully, but not exclusively, to the spring compliance (see *active coils* below). The spring in Figure 2 has $N_F = 1$ free coils ($dee'd'$).



(a) (b)

Figure 2. Reference spring with squared and ground ends: a) load-free configuration; b) compressed solid configuration (with a detail of the tip section of the wire).

*Number of **end coils*** (N_E) – Refers to the coils at each end of the spring, which are subjected to one or more of the following constraints: a) contact with the supporting plates (abc and $a'b'c'$ in Figure 2a), b) mutual contact between them (in case of multiple end coils), c) interaction with the free coils (contact points ad and $a'd'$ in Figure 2a). The end coils are frequently identified as dead coils, meaning that they do not contribute to the spring compliance. This is only partly true as explained in the definition of active coils below. The spring in Figure 2 has $N_E = 2$ end coils ($abcd$ e $a'b'c'd'$). In general, the relationship $N_T = N_F + N_E$ holds true.

*Number of **active coils*** (N_A) – This is the overall number of coils which effectively contribute to the deflection, δ, of the spring according to the well-known equation [1-3,7]

$$\delta = N_A \frac{8FD^3}{Gd^4} \tag{1}$$

where F is the applied force, D is the mean coil diameter, d is the wire diameter and G is the shear modulus of the material. The number of active coils, N_A, to be used in (1) cannot be measured directly on the spring but must be estimated theoretically, computationally or experimentally. The number of active coils is linked to the number of free coils as follows

$$N_A = N_F + 2\varphi \tag{2}$$

where φ is the fraction of each end coil (segments cd and $c'd'$ in Figure 2a) which contributes to the overall compliance due to torsional strain. For squared and ground end coils, Vogt [10] calculates theoretically $\varphi \approx 0.25$, meaning that about one quarter of each end coil contributes to the spring deflection. This value, validated experimentally by Vogt himself [10], has been here confirmed computationally with a finite element analysis illustrated in the Appendix. The spring in Figure 2 has $N_A \approx 1 + 2 \times 0.25 = 1.5$ active springs (path $cdee'd'c'$). It is remarkable that no modern textbooks of machine design acknowledge Vogt's contribution and only a few [2,3] point to a distinction between free coils and active coils. Otherwise, the two terms are used interchangeably.

Number of ***dead coils*** (N_0) – The number of coils which, though taking up space, contribute nothing to the flexibility of the spring. The dead coils are obtained by subtracting the active coils from the total coils: $N_0 = N_T - N_A$. The spring in Figure 2 has $N_0 \approx 3 - 1.5 = 1.5$ dead coils (segments abc and $a'b'c'$ in Figure 2a).

Number of ***solid coils*** (N_S) – This term represents the number of wire cross sections (i.e. the number of virtual flat coils) which, if put one on top of the other, would give the solid length of the spring as $L_P = N_S\, d$. This is the minimum axial space taken up by the spring. By generalizing the coil count implicit in Figure 2, we can write,

$$N_S = N_T - 1 + 2\xi \tag{3}$$

where $\xi = s/d$ is the ratio between the thickness, s, of the wire tip (locations a and a' in Figure 2a) and the nominal wire diameter, d (see upper detail in Figure 2b). The spring in Figure 2, in which $\xi = 0.25$ (typical value), has $N_S = 3 - 1 + 2 \times 0.25 = 2.5$, which is easily verified in the figure.

Table 2. Relationships for coil counts and numerical example for a spring with squared and ground ends and wire diameter d. The centre columns refer to the present theory. The last columns refer to the classical textbook procedures. Typical values for parameters in the new relationships: $\varphi = 0.25$, $\xi = 0.25, 0.5, 0.75, 1.0$.

		Present theory		Textbook procedures[2,3]	
Term	Symbol	Relationship	Example (Figure 2)	Relationship	Example (Figure 2)
Total coils	N_T	$N_F + N_E = N_A + N_0$	3	N_T	3
Free coils	N_F	$N_T - N_E$	1	$N_T - 2$	1
End coils	N_E	$N_T - N_F$	2	2	2
Active coils	N_A	$N_F + 2\varphi$	1.5	$N_T - 2$	1
Dead coils	N_0	$N_T - N_A$	1.5	2	2
Solid coils	N_S	$N_T - 1 + 2\xi$	2.5	N_T	3
Solid length	L_S	$N_S\, d$	$2.5d$	$N_S\, d$	$3d$

The number of coils defined above, their mutual relationships and the actual coil counts for the spring in Figure 2 are summarized in the centre two columns of Table 2. For the sake of comparison, the last two columns in Table 2 list the characteristic spring numbers obtained for the same spring by applying standard textbook procedures[2,3] (e.g. Table 1). Starting from the same number of total coils ($N_T = 3$), which is the most objective coil count in a helical spring, the standard procedure underestimates the number of active coils ($N_A = 1$ instead of 1.5) and overestimates the solid coils ($N_S = 3$ instead of 2.5). Especially for short springs with few active coils, both differences can be much detrimental to the design accuracy.

Within the category of springs with squared and ground ends, it is easily seen that the relationships in Table 2 apply to any other combination of end coils and tip thickness. This is especially true for the expression of the solid coils and the corresponding solid length. For example, for the spring in Figure 3a, in which $\xi = 0.5$, we have $N_T = 3$, $N_L = 1$, $N_E = 2$, $N_S = 3 - 1 + 2 \times 0.5 = 3$. Similarly, for the spring in Figure 3b, in which $\xi = 0.25$, we have $N_T = 3.5$, $N_L = 1$, $N_E = 2.5$, $N_S = 3.5 - 1 + 2 \times 0.25 = 3$. In both cases, the spring count is easily verified to be correct from the drawings.

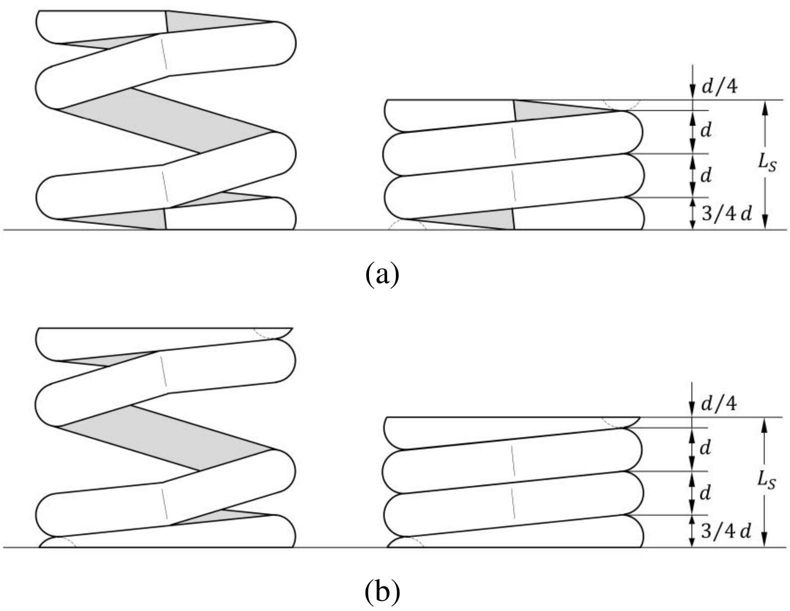


Figure 3. Springs with squared and ground ends with: a) 2 end coils; b) 2.5 end coils.

Other spring ends

The expressions in Table 2 can be applied also to the other types of spring end shown in Figure 1. For example, for the plain ends in Figure 1a, in which $\xi = 1.0$, we have $N_T = 5$, $N_F = 5$, $N_E = 0$, $N_S = 5 - 1 + 2 \times 1.0 = 6$, in line with Table 1. For the end set-up in Figure 1b, in which $\xi = 0.25$, it results $N_T = 5$, $N_F \approx 4$, $N_E = 1$, $N_S = 5 - 1 +$

$2 \times 0.25 = 4.5$, a little less than reported in Table 1. For Figure 1c, in which $\xi = 1.0$, we have $N_T = 5$, $N_S \approx 4.5$, $N_E = 1.5$, $N_S = 5 - 1 + 2 \times 1 = 6$, in keeping with Table 1.

Conclusions

Apart from insightful considerations reported by Wahl [1] and Vogt [10], the published values for the coil count in cylindrical helical compression springs do not differentiate between free and active coils, between end coil and dead coils, and overestimate the solid length of the spring. This technical note clarifies the meaning of the relevant coil counts and provides formal relationships that help the designer improve the prediction of the spring rate and of the solid length. This is particularly important for the squared and ground end design, which is most encountered in practice. For the normal proportions of this widespread geometry ($\xi = 0.25$ and $N_E = 2$), the following values are recommended for N_T total coils: dead coils $N_0 = 1.5$ (correct value) , active coils $N_A = N_T - N_0$ (correct value), solid length $L_S = N_T d$ (in slight excess of the correct value).

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Appendix

Finite element modelling

Figure 4a shows the finite element model of a single steel end coil of the squared and ground type, resting on a rigid flat surface. The circular cross section at the beginning of the taper was loaded remotely by a force F, acting along the spring central axis. The coil has a wire diameter $d = 10$ mm and a mean diameter $D = 50$ mm. The tip thickness below the loaded full section is $s = 2.5$ mm (hence $\xi = s/D = 0.25$, see also Figure 2b). The coil was discretized in 17 832 tetrahedral elements, resulting in 28 927 nodes. The grid used was the stabilized result of a mesh convergence process comprising three models. Starting from an initial coarse mesh, the number of nodes was approximately doubled in two steps, upon which it was seen that the second and the third meshes gave more or less the same displacements. The coil material was assumed elastic, with the properties of steel (Young's modulus = 206 000 N/mm^2, Poisson's ratio = 0.3). The analysis was non-linear due to unilateral contact between loaded section and tip section and between ground surface of the coil and counterplane. The force was applied by assuming rigid the loaded section (colored light gray in Figure 4a) and by applying to its nodes a system of forces equivalent to a single axial force F acting along the spring axis and pushing the spring against the supporting plane. The value of F was increased incrementally from zero up to 4 kN (a value capable of producing in the full section of the wire a maximum torsional stress of about 500 N/mm^2). The interaction between the coil and the countersurface was described as unilateral contact, as well as the interaction between the loaded section and the adjacent end section of the tapered coil (resting on the plane). To avoid rigid body motions, the end section was constrained minimally in two points of its straight edge to suppress rotations and translations along the plane. The analysis was performed using the general purpose LUSAS package (version 21.0).

The results were analysed in terms of axial deflection, δ, (Figure 4a) and contact area (Figure 4b). By inverting equation (1), the equivalent number of active coils, $\varphi = N_A$, contributed by the end coil to the flexibility of the spring was obtained from the axial deflection as

$$\varphi \equiv \delta \frac{Gd^4}{8FD^3} \tag{4}$$

Using (4), values for φ ranging from 0.24 to 0.27 were found, increasing with the applied force F. This range brackets the value $\varphi \approx 0.25$ reported by Vogt [10] and testifies to the relevance of Vogt's result, although much disregarded in the technical literature. The slight increase of φ detected computationally can be explained with the stabilization of the contact between the ground surface and the counterplane as shown in Figure 4b. These contact patterns are in line with the experimental findings reported by Gevorgyan and Kletzin [11].

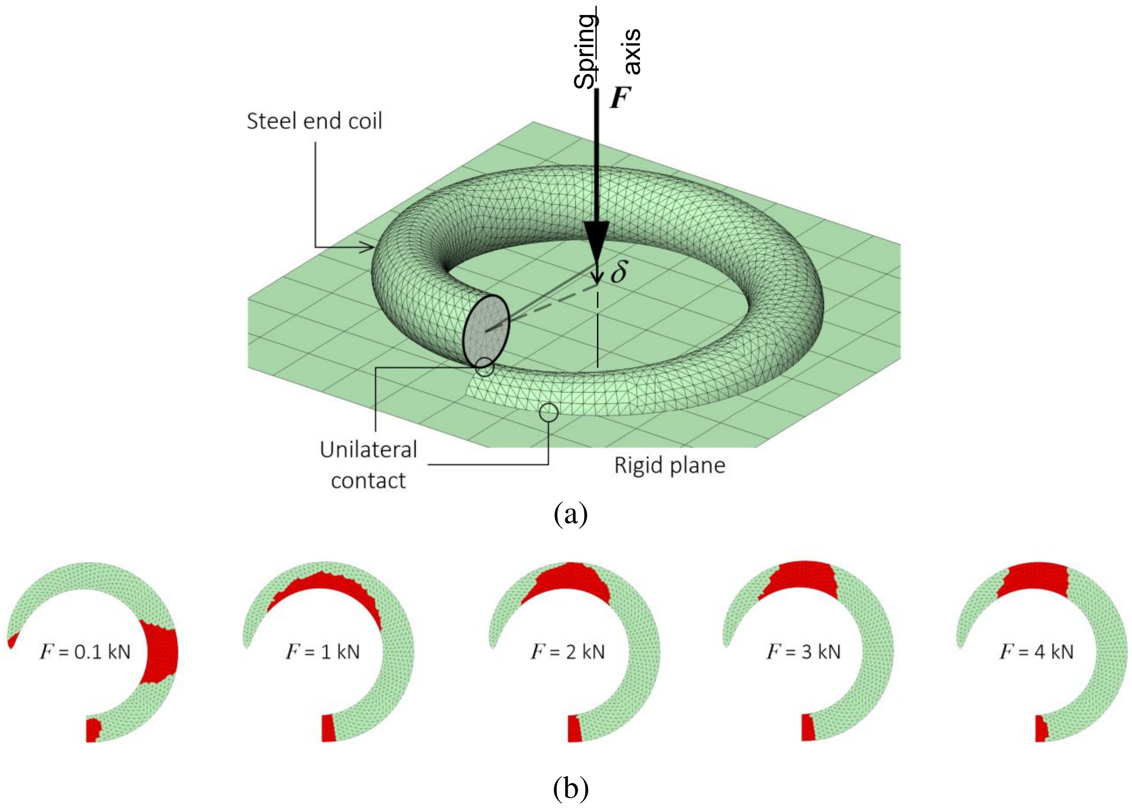

Figure 4. Finite element model of a square and ground steel end coil resting on a rigid plane: a) overall model; b) true contact areas (in red) between coil and plane.

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