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Free and forced morphodynamics of river bifurcations

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Abstract

Water and sediment distribution by river bifurcations is often highly unbalanced. This may result from a variety of factors, like migration of bars, channel curvature, backwater effects, which promote an uneven partition of flow and sediment fluxes in the downstream branches, which we call “forcings”. Bifurcations also display an intrinsic instability mechanism that leads to unbalanced configurations, as it occurs in the idealized case of a geometrically symmetric bifurcation, which we call “free”, provided the width-to-depth ratio of the incoming flow is large enough. Most frequently, these free and forced mechanisms coexist, however their controlling roles on bifurcation dynamics has not been investigated so far. In this paper we address such question by proposing a unified free-forced modelling framework for bifurcation morphodynamics. Upstream channel curvature and different slopes of downstream branches (slope advantage) are specifically investigated as forcing effects typically occurring in bifurcations of alluvial channels. The modelling strategy is based on the widely used two-cell model of *Bolla Pittaluga et al.* (2003) here extended to account for the spatially non-uniform fluxes entering the bifurcation node. Results reveal that the relative role of free and forced mechanisms depends on the width to depth ratio falling above or below the resonant threshold that controls the stability of free bifurcations: when the main channel is relatively wide and shallow (super-resonant regime) the bifurcation invariably evolves towards unbalanced configurations, whatever the combination of curvature and slope advantage values, which instead control the bifurcation response under sub-resonant conditions. Detection of the resonant aspect ratio as a key threshold also releases the modelling approach from the need of parameter calibration that characterized previous approaches, and allows for interpreting under a unified framework the opposite behaviours shown by gravel bed and sand bed bifurcations for increasing Shields parameter values.

1 Introduction

Channel bifurcations control the downstream distribution of water and sediments in a variety of fluvial environments, such as deltas, alluvial fans, braided and anabranching rivers (*Slingerland and Smith, 2004; Kleinhans et al., 2013*). Understanding their dynamics is therefore important for managing water resources and the flooding risk, predicting the long-term morphological evolution of channel networks and evaluating the effectiveness over time of river restoration projects aimed at reactivating a multi-thread configuration (e.g., *Habersack and Piégay, 2007*). Bifurcation dynamics also control instream processes in meander bends that mitigate the development of channel sinuosity through the occurrence of short cuts through point bars (*Grenfell et al., 2012; van Dijk et al., 2014*).

River bifurcations have been extensively studied through laboratory-scale physical models (*Federici and Paola, 2003; Bertoldi and Tubino, 2007; Bertoldi et al., 2009; Le et al., 2018b*), and mathematical models based on 1D (*Wang et al., 1995; Bolla Pittaluga et al., 2003; Kleinhans et al., 2013; Salter et al., 2018*), 2D (*Edmonds and Slingerland, 2008; Siviglia et al., 2013; Le et al., 2018b,a*) and 3D approaches (*Kleinhans et al., 2008*). Along with field observations (e.g., *Zolezzi et al., 2006; Kleinhans et al., 2012*), these studies highlight the almost invariable tendency of bifurcations to produce an uneven distribution of flow and sediment transport, which results in a strong asymmetry of the channel width and bed elevation of downstream anabranches.

This type of unbalanced configuration is often promoted by various “forcing” effects that drive the bifurcation towards an unbalanced state, sometimes leading to the complete closure of one of the anabranches. Forcing factors include both upstream and downstream effects. Upstream effects comprise mechanisms that feed the bifurcating node with a topographically-driven uneven distribution of flow and transport rate, like the curvature of the upstream channel (*Kleinhans et al., 2008; Hardy et al., 2011; Sloff and Mosselman, 2012*) and the occurrence of migrating bars (*Bertoldi et al., 2009; Bertoldi, 2012*) or steady bars (*Le et al., 2018b*). Downstream effects include mechanisms that provide, locally or through backwater effects, a slope advantage to one of the distributaries (*Edmonds, 2012; van Dijk et al., 2014; Zhang et al., 2017; Salter et al., 2018*). For purely illustrative purposes, Figure 1 shows real-world bifurcations with examples of these

53 concepts. Figures 1a and 1b provide examples of bifurcations where one of these forcing factors is
 54 likely dominant: the upstream channel curvature (Figure 1a) and the slope advantage for the right
 55 bifurcate (Figure 1b). In Figures 1c and 1d these two forcings likely have comparable relevance.
 56 They might cooperate in the case of Figure 1c, because the chute channel detaches from the outer
 57 bank of the upstream channel bend, thus receiving most of the water and sediment input from
 58 upstream, and is also shorter than the other (left) bifurcate, thus having a slope advantage. On
 59 the contrary, they likely compete in the case of Figure 1d, because the shorter chute channel occurs
 60 on the inner bank of the upstream channel bend.

61 Interestingly, the unbalanced configuration can also result from an inherent instability mecha-
 62 nism, even in the absence of external forcings, as shown theoretically by *Wang et al.* (1995) and
 63 *Bolla Pittaluga et al.* (2003), and later demonstrated through laboratory and numerical studies
 64 (*Bertoldi and Tubino*, 2007; *Edmonds and Slingerland*, 2008; *Siviglia et al.*, 2013; *Salter et al.*,
 65 2018). More recently *Redolfi et al.* (2016) provided an interpretation of such “free” bifurcations
 66 instability within the framework of the theory of morphodynamic influence of *Zolezzi and Sem-*
 67 *inara* (2001), showing that the unbalanced configuration arises when the bifurcation is able to
 68 exert an upstream morphodynamic influence that allows for the formation of an upstream steady
 69 bar. Such upstream morphodynamic influence theoretically occurs when the width to depth ratio
 70 exceeds a threshold “resonant” value, as originally defined by *Blondeaux and Seminara* (1985) in
 71 the theory of regular meanders. For small (sub-resonant) values of the width to depth ratio a
 72 symmetric free bifurcation keeps stable and equally distributes water and sediment fluxes in the
 73 downstream branches, while in the super-resonant regime such balanced configuration is no longer
 74 stable and the bifurcation invariably evolves towards an unbalanced configuration.

75 The above scenario suggests that in sub-resonant conditions the tendency towards unbalanced
 76 states observed in real rivers is mainly driven by external forcing factors, while in super-resonant
 77 conditions both free and forced mechanisms are likely interacting, though their respective roles
 78 have not been investigated so far. The question therefore arises to which extent the autogenic, free-
 79 instability mechanism or instead the external forcings affect the behaviour of natural bifurcations,
 80 and under which conditions those effects cooperate or compete to produce what is observed in

81 such complex settings.

82 We aim at answering this question by taking the viewpoint of river bifurcations as dynamical
83 systems for which a distinct role of the free and forced responses can be identified. This method-
84 ological distinction is based on the recognition that free and forced mechanisms display substantial
85 differences in their evolutionary temporal and spatial scales. Similar approaches have proven to
86 provide thorough insight in the study of other morphodynamic processes, like the dynamics of
87 river bars in curved or meandering channels (*Seminara and Tubino, 1989*), where migrating free
88 bars develop on a much faster scale than that required to shape the meander planform (*Tubino*
89 *and Seminara, 1990*).

90 In this paper we cast within a unified theoretical framework previous results on free and
91 forced bifurcations and consider two main forcing factors: the upstream effect exerted by an
92 incoming curved channel and the downstream effect of slope advantage of one of the distributaries,
93 which can derive from the different length of the distributaries, from differential downstream
94 degradation/deposition (*Salter et al., 2018*), or from backwater effects (e.g. *Edmonds, 2012*).

95 The analysis is based on the two-cell model originally proposed by *Bolla Pittaluga et al. (2003)*,
96 as extended by *Kleinhans et al. (2008)* to account for the curvature-driven secondary flow, and on
97 the theoretical results of *Redolfi et al. (2016)*. The analytical model prescribes physically-based,
98 simplified nodal point relationships that enables us to explore the basic mechanisms that drive the
99 water and sediment distribution at the node. As highlighted by *Wang et al. (1995)*, the behaviour
100 of the bifurcation depends on how the sediment is distributed with respect to the downstream
101 transport capacity; sediment distribution is in turn determined by the transverse flow-exchange
102 and gravitational effects on bed load transport just upstream the bifurcation node, as explained
103 by *Bolla Pittaluga et al. (2003)*.

104 2 Methods

105 Our model stems from the *Bolla Pittaluga et al. (2003)* two-cell approach, and is formulated to
106 incorporate both free and forced bifurcation responses, and their interaction. It allows predicting
107 how water and sediment fluxes delivered from the upstream main channel are drained by the

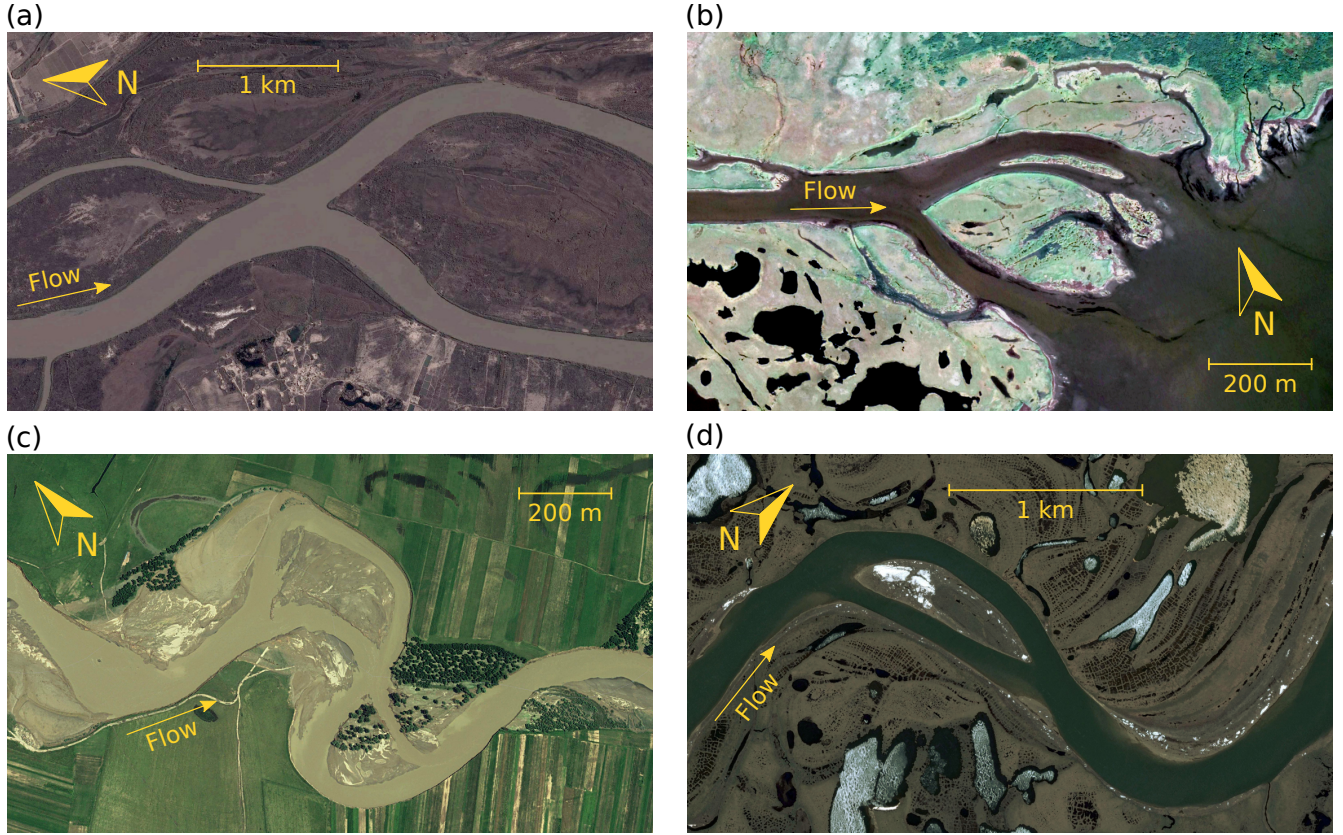


Figure 1: Satellite images showing illustrative examples of river bifurcations: (a) Tigris River near Baghdad (Iraq), $34^{\circ}16' N$, $43^{\circ}50' E$, with a curved upstream channel and downstream bifurcates having a nearly symmetrical channel geometry; (b) estuary in the Kamchatka Peninsula (Russia), $60^{\circ}02' N$, $163^{\circ}40' E$, where the left bifurcate likely covers a longer distance for the same elevation gap from the bifurcation node to the sea, suggesting a possible slope advantage for the left bifurcate; (c) bends with chute cutoffs in the meandering Siret River (Romania), $47^{\circ}39' N$, $26^{\circ}30' E$, with the cutoff channel initiating on the outer bank of an upstream channel bend and being shorter than the left bifurcate; (d) meandering River in the Ust-Chaun area (Russia), $68^{\circ}42' N$, $170^{\circ}35' E$, with the cutoff channel initiating on the inner bank of a bend and being shorter than the left bifurcate; from *Google Earth*, *Digital Globe* (2018).

108 downstream anabranches, under different combination of external forcings.

109 The bifurcation geometry is sketched in Figure 2a, having an upstream, curved main channel
 110 of width W_a , slope S_a and radius of curvature R , which bifurcates in two downstream channels
 111 having width W_b and W_c and slope S_b and S_c , respectively. In the model, flow and sediment
 112 balances applied to two cells of length αW_a , which also accounts for transverse exchanges, rules
 113 the distribution of water (Q) and sediment (Qs) fluxes between the bifurcates, as represented in
 114 Figures 2b and 2c.

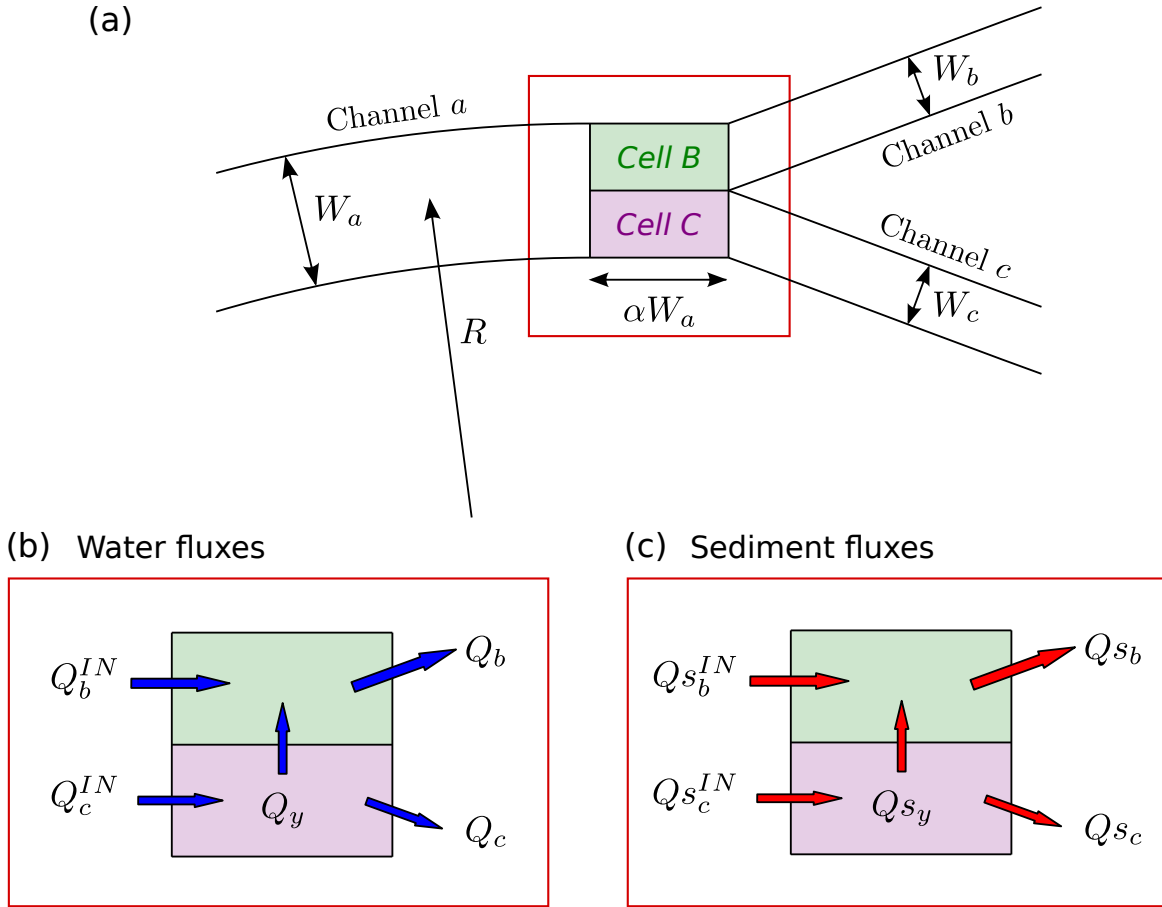


Figure 2: Bifurcation geometry and notation used in the mathematical model formulation: (a) planform view, showing the curved main channel, the two distributaries and the two cells; (b) and (c) water and sediment fluxes through the cells.

115 The model considers the effect of secondary flows associated with streamline curvature within
 116 the node cells (*Kleinhans et al.*, 2008), and includes an extension of such previous model to account
 117 for the non-uniform distribution for the entering water and sediment fluxes.

2.1 The free bifurcation

The core of the method is a model for a free bifurcation with perfectly symmetrical geometry and boundary conditions (i.e. with no curvature nor slope advantage) that follows the classic approach of *Bolla Pittaluga et al.* (2003). The flow and bed topography in the three channels result from a 1D mobile bed model, which can be solved once the following five matching conditions at the bifurcation node are specified: conservation of sediment and water fluxes (two conditions), energy conservation (i.e. water surface elevation) along each cell (two conditions), and a physically-based relation that prescribes how sediment fluxes are partitioned at the bifurcation node. This type of nodal point relation is the key to incorporate bifurcation morphodynamics within a simple 1D scheme, and accounts for the exchange of sediment between the two cells through the following relationship:

$$\underbrace{\frac{Qs_y/(\alpha W_a)}{Qs_a/W_a}}_{\text{sediment flux direction}} = \underbrace{\frac{Q_y/(\alpha W_a D_{abc})}{Q_a/(W_a D_a)}}_{\text{velocity direction}} - \underbrace{\frac{2r}{\sqrt{\theta_a}} \frac{\eta_b - \eta_c}{W_b + W_c}}_{\text{gravitational effect}}, \quad (1)$$

where the direction of the sediment flux is determined by the direction of velocity and by the gravitational effect induced by the transverse gradient of the bed elevation. The last term of Equation (1) is estimated according to the *Ikeda* (1982) formulation, where r is a dimensionless coefficient (e.g., *Baar et al.*, 2018), θ_a is the Shields stress in the main Channel a , η_b and η_c indicate the bottom elevation at the inlet of the distributary Channels b and c , respectively. The mean depth within the cell, defined as $D_{abc} = (2D_a + D_b + D_c)/4$, can be simplified to $D_{abc} = D_a$ as proposed by *Salter et al.* (2018).

The flow in the three anabranches is modelled using a classic 1D shallow water and Exner model, whose steady solution is simply an uniform flow. The water flow in each channel $i = \{a, b, c\}$ is given by:

$$Q_i = W_i c_i \sqrt{g S_i} D_i^{3/2}, \quad (2)$$

where g is the gravitational acceleration, D_i is the water depth and c_i is the dimensionless Chézy

140 coefficient, which can be calculated as (*Engelund and Fredsoe*, 1982):

$$c_i = 6 + 2.5 \log \left(\frac{1}{2.5} \frac{D_i}{d_{50}} \right), \quad (3)$$

141 with d_{50} indicating the median grain size.

142 Similarly, the volumetric sediment flux is computed as:

$$Q_{s_i} = W_i \sqrt{g \Delta d_{50}^3} \Phi \left(\theta_i, \frac{D_i}{d_{50}} \right), \quad \theta_i = \frac{S_i D_i}{\Delta d_{50}}, \quad (4)$$

143 where Δ is the relative submerged density of the sediment, θ_i is the Shields stress and Φ is a
 144 function given by the sediment transport formula. Specifically, we used the *Parker* (1990) formula
 145 for gravel bed channels and the *Engelund and Hansen* (1967) formula for sand bed cases.

146 The free character of the bifurcation manifests itself in the symmetrical configuration of up-
 147 stream and downstream channels, which determines water and sediment fluxes that enter and exit
 148 the cells (Figures 2b and 2c). First, the input fluxes Q_i^{IN} and $Q_{s_i}^{IN}$ that are delivered by the
 149 upstream channel into the node cells are uniform. Second, the absence of slope advantage (i.e.
 150 $S_b = S_c$) leads to the same water and sediment rating curves for the two bifurcates. Therefore,
 151 possible asymmetries of the output fluxes are not driven by the upstream/downstream conditions
 152 but can only derive from an uneven redistribution by the bifurcation node.

153 2.2 The forced bifurcation

154 Different forcing effects can be further incorporated in the free bifurcation model, to increase its
 155 ability of the model to represent real bifurcation configurations. The key to model those effects
 156 is to act on the upstream and boundary conditions imposed at the node cells, which practically
 157 implies considering non-uniform water and sediment fluxes entering the two node cells, and/or
 158 imposing different water and sediment rating curves for the two bifurcates. Such an approach has
 159 been already exploited by *Bertoldi et al.* (2009) when modelling how bifurcations dynamics can
 160 be affected by migrating bars, which were schematised as periodic temporal oscillations of water
 161 and sediment fluxes delivered to the node cells. Here we consider two different forcing effects: (i)

162 a downstream slope advantage of one bifurcate with respect to the other one and (ii) the presence
 163 of a curved upstream channel.

164 A downstream slope advantage increases the probability for an unbalanced water and sediment
 165 flux towards the “advantaged” distributary. This effect can be taken into account by setting
 166 different values of S_b and S_c in Equations (2) and (4). This breaks the symmetry in the downstream
 167 rating curves, even if the other geometrical parameters remain equal between the two downstream
 168 branches. We quantify the slope advantage through the following parameter:

$$\Delta S = \frac{S_b - S_c}{S_b + S_c}, \quad (5)$$

169 with positive values of ΔS indicating that the outer bend bifurcate (Channel b of Figure 2a) is
 170 steeper than the inner bend bifurcate (Channel c).

171 We model the presence of an upstream bend (Channel a) feeding the bifurcation as does a
 172 channel with constant radius of curvature R . The curvature of the main channel leads to the
 173 formation of a spiral flow (Figure 3a), which in turn produces a shear stress in the transverse
 174 direction. Therefore, the bottom stress ends up being deflected by an angle ϕ_τ , which can be
 175 computed as (*Struiksmma et al.*, 1985):

$$\tan(\phi_\tau) = -A \frac{D}{R}, \quad (6)$$

176 where D is the local water depth and A is the coefficient that defines the intensity of the secondary
 177 flow, given by:

$$A = \frac{2}{\kappa^2} \left(1 - \frac{1}{\kappa c} \right) \quad (7)$$

178 with $\kappa = 0.4$ indicating the Von Karman constant.

179 In the region where the flow is fully developed (i.e., far enough from the bend entrance),
 180 the flow characteristics do not vary along the channel, and the depth-averaged velocity is purely
 181 longitudinal. In these conditions, the deflection of the bed shear stress is compensated by a

transverse gradient of the bed elevation (see Figure 3a) that is given by:

$$\frac{d\eta}{dy} = \frac{\sqrt{\theta}}{r} \tan(\phi_\tau), \quad (8)$$

(for mathematical details see Appendix A), which in turn generates non-uniform transverse profiles of water depth, longitudinal velocity and shear stress. Consequently, water and sediment fluxes feeding the two cells are not uniform but are mainly delivered towards the cell positioned at the outer bend (Cell B of Figure 3b).

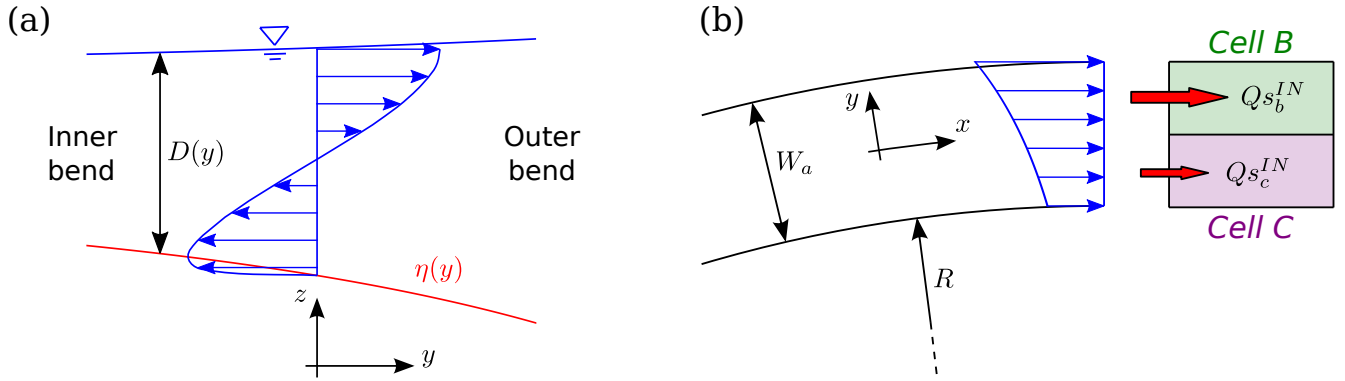


Figure 3: Illustration of the secondary flow solution in the upstream channel: (a) cross-sectional view, indicating the vertical profile of the transverse velocity (arrows), and the transverse profiles of bed elevation and water depth; (b) planimetric view, with the transverse velocity profile (arrows) and the associated sediment fluxes entering the cells (Qs_b^{IN} and Qs_c^{IN}).

Furthermore, as suggested by *Kleinhans et al.* (2008) the deviation of the bed shear stress given by Equation (6) is also active within the cells, so that the nodal point relation (1) needs to be extended as follows:

$$\underbrace{\frac{Qs_y}{\alpha Qs_a}}_{\text{sediment flux direction}} = \underbrace{\frac{Q_y}{\alpha Q_a}}_{\text{velocity direction}} - \underbrace{\frac{2r}{\sqrt{\theta_a}} \frac{\eta_b - \eta_c}{W_b + W_c}}_{\text{gravitational effect}} - \underbrace{A \frac{D_a}{R}}_{\text{spiral flow effect}}, \quad (9)$$

where D_a and θ_a are the average values of water depth and Shields stress in the upstream Channel a .

The above formulation is generally suitable for modelling bifurcations with arbitrary channel widths. However, for the purpose of analysing the interaction between the different mechanisms, also in comparison with previous works (e.g., *Bolla Pittaluga et al.*, 2015), we restrict our attention

195 to the basic case where both the bifurcates have half the width of the main channel ($W_b = W_c =$
196 $W_a/2$).

197 **3 Results**

198 The model solution can be expressed as a function of a few dimensionless parameters. First,
199 the solution depends on the reference conditions, which are defined as the uniform flow and
200 sediment transport in a straight channel with same slope, width and discharge of the main channel.
201 Specifically, we need to prescribe three main parameters, namely the aspect ratio, the Shields stress
202 and the relative roughness:

$$\beta_0 = \frac{1}{2} \frac{W_a}{D_0}, \quad \theta_0 = \frac{S_a D_0}{\Delta d_{50}}, \quad d_{s0} = \frac{d_{50}}{D_0}, \quad (10)$$

203 where zero subscript (e.g., D_0) indicates the reference conditions.

204 Second, the solution depends on the intensity of the forcing effects, which is specified through
205 the normalized curvature W_a/R and the slope advantage ΔS .

206 In the following, we analyse the model outputs in terms of discharge asymmetry (*Bertoldi*
207 *et al.*, 2009), which can be taken as a representative indicator of the bifurcation response:

$$\Delta Q = \frac{Q_b - Q_c}{Q_a}, \quad (11)$$

208 which may range from -1 (no flow in Channel b) to $+1$ (no flow in Channel c), with $\Delta Q = 0$
209 indicating balanced bifurcations.

210 **3.1 The free bifurcation**

211 The free bifurcation configuration is made by three straight channels without any slope advantage
212 nor width or angle asymmetry, thus without external effects that may force an unbalanced config-
213 uration. In this condition the intuitive expectation would be a symmetrical bifurcation response,
214 with an even distribution of downstream water and sediment fluxes. As illustrated in Figure 4,

the balanced configuration ($\Delta Q = 0$) is indeed an equilibrium solution of the system. However, for relatively high values of the aspect ratio the balanced configuration becomes unstable, and one of the channels, indifferently, tends to dominate.

As highlighted by *Bolla Pittaluga et al.* (2003) this type of instability of the symmetric equilibrium solution does not necessarily lead to a complete closure of one branch, but new, unbalanced and stable equilibrium states are possible. In Figure 4, this is represented by the formation of a so called “pitchfork bifurcation” (e.g., *Wiggins*, 2003) in the equilibrium diagram, which occurs at β_0 near 13.5. The two unbalanced equilibrium configurations are physically sustained by the formation of an inlet step, i.e. a localized steep reach at the head of the largest flow-carrying bifurcate (*Bertoldi et al.*, 2009), which steers the sediment flux and thus satisfies the balance between sediment supply and transport capacity of the bifurcates.

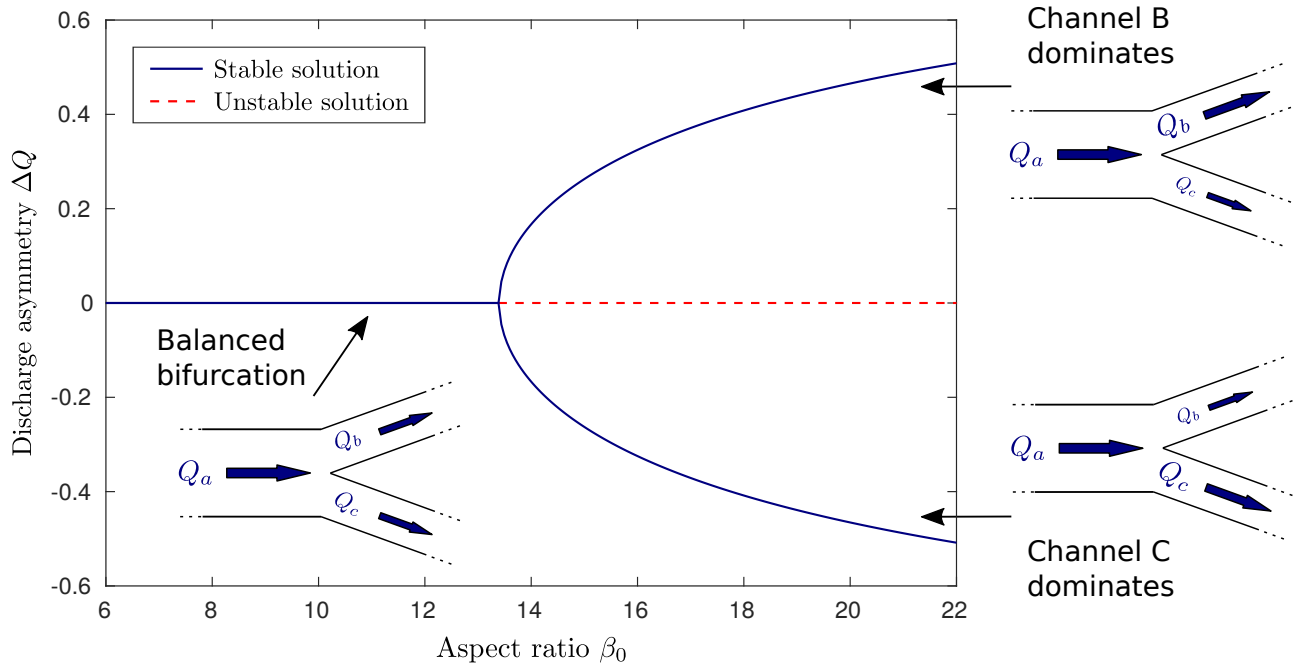


Figure 4: Equilibrium solutions of a free bifurcation (i.e. straight channel with no slope advantage) according to the two-cell model. Solid lines indicates stable solutions, while the dashed line represents an unstable equilibrium configuration. Parameters are $\theta_0 = 0.1$, $d_{s0} = 0.02$, $r = 0.5$; the *Parker* (1990) transport formula is used.

The critical point at which the pitchfork bifurcation appears is determined through a linear stability analysis (*Bolla Pittaluga et al.*, 2015), whose result can be expressed in the following

228 general form:

$$\beta_C = \frac{r\alpha}{\sqrt{\theta_0}} \frac{4}{\Phi_D + \Phi_T - (3/2 + c_D)}, \quad (12)$$

229 whose coefficients are defined as:

$$c_D := \frac{D_0}{c_0} \frac{\partial c}{\partial D} \Big|_{D_0}, \quad \Phi_D := \frac{D_0}{\Phi_0} \frac{\partial \Phi}{\partial D} \Big|_{D_0, \theta_0}, \quad \Phi_T := \frac{\theta_0}{\Phi_0} \frac{\partial \Phi}{\partial \theta} \Big|_{D_0, \theta_0}, \quad (13)$$

230 and represent the sensitivity of Chézy coefficient and of the dimensionless sediment transport rate
 231 to variations of water depth and Shields stress. The algebraic expressions of the coefficients c_D ,
 232 Φ_D and Φ_T for the used formulae of *Parker* (1990) and of *Engelund and Hansen* (1967), as well
 233 as for the classical relation of *Meyer-Peter and Muller* (1948), are reported in Appendix B for
 234 the sake of clarity. Equation (12) represents a generalization of the formula proposed by *Bolla*
 235 *Pittaluga et al.* (2015) for arbitrary transport and friction formulae.

236 According to Equation (12), the critical aspect ratio is directly proportional to the cell length
 237 represented by α . In a 1D formulation, the parameter α needs to be empirically calibrated,
 238 resulting in rather different literature values, ranging from 1 (*Bolla Pittaluga et al.*, 2003) to
 239 6 (*Bertoldi and Tubino*, 2007). This limitation has been solved by *Redolfi et al.* (2016), who
 240 developed an analytical linear solution of the fully 2D problem; in that formulation, the length of
 241 the upstream cells is resolved by the model itself, so that no specific calibration is needed.

242 The analysis of *Redolfi et al.* (2016) demonstrated that the emergence of an unbalanced solution
 243 in a free bifurcation depends on the formation of an upstream steady bar, which occurs when the
 244 bifurcation is able to exert a morphodynamic influence in the upstream direction (see Figure
 245 5). As theoretically derived by *Zolezzi and Seminara* (2001) and experimentally observed by
 246 *Zolezzi et al.* (2005) any fixed geometrical disturbance can produce a permanent upstream bed
 247 deformation, usually taking the form of a steady bar, when the channel is wide and shallow enough
 248 for the aspect ratio of the main channel to exceed the resonant threshold, β_R , as originally defined
 249 in the theory of regular meanders by *Blondeaux and Seminara* (1985). Therefore, under super-
 250 resonant conditions ($\beta_0 > \beta_R$) the bifurcation node - as a fixed geometrical disturbance - can
 251 trigger such an upstream morphodynamic influence. This is essentially the physical mechanism

252 behind the mathematical instability of the balanced equilibrium solution, which therefore makes
 253 a free bifurcation unbalanced.

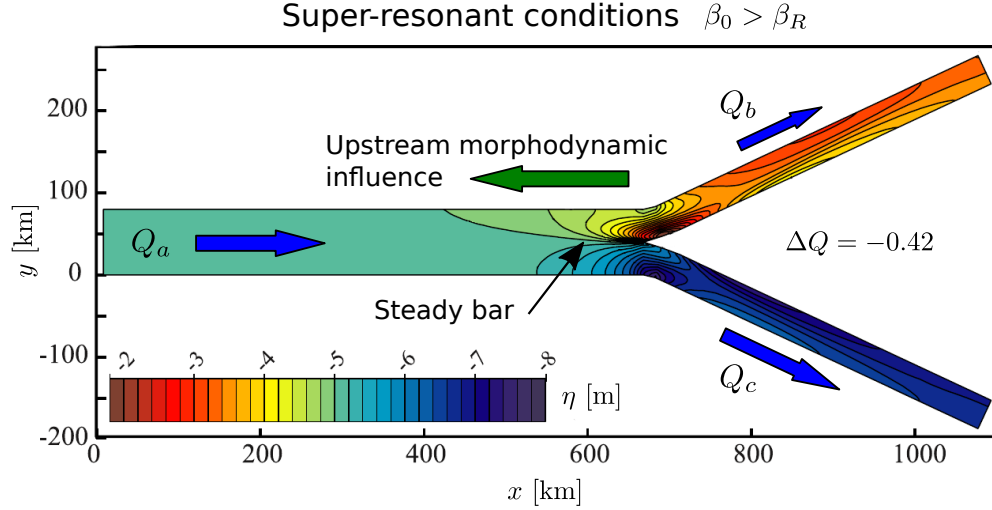


Figure 5: Map of bed elevation and water distribution in a free bifurcation from the numerical simulations of *Edmonds and Slingerland* (2008), adapted from their Figure 5b. The equilibrium configuration is made unbalanced by the formation of an upstream steady bar, which deviates most water and sediment fluxes towards the right bifurcate. According to *Redolfi et al.* (2016) this is an effect of the upstream morphodynamic influence exerted by the bifurcation node, which occurs when the aspect ratio of the upstream channel β_0 exceeds the resonant threshold β_R .

254 The computation of the resonant aspect ratio requires the solution of a fourth-order polynomial,
 255 which can be readily obtained using the available Matlab code (see Acknowledgements Section).
 256 The resulting values for gravel bed and sand bed river channels as a function of the relevant
 257 dimensionless parameters are reported in Figures 6a and 6b, respectively. For sand bed rivers
 258 the Chézy coefficient is independently fixed, rather than derived from Equation (3), to account
 259 for the higher drag exerted by bedforms, also consistently with previous applications (*Edmonds*
 260 *and Slingerland*, 2008). In gravel bed channels β_R increases with both the Chézy coefficient and
 261 the Shields stress, which explains the tendency of bifurcations to stabilize when increasing θ_0
 262 (*Bolla Pittaluga et al.*, 2015). Conversely, in sand bed channels the resonant threshold tends to
 263 decrease with the Shields stress, which explains the opposite effect of θ_0 observed by *Edmonds*
 264 *and Slingerland* (2008). Consistently with *Bolla Pittaluga et al.* (2015) the different behaviour of
 265 sand bed and gravel bed channels is related to the different response of the dimensionless sediment
 266 transport rate to variations of the Shields stress (which depends on the transport formula) rather

Table 1: List of datasets used to evaluate the resonance criterion for bifurcation instability.

Dataset	Method	Bed material	# of cases	β_0	θ_0
<i>Bertoldi and Tubino</i> (2007)	Laboratory	gravel	25	4.9 – 26.3	0.042 – 0.099
<i>Edmonds and Slingerland</i> (2008)	Numerical	sand	11	8	0.80 – 2.19
<i>Siviglia et al.</i> (2013)	Numerical	gravel	18	3.5 – 24.0	0.060 – 0.200
<i>Zolezzi et al.</i> (2006)	Field	gravel	6	9.5 – 14.5	0.053 – 0.088
<i>Bolla Pittaluga et al.</i> (2015)	Field	sand	11	16.9 – 77.1	0.30 – 1.16

than an effect of a gradient in the water surface elevation near the bifurcation, as suggested by *Edmonds and Slingerland* (2008).

Alternatively, the resonant aspect ratio can be calculated though the approximate expression by *Camporeale et al.* (2007):

$$\beta_R = \frac{\pi}{2\sqrt{2}} \frac{c_0\sqrt{r}}{\theta_0^{1/4}} \frac{1}{\sqrt{\Phi_D + \Phi_T - (3/2 + c_D)}}, \quad (14)$$

which provides rather accurate estimates of β_R for sand bed channels, while it gives slightly underestimated values for gravel bed cases (up to -13% for the range of parameters in Figure 6a).

The resonant threshold provides a simple criterion to determine if a free bifurcation remains balanced or tends to evolve towards unbalanced states. To test this criterion we used the literature datasets listed in Table 1, which include gravel and sand bifurcations measured in the field, modelled numerically and reproduced in laboratory-scale physical models. For each of the 71 bifurcations the different authors provided observation of their balanced/unbalanced state and the basic flow parameters, which were used to compute the resonant threshold and the parameter $(\beta_0 - \beta_R)/\beta_R$ representing the relative distance to resonance. We set $r = 0.5$ for all cases except for the *Edmonds and Slingerland* (2008) experiments, for which an equivalent $r \simeq 0.35$ value was needed to match the default $\alpha_{bn} = 1.5$ value of their Delft3D formulation (e.g., *Lesser et al.*, 2004). Results reported in Figures 6c and 6d show that balanced bifurcations (closed markers) tend to stay below the dashed line (i.e. sub-resonant conditions), while all the unbalanced bifurcations (open markers) are located above the dashed line (i.e. super-resonant conditions), independently of the Shields stress. It is worth noting that the resonant criterion also captures the numerical results of *Edmonds and Slingerland* (2008), who demonstrated that balanced solutions in sand bed channels tends to become unstable with increasing θ_0 . The above analysis confirms that the reso-

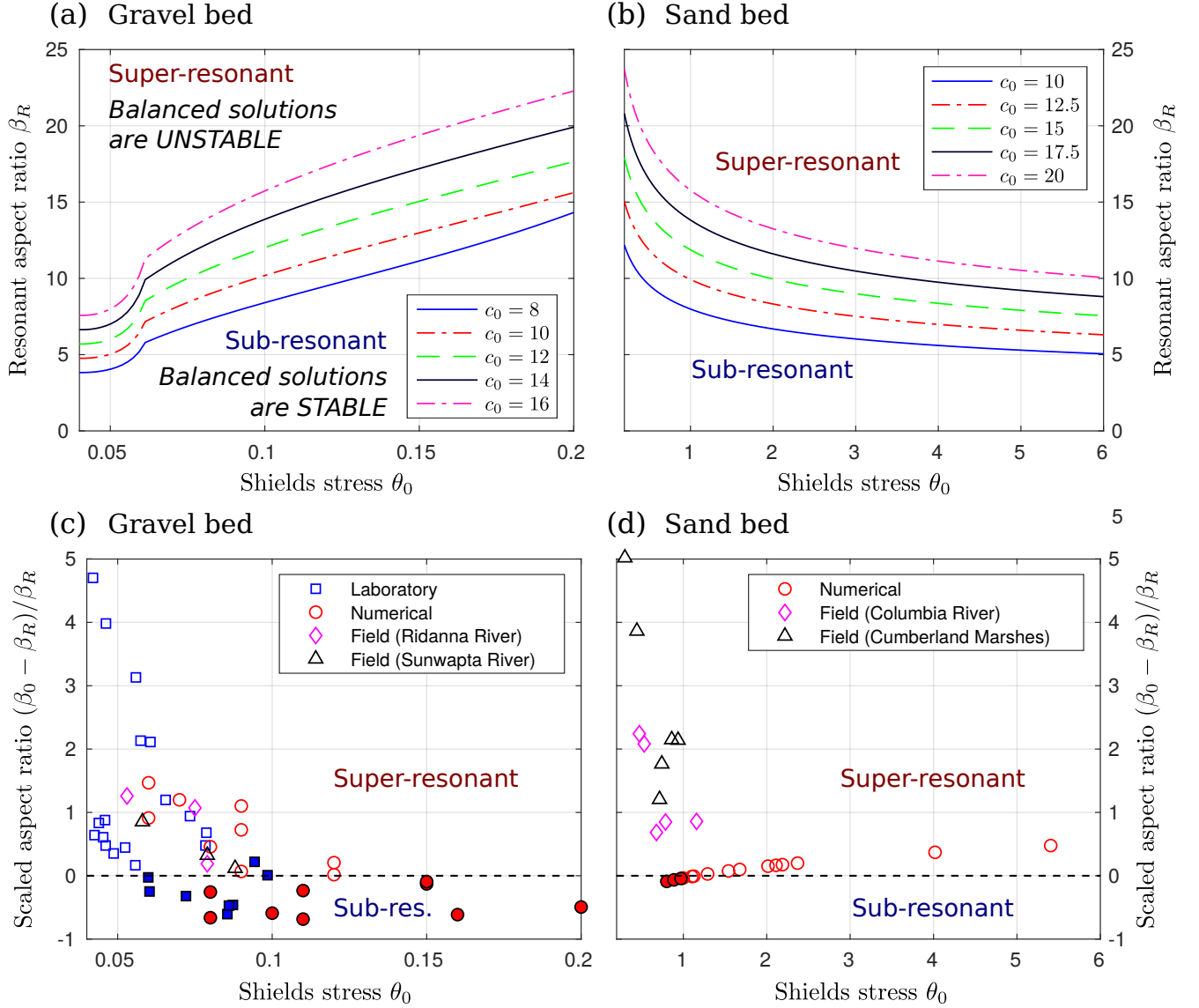


Figure 6: (a) and (b) Resonant aspect ratio β_R for gravel and sand bed channels as a function of Shields stress and Chézy coefficient. Under sub-resonant conditions (i.e. $\beta_0 < \beta_R$) the balanced bifurcation configuration is stable, while in super-resonant channels ($\beta_0 > \beta_R$) the instability mechanism makes the bifurcation unbalanced. (c) and (d) Observations of the balanced (close markers) or unbalanced (open markers) state of the bifurcation as a function of Shields stress and relative distance from the resonant threshold (scaled aspect ratio), for each of the 71 bifurcations listed in Table 1.

288 nant threshold correctly predicts stability of both gravel and sand bed bifurcations, with the key
 289 advantage of avoiding the need of calibrating a specific parameter like the α required by previous
 290 theoretical models (e.g., *Bolla Pittaluga et al.*, 2003, 2015).

291 The value of α that makes the nonlinear two-cell model consistent with the 2D theory can
 292 be determined by simply setting $\beta_C = \beta_R$ in Equation (12). Results reported in Figure 7 show
 293 a significant dependence of α on the reference conditions (θ_0 and c_0), which explains the large
 294 variability emerging in the literature. We notice that this estimate is strictly valid in the neighbour
 295 of the instability threshold, while for larger β it provides an upper limit of the optimal value for
 296 predicting the discharge asymmetry, as suggested by both laboratory observations and theoretical
 297 considerations (*Bertoldi and Tubino*, 2007; *Redolfi et al.*, 2016).

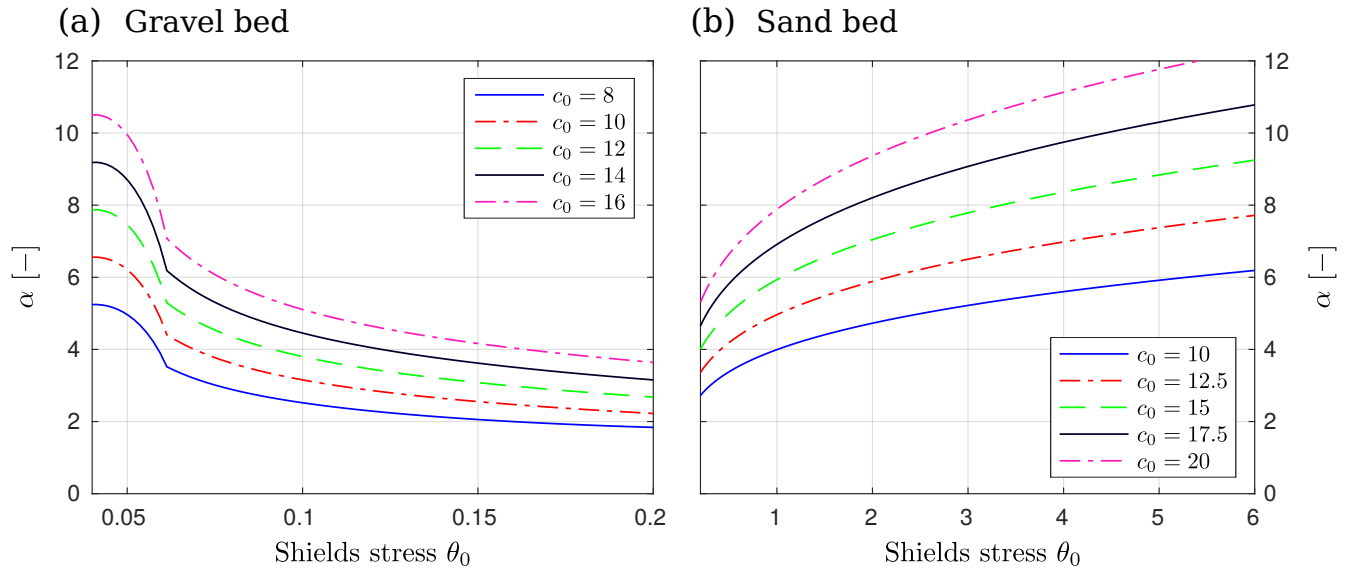


Figure 7: Values of the parameter α that make the two-cell model consistent with the linear 2D theory of *Redolfi et al.* (2016): (a) gravel bed channels, (b) sand bed channels.

298 Data of Table 1 include field measurements of natural bifurcations, which are not necessarily
 299 free. In this case the observed asymmetry may be not fully ascribed to the free instability mecha-
 300 nism but it may also be enhanced by the presence of the external forcings. The analysis of forcing
 301 effects and their interaction with the free instability mechanism is the main subject of the next
 302 section.

3.2 The forced bifurcation

Natural bifurcations are rarely free because of a number of forcing effects, including curvature of the main channel, different bifurcation angles, slope advantages, migration of bars, presence of obstacles, differential downstream sedimentation (e.g. *Van der Mark and Mosselman*, 2013; *van Dijk et al.*, 2014; *Le et al.*, 2018b,a; *Salter et al.*, 2018).

In the presence of a forcing effect, for example a curvature of the main channel, the equilibrium diagram of Figure 4 changes its topology. Specifically, as illustrated in Figure 8a, the equilibrium solution at relatively low values of β_0 is not balanced (i.e. $\Delta Q = 0$) and as expected more water is flowing towards the outer bend. When β_0 increases, the effect of the channel curvature tends to be amplified by the bifurcation, resulting in a more and more unbalanced configuration. However, at a given β value two additional equilibrium solutions form (a so called “imperfect pitchfork bifurcation”, see for example *Golubitsky and Schaeffer*, 1979). One of them is unstable (dashed line in Figure 8a), while the more unbalanced solution is stable. This suggests the possibility for the bifurcation to attain a different, stable equilibrium point, where most of water and sediment fluxes are deviated towards the inner bend bifurcate.

The behaviour of the equilibrium solutions for different values of the dimensionless channel curvature W_a/R is illustrated in Figure 8b. When $W_a/R = 0$ (straight channel) we obtain again the solution of Figure 4, here represented in terms of the distance from the resonant point, so that the pitchfork “bifurcation” appears at $(\beta_0 - \beta_R)/\beta_R = 0$. By analysing the effect of increasing curvature two relevant aspects emerge. First, the increase of the discharge asymmetry with channel curvature is significantly more pronounced at relatively small values of the channel aspect ratio (i.e. in the sub-resonant regime), where lines corresponding to different curvature values are more spaced apart, while at higher (i.e., super-resonant) aspect ratios the effect of curvature is minimal. Second, the value of the aspect ratio at which the second stable solution forms increases with channel curvature. For example, when $W_a/R = 0.1$ an aspect ratio 50% higher than the resonant value is needed to allow for the existence of multiple stable solutions.

The possibility of obtaining multistable solutions depending on channel curvature and aspect ratio is better illustrated in Figure 9a. While under sub-resonant conditions the equilibrium

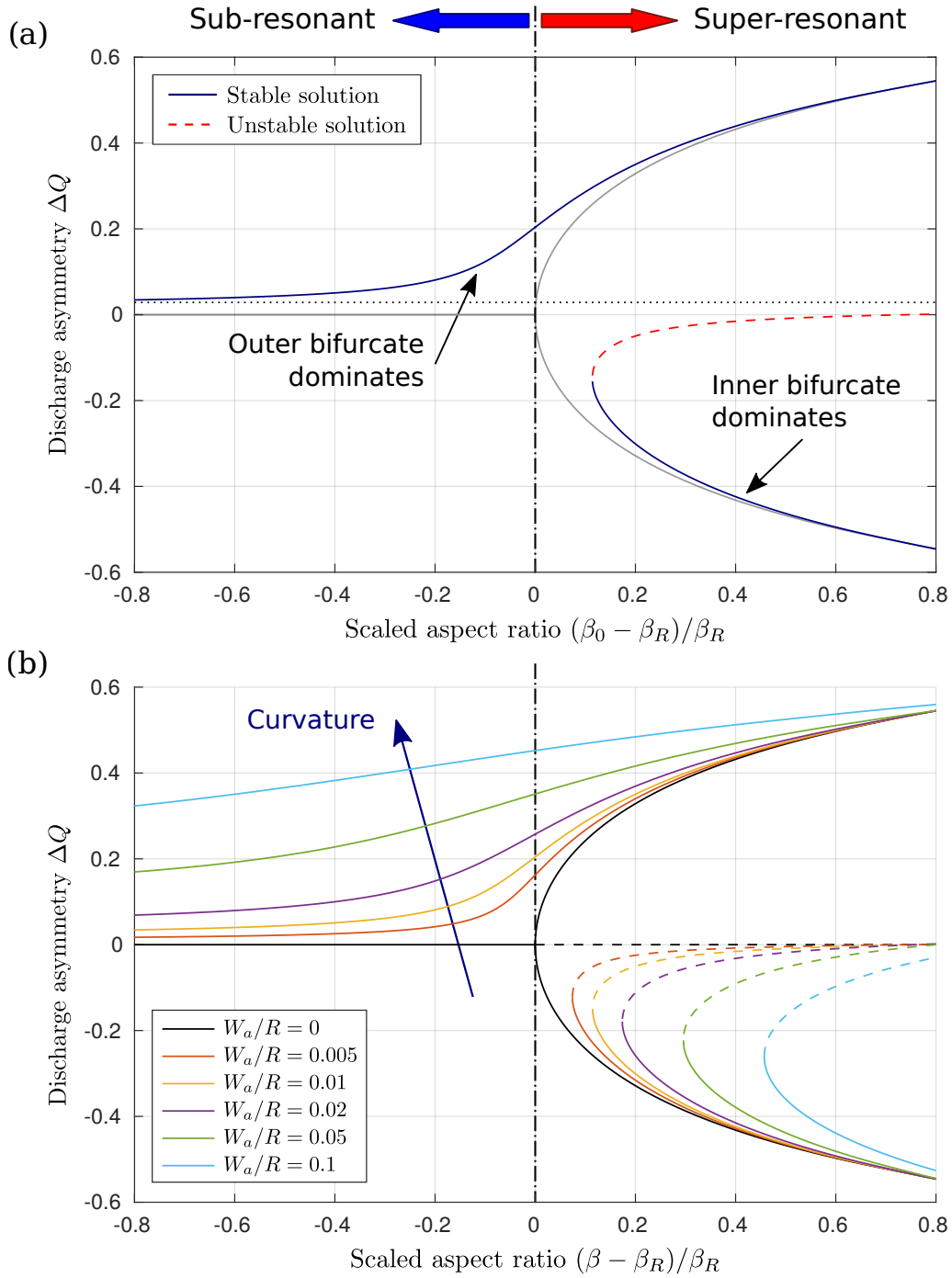


Figure 8: Stable (solid lines) and unstable (dashed lines) equilibrium solutions for a bifurcation with a curved upstream channel, as a function of the scaled aspect ratio. (a) Example with fixed curvature ($W_a/R = 0.01$), where the dotted line indicates the discharge ratio at the cell entrance and the grey lines represent the reference, "free" solution. (b) Effect of increasing curvature values. The vertical dashed line separates the sub-resonant (left) from the super-resonant (right) region.

331 solution is unique, two stable equilibrium solutions exist in super-resonant conditions. However,
 332 if the curvature is sufficiently large (depending on $(\beta_0 - \beta_R)/\beta_R$) only the solution for which the
 333 outer channel dominates is possible.

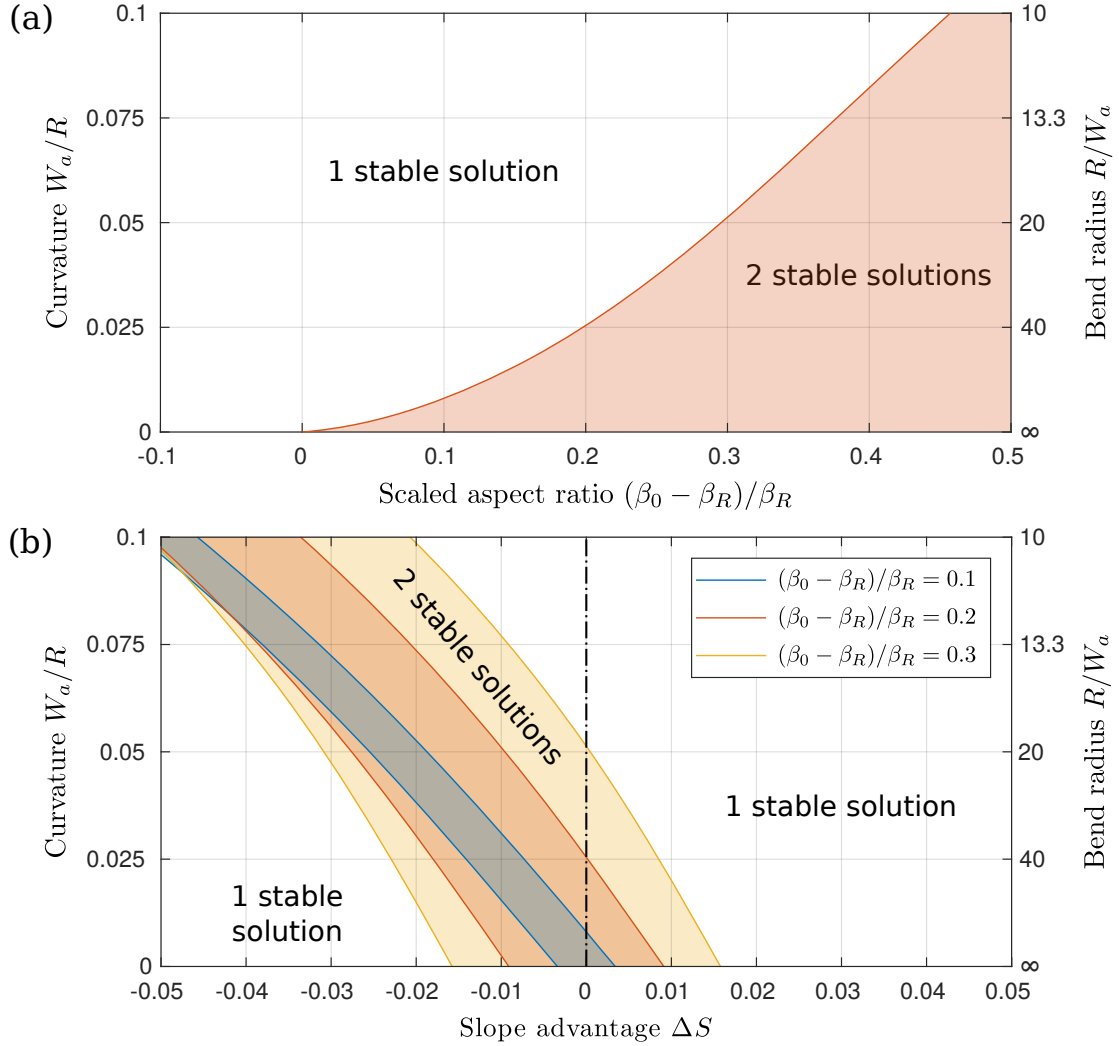


Figure 9: Stability diagram indicating regions where two stable equilibrium solutions exist depending on: (a) scaled aspect ratio and channel curvature (no slope advantage); (b) the combined effect of channel curvature and slope advantage, for different values of the scaled aspect ratio.

334 The above depicted scenario is characteristic of imperfect systems, and turns out to be similar
 335 when analysing different kind of forcings. Here we do not specifically report on the effect of the
 336 slope advantage but we directly focus on the more interesting analysis of its interaction with the
 337 curvature of the upstream channel.

338 In some cases the effect of main channel curvature can be compensated by a slope advantage
 339 that tends to steer water and sediment flows towards the steeper inner bend bifurcate (e.g., *Klein-*

hans et al., 2008; van Dijk et al., 2014). Analysis of the combined effect of the different forcings
 on the discharge asymmetry gives the results illustrated in Figure 10a, which confirms that for a
 sub-resonant bifurcation a channel curvature can be compensated by a gradient advantage. When
 $\Delta S = -0.01$ the discharge asymmetry is the same as the upstream asymmetry independently of
 β_0 , while for higher slope advantages the bifurcation tends to distribute water towards the inner
 bend channel, in a greater proportion when β_0 increases. However, the scenario dramatically
 changes when the aspect ratio exceeds the resonant threshold (i.e. super-resonant conditions).
 In this case, equilibrium solutions are never balanced, with one of the two bifurcates becoming
 dominant for any combination of curvature and slope advantage. In Figure 10b one sees that un-
 der sub-resonant conditions the equilibrium ΔQ varies smoothly with the slope advantages, while
 when β_0 exceeds β_R sharp transitions and hysteresis are expected when varying ΔS .

In general the discharge asymmetry depends on slope advantage and curvature as illustrated in
 Figure 11. Under sub-resonant conditions (Figure 11a) variations of ΔQ are always smooth, and
 the effect of channel curvature can be always compensated by negative values of ΔS . Conversely,
 under super-resonant conditions (Figure 11b) the bifurcation is never balanced, and there is a well-
 defined region (black oblique stripes) in the forcing parameters space where two stable solutions
 coexist. The width of such bi-stable region depends on the distance from the resonant point as
 depicted in Figure 9b.

It is important to remark that all our diagrams have been obtained by considering fixed values
 of relative roughness ($d_{s0} = 0.02$), Shields stress ($\theta_0 = 0.1$) and Ikeda (1982) coefficient ($r = 0.5$),
 and the same transport (*Parker*, 1990) and friction (Equation (3)) formulae. Nevertheless, from a
 qualitative point of view, model results, and therefore their interpretation, are fully independent
 of the specific choice of flow parameters values and closure relations for sediment transport.

4 Discussion

The present work has built on previous analyses to propose a theoretical framework for river bifur-
 cations within the context of 1D morphodynamic modelling that accounts for key 2D ingredients
 at the entrance of the bifurcation node and in the fluxes delivered by the upstream channel.

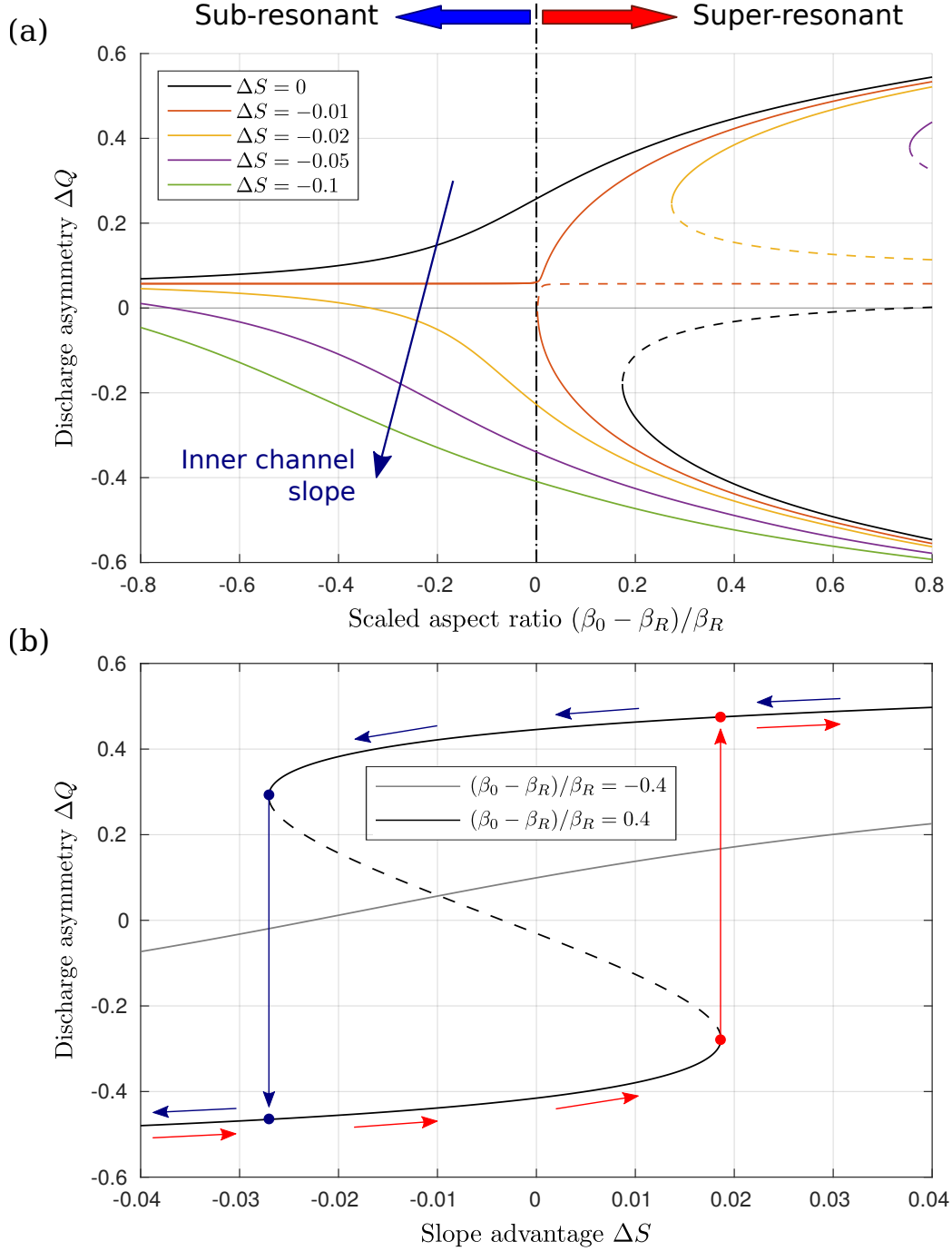


Figure 10: Effect of the slope advantage on a curved ($W_a/R = 0.02$) bifurcation. (a) Equilibrium solutions as a function of the aspect ratio β_0 . (b) Equilibrium solutions as a function of the slope advantage under sub-resonant (grey line) and super-resonant (black lines) conditions. Solid and dashed lines indicate stable and unstable solutions respectively, while arrows indicate possible trajectories when increasing (red arrows) or decreasing (blue arrows) ΔS .

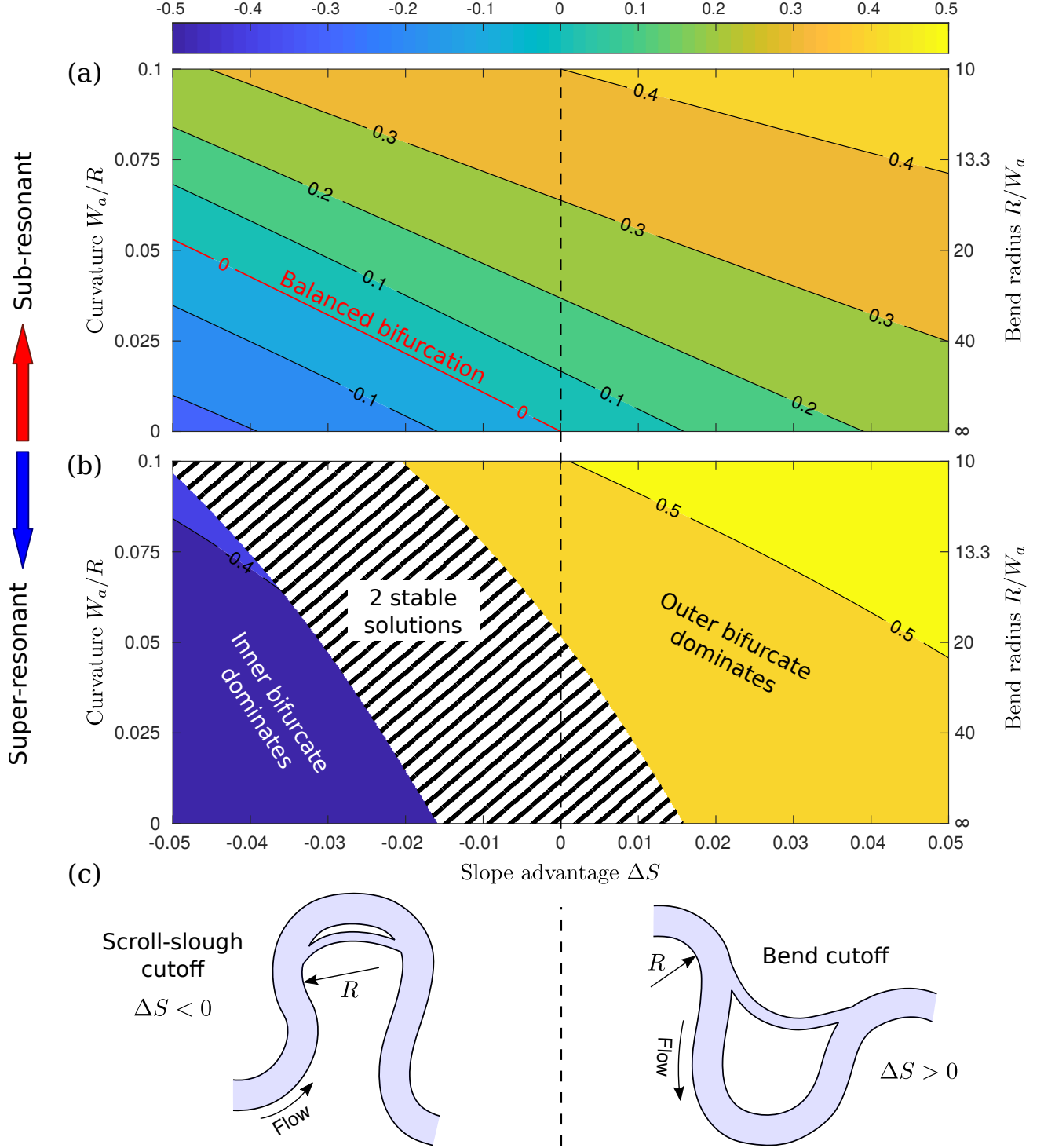


Figure 11: Discharge asymmetry as a function of curvature and slope asymmetry: (a) sub-resonant conditions ($(\beta_0 - \beta_R)/\beta_R = -0.3$); (b) super-resonant conditions ($(\beta_0 - \beta_R)/\beta_R = 0.3$), with black oblique stripes indicating the region where two stable solutions exists. (c) example of different slope advantages in meanders chute cutoff. Left: steeper inner channel ($\Delta S < 0$); right: steeper outer channel ($\Delta S > 0$).

367 Here we discuss (i) the main implications of our findings and the potential of the proposed
368 approach for the interpretation of bifurcation dynamics, as it emerges from both observations and
369 modelling; (ii) the significance of the equilibrium analysis for time-dependent processes; (iii) the
370 need to clarify the specific use of the wording “instability” when addressing bifurcation dynamics,
371 in the light of our findings and in the context of previous studies; (iv) applicability and limitations
372 of our approach.

373

374 *Enhanced insight on bifurcation morphodynamics*

375 The core of the model lies in incorporating in the 1D scheme of *Bolla Pittaluga et al.* (2003)
376 (i) the resonant aspect ratio as threshold for bifurcation stability (*Redolfi et al.*, 2016) and (ii)
377 the effect of the forcing factors, through a proper modelling of the water and sediment fluxes
378 delivered from the upstream channel and accepted by the downstream bifurcates. These fluxes
379 are laterally symmetrical in the case of a purely free bifurcation, while the opposite occurs when
380 adding forcing effects, which are almost invariably observed in real settings. Among the broad
381 variety of forcing factors that characterize natural river bifurcations, here we addressed the isolated
382 and the combined effect of an upstream channel curvature and a gradient advantage between the
383 downstream branches.

384 A first important insight allowed by the proposed approach is the confirmation of the key role
385 of the resonant aspect ratio on bifurcation behaviour, and an in-depth quantitative understanding
386 of how the bifurcation response depends on the relative distance of the channel-forming aspect
387 ratio value from such resonant threshold. This is supported with an analysis of an unprecedented
388 number of laboratory, numerical and field data, for both gravel-bed and sand bed streams. The
389 stability criterion based on the resonant threshold explains the loss of stability with increasing
390 Shields stress of balanced sand bed bifurcations observed by *Edmonds and Slingerland* (2008),
391 which is simply the consequence of adopting different transport and friction formulae. Compared
392 with previous stability criteria, the analysis based on resonance offers the key advantage that it
393 does not require the calibration of a specific parameter like the k exponent of *Wang et al.* (1995)
394 or the α length parameter of *Bolla Pittaluga et al.* (2003, 2015).

395 Interestingly, the role of the free instability mechanism is not limited to ideal, geometrically
396 symmetric bifurcations with symmetrical boundary conditions, but it is also a key controlling
397 factor for complex, forced bifurcations. Consequently, also the response of bifurcation to the
398 external forcings highly depends on channel conditions with respect to resonance. Under sub-
399 resonant conditions, the bifurcation behaviour is relatively simple, as it is perfectly balanced for
400 free, symmetrical bifurcations and mostly dominated by the forcing effects when they are present.
401 On the contrary, under super resonant conditions balanced equilibrium configurations are never
402 stable, so that the bifurcation always tends to highly asymmetric equilibrium states. Here, multiple
403 stable solutions are possible, including counter-intuitive configurations where the inner bifurcate
404 prevails, which suggests the possibility of complete shifts and hysteresis in the bifurcation response
405 to changing conditions (*Scheffer et al.*, 2001).

406 The whole picture yields a clear, physically-based key to interpreting results of field observa-
407 tions and numerical models, which at times displayed behaviours that could not be given a fully
408 exhaustive explanation. An example is provided by the result of *Kleinhans et al.* (2008), based on
409 a three-dimensional model of a curved bifurcation with different channel gradients. These results
410 revealed a “dramatic effect” of the width to depth ratio on discharge distribution and overall bed
411 morphology, with the bifurcation switching from a dominant inner-bend bifurcate to a dominant
412 outer-bend bifurcate, which indicates that the bifurcation behaviour “bifurcates” at a certain
413 width between $W_a = 288$ m and 378 m (see Section 4.5 of *Kleinhans et al.*, 2008). By applying our
414 proposed modelling framework, and using the same closure relation and flow conditions as in the
415 numerical experiments, it turns out that the observed range of widths correspond to the transition
416 from from sub-resonant (Figure 12a) to super-resonant (Figures 12b and 12c) conditions. This
417 transition can explain the fairly different morphological evolution upstream the bifurcation: under
418 sub-resonant conditions the bifurcation does not significantly affect the upstream bed elevation
419 (dashed line of Figure 12a), while under super-resonant conditions the upstream morphodynamic
420 influence triggered by the bifurcation induces the formation of steady bars in the upstream channel
421 (Figures 12b and 12c), which affect how discharge is downstream distributed.

422 Super-resonant conditions are not rare in nature, insofar as gravel bed rivers tends to behave

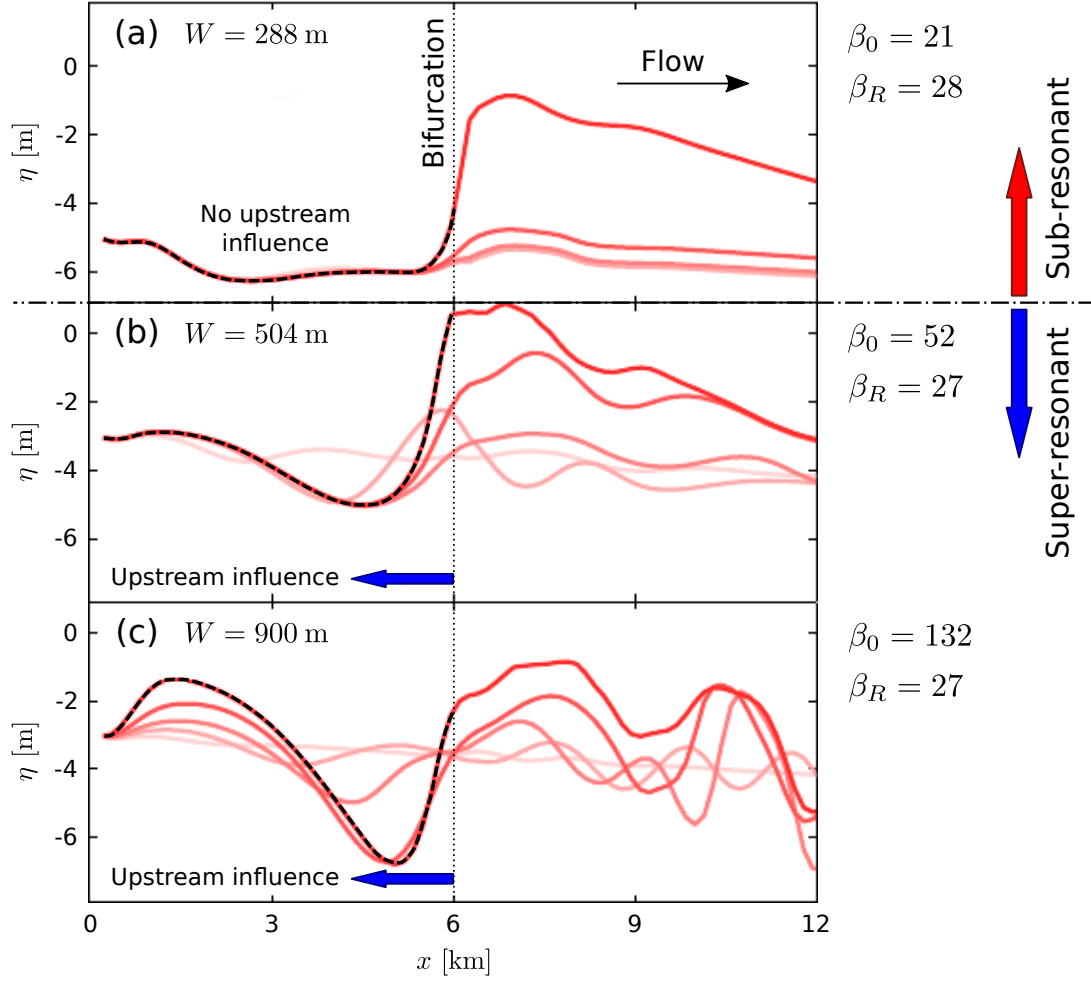


Figure 12: Profiles of bed elevation at the outer bank of a curved channel that bifurcates at $x = 6$ km, according to the numerical simulations of *Kleinhans et al.* (2008), adapted from their Figure 12 (long bend series). (a) Sub-resonant conditions; (b) and (c) super-resonant conditions. Lines from light to dark indicate advancing time. The dashed line indicates the final bed profile upstream the bifurcation, which shows significant oscillations only when the upstream channel falls under super-resonant conditions.

super-resonantly, especially for near-threshold (i.e. low Shields stress) channels (*Zolezzi et al.*, 2009). This confirms the tendency of balanced bifurcations to become unstable when the fixed bank hypothesis is released so that channels are free to reach their regime width (*Miori et al.*, 2006).

When different forcing effects are simultaneously acting they can compensate themselves (e.g. *Kleinhans et al.*, 2008; *van Dijk et al.*, 2014; *Kleinhans et al.*, 2013), leading to a nearly balanced configuration. For example a gradient advantage in a scroll-slough chute cutoff of a meander bend (Figure 11c) can balance the opposite effect of the channel curvature. However, this is possible only if the upstream channel is in sub-resonant conditions, so that the bifurcation is intrinsically stable. On the contrary, under super-resonant conditions the bifurcations tends to propagate its morphodynamic influence in the upstream direction, so that the discharge partition is always highly unbalanced, independently of the upstream and downstream conditions.

Similar results can be found when other kind of forcing effects are interacting with the free instability mechanism, as emerging in the analysis of *Bertoldi et al.* (2009) about the effect of downstream-migrating alternate bars on the bifurcation dynamics. In that context, different characteristic regimes have been identified depending on the channel aspect ratio with respect to a critical threshold. For relatively small β_0 the bifurcation is essentially balanced with discharge oscillations around $\Delta Q = 0$ caused by the passage of bars, while for higher β_0 unbalanced states are observed. In this latter case the bifurcation can be either “bar perturbed” (small oscillations around a stable unbalanced state) or “bar dominated” (frequent switching of the bifurcation between opposite, highly unbalanced states). A diagram similar to Figure 10b can be used to interpret the different scenarios: essentially balanced solutions occur under sub-resonant conditions, while super resonant conditions yield highly unbalanced solutions, which are “bar perturbed” when the variations of the forcing factor are relatively weak, or “bar dominated” when forcing effect is strong enough to make the solution jumping between opposite states.

448

Quasi-equilibrium and temporal scales: the present work in a broader context

Our analysis is focused on steady equilibrium configurations, where both upstream and down-

stream channels are considered in planimetric and altimetric equilibrium. Strictly speaking, this is rarely the case of natural bifurcation, because forcing effects are usually varying in time. Nevertheless, as long as their rate of change is slow as compared with the intrinsic time scale of the bifurcation, the response of the system can be studied as a sequence of quasi-equilibrium states. This allows us to interpret the action of downstream migrating bars, as well as the analogous effect of the migration of the upstream meander (*Kleinhans et al.*, 2011), by means of equilibrium diagrams like those of Figure 10.

Similarly, the quasi-steady analysis can be applied for interpreting the effect of downstream variations, provided they are comparatively slow. For example, in the depositional systems investigated by *Salter et al.* (2018), the interaction between the bifurcation and the downstream bifurcates leads to autogenic temporal oscillations of channel slope and discharge asymmetry. This process evolves on a time scale that is proportional to the length of the downstream bifurcates, which is usually much longer than the intrinsic time scale of the bifurcation evolution (see *Miori et al.*, 2006). Therefore, focusing on the local behaviour of the bifurcation node, such downstream mechanisms can be considered as an external forcing effect, coherently with the definition adopted in the present work.

This provides a worthwhile example of how the definition of the forcing factors depends in general on the spatial and temporal scales under consideration. An analogous concept is at the core of classical theoretical studies on bar-bend interactions in river meanders (e.g., *Tubino and Seminara*, 1990), which pointed out how the dynamics of sediment bars inside meandering channels depends on the interaction between the free instability mechanisms that causes spontaneous development of migrating bars, and the effect of the variable meander curvature. In general also the channel curvature is not fixed but changes in time as the meander develops. However, as long as the two mechanisms act at different time scales, the planform evolution being a much slower process, when focusing on bar dynamics the meander curvature can be considered as a fixed forcing factor.

477

The meaning of “instability” within bifurcation dynamics

479 The outcomes of this work also suggest revisiting the use of the key wording “instability”,
 480 which has been often used in previous studies of river bifurcation morphodynamics, though in
 481 many times with different meanings. “Instability” has been indeed used to indicate: (i) the sit-
 482 uation whereby an equilibrium bifurcation configuration is unstable; (ii) a systematic change of
 483 the discharge distribution over time (e.g., *Kleinhans et al.*, 2013); (iii) a bifurcation that evolves
 484 towards an highly unbalanced configuration and eventually produces the complete closure of one
 485 of the two bifurcates (e.g., *Burge*, 2006; *Le et al.*, 2018b). In this paper we have used “instability”
 486 in its mathematical meaning (i), thus indicating the mathematical instability of an equilibrium
 487 configuration (mathematical solution), which in itself could be either symmetrical or asymmetri-
 488 cal, and therefore may become inconsistent with meanings (ii) or (iii). Moreover, though meanings
 489 (ii) and (iii) might be interchangeable under some circumstances, this does not apply in general,
 490 and they may not be of help in disentangling the role of the free and of the forced bifurcation
 491 mechanisms when analysing a specific situation. For example, the instability of the balanced so-
 492 lution in the super-resonant regime does not necessarily lead to the closure of one bifurcate, but
 493 often leads to a stable, unbalanced configuration. Similarly, a partial or complete channel closure
 494 may be caused by a forcing factor rather than an instability of an equilibrium configuration, which
 495 in itself could be symmetrical or asymmetrical. This is, for instance, the case when a localized
 496 obstacle deviates the flux towards one preferred bifurcate (*Le et al.*, 2018b).

497

498 *Applicability and limitations of the present approach*

499 In this paper we have adopted a local viewpoint of the bifurcation morphodynamics, which
 500 focuses on a tile of a complex mosaic of processes where several autogenic mechanisms interact
 501 at different spatial and temporal scales (e.g., in the case of bifurcations coupled with aggrading
 502 downstream channels or embedded in braided networks).

503 The methodological approach can be broadly applied to analyse river bifurcations in real
 504 settings. The key ingredient required by the two-cell model to account for the upstream forcings
 505 is the availability of suitable transverse distributions of flow and sediment transport to compute
 506 water and sediment fluxes that enter the bifurcation node. Our results refer to the simple case of a

relatively long channel of constant curvature, and are based on the assumption of fully developed flow, which is not satisfied in short bends and in general when the curvature is spatially varying, as in meandering channels. Extending the model to treat such complex configurations would require coupling the two-cell nodal point conditions with a sound model for flow and bed topography in meandering channels (e.g., *Zolezzi and Seminara, 2001*). This analysis is beyond the scope of the present paper; however, we may expect that in this case bifurcation stability will depend not only on local curvature, but also on the position of the bifurcation node with respect to the steady pattern of point bars forming in the upstream channel (*Le et al., 2018a*).

Further investigation is needed to understand to what extent the key mechanisms driving the bifurcation instability, and in particular the upstream morphodynamic influence, can be affected by processes that are often not reproduced by mathematical and physical models. Specifically, two fundamental processes would probably need more consideration in future research. The first is sediment sorting: despite some indications of a relatively weak effect of grain sorting on the stability of migrating bars (*Lanzoni and Tubino, 1999*), its role in determining the bifurcation stability is not clear, especially in gravel bed channels (*Burge, 2006*). The second process is suspended sediment transport, which is often dominant in large, multi-thread, sand bed rivers (*Szupiany et al., 2012*): when suspended load is the dominant mode of sediment transport the gravitational pull towards the deeper channel is probably weaker, so that the bifurcation may be even more unstable than currently predicted (*Kleinhans et al., 2006*).

5 Conclusions

The present work offers a viewpoint of river bifurcations as dynamical systems for which a distinct role of the free and forced responses can be identified. We propose a theoretical framework based on the 1D model with two-cell bifurcation node originally developed by *Bolla Pittaluga et al. (2003)*, as extended by *Kleinhans et al. (2008)* to account for the curvature-driven secondary flow. Furthermore, we incorporate the key outcomes of the fully 2D analytical approach of *Redolfi et al. (2016)* within the classical 1D scheme for river bifurcations. Two main forcing factors are considered, the curvature of the upstream channel and a slope advantage of one of the bifurcates,

534 though the approach could be easily extended to account for other factors (e.g., the presence of a
535 local obstacle upstream the bifurcation).

536 The key advantage of the proposed approach is its ability to clearly isolate the different free
537 and forced mechanisms that may control the bifurcation dynamics in a complex setting, like that
538 of real-world bifurcations, thus resulting in a suitable tool to gain clear insight in the analysis and
539 interpretation of numerical model outcomes and of field observations. The key novel outcomes for
540 bifurcation dynamics can be summarized in the following four items.

- 541 1. The bifurcation stability criterion based on the resonant aspect ratio threshold has been
542 successfully tested against data of both gravel bed and sand bed channels, and it allows
543 for capturing the opposite effects of Shields stress, which tend to promote more balanced
544 bifurcations in gravel bed rivers and more unbalanced bifurcations in sand bed streams.
545 This criterion can then be used for predicting the balanced/unbalanced character of free
546 bifurcations, and it allows incorporating the fully-2D solution of *Redolfi et al.* (2016) within
547 the classical 1D theory (*Wang et al.*, 1995; *Bolla Pittaluga et al.*, 2003) with no need to
548 calibrate specific bifurcation parameters.
- 549 2. The role of the free instability mechanism is not limited to purely free bifurcations, but is
550 also fundamental in the dynamics of the forced bifurcations that are more representative of
551 real-world settings. Therefore, river bifurcations with super-resonant upstream channels are
552 dominated by the free mechanism, characterized by multiple, highly unbalanced equilibrium
553 configurations. This remarkable behaviour might lead to counter-intuitive outcomes, where
554 for example water and sediment fluxes are mainly delivered towards the bifurcate located at
555 the inner bank of a channel bend.
- 556 3. Analysis of the interaction between two of the most common forcing effects (slope advantage
557 and curvature) allows us to quantify the parameters range where free and forcing effects
558 cooperate or compete in determining the overall bifurcation dynamics. Under sub-resonant
559 conditions, the interaction between upstream curvature and slope advantage is smoothly
560 dependent on the relative intensity of the forcings (*Kleinhans et al.*, 2008; *van Dijk et al.*,

2014), while this is not the case under super-resonant conditions, for which abrupt transitions between opposite, highly unbalanced equilibrium states are expected.

4. The above results highlight how river bifurcations behave as dynamical systems like many other eco-morphological processes in rivers and freshwater bodies (e.g. *Scheffer et al.*, 2001), where the nonlinear interaction among internal and external mechanisms gives rise to a complex response, characterized by sensitivity to the initial conditions, multistable states and hysteresis cycles.

Acknowledgements

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Appendix A: Fully developed flow in a constant curvature channel

Here we provide a detailed derivation of the flow field in a channel of constant curvature, obtained by following the *Struiksmā et al.* (1985) approach.

The model is formulated in a curvilinear system of reference $\{x, y\}$, where x is pointing in the downstream direction and y represents the transverse coordinate (see Figure 3). Assuming a fully developed flow, all dependent variables vary only with y , and therefore the x -derivatives vanish; in such conditions, the continuity equation gives zero transverse fluxes of water and sediment, while the longitudinal momentum equation reduces to an uniform flow relation for the depth-averaged longitudinal velocity U , namely:

$$U = c \sqrt{g S D}, \quad (\text{A1})$$

584 where the longitudinal slope S depends on y as:

$$S = S_0 \frac{R}{R + y}, \quad (\text{A2})$$

585 with S_0 indicating the slope of the channel centreline (i.e. at $y = 0$).

586 The longitudinal velocity generates a shear stress τ_x , which can be computed as:

$$\tau_x = \rho \frac{U^2}{c^2}, \quad (\text{A3})$$

587 while the secondary flow produces a shear stress in the transverse direction, given by (see *Struiksm*
588 *et al.*, 1985):

$$\tau_y = -\rho A \frac{DU^2}{c^2} \frac{1}{R}, \quad (\text{A4})$$

589 where ρ indicates the water density.

590 The transverse stress τ_y needs to be compensated by a gradient of the bed elevation. There-
591 fore, considering the *Ikeda* (1982) formulation for the effect of gravity on the sediment transport
592 direction, the following relation arises:

$$\frac{r}{\sqrt{\theta}} \frac{d\eta}{dy} = \frac{\tau_y}{\tau_x} = -A \frac{D}{R}. \quad (\text{A5})$$

593 Transverse profiles of bed elevation can be obtained by integrating Equation (A5). To this
594 aim, we need to specify how the water depth varies along the cross section through the following
595 geometrical relation:

$$\frac{dD}{dy} = \frac{dH}{dy} - \frac{d\eta}{dy}, \quad (\text{A6})$$

596 where H indicates the water surface elevation.

597 Under the hypothesis of horizontal free surface, Equation (A6) reduces to $dD/dy = -d\eta/dy$,
598 so that the differential equation (A5) can be easily solved in terms of D . More generally, the

599 gradient of free surface elevation can be derived from the equation of transverse momentum:

$$g \frac{dH}{dy} + \frac{\tau_y}{\rho D} = \frac{U^2}{R}, \quad (\text{A7})$$

600 which, when combined with Equations (A5) and (A6), gives an expression of the type:

$$\frac{dD}{dy} = fct(y, D), \quad (\text{A8})$$

601 which can be easily solved by numerical integration.

602 The effect of the channel curvature on the transverse profiles of bed and water surface elevation
 603 is illustrated in Figure 13. The spiral flow induces higher bed elevation and slightly lower water
 604 surface elevation at the inner bend. Consequently, water depth, velocity, and water and sedi-
 605 ment fluxes are higher at the outer bend. The resulting transverse profiles are clearly nonlinear,
 606 especially when the channel is highly curved.

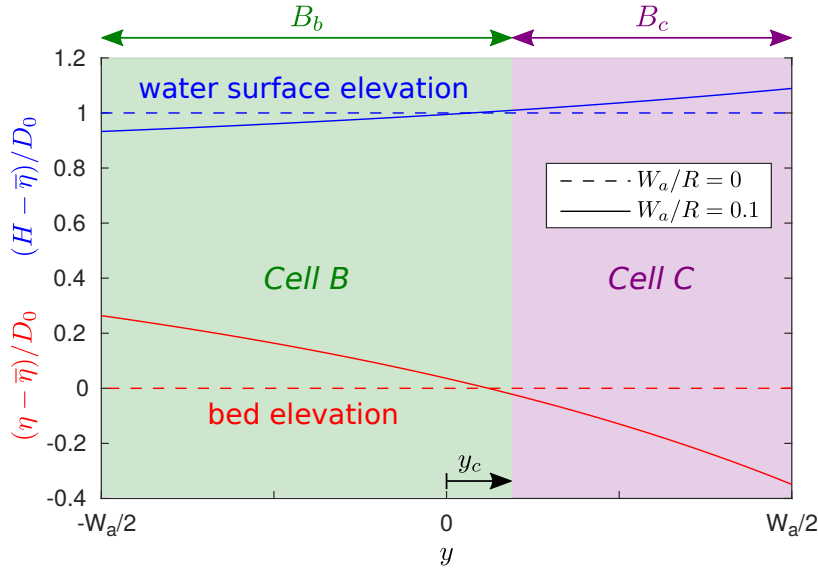


Figure 13: Transverse profiles of (scaled) bed and water surface elevation in the main channel, where $\bar{\eta}$ is the mean bed elevation. Dashed lines: straight channel; solid lines: curved channel. Parameters are $\theta_0 = 0.1$, $d_s = 0.02$, $r = 0.5$. Background colours indicate the position of the two cells of size B_b and B_c .

607 Once the transverse profiles are known, the input fluxes for the two-cell model can be com-
 608 puted by integrating along their respective domain. In the general case of different width of the

downstream bifurcates (i.e. $W_b \neq W_c$), the width of the two cells (see Figure 13) can be calculated as:

$$B_b = W_a \frac{W_b}{W_b + W_c}, \quad B_c = W_a \frac{W_c}{W_b + W_c}, \quad (\text{A9})$$

so that the transverse position of the interface between the two cells reads:

$$y_c = B_b - \frac{W_a}{2} = \frac{W_a}{2} \left(\frac{W_b - W_c}{W_b + W_c} \right), \quad (\text{A10})$$

which vanishes when $W_b = W_c$ as assumed in the paper. Finally, water and sediment fluxes feeding the two cells are given by the following relations:

$$Q_b^{IN} = \int_{-W_a/2}^{y_c} U D dy, \quad Q_c^{IN} = 1 - Q_b^{IN}, \quad (\text{A11})$$

$$Q_{s_b}^{IN} = \sqrt{g \Delta d_{50}^3} \int_{-W_a/2}^{y_c} \Phi \left(\theta, \frac{D}{d_{50}} \right) dy, \quad Q_{s_c}^{IN} = 1 - Q_{s_b}^{IN}. \quad (\text{A12})$$

Appendix B: Algebraic expression for c_D , Φ_D and Φ_T coefficients

In this section we provide an explicit expression of the coefficients arising from linear stability analysis, which are needed to evaluate the critical and the resonant aspect ratio through Equations (12) and (14).

The c_D coefficient, which defines the response of the Chézy coefficient to variations of water depth, is defined as:

$$c_D := \frac{D_0}{c_0} \frac{\partial c}{\partial D} \bigg|_{D_0}. \quad (\text{B1})$$

When considering the logarithmic formula of *Engelund and Fredsoe* (1982) (Equation (3)), we obtain:

$$c_D = \frac{2.5}{c_0}. \quad (\text{B2})$$

where c_0 is the Chézy coefficient evaluated at reference conditions, namely:

$$c_0 = 6 + 2.5 \log \left(\frac{1}{2.5 d_s} \right). \quad (\text{B3})$$

624 Similarly, the coefficients Φ_D and Φ_T , which specify the sensitivity of the sediment transport
 625 to variations of water depth and Shields stress, are defined as:

$$\Phi_D := \frac{D_0}{\Phi_0} \frac{\partial \Phi}{\partial D} \Big|_{\theta_0, D_0}, \quad \Phi_T := \frac{\theta_0}{\Phi_0} \frac{\partial \Phi}{\partial \theta} \Big|_{\theta_0, D_0}, \quad (\text{B4})$$

626 and their explicit expression depends on the sediment transport formula used.

627 The *Engelund and Hansen* (1967) transport formula reads:

$$\Phi = 0.05 c^2 \theta^{2.5}, \quad (\text{B5})$$

628 and gives the following coefficients:

$$\Phi_D = 2 c_D, \quad \Phi_T = 2.5. \quad (\text{B6})$$

629 Transport formulae designed for bed load are often expressed in terms of θ only, and therefore
 630 Φ_D vanishes as the bed load function does not depend explicitly on water depth. For example
 631 when using the *Meyer-Peter and Muller* (1948) relation

$$\Phi = 8 (\theta - \theta_{cr})^{1.5} \quad (\text{B7})$$

632 the coefficients reads:

$$\Phi_D = 0, \quad \Phi_T = 1.5 \frac{\theta_0}{\theta_0 - \theta_{cr}}. \quad (\text{B8})$$

633 The *Parker* (1990) formula can be expressed as:

$$\Phi = 0.00218 \theta^{1.5} G(\xi), \quad \xi := \frac{\theta}{0.0386}, \quad (\text{B9})$$

634 where $G(\xi)$ is a piecewise-defined function:

$$G(\xi) = \begin{cases} 5474 (1 - 0.853/\xi)^{4.5} & \xi > 1.59 \\ \exp [14.2(\xi - 1) - 9.28(\xi - 1)^2] & 1 \leq \xi \leq 1.59 \\ \xi^{14.2} & \xi < 1 \end{cases} . \quad (\text{B10})$$

635 In this case we obtain:

$$\Phi_D = 0, \quad \Phi_T = 1.5 + \frac{G_\xi}{0.0386}, \quad G_\xi := \frac{\xi_0}{G_0} \frac{dG}{d\xi}, \quad (\text{B11})$$

636 where G_ξ can be expressed by deriving Equation (B10), which gives:

$$G_\xi = \begin{cases} \frac{4.5}{\xi_0/0.853 - 1} & \xi_0 > 1.59 \\ -18.56 \xi_0^2 + 32.76 \xi_0 & 1 \leq \xi_0 \leq 1.59 \\ 14.2 & \xi_0 < 1 \end{cases} . \quad (\text{B12})$$

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