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Accounting for controversies in ESG scores: a Fuzzy TOPSIS approach

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Accounting for controversies in ESG scores: a Fuzzy TOPSIS approach

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Abstract

The aim of this paper is to provide a ranking of companies, based on ESG scores, which accounts for the greenwashing phenomenon. LSEG publishes an index of controversies that measures the extent to which a company is exposed to scandals or legal disputes or fines. In order to account for the uncertainty arising from possible greenwashing, we propose the use of Triangular fuzzy numbers to model the ESG score adjusted for controversies. Fuzzy TOPSIS is used for the final ranking of companies. Results are benchmarked to the LSEG ranking adjusted for controversies and a standard TOPSIS ranking.

The results show that the ranking that accounts for greenwashing in a fuzzy setting offers a more balanced and resilient ranking against phenomena such as greenwashing. By comparing the rankings obtained with the one developed by LSEG, significant differences emerge, attributable to the methodology used and data biases. The study highlights how both methods are effective tools for classifying corporate ESG performance, with fuzzy TOPSIS standing out for its innovation in addressing the limitations of traditional data.

Keywords: *ESG (Environmental, Social, Governance), Greenwashing, MCDM (Multi-Criteria Decision Making), fuzzy TOPSIS, Spearman Correlation.*

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1. Introduction

ESG criteria (Environmental, Social, Governance) represent a set of parameters used to assess and measure the sustainability and ethical impact of a company. In recent years, they have become increasingly important in the business and financial landscape, transforming into essential metrics for guiding the strategic choices of companies, investors, and governments, thereby promoting sustainable and long-term business practices. They encompass three fundamental areas: environmental impact management, social engagement, and corporate governance stability. Environmental aspects consider policies to reduce pollution, optimize the use of natural resources, and address climate change; social aspects evaluate working conditions, the positive impact on communities, and the respect for human rights; finally, governance focuses on issues such as transparency, board composition, and conflict-of-interest management. Companies that achieve strong performance in these areas are considered more resilient and capable of facing future challenges and risks, as they tend to build trust-based relationships with investors and stakeholders and mitigate reputational and regulatory risks.

LSEG is one of the main ESG data providers and publishes also an ESG Controversy Score that indicates the level of a company's exposure to controversies and adverse events related to environmental, social, and governance issues, based on reports from global media sources. This score is determined according to 23 categories of potential ESG controversies, although these specific categories are not explicitly disclosed by LSEG. If a company becomes involved in a scandal during the year, it is penalized, which directly impacts its score and its overall ESGC evaluation.

The ESG Controversy Score serves as a tool to address the issue of greenwashing, a deceptive practice in which companies manipulate or falsify sustainability-related data, such as those found in corporate reports, to appear more environmentally responsible than they truly are. The goal of this practice is often to secure higher ESG ratings, responding to the growing demand from stakeholders for greater corporate commitment to sustainability. However, greenwashing represents a serious risk, as it can mislead investors into making decisions based on distorted information and weaken overall market confidence in ESG ratings and corporate transparency.

Within this framework, uncertainty on ESG scores represents one of the main challenges, as the available data are often subject to variability, ambiguity, and diverse interpretations.

Various papers use Multi Criteria Decision Making tools, in particular the TOPSIS method, to rank companies according to ESG performance (see e.g. Korzeb and Samaniego-Medina (2019), Diao et al. (2023), Xu and Zhao (2024)); however, they do not address the greenwashing problem that might affect the ranking.

In order to model uncertainty in rankings, some papers use fuzzy TOPSIS to model perception from different gorups of Decision Makers (see e.g. Escrig-Olmedo et al. (2017), Carnero (2020), Awasthi et al. (2010)). Other papers use hybrid methods in order to address the sensitivity of the final outcome of a MCDM ranking to perturbation in the weights. To this end they either use fuzzy weights (see e.g. Su and Sun (2023), Dincer et al. (2016)) or Rus (see e.g. Song and Chung (2016)). Therefore, fuzzy numbers are used in the literature eather to model the views of different decision makers or to model the fuzzy weights of the different criteria no contribution in the literature uses fuzzy numbers to model uncertainty in the ESG score arising from controversies.

In order to cope with the uncertainty arising from controversies, we propose the use of Triangular fuzzy numbers to model the ESG score. Fuzzy TOPSIS is then used for the ranking of companies. Results are benchmarked to the LSEG ranking adjusted for controversies and a standard TOPSIS approach. To the best of our knowledge this is the first paper that uses fuzzy Triangular numbers to model the uncertainty inherent in ESG scores. In this context, the use of fuzzy TOPSIS proves especially advantageous, as it allows for the handling of the inherent uncertainty in the data.

2. Literature Review

In this section we present several studies concerning the application of the TOPSIS and fuzzy TOPSIS methods in ESG contexts, highlighting how the integration of different approaches can be used to solve complex problems related to sustainability and social responsibility in various situations and specific cases.

Some papers use the TOPSIS approach to rank companies or countries according to ESG scores. For companies, Korzeb and Samaniego-Medina (2019) examine the involvement of Polish banks in sustainable development through a multidimensional analysis based on the TOPSIS method to determine the sustainability level of 14 banks analyzed between 2015 and 2017, using 10 evaluation criteria divided into three categories: financial sustainability disclosure, energy consumption and savings, and product responsibility. The analysis conducted by the authors shows that the level of commitment of Polish banks towards sustainable development is generally low, with worse results for banks with foreign capital compared to those with domestic shareholders. Xu and Zhao (2024) study the sustainability level of 79 Chinese energy companies listed on the stock market and classified as "major polluters" in 2021, using 40 indicators categorized into the three macro-criteria: Environmental, Social, and Governance (E, S, and G). After determining the weights of each criterion using the AHP (Analytic Hierarchy Process) method, they use the TOPSIS method to classify these companies. The results show how this application proves effective in providing an objective assessment of companies, while also emphasizing the need to improve ESG regulations, strengthen oversight of disclosed information, and encourage transparency to ensure a more sustainable future. The analysis further reveals an overall moderate ESG performance, with strong disparities among the firms and particularly weak outcomes in the governance dimension, which emerged as the least developed among the three. For countries, Diao et al. (2023) analyze the sustainability performance of countries along the Arctic route between 2013 and 2019, known as the "Silk Road on Ice," using an ESG evaluation method and combining the entropy method (for assigning weights to each criterion) with the TOPSIS model. The authors' goal, in addition to assessing the sustainability of the countries along this route, is to propose recommendations for strengthening the sustainable development of the project.

In order to account for different perceptions or judgements about sustainability, the following papers analyse fuzzy TOPSIS methodologies on different domain of applications. In their study, Escrig-Olmedo et al. (2017) employ a comprehensive set of both financial and ESG criteria to evaluate 52 listed companies in the apparel sector. The fuzzy TOPSIS method is applied under two distinct scenarios: the first emphasizes financial criteria, representing investors with economic priorities, while the second assigns greater weight to ESG factors, reflecting the preferences of investors more focused on sustainability. This comparative analysis allows for observing variations in company rankings and portfolio selections based on differing investor preferences, thereby demonstrating the flexibility and effectiveness of the proposed methodology in adapting to diverse investment profiles. Furthermore, the authors validate their results by comparing them with established ESG ratings, such as those from Thomson Reuters ASSET4, revealing a strong correlation between the fuzzy TOPSIS rankings and existing ratings, which supports the robustness and reliability of the proposed approach. Carnero (2020) presents an innovative model to assess environmental sustainability in hospital settings, applied to a public hospital in Spain, considering the opinions of both patients and experts. The study utilizes eight criteria for evaluating environmental sustainability, including resource consumption such as water and energy, management of environmental incidents, and waste production and treatment. The results show that, on average, the experts' judgments are stricter than those of the patients, who perceive the hospital's environmental sustainability in a more positive light. Awasthi et al. (2010) propose a fuzzy multi-criteria approach for evaluating supplier performance within a supply chain using 12 environmental sustainability criteria, including the use of eco-friendly technologies, environmental certifications, pollution reduction, and sustainable research and development initiatives. After applying 17 different scenarios, the analysis revealed that the choice of the best supplier remained stable across most scenarios, underscoring the applicability of fuzzy TOPSIS in real business decisions and enabling the integration of environmental criteria into procurement policies.

Last, some papers use a hybrid approach that combines the TOPSIS or the fuzzy TOPSIS method with other techniques to evaluate complex problems related to sustainability and social responsibility. Su and Sun (2023) propose an innovative approach for evaluating ESG performance within the context of state-owned mining companies in China, combining Cumulative Prospect Theory (CPT) with the TOPSIS method. Their study addresses the limitations of traditional methods, which tend to overlook the psychological factors that influence investor decisions. The CPT-TOPSIS model seeks to provide a more comprehensive evaluation by acknowledging that financial decisions are not always based on pure rationality, but are often influenced by emotions and perceptions. Dincer et al. (2016) propose a hybrid model combining fuzzy AHP with fuzzy TOPSIS to assess investor preferences in the selection of industries. Their model is applied to the BIST 100 index of the Istanbul Stock Exchange and aims to identify the best industries for individual investors, evaluating both financial and behavioral criteria in an emerging market context. Song and Chung (2016) use a combination of TOPSIS and the RUS (Robustness, Uncertainty, and Sensitivity Analysis) method, that analyzes how variations in the weights of criteria or in the evaluations of attributes affect the final outcomes, allowing the identification of robust solutions and mitigating the impact of uncertainties in quantitative data. Their focus is on vulnerability to climate change within the environmental sustainability framework, a significant component of ESG criteria. Their study represents an important contribution to multi-criteria evaluation of climate vulnerability, with a particular emphasis on the damage caused by floods.

Authors	Data	Methodology	Results
Korzeb, Z.; Samaniego-Medina, R.	They study the sustainability level of 14 banks analyzed between 2015 and 2017, using 10 evaluation criteria divided into three categories: financial sustainability disclosure, energy consumption and savings, and product responsibility	TOPSIS	The results show that the level of commitment of Polish banks towards sustainable development was generally low, with worse results for banks with foreign capital compared to those with domestic shareholders.
Diao, M.; Fang, H.; Zhou, X.	They study countries along the Arctic route, known as the "Silk Road on Ice," using ESG evaluation criteria, between 2013 and 2019.	TOPSIS	The authors aim not only to assess ESG sustainability along the Ice Silk Road, but also to provide recommendations for improving it. Using entropy weight TOPSIS, they find significant disparities: Nordic and East Asian countries perform best, while China ranks lowest, especially in environmental and social aspects. They suggest China should strengthen cooperation to support more balanced sustainable development

Xu, X.; Zhao, H.	They study the sustainability level of 79 Chinese energy companies listed on the stock market and classified as "major polluters" in 2021, using 40 indicators categorized into the three macro-criteria: Environmental, Social, and Governance	TOPSIS	The results show how this application proves effective in providing an objective assessment of companies, while also emphasizing the need to improve ESG regulations, strengthen oversight of disclosed information, and encourage transparency to ensure a more sustainable future. The analysis further reveals an overall moderate ESG performance, with strong disparities among the firms and particularly weak outcomes in the governance dimension, which emerged as the least developed among the three.
Escrig-Olmedo, E.; et al.	The authors applied their methodology to a sample of 52 companies in the clothing industry, evaluating them according to four main criteria: Financial, Environmental, Social and Governance criterion.	Fuzzy TOPSIS	The results demonstrate that the fuzzy TOPSIS method is effective in ranking companies in a manner that reflects the diverse preferences of investors regarding financial and ESG criteria. The rankings vary significantly depending on the weighting scenario applied to each criterion, highlighting the method's ability to adapt to different investment profiles. Furthermore, comparison with established ESG ratings, such as Thomson Reuters ASSET4, reveals a strong correlation, confirming the reliability and validity of the proposed approach.
Carnero, M. C.	The author analyzes environmental sustainability in hospital settings, applied to a public hospital in Spain, considering the opinions of both patients and experts, usingeight criteria for evaluating environmental sustainability.	Fuzzy TOPSIS	The results show that, on average, the experts' judgments are stricter than those of the patients, who perceive the hospital's environmental sustainability in a more positive light.
Awasthi, A.; et al.	They analyze supplier performance within a supply chain using 12 environmental sustainability criteria, including the use of eco-friendly technologies, environmental certifications, pollution reduction, and sustainable research and development initiatives.	Fuzzy TOPSIS	After applying 17 different scenarios, the analysis revealed that the choice of the best supplier remained stable across most scenarios, underscoring the applicability of fuzzy TOPSIS in real business decisions and enabling the integration of environmental criteria into procurement policies.
Su, J.; Sun, Y.	They analyze five state-owned mining companies in China using ESG criteria.	CPT, TOPSIS	Their study addresses the limitations of traditional methods, which tend to overlook the psychological factors that influence investor decisions. The CPT-TOPSIS model seeks to provide a more comprehensive evaluation by acknowledging that financial decisions are not

			always based on pure rationality, but are often influenced by emotions and perceptions.
Dincer, H.; et al.	They analyze the BIST 100 index of the Istanbul Stock Exchange to identify the best industries for individual investors, using financial and behavioral criteria.	Fuzzy AHP, Fuzzy TOPSIS	The results of the study indicate that investors' perceptions of market conditions and the global financial situation influence their selection of industry sectors for equity investments and that investors' investment decisions are strongly based on the performance and risk levels of individual assets/shares.
Song, J.Y.; Chung, E.	They assess the climate change vulnerability of seven metropolitan cities in South Korea in the context of environmental sustainability, a significant component of ESG criteria, using 20 evaluation criteria.	TOPSIS, RUS	Applying the TOPSIS method, the authors calculate a vulnerability ranking for the seven cities. Subsequently, they use the RUS approach to analyse the robustness of the obtained rankings, identify minor variations in the criteria weights that could cause ranking inversions between cities, and assess the sensitivity of the performance values to the vulnerability rankings. This approach highlighted the effectiveness of the RUS method in analysing vulnerability to climate change.

Table 1. Literature Review

3. The Dataset

For the analysis conducted in this study, the data is downloaded from LSEG for the year 2022. Regarding the alternatives considered, a total of **2307 companies** are included in the analysis. For each company we downloaded the ESG Score (ESG), ESG Controversy Score (ESG_{contr}), ESG Combined Score (ESGC), Environmental Pillar Score (E), Social Pillar Score (S), Governance Pillar Score (G).

The **ESG Score** assesses a company's overall performance across the three main pillars: environmental, social, and governance, using self-reported information from the companies themselves. *"LSEG captures and calculates over 630 ESG measures at the corporate level, of which a subset of 186 of the most comparable and sector-relevant metrics feed into the overall company assessment and scoring process."* These measures are organized into 10 categories that aggregate the three pillar scores and contribute to defining the overall ESG score, thereby reflecting the company's commitment and effectiveness based on publicly available information.

The **ESG Controversy Score** indicates the level of a company's exposure to controversies and adverse events related to environmental, social, and governance issues, based on reports from global media sources. This score is determined according to 23 categories of potential ESG controversies, although these specific categories are not explicitly disclosed by LSEG. If a company becomes involved in a scandal during the year, it is penalized, which directly impacts its score and its overall ESG evaluation. Furthermore, if new developments related to the event arise, such as legal actions, ongoing legislative disputes, or sanctions, the impact of the controversy may extend into the following year.

Companies with no recorded controversies are assigned a controversy score of 100; companies with recorded controversies are assigned a controversy score lower than 100. Additionally, the controversy score accounts for market capitalization bias, which can disproportionately disadvantage large companies, as they are often subject to greater media scrutiny compared to smaller firms.

The **ESG Combined Score (ESGC)** represents the company's overall assessment based on the information provided under the environmental, social, and corporate governance pillars (ESG Score), with the inclusion of an ESG Controversy component. If a company is involved in controversies and their impact is lower than the ESG Score, the ESGC Score is calculated as a weighted average of the ESG Score and the Controversy Score for each fiscal period. Particular emphasis is placed on the most recent controversies, which are reflected in the latest completed period. On the other hand, if a company's Controversy Score is greater than or equal to its ESG Score, the ESGC Score coincides with the ESG Score.

$$ESGC = \begin{cases} ESG_{score} & if \quad ESG_{contr} \geq ESG_{score} \\ \frac{ESG_{score}+ESG_{contr}}{2} & if \quad ESG_{score} > ESG_{contr} \end{cases} \qquad (1)$$

The **Environmental Pillar Score** assesses a company's impact on the environment, considering factors such as CO_2 emissions, waste management, resource usage, and sustainability policies.

The **Social Pillar Score** evaluates the company's relationship with employees, customers, and communities, including aspects like diversity and inclusion, working conditions, human rights, and social responsibility.

The **Governance Pillar Score** examines the quality of corporate management, transparency, business ethics, and board composition to ensure fair and responsible operations.

4. Modelling uncertainty on the ESG score

In order to account for different perceptions or judgements about the real sustainability of the companies, arising from controversies on the ESG score, we use fuzzy numbers to model uncertainty. Fuzzy numbers are an extension of classical numbers and are used to represent uncertain or approximate quantities. They are based on fuzzy sets and are characterised by several fundamental elements, including the membership function $\mu_A(x)$[3] which defines the degree of memebership of a value *x* to the fuzzy set A[4]. In particular, in our setting, we use triangular fuzzy numbers. A triangular fuzzy number is a particular representation of a fuzzy number and is defined by a triplet **(a, b, c)**. These parameters represent the x-coordinates of the three vertices of the membership function $\mu_A(x)$: **a** represents the lower bound, **b** is the central value, corresponding to the element with the highest degree of membership, **c** represents the upper bound as in Figure 1.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a \\ \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \dfrac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x \geq c \end{cases}$$

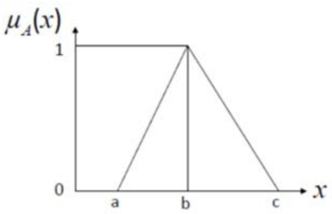

Figure 1. Triangular membership function.

Specifically, for each of the three ESG criteria (Environmental, Social, and Governance) a triangular fuzzy number is defined, consisting of a lower bound, a most possible value, and an upper bound, within a scale ranging from 0 to 100.

[3] A membership function $\mu_A(x)$ takes value in the range [0,1]: if $\mu_A(x) = 0$ the value *x* doesn't belong to the fuzzy set A; if $\mu_A(x) = 1$ the value *x* belongs completely to the fuzzy set; if $0 < \mu_A(x) < 1$ the value *x* partially belongs to the fuzzy set.

[4] Fuzzy Set is defined as follows: if X is a universe of discourse and *x* is a particular element of *X*, then a fuzzy set *A* defined on *X* can be written as follows: $A = \{(x, \mu_A(x)), x \in X\}$.

In order to compute the triangular fuzzy number representing the fuzzy ESG score we distinguish three cases. First case is if the controversy score is lower than the ESG score. Second case is if the controversy score is equal to or higher than the ESG score but lower than 100. Third case is if the controversy score is equal to 100.

In the first case, by looking to equation (1) we see that for LSEG, if a company is involved in controversies and the ESG_{contr} score is lower than the ESG Score, then the ESG combined Score (ESGC) is calculated as a weighted average of the ESG Score and the ESG Controversy Score for each fiscal period. Therefore, we can assume that the difference between the ESG score and the ESGC score represents the uncertainty about the true ESG rating. Therefore, we use this difference to determine the left and right spreads of the triangular fuzzy numbers used to model uncertainty for the ESG score. Moreover, given that we do not know which pillar is mostly affected by the controversies, we can assume that the uncertainty is the same for any pillar.

The triangular fuzzy number for each pillar score i = *E, S, G* (E: Environmental Pillar Score, S: Social Pillar Score and G: Governance Pillar Score) is defined by a triplet (a_i, b_i, c_i) as follows:

$$\begin{cases} a_i = \ max(0, b_i - (ESG - ESGC)) \\ b_i = X_i - (ESG - ESGC) \\ c_i = X_i \end{cases} \tag{2}$$

where X_i is the pillar score with i = *E, S, G*, *ESGC* is the ESG Combined Score, and *ESG* is the ESG Score. The lower bound is constrained to be greater than or equal to zero given that the scale for the fuzzy number is [0,100].

In the second case, if the ESG Controversy score is lower than 100, but higher than the ESG score, then in equation (1) the ESG score is not affected. However, in our modelization of uncertainty on the ESG score, we account for the presence of some controversies that do not change the most possible value, but decrease the lower bound of the triangular fuzzy number as follows:

$$\begin{cases} a_i = \ b_i - (ESG_{contr} - ESGC) \\ b_i = X_i \\ c_i = X_i \end{cases} \tag{3}$$

where X_i is the pillar score with *i = E, S, G* , *ESGC* is the ESG Combined Score, ESG_{contr} is the ESG Controversy Score.

In the third case, if the ESG Controversy Score is equal to 100 (no controversies), then in our setting, there is no fuzziness in the ESG evaluation, therefore the triangular fuzzy number coincides with a crisp number for each pillar *i*:

$$\begin{cases} a_i = X_i \\ b_i = X_i \\ c_i = X_i \end{cases} \tag{4}$$

where X_i is the pillar score with *i = E, S, G*.

To be more precise we provide a numerical example. We consider three companies (X1, X2 e X3) with the respective crisp values for the *E*, *S*, *G* pillars and the *ESG*, *ESGC*, ESG_{contr} scores, as retrieved from LSEG. In this example, we illustrate in *Table 2* how to model the *E, S, G* pillars for each company with triangular fuzzy numbers (a_i, b_i, c_i) where i = *E, S, G*.

Company	*ESG*	*ESGC*	ESG_{contr}	*E*	*S*	*G*	(a_E, b_E, c_E)	(a_S, b_S, c_S)	(a_G, b_G, c_G)
X1	95,57	48,15	56,62	93,02	97,24	94,94	(0; 45,6; 93,02)	(2,40; 49,82; 97,24)	(0,10; 47,52; 94,94)
X2	74,07	74,07	83,33	75,04	79,99	62,04	(65,78; 75,04; 75,04)	(70,73; 79,99; 79,99)	(52,78; 62,04; 62,04)
X3	93,99	93,99	100,00	97,15	94,05	90,14	(97,15; 97,15; 97,15)	(94,04; 94,04; 94,04)	(90,14; 90,14; 90,14)

Table 2. A numerical example of triangular fuzzy numbers.

For company X1, since ESG_{contr} is lower than *ESG*, the values of a_E, b_E, c_E, are calculated using equation *(2)*; for company X2, where ESG_{contr} is higher than *ESG* but not equal to 100, the values of a_E, b_E, c_E, are determined according to equation *(3)*; finally, for company X3, whose ESG_{contr} value equals 100, indicating the absence of controversies, the values of a_E, b_E, c_E, are equal to *E*, as shown in equation *(4)*. The same procedure applies to the *S* and *G* pillars.

5. Application of the TOPSIS and Fuzzzy TOPSIS Methods in the ESG Context

This section presents the ranking based on the TOPSIS method in subsection 5.1 and the ranking based on the fuzzy TOPSIS method in subsection 5.2. Both the TOPSIS and the fuzzy TOPIS are applied to the 2307 companies (alternatives) with respect to three criteria given by the three pillars (E, S, G). For simplicity, an equal weighting approach is used for the three pillars. Section 5.3 provides a comparison among the two methods.

5.1 Results based on TOPSIS

This section presents the results of the analysis conducted using the TOPSIS method. To ensure a more effective representation, we provide in Table 3 and Table 4 the **top 10 and bottom 10** companies in the ranking respectively. This targeted selection allows for a focus on both companies that demonstrate the best performance and those with the worst.

Company Name	Euclidean Distance PIS	Euclidean Distance Nis	Closeness Coeff	Industry Group
AstraZeneca PLC	0.000768	0.019895	0.962847	Pharmaceuticals
Snam SpA	0.000878	0.019793	0.957526	Oil & Gas Related Equipment and Services
Shell PLC	0.000933	0.019638	0.954638	Oil & Gas
Roche Holding AG	0.000947	0.019770	0.954309	Pharmaceuticals
Mercedes-Benz Group AG	0.001108	0.019714	0.946771	Automobiles & Auto Parts
Assicurazioni Generali SpA	0.001349	0.019502	0.935320	Insurance
BNP Paribas SA	0.001349	0.019367	0.934864	Banking Services
Intesa Sanpaolo SpA	0.001419	0.019364	0.931709	Banking Services
STMicroelectronics NV	0.001447	0.019218	0.929978	Semiconductors & Semiconductor Equipment
Allianz SE	0.001544	0.019047	0.925021	Insurance

Table 3. Ranking of the top 10 companies in the TOPSIS method performed on the original dataset.

The results for the top 10 companies yields noteworthy and significant insights. A key observation is that the top four companies originate from sectors traditionally not associated with sustainable practices, such as energy and pharmaceuticals. However, their high ranking suggests a positive effort of these industries, indicating that companies are striving to align with ESG principles and improve their sustainability performance.

Company Name	Euclidean Distance PIS	Euclidean Distance NIS	Closeness Coeff	Industry Group
ENR Russia Invest SA	0.019884	0.001002	0.047988	Investment Banking & Investment Services
Tivoli A/S	0.019939	0.000774	0.037357	Hotels & Entertainment Services
TraWell Co SpA	0.020119	0.000739	0.035413	Transport Infrastructure
Arundel AG	0.020002	0.000712	0.034384	Investment Holding Companies
Syn hf	0.020040	0.000640	0.030957	Telecommunications Services
Airesis SA	0.020116	0.000596	0.028779	Investment Banking & Investment Services
Kbc Ancora NV	0.020150	0.000539	0.026047	Investment Banking & Investment Services
Eeii AG	0.020180	0.000469	0.022701	Investment Holding Companies
Gevelot SA	0.020207	0.000432	0.020916	Machinery, Tools, Heavy Vehicles, Trains & Ships
Traction AB	0.020313	0.000412	0.019874	Investment Holding Companies

Table 4. Ranking of the bottom 10 companies in the TOPSIS method performed on the original dataset.

In contrast, the positioning of the bottom 10 companies reveals markedly different results. These companies exhibit extremely low closeness coefficients and the greatest distances from the Positive Ideal Solution (PIS), confirming their misalignment with optimal ESG practices. The results highlight the urgent need for these companies to undertake decisive actions to improve their environmental, social, and governance practices. Their current ranking reflects unsustainable operational models in the long term. The primary challenge for these firms lies in transforming existing difficulties into growth opportunities, thereby enhancing their reputation and creating sustainable value for all stakeholders.

In addition to examining part of the ranking results of companies based on Environmental, Social, and Governance (ESG) criteria, this study also focuses on interpreting the graphical representations obtained from the analysis, highlighting the previously discussed findings. By analyzing the graph displaying the distances of each alternative (2,307 in total) from the ideal solution (red graph) and the anti-ideal solution (blue graph), the following insights can be drawn:

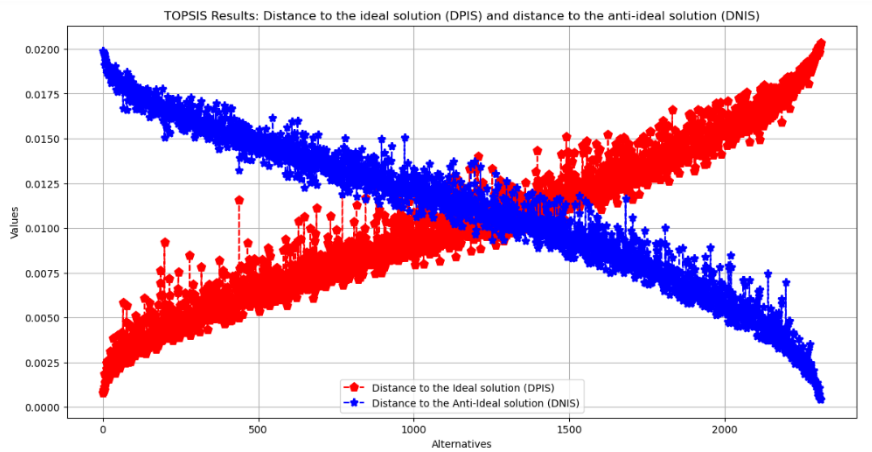

Figure 2. TOSPIS RESULTS: Distance to ideal solution (DPIS) and distance to anti-ideal solution (DNIS).

The graph exhibits opposing distance trends for the two curves. The variations and fluctuations in distance values suggest that, while many alternatives exhibit similar performance levels, some alternatives stand out due to their significant deviations from both the ideal and anti-ideal solutions, indicating greater variability in their performance. Alternatives with very low distances from the Positive Ideal Solution (PIS) and, at the same time, very high distances from the Negative Ideal Solution (NIS) show the best performance. This aligns with the TOPSIS methodology, where optimal solutions are those closest to the ideal solution (low PIS distance) and farthest from the anti-ideal solution (high NIS distance). In the central portion of the graph, the two curves tend to overlap, indicating alternatives that are equally distant from both the ideal and anti-ideal solutions. This suggests that these alternatives occupy an intermediate position in terms of performance. The graph representing the relative closeness coefficient is depicted in Figure 3.

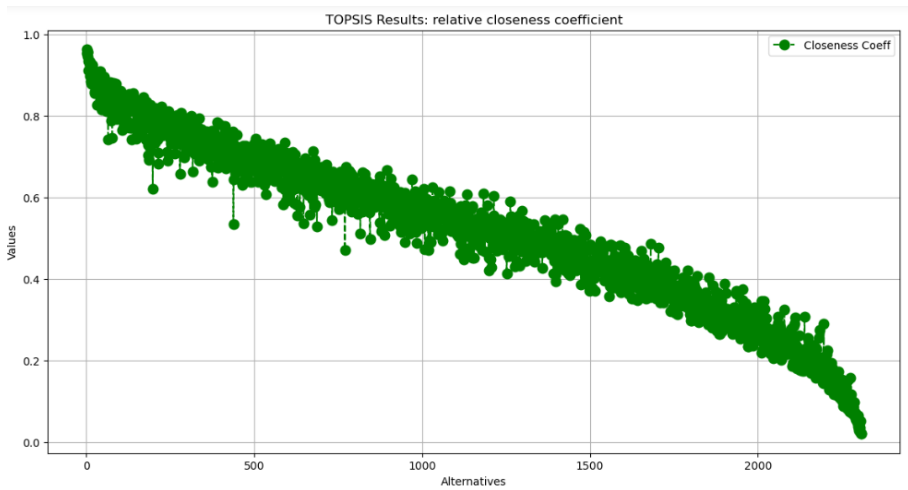

Figure 3. TOPSIS results: relative closeness coefficient.

This graph exhibits a decreasing trend. As previously highlighted, the relative closeness coefficient, calculated based on the PIS and NIS distances, clearly identifies the superior alternatives**:** higher coefficient values indicate a closer proximity to the ideal solution**.** The best-performing alternatives are concentrated at the beginning of the graph, where the closeness coefficient approaches 1. Conversely, towards the end of the graph, where the values decrease, the alternatives with lower performance levels are located, as their closeness coefficients approach 0.

5.2 Results based on fuzzy TOPSIS

This section presents the results of the analysis conducted using the fuzzy TOPSIS method. The objective of the use of fuzzy TOPSIS is to account for uncertainty in the E, S, and G scores represented by the number of controversies, aiming to achieve a more in-depth analysis of corporate performance with respect to ESG criteria. To ensure a more effective representation, we provide in Table 5 and Table 6 the **top 10 and bottom 10** companies in the ranking respectively.

Company Name	Euclidean Distance PIS	Euclidean Distance NIS	Closeness Coeff	Industry Group
STMicroelectronics NV	0,196022793	0,963632105	0,830964545	Semiconductors & Semiconductor Equipment
SIG Group AG	0,203128712	0,957327014	0,82495781	Containers & Packaging
Unibail-Rodamco-Westfield SE	0,204828827	0,955755272	0,823512293	Residential & Commercial REITs
Aegon Ltd	0,208146537	0,952880263	0,820722022	Insurance
Banco Bilbao Vizcaya Argentaria SA	0,209687775	0,951325616	0,819392458	Banking Services
alstria office REIT AG	0,214192204	0,947509536	0,815622034	Residential & Commercial REITs
Storebrand ASA	0,215248645	0,947001008	0,814799992	Investment Banking & Investment Services
Roche Holding AG	0,216265555	0,946325685	0,813979714	Pharmaceuticals
Granges AB	0,216187728	0,945423434	0,813889763	Metals & Mining
Ucb SA	0,21955564	0,94343542	0,811214679	Pharmaceuticals

Table 5. Ranking of the top 10 companies in the fuzzy TOPSIS method performed on the original dataset.

Similarly, the ranking of the bottom 10 companies is presented in Table 6:

Company Name	Euclidean Distance PIS	Euclidean Distance NIS	Closeness Coeff	Industry Group
Bank of Greece	0,711030085	0,19128948	0,211997486	Banking Service
Tivoli A/S	0,712173263	0,185832312	0,20693893	Hotels & Entertainment Services
Arundel AG	0,713721237	0,178447213	0,200015158	Investment Holding Companies
Syn hf	0,715084868	0,171637004	0,193563517	Telecommunications Services
TraWell Co SpA	0,716704138	0,163509016	0,185760705	Transport Infrastructure
Airesis SA	0,717646085	0,158213729	0,180638187	Investment Banking & Investment Services
Kbc Ancora NV	0,718620284	0,152737619	0,175286892	Investment Banking & Investment Services
Eeii AG	0,719355178	0,148441037	0,171055179	Investment Holding Companies
Gevelot SA	0,720157793	0,143660786	0,166308979	Machinery, Tools, Heavy Vehicles, Trains & Ships
Traction AB	0,722799073	0,126787831	0,149234681	Investment Holding Companies

Table 6. Ranking of the last 10 companies in the fuzzy TOPSIS method performed on the original dataset.

Analyzing the results of the top and bottom 10 companies in the fuzzy TOPSIS method, we observe that, as in the TOPSIS method, the relative proximity coefficients exhibit significant variability. Companies at the top of the ranking have coefficients close to 1, while those at the bottom have values near 0. These results indicate that the fuzzy TOPSIS method, by introducing uncertainty into the calculations through the use of triangular fuzzy numbers, has generated a ranking with slightly lower values for the top firms and higher values for the bottom ones, if compared to the TOPSIS method. This means that if we account for uncertainty, given by the number of controversies, the top firms are less performing and the bottom firms are more performing than in the case in which we do not account for uncertainty, as in the TOPSIS method.

In Figure 4 we report the distance from the ideal solution (DPIS) and the distance from the anti-ideal solution (DNIS), as well as the relative proximity coefficient. With respect to the TOPSIS ranking, this chart shows more variability in the results and the distance from the ideal solution crosses the distance from the anti-ideal solution in a point that is closer to the end of the chart. This means that for bottom performing firms, the difference between the ideal and anti-ideal solution is lower than in the TOPSIS ranking. This is evident also in Figure 5 where we report the closeness coefficient that attains values higher than the ones in the TOPSIS ranking for bottom firms. If we compare Figure 3 with Figure 5 we see that the closeness coefficient decreases less sharply in the fuzzy TOPSIS case w.r.t. the TOPSIS one.

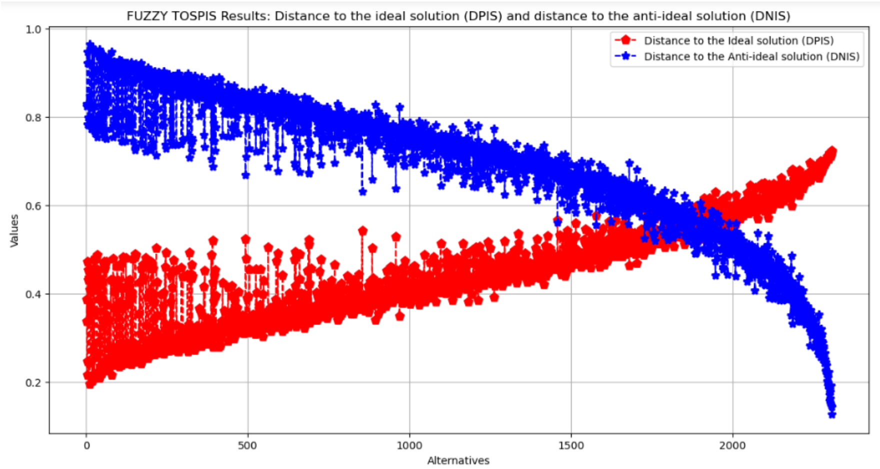

Figure 4. FUZZY TOSPIS RESULTS: Distance to the ideal solution (DPIS) and distance to the anti-ideal solution (DNIS).

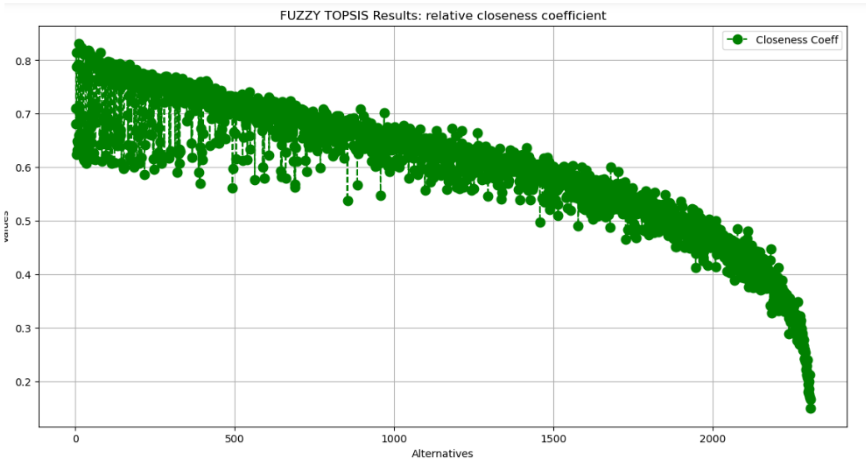

Figure 5. FUZZY TOPSIS results: relative closeness coefficient.

5.3 Comparison of the results with LSEG rankings

In this section we analyze the correlation between the ranking obtained with the TOPSIS method, the fuzzy TOPSIS method and the ranking reported by LSEG, based on the ESG score and the ESG combined score that accounts for controversies as in equation (1). To assess the similarity among rankings we employ the Spearman's rank correlation coefficient and the Kendall's tau correlation coefficient.

Spearman's rank correlation coefficient measures the monotonic relationship between two variables. It is a non-parametric test, meaning that it does not require the data to follow a normal distribution.

Unlike Pearson's correlation coefficient, which assesses the linearity of the relationship between two quantitative variables, Spearman's coefficient is based on data ranks, making it particularly useful when normality assumptions are not met or when working with ordinal data. Spearman's coefficient, denoted by the **ρ (rho)**, takes values between -1 and +1:

- **+1**: indicates a perfect positive correlation between the ranks of the two variables;
- **-1**: indicates a perfect negative correlation;
- **0**: indicates the absence of correlation (see e.g. J. Hauke and T. Kossowski (2011)).

In the context of our analysis, the use of Spearman's correlation coefficient allowed us to determine the extent to which different multicriteria methods produce similar rankings of companies based on ESG criteria. High coefficient values suggest that the methods tend to rank companies in a similar manner, whereas values close to zero indicate a low level of agreement between the generated rankings. This correlation analysis is essential for assessing the consistency and reliability of different evaluation approaches, helping to identify potential discrepancies and improve corporate sustainability assessment methodologies.

Spearman Index	*TOPSIS*	*Fuzzy TOPSIS*	*LSEG - ESG Score*	*LSEG - ESGC Score*
TOPSIS		0,97394403	0,9452648	0,85572606
Fuzzy TOPSIS	0,97394403		0,92552885	0,89100112
LSE - ESG Score	0,9452648	0,92552885		0,913432
LSEG - ESGC Score	0,85572606	0,89100112	0,913432	

Table 7. Spearman Index (General Case).

An examination of the Spearman correlation table reveals that all combinations of the methodologies analyzed exhibit high correlation values, approaching perfect correlation (+1). This consistency among the rankings persists despite significant methodological differences, such as the distinct use of information and the uniform weight assignment (1/3, 1/3, 1/3) applied in the TOPSIS and fuzzy TOPSIS methods.

The main reason for this strong correlation lies in the predominance of companies without controversies: out of a total of 2,307 companies, as many as 1,904 (corresponding to 82.5%) report an ESG Controversy Score of 100. In these cases, the ESG Score coincides with the ESGC Score, and the triangular fuzzy numbers used to represent the E, S, and G criteria align with the corresponding crisp values (see Section 4.2). As a result, the rankings derived from LSEG (ESG Score and ESGC Score) and those based on TOPSIS and fuzzy TOPSIS appear highly similar.

Consequently, the differences between methodologies that account for controversies and those that do not are primarily concentrated within the subset of 403 companies for which the ESG Controversy Score is different from 100. For this reason, a targeted analysis of this subset was deemed appropriate.

Kendall's τ (tau) is a **non-parametric correlation coefficient** that measures the strength and direction of the association between two ordinal variables. Unlike Pearson's correlation coefficient, which assesses linear relationships and requires normally distributed data, Kendall's tau relies on the ranking of the data, making it particularly suitable for ordinal data or situations where parametric assumptions are not met. It was introduced by **Maurice G. Kendall in 1938** as a robust alternative to parametric correlation measures (see e.g. M. G. Kendall and J. D. Gibbons (1990)).

The tau coefficient ranges from -1 to +1, where:

- **+1** indicates perfect positive correlation between the two classifications
- **0** indicates no correlation
- **–1** indicates perfect negative correlation

The basic formula for Kendall's tau is:

$$\tau = \frac{(n_c - n_d)}{\frac{1}{2}n(n-1)} \tag{5}$$

Where n_c is the number of **concordant pairs**, n_d is the number of **discordant pairs**, n is the total number of observation. A concordant pair is a pair of observation (x_i, y_i) and (x_j, y_j) such that if $x_i > x_j$ then $y_i > y_j$, or vice versa. A discordant pair, on the other hand, is one in which the ordering of *x* and *y* is opposite. (see e.g. M. G. Kendall and J. D. Gibbons (1990)).

The results obtained from the analysis are reported in Table 8:

Kendall's Tau (General Case)	***Value***
LSEG ESG - LSEG ESGC	0,8931
LSEG ESG - TOPSIS	0,9093
LSEG ESG - FUZZY TOPSIS	0,8769
LSEG ESGC - TOPSIS	0,8152
LSEG ESGC - FUZZY TOPSIS	0,8688
TOPSIS - FUZZY TOPSIS	0,9029

Table 8. Kendall's Tau (General Case).

The results show a high correlation for all pairs of methodologies analyzed, thereby confirming and reinforcing the findings and rationale discussed in Spearman analysis.

A final metric used to evaluate the performance of the obtained ranking algorithms is **Precision@K**. Precision@K measures how many of the **top K** ranked items are relevant; it takes values in the range [0,1], where higher values indicate better performance. However, Precision only reflects the number of relevant items in the top K positions without assessing the quality of the ranking within the list. A relevant item is defined as an alternative that appears in the top K positions of both rankings being compared, regardless of its specific position (see e.g. S. Niu et al. (2012)).

The basic formula for Precision@K is:

$$Precision@K = \frac{Number\ of\ relevant\ items\ in\ K}{Total\ number\ of\ items\ in\ K} \tag{6}$$

Where **K** is an arbitrarily chosen cut-off level used to limit the scope of the evaluation. By varying the value of K, it is possible to perform increasingly focused analyses of the rankings, concentrating on the top K alternatives while disregarding the remaining.

Precision answer the question: *Among the top K suggested items, how many are actually relevant?*

Considering the full dataset of 2307 companies, five scenarios were analyzed in which **K** was set to the following values: **25, 50, 100, 250** and **500**.

Precision@K	25	50	100	250	500
ESG-ESGC	0,24	0,3	0,45	0,61	0,76
ESG-TOPSIS	0,76	0,78	0,77	0,86	0,9
ESG-FUZZY TOPSIS	0,36	0,4	0,49	0,63	0,81
ESGC-TOPSIS	0,24	0,28	0,47	0,6	0,75
ESGC-FUZZY TOPSIS	0,64	0,68	0,73	0,83	0,88
TOPSIS-FUZZY	0,36	0,4	0,53	0,66	0,83

Table 9. Precision@K (General Case).

The results show that, with the expection of the ESG-TOPSIS and ESGC-Fuzzy TOPSIS pairs, which already exhibit relatively high values at Precision@25, due to the similarity in their underlying objectives (as ESG and TOPSIS do not account for controversies, unlike ESGC and fuzzy TOPSIS), the remaining pairs become increasingly similar as the number of alternatives considered increases.

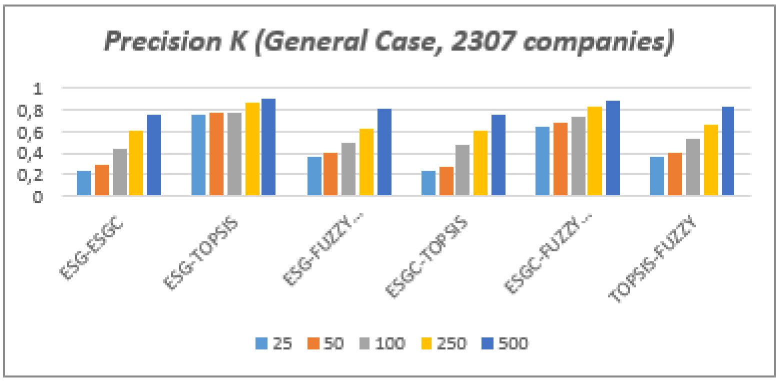

Figure 6. Precision@K (General Case): Bar Chart.

6. Application of the TOPSIS and fuzzy TOPSIS Methods to the set of companies with controversies

In this chapter, the implementation of the TOPSIS and fuzzy TOPSIS methods follows the same methodology described in Chapter 5, utilizing the same variables, criteria, and corresponding weights to ensure analytical consistency. However, in this case, the application is performed on a reduced version of the dataset (403 companies). The decision to reduce the dataset stems from the exclusion of companies with an ESG Controversies Score of 100, meaning those without controversies. This adjustment allows for a closer examination of the impact of this modification and the differences in results compared to the previous applications conducted on the original dataset (2,307 alternatives).

6.1 TOPSIS Results

The TOPSIS analysis conducted on the dataset of 403 companies, despite following the same methodology as the previous application, yields different results. These findings are presented below, maintaining the same structure used in previous implementations. The ranking of the top 10 companies is as follows:

Company Name	Euclidean Distance PIS	Euclidean Distance NIS	Closeness Coeff	Industry Group
AstraZeneca PLC	0.001422	0.036913	0.962901	Pharmaceuticals
Snam SpA	0.001727	0.036673	0.955020	Oil & Gas Related Equipment and Services
Shell PLC	0.001749	0.036423	0.954192	Oil & Gas
Roche Holding AG	0.001835	0.036640	0.952315	Pharmaceuticals
Mercedes-Benz Group AG	0.002151	0.036534	0.944389	Automobiles & Auto Parts
Assicurazioni Generali SpA	0.002569	0.036183	0.933704	Insurance
BNP Paribas SA	0.002654	0.035842	0.931051	Banking Services
Intesa Sanpaolo SpA	0.002809	0.035822	0.927286	Banking Services
Allianz SE	0.002978	0.35262	0.922121	Insurance
Abb Ltd	0.003131	0.035512	0.918975	Machinery, Tools, Heavy Vehicles, Trains & Ships

Table 10. Ranking of the top 10 companies in the TOPSIS method carried out on the reduced dataset.

It is observed that the top positions in this ranking are largely occupied by the same companies that appeared in the ranking obtained from the application of the TOPSIS method on the original dataset. The proximity coefficients of these companies remain very similar, with minimal differences often limited to decimal variations. The ranking of the bottom 10 companies is presented in Table 11:

Company Name	Euclidean Distance PIS	Euclidean Distance NIS	Closeness Coeff	Industry Group
Anglo-Eastern Plantations PLC	0.030326	0.008261	0.214090	Food & Tobacco
CPH Chemie und Papier Holding AG	0.030330	0.008168	0.212165	Paper & Forest Products
Gateley (Holdings) PLC	0.030972	0.007441	0.193719	Professional & Commercial Services
Pinewood Technologies Group PLC	0.030892	0.007256	0.190208	Specialty Retailers
Angling Direct PLC	0.032404	0.006064	0.157631	Specialty Retailers
CureVac NV	0.034553	0.005343	0.133923	Biotechnology & Medical Research
Trivago NV	0.034783	0.005373	0.133799	Software & IT Services
Pharol SGPS SA	0.033800	0.004973	0.128258	Telecommunications Services
SDI Group PLC	0.035832	0.002990	0.077028	Healthcare Equipment & Supplies
Dewhurst Group PLC	0.036200	0.002932	0.074921	Machinery, Tools, Heavy Vehicles, Trains & Ships

Table 11. Ranking of the bottom 10 companies in the TOPSIS method performed on the modified dataset.

On the other hand, the ranking of the bottom 10 companies exhibits significant differences compared to the previous implementation. The lowest positions are occupied by different companies than those in the ranking obtained from the original dataset.

6.2 Fuzzy TOPSIS Results

The fuzzy TOPSIS analysis conducted on the reduced dataset (403 companies) was performed using the same approach as in the application on the original dataset. The results, which also differ from those obtained in the corresponding implementation on the original dataset, are presented below. The ranking of the top 10 companies is as follows:

Company Name	Euclidean Distance PIS	Euclidean Distance NIS	Closeness Coeff	Industry Group
Roche Holding AG	0.209284	0.952355	0.819838	Pharmaceuticals
Eurofins Scientific SE	0.227926	0.934850	0.803981	Healthcare Providers & Services
Coca-Cola Europacific Partners PLC	0.234073	0.929872	0.798897	Beverages
Assicurazioni Generali SpA	0.236015	0.928898	0.797397	Insurance
Banco de Sabadell SA	0.238314	0.926271	0.795366	Banking Services
Snam SpA	0.240409	0.925983	0.793887	Oil & Gas Related Equipment and Services
Qiagen NV	0.242271	0.921729	0.791863	Biotechnology & Medical Research

J Sainsbury PLC	0.247142	0.918212	0.787926	Food & Drug Retailing
Hugo Boss AG	0.249588	0.915053	0.785695	Textiles & Apparel
Muenchener Rueckversicherungs Gesellschaft in ...	0.250037	0.914633	0.785315	Insurance

Table 12. Ranking of the top 10 companies in the fuzzy TOPSIS method performed on the reduced dataset.

The results reveal a high variability in values. An important factor to consider is that, compared to the previous analysis, all companies with an ESG Controversies Score of 100 have been removed. This filtering has excluded companies without critical ESG issues. However, the results do not indicate lower scores than those obtained with fuzzy TOPSIS on the original dataset; instead, they are similar, with the only difference being the altered positioning of the companies. The ranking of the bottom 10 companies is presented in Table 13:

Company Name	**Euclidean Distance PIS**	**Euclidean Distance NIS**	**Closeness Coeff**	**Industry Group**
Schweizerische Nationalbank	0.643764	0.437126	0.404413	Banking Services
Pinewood Technologies Group PLC	0.648075	0.424915	0.396010	Specialty Retailers
Gateley (Holdings) PLC	0.649550	0.421655	0.393627	Professional & Commercial Services
Porsche Automobil Holding SE	0.655835	0.417003	0.388692	Automobiles & Auto Parts
Angling Direct PLC	0.661153	0.390377	0.371247	Specialty Retailers
Pharol SGPS SA	0.674856	0.351179	0.342268	Telecommunications Services
CureVac NV	0.679455	0.338165	0.332310	Biotechnology & Medical Research
Trivago NV	0.681201	0.332775	0.328189	Software & IT Services
SDI Group PLC	0.695002	0.282376	0.288912	Healthcare Equipment & Supplies
Dewhurst Group PLC	0.698899	0.266064	0.275724	Machinery, Tools, Heavy Vehicles, Trains & Ships

Table 13. Ranking of the bottom 10 companies of the fuzzy TOPSIS method performed on the reduced dataset.

6.3 Comparison of Results with LSEG (Reduced Case)

In this analysis, which includes only companies with reported controversies, a correlation analysis between methodologies (including the LSEG ratings) was once again conducted using Spearman's correlation coefficient.

Spearman Index	*TOPSIS*	*Fuzzy TOPSIS*	*LSEG - ESG Score*	*LSEG - ESGC Score*
TOPSIS		0,65071059	0,973942476	0,482028718
Fuzzy TOPSIS	0,65071059		0,621976047	0,9481156
LSEG - ESG Score	0,973942476	0,621976047		0,480731187
LSEG - ESGC Score	0,482028718	0,9481156	0,480731187	

Table 14. Spearman Index (Reduced Case).

The highest value (ρ = 0.9739) is observed between the ranking produced by the TOPSIS method and that based on the LSEG ESG Score, indicating a very strong consistency between the two rankings. This result is fully expected, given that both methods do not account for corporate controversies. The similarity can thus be attributed to a shared methodological approach that evaluates ESG performance without applying penalties for critical events.

A similarly strong correlation is found between fuzzy TOPSIS and the ESGC Score (ρ = 0.9481), suggesting a high degree of correlation between the resulting rankings. This finding confirms that methods incorporating reputational risk tend to produce comparable outcomes.

The correlation between TOPSIS and fuzzy TOPSIS (ρ = 0.6507) appears moderate, indicating partial convergence. Although both techniques belong to the family of Multi-Criteria Decision-Making (MCDM) methods, the fact that only the fuzzy variant integrates controversy-related factors introduces a shift in company rankings, leading to a partially divergent ordering. It is plausible that companies exhibiting high ESG performance yet involved in scandals are downgraded only by fuzzy TOPSIS, resulting in a ranking less aligned with that of standard TOPSIS.

Lower correlation coefficients are observed when comparing methodologies that adopt opposing approaches regarding the treatment of controversies. The correlation between the ESG Score and fuzzy TOPSIS (ρ = 0.6220), although above the threshold for a moderate relationship, suggests that the inclusion of reputational components introduces notable discrepancies, particularly in the top and bottom positions of the ranking, those most sensitive to changes. An even lower correlation is found between ESGC and TOPSIS (ρ = 0.4820), indicating only partial alignment and a clear methodological divergence, consistent with the absence of reputational penalties in the standard TOPSIS model.

Lastly, the coefficient between the ESG Score and the ESGC Score (ρ = 0.4807) further confirms that incorporating controversies into ESG evaluations significantly impacts the resulting rankings, substantially altering company positions. This effect is particularly pronounced in the present case study, as the dataset is composed exclusively of companies involved in controversies, making the reputational dimension a non-negligible factor in assessing overall sustainability.

In conclusion, the results obtained through the Spearman correlation index reinforce the empirical evidence that methods with similar approaches to the treatment of controversies tend to generate more consistent rankings. Conversely, methodological differences in handling such critical factors lead to marked divergences. The inclusion of the "controversies" variable thus emerges as a key determinant for a more realistic and reliable evaluation of corporate ESG performance.

To support and validate the results obtained through the Spearman correlation index, an additional analysis is conducted using **Kendall's Tau correlation coefficient**, which is particularly well-suited for ordinal data and effective in assessing the consistency between rankings. In this case as well, the results confirm the main findings previously observed.

Kendall's Tau (Reduced Case)	***Value***
LSEG ESG - LSEG ESGC	0,3872
LSEG ESG - TOPSIS	0,8722
LSEG ESG - FUZZY TOPSIS	0,4748
LSEG ESGC - TOPSIS	0,3684
LSEG ESGC - FUZZY TOPSIS	0,8205

Table 15. Kendall's Tau (Reduced Case).

In particular, the highest correlation is observed between the rankings obtained through LSEG ESG and TOPSIS (τ = 0.8722), demonstrating the strong similarity between two methods that do not incorporate the "controversies" factor in their evaluations. Similarly, a high correlation is also found between ESGC and fuzzy TOPSIS ($\tau = 0.8205$), confirming the consistency between approaches that penalize companies involved in critical events.

Lower values, on the other hand, emerge in cross-comparisons between methods based on differing assumptions regarding the treatment of controversie, for example, between ESGC and TOPSIS ($\tau = 0.3684$), or between ESG and fuzzy TOPSIS ($\tau = 0.4748$). These results reflect the significant impact that reputational components have on the final ranking.

Overall, the Kendall's Tau index also confirms that the inclusion or exclusion of controversy-related evaluations represents the main discriminating factor among the analyzed rankings.

The final analysis focuses on the application of the Precision@K metric, with the aim of evaluating the overlap between rankings obtained through different methodologies with respect to the top K positions (with K ranging from 10 to 250). This type of metric is particularly useful when the interest lies in top-tier selections, namely the companies considered most sustainable according to each method.

Precision@K	***10***	***25***	***50***	***100***	***250***
ESG-ESGC	0,3	0,2	0,22	0,28	0,75
ESG-TOPSIS	0,8	0,76	0,82	0,85	0,95
ESG-FUZZY TOPSIS	0,3	0,2	0,22	0,3	0,82
ESGC-TOPSIS	0,3	0,16	0,24	0,31	0,75
ESGC-FUZZY TOPSIS	0,8	0,76	0,78	0,94	0,9
TOPSIS-FUZZY	0,3	0,16	0,24	0,34	0,82

Table 16. Precision@K (Reduced Case).

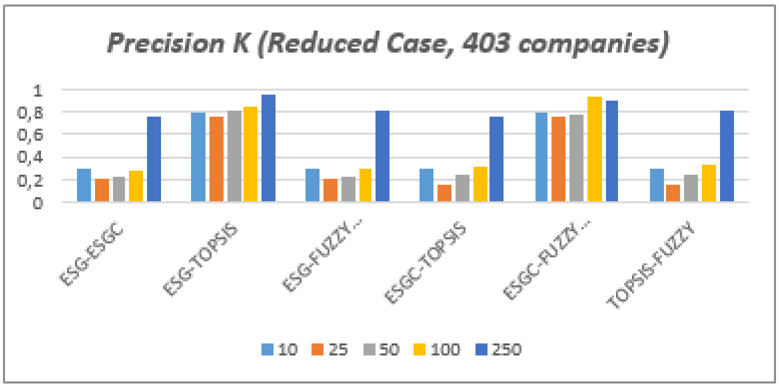

Figure 7. Precision@K (Reduced Case): Bar Chart.

The results clearly show that the ESG–TOPSIS and ESGC–Fuzzy TOPSIS pairs exhibit significantly higher Precision@K values starting from K = 10, with scores reaching 0.8 and 0.76, respectively. This high level of correlation in the top positions remains consistent as K increases, reaching values close to or exceeding 0.9. These findings align with previous observations: TOPSIS tends to produce rankings similar to ESG (as neither penalizes corporate controversies), while fuzzy TOPSIS closely aligns with ESGC (as both incorporate reputational factors into their evaluations).

Conversely, the remaining combinations, particularly ESG–ESGC, ESGC–TOPSIS, and TOPSIS–Fuzzy TOPSIS, show substantially lower values in the top positions (Precision@10 and @25 generally ≤ 0.3), indicating limited agreement among the top-K ranked companies. However, as K increases, these values tend to rise, suggesting that the methodological divergences are concentrated mainly at the top of the rankings, while similarity increases when the analysis is extended to a broader set of alternatives.

Overall, the Precision@K analysis confirms that the inclusion or exclusion of controversy treatment remains the primary factor driving divergence among the rankings. Moreover, it highlights that agreement between methods is significantly stronger within the top lists only when they share the same evaluative approach, whereas in the absence of such alignment, overlaps are minimal, particularly among the top K elements.

Conclusions

The objective of this study is to propose a more robust and realistic ESG classification system, capable of capturing the nuances associated with decision-making uncertainty and the impact of corporate controversies. To this end, a model is developed based on the integration of fuzzy logic with the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution), applied to a dataset of publicly listed companies and compared with the ESG rankings provided by LSEG, both in the standard version (ESG Score) and in the variant that includes penalties for controversies (ESGC Score).

The results obtained confirm that the fuzzy TOPSIS approach constitutes a methodologically more flexible and expressive solution compared to the direct use of ESG scores provided by external rating agencies. In particular, within the subset consisting of 403 companies involved in controversies, significant differences emerged in the rankings, with low consistency observed between methods that include reputational components and those that do not.

Spearman and Kendall's Tau correlation coefficients indicated a strong alignment between fuzzy TOPSIS and ESGC ($\rho \approx 0.95$; $\tau \approx 0.82$), while cross-comparisons with the standard ESG rankings yielded weak to moderate correlations, thereby confirming the discriminative relevance of including controversies in the evaluation process.

Further supporting this evidence, the application of the Precision@K index showed that rankings based on similar methodological approaches (e.g., fuzzy TOPSIS and ESGC, or TOPSIS and ESG) tend to overlap in the top positions. In contrast, combinations involving "mixed" methods produce marked discrepancies. This finding is particularly relevant, as the top-ranked positions in an ESG classification are those that most strongly influence investment decisions and policy-making processes (see e.g. N. Semenova and L. G. Hassel (2015)).

The choice to adopt the fuzzy TOPSIS method, rather than relying solely on the ESGC score, may be influenced by several factors, primarily related to substantial differences in methodological structure, the ability to manage uncertainty, and adaptability to specific decision-making contexts.

Unlike the fuzzy approach, the ESGC score provided by LSEG merely penalizes companies involved in controversies through a predefined reduction mechanism applied to the aggregated ESG score. However, it does not offer any explicit representation of the degree of uncertainty associated with the assessment. In other words, the ESGC system follows a deterministic logic, whereby the presence of controversies leads to a fixed downgrade in the rating, without disclosing the variability of the judgment, the reliability of the sources used, or the specific impact of such controversies on the individual ESG pillars under evaluation. This kind of approach, typical of proprietary models, is methodologically opaque and does not allow for the customization of how ambiguous cases are handled, nor for adaptation to complex or incomplete informational scenarios.

By contrast, a model based on fuzzy logic allows uncertainty to be formalized through the use of triangular fuzzy numbers, thereby providing a more realistic representation of the vagueness inherent in the data and evaluations. This enhances the transparency of the decision-making process and allows for greater adaptability to the specific informational conditions of the context being analyzed (see e.g. Zimmermann (2001)).

Unlike LSEG's ESGC index, which represents a monolithic aggregated score based on a proprietary methodology that cannot be modified by the end user, the fuzzy TOPSIS method belongs to the family of Multi-Criteria Decision Making (MCDM) approaches. This framework enables the construction of flexible decision models in which the weights assigned to different ESG indicators can be dynamically adjusted according to the explicit or implicit preferences of the decision-maker. As noted by Triantaphyllou (2000), MCDM methods are particularly well suited for real-world contexts, where decision alternatives are finite, criteria are multiple and often heterogeneous, and absolute optimality is less important than the ability to represent credible and context-sensitive trade-offs. The incorporation of fuzzy logic into the TOPSIS method also allows for the expression of qualitative and ambiguous judgments, enhancing the model's adaptability to incomplete or uncertain information scenarios. In contrast, ESGC does not allow for the customization of evaluation criteria or for the exploration of sensitivity to weight variations, making it less appropriate for conducting transparent and replicable assessments of ESG performance.

Moreover, as highlighted by Berg, Kölbel, and Rigobon (2022), the divergence among commercial ESG ratings is largely attributable to methodological opacity and the lack of shared standards across providers. Although, for instance, Refinitiv employs a very large number of indicators, including several unconventional ones, and demonstrates high internal consistency in its model ($R^2 = 0.92$), this reliability is confined to the internal workings of the system itself. For external users, the evaluation process remains inaccessible and essentially functions as a “black box,” neither replicable nor verifiable. In contrast, the adoption of a fuzzy-based model allows for the customization of the decision-making structure and the linguistic values associated with ESG criteria, thereby making the classification process more transparent, controllable, and replicable, particularly in contexts characterized by informational uncertainty and the presence of multiple evaluation dimensions.

In conclusion, the integration of the fuzzy TOPSIS method has proven to be an effective strategy for overcoming the limitations of traditional ESG metrics, offering a model that is more responsive to controversies, capable of managing uncertainty in a structured manner, and well-suited for multi-criteria analysis. In a context where the credibility of ESG assessments is increasingly under scrutiny (see, e.g., S. Kotsantonis and G. Serafeim (2019)), decision-making tools of this nature are essential for strengthening investor confidence and for guiding sustainable strategies grounded in transparent and methodologically sound evidence.

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Tables

Figures

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