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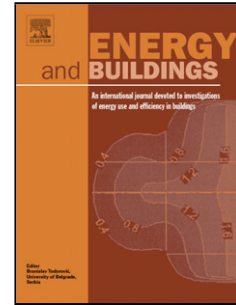
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Author: <ce:author id="aut0005"  
author-id="S0378778817300348-  
b051e3d16a0cd03a190ce9df985b4d61"> Alberto  
Muscio<ce:author id="aut0010"  
author-id="S0378778817300348-  
d47a8bf234aff3379ea2854e135280f1"> Hashem  
Akbari



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# An index for the overall performance of opaque building elements subjected to solar radiation

Alberto Muscio<sup>1</sup>, Hashem Akbari<sup>2</sup>

<sup>1</sup>*University of Modena and Reggio Emilia, Modena (Italy)*

<sup>2</sup>*Concordia University, Montreal (Canada)*

## HIGHLIGHTS

- The thermal behavior of an external opaque building element depends on several performance parameters.
- Commonly considered parameters related to insulation cannot describe the element behavior when it is subjected to the solar cycle.
- Parameters related to thermal inertia do not include the capability of reflecting solar energy toward the atmosphere.
- External radiative properties are unrelated to both insulation and inertia.
- The "solar transmittance index" (STI) takes into account both the radiative properties of the external surface and the thermo-physical properties of the materials under the surface

## ABSTRACT

The thermal behavior of an external opaque building element depends on the combination of several physical characteristics related to insulation level, thermal inertia, external radiative properties. Concerning the insulation level, parameters like the R-value and its inverse, the U-value or thermal transmittance, are commonly considered in building codes, but they are defined with reference to steady-state conditions and cannot describe the behavior of the element when it is subjected to the cycle of solar radiation. On the other hand, parameters like periodic thermal transmittance, decrement factor and time shift represent the dynamic response of the element resulting from its thermal inertia, so they are often considered but do not include the capability of returning solar energy to the atmosphere. In this regard, a few additional parameters are relevant such as solar reflectance and thermal emittance of the external surface, which are unrelated to both insulation and inertia.

In order to rate the overall dynamic behavior of an external opaque building element subjected to the cycle of solar radiation and constant indoor temperature, a "solar transmittance index" (STI) is proposed. STI includes in a single performance parameters the effects of both the radiative properties of the external surface and the thermo-physical properties of the materials under the surface. The utilization of such single performance parameter may be greatly helpful in defining requirements and policies to prevent building overheating, reduce cooling energy demand and mitigate the fallouts of the urban heat island effect.

**KeyWords:** solar reflectance; thermal insulation; thermal inertia; roof; building cooling

## 1. Introduction

The fallouts of the urban heat island (UHI) effect are made more and more evident by the progressive urbanization of world population, and they are exacerbated further by global warming. All this, combined with the economic development, is expected to significantly increase the cooling energy demand of buildings and make cooling the dominant energy component [1]. As a result, in many countries the attention of policy makers and other stakeholders of the construction sector is shifting from the cold season to the hot season, in order to counter the overheating of buildings and urban areas and mitigate its impact on both individuals and communities in terms of economic and environmental sustainability, discomfort and health issues. The last decades have thus seen the research and development of a wide set of countermeasures such as cool roofs, cool pavements and

urban vegetation [2,3], promoting and supporting the implementation of innovative building practices and policies.

Preventative measures such as installing cool materials, green roofs and shading devices provide protection from heat gains. Whereas, modulation and heat dissipation techniques such as thermal mass and free or forced ventilation allow the building to store and then dissipate heat gains. Public administrations of most countries have thus set, and progressively raised, requirements about building elements, especially opaque ones. In the past the requirements were mostly set with regard to thermal insulation, formulated in terms of limits to the R-value or its inverse, the U-value or thermal transmittance. However, the thermal behavior of an opaque building element with layered structure does not depend only on the insulation level, but also on parameters related to rejection of solar radiation, or thermal capacity of the element layers. In fact, the R-value or the U-value are defined with reference to steady-state conditions and cannot describe the element behavior induced by the cycle of solar radiation. The dynamic response of the element is more effectively described by parameters related to thermal inertia such as periodic thermal transmittance, thermal admittance, decrement factor, or time shift as defined by ISO 13786 [4]. On the other hand, rejection of solar radiation directly at the external surface can be more effective than thermal insulation and thermal inertia in many cases, therefore, limits and/or incentives are being set also for radiative surface properties such as solar reflectance and thermal emittance.

Requirements for opaque building elements must often be satisfied contemporarily on a wide range of unrelated or weakly related parameters: R-value or U-value to prevent heat loss in the heating seasons, surface mass, decrement factor, time shift, modulus of the periodic thermal transmittance, solar reflectance and thermal emittances to avoid overheating in the hot season. This makes difficult for designers, constructors and product manufacturers to compare different building solutions and operate choices, especially when a relatively low thickness and mass are available to accommodate additional layers in roofs and walls. On the other hand, the need to consider the combined effect of both mass properties and surface properties has been evidenced by several means, aimed to estimate the indoor temperature based on numerical [5-8] or even experimental techniques [9]. Nonetheless, such relatively complex calculation tools or measurement approaches can seldom be used for the preliminary selection of building solutions and they are also out of reach for many designers and construction firms.

A very interesting approach to classify roof and wall elements that takes into account both surface and mass properties is that of the “thermal performance index” (TPI), in which the rating 100 (the lower the better) is assigned for a given excess of peak temperature at the ceiling of an unconditioned space, or for a given peak of heat flow rate per unit of ceiling surface of a conditioned space [10]. A modification of the approach was recently proposed [11] with the “new” thermal performance index (\*TPI), in which the rating 100 (the higher the better) is assigned for the decrease of ceiling temperature of an optimal roof solution with respect to the ceiling temperature achieved in the worst case of a galvanized iron roof. In those and other analogous studies [7], however, temperature and heat flow rate are calculated numerically for assigned boundary conditions, with an approach that again may result impractical.

In the present work, a “solar transmittance index” (STI) is proposed to rate the overall dynamic thermal behavior of an opaque building element when its external surface is subjected to the cycle of solar radiation and the indoor temperature is kept constant. STI includes in a single performance parameters both the radiative properties at the external surface and the thermo-physical properties of the materials under the surface. It is developed by a procedure similar to that of the \*TPI, but with an approach independent of the installation site and aimed to obtain the maximum ease of calculation.

## **Nomenclature**

### *Latin symbols*

$c$	specific heat (J/(kg K))
$d$	mass density (kg/m <sup>3</sup> )
$DDH_h$	hot discomfort degree hours index
$f$	decrement factor ( $0 < f < 1$ )
$f_{ST}$	solar transmittance factor ( $0 < f_{ST} < 1$ )
$f_{ST(\text{tested})}$	solar transmittance factor of the tested surface ( $0 < f_{ST(\text{tested})} < 1$ )
$f_{ST,\text{optimal}}$	solar transmittance factor of the optimal reference case ( $0 < f_{ST,\text{optimal}} < 1$ )
$f_{ST,\text{worst}}$	solar transmittance factor of the worst reference case ( $0 < f_{ST,\text{worst}} < 1$ )
$h$	heat transfer coefficient (W/(m <sup>2</sup> K))
$h_{ce}$	external convective heat transfer coefficient (W/(m <sup>2</sup> K))
$h_{ci}$	internal convective heat transfer coefficient (W/(m <sup>2</sup> K))
$h_e$	external heat transfer coefficient (W/(m <sup>2</sup> K))
$h_i$	internal heat transfer coefficient (W/(m <sup>2</sup> K))
$h_{re}$	external radiative heat transfer coefficient (W/(m <sup>2</sup> K))
$h_{ri}$	internal radiative heat transfer coefficient (W/(m <sup>2</sup> K))
$I_{sol}$	solar irradiance (W/m <sup>2</sup> )
$\bar{I}_{sol}$	average value of solar irradiance oscillation (W/m <sup>2</sup> )
$\hat{I}_{sol}$	(first harmonic of) solar irradiance oscillation (W/m <sup>2</sup> )
$I_{sol,\text{max}}$	maximum solar irradiance (W/m <sup>2</sup> )
$\hat{I}_{sol,n}$	n <sup>th</sup> harmonic of solar irradiance oscillation (W/m <sup>2</sup> )
$I_{sol,\lambda}$	reference spectral solar irradiance (W/(m <sup>2</sup> nm))
$k$	thermal conductivity (W/(m K))
$k_j$	thermal conductivity of the j <sup>th</sup> homogeneous solid layer (W/(m K))
$L$	thickness of a solid layer (m)
$L_j$	thickness of the j <sup>th</sup> homogeneous solid layer (m)
$n$	order of the n <sup>th</sup> harmonic
$p_{v,e}$	external water vapor pressure (Pa)
$q$	heat flow rate per unit surface (W/m <sup>2</sup> )
$\hat{q}_e$	(first harmonic of) heat flow rate oscillation on the outer side (W/m <sup>2</sup> )
$q_i$	heat flow rate on the inner side (W/m <sup>2</sup> )
$\bar{q}_i$	average value of heat flow rate oscillation on the inner side (W/m <sup>2</sup> )
$\hat{q}_i$	(first harmonic of) heat flow rate oscillation on the inner side (W/m <sup>2</sup> )
$\hat{q}_{i,n}$	n <sup>th</sup> harmonic of heat flow rate oscillation on the inner side (W/m <sup>2</sup> )
$q_{i,\text{peak}}$	peak heat flow rate on the inner side (W/m <sup>2</sup> )
$R$	thermal resistance (m <sup>2</sup> K/W)
$R_a$	thermal resistance of an airspace (m <sup>2</sup> K/W)
$R_k$	thermal resistance of the k <sup>th</sup> non-homogeneous solid layer or airspace (m <sup>2</sup> K/W)
$R_{se}$	external surface or film resistance (m <sup>2</sup> K/W)
$R_{si}$	internal surface or film resistance (m <sup>2</sup> K/W)
<b>SRI</b>	solar reflectance index (%)
<b>STI</b>	solar transmittance index (%)
$T$	temperature (°C or K)
$T_{air}$	external air temperature (°C or K)
$\bar{T}_{air}$	average external air temperature (°C or K)
$T_e$	external temperature (°C or K)
$\bar{T}_e$	average external temperature (°C)
$T_i$	internal temperature (°C or K)
$T_{me}$	mean of surface and ambient temperature on the outer side (°C or K)
$\bar{T}_{me}$	average mean of surface and ambient temperature on the outer side (°C or K)

$T_{mi}$	mean of surface and ambient temperature on the inner side ( $^{\circ}\text{C}$ or $\text{K}$ )
$T_{sb}$	temperature of the black reference surface ( $^{\circ}\text{C}$ or $\text{K}$ )
$T_{se}$	external surface temperature ( $^{\circ}\text{C}$ or $\text{K}$ )
$\bar{T}_{se}$	average external surface temperature ( $^{\circ}\text{C}$ or $\text{K}$ )
$T_{si}$	internal surface temperature ( $^{\circ}\text{C}$ or $\text{K}$ )
$T_{si,peak}$	peak of internal surface temperature ( $^{\circ}\text{C}$ )
$T_{sky}$	sky temperature ( $^{\circ}\text{C}$ or $\text{K}$ )
$T_{sol-air}$	sol-air temperature ( $^{\circ}\text{C}$ or $\text{K}$ )
$\bar{T}_{sol-air}$	average sol-air temperature ( $^{\circ}\text{C}$ or $\text{K}$ )
$T_{sw}$	temperature of the white reference surface ( $^{\circ}\text{C}$ or $\text{K}$ )
$t$	time in the day (s)
$t_0$	day period ( $24\text{h} = 86'400\text{ s}$ )
$\text{TPI}$	thermal performance index (%)
$*\text{TPI}$	“new” thermal performance index (%)
$\text{TPI}_h$	“hot” thermal performance index (%)
$U$	thermal transmittance ( $\text{W}/(\text{m}^2\text{K})$ )
$v_{wind}$	wind velocity (m/s)
$Y_{ie}$	periodic thermal transmittance ( $\text{W}/(\text{m}^2\text{K})$ )
$Z$	heat transfer matrix ( $\text{m}^2\text{K}/\text{W}$ )
$Z_a$	heat transfer matrix of an airspace ( $\text{m}^2\text{K}/\text{W}$ )
$Z_{n=1..N}$	heat transfer matrix of the $n^{\text{th}}$ layer ( $\text{m}^2\text{K}/\text{W}$ )

#### *Greek and mixed symbols*

$\beta$	angle between vertical direction and normal to the considered surface (rad)
$\Delta T$	temperature difference ( $^{\circ}\text{C}$ )
$\Delta t_f$	time shift or lag time of the periodic thermal transmittance (s or h)
$\Delta T_{si,peak}$	peak of internal surface temperature increase ( $^{\circ}\text{C}$ )
$\Delta T_{si,peak(tested)}$	peak of internal surface temperature increase for the tested case ( $^{\circ}\text{C}$ )
$\Delta T_{si,peak,optimal}$	peak of internal surface temperature increase for the optimal reference case ( $^{\circ}\text{C}$ )
$\Delta T_{si,peak,worst}$	peak of internal surface temperature increase for the worst reference case ( $^{\circ}\text{C}$ )
$\delta$	thermal penetration depth (m)
$\varepsilon_e$	thermal emittance of the external surface ( $0 < \varepsilon_e < 1$ )
$\varepsilon_i$	thermal emittance of the internal surface ( $0 < \varepsilon_i < 1$ )
$\theta_e$	sol-air temperature ( $^{\circ}\text{C}$ )
$\bar{\theta}_e$	average sol-air temperature ( $^{\circ}\text{C}$ )
$\hat{\theta}_e$	(first harmonic of) sol-air temperature oscillation ( $^{\circ}\text{C}$ )
$\hat{\theta}_{e,n}$	$n^{\text{th}}$ harmonic of sol-air temperature oscillation ( $^{\circ}\text{C}$ )
$\hat{\theta}_i$	(first harmonic of) internal temperature oscillation ( $^{\circ}\text{C}$ )
$\lambda$	wavelength (nm)
$\rho_{sol}$	solar reflectance ( $0 < \rho_{sol} < 1$ )
$\rho_{sol,b}$	solar reflectance of the reference black surface ( $0 < \rho_{sol,b} < 1$ )
$\rho_{sol,w}$	solar reflectance of the reference white surface ( $0 < \rho_{sol,w} < 1$ )
$\rho_{\lambda}$	spectral reflectivity ( $0 < \rho_{\lambda} < 1$ )
$\sigma_0$	Stefan-Boltzmann constant ( $5.67 \cdot 10^{-8} \text{ W}/(\text{m}^2\text{K}^4)$ )
$\varphi$	phase (of the first harmonic) of heat flow oscillations (rad)
$\varphi_n$	phase of the $n^{\text{th}}$ harmonic of heat flow oscillations (rad)
$\psi$	phase (of the first harmonic) of temperature oscillations (rad)
$\psi_n$	phase of the $n^{\text{th}}$ harmonic of temperature oscillations (rad)
$\omega$	fundamental angular frequency of temperature oscillations (rad/s)

## 2. Parameters relevant to the thermal behavior of opaque elements

The thermal behavior of an opaque building element is influenced by the materials from which the element is built, the properties of its external and internal surfaces and the boundary conditions at such surfaces. Roofs and walls are mostly made of stacked homogenous layers such as bricks or concrete, insulation materials, wooden or gypsum panels, plaster, possibly airspaces, membranes and tile mantles, therefore parameters relevant to the thermal behavior of layered roofs and walls are analyzed in the followings.

### 2.1. Parameters related to thermal insulation

The thermal insulation provided by a building element is evaluated in terms of R-value or thermal resistance ( $\text{m}^2\text{K}/\text{W}$ ). This is defined for steady-state conditions as the ratio of the temperature difference across the element  $\Delta T$  (K) and the rate of heat transfer per unit area  $q$  ( $\text{W}/\text{m}^2$ ). The inverse of the thermal resistance, the U-value or thermal transmittance ( $\text{W}/(\text{m}^2\text{K})$ ), is often used.

$$R = \frac{\Delta T}{q} \equiv \frac{1}{U} \quad (1)$$

The thermal resistance of a building element is calculated according to standards such as those by ISO [12] and ASHRAE [13] as the sum of the conductive resistances of the element layers and the surface or film resistances associated to convection and radiation at the internal and external surfaces:

$$R = \frac{1}{h_i} + \sum_j \frac{L_j}{k_j} + \sum_k R_k + \frac{1}{h_e} \equiv \frac{1}{U} \quad (2)$$

where,

$h_i$  heat transfer coefficient by convection and radiation at the internal surface ( $\text{W}/(\text{m}^2\text{K})$ )

$L_j$  thickness of the  $j^{\text{th}}$  homogeneous solid layer (m)

$k_j$  effective thermal conductivity of the material in the  $j^{\text{th}}$  homogeneous solid layer  $\text{W}/(\text{m K})$

$R_k$  thermal resistance of the  $k^{\text{th}}$  nonhomogeneous solid layer or airspace ( $\text{m}^2\text{K}/\text{W}$ )

$h_e$  heat transfer coefficient by convection and radiation at the external surface ( $\text{W}/(\text{m}^2\text{K})$ )

Thermal resistances vary with temperature but it is a common practice in construction to treat them as constant values. The internal and external heat transfer coefficients, inverse of the internal and external surface or film thermal resistances  $R_{si}$  and  $R_{se}$  ( $\text{m}^2\text{K}/\text{W}$ ), respectively, are given by the combination of a convection (heat transfer) coefficient,  $h_{ci}$  or  $h_{ce}$  ( $\text{W}/(\text{m}^2\text{K})$ ), and a radiation (heat transfer) coefficient,  $h_{ri}$  or  $h_{re}$  ( $\text{W}/(\text{m}^2\text{K})$ ):

$$h_i = h_{ci} + h_{ri} \equiv \frac{1}{R_{si}} \quad \text{and} \quad h_e = h_{ce} + h_{re} \equiv \frac{1}{R_{se}} \quad (3)$$

When the heat flow through a building element is calculated,  $\Delta T$  is the difference between external temperature  $T_e$  and internal temperature  $T_i$ . The former is about that of the outdoor air, but it can be significantly lower with clear sky conditions and low humidity due a lower temperature of the sky. In fact, the external temperature relevant to heat transfer can be estimated as the average of the air temperature  $T_{air}$  and the effective sky temperature  $T_{sky}$  weighted by the external convection and radiation coefficients,  $h_{ce}$  and  $h_{re}$ :

$$T_e \approx \frac{h_{ce} T_{air} + h_{re} T_{sky}}{h_{ce} + h_{re}} \quad (4)$$

$T_{\text{sky}}$  is usually evaluated from parameters such as the near-ground air temperature (that is  $T_{\text{air}}$ ) and the partial pressure of water vapor  $p_{v,e}$  (Pa) through formulas such as [14]

$$T_{\text{sky}} = T_{\text{air}} - \left[ T_{\text{air}} - 18 + 51.6 \exp\left(-\frac{p_{v,e}}{1000}\right) \right] \frac{(1 + \cos \beta)}{2} \quad (5)$$

where  $\beta$  is the angle between the vertical direction and the normal to the considered surface, and the correction applied through  $\beta$  to the temperature difference between brackets allows to take into account the view-factor between surface and sky.

The internal convection coefficient  $h_{ci}$  is generally given by free convection phenomena and it is a constant value, depending on the direction of heat flow. For example, values of 0.7, 2.5 and 5.0  $\text{W}/(\text{m}^2\text{K})$  are proposed in ISO 6946 [12] for downward, horizontal and upward heat flow, respectively. The external convection coefficient  $h_{ce}$  is given by superposition of free convection phenomena and wind effects, so it depends also on wind velocity  $v_{\text{wind}}$  (m/s). It can be estimated by formulas like the simplified one proposed in ISO 6946 [12]:

$$h_{ce} = 4 + 4v_{\text{wind}} \quad (6)$$

The radiation heat transfer coefficient is related to the thermal emittance of the considered surface through the following formula [12]:

$$h_{ri} = \varepsilon_i \sigma_0 4T_{mi}^3 \quad \text{and} \quad h_{re} = \varepsilon_e \sigma_0 4T_{me}^3 \quad (7)$$

where

- $\varepsilon_i$  thermal emittance of the inner surface ( $0 < \varepsilon_i < 1$ )
- $\varepsilon_e$  thermal emittance of the outer surface ( $0 < \varepsilon_e < 1$ )
- $\sigma_0$  Stefan-Boltzmann constant ( $5.67 \times 10^{-8} \text{ W}/(\text{m}^2\text{K}^4)$ )
- $T_{mi}$  mean of surface and ambient temperature on the inner side (K)
- $T_{me}$  mean of surface and ambient temperature on the outer side (K)

The thermal emittance of built surfaces can be measured by standard test methods such as ASTM C1371 [15] or EN 15976 [16], but it is common practice in construction to take it equal to 0.9 for nonmetallic or coated elements. Thermal emittance is usually around 0.9 for non-metallic surfaces, however, it can be significantly lower with uncoated metal surfaces, metal surfaces with very thin coatings, or coatings with metallic pigments. It is also a common practice to take constant the mean of surface and ambient temperatures: for example,  $T_{mi}$  is set equal to the internal set-point temperature  $T_i$  of the HVAC system to calculate the internal radiation coefficient  $h_{ri}$ , whereas  $T_{me}$  is set equal to a value close to that of the external ambient temperature  $T_e$  to calculate the external radiation coefficient  $h_{re}$ .

Lower limits to the R-value, or upper limits to the U-value, have been made compulsory in most countries, varying with the local climate, the type of building element (roof, wall, window, etc.), possibly the orientation (north), the age and use of the building, the cases of new building or simple retrofit.

## 2.2. Parameters related to thermal inertia

The R-value and the U-value are defined with reference to steady-state conditions since they are intended to evaluate the heat loss in the heating season, when the external temperature is substantially steady during the day and solar radiation is low. They cannot describe the dynamic behavior of a building element when this is subjected to the cycle of solar radiation and the relevant external temperature is not the true ambient one, but an equivalent quantity known as the sol-air temperature:

$$T_{\text{sol-air}}(t) = T_e(t) + \frac{(1 - \rho_{\text{sol}})}{h_e} I_{\text{sol}}(t) \quad (8)$$

where,

$t$  time in the day (s)

$\rho_{\text{sol}}$  solar reflectance, i.e. the ratio of reflected and incident solar radiation ( $0 < \rho_{\text{sol}} < 1$ )

$I_{\text{sol}}$  solar irradiance, i.e. the heat rate of incident solar radiation per unit surface ( $\text{W/m}^2$ )

The sol-air temperature has a periodic time-evolution pattern that follows the cycle of solar irradiance. Such cycle peaks at solar noon on a horizontal surface and it is nil in the night. The external temperatures of the air and the sky may have a periodic time-evolution pattern as well, following the cycle of solar irradiance with a short time lag. When a strong cycle of sol-air temperature occurs, the thermal problem becomes unsteady and the thermal inertia of the building elements must be taken into account. In this case, direct solutions to the heat transfer equation are generally unavailable and numerical methods can be used. Nonetheless, the relatively simple approach to the dynamic analysis of building elements provided by ISO 13786 [4] is widely used. It is based on that any periodic function such as the time-evolution pattern of sol-air temperature can be decomposed into a Fourier series made of the sum of a (possibly infinite) set of simple oscillating functions, namely sines or cosines (or, equivalently, complex exponentials):

$$T_{\text{sol-air}}(t) = \bar{\theta}_e + \sum_{n=1}^{\infty} |\hat{\theta}_{e,n}| \cos(n\omega t + \psi_n) \cong \bar{T}_e + \frac{(1 - \rho_{\text{sol}})}{h_e} \bar{I}_{\text{sol}} + \frac{(1 - \rho_{\text{sol}})}{h_e} \sum_{n=1}^{\infty} |\hat{I}_{\text{sol},n}| \cos(n\omega t + \psi_n) \quad (9)$$

where

$n$  order of the  $n^{\text{th}}$  harmonic

$\bar{\theta}_e$  average value of sol-air equivalent temperature ( $^{\circ}\text{C}$ )

$|\hat{\theta}_{e,n}|$  oscillation amplitude of the  $n^{\text{th}}$  harmonic of sol-air equivalent temperature ( $^{\circ}\text{C}$ )

$\omega$  fundamental angular frequency of the temperature oscillations (rad/s)

$\psi_n$  phase of the  $n^{\text{th}}$  harmonic of temperature oscillations (rad)

$\bar{T}_e$  daily average value of the external temperature ( $^{\circ}\text{C}$ )

$\bar{I}_{\text{sol}}$  average value of solar irradiance ( $\text{W/m}^2$ )

$|\hat{I}_{\text{sol},n}|$  oscillation amplitude of the  $n^{\text{th}}$  harmonic of solar irradiance ( $\text{W/m}^2$ )

The angular frequency  $n\omega$  of the  $n^{\text{th}}$  harmonic is correlated to the period  $t_0$  of the oscillation, taken equal to  $24\text{h} \equiv 86'400\text{ s}$  for a day cycle:

$$n\omega = n \frac{2\pi}{t_0} \quad (10)$$

The air temperature has a relatively weak cycle over urban and suburban areas, especially under a humid and polluted atmosphere, with an oscillation amplitude as low as a few Celsius degrees. An even narrower oscillation amplitude can consequently be calculated through eq. (4) for the external temperature  $T_e$  including both the air and sky temperature, which was therefore taken constant in the final approximation of eq. (9) in view of the much larger oscillation predictable for the term induced by the cycle of solar irradiance. The group of terms multiplying  $I_{\text{sol}}$ , in which  $h_e$  is also assumed constant due to its weak dependence on temperature, clearly affects linearly all harmonics of the Fourier series.

The oscillation of sol-air temperature penetrates the building element and it eventually contributes to a periodic heat flow entering the internal ambient:

$$q_i(t) = \bar{q}_i + \sum_{n=1}^{\infty} |\hat{q}_{i,n}| \cos(n\omega t + \varphi_n) \quad (11)$$

where

$\bar{q}_i$  average value of heat flow rate on the inner side (W/m<sup>2</sup>)

$|\hat{q}_{i,n}|$  oscillation amplitude of the n<sup>th</sup> harmonic of heat flow rate on the inner side (W/m<sup>2</sup>)

$\varphi_n$  phase of the n<sup>th</sup> harmonic of heat flow oscillations (rad)

The length of penetration of the harmonics decreases with their order n, moreover the first-order term is often dominant. As a result, the analysis can be limited to the first order term (n=1), which yields the most significant effects in the indoor space. The deviation arising by the use non sinusoidal boundary conditions with respect to the sinusoidal ones considered in ISO 13786 was found to be small and generally precautionary [17]. The oscillation of the external equivalent temperature  $\theta_e$  and that of the internal heat flow rate  $q_i$  can thus be simplified as

$$\theta_e(t) = \bar{\theta}_e + |\hat{\theta}_e| \cos(\omega t + \psi) \equiv \bar{\theta}_e + \frac{1}{2} [\hat{\theta}_{+e} e^{j\omega t} + \hat{\theta}_{-e} e^{-j\omega t}] \quad (12)$$

$$q_i(t) = \bar{q}_i + |\hat{q}_i| \cos(\omega t + \varphi) \equiv \bar{q}_i + \frac{1}{2} [\hat{q}_{+i} e^{j\omega t} - \hat{q}_{-i} e^{-j\omega t}] \quad (13)$$

where the complex amplitudes of external temperature and internal heat flow rate are defined by the following relationships:

$$\hat{\theta}_{\pm e} = |\hat{\theta}_e| e^{\pm j\psi} \quad (14)$$

$$\hat{q}_{\pm i} = |\hat{q}_i| e^{\pm j\varphi} \quad (15)$$

Analogous formulas can be built for the oscillation of the heat flow  $q_e(t)$  at the external surface, as well as for the oscillation of temperature  $\theta_i(t)$  in the internal ambient. Complex temperature and heat flow oscillations are interrelated by a complex heat transfer matrix  $Z$ , whose terms depend on the material and thickness of each element layer and on the boundary conditions at the external and internal surfaces:

$$\begin{pmatrix} \hat{\theta}_e \\ \hat{q}_e \end{pmatrix} = \begin{pmatrix} Z_{ii} & Z_{ie} \\ Z_{ei} & Z_{ee} \end{pmatrix} \begin{pmatrix} \hat{\theta}_i \\ \hat{q}_i \end{pmatrix} \quad (16)$$

The heat transfer matrix from ambient to ambient is the product of the heat transfer matrices associated to the various layers of the building element, conventionally starting from the innermost one:

$$Z = \begin{pmatrix} Z_{ii} & Z_{ie} \\ Z_{ei} & Z_{ee} \end{pmatrix} = Z_N Z_{N-1} \dots Z_3 Z_2 Z_1 \quad (17)$$

The heat transfer matrices of the boundary layers are as follows:

$$Z_1 = \begin{pmatrix} 1 & 1/h_i \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad Z_N = \begin{pmatrix} 1 & 1/h_e \\ 0 & 1 \end{pmatrix} \quad (18)$$

The heat transfer matrix of a homogeneous solid layer, omitted here for sake of brevity, is basically a function of the ratio  $\delta/L$  of thickness  $L$  (m) and penetration depth  $\delta$  (m) associated to the layer, the latter defined as follows:

$$\delta = \sqrt{\frac{k t_0}{\pi d c}} \quad (19)$$

where

d mass density (kg/m<sup>3</sup>)  
c specific heat (J/(kg K))

Several test methods are available to determine the thermal conductivity and the specific heat of the materials, which must be known with proper accuracy to calculate the penetration depths and, from these, to build the heat transfer matrix  $Z$ .

The heat transfer matrix of an airspace with negligible inertia and thermal resistance  $R_a$ , depending on its thickness, inclination, and the thermal emittance of the bounding surfaces, is:

$$Z_a = \begin{pmatrix} 1 & R_a \\ 0 & 1 \end{pmatrix} \quad (20)$$

Parameters describing the dynamic behavior of a building element can eventually be extracted from  $Z$ . Among these, the periodic thermal transmittance  $Y_{ie}$  relates to the oscillating external temperature the oscillating heat flow entering the internal ambient when the temperature of that ambient is kept constant:

$$\hat{q}_i = Y_{ie} \hat{\theta}_e \cong Y_{ie} \frac{(1 - \rho_{sol})}{h_e} \hat{I}_{sol} \quad (21)$$

with

$$Y_{ie} = \frac{1}{Z_{ie}} \quad \text{and} \quad \hat{\theta}_i = 0 \quad (22)$$

The modulus of the (complex) periodic thermal transmittance is a representation of the heat flow cycle produced at the inner surface by a temperature oscillation at the outer side when the temperature of the internal ambient is kept constant.

The decrement factor  $f$  is the ratio of modulus of  $Y_{ie}$  and steady state thermal transmittance  $U$ :

$$f = \frac{|Y_{ie}|}{U} \quad (23)$$

With constant internal temperature,  $f$  expresses the ratio of the maximum heat flow rate entering the inner space and that through an element with same thermal transmittance  $U$  but negligible inertia.

The time shift or lag time of the periodic thermal transmittance  $\Delta t_f$  is related to the argument of  $Z_{ie}$  evaluated in the range  $-2\pi$  to  $0$ :

$$\Delta t_f = \frac{t_0}{2\pi} \arg(Z_{ie}) \quad (24)$$

It represents the time shift with which a temperature cycle at the outer side reaches the inner side while travelling through the building element.

Decrement factor and time shift are highly considered in construction since they can be easily understood. In passive cooling techniques a properly-high time shift may be desirable, and it is sometimes required, so that the temperature cycle induced by the external temperature oscillation reaches the internal space during nighttime, when it can be mitigated by free or forced ventilation with cooler external air. This concept may unfortunately fail when the urban heat island effect occurs, since the external air temperature may be too high in the nighttime to allow night cooling and one has to rely upon air conditioning. On the other hand, a proper thermal inertia can significantly reduce the heat flow to the inner ambient that is induced by the solar cycle, so upper

limits are generally set for  $|Y_{ie}|$  alongside  $U$ . In this regards, as far as thermal bridges can be neglected, the daily average heat flow rate per unit area entering the indoor space with a constant temperature  $T_i$  is:

$$\bar{q}_i = U \times (\bar{\theta}_e - T_i) \equiv U \left[ \bar{T}_e + \frac{(1 - \rho_{sol})}{h_e} \bar{I}_{sol} - T_i \right] \cong U \frac{(1 - \rho_{sol})}{h_e} \bar{I}_{sol} \quad (25)$$

The final approximation is possible if  $\bar{T}_e \cong T_i$ , as it is often the case in humid climates or when the UHI effect occurs. In fact, ambient conditions are considered as specified in the ASTM E1980 standard, in which a peak air temperature of 310 K = 37°C and a sky temperature of 300 K = 27°C are proposed. Those values are well representative of typical peak conditions in urban and suburban areas of humid climates, and the peak external temperature resulting from eq. (4) is somewhere in the mid. In the above mentioned areas the oscillation amplitude of the air temperature and, consequently, of the external temperature is a few Celsius degrees, so the average external temperature is close to the usual internal temperature of air conditioned spaces, set to 26-27°C in Europe or North America. This supports the assimilation of average external temperature and internal temperature for sake of simplicity. As a result, the daily average cooling power to be provided by an air conditioning system to offset the transmitted heat is related to the quantity calculated in eq. (25).

Some supplementary power, however, may be required to maintain a constant operative temperature despite the daily variation of sol-air temperature. Considering only the first order term, such supplementary power must compensate the upper peak of sol-air temperature oscillations as expressed by eq. (21), so that one has:

$$q_{i,peak} = \bar{q}_i + |\hat{q}_i| \quad (26)$$

As already mentioned, high values of  $\Delta t_f$  and low values of  $f$  or  $|Y_{ie}|$  are often required by building codes, asking for a high thermal capacity of wall and roof layers. Proper values of those parameters are generally yielded by a properly high mass of the building element. So a simplistic requirement was introduced in Italy in terms of a lower limit for the surface mass (the ratio of mass and frontal surface area of the element), set equal to of 230 kg/m<sup>2</sup> for vertical walls. A high thermal capacity is also useful to achieve steady temperatures in the built environment, nevertheless similar results can be achieved by a proper control of internal and solar heat gains and an efficient air conditioning system.

Altogether, requirements related to the thermal mass or thermal inertia are added up to those related to thermal insulation. Satisfying contemporarily such requirements and comparing different building solutions may result difficult to designers. Moreover, and above all, the actual oscillation amplitude of the sol-air temperature that results from the solar reflectance of the external surface is not taken into account by parameters related to thermal inertia such as those obtained from the heat transfer matrix, therefore the above mentioned requirements may result in thick and heavy building elements and, at the same time, in an unsatisfactory performance during the cooling season.

### 2.3. Surface radiative properties

The sol-air temperature introduced with eq. (8) depends on the solar reflectance or albedo of the surface  $\rho_{sol}$  and, through the radiative component  $h_{re}$  of the external heat transfer coefficient  $h_e$ , on the external thermal emittance  $\varepsilon_e$ . The higher is solar reflectance, the lower is the increase of the sol-air temperature with respect to the actual external temperature. Moreover, in low or intermediate wind conditions the radiative component of the external heat transfer coefficient, which is directly related to thermal emittance, may be higher than, or at least comparable to, the convective component. Indeed cool materials such as cool roofs and cool pavements are identified by such technical designations when they show high values of both solar reflectance and thermal emittance.

The solar reflectance  $\rho_{sol}$  of a surface can be calculated by integrating over the range of solar radiation (from 300 nm to 2500 nm) the measured spectrum of reflectivity  $\rho_{\lambda}$ , defined as the ratio of reflected part and total amount of incident radiation at the considered wavelength  $\lambda$  (nm), weighted by the standard spectral irradiance of the sun at the earth surface,  $I_{sol,\lambda}$  (W/(m<sup>2</sup>nm)):

$$\rho_{sol} = \frac{\int_{300}^{2500} \rho_{\lambda} I_{sol,\lambda} d\lambda}{\int_{300}^{2500} I_{sol,\lambda} d\lambda} \quad (27)$$

Based on the above concept, solar reflectance can be measured by several methods [18-21]. Minimum values of solar reflectance and sometimes thermal emittance are set in many countries by building codes such as Title 24 of California [22], as well as by voluntary programs incentivizing energy efficiency like the Energy Star of EPA [23], which asks for a minimum solar reflectance of 0.65 for horizontal or low-slope roofs and 0.30 for high-slope roofs, as well as a minimum thermal emittance of 0.80. In some cases, only the solar reflectance or its complement to unity, the solar absorptance, are considered for opaque building elements. Initial radiative properties or those after natural or accelerated aging can also be certified through bodies like the Cool Roof Rating Council of U.S. [24] or the European Cool Roof Council [25].

The combined effects of solar reflectance and thermal emittance can be expressed through the “solar reflectance index” (SRI), a parameter calculated as [26]

$$SRI = 100 \times \frac{T_{sb} - T_{se}}{T_{sb} - T_{sw}} \quad (28)$$

where  $T_{se}$  (K) is the temperature that the analyzed surface would steadily reach when irradiated by a reference solar flux  $I_{sol,max} = 1000$  W/m<sup>2</sup> at atmospheric air temperature  $T_{air} = 310$  K, sky temperature  $T_{sky} = 300$  K, and with convection heat transfer coefficient  $h_{ce}$  to which three different values can be assigned, equal to 5, 12, and 30 W/(m<sup>2</sup>K) for, respectively, low ( $v_{wind} < 2$  m/s), intermediate ( $2$  m/s  $< v_{wind} < 6$  m/s), and high ( $6$  m/s  $< v_{wind} < 10$  m/s) wind speed.  $T_{sb}$  (K) and  $T_{sw}$  (K) are the temperatures that would be reached by two reference surfaces, respectively a black one ( $\rho_{sol,b} = 0.05$ ) and a white one ( $\rho_{sol,w} = 0.80$ ), both ones having high thermal emittance ( $\epsilon_e = 0.90$ ). SRI represents the decrement of surface temperature that the analyzed surface would allow with respect to the reference black one in the reference conditions, divided by the analogous decrement allowed by the reference white surface and eventually given in percentage terms. The surface temperature  $T_{se}$  (as well as  $T_{sb}$  and  $T_{sw}$ ) is determined by iteratively solving the following relationship:

$$(1 - \rho_{sol}) I_{sol} = \epsilon_e \sigma_0 (T_{se}^4 - T_{sky}^4) + h_{ce} (T_{se} - T_{air}) \quad (29)$$

SRI is contemplated by voluntary rating systems such as LEED [27] when dealing with the summer performance of opaque building elements, so it matches the need of a single performance parameters and allows comparing the performance of different cool roofing solutions. Its main limitation is that it is based on the hypothesis of adiabatic roof or wall external surface and it does not consider neither the insulation nor the inertia of the materials below. On the other end, the SRI works very well even with non-adiabatic surfaces since the heat flow rate conducted inside can be lower by one or two orders of magnitude than solar irradiance.

### 3. The solar transmittance index (STI)

The thermal behavior of a building element subjected to the cycle of solar radiation or, in analogous term, to the cycle of sol-air temperature depends on the combined effects of thermal insulation, thermal inertia and radiative properties. In this regard, many parameters must be contemporarily considered such as R-value or U-value, decrement factor, time-shift, modulus of the

periodic thermal transmittance, surface mass, solar reflectance or solar absorptance, thermal emittance, and SRI, therefore minimum or maximum values of such parameters are generally specified by building codes and incentive programs of the construction sector. However, the comparison of different building solutions may result quite difficult: for example, an equivalent performance may be provided by a roof with high thermal mass but dark surface and one with relatively low mass but highly reflective external surface. A very high solar reflectance may almost cancel the cycle of sol-air temperature, whichever is the thermal transmittance or the thermal internal, but it would be difficult to preserve due to soiling of bio-fouling, so the lower solar reflectance of the aged surface should be considered. Moreover, highly reflective surfaces are white, but different colors may be compulsory for historical or traditional buildings or even whole urban areas. On the other hand, a high thermal inertia has an effective and unchanging thermal response, but it may conflict with seismic requirements, maximum allowed thicknesses, or costs. A mix of properly chosen surface and mass properties is probably the most effective approach, but it was already pointed out that comparing different building solutions on the basis of several unrelated parameters may be tricky and viable only through numerical simulation by dynamic numerical methods, possibly taking into account local ambient conditions and the building as a whole. As a result, requirements to prevent building overheating are difficult to identify and verify, be they conceived as a mere set of limits to the above-mentioned unrelated parameters or in terms of synthetic comfort indicators such as the “hot thermal performance index” (TPI<sub>h</sub>) or the “hot discomfort degree hours” index (DDH<sub>h</sub>) analyzed in [7], or the “thermal deviation index” (TDI) proposed in [28]. An arbitrary set of limiting values could also be distortive of the market.

In view of the considerations above, a single index is proposed here, including both radiative properties of the external surface and thermo-physical properties of the materials below. It is conceived to be applied to single opaque building elements when their external surface is subjected to the cycle of solar radiation and the indoor temperature is kept constant by an air conditioning system. The approach moves from the evidence that the cycle of sol-air temperature, to which the heat flow entering the inner spaces can be related through the heat transfer matrix  $Z$ , is in turn related to the solar reflectance and the surface heat transfer coefficient. Approximating the solar irradiance cycle to a perfectly sine oscillation, in which the average irradiance is equaled to the amplitude of oscillation, the peak cooling power to be provided by an air conditioning system as expressed by eq. (26) combined with eq. (25) and eq. (21) is

$$q_{i,\text{peak}} = \bar{q} + |\hat{q}_i| \cong U \frac{(1-\rho_{\text{sol}})}{h_e} \bar{I}_{\text{sol}} + |Y_{\text{ie}}| \frac{(1-\rho_{\text{sol}})}{h_e} |\hat{I}_{\text{sol}}| \cong \frac{(1-\rho_{\text{sol}})}{h_e} (U + |Y_{\text{ie}}|) \frac{I_{\text{sol,max}}}{2} \quad (30)$$

where one can set  $|\hat{I}_{\text{sol}}| \cong \bar{I}_{\text{sol}} \cong I_{\text{sol,max}}/2$  in view of the sine approximation, with  $I_{\text{sol,max}}$  being the peak solar irradiance.

A comprehensive index has thus surfaced, including both the radiative properties at the external surface and the thermo-physical properties of the materials under the surface, which we will tentatively call “solar transmittance factor” ( $f_{\text{ST}}$ ):

$$f_{\text{ST}} = \frac{(1-\rho_{\text{sol}})}{h_e} (U + |Y_{\text{ie}}|) \cong (1-\rho_{\text{sol}}) \mathcal{R}_{\text{se}} U (1+f) \quad (31)$$

A sketch of the thermal process is depicted in fig. 1, which shows how the cycle of the external surface temperature induced by the solar cycle and controlled by the solar reflectance propagates through a building element and induces a cycle of the internal surface temperature with peak value well above the internal ambient temperature, thus yielding a peak of operative temperature and entering heat flux. The increase of the average internal surface temperature with respect to the internal ambient temperature is controlled by the steady-state thermal transmittance, whereas the oscillation amplitude of and the resulting surface temperature peak is controlled by the modulus of the periodic thermal transmittance.

In the formula in eq. (12), the radiative component  $h_{re}$  included in the external heat transfer coefficient  $h_e$  or its inverse, the external surface resistance  $R_{se}$ , depends on the external surface temperature. In principle,  $h_{re}$  could be evaluated by some recursive approach exploiting the full Fourier series of the sol-air temperature and the heat transfer matrix approach, possibly taking into accounts also the weak dependence of  $U$  and  $|Y_{ie}|$  on  $h_{re}$ . The use of an approximated value may however be preferable, also in view of the approximation with which the convective component  $h_{ce}$  is generally known. More specifically, the mean external temperature  $T_{me}$  from which  $h_{re}$  is calculated through eq. (7) can be set equal to the mean of effective sky temperature and average external surface temperature, as proposed in [12]:

$$\bar{T}_{me} = \frac{T_{sky} + \bar{T}_{se}}{2} \quad (32)$$

The average external surface temperature is calculated decreasing the average sol-air temperature by the average temperature drop across the external surface resistance, which is the inverse of  $h_e$ :

$$\begin{aligned} \bar{T}_{se} &\approx \bar{T}_{sol-air} - \frac{\bar{q}}{h_e} \approx \left[ T_e + \frac{(1-\rho_{sol})\bar{I}_{sol}}{h_e} \right] - \frac{U}{h_e} \left[ T_e + \frac{(1-\rho_{sol})\bar{I}_{sol}}{h_e} - T_i \right] \equiv \\ &\equiv \left[ \frac{h_{ce}T_{air} + h_{re}T_{sky}}{h_{ce} + h_{re}} + \frac{(1-\rho_{sol})I_{sol,max}}{h_{ce} + h_{re}} \frac{1}{2} \right] \left( 1 - \frac{U}{h_{ce} + h_{re}} \right) + \frac{U}{h_{ce} + h_{re}} T_i \end{aligned} \quad (33)$$

A simple recursive approach can be used to calculate  $T_{me}$  and, from this,  $h_{re}$ . Capitalizing on the work made to develop the SRI, same values can be used for  $h_{ce}$ , related to different wind velocities. The same air temperature ( $T_{air}=310$  K) and sky temperature ( $T_{sky}=300$  K) can also be considered, as well as the same peak solar irradiance ( $I_{sol,max}=1000$  W/m<sup>2</sup>), which is assumed to be twice the amplitude and the average value of the solar irradiance sine cycle.  $T_i$  can be set equal to 27°C≅300 K, a value usually considered for indoor spaces in the hot season. An alternative choice with similar values can be provided by the neutral temperature  $T_n$  (°C) as proposed in [29], corresponding to maximum comfort conditions and correlated to the daily average temperature of the outdoor air  $\bar{T}_{air}$  (°C):

$$T_n = 13.5 + 0.54 \cdot \bar{T}_{air} \quad (34)$$

Values of  $h_e$  eventually calculated by the above outlined approach are plotted with respect to  $\rho_{sol}$  and  $\varepsilon_e$  in fig. 2 for low and intermediate wind conditions and  $U=1$  W/(m<sup>2</sup>K). The relatively weak dependence of  $h_e$  on  $T_{me}$  for assigned values of  $\varepsilon_e$ , which justifies an approximate evaluation of  $T_{me}$ , is then shown in fig. 3. A very weak dependence of the external heat transfer coefficient may also be observed in fig. 4 with respect to the thermal transmittance  $U$ , decreasing further with increasing solar reflectance.

The solar transmittance factor  $f_{ST}$  as defined by eq. (31) correlates the peak of heat flow rate per unit area that enters the inhabited space to the peak of solar irradiance onto the external surface, where the solar irradiance has been approximated to a sine cycle. From the entering heat flow rate, the peak of radiant temperature of the ceiling surface, which is a main source of thermal discomfort below and may affect cooling energy demand by asking for a lower temperature of the indoor air, can be estimated as follows:

$$T_{si,peak} = T_i + \Delta T_{si,peak} \quad (35)$$

where the peak of surface temperature increase  $\Delta T_{si,peak}$  with respect to  $T_i$  is

$$\Delta T_{si,peak} = \frac{q_{i,peak}}{h_i} \approx \frac{1}{h_i} f_{ST} \frac{I_{sol,max}}{2} \quad (36)$$

Pre-calculated values of  $h_i$  such as those suggested by ISO 6946 [12] and analogous standards can be used, or a more precise value can be estimated by a recursive approach from eq. (3) combined with eq. (7).  $I_{sol,max}$  can again be taken equal to  $1000 \text{ W/m}^2$  as suggested by ASTM E1980 [26]. From the peak of surface temperature increase, measured from a reference temperature depending on comfort requirements, the “new” thermal performance index \*TPI was proposed by [11]:

$$*TPI = 100 \frac{\Delta T_{si,peak,worst} - \Delta T_{si,peak(tested)}}{\Delta T_{si,peak,worst} - \Delta T_{si,peak,optimal}} \quad (37)$$

It is shown in next section that this definition, which is clearly analogous to SRI in eq. (28), yields an accumulation of moderately to high performing solutions close to the best performance of 100% if one choses the worst case among the less performing ones, for example an uninsulated metal roof with uncoated surface like that proposed in [11]. Moreover, in eq. (37) the peak of internal surface temperature increase for the optimal case scenario is made dependent on the local weather conditions and the characteristics of the roof, but this may result impractical and somewhat arbitrary. On the other hand, the solar transmittance factor as defined by eq. (31) also yields an accumulation close to the best performance of 0. Therefore, an alternative index is proposed here, inspired by SRI and \*TPI and tentatively called the “solar transmittance index” (STI):

$$STI = 100 \frac{f_{ST,worst} - f_{ST(tested)}}{f_{ST,worst} - f_{ST,optimal}} \quad (38)$$

In view of eq. (36), where  $h_i$  and  $I_{sol,max}$  are constant values, one can reason in terms of either inner surface temperature increase or entering heat flow rate per unit surface, so the comparison of building solutions can be directly based on  $f_{ST}$  rather than  $\Delta T_{si,peak}$  if a same  $h_i$  value is used for all cases. STI thus represents the percent fraction of the peak of transmitted heat flow rate per unit surface in the worst reference case, and in excess to the optimal reference case, that is cancelled by means of the considered building solution.

#### 4. Discussion

The lower  $f_{ST}$ , the lower the peak of heat flow rate with its negative effects on thermal comfort and air conditioning costs. A low value of  $f_{ST}$  can be achieved by intervening on any of the parameters it involves. A low (steady state) thermal transmittance allows to limit the average heat flow, whereas a low periodic thermal transmittance allows to limit the oscillation amplitude of the heat flow. Their combination limits the peak of heat flow rate, but the same effect can alternatively or complementary be provided by a high value of the solar reflectance. By contrast, a low solar reflectance, that is a high absorption of solar radiation, may cancel most of the benefits provided by low values of both steady and periodic thermal transmittances. The thermal emittance of the external surface is also taken into account in  $f_{ST}$  through the radiative component of the external heat transfer coefficient in eq. (31). More specifically, a low value of the thermal emittance has a negative impact on  $f_{ST}$ , as well known with cool roof materials.

With regard to STI, a truly optimal case would be  $f_{ST,optimal}=0$ . With this setting in eq. (38), values of STI in excess of 100% are avoided, and the index consequently represents the fraction of the peak of surface temperature increase for the reference worst case that is cancelled. A 100% performance, however, is practically impossible to reach and a different choice may be preferable for the reference optimal case, for example a solution with high insulation level, high thermal inertia, and high solar reflectance at the same time. A reference worst case with moderate

performance instead of a very low performing one can also be chosen to avoid accumulation of different building solutions close to the best performance value of 100% and thus increase differentiation.

Values of  $f_{ST}$  for some layer structures of building elements have been calculated and are reported in tab. 1, with reference values of the materials properties given in tab. 2, and surface properties given in tab. 3. The two opposite cases of a heavy concrete slab and a very light panel have been considered, with different level of insulation and surface coating. Hollow core slabs and wooden roofs with or without tile mantle are expected to show intermediate behavior. The comparison is made for the most demanding case of low wind conditions and  $h_{ce}=5 \text{ W}/(\text{m}^2\text{K})$ . For the heat transfer coefficient  $h_i$  at the inner surface, values of 7.69 and 5.88  $\text{W}/(\text{m}^2\text{K})$  as suggested in ISO 6946 [12] can be used in case of, respectively, downward entering heat flow (through ceilings) or horizontal heat flow (through walls), based on natural convection as well as radiation, with internal thermal emittance  $\varepsilon_i$  around 0.9. Here the value for downward entering heat flow is used since the focus is on roofing solutions. The influence of thin surface layers such as plaster or gypsum boards is neglected for sake of simplicity.

One can observe in tab. 1 that the best performing solution n. 09 of a 20 cm thick concrete slab with 10 cm of external foam insulation and a cool coating on the external surface yields  $f_{ST}=0.007$ . With  $h_i=5.88 \text{ W}/(\text{m}^2\text{K})$  and  $I_{sol,max}=1000 \text{ W}/\text{m}^2$ , a peak ceiling temperature increase of less than one kelvin is obtained. On the other hand, the concrete slab with dark coating (e.g. a dark waterproofing membrane) of solution n. 01 yields  $f_{ST}=0.277$  and, consequently, the peak ceiling temperature would rise by a few tens of kelvins. An uninsulated metal slab with uncoated (n. 10) or dark coated (n. 11) external surface yields  $f_{ST}=0.381$  or  $f_{ST}=0.589$ , respectively, and the ceiling temperature would rise to several tens of kelvins. Interestingly, very low values of  $f_{ST}$  are yielded by some heavy solutions with moderate to high insulation (n. 05, 06, 08, 09), but also by light solutions such as insulated metal panels with cool external coating (n. 18, 19).

An accumulation of  $f_{ST}$  close to very low values is generally observed in tab. 1. As already, underlined, this may make the solar transmittance factor unappealing for product manufacturers since it would not provide an adequate differentiation between roofing solutions. In this perspective, STI has been introduced. Values of STI for different values of  $f_{ST,worst}$  and  $f_{ST,optimal}$  are given in tab. 4, considering alternative reference worst cases such as the weakly or heavy insulated concrete slab with dark coating (n. 02 or n. 03), for which  $f_{ST}=0.051$  or  $f_{ST}=0.028$ , or even the weakly insulated metal slab with dark coating (n. 12), with  $f_{ST}=0.095$ . By choosing  $f_{ST}\approx 0.381$  (n. 10) one would obtain for STI the same order of magnitude of \*TPI, however an accumulation close to 100% is achieved and an adequate differentiation between roofing solutions is again prevented.

## 5. Concluding remarks

The “solar transmittance index” (STI) has been developed to take into account both the radiative properties of the external surface and the thermo-physical properties of the materials under the surface; the latter ones including the effects of either thermal insulation or thermal mass. STI can allow a quick and easy comparison of different building solutions by looking at a single performance parameter. This may help preliminary choices of designers and constructors, as well as the definition of requirements and policies of public institutions aimed to mitigate cooling energy demand. The STI is based on a “solar transmittance factor” ( $f_{ST}$ ) that is easy to calculate from the thermal transmittance (or its inverse, the R-value), the modulus of the periodic thermal transmittance or the decrement factor, and the solar reflectance. Thermal emittance and external wind conditions are also taken into account through the heat transfer coefficient at the external surface, which can be calculated by a recursive procedure or by some approximating polynomials.

The choice of the worst reference case and, above all, the optimal reference case, that is the worst and optimal values of  $f_{ST}$ , is crucial to sharply differentiate the STI values for different building solutions, so it must be carefully thought. A working hypothesis about the worst reference case is to analyze data of existing roofs and their thermal characteristics, in order to select the median or the 3<sup>rd</sup> quartile as a reference value. Deeper analyses are planned on this topic. The focus was on roofing solutions, but wall elements will also be considered in future work. Moreover, the influence of a significant swing of the air temperature will be analyzed, and the relationship between STI and actual building performance in terms of thermal comfort and cooling energy demand will be investigated by dynamic simulation.

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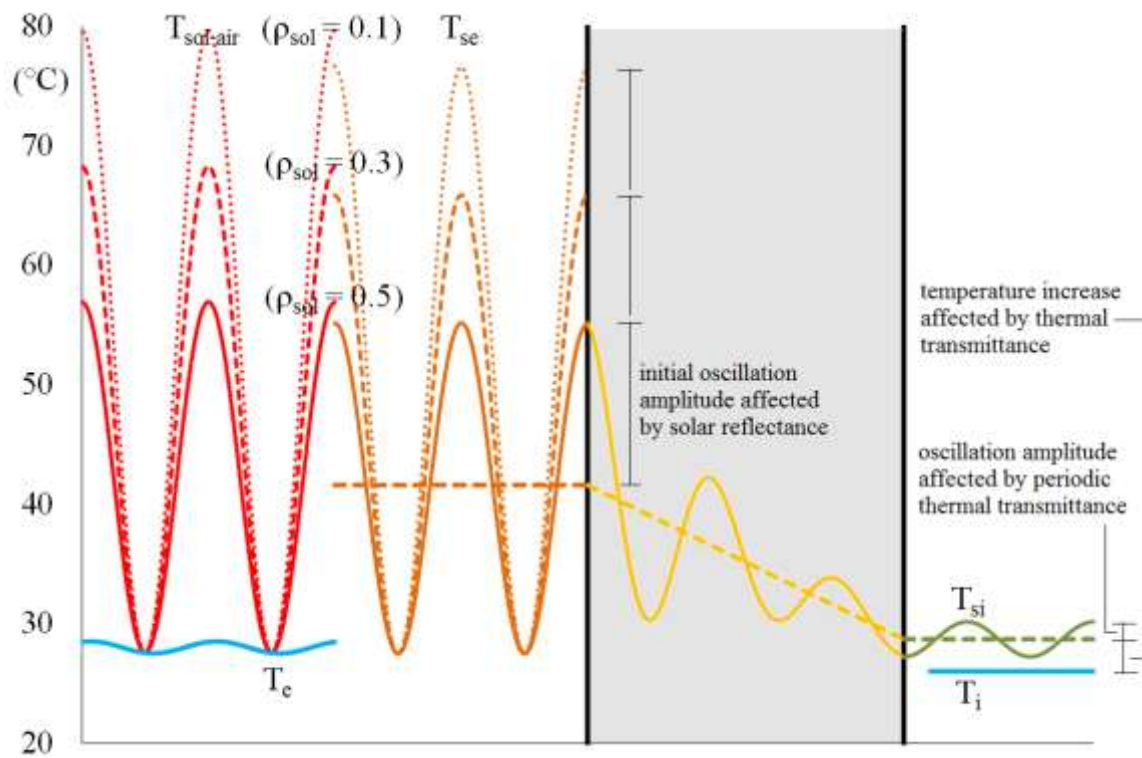


Figure 1. Cycle of the external surface temperature  $T_{se}$  induced by the sol/air temperature  $T_{sol-air}$  and controlled by the solar reflectance  $\rho_{sol}$ : the cycle propagates through a building element and induces a cycle of the internal surface temperature  $T_{si}$  having a peak value well above the internal ambient temperature  $T_i$ , thus yielding a peak of operative temperature and entering heat flux.

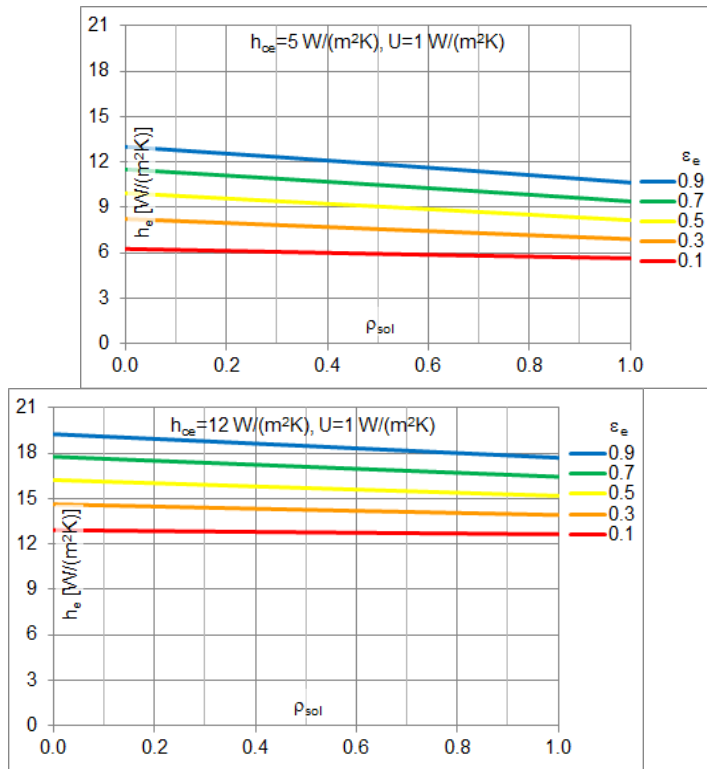


Figure 2. External heat transfer coefficient vs. solar reflectance and thermal emittance for low (left,  $0 < v_{wind} < 2$  m/s) and intermediate (right,  $2 < v_{wind} < 6$  m/s) wind conditions,  $U = 1$  W/(m<sup>2</sup>K).

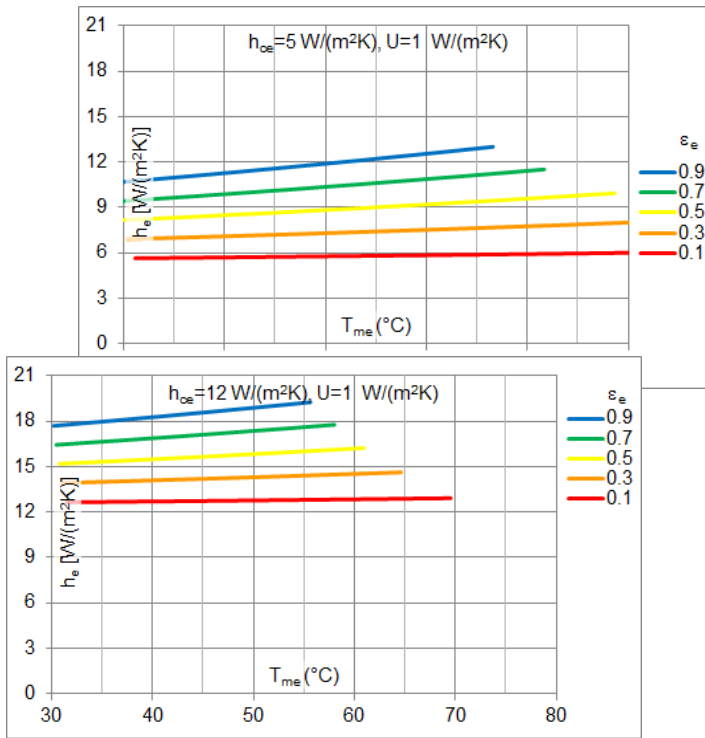


Figure 3. External heat transfer coefficient vs. mean external temperature and thermal emittance for low (left,  $0 < v_{wind} < 2$  m/s) and intermediate (right,  $2 < v_{wind} < 6$  m/s) wind conditions,  $U = 1$  W/(m<sup>2</sup>K).

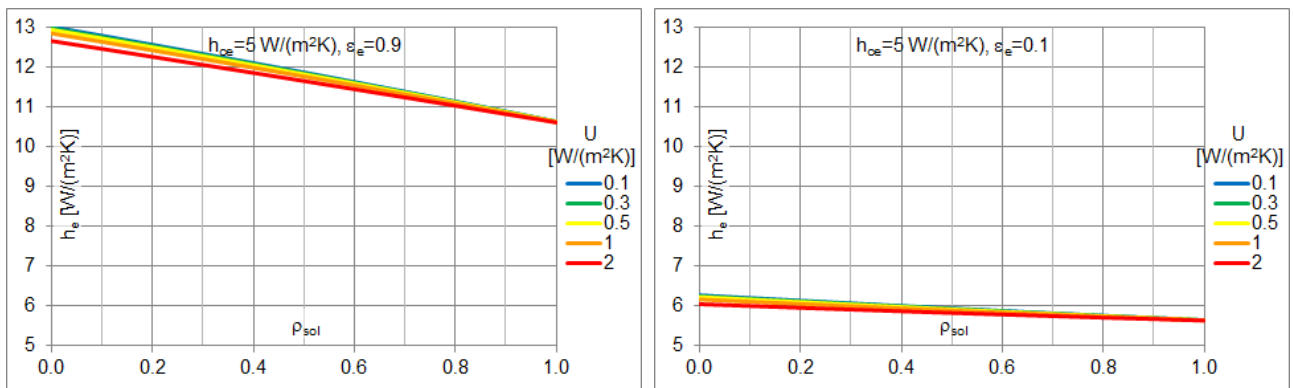


Figure 4. External heat transfer coefficient vs. solar reflectance and thermal transmittance for low wind conditions ( $0 < v_{wind} < 2$  m/s), and upper (left,  $\epsilon_e = 0.9$ ) and lower values of the thermal emittance (right,  $\epsilon_e = 0.1$ ).

Table 1. Analysed roofing solutions and performance parameters (low wind conditions).

Type	Element description*	External surface		U	$ Y_{ie} $	$f_{ST}$
		$\rho_{sol}$	$\varepsilon_e$	(W/(m <sup>2</sup> K))	(-)	(-)
01	C20, dark	0.1	0.9	2.760	1.036	0.277
02	C20+fp05, dark	0.1	0.9	0.621	0.093	0.051
03	C20+fp10, dark	0.1	0.9	0.350	0.049	0.028
04	C20, light	0.45	0.9	2.726	0.963	0.174
05	C20+fp05, light	0.45	0.9	0.619	0.093	0.033
06	C20+fp10, light	0.45	0.9	0.349	0.048	0.018
07	C20, cool	0.8	0.9	2.688	0.963	0.066
08	C20+fp05, cool	0.8	0.9	0.617	0.093	0.013
09	C20+fp10, cool (best)	0.8	0.9	0.348	0.048	0.007
10	M, bare (low $\varepsilon_e$ )	0.6	0.2	3.078	3.078	0.381
11	M, dark	0.1	0.9	3.957	3.957	0.589
12	fp05+M, dark (worst)	0.1	0.9	0.667	0.667	0.095
13	fp10+M, dark	0.1	0.9	0.364	0.362	0.051
14	M, light	0.45	0.9	3.893	3.893	0.372
15	fp05+M, light	0.45	0.9	0.665	0.665	0.061
16	fp10+M, light	0.45	0.9	0.363	0.361	0.033
17	M, cool	0.8	0.9	3.823	3.823	0.140
18	fp05+M, cool	0.8	0.9	0.662	0.662	0.024
19	fp10+M, cool	0.8	0.9	0.362	0.360	0.013

\*Layers are listed from the innermost one; material and surface types are referred to tabs. 2-3.

Table 2. Reference material properties (solid layers).

Type	Material	Density d (kg/m <sup>3</sup> )	Conductivity k (W/(m K))	Specific Heat c (J/(kg K))
C20	concrete slab	2400	1.8	1000
M	metal coil	7800	50	500
fp05	foam insulation panel 5 cm	30	0.04	1400
fp10	foam insulation panel 10 cm	30	0.04	1400

Table 3. Reference radiative properties (external surface).

Type	Surface	Solar Reflectance ( $0 < \rho_{sol} < 1$ )	Thermal Emittance ( $0 < \varepsilon_e < 1$ )
dark	dark coating	0.10	0.90
light	light color coating	0.45	0.90
cool	cool white coating	0.80	0.90
bare	bare metal (low emissivity)	0.60	0.20

Table 4. STI values for different roofing solutions and reference cases (low wind conditions).

			$f_{ST,optimal} = 0$				$f_{ST,optimal} = 0.007$ (n.09)			
			$f_{ST,worst}$				$f_{ST,worst}$			
			0.381 (n.10)	0.095 (n.12)	0.051 (n.02)	0.028 (n.03)	0.381 (n.10)	0.095 (n.12)	0.051 (n.02)	0.028 (n.03)
Type	Element	$f_{ST,tested}$ (-)	STI (%)	STI (%)	STI (%)	STI (%)	STI (%)	STI (%)	STI (%)	STI (%)
01	C20, dark	0.277	27	< 0	< 0	< 0	28	< 0	< 0	< 0
02	C20+fp05, dark	0.051	87	47	0	< 0	88	50	0	< 0
03	C20+fp10, dark	0.028	93	70	44	0	94	76	52	0
04	C20, light	0.174	54	< 0	< 0	< 0	55	< 0	< 0	< 0
05	C20+fp05, light	0.033	91	65	35	< 0	93	71	41	< 0
06	C20+fp10, light	0.018	95	81	64	35	97	87	74	47
07	C, cool	0.066	83	30	< 0	< 0	84	32	< 0	< 0
08	C20+fp05, cool	0.013	97	86	75	55	98	94	87	73
09	C20+fp10, cool	0.007	98	92	86	75	100	100	100	100
10	M, bare (low $\epsilon_e$ )	0.381	0	< 0	< 0	< 0	0	< 0	< 0	< 0
11	M, dark	0.589	< 0	< 0	< 0	< 0	< 0	< 0	< 0	< 0
12	fp05+M, dark	0.095	75	0	< 0	< 0	77	0	< 0	< 0
13	fp10+M, dark	0.051	87	46	< 0	< 0	88	50	< 0	< 0
14	M, light	0.372	2	< 0	< 0	< 0	3	< 0	< 0	< 0
15	fp05+M, light	0.061	84	35	< 0	< 0	86	38	< 0	< 0
16	fp10+M, light	0.033	91	65	34	< 0	93	70	40	< 0
17	M, cool	0.140	63	< 0	< 0	< 0	65	< 0	< 0	< 0
18	fp05+M, cool	0.024	94	75	53	15	96	81	62	20
19	fp10+M, cool	0.013	97	86	74	54	98	93	87	72

\*Layers are listed from the innermost one; material and surface types are referred to tabs. 2-3.